DEPARTAMENTO DE FÍSICA TEÓRICA
UNIVERSIDAD AUTÓNOMA DE MADRID

INSTITUTO DE FÍSICA TEÓRICA
(UAM/CSIC)

Non-perturbative aspects of type II string compactifications

Memoria de Tesis Doctoral presentada ante la Facultad de Ciencias,
Sección de Ciencias Físicas, de la Universidad Autónoma de Madrid,
por **Pablo Soler Gomis**

Trabajo dirigido por el **Dr. D. Ángel M. Uranga Urteaga**
Profesor de Investigación del Instituto de Física Teórica
del Consejo Superior de Incestigaciones Científicas,

Madrid, Mayo de 2012.

Contents

Chapter 1

Introduction

The Standard Model (SM) of particle physics (supplemented by masses for the neutrinos to account for oscillations) has been tested to a high level of precision in a wide range of energies by different experiments. Most notably, particle accelerators have confirmed its predictions up to energies of the order of the TeV with no significant deviation found so far. The last piece of the model, the Higgs boson, is expected to be found at the LHC in the very near future. In fact, strong hints towards the existence of a Higgs-like particle with an approximate mass of 125 GeV have been already reported by the Atlas and CMS collaborations [1, 2].

The model has resisted exhaustive scrutiny and, as long as no evidence against it is presented (perhaps by new discoveries at the LHC), there is no apparent need to modify it. Nevertheless, there are strong theoretical reasons to expect that the SM is not a complete theory and must be supplemented by new physics at higher energies (probably not much higher than the TeV).

Most prominently, gravity cannot be consistently accommodated in the framework of the model. The SM is formulated in terms of a quantum field theory with a gauge group $SU(3) \times SU(2) \times U(1)$ that can account for all nuclear and electromagnetic interactions, see e.g. [3–5]. A complete formulation of gravity at the quantum level does not seem possible in the context of field theory. The reason is that gravity is non-renormalizable, and its mere formulation requires specifying an infinite number of parameters, the coupling constants of the irrelevant terms of the lagrangian. These couplings become increasingly important at high energies, and hence the short distance behavior of the theory is ill-defined.

Our present understanding of non-renormalizable field theories indicates, rather than a failure of gravity at the quantum level, that we should interpret our models as low energy effective theories valid up to some high energy scale at which new physics should come into play to render the ultraviolet (UV) behavior consistent. The cutoff scale at which our effective field theory breaks down should be no higher than the Planck scale $\sim 10^{19}$ GeV at which quantum gravitational effects become important.

Since, as we have mentioned, no consistent embedding of gravity into a quantum field theory seems possible; a complete description of Nature valid up to arbitrary scales will require new theoretical frameworks, such as string theory[1].

[1] Nevertheless, string theory in certain backgrounds has been proposed to have dual descriptions in terms of quantum field theories, along the lines of the AdS/CFT correspondence [6–8].

Somewhat more subjectively, physicists are usually incommoded by the apparent arbitrariness of the SM content. Ultimately, one would like to understand what determines the particular pattern of gauge fields and matter multiplets of the theory, together with the values of the approximately 30 external parameters that enter the lagrangian. Particularly awkward are the values of the couplings in the Higgs-Yukawa sector (incidentally the least tested one). Quark masses differ by a factor of $\sim 10^5$ for no apparent reason, from the *up* quark with $\sim 2 \times 10^{-3}$ GeV to the *top* quark of ~ 173 GeV. Taking into account leptons and specially the tiny masses of neutrinos makes the situation much worse. Moreover, while the mixing matrices of the quark and lepton sectors (the CKM and PMNS matrices) do seem to follow some rough patterns, the fundamental origin of their completely different structures is not understood.

On a similar line of thinking, but even more dramatically, some of the parameters of the SM take values that seem *unnaturally* [9] small from the perspective of effective field theories:

- The observed expansion of the Universe points towards the existence of a cosmological constant (or vacuum energy) of the order $\Lambda_{\text{c.c.}} \sim 10^{-12}$ (eV)^4. In the SM, the cosmological constant receives contributions from several sectors, most prominently by the Higgs scalar potential. In addition, loop corrections of order of the cutoff scale to the vacuum energy are expected to lift its theoretical value to something as large as $\Lambda_{\text{c.c.}} \sim M_p^4 \sim 10^{112}$ (eV)^4.

  The difference of 124 orders of magnitude between the measured value and the theoretical expectations is one of the most fundamental and less understood problems that theoretical physics faces nowadays. One could in principle introduce a bare cosmological constant Λ_0 in the lagrangian of the SM, so that the total theoretical value agrees with the experimental one. However, this cancellation of widely different contributions at different scales requires a spectacular fine-tuning which cries out for a deeper theoretical explanation (perhaps of the anthropic kind [10]).

- A different problem of hierarchies arises in the electro-weak (EW) sector of the SM. At present, we do not understand why the scale of EW symmetry breaking (or equivalently the Higgs squared mass) differs so much from the Planck scale. Just like the cosmological constant, the Higgs mass should receive radiative corrections of order of the cutoff scale of the theory, which would push it far away from the expected experimental value $M_H^2 \sim (125 \text{ GeV})^2$. Again, a bare parameter in the lagrangian could reconcile theory and observations, but would require an unnatural fine-tuning of about 10^{-34}.

- A milder problem arises in the sector of strong interactions of the SM. This sector admits a CP violating phase θ entering the action in the form

  $$S_\theta = \frac{\theta}{32\pi^2} \int F \wedge F$$

  Such a phase would lead to an electric dipole moment of the neutron which has not been observed. The experimental bound is roughly $\theta < 10^{-10}$, which apparently introduces a new fine-tuning problem.

  Unlike the cosmological and the EW hierarchy problems, the strong CP problem admits a relatively satisfactory answer in a simple extension of the SM [11,12]. The rough idea is to promote the phase θ from a constant parameter to a dynamical field, an axion, whose vacuum expectation value would naturally be zero.

Altogether, the absence of gravity and the fine-tuning problems of the SM indicate that it should be taken as an effective theory that must be supplemented at high energies by new physics. The EW hierarchy problem points towards de TeV scale as the scale in which the extensions of the theory should come into play. On the other hand, gravity suggests that, at least at the Planck scale, our description of Nature is most probably given by something more general than a standard quantum field theory.

Several proposals have been made in the context of quantum field theory to address the EW hierarchy problem. Among them, the most popular ones are low energy supersymmetry, which cancels the quadratically divergent corrections to the Higgs mass; extra dimensions, either large or warped, which allow for a drastic lowering of the fundamental scale of gravity; and *technicolor*-like theories, in which the EW scale is dynamically generated by the strong dynamics of a new gauge sector. Some of these proposals include furthermore extra matter fields which, in the appropriate situations, are good candidates to account for the dark matter observed in the Universe.

The task of embedding gravity in a quantum theory seems a more fundamental and difficult one. Up to now very few candidates have been proposed, and among them, string theory is by far the most successful one. String theory provides a UV consistent framework that includes quantum gravity, and which reduces at low energies to familiar quantum field theories. Furthermore, all the mechanisms proposed to address the EW hierarchy problem, as well as the axionic solution of the strong CP problem, can be naturally included into this framework.

Formal aspects of string theory

Of course, a review of the vast subject of string theory is not appropriate here. We refer the reader to some of the many texts that have been written on the subject [13–20]. Let us, however, make a few general comments regarding broad features of the theory.

Given such a good candidate for physics beyond the SM, the natural question is: *if it is not a usual quantum field theory, then what IS string theory?* Surprisingly enough, we do not have a complete answer to this apparently simple question yet. What we know, instead, is how the theory *looks like* in certain corners of its parameter space.

In these regions, string theory is indeed described by a theory of one-dimensional extended objects (the strings) that live in a ten dimensional space-time. Different string excitations represent different particles of spacetime, and their interaction amplitudes are given in terms of perturbative series in a small coupling constant g_s. Each term of the expansion can be computed as a two dimensional (2d) correlator in a Riemann surface, see fig. 1.1. The power of the coupling g_s at which each 2d correlator contributes to the physical 10d amplitude is determined by the genus of the corresponding surface. Our focus in this work will be on the corner described by type II theories.

In a certain sense, this worldsheet formulation of string theory, which is only valid perturbatively, gives us "Feynman rules" with which we can compute physical amplitudes. However, it does not provide a fundamental action from which these rules could be derived. Such an

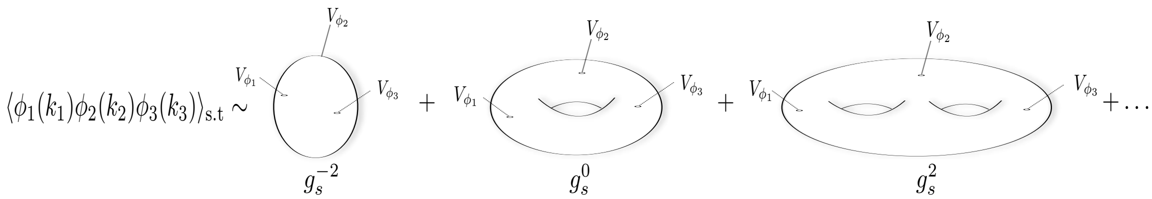


Figure 1.1: Diagramatic representation of the worldsheet perturbative computation of a spacetime three-point function.

action, should be in principle valid for arbitrary values of g_s and include non-perturbative terms of the general form e^{-1/g_s} that are not contained in the worldsheet expansion.

The area of string theory that looks for such a non-perturbative formulation is string field theory [21, 22], see e.g. [23, 24] for reviews. The subject is technically very demanding and, despite some successes, concrete results are quite hard to obtain. Part of my work during the last four years has focused on the study of analytic solutions of a specific type of string field theories. The methods and scope of this work are unfortunately quite different from the ones presented in this thesis. Therefore, I simply refer the interested reader to the original publications [25, 26] where the results of my work were presented.

Despite the lack of a complete formulation of the theory, a lot of its non-perturbative structure has been revealed by semiclassical techniques. A prominent role in such studies is played by D-branes, whose microscopic understanding and connection to non-perturbative solutions of supergravity were first presented in [27]. Dp-branes are extended dynamical objects whose worldvolume spans p spatial dimensions, and whose quantum fluctuations can be described by open strings attached to the brane.

The proper interpretation of D-branes has led to a better, yet not complete, understanding of string theory as a whole. In particular, we know now that the five different perturbative formulations of string theory are related to one another, as well as to the more mysterious 11d M-theory, in a web of dualities depicted in figure 1.2.

An even more striking development of string theory was the precise implementation of the principle of holography to theories of quantum gravity [28]. In its simplest form [6–8], he AdS/CFT correspondence proposes the equivalence of type IIB string theory living in $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills field theory in four dimensions (the frontier of AdS_5). Again, D-branes played a crucial role in the formulation of this mayor breakthrough.

Finally, let us mention that the formal study of string theory has developed into an area of common interest for physicists and mathematicians. A particular instance in which this is more manifest is the formulation of topological string theories, see e.g. [29–31] for reviews. Roughly stated, these are simplified versions of string theory, simple enough as to be analytically manageable to a large extent, yet rich enough to provide interesting physical and mathematical results on string and field theory. Some aspects of topological strings will be dealt with in chapter 4 of this thesis. Unfortunately, we will not have time to present a general review of the subject here, and we will mostly limit ourselves to borrow some results when necessary.

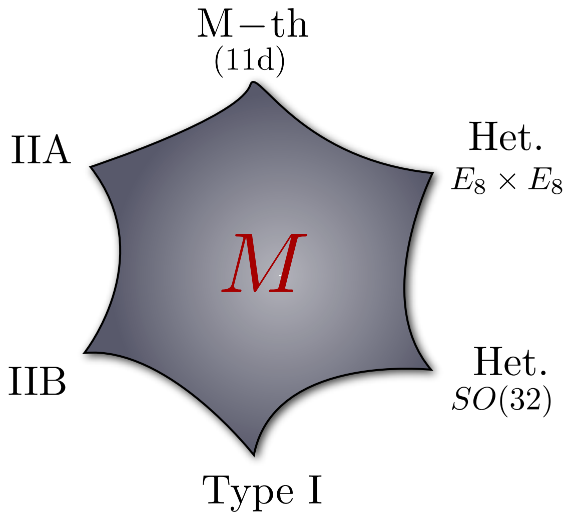


Figure 1.2: Web of dualities of perturbative string theories, depicted as limits of a fundamental unknown theory, M-theory.

