# From N=2Z in $^{60}$Ca to N=Z in $^{80}$Zr: Connecting the driplines

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**Abstract.** We present Large Scale Shell Model calculations using the effective interaction LNPS, which reproduce nicely the experimental data available in the N=40 island of deformation. We deal as well with the triple (shape?) coexistence in the low energy spectrum of $^{68}$Ni, showing that the highly deformed band based upon the excited $0^+$ state at 2627 keV is in fact the doorway to the island. The onset of prolate quadrupole collectivity and shape coexistence in the heavy N=Z nuclei, is explained by means of the realizations of the pseudo plus quasi SU3 symmetry, with special emphasis on the shape mixing in the ground state of $^{72}$Kr.

## 1. Introduction

Why do the quadrupole correlations thrive in the nucleus? The fact that the spherical nuclear mean field is close to the harmonic oscillator has profound consequences, because its dynamical symmetry, responsible for the accidental degeneracies of its eigenvalues, is SU(3), which has among its generators the quadrupole operator. When valence protons and neutrons occupy the degenerate orbits of a major oscillator shell, and for an attractive Q·Q interaction, the many body problem has an analytical solution in which the ground state of the nucleus is maximally deformed (Elliott's model) [1]. Where else do the quadrupole correlations thrive? In cases when both valence neutrons and protons occupy quasi-degenerate orbits with $j_r - j_s$=2 and $l_r - l_s$=2, (quasi-SU3) [2], or quasi-spin multiplets (pseudo-SU3) [3, 4]. For example, $0f_{7/2}$ and $1p_{3/2}$, or $0g_{9/2}$, $1d_{5/2}$ and $2s_{1/2}$ form quasi-SU3 multiplets and $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ a pseudo-SU3 triplet.

## 2. The minimal valence space for the N=40 isotones

In this region of the chart of nuclei, the valence space should ideally include the full $pf$ shell for neutrons and protons as well as (at least) the quasi-SU3 sequence $0g_{9/2}$, $1d_{5/2}$, $2s_{1/2}$. However, the dimensions of this space are out of reach for the moment. Thus we shall enforce the following restrictions which are physically sound: for the very neutron rich isotopes we adopt a $^{48}$Ca core and exclude the $gds$ proton orbits and the $2s_{1/2}$ neutron orbit. For the proton rich isotopes beyond $^{68}$Se, we take a $^{56}$Ni core and exclude the $2s_{1/2}$ orbit both for protons and neutrons. We use the interaction LNPS [5], whose effective single particle energies in the neutron rich sector are plotted in Figure 1.

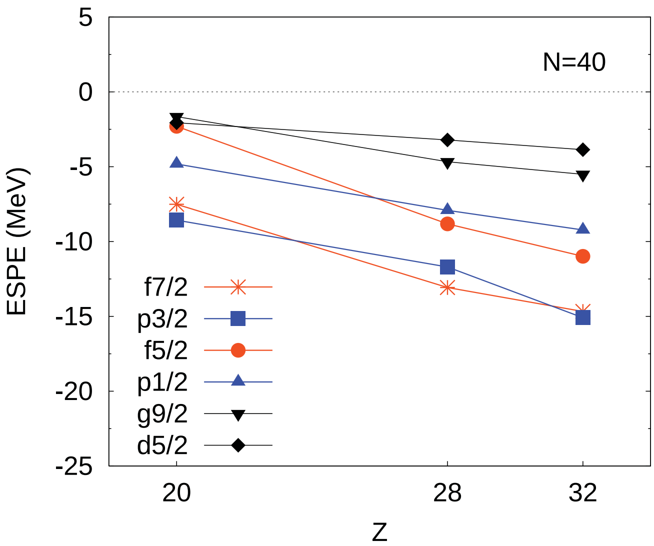


**Figure 1.** Effective neutron single particle energies of the N=40 isotopes from $^{60}$Ca to $^{72}$Ge.

These ESPE's are the phenomenological input of the Large Scale Shell Model (LSSM) calculations and are dictated by the experiment. They should provide a meaningful meeting point with the "ab initio" calculations. Notice the unorthodox spherical mean field at Z=20 with the orbits $0g_{9/2}$, $1d_{5/2}$ and $0f_{5/2}$ nearly degenerated. Similar situations are found for the N=20 and N=28 isotones toward Z=8, and for the N=8 isotopes approaching Z=2.