String phenomenology

Besides the study of formal aspects of string theory, which is interesting in its own, our main goal is the application of string theory to the understanding of observed physical phenomena. We could say that the string framework and techniques can been applied to three vast different areas of more *conventional* physics: cosmology, with the interpretation of black holes and inflation in terms of D-branes; collective phenomena in strongly coupled systems, like condensed matter or gauge theory plasmas studied in terms of dual gravitational theories by application of the AdS/CFT correspondence; and particle physics, on which we focus next.

At low energies, the physical amplitudes of string states computed by worldsheet diagrams as in figure 1.1, can be reproduced by 10d supersymmetric quantum field theories. The connection to conventional 4d physics and to the SM requires hence the compactification of six dimensions. As we see, string theory encodes naturally the two most popular proposals of physics beyond the Standard Model, namely extra dimensions and supersymmetry. At the same time, it provides an UV completion or these low energy theories, which break down at energies of the order of the string scale.

String phenomenology consists in the study of compactifications of string theory that lead to effective low energy physics as close as possible to the SM, or at least to some of its supersymmetric extensions, see [20] for a recent review of the subject. The task is extremely difficult for two main reasons. On the one hand, string theory seems to admit an enormous number of solutions, even if we restrict to those with four non-compact dimensions. On the other, the analysis of the effective action of string compactifications requires advanced mathematical tools which can often be only applied in certain simplified setups.

Accordingly, a compromise between theoretical tractability and phenomenological interest needs to be found. Our guiding principles in this task are two, namely the preservation of supersymmetry in four dimensions, and the appearance of non-abelian gauge symmetries with charged chiral fields. The first feature simplifies the treatment of the theories enormously, and is appealing for phenomenology. The second is probably the most fundamental property of the SM.

Our interest in this work is in type II string compactifications. The relevant compactification spaces for this cases are Calabi-You (CY) manifolds, which lead to effective theories with $\mathcal{N} = 2$ supersymmetry in four dimensions. These theories, although enormously interesting from a formal point of view [32, 33], have too much supersymmetry to be directly applicable to phenomenology, since they do not admit chiral fermions. The problem is solved by the introduction of gauge D-brane (together with orientifold planes required for consistency).

Dp-branes can wrap internal $p-3$ dimensional cycles of the CY space, and span the four non-compact dimensions. The corresponding effective field theory in 4d has reduced $\mathcal{N} = 1$ supersymmetry, and furthermore contains non-abelian gauge bosons that arise from open strings attached to the brane. Furthermore, chiral representations of the gauge group appear if the branes are either magnetized, sitting at singular points of the internal space, or wrapping different intersecting cycles of the CY.

These ingredients, CY spaces and gauge branes, have provided extremely useful tools to study phenomenologically viable models based on type II strings. Their effective theories, as computed by perturbative worldsheet techniques can indeed come quite close to that of the SM, or one of its supersymmetric versions, such as the Minimal Supersymmetric Standard Model (MSSM). The type II models can have massless spectra and effective actions similar to that of the (MS)SM, but suffer in general from important phenomenological problems.

The models generically include massless (or even tachyonic) scalar fields (the *moduli*) whose appearance is in contradiction with cosmological observations. Furthermore, many of them lack, at the perturbative level, some couplings such as neutrino Majorana masses or certain Yukawa couplings, whose presence is required for phenomenology. Finally, the implementation of controlled mechanisms of supersymmetry breaking is very complicated in specific setups. These are some of the most important drawbacks that type II model building has to face.

The introduction of background fluxes can alleviate some of these problems, see e.g. [34]. In particular, they can give masses to some of the moduli, and can serve as means to break supersymmetry. In this work, however, we will not consider these fluxes, but will rather focus on the appearance of non-perturbative effects, not contained in the worldsheet expansion of perturbative string theory.

Non-perturbative effects

An important question arises naturally: if string theory is formulated only at the perturbative level, how can one obtain information on its non-perturbative effects? Again, D-branes play a key role in the answer [35–38]. This is quite natural, since the tension of these objects scale as $1/g_s$, and their partition functions are proportional to e^{-1/g_s}. They indeed disappear from the dynamics of theory in the perturbative limit $g_s \to 0$.

Consider a Dp-brane whose worldvolume, rather than spanning the non-compact Minkowski directions, is wrapped entirely around a $p+1$-cycle of the internal space. This configuration is localized in spacetime just like the usual instantons of non-abelian gauge field symmetries, see e.g. [39]. In analogy with these, they are expected to contribute non-perturbatively to the effective action.

Since we lack a non-perturbative formulation of string theory, D-brane instanton effects cannot be derived from first principles. Instead, an instanton calculus has been developed in the last few years by analogy with the non-perturbative effects of gauge theories, which are well understood [40–42].

It has been shown that D-brane instantons do indeed contribute non-perturbatively to the effective actions of type II string compactifications. They have become nowadays crucial mechanisms to solve some of the aforementioned problems of perturbative string phenomenology. We can conclude that D-brane instantons are an extremely important part of string theory, both at a formal and phenomenological level, that deserves a thorough study.

On the one hand, they play a crucial role in making the famous web of string dualities (figure 1.2) consistent since, in many cases, strong coupling regimes are related weak coupled ones. Moreover, these effects should arise naturally in a complete non-perturbative formulation of string theory that can tell us *what is string theory.* One can expect that D-brane instantons may provide important guidelines in the search for an answer.

On the other hand, instanton effects have extremely important consequences in the phenomenology of string theory models of particle physics. They provide mechanisms to generate couplings which are forbidden at a perturbative level and are necessary for viable phenomenological models. These include stabilization superpotentials for scalar moduli [43], as well as matter couplings such as neutrino Majorana masses and Yukawa couplings [40–42]. Furthermore, their exponential suppression can be used in many situations to explain hierarchies of the Standard Model.

The results of this work

The goal of this thesis is to clarify several aspects of the complicated and sometimes obscure instanton calculus, with special interest in processes of multiple instantons. During the last four years I have worked on a simple method which allows the computation of non-perturbative effects of type II models in terms of the BPS particle spectrum of dual theories. In the situations in which it can be applied, the procedure permits the resummation of several sectors of multi-instanton effects with almost no effort.

Besides a powerful computational tool, the method provides us with an advantageous point of view that sheds light on many qualitative aspects of usually obscure multi-instanton processes. In particular our results have revealed in an explicit way the strong connection between D-brane instantons and topological strings which underlies in an elegant manner the consistency of non-perturbative effects across lines of BPS stability in moduli space [44, 45]. A different application of the method we present here is the correct interpretation of so-called poly-instanton processes which have puzzled the string community for some time [46]. Finally, our results provide a further piece of evidence that the formulation of D-brane instanton calculus mentioned above is indeed correct.

On a somewhat more phenomenological direction, my studies have also led to a systematic understanding of the appearance of discrete gauge symmetries in large classes of string vacua, concretely those based on intersecting D-branes in type II orientifolds. The general idea is that D-brane instanton effects that break global $U(1)$ symmetries of the effective theory, still preserve in quite generic situations some discrete subgroup thereof which remains exact even at the non-perturbative level. We can explicitly show that, in such cases, no instanton configuration whatsoever, either simple or multi-instantonic, BPS or not, etc. can violate these remnant symmetries.

The analysis is quite general and simple, and shows that discrete gauge symmetries arise naturally, preventing certain couplings from appearing in the effective action even at the non-perturbative level. The application of our results to phenomenological type II models is crucial, since it explains neatly the stability of the proton in string constructions, which was so far not systematically understood.

As mentioned before, the part of my work dealing with analytical solutions of the string field theory equations of motion lies outside the main line of reasoning of this thesis, and has not been presented here for clarity. The results of this research can be found in [25, 26].

Structure of this thesis

In chapters 2 and 3 we review some general results on type II orientifold compactifications. We study the massless spectrum of these models and the effective field theories that govern their dynamics in four dimensions. Special emphasis is given to the separation of perturbative and non-perturbative terms, and to non-renormalization theorems which are presented here as simple consequences of periodicity properties of axioins. In the end we introduce D-brane instantons and D-brane instanton calculus, and stress the crucial formal and phenomenological roles they play.

Chapter 4 presents the results on multi-instanton calculations of [47,48]. In section 4.1 we present the generalities of the method which is applied in later sections. In 4.2 we explicitly compute contributions from D1/D(-1) multi-instantons to the hypermultiplet moduli spaces of type IIB CY compactifications [47]. We argue there that the connection with topological strings which is manifest in our computations underlies the continuity of non-perturbative effects across lines of BPS stability. As an application of the general method to a setup with gauge branes and orientifold planes, we present in section 4.3 the computation of quartic gauge and gravitational couplings in certain 8d compactifications of type I' theory [48]. The results are then used to propose an interpretation of the obscure poly-instanton effects of [46].

In chapter 5 we present our results on discrete gauge symmetries in type II orientifold models [49]. In section 5.1 we describe the general analysis, while in 5.2 we apply it to some explicit constructions of (MS)SM and $SU(5)$ grand unified models with intersecting D6-branes. Some aspects of the generalization of the analysis to F-theory setups are presented in 5.3.

Finally, in chapter 6 we present our conclusions and some proposals for future investigations.

Introducción

(Spanish translation of chapter 1)

El Modelo Estándar (ME) de física de partículas (suplementado por masas de neutrinos para dar cabida a sus oscilaciones), ha sido probado a un alto nivel de precisión en un amplio rango de energías por diversos experimentos. Más notablemente, los aceleradores de partículas han confirmado sus predicciones hasta energías del orden del TeV sin encontrar desviaciones significativas hasta la fecha. Esperamos que la última pieza del modelo, el bosón de Higgs, sea encontrada en el LHC en un futuro muy cercano. De hecho, fuertes indicaciones de la existencia de una partícula similar al Higgs, con una masa de aproximadamente 125 GeV han sido ya comunicadas por las colaboraciones Atlas y CMS [1, 2].

El modelo ha resistido un exhaustivo escrutinio y, mientras no se presenten evidencias contra él (quizás por nuevos descubrimientos en el LHC), no hay razón aparente para modificarlo. No obstante, hay razones teóricas muy fuertes para esperar que el ME no es una teoría completa y debe ser complementado con nueva física a energías mayores (probablemente no mucho mayores que el TeV).

Más notablemente, la gravedad no puede ser embebida consistentemente en el contexto del modelo. El ME es formulado en términos de una teoría cuántica de campos con grupo gauge $SU(3)\times SU(2)\times U(1)$ que puede explicar todas las interacciones nucleares y electromagneticas, c.f. [3–5]. Una formulación completa de gravedad a nivel cuántico no parece posible en el marco de teoría de campos. La razón es que la gravedad es no-renormalizable, y su mera formulación requiere especificar un número infinito de parámetros, las constantes de acoplo de los términos irrelevantes del lagrangiano. Estos acoplos se vuelven cada vez más importantes a altas energías, y por tanto el comportamiento de la teoría a escalas cortas está mal definido.

Nuestro entendimiento presente de teorías de campos no-renormalizables indica, más que un fallo de la gravedad a nivel cuántico, que deberíamos interpretar nuestros modelos como teorías efectivas a bajas energías, válidas sólo hasta una escala alta determinada, a la que nueva física debería entrar en juego hacer el comportamiento ultravioleta (UV) consistente. La escala de corte a la que nuestra teoría efectiva se rompe no debería ser mayor que la escala de Planck $\sim 10^{19}$ GeV donde los efectos cuánticos gravitacionales se vuelven importantes.

Ya que, como hemos visto, no parece posible embeber la gravedad en una teoría cuántica de campos, una descripción completa de la Naturaleza válida a energías arbitrarias necesitará de nuevos marcos teóricos, tales como la teoría de cuerdas[2]. .

Algo más subjetivamente, los físicos tienden a sentirse incomodados por la aparente arbi-

[2] No obstante, se ha propuesto que teoría de cuerdas en ciertos espacios ambiente tienen descripciones duales en términos de teorías cuánticas de campos, en la línea de la correspondencia AdS/CFT [6–8].

trariedad del contenido del ME. En última instancia, nos gustaría entender qué determina el particular patrón de campos gauge y multipletes de materia de la teoría, además de los valores de los aproximadamente 30 parámetros que aparecen en el lagrangiano. Particularmente embarazosos son los valores de los acoplos en el sector de Higgs-Yukawa (incidentalmente el menos probado de todos). Las masas de los quarks difieren en factores de hasta 10^50 sin razón aparente, desde el quark *up* con $\sim 2 \times 10^{-3}$ GeV hasta el quark *top* con ~ 173 GeV. Teniendo en cuenta los leptones, y en especial las minúsculas masas de los neutrinos, la situación se vuelve aún mucho peor. Más aún, mientras que las matrices de mezcla de los sectores de quarks y leptones (las matrices CKM y PMNS) sí siguen aparentemente un patrón, el origen fundamental de sus estructuras completamente distintas no está entendido.

En una linea de razonamiento similar, pero incluso más dramática, algunos de los parámetros del ME toman valores *antinaturalmente* [9] pequeños desde el punto de vista de teorías efectivas.

- La expansión acelerada del Universo observada apunta hacia la existencia de una constante cosmológica (o energía de vacío) del orden $\Lambda_{\text{c.c.}} \sim 10^{-12}$ (eV)^4. En el ME, la constante cosmológica recibe contribuciones de distintos sectores, principalmente del potencial escalar del Higgs. Además, se espera que correcciones de 'loops' a la energía de vacío del orden de la escala de corte eleven su valor hasta algo tan grande como $\Lambda_{\text{c.c.}} \sim M_p^4 \sim 10^{112}$ (eV)^4.

  La diferencia de 124 órdenes de magnitud entre el valor observado y la expectación teórica es uno de los problemas más fundamentales y menos comprendidos que la física teórica afronta en nuestros días. Uno podría en principio introducir una constante cosmológica 'desnuda' Λ_0 en el lagrangiano del ME, de manera que el valor total teórico esté de acuerdo con el experimental. Sin embargo, esta cancelación de contribuciones ampliamente distintas y a diferentes escalas, requiere un espectacular 'ajuste-fino' que clama por una mejor explicación teórica (quizás algo en las líneas antrópicas de [10]).

- Un problema distinto de jerarquías aparece en el sector electrodébil (ED) del ME. Hoy en día, no entendemos por qué la escala de ruptura de simetrá ED (o equivalentemente la masa al cuadrado del bosón de Higgs) difiere tanto de la escala de Planck. Al igual que la constante cosmológica, la masa del Higgs debería recibir correcciones radiativas del orden de la escala de corte de la teoría, que la empujaría muy lejos del valor experimental esperado de $M_H^2 \sim (125\ \text{GeV})^2$. De nuevo, un parámetro desnudo en el lagrangiano podría reconciliar teoría y observaciones, pero requeriría de un 'ajuste-fino' forzado de aproximadamente 10^{-34}.

- Un problema más leve de jerarquías aparece en el sector de interacciones fuertes del ME. Este sector admite una fase de violación de CP que entra en la acción en la forma

  $$S_\theta = \frac{\theta}{32\pi^2} \int F \wedge F$$

  Dicha fase conllevaría un momento dipolar eléctrico del neutrón que no ha sido observado. La cota experimental es de aproximadamente $\theta < 10^{-10}$, lo que aparentemente lleva de nuevo a un problema de 'ajuste-fino'.

  Al contrario que los problemas de la constante cosmológica y de jerarquía ED, el problema CP fuerte admite una respuesta relativamente satisfactoria en una extensión simple del ME [11,12]. A *grosso modo*, la idea consiste en promover la fase θ de un parámentro constante a un campo dinámico, un axión, cuyo valor esperado en el vacío sería naturalmente cero.

En conjunto, la ausencia de gravedad y los problemas de 'ajuste-fino' del ME indican que éste debería ser considerado una teoría efectiva que tentría que ser suplementada a altas energías por nueva física. El problema de jerarquía ED apunta hacia la escala del TeV como la escala a la que la extensión de la teoría debería entrar en juego. Por otro lado, la gravedad sugiere que, al menos a la escala de Planck, una descripción consistente de la Naturaleza estará dada con toda probabilidad por algo más general que una teoría cuántica de campos.

Varias propuestas han sido hechas en el contexto de teorías cuánticas de campos para afrontar el problema de jerarquía ED. Entre ellas, las más populares son teorías del tipo *technicolor*, en la que la escala ED se genera por la dinámica fuerte de un nuevo sector gauge; dimensiones extra, ya sea grandes o combadas, que permiten una drástica reducción de la escala fundamental de gravedad; y supersimetría a bajas energías, que cancela las divergencias cuadráticas de la masa de Higgs. Algunas de estas propuestas incluyen además campos de materia extra que, en circunstancias adecuadas, son buenos candidatos para explicar la materia oscura observada en nuestro Universo.

La tarea de embeber la gravedad en una teoría cuántica parece una pregunta más difícil y fundamental. Hasta la fecha, muy pocas teorías candidatas han sido propuestas, y de entre ellas, la más exitosa de lejos es la teoría de cuerdas. Esta teoría provee de un marco consistente UV que incluye gravedad, y que se reduce a bajas energías a las teorías cuánticas de campos familiares. Más aún, todos los mecanismos propuestos para afrontar el problema de jerarquías ED, así como la solución axiónica del problema CP fuerte, pueden ser acomodadas naturalmente en el contexto de las cuerdas.

Aspectos formales de teoría de cuerdas

Este no es el sitio adecuado para revisar el vasto campo de la teoría de cuerdas. Referimos al lector a algunos de los muchos textos que se han escrito sobre el tema [13–20]. En su lugar, haremos algunos comentarios sobre las características generales de la teoría.

Dada una candidata tan buena para física más allá del ME, la pregunta natural es: *si no es una teoría cuántica de campos usual, entonces ¿que ES la teoría de cuerdas?* Sorprendentemente, no tenemos una respuesta completa a esta pregunta aparentemente sencilla. Lo que sabemos, en su lugar, es a qué se parece en ciertos rincones de su espacio de parámetros.

En dichas regiones, la teoría de cuerdas es descrita de hecho por una teoría de objetos extensos de una dimensión (las cuerdas) que viven en un espacio-tiempo de diez dimensiones (10d). Distintas excitaciones de las cuerdas representan distintas partículas del espacio-tiempo, y sus amplitudes de interacción vienen dadas en términos de series perturbativas en un acoplo débil g_s. Cada término de la expansión puede ser calculado como un correlador en una superficie de Riemann en 2d, ver fig. 1.3. La potencia del acoplo g_s a la que el correlador 2d contribuye a la amplitud física en 10d viene determinado por el género de la superficie correspondiente. Nuestra atención en este trabajo se centra en la región descrita por teorías de cuerdas de tipo II.

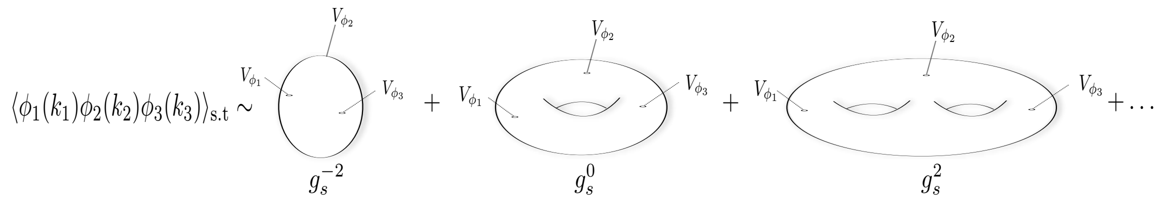


Figure 1.3: Diagramatic representation of the worldsheet perturbative computation of a spacetime three-point function.

En cierto sentido, esta formulación de teoría de cuerdas, que es sólo válida perturbativamente, nos da las "reglas de Feynman" con las que calcular amplitudes físicas. Sin embargo, no nos proporciona una acción fundamental a partir de las cual podrían derivarse dichas reglas. Una acción como esa debería ser en principio válida para valores arbitrarios de g_s, e incluir términos no-perturbativos de la forma general e^{-1/g_s} que no aparecen en la expansión perturbativa.

El área de teoría de cuerdas que busca tal formulación no-perturbativa es la teoría de campos de cuerdas [21, 22], veanse por ejemplo los resúmenes [23, 24]. El tema es bastante comoplejo a nivel técnico y, a pesar de ciertos éxitos, es difícil obtener resultados concretos. Parte de mi trabajo durante estos últimos cuatro años se ha centrado en el estudio de soluciones analíticas de cierto tipo de teorías de campos de cuerdas. El rango y los métodos de ese trabajo son, desafortunadamente, muy diferentes de los presentados en esta tésis. Por lo tanto, me limito a dirigir al lector interesado a las publicaciones originales donde los resultados de mi trabajo fueron presentados [25, 26].

A pesar de la falta de una formulación completa de la teoría, mucha información sobre su estructura a sido revelada mediante técnicas semiclásicas. Un papel fundamental en dichos estudios es jugado por las D-branas, cuya comprensión microscópica y conexión con soluciones no-perturbativas de gravedad fueron presentadas originalmente en [27]. Las Dp-branas son objetos dinámicos extensos cuyo 'mundo de volumen' se extiende por p dimensiones espaciales, y cuyas fluctuaciones cuánticas pueden ser descritas en término de cuerdas abiertas ligadas a la brana.

La correcta interpretación de las D-branas ha llevado a una mejor, aunque no completa, comprensión de la teoría de cuerdas en su conjunto. En particular, ahora sabemos que las cinco distintas formulaciones perturbativas de la teoría de cuerdas están conectadas unas con otras, así como a la más misteriosa teoría M en 11d, en una red de dualidades representada en la figura 1.4.

Un desarrollo aún más llamativo de la teoría de cuerdas fue la implementación concreta del principio de holografía para teorías de gravidad cuántica [28]. En su forma más simple [6–8], la correspondencia AdS/CFT propone la equivalencia de la teoría de cuerdas tipo IIB en $AdS_5 \times S^5$ y teorías de Yang-Mills supersimétricas $\mathcal{N} = 4$ en cuatro dimensiones (la frontera de AdS_5). De nuevo, las D-branas juegan un papel crucial en la formulación de este gran avance.

Finalmente, mencionaremos que el desarrollo formal de la teoría de cuerdas se ha convertido en un área de interés común para físicos y matemáticos. Un caso particular en el

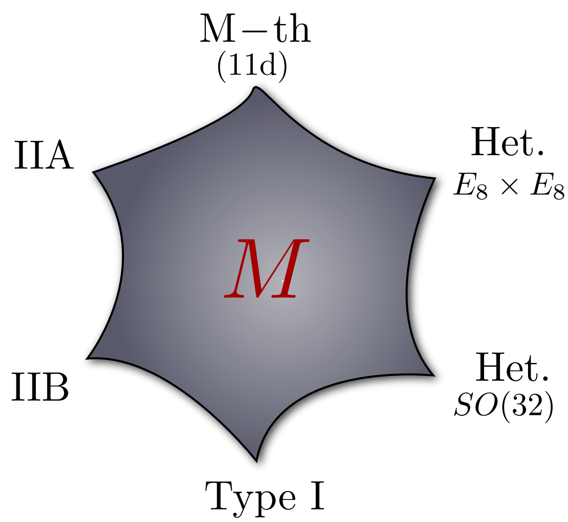


Figure 1.4: Web of dualities of perturbative string theories, depicted as limits of a fundamental unknown theory, M-theory.

que esto es manifiesto es la formulación de teorías topológicas de cuerdas, c.f. [29–31]. En términos generales, éstas son versiones simplificadas de teorías de cuerdas, suficientemente sencillas como para ser manejadas analíticamente, y al mismo tiempo, suficientemente complejas como para aportar resultados interesantes en teorías de campos y de cuerdas. Trataremos algunos aspectos de cuerdas topológicas en el capítulo 4 de esta tesis. Desafortunadamente, no tendremos tiempo aquí de presentar una revisión general del campo, y nos limitaremos en gran medida a tomar ciertos resultados de la bibliografía cuando sea necesario.

Fenomenología de cuerdas

Ademá del estudio de aspectos formales de la teoría de cuerdas, que es interesante en sí mismo, nuestro mayor objetivo es la aplicación de la teoría a los fenómenos físicos observados. Podría decirse que el marco de trabajo y las técnicas de las cuerdas han sido aplicados a tres grandes áreas de física *convencional*: cosmología, con la interpretación de los agujeros negros y de inflación en términos de D-branas; fenómenos colectivos en teorías a acoplo fuerte, tales como materia condensada y plasmas de teorías gauge, estudiadas en terminos de teorías gravitacionales duales gracias a la correspondencia AdS/CFT; y la física de partículas, en la que nos centramos a partir de ahora.

A bajas energías, las amplitudes físicas de estados de cuerdas calculadas mediante diagramas de superficie como en la figura 1.3, pueden reproducirse a partir de una teoría cuántica de campos supersimétrica en 10d. La conexión con física convencional en 4d y con el ME requiere por tanto la compactificación de seis dimensiones. Como vemos, la teoría de cuerdas engloba las dos propuestas más populares de física más allá del modelo estándar, es decir, dimensiones extra y supersimetría. Al mismo tiempo, aporta una terminación UV de estas teorías efectivas, que pierden validez a energías del orden de la escala de la cuerda.