## 3. Triple coexistence in $^{68}$Ni

In a first approximation, the ground state of $^{68}$Ni behaves as double magic. But its low energy spectrum is much more complex, as three coexisting $0^+$ states appear between 0 and ∼2.5 MeV [6]. This is clearly seen in Figure 2. The excitation energy of the $0^+_2$ state has been recently remeasured and now the state is placed at 1.6 MeV [7, 8]. This state is dominated by 2p-2h neutron excitations and is mildly oblate according to the calculations. The $0^+_3$ is predicted by the LSSM calculations to be the band head of a very low-lying superdeformed band of 6p-6h nature. These calculations describe transition rates ranging over more than two orders of magnitude. One can understand the lowering of the $0^+_3$ state in terms of the variants of SU(3) already mentioned. The neutron 4p-4h configuration takes full advantage of the quadruple correlations (quasi-SU3 for the particles and pseudo-SU3 for the holes) if the corresponding proton configuration is a 2p-2h across Z=28, as it is the case in the calculations, because the two protons excited are in a pseudo-SU3 configuration as well.

## 4. The island of deformation south of $^{68}$Ni

For Z<28, the protons in the $0f_{7/2}$ orbit contribute efficiently to the build-up of the quadrupole collectivity of the configurations with np-nh neutron excitations across N=40, which take advantage of the quasi-SU3 coherence of the doublet $0g_{9/2}$- $1d_{5/2}$. Our LSSM calculations in the valence space of the full $pf$-shell for the protons and the $0f_{5/2}$ $1p_{3/2}$ $1p_{1/2}$ $0g_{9/2}$ and $1d_{5/2}$ orbits for the neutrons, predict another region of deformation centered in $^{64}$Cr.

The mechanism at play is similar to the one that produces the other "islands of inversion" at N=20 and N=28. On the one side, the N=40 gap disappears at Z=20 (as it does at Z=8 for the N=20 isotopes). In addition, when the proton $pf$-shell is not completely filled, the quadrupole

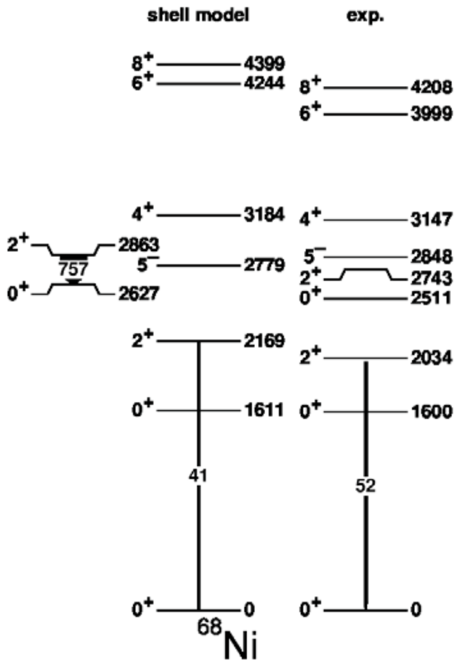


**Figure 2.** Level scheme of $^{68}$Ni: Experiment [6, 7] compared to the results of the LSSM calculations with the LNPS interaction [9].

energy gains of the intruder configurations with several neutrons excited across N=40 is very large. The final result is that configurations alike to that of the third $0^+$ state in $^{68}$Ni become the ground states of $^{66}$Fe, $^{64}$Cr and $^{62}$Ti. This can be seen in the behavior of the $2^+$ excitation energies and B(E2) values depicted in Figure 3. In this sense we can submit that the $0_3^+$ state in $^{68}$Ni is the precursor of the N=40 island of deformation.