La fenomenología de cuerdas consiste en el estudio de compactificaciones de teorías de cuerdas que reproducen física a bajas energías tan próxima al ME (o al menos a una de sus extensiones supersimétricas) como sea posible. Una revisión reciente de la situación de esta materia puede encontrarse en [20]. La tare es extremadamente difícil por dos razones fundamentales. Por un lado, la teoría de cuerdas parece admitir un número enorme de soluciones, incluso si nos restringimos a aquellas con cuatro dimensiones no-compactas. Por el otro, el análisis de las acciones efectivas de las compactificaciones de cuerdas requiere herramientas matemáticas avanzadas que, a menudo, pueden ser utilizadas tan sólo en situaciones simplificadas.

En consecuencia, es necesario encontrar un compromiso entre manejabilidad teórica e interés fenomenológico. Nuestras guías principales en esta tarea son dos: la preservación de supersimetría en cuatro dimensiones, y la aparición de simetrías gauge no-abelianas con materia quiral cargada. La primera propiedad simplifica el tratamiento de las teorías enormemente, además de ser atractiva a nivel fenomenológico. La segunda es probablemente la propiedad más fundamental del ME.

Nuestro interés en este trabajo se centra en compactificaciones de cuerdas del tipo II. Los espacios de compactificación relevantes en estos casos son variedades Calabi-Yau (CY), que llevan a teorías efectivas en 4d con supersimetría $\mathcal{N} = 2$. Estas teorías, pese a ser enormemente interesantes desde un punto de vista formal [32, 33], tienen demasiada supersimetría para ser aplicadas en modelos fenomenológicos, ya que no permiten la existencia de fermiones quirales. El problema se resuelve con la introducción de D-branas gauge (junto con los planos *orientifold* necesarios por consistencia)

Las Dp-branas pueden enrollar ciclos internos de $p - 3$ dimensiones del CY, y expandirse a lo largo de las cuatro dimensiones no-compactas. La acción efectiva correspondiente tiene supersimetría reducida $\mathcal{N} = 1$, y además contiene bosones gauge no-abelianos que surgen por las cuerdas abiertas ligadas a las branas. Además, representaciones quirales del grupo gauge aparecen si las branas están bien magnetizadas, bien localizadas en puntos singulares del espacio interno, o bien enrolladas a lo largo de ciclos intersecantes del CY.

Estos ingredientes, espacios CY y branas gauge, han aportado herramientas extraordinariamente útiles en el estudio de modelos fenomenológicamente viables basados en cuerdas tipo II. Sus teorías efectivas, calculadas mediante técnicas perturbativas en superficies 2d, pueden llegar a ser de hecho muy parecidas a las del ME, o a alguna de sus versiones supersimétricas tal como el Modelo Estándar Mínimamente Supersimétrico (MEMS). Los modelos tipo II pueden tener espectros no masivos y acciones efectivas similares a los del ME(MS), pero adolecen en general de importantes problemas fenomenológicos.

Los modelos incluyen genéricamente campos escalares sin masa (o incluso taquiónicos), los modulos, cuya existencia entra en contradicción con observaciones cosmológicas. Es más, muchos de ellos carecen, a nivel perturbativo, de algunos acoplos como masas de Majorana para neutrinos, o ciertos acoplos de Yukawa, cuya presencia es necesaria para una fenomenología adecuada. Finalmente, la implementación de mecanismos controlados de ruptura de supersimetría es muy complicada en situaciones concretas. Estos son los principales inconvenientes que la construcción de modelos tipo II debe afrontar.

La introducción de flujos de fondo puede aliviar algunos de estos problemas, c.f. [34]. En particular, pueden dar masa a algunos de los módulos, y pueden servir como mecanismos de ruptura de supersimetría. En este trabajo, sin embargo, no consideraremos estos flujos, sino que nos concentraremos en la aparición de efectos no-perturbativos, no contenidos en la formulación perturbativa de teoría de cuerdas.

Efectos no-perturbativos

Una pregunta fundamental surge naturalmente: si la teoría de cuerdas está formulada exclusivamente a nivel perturbativo, ¿cómo puede uno obtener información de sus efectos no-perturbativos? De nuevo, las D-branas juegan un papel central en la respuesta [35–38]. Esto es bastante natural, ya que la tensión de estos objetos escalea como $1/g_s$, y sus funciones de partición son proporcionales a e^{-1/g_s}. De hecho, las D-branas desaparecen de la dinámica de la teoría en el límite perturbativo $g_s \to 0$.

Consideremos una Dp-brana cuyo volumen, en vez de expandirse a lo largo de las dimensiones Minkowski no-compactas, está enrollado en su totalidad alrededor de un $(p+1)$-ciclo del espacio interno. Esta configuración está localizada en el espacio-tiempo, al igual que los instantones familiares de teorías gauge no-abelianas, c.f. [39]. En anaogía con éstos, se espera que contribuyan a la acción efectiva no-perturbativamente.

Dado que carecemos de una formulación no-perturbativa de la teoría de cuerdas, los efectos de instantón de D-brana no pueden obtenerse por primeros principios. En su lugar, técnicas de cálculo de instanón han sido desarrolladas en años recientes en analogía con los efectos no-perturbativos de teorías gauge, que son bien comprendidos [40–42].

Se ha comprobado que los instantones de D-brana de hecho contribuyen no-perturbativamente a las acciones efectivas de compactificaciones de cuerdas tipo II. Hoy en día se han convertido en mecanismos cruciales para resolver algunos de los problemas de la fenomenología perturbativa de cuerdas mencionados arriba. Podemos concluir que los instantones de D-brana son una parte extremadamente importante de la teoría de cuerdas, tanto a nivel formal como a nivel fenomenológico

Por un lado, juegan un papel crucial en la consistencia de la famosa red de dualidades de cuerdas (figura 1.4) ya que, en muchos casos, los regímenes de acoplo fuerte de una teoría están relacionados con acoplos débiles en otra. Más aún, estos efectos deberían aparecer naturalmente en una formulación no-perturbativa de la teoría de cuerdas que pueda decirnos *qué es la teoría de cuerdas.* Es de esperar que los instantones de D-brana aporten importantes pistas a la búsqueda de una respuesta.

Por otro lado, los efectos de instanton tienen consecuencias extremadamente importantes en la fenomenología de models de cuerdas de física de partículas. Aportan mecanismos para generar acoplos que están prohibidos a nivel perturbativo, y que son necesarios en modelos fenomenológicos viables. Estos incluyen superpotenciales de estabilización de modulos [43], así como acoplos de materia tales como masas de Majorana y acoplos de Yukawa [40–42].

Los resultados de este trabajo

El objetivo de esta tesis es clarificar ciertos aspectos del complicado y en ocasiones oscuro cálculo de instantones, con especial interés en procesos con múltiples instantones. Durante los últimos cuatro años he trabajado en un simple método de cálculo que permite el cómputo de efectos no-perturbativos de modelos tipo II en términos del espectro BPS de teorías duales. En las situaciones en las que puede ser aplicado, el procedimiento permite la resumación de varios sectores de efectos de multi-instantón sin apenas esfuerzo.

Además de una herramienta potente de computacíon, el método aporta una perspectiva aventajada que arroja luz sobre varios aspectos cualitativos de los generalmente oscuros procesos de multi-instantón. En concreto, nuestros resultados han revelado de forma explícita una fuerte relación entre instantones de D-brana y las cuerdas topológicas, que subyace elegantemente la consistencia de los efectos no-perturbativos a través de muros de estabilidad BPS en el espacio de módulos. Una aplicación distinta del método que presentamos aquí es la interpretación correcta de los llamados efectos de poli-instantón que han desconcertado a la comunidad de teóricos de cuerdas por un tiempo [46]. Finalmente, nuestros resultados aportan pruebas adicionales de que el cálculo estándar de efectos de instantón de D-brana es de hecho correcto.

En una línea de investigación más fenomenológica, mis estudios han conducido también a una comprensión sistemática de la aparición de simetrías gauge discretas en amplias clases de vacíos de cuerdas, concretamente en aquellos basados en branas intersectantes en *orientifolds* tipo II. El análisis es muy general y simple, y muestra que las simetrías gauge discretas surgen naturalmente, impidiendo la aparición de ciertos acoplos en la acción efectiva incluso a nivel no-perturbativo. La aplicación de nuestros resultados a modelos fenomenológicos tipo II es crucial, ya que explica claramente la estabilidad del proton, que hasta ahora no era comprendida de manera sistemática.

Como ya hemos mencionado, la parte de mi trabajo relacionada con soluciones analíticas de teorías de campos de cuerdas queda fuera del rango de estudio de esta tesis, y no ha sido presentado aquí por claridad. Los resultados de esta investigación pueden ser encontrados en [25, 26].

Estructura de esta tesis

En los capítulos 2 y 3 resumimos algunos resultados generales sobre compactificationes orientifold tipo II. Estudiamos el espectro sin masa de estos modelos y las teorías efectivas que gobiernan su dinámica en cuatro dimensiones. Ponemos especial énfasis en la separación entre efectos perturbativos y no-perturbativos, y a los teoremas de no-renormalización, que se presentan aquí como simples consecuencias de las propiedades de periodicidad de campos axiónicos. Finalmente introducimos los instantones de D-brana y el método estándar de cálculo de sus efectos. Recalcamos especialmente el papel crucial que juegan tanto a nivel fenomenológico como a nivel formal.

En el capítulo 4 presentamos los resultados de los cálculos de multi-instanton de [47, 48]. En la sección 4.1 presentamos las generalidades del método que aplicamos and las secciones subsiguientes. En 4.2calculamos explícitamente las contribuciones de multi-instantones D1/D(-1) al espacio de hipermultipletes de compactificaciones CY tipo IIB [47]. Sostenemos que la conexión con la cuerda topológica que es manifiesta en nuestros cálculos subyace a la continuidad de efectos no perturbativos a través de lineas de estabilidad BPS. A manera de aplicación del método general a una situación con branas gauge y planos orientifold, presentamos en la sección 4.3 el cómputo de acoplos cuárticos gauge y gravitacionales en ciertas compactificaciones en 8d de teorías tipo I' [48]. Los resultados son utilizados finalmente para proponer una interpretación de los oscuros procesos de poli-instanton de [46].

En el capítulo 5 presentamos nuestros resultados sobre simetrías gauge discretas en modelos orientifold tipo II [49]. En la sección 5.1 describimos el análisis general, mientras que en 5.2 lo aplicamos a algunas construcciones explícitas similares al ME(MS) y a teorías de granunificación $SU(5)$ con D6-branas intersecantes. Algunos aspectos de la generalización del análisis a construcciones de teoría F están recogidos en 5.3.

Finalmente, presentamos nuestras conclusiones y propuestas para futuras líneas de investigación en el capítulo 6.

Chapter 2

Type II string compactifications

In this and the following chapters we review some of the basic tools that will be needed to present the results of this thesis. Most of the results are quite standard and can be found in general textbooks such as [13–20]. We study here the low energy effective actions of type IIA superstring theories and their compactifications to four dimensions. We focus on internal Calabi-Yau manifolds which lead to supersymmetric effective theories that are under better control than non-supersymmetric ones, see e.g. [14, 50–52]. We present the massless spectrum of closed string fields in these setups. The scalar components of the supermultiplets play a crucial role in our work. As we emphasize, their real parts parametrize the geometry of the internal space, while the imaginary parts are generalized Wilson-lines whose (axionic) properties we review.

The introduction of D-branes and orientifold planes is described next, with special interest in configurations of intersecting D6-branes which preserve $\mathcal{N} = 1$ supersymmetry and lead to chiral spectra, see e.g. [53, 55–57]. The crucial Green-Schwarz anomaly cancellation mechanism [58, 59] and its interplay with axions is described in detail. Finally, a toroidal model with a Standard Model like spectrum is introduced. This type of models will be the main focus of chapter 5.

2.1 Type II theories in ten dimensions

Ultimately, we will be interested in compactifications of type II string theory to four dimensions and their behavior at energies that can be tested experimentally at particle physics laboratories. The current energy bound on these experiments is set by the LHC to be of the order of a few TeV. This is, for the vast majority of string theory scenarios, much lower than the string mass scale $M_s \propto (\alpha')^{-1/2}$, which is typically taken to lie at the Planck scale ($\sim 10^{19}$ GeV) or at a Grand Unification Theory (GUT) scale ($\sim 10^{15} - 10^{16}$ GeV). This means that massive excited states of the string, whose masses are of order M_s, will never be produced at our colliders and can be integrated out from the low energy effective actions that we are interested in. The starting point of our analysis will be the effective field theories describing type IIA and IIB superstring theories at low energies in ten flat dimensions. In the next section, we will study their compactifications to four dimensions.

The 10d bosonic massless spectra of type IIA and IIB theories share common Neveu

Schwarz-Neveu Schwarz (NS-NS) sectors containing a dilaton scalar ϕ, a two-form B_2 and a two-index traceless symmetric tensor G_{MN} which plays the role of the graviton.[1] The dilaton plays a distinguished role in perturbative string theory, since its expectation value determines the strength of string interactions and serves as an expansion parameter for a perturbative series $g_s = e^{\phi}$.

The GSO projection acts differently on the Ramond (R) sectors of the two theories, leading to different massless spectra. For the type IIA theory the massless bosons in the RR sector are the odd degree 1- and 3-forms C_1 and C_3, as well as their magnetic or hodge duals 7- and 5-forms C_7 and C_5. For the type IIB theory they are the even degree 0-, 2-, and 4-forms C_0, C_2 and C_4, and their duals C_8 and C_6. The 4-form C_4 has self-dual field strength $*dC_4 = dC_4$.

Type II string theories enjoy several local spacetime symmetries:

- Diffeomorphism invariance in ten dimensions, since they include the graviton G_{MN} and therefore gravity.
- Abelian gauge transformations of the p-forms (both RR and NS-NS forms),

$$A_p \to A_p + d\Lambda_{p-1}, \tag{2.1}$$

  which generalize to $p > 1$ the usual gauge symmetries of 1-forms in four dimensions.
- Ten dimensional $\mathcal{N} = 2$ local supersymmetry (32 supercharges). The bosonic fields, together with the fermions that arise from the R-NS and NS-R sectors (gravitinos and dilatinos), fill out the gravity multiplets of the chiral and non-chiral $\mathcal{N} = 2$ superalgebra for type IIB and type IIA theories, respectively.

The effective actions governing the dynamics of the fields in 10d spacetime are severely constrained by these symmetries. We will briefly comment on how they can be obtained through worldsheet 2d conformal field theory (CFT) computations in the next chapter, but for the moment, let us just write down the results for their bosonic sectors. We include terms up to two derivatives, which are the most relevant ones at low energies. For the type IIA theory the action reads

$$\begin{aligned} 2\kappa_{10}^2 S_{IIA} &= \int d^{10}x\,(-G)^{\frac{1}{2}}\Big[e^{-2\phi}\Big(R + 4\,\partial_M\phi\,\partial^M\phi - \tfrac{1}{2}|H_3|^2\Big) - \tfrac{1}{2}|F_2|^2 - \tfrac{1}{2}|\tilde{F}_4|^2\Big] \\ &\quad -\frac{1}{2}\int_{10d} B_2 \wedge F_4 \wedge F_4 \end{aligned} \tag{2.2}$$

while for the type IIB theory we have

$$\begin{aligned} 2\kappa_{10}^2 S_{IIB} &= \int d^{10}x\,(-G)^{\frac{1}{2}}\Big[e^{-2\phi}\Big(R + 4\,\partial_M\phi\,\partial^M\phi - \tfrac{1}{2}|H_3|^2\Big) - \tfrac{1}{2}|F_1|^2 - \tfrac{1}{2}|\tilde{F}_3|^2 - \tfrac{1}{2}|\tilde{F}_5|^2\Big] \\ &\quad -\frac{1}{2}\int_{10d} C_4 \wedge H_3 \wedge F_3. \end{aligned} \tag{2.3}$$

[1] 10d indices will be labeled by upper case latin letters $M, N = 0, \ldots, 9$. 4d quantities in the compactified theories introduced in the next section will be labeled by lower case greek letter indices $\mu, \nu = 0, \ldots, 3$. A single numerical subindex in a cohomology form denotes its degree, e.g. $B_2 \equiv B_{MN}\, dx^M \wedge dx^N$. Einstein's convention for summation over repeated indices is in force.

Here $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$ encodes Newton's constant in terms of the string scale. $F_p = dC_{p-1}$ and $H_3 = dB_2$ are the field strengths of the generalized gauge fields and their kinetic terms are of the form

$$\int d^{10d}x\,(-G)^{\frac{1}{2}}\,|F_p|^2 \equiv \int d^{10}x\,(-G)^{\frac{1}{2}}\,\frac{1}{p!}F_{M_1\ldots M_p}F^{M_1\ldots M_p} = \int_{10d} F_p \wedge *F_p. \tag{2.4}$$

We have also defined the following modified field strengths

$$\begin{aligned} &\text{Type IIA:} && \tilde{F}_4 = F_4 - C_1 \wedge H_3 \\ &\text{Type IIB:} && \tilde{F}_3 = F_3 - C_0\,H_3\,, \qquad \tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3. \end{aligned} \tag{2.5}$$

The type IIB action should be supplemented with a self-duality condition for the 4-form field strength. Notice also that the above actions have been written in the string frame, with a dilaton factor accompanying the Einstein-Hilbert action. The usual spacetime Einstein frame action with standard kinetic terms for the gravitons can be recovered by a suitable redefinition of the fields.

2.2 Calabi-Yau compactifications

After introducing the effective actions of type II theories in ten dimensions, we proceed now to their Kaluza-Klein (KK) compactifications to four dimensions. We consider the simplest setups in which the ten dimensional spacetime background is described by a factorized geometry $\mathbf{M}_4 \times \mathbf{X}_6$, i.e. the direct product of four dimensional Minkowski space $\mathbf{M}_4$ and a six dimensional compact internal manifold $\mathbf{X}_6$. This ansatz assumes that no background fields other than the metric (e.g. p-form fields) have been turned on. More general compactifications require more advanced techniques and will not play a crucial role in this work.

Since no experimental evidence for extra dimensions has been found so far, we must take the compactification space $\mathbf{X}_6$ to have a typical size R small enough so that KK replicas of massless particles decouple from observable physics. On the other hand, in order to validate the field theory KK approximation, we must assume that this size is much smaller than the typical string size $\alpha'/R^2 \ll 1$. This guarantees that winding modes of the string, and in general states coming from extended objects that wrap internal cycles of the compactification space, acquire large masses and decouple from the low energy theory.

Now, what manifold should be chosen for $\mathbf{X}_6$ is one of the most important and most difficult questions that the area of string phenomenology has to face. Together with a choice of spacetime filling D-branes (to be introduced in the next section) and background fluxes, it determines the low energy theory that describes the physics observed in four dimensions. In particular, just the topology of $\mathbf{X}_6$ gives such important information as the number of supersymmetries or the massless closed string fields that survive the compactification.

Ideally we would like to find a compactification that leads to 4d physics as close as possible to the Standard Model, or at least to one of its proposed generalizations such as the Minimal Supersymmetric Standard Model (MSSM). The task is extremely difficult, however, due to the complicated analysis of the theory and its equations of motion for general $\mathbf{X}_6$, which is out of reach for our present mathematical tools. Not a single completely satisfactory example has been found so far.

It has become clear, instead, that a compromise between theoretical tractability and phenomenological interest is necessary. We should find general compactification setups that can be studied with certain level of detail, and are still flexible enough to accommodate at least some of the properties of the Standard Model or the MSSM.

Our guiding principles in fulfilling this objective will be two, namely, the preservation of some amount of supersymmetry in four dimensions, and the appearance of non-abelian gauge symmetries and fermions in chiral representations thereof. We will focus on the former feature in this section, while the latter will be accomplished in section 2.3 with the introduction of orientifold planes and D-branes.

2.2.1 Supersymmetry in four dimensions

The reasons to study supersymmetric compactifications of string theory are twofold. On the theoretical side, they lead to stable and tachyon-free string vacua, that automatically satisfy the supergravity (second order) equations of motions. This is most welcome since one does not need to explicitly solve them, once the much simpler first order supersymmetry constraints are fulfilled. Supersymmetric field theories are in general more constrained and easier to handle than non-supersymmetric ones. Moreover, it has been realized that supersymmetric field theories (specially $\mathcal{N} = 2$ theories) and supersymmetric compactifications of string theories have extremely rich mathematical structures [32, 33] and are useful tools to study problems of even pure mathematics. As an example, topological strings, which will be briefly discussed in chapter 4 and play a crucial role in [47], lie indeed in that region of tight overlap between string theory and mathematics.

On the phenomenological side, supersymmetric theories have been arguably the most thoroughly studied extensions of the Standard Model for several reasons, but mainly because of their capacity to alleviate the electroweak hierarchy problem, see e.g. [60–62]. In this respect, finding a superstring compactification that leads to some of the proposed supersymmetric extensions of the Standard Model, such as the MSSM, may serve as good an objective as looking for the Standard Model itself. Furthermore, even if supersymmetry does not show up at the LHC and is not realized at the TeV scale, supersymmetric compactifications of string theory serve as good toy models to understand the behavior of more general non-supersymmetric string setups.

With these motivations in mind, let us discuss what type of compactification manifolds $\mathbf{X}_6$ preserve some of the supersymmetries of the ten dimensional theory. In the original theory in flat spacetime, these are generated by operators $Q = \epsilon_L Q_L + \epsilon_R Q_R$, with 32 supercharges organized into two $SO(10)$ spinors Q_L and Q_R of opposite chirality. The transformation coefficients ϵ_L and ϵ_R are constant spinors of $SO(10)$ with the same chirality as the corresponding Q's. These will survive as supersymmetries of the compactified theory if one can still globally define *constant* spinors on the $\mathbf{M}_4 \times \mathbf{X}_6$ background. Since the compact space is curved, what we actually mean is covariantly constant spinors globally defined over the whole space, and in particular over $\mathbf{X}_6$.[2]

Now, covariantly constant spinors (and more in general, covariantly constant objects under any representation of the Lorentz group) are those that do not transform under the

[2]Of course, locally the total space looks flat and one can define constant spinors in a sufficiently small neighborhood of any point p.

holonomy group of the manifold, i.e. those that remain invariant under parallel transport along any closed path on the manifold. For general varieties $\mathbf{X}_6$, this group is the whole of $SO(6)$ and no covariantly constant spinor can be defined globally. What we need is a manifold whose holonomy is a subgroup $H \subset SO(6)$ such that the decomposition of the spinor representations $\mathbf{4}$ and $\bar{\mathbf{4}}$ of $SO(6)$ under it contains a singlet. This is achieved for holonomy groups $H \subseteq SU(3)$.

A ten dimensional spinor decomposes in the following manner under such a group

$$\begin{array}{ccccc} SO(10) & \to & SO(6)\times SO(1,3) & \to & SU(3)\times SO(1,3) \\ \mathbf{16} & & (\mathbf{4},\mathbf{2})+(\bar{\mathbf{4}},\mathbf{2}') & & (\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2}')+(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2}') \end{array} \tag{2.6}$$

Here, $\mathbf{2}$ and $\mathbf{2}'$ denote left- and right-handed chiral spinors under the 4d Lorentz group. The $SU(3)$ singlets in the last column represent the covariantly constant spinors we were looking for. They lead to four unbroken supercharges Q_α, $\bar{Q}_{\dot\alpha}$ for each 10d spinor supercharge. Since the type II theories we are interested in have 10d $\mathcal{N}=2$ supersymmetry, compactifications on manifolds of strict $SU(3)$ holonomy lead to 4d effective field theories with eight supercharges and hence $\mathcal{N}=2$ supersymmetry. Manifolds with a holonomy group smaller than $SU(3)$ can host several linearly independent covariantly constant spinors, and therefore lead to effective theories with higher supersymmetries ($\mathcal{N}=4$ and $\mathcal{N}=8$ for type II theories).

2.2.2 Calabi-Yau manifolds and their moduli spaces

Finding six-dimensional compact manifolds with metrics of $SU(3)$ holonomy seems like an extremely difficult task. Indeed, no analytic expression has been found for a single such metric so far. However, a powerful mathematical theorem conjectured by E. Calabi, and subsequently proved by S.-T. Yau, ensures that any $2N$-dimensional manifold that satisfies certain simple conditions admits a metric with $SU(N)$ holonomy. This theorem is extremely useful since, even if it does not tell us the explicit form of the metric, it allows us to construct large classes of varieties that can be used for supersymmetric compactifications of string theory. Happily, a vast amount of information about the 4d effective theory can be obtained from global properties of the internal space, without the need for the explicit form of the metric.