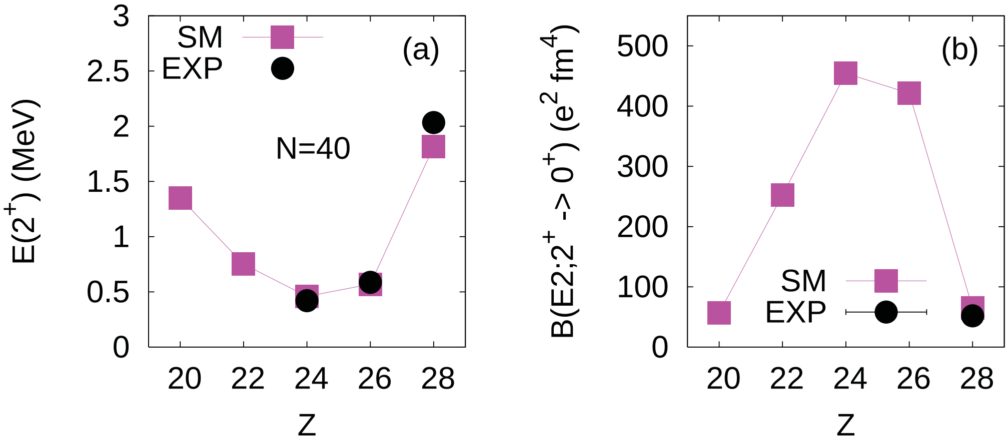


**Figure 3.** Excitation energies and B(E2) values for the N=40 isotopes, LNPS results *vs* experiment [10].

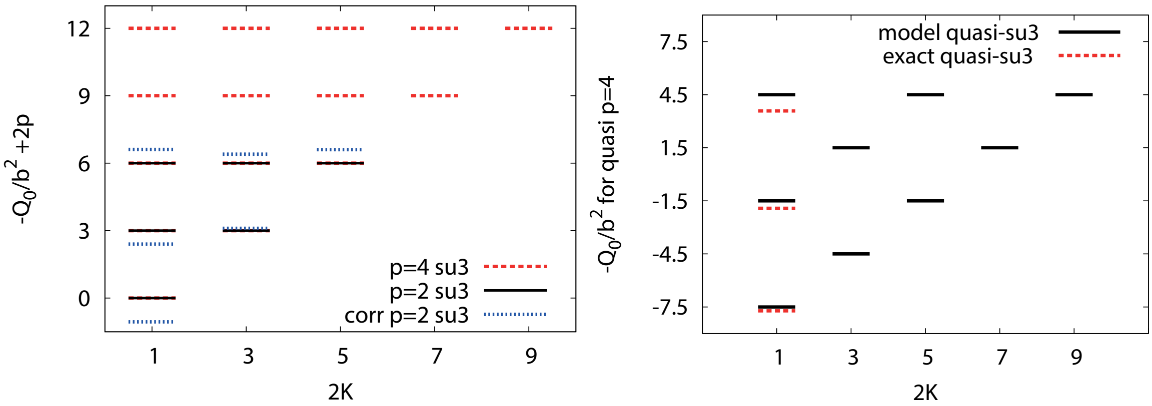


**Figure 4.** The intrinsic orbits of SU3 for p=2 and p=4 (p is the principal quantum number of the HO and b its length parameter) and the pseudo-SU3 (p=2) ones for the $1p_{3/2}$-$1p_{1/2}$-$0f_{5/2}$ (left panel). The quasi-SU3 orbits for the *gds* space ($0g_{9/2}$, $1d_{5/2}$ and $2s_{1/2}$) (right panel).