The Calabi-Yau (CY) theorem applies to manifolds $\mathbf{X}$ of dimension $2N$ which are complex and Kähler, and have vanishing first Chern class. These are called CY N-folds [14, 50–52]. Being complex means that they admit a globally defined complex structure tensor $I^i_{\bar{j}}$ which can be used to define complex coordinates $\{z^i, \bar{z}^{\bar{j}}\}$, that transform holomorphically under changes of patch. Given such a complex structure, there exists a form J of degree $(1,1)$ out of which a metric with mixed indices can be constructed

$$J = g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}. \tag{2.7}$$

If this form is closed $dJ = 0$, it is called the Kähler form, and the manifold is said to be Kähler. This last condition ensures that under parallel transport holomorphic and anti-holomorphic indices do not mix, and hence the holonomy lies in $U(N) = SU(N) \times U(1) \subset SO(2N)$. The final condition on the vanishing of the first Chern class ensures that the $U(1)$ factor decouples and the holonomy is indeed $SU(N)$ as desired. Finally, these properties also ensure that the

CY metric is Ricci flat, so the supergravity equations of motion (in the absence of other ingredients such as background fluxes) are automatically satisfied.

Focusing now on the specific case of CY threefolds let us present their hodge diamond, i.e. the dimensions of their Dolbeault cohomology groups. It can be shown that, for a manifold of strict $SU(3)$ holonomy, these take the form

$$\begin{array}{ccccccc} & & & h_{0,0} & & & \\ & & h_{1,0} & & h_{0,1} & & \\ & h_{2,0} & & h_{1,1} & & h_{0,2} & \\ h_{3,0} & & h_{2,1} & & h_{1,2} & & h_{0,3} \\ & h_{3,1} & & h_{2,2} & & h_{1,3} & \\ & & h_{3,2} & & h_{2,3} & & \\ & & & h_{3,3} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h_{1,1} & & 0 & \\ 1 & & h_{2,1} & & h_{2,1} & & 1 \\ & 0 & & h_{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.8}$$

Of particular importance is the unique cohomology $(3,0)$ class which we will denote by Ω.

The CY theorem also ensures that, for a Kähler manifold with vanishing first Chern class, the metric of $SU(N)$ holonomy is unique for any given choice of complex structure and Kähler class ($J \in H^{(1,1)}(\mathbf{X}, \mathbb{R})$). This means that the parameters that determine the CY metric of a specific manifold are

- **Kähler moduli:** the Kähler class J can be expanded in a basis of $(1,1)$-forms $\{\omega_a\}$:

$$J = \sum_{a=1}^{h_{1,1}} t_a \, \omega_a. \tag{2.9}$$

  The real coefficients t_a determine uniquely the Kähler class of the manifold.

- **Complex structure moduli:** The complex structure tensor can be uniquely related to a $(2,1)$-form with the aid of the holomorphic $(3,0)$-form, $I_{ij\bar{l}} = \Omega_{ijk} I^k_{\bar{l}}$. Hence, a choice of complex structure is equivalent to the choice of a (2,1)-form class, and is parametrized by $2h_{2,1}$ real moduli or $h_{2,1}$ complex ones.

These moduli will show up as massless scalar fields of the 4d effective field theory. This is intuitively clear since their values (i.e. their vacuum expectation values (vev)) parametrize internal space geometries that respect supersymmetry and hence all yield zero vacuum energy (remember that the 4d background is taken to be Minkowski). Therefore, they describe flat directions of the effective potential. Geometric moduli will play a very important role in this work.

2.2.3 Low energy spectrum and effective action

After having described in some detail the topology and geometry of the compactification spaces that we will use, let us study the corresponding KK reduction of the ten dimensional supergravities introduced in section 2.1. We focus our attention on the bosonic sector of the type IIA theory. The fermionic sector can be inferred by supersymmetry, and the results we will present can be applied to the type IIB theory using mirror symmetry (which will be briefly discussed later on).

The low energy bosonic spectrum: generalities

As a first step we introduce the fields that will come into play, i.e. those that remain massless in four dimensions. These come from the ten dimensional p-form fields (including the case $p = 0$, i.e. scalar fields) and the gravitons.

We have already argued that there will be massless scalar fields that parametrize the geometry of the compactification space. These arise from components of the 10d graviton whose indices lie in the internal space G_{mn}. They are the complex structure and Kähler moduli introduced before. Components with one index in the compact manifold and on in Minkowski space $G_{\mu i}$ would lead to massless 4d bosons if they behaved as Killing vectors of the 6d manifold. However, one can show that for CY manifolds, no such Killing vector exists (i.e. there are no continuous isometries), so no 4d vectors arise from the 10d metric in this case. This fact is related to the absence of 1-forms in a CY, as can be seen in (2.8). Finally, the components with both indices pointing in the Minkowski directions $G_{\mu\nu}$ lead to the expected 4d gravitons.

To apply the KK procedure to a p-form $C_p(x^M)$ we first have to separate internal 6d coordinates x^m from the 4d ones x^μ and propose an ansatz which will include terms of the form[3]

$$C_p(x^M) = c_q(x^m) \wedge C_{p-q}(x^\mu). \tag{2.10}$$

Here $c_q(x^m)$ is a q-form in $\mathbf{X}_6$ and $C_{p-q}(x^\mu)$ represents a 4d $(p-q)$-form field. Since the kinetic term operator, the Laplacian $\Delta = dd^\dagger + d^\dagger d$, decomposes as

$$\Delta_{10} = \Delta_6 + \Delta_4, \tag{2.11}$$

massless 4d form fields of degree $(p-q)$ arise from q-form zero modes of the 6d Laplacian Δ_6, i.e. from harmonic q-forms. The number of independent harmonic forms in a manifold $\mathbf{X}_6$ is counted by the Betti numbers $b_q(\mathbf{X}_6)$. Hence, from every 10d p-form field we expect to obtain $b_q(\mathbf{X}_6)$ independent 4d $(p-q)$-form massless fields for $q = 0, \ldots p$.

Now an important question is what happens to the original gauge symmetries of the original 10d fields $C_p \to C_p + d\Lambda_{p-1}$. For 4d q-form fields of degree $1 \le q \le p$, it is easy to see that the symmetry remains in the expected way $C_q \to C_q + d\Lambda_{q-1}$. The story is more subtle for scalar fields ($q = 0$) arising from p-forms with a gauge symmetry.[4] We study for simplicity a toy example of a circle compactification of a $U(1)$ gauge field A_μ with symmetry $A_\mu \to A_\mu + d\lambda$ minimally coupled to some matter fields ψ_q of charge q. That is, we study five dimensional quantum electrodynamics compactified on a circle.

We are interested in the zero-modes of the internal component A_4, i.e. the constant modes $\langle A_4 \rangle$. Locally, it can be gauged away by a transformation

$$A_\mu \quad \to \quad A_\mu + \partial_\mu \lambda \qquad \text{with} \quad \lambda = -\langle A_4 \rangle x^4 \, . \tag{2.12}$$

However, the transformation parameter is not globally well defined due to the non-trivial topology of the internal $\mathbf{S^1}$. $\langle A_4 \rangle$ is thus physical and can indeed be measured by the gauge

[3] We recall that upper case latin indices label ten dimensional quantities $M, N = 0, \ldots, 9$, lower case indices are for internal 6d coordinates $m, n = 4, \ldots 9$ and greek indices are for the 4d ones $\mu, \nu = 0, \ldots 3$.

[4] Scalars in 10d have no special symmetry, as well as the 4d scalars they lead to. The dilaton reduces to a single dilaton (since $b_0(\mathbf{X}) = 1$ for any connected manifold) with no particular symmetry.

invariant Wilson line

$$W = e^{i\int A_4} = e^{2\pi\, i\, R\, \langle A_4 \rangle}\,. \tag{2.13}$$

From this object, we see that the quantity $\langle A_4 \rangle$ is periodic with period $1/R$.[5] Another way to see the periodicity of the scalars $\langle A_4 \rangle$ is to take a 4d perspective and ask what is the consequence of turning on non-zero value of these fields. The minimal coupling of a field of charge q to the gauge bosons tells us that the effect of the Wilson line is to shift the internal momentum (i.e. the KK momentum $k \in \mathbb{Z}$) of a given state by

$$k \quad \to \quad k + q\,\langle A_4 \rangle R\,, \qquad \text{with} \quad k \in \mathbb{Z}\,. \tag{2.14}$$

It is clear that the spectrum of KK states will not be affected by the shift $A_4 \to A_4 + 1/R$, as long as we consider quantized charges appropriately normalization so that the minimal charge in the spectrum is one.

This result can be intuitively understood in a setup in which the gauge field comes from open strings that live on a D-brane that wraps the internal circle. The Wilson line scalar A_4 is related by T-duality, to a scalar ϕ whose vev parametrizes the location of the T-dual brane on the internal (T-dual) circle, and has obviously the desired identification property $\phi \sim \phi + 2\pi\tilde{R}$ (where the dual radius is $\tilde{R} \sim 1/R$). The lesson, however, is more general, and in fact scalars that arise from the reduction of p-form fields with a gauge symmetry always parametrize circles. We call them *axionic* fields for reasons that will become apparent in chapter 3. This property has extremely important consequences and will play a crucial role in the rest of this work.

The type IIA spectrum

Let us apply the KK reduction to type IIA string theory compactified on a CY threefold. The 10d bosonic fields that we have to consider are the dilaton ϕ, the graviton G_{MN}, the NS-NS 2-form B_2 and the RR 1- and 3-form fields C_1 and C. With the above remarks, and with the help of the hodge numbers of CY threefolds (2.8), this is straightforward. The result is summarized in the following table

		Gravity	$h_{1,1}$ Vector	$h_{2,1}$ Hyper	Hyper
G	$\to$	$G_{\mu\nu}$	$h_{1,1}$ Kähler	$2h_{2,1}$ Compl. Str.	
B_2	$\to$		$B_{i\bar{j}}$		$B_{\mu\nu}$
ϕ	$\to$				ϕ
C_1	$\to$	C_μ			
C_3	$\to$		$C_{i\bar{j}\mu}$	$C_{ij\bar{k}}$, $C_{\bar{i}\bar{j}k}$	C_{ijk}, $C_{\bar{i}\bar{j}\bar{k}}$

The fields are organized in supermultiplets of the 4d $\mathcal{N} = 2$ theory which are represented by columns in the table. The first one represents the supergravity multiplet which contains

[5] In the rest of this work we will usually redefine the Wilson line scalars $\tilde{A}_4 \equiv RA_4$ so that their period is just 1.

the graviton and a graviphoton vector field. Next, there appear $h_{1,1}$ vector multiplets. They contain the Kähler moduli of the internal CY manifold, that pair up with scalars coming from the NS-NS 2-form to form complex fields. Recall that, as argued above, the latter parametrize circles, i.e. $B_{i\bar{j}} \sim B_{i\bar{j}} + 1$ with the appropriate normalization. The third column represents the geometric hypermultiplets which contain the complex structure moduli, which pair up with scalars coming from the RR 3-form. Again, the latter are defined modulo $\mathbb{Z}$ (with the appropriate normalization). Finally, there is a universal hypermultiplet whose scalar contain the dilaton and the NS-NS 2-form (which can be dualized to a scalar in 4d). These pair up with scalars coming from the RR 3-form which have the by now familiar axionic periodicity properties.

The low energy effective action

Let us briefly discuss some aspects of the low energy effective action that describes the dynamics of these fields. Since we will study effective actions in some detail after introducing orientifold planes and D-branes in the next chapter, we will be rather sketchy here.

Supersymmetric effective actions are most naturally written in the language of $\mathcal{N} = 1$ multiplets. A vector multiplet of $\mathcal{N} = 2$ decomposes as an $\mathcal{N} = 1$ vector multiplet V whose vector component we denote by A_μ, and a chiral multiplet Φ (in the adjoint representation of the gauge group) whose complex scalar component we label also by Φ. Hypermultiplets of $\mathcal{N} = 2$ theories, on the other hand, decompose into two $\mathcal{N} = 1$ chiral multiplets of opposite chirality, whose scalars we denote by Q and $\tilde{Q}$.

Now, it can be shown that the effective action of these theories have completely decoupled kinetic terms for vector and hypermultiplets. For vector multiplets, there is a term of the form

$$S_{4d} \sim \int_{\mathbf{M}_4} G_{a\bar{b}}(\Phi, \bar{\Phi})\, d\Phi^a \wedge *d\Phi^{\bar{b}} \tag{2.15}$$

This is the action of a non linear sigma model (NLSM) with a target space parametrized by the Kähler moduli Φ and with a metric $G_{a\bar{b}}(\Phi, \bar{\Phi})$. Constraints from $\mathcal{N} = 2$ supersymmetry restrict this to be *special Kähler*, i.e. a Kähler metric which is completely determined by a holomorphic function $\mathcal{F}(\Phi)$ called the prepotential. In fact, the prepotential determines not only the Kähler term for the adjoint chiral multiplets, but also the gauge kinetic function for the vector ones V (whose field strength chiral multiplets we denote by W_α). These can be written in $\mathcal{N} = 1$ language as

$$\mathcal{L}_{4d} \sim \mathrm{Im}\left[\int d^2\theta\, d^2\bar{\theta}\, (\partial_a \mathcal{F})\bar{\Phi}_a + \int d^2\theta\, \tfrac{1}{2}(\partial_a\partial_b\mathcal{F})(W^\mu)_a(W_\mu)_b\right]. \tag{2.16}$$

On the other hand, the kinetic term for the hypermultiplets Q and $\tilde{Q}$ defines a metric on the moduli space of complex structures which is restricted by supersymmetry to be a much more complicated *quaternionic Kähler* metric. In the limit in which gravity decouples, the restriction reduces to the somewhat simpler *hyperkähler* condition. Despite their interest, we will not deal explicitly with the geometric properties of these type of manifolds.

Mirror symmetry and type IIB

The results we have obtained for the type IIA CY compactifications relate to those of the type IIB theory by mirror symmetry [29]. Roughly speaking, this duality states that compactification of type IIA on a CY manifold $\mathbf{X}_6$ is completely equivalent to compactification of type IIB on a *mirror* CY $\tilde{\mathbf{X}}_6$. The hodge numbers of dual manifolds are related through

$$h_{1,1}(\mathbf{X}_6) = h_{2,1}(\tilde{\mathbf{X}}_6)\,, \qquad h_{2,1}(\mathbf{X}_6) = h_{1,1}(\tilde{\mathbf{X}}_6). \tag{2.17}$$

We have seen that the massless spectrum of type IIA on $\mathbf{X}_6$ contains, besides the gravity multiplet and the universal hypermultiplet, $h_{1,1}(\mathbf{X}_6)$ vector multiplets and $h_{2,1}(\mathbf{X}_6)$ hypermultiplets. Mirror symmetry states, through (2.17) that compactification of type IIB on the mirror CY manifold $\tilde{\mathbf{X}}_6$ will contain, besides the gravity multiplet and the universal hypermultiplet, $h_{2,1}(\tilde{\mathbf{X}}_6)$ vector multiplets and $h_{1,1}(\tilde{\mathbf{X}}_6)$ hypermultiplets. This can be easily checked by direct KK reduction of the ten dimensional fields of type IIB theory.

Mirror symmetry is a very efficient tool for the computation of effective actions of type II compactifications. It relates the complex structure moduli space of a compactification manifold to the Kähler moduli space of the mirror one. A quantity that is be very complicated (e.g. receive quantum corrections) on one side, may be related through mirror symmetry to a simple term on the other. We will not make explicit use of this duality in this work, but it is always useful to have it in the back of our minds. As an example, in chapter 4 we will study type IIB CY compactifications and will use some results of this section which have been originally derived for type IIA. Mirror symmetry provides the bridge between both theories.

2.3 Gauge D-branes and orientifold planes

The setups discussed so far have very rich mathematical structures and are interesting from a theoretical point of view. However, they have no direct application to phenomenology for two reasons. First, they do not contain non-abelian gauge symmetries and hence cannot embed the $SU(3) \times SU(2) \times U(1)$ gauge group of the Standard Model. Second, even if such a group appeared, the amount of supersymmetry ($\mathcal{N} = 2$ in four dimensions) is too large to accommodate fermions in chiral representations, which is one of the most important features of the Standard Model. Both problems can be overcome with the introduction of D-branes and orientifold planes (O-planes).

2.3.1 D-branes in flat spacetime: spectrum and effective action

Dp-branes are $(p+1)$-dimensional extended objects which are embedded in spacetime and can span non-compact dimensions and wrap non-trivial cycles of compact spaces. They are dynamical objects whose fluctuations can be described at weak coupling by open strings with endpoints attached to the branes, and whose massless spectrum includes gauge bosons [27].

The case of a single D-brane

For the simplest case of a single Dp-brane in flat space, the bosonic massless spectrum of the worldvolume $(p+1)$-dimensional theory includes a $U(1)$ gauge boson A_μ and $9-p$ real scalars ϕ^i. The latter parametrize the position of the brane in the transverse directions, and hence the embedding of the worldvolume in spacetime. Together with the fermionic spectrum, the fields form a vector multiplet of a supersymmetric theory in $p+1$ dimensions with 16 supercharges. This implies that D-branes are BPS objects that kill half of the 32 supercharges of the original type II theories. The explicit supersymmetries preserved by the brane are of the form $Q = \epsilon_L Q_L + \epsilon_R Q_R$, with coefficients satisfying

$$\epsilon_L = \Gamma^0 \dots \Gamma^p \epsilon_R, \tag{2.18}$$

where $\{\Gamma^0, \dots, \Gamma^p\}$ are the ten dimensional gamma matrices along the directions spanned by the brane. This feature is most welcome since, besides containing gauge symmetries which can be easily enhanced to non-abelian groups (as we will shortly describe), being BPS, they can be used to reduce the supersymmetry of CY compactifications to $\mathcal{N} = 1$, which admits chiral representations for fermions in four dimensions.

The effective action describing the dynamics of the fields that live in the worldvolume W_{p+1} of the brane, and their interactions with closed string fields that propagate in the 10d bulk, contains two terms. The first one is known as the Dirac-Born-Infeld (DBI) action and reads

$$S_{DBI} = -\mu_p \int_{W_{p+1}} d^{p+1}x \, e^{-\phi} \sqrt{-\det(G + B - 2\pi\alpha' F)}. \tag{2.19}$$

Here G and B represent the 10d metric and NS-NS 2-form field.[6] F is the field strength of the worldvolume $U(1)$ gauge bosons and μ_p is a simple power of the string tension α'. The appearance of the dilaton implies that the D-brane tension scales as μ_p/g_s, indicating that D-branes are intrinsically non-perturbative objects in g_s. The DBI action describes the coupling of the worldvolume fields to the 10d NS-NS fields. Freezing the dilaton ϕ to a constant value, and ignoring momentarily the 2-form B, an expansion in powers of F describes first of all the volume of the brane, and second the standard kinetic Yang-Mills action for the gauge fields, with $g_{YM}^2 \sim g_s$.

The other term in the effective action, the Chern-Simons (CS) term, describes the topological coupling of the brane to the closed string RR fields

$$S_{CS} = \mu_p \int_{W_{p+1}} \sum_q C_q \wedge e^{2\pi\alpha' F - B} \wedge \hat{A}(R). \tag{2.20}$$

C_q are the (pullbacks of the) 10d RR q forms, with q running through the appropriate values for type IIA and IIB theories, and the curvature R appears through the so called A-roof polynomial whose explicit form will be presented in appendix 4.B of chapter 4, but will not play an important role for the moment. The CS action implies that Dp-branes source RR $(p+1)$-form fields (and possibly forms of lower degree) and are charged under the corresponding $U(1)$ gauge groups. It can be easily inferred from this action, that Dp-branes exist only for even values of p in the type IIA theory, while odd values of p are the ones that appear in type IIB.

[6] To be precise, the pullbacks $\phi^*(G)$ and $\phi^*(B)$ of these 10d fields to the worldvolume of the brane should appear in this and the following equations. This is the way in which the scalar fields ϕ^i which embed the brane worldvolume into spacetime appear in the effective action. We omit this technicality for clarity.

Stacks of intersecting D6-branes

We want to discuss now several important features that arise when one considers configurations with multiple branes, namely the appearance of non-abelian gauge groups and of fermions in chiral representations thereof. As in the previous section, we focus in the case of type IIA superstring theory. Analogous results can be obtained for type IIB with similar techniques, or inferred in many cases via mirror symmetry (or T-duality). Furthermore, since D6-branes play a prominent role in phenomenological approaches to the Standard Model from type IIA strings, and in particular in the work presented in chapter 5, we focus in this case in the following [53, 55–57].

The first configuration of multiple branes (say N) to consider is that in which their worldvolumes coincide. In this case, massless open strings can have one extreme in one brane of the stack, and the other in another brane. The net effect is that the gauge group of the worldvolume theory is enhanced from $U(1)^N$ to $SU(N)$. The open string fields now form a vector supermultiplet in the adjoint representation, with the scalar components playing the role of Higgs fields. Giving vev's to these fields generically breaks the group $SU(N) \to U(1)^N$, as is natural since their values can be interpreted as the distance between the branes of the stack.

The second possible configuration is that in which two stacks of N_1 and N_2 coincident branes are not parallel, but intersecting [63]. For the case of D6-branes, the intersection locus is four dimensional. Besides the already described spectrum of $\mathcal{N} = 2$ vector supermultiplets of the $U(N_1) \times U(N_2)$ gauge theory coming from open strings with both ends on the same stack, there arise new massless states from open strings that stretch between the two stacks. These states are only massless if they are located at the 4d intersection locus of the branes. Their spectrum contains chiral fermions in the bifundamental representation $(\mathbf{N_1}, \overline{\mathbf{N_2}})$ of $SU(N_1) \times SU(N_2)$. These will pair up with massless scalars to form chiral multiplets of 4d $\mathcal{N} = 1$ supersymmetry if the brane configuration preserves supersymmetry, i.e. if there exist spinors satisfying conditions (2.18) for both stacks simultaneously. If there remain no supersymmetries, the corresponding scalars will be massive or even tachyonic, signaling an instability of the brane configuration.

2.3.2 Compactifications to four dimensions

We have described a mechanism to introduce non-abelian gauge groups and chiral fermions in type IIA string theory. Now we would like to implement this mechanism in the compactification setups presented in section 2.2. Along the way we will see that in order to preserve supersymmetry we need to introduce, together with D-branes, orientifold planes.

Supersymmetric configurations

We are interested in CY compactification backgrounds of type IIA theory in which D6 branes wrap non-trivial 3-cycles Π_a of the internal space, and span the four non-compact dimensions. These backgrounds preserve 4d Lorentz invariance and are therefore the ones of most direct interest for phenomenology. The particular wrapping cycles of interest for us are the so-called *special Lagrangian* 3-cycles, defined by the conditions [35, 64]

$$J|_{\Pi} = 0\,, \qquad \mathrm{Im}\,(e^{-i\varphi}\,\Omega_3)|_{\Pi} = 0\,. \tag{2.21}$$

Recall that J is the Kähler $(1,1)$-form of the CY, and Ω_3 its unique holomorphic $(3,0)$-form. φ is some fixed constant angle. These conditions ensure that the tangent spaces at different points of Π are related through $SU(3)$ rotations. This ensures the existence of covariantly constant spinors and hence the persistence of an $\mathcal{N}=1$ subalgebra of the $\mathcal{N}=2$ supersymmetry preserved by the CY compactification. As we have already remarked, backgrounds that preserve supersymmetry are automatically stable and free of tachyons. The particular linear combination of supercharges that survive is determined by the angle φ. Hence, different stacks of D6-branes can wrap different special Lagrangian 3-cycles and still preserve a common supersymmetry, as long as their phases φ are aligned. Let us finally notice that special Lagrangian cycles are volume minimizing, and therefore satisfy the equations of motion derived from the DBI action (2.19).

Massless spectrum

Consider a supersymmetric configuration with k stacks of D6-branes that wrap mutually supersymmetric (but intersecting) special Lagrangian 3-cycles $\{\Pi_a\}$ of the CY. The massless spectrum of open strings is similar to the one described in section 2.3.1. Open strings with both ends attached to the same stacks realize the $\mathcal{N}=1$ vector multiplets of the $U(N_1)\times\ldots\times U(N_k)$ gauge theory. This sector may also contain chiral multiplets in the adjoint representation of some $U(N_a)$ factor if the corresponding cycle Π_a has deformation parameters consistent with the special Lagrangian condition. The number of such multiplets is counted by the Betti number $b_1(\Pi_a)$. On the other hand, open string that live in the intersection of two stacks Π_a and Π_b and stretch between them lead to chiral multiplets in the bifundamental representation $(\mathbf{N}_a, \overline{\mathbf{N}}_b)$. The number of such fields is counted by the topological intersection number of the 3-cycles $I_{ab} = [\Pi_a]\cdot[\Pi_b]$.[7] To summarize, the gauge symmetry and chiral matter content of these models are

$$\begin{array}{ll} \text{Gauge group} & \otimes_a U(N_a)\,, \\ \text{Chiral fermions} & \displaystyle\sum_{a,b} I_{ab}(\mathbf{N}_a, \overline{\mathbf{N}}_b)\,. \end{array} \tag{2.22}$$

Branes that wrap cycles which intersect one another multiple times are useful in approaches to the Standard Model since they lead to replicated families of fermions with the same quantum numbers.