## 5. Deformation in heavy N=Z nuclei; The Quasi+Pseudo SU3 framework

The ground states of the N=Z nuclei above $^{68}$Se and below $^{88}$Ru, (and their neighbors) are dominated by configurations with np-nh jumps across N=Z=40. Why? Because the **n** particles sit in quasi-SU3 orbits and the **n** holes in pseudo-SU3, thus maximizing their quadrupole moments and, a fortiori, their quadrupole correlation energy. The gains in correlation energy overwhelm the monopole energy cost of crossing the gap between the upper *pf*-shell orbits and the intruder $0g_{9/2}$ and $1d_{5/2}$. This is clearly seen in the diagrams of the intrinsic SU3-like states in Figure 4. Filling the lowest quasi and pseudo SU3 intrinsic orbits we can determine for each nucleus which are the most favorable configurations from the quadrupole point of view. They are the following (b is the harmonic oscillator length):

- $^{72}$Kr
  - 4p-4h; $Q_0$= 60 $b^2$
  - 8p-8h; $Q_0$= 73 $b^2$
  - 12p-12h; $Q_0$= 74 $b^2$
- $^{76}$Sr
  - 4p-4h; $Q_0$= 51 $b^2$
  - 8p-8h; $Q_0$= 77 $b^2$
  - 12p-12h; $Q_0$= 79 $b^2$
- $^{80}$Zr
  - 8p-8h; $Q_0$= 72 $b^2$
  - 12p-12h; $Q_0$= 83 $b^2$
  - 16p-16h; $Q_0$= 85 $b^2$

For instance the quadrupole moment of the 8p-8h configuration in $^{80}$Zr is obtained from the figure as follows; put 8 particles in Quasi-SU3 to get $Q_0$= 4 x 7.5 + 4 x 4.5 = 48 $b^2$ and 16 in Pseudo-SU3 $Q_0$=5.06 x 4 + 1.41 x 4 + 1.08 x 4 + (− 2.35) x 4 = 20.72 $b^2$ for a total $Q_0$= 48 + 21 + 3 = 72 $b^2$ (the extra three is an SU3 technicality).
We have recently shown that these quadrupole moments are very resilient to departures of the single particle energies from their degenerated limit [11]. The key point here is that the

quadrupole energy gains grow with the square of the quadrupole moment whereas the monopole losses are at most proportional to the number of particle-hole jumps (modulo the N=40 gap). In $^{76}$Sr and $^{80}$Zr the deformed (prolate) configurations, 8p-8h and 12p-12h, win comfortably. In $^{72}$Kr the 4p-4h prolate and oblate solutions (oblate corresponding to $(0g_{9/2})^4$ instead of $(gds)^4$) are degenerated as we shall discuss next. From the $Q_0$'s one can deduce the B(E2)'s. The $2^+ \to 0^+$ is equal to ${Q_0}^2/50.3$ and the $4^+ \to 2^+$ a factor 1.43 larger. To compare with experiment we set $b^2$=4.5 fm$^2$ and obtain the following B(E2) values:

- $^{72}$Kr; $2^+ \to 0^+$; 1470 e$^2$fm$^4$; $4^+ \to 2^+$; 2100 e$^2$fm$^4$
- $^{76}$Sr; $2^+ \to 0^+$; 2380 e$^2$fm$^4$; $4^+ \to 2^+$; 3410 e$^2$fm$^4$
- $^{80}$Zr; $2^+ \to 0^+$; 2800 e$^2$fm$^4$; $4^+ \to 2^+$; 4000 e$^2$fm$^4$

The available experimental results are:

- $^{72}$Kr; $2^+ \to 0^+$; 810(150) e$^2$fm$^4$; $4^+ \to 2^+$; 2720(550) e$^2$fm$^4$ [12, 13]
- $^{76}$Sr; $2^+ \to 0^+$; 2200(270) e$^2$fm$^4$ [14]
- $^{80}$Zr; no data yet.

Notice that the agreement is excellent except for the B(E2) $2^+ \to 0^+$ of $^{72}$Kr. But this is a blessing in disguise because it may led us to understand better the prolate oblate coexistence in this nucleus.

## 6. $^{72}$Kr, a case of full prolate oblate mixing

It is common lore to speak of prolate-oblate or prolate-spherical coexistence when an excited $0^+$ state appears at very low energy. This is the case in $^{72}$Kr, whose first excited state is a $0^+$ at 671 keV followed by a $2^+$ at 710 keV. The very large B(E2) of the transition $4^+ \to 2^+$ strongly suggest that the $2^+$ belongs to a prolate band which extends up to J=$16^+$ [15]. But, if it is so, where is the band-head? If we follow the J(J+1) sequence from the upper part of the band we should expect it to be 250 keV below the $2^+$, which is very close to the experimental excitation energies of the $2^+$'s in $^{76}$Sr and $^{80}$Zr [16, 17]. Obviously the distortion must be due to the mixing of the prolate band-head with a close-lying oblate state. This is borne out by a recent Coulex measurement of the spectroscopic quadrupole moment of the $2^+$ state, made at Isolde [18], which finds it prolate.

We can thus propose a very simple model to explain the prolate oblate mixing in $^{72}$Kr. The first element to take into account is that the oblate and prolate 4p-4h states do not mix directly; *i.e.* $\langle p|H|o\rangle \approx 0$. The mixing should then proceed through 2p-2h or 6p-6h states. Lets take these to be represented by an auxiliary state $|I\rangle$, and further assume that it lies at about $\Delta$E=4 MeV (as our calculations indicate) and that its coupling to both prolate and oblate states is equal to $\delta$. Taking the prolate and oblate states degenerated for simplicity, the mixing matrix reads:

$$\begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & \delta \\ \delta & \delta & \Delta E \end{pmatrix}$$

The mixing can proceed as well through a cloud of N states. In this, more realistic, case the matrix would be:

$$\begin{pmatrix} 0 & 0 & \beta & \beta & \beta & \dots \\ 0 & 0 & \beta & \beta & \beta & \dots \\ \beta & \beta & \Delta E & 0 & 0 & \dots \\ \beta & \beta & 0 & \Delta E & 0 & \dots \\ \beta & \beta & 0 & 0 & \Delta E & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

Which has the same two lowest eigenvalues and eigenvectors than the previous one, provided $\delta=\sqrt{N}\ \beta$. Taking $\delta \sim 1$ MeV, or $\beta$=0.4 and N=6, as our calculations suggest, the eigenvalues are: –0.5 MeV, 0.0 MeV and +4.5 MeV. They fit nicely the experimental energies. The eigenstates corresponding to the two lower eigenvalues are:

$$|0^+_1\rangle = 43\%\ |p\rangle + 43\%\ |o\rangle + 14\%\ |I\rangle \text{ and } |0^+_2\rangle = 50\%\ |p\rangle + 50\%\ |o\rangle$$

The mixing of the oblate and prolate $2^+$'s is bound to be much smaller, and we disregard it. Therefore, the B(E2)($2^+\rightarrow 0^+_1$) will be approximately one half of the expected value for the prolate band in full accord with the experimental data.

How to name the shape of an object which is an even mixture of prolate and oblate? What is the nature of this mixing of shapes? Or should we rather speak of a shape entangled state?

## 7. Conclusions

The onset of the different modes of quadrupole collectivity depend on the structure of the spherical mean field. In the valence space comprising the $pf$ shell and the $0g_{9/2}$ and $1d_{5/2}$ orbits we have explained the appearance of large prolate deformation at N=Z in terms of different realizations of SU3. Similar arguments explain the onset of deformation in the new island of inversion around $^{64}$Cr and its precursor, the superdeformed band of $^{68}$Ni.

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