Notice that given the Betty numbers of a CY threefold (2.8), there are no non-trivial 1- or 5-cycles on which to wrap D4- or D8-branes that span the non-compact dimensions in a stable way. This is the main reason for our interest in D6-branes in this section.

[7] The topological intersection number can be defined with the aid of the 3-form $\delta(\Pi_a)$ Poincare dual to the 3-cycle Π. Intuitively, this is a 3-form living in the CY $\mathbf{X}_6$ which vanishes outside the corresponding cycle Π. The intersection number is then defined as $I_{ab} = \int_{\mathbf{X_6}} \delta(\Pi_a)\wedge\delta(\Pi_b)$.

2.3.3 Tadpole cancellation and orientifold planes

The models we have presented are inconsistent as they stand. The reason is that, as we have remarked, D-branes are sources of gauge fields (RR p-form fields). Gauss law implies that in a compact space, the total charge for these fields must add up to zero, but flux lines sourced at the branes have nowhere to escape. In other words, the theory is inconsistent since there are no sinks for RR fields, i.e. objects of negative charge. This is usually stated by equivalently saying that there are RR-tadpoles, computed through worldsheet disc diagrams with boundaries on the branes, which prevent the equations of motion from being satisfied. Indeed, these equations would imply

$$[\Pi_{\text{tot}}] \equiv \sum_a N_a \, [\Pi_a] = 0 \tag{2.23}$$

If we were to satisfy this equation with the elements presented so far, we would need to introduce anti D6-branes, which have indeed negative RR charge. However, their presence would break supersymmetry and lead to potential instabilities.

The way to preserve supersymmetry while fulfilling the RR tadpole cancellation conditions is to introduce non-orientable strings through an orientifold quotient of the theory. This symmetry operator by which we quotient takes the form $\Omega\mathcal{R}(-1)^{F_L}$, where Ω is the worldsheet parity operator and $\mathcal{R}$ is an antiholomorphic $\mathbb{Z}_2$ symmetry of the CY space $\mathbf{X}_6$. The latter acts as $(z_1, z_2, z_3) \to (\bar{z}_1, \bar{z}_2, \bar{z}_3)$ on local complex coordinates and hence implements the changes $J \to -J$ and $\Omega_3 \to \overline{\Omega}_3$ on the Kähler and holomorphic forms of the CY. The factor $(-1)^{F_L}$, with F_L being the left-moving spacetime fermion number, is introduced so that the orientifold projector squares to unity. The spacetime locus invariant under $\mathcal{R}$ is a 3-cycle locally defined by $\text{Im}\,(z_i) = 0$, together with the noncompact dimensions which are trivially invariant under $\mathcal{R}$. Spanning this locus there is a non-dynamical object called an orientifold 6-plane (O6-plane for short), which carries nonetheless tension and RR 7-form charge (i.e. it is a source of gravitons and gauge fields). In fact, through worldsheet crosscap computations, one can see that their tension can be negative and their RR charge is opposite to that of a D6-brane, with

$$Q_{O6} \;=\; -4Q_{D6}. \tag{2.24}$$

Notice that, from the way that $\mathcal{R}$ acts on J and Ω_3, the cycle wrapped by the O6-plane is in fact an special Lagrangian 3-cycle, with $\varphi = 0$ taken by convention. This means that O6-planes also break half of the supersymmetries preserved by the CY compactification, and preserve those determined by the BPS angle $\varphi = 0$. In order to combine D6-branes and O-planes in a supersymmetric compactification, we need that all these objects preserve the same particular supersymmetry, which we have chosen to be parametrized by $\varphi = 0$.

The solution to the RR tadpole cancellation condition is now clear. O-planes are the objects we were looking for to play the role of sinks for the RR flux lines in the compact manifold. Consistent models will include O6-planes wrapped on special Lagrangian cycles $[\Pi_{O6}]$, stacks of D6-branes wrapped on cycles $[\Pi_a]$, as well as stacks of D6-branes wrapped on cycles $[\Pi_{a'}]$ which are symmetric images of the $[\Pi_a]$ under the orientifold symmetry. The latter are needed to render the orientifold projection consistent. The explicit condition of tadpole cancellation is now written as

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4\,[\Pi_{O6}] = 0. \tag{2.25}$$

This is simply the statement that the RR charges on the compact space must add up to zero.

Orientifolded spectrum

The introduction of O6-planes modifies the spectrum of both closed and open strings in a substantial manner. We will study the effect on closed string states in section 3.1. Here we focus on the open string sector.

First, consider a stack of N D6-branes on top of an O6-plane. Before the orientifold action, the open string spectrum is a $U(N)$ vector multiplet. There are two types of orientifold projections, labeled by $O^{\pm}$, depending of how they act on the Chan-Paton indices. The one of present interest for us is the O^- projection, which leads to O-planes with negative RR charge (as described above), and reduce the gauge group to $SO(N)$.

For branes that do not coincide with the O-plane (parallel or intersecting), open strings with both ends on the same stack do not feel locally the O-plane. Before the orientifold projection, the configuration of interest is that of a stack of N D6-branes and their N image D6'-branes. The massless states form a vector multiplet of the $U(N) \times U(N)'$ gauge group. The orientifold identifies both stacks and projects the gauge group to a single linear combination $U(N)$. Notice that, because of the worldsheet parity projection Ω, the fundamental representation $\square$ of $U(N)$ is identified with the anti-fundamental $\overline{\square}'$ of the image $U(N)'$.

Finally, a stack of N D6-branes orthogonal to the O6-plane can also be invariant under the projection. The action of the orientifold on the open strings reduces the gauge group from $U(N)$ to $USp(N)$, hence N must be even in this case.

In the intersection of different stacks of branes there will still be chiral multiplets transforming under chiral representation of the orientifolded gauge groups. Their number and specific representation can be obtained by carefully taking into account the mentioned identification of groups and representations under the orientifold. As an example, consider a stack a of N_a branes intersecting with its own orientifold image a'. The original gauge group is $U(N_a) \times U(N_a)'$, and there are $I_{aa'}$ massless chiral multiplets in the bifundamental $\square_a \otimes \overline{\square}_{a'}$ living in the intersections.[8] The orientifold reduces the group to a single $U(N_a)$ and identifies representations as $\square_a \otimes \overline{\square}_{a'} = \square_a \otimes \square_a = \square\!\square_a + \begin{smallmatrix}\square\\\square\end{smallmatrix}_a$. This is the representation of the chiral superfields if the intersections lie away from the O-plane. For intersections that occur on top of the O-plane, the projection eliminates the symmetric $\square\!\square$ part of the representation and only states in the antisymmetric representation $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ survive. The total number of chiral multiplets in the symmetric $n_{\rm sym}$ and antisymmetric $n_{\rm asym}$ representations can finally be written as

$$n_{\rm sym} = \tfrac{1}{2}(I_{aa'} - I_{a,O6})\,, \qquad n_{\rm asym} = \tfrac{1}{2}(I_{aa'} + I_{a,O6})\,. \tag{2.26}$$

These are, respectively, the number of intersections of the a and a' stacks away from the O-plane, and the total number of them, as counted on the quotient space $\mathbf{X}_6/\mathcal{R}$.

Other examples of intersecting branes in the presence of an O-plane are simpler and their chiral spectra can be easily obtained in a similar way.

[8] It is implicit here and in what follows that, under $U(N) = SU(N) \times U(1)$, states that transform in the fundamental $\square$ of $SU(N)$ have charge $+1$ under $U(1)$, while those on the antifundamental $\overline{\square}$ have charge -1.

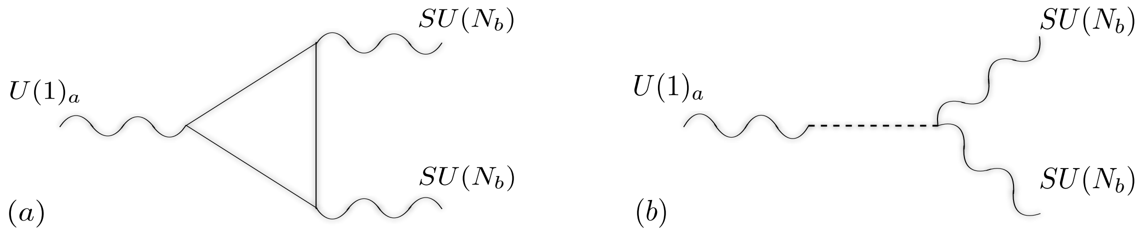


Figure 2.1: Diagrams contributing to the $U(1)_a - SU(N_b)^2$ anomaly.

2.3.4 Anomaly cancellation: the Green-Schwarz mechanism

We have presented general setups which lead to four-dimensional field theories that include, besides gravity, non-abelian gauge symmetries and fermions transforming under chiral representations thereof. They have therefore the potential to reproduce the Standard Model or the MSSM which is the main goal of the area of string phenomenology. We will present shortly a specific model that points in that direction. Before doing that, let us notice that chiral gauge theories are usually plagued with anomalies (both gravitational and gauge), and only very restricted fermion contents lead to consistent anomaly-free theories. Since we have obtained our models from well-defined superstring theories, and have been careful about possible inconsistencies or instabilities in the compactification process, we expect our effective actions to be free of anomalies. Checking that this is indeed so is a very instructive exercise from which we obtain general results that will be extensively used in the following chapters.

The $SU(N)^3$ anomaly

First of all, consider the $SU(N_a)^3$ anomaly that may arise for the non-abelian part of the gauge group living on a stack a of D6-branes. The contributions to the triangle anomalous diagram for fermions in a representation R of $SU(N_a)$ are weighted by the so-called anomaly coefficients $A(R)$. For the representations obtained in the models at hand these are

$$A(\square_a) = 1\,, \qquad A(\overline{\square}_a) = -1\,, \qquad A(\square\!\square_a) = N_a + 4\,, \qquad A\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}_a\right) = N_a - 4\,. \qquad (2.27)$$

Summing up the contributions from all charged chiral fermions, we realize that the $SU(N_a)^3$ anomaly Δ_a vanishes as a consequence of the RR tadpole cancellation condition (2.25):

$$\begin{aligned}\Delta_a \sim\ & A(\square_a) \sum_b N_b\, I_{ab} + A(\square_a) \sum_{b' \neq a'} N_{b'}\, I_{ab'} + A\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}_a\right) \tfrac{1}{2}(I_{aa'} + I_{a,O6}) + A(\square\!\square_a)\tfrac{1}{2}(I_{aa'} - I_{a,O6}) \\ =\ & [\Pi_a] \cdot \left(\sum_b N_b\, [\Pi_b] + \sum_{b'} N_{b'}\, [\Pi_b'] - 4[\Pi_{O6}] \right) = 0 \qquad (2.28)\end{aligned}$$

The mixed $U(1) - SU(N)^2$ anomaly

Besides the purely non-abelian anomalies, there may arise mixed $U(1)_a - SU(N_b)^2$ anomalies Δ_{ab}. The usual contribution from chiral fermions to the triangle diagrams controlling these anomalies (Fig. 2.1(a)) does not cancel in general. It is given by

$$\Delta_{ab} \sim N_a\,[\Pi_a]\cdot[\Pi_b] - N_a\,[\Pi_{a'}]\cdot[\Pi_b]\,, \tag{2.29}$$

which is generically non-zero. The anomalies vanish, however, due to the so-called Green-Schwarz (GS) mechanism that we review next [58, 59]. The crucial point is that there are couplings in the effective action from which one can construct diagrams that contribute to the same anomalous amplitude. These arise from the D6 worldvolume Chern-Simons action (2.20) and involve 4d scalars and their dual 2-forms.

Let us take a symplectic basis of 3-cycles $\{[\alpha_k],[\beta^k]\}_{k=0,\ldots,h_{2,1}}$ of the CY that satisfy $[\alpha_k]\cdot[\beta^l]=\delta_k^l$. The $[\alpha_k]$ and $[\beta^k]$ cycles can be taken to be even and odd, respectively, under the orientifold action. We can expand the cycles $[\Pi_a]$ wrapped by the branes (and their orientifold images) on this basis as

$$[\Pi_a] = (r_a^k\,[\alpha_k] + s_{ak}\,[\beta^k])\,, \qquad [\Pi_{a'}] = (r_a^k\,[\alpha_k] - s_{ak}\,[\beta^k])\,. \tag{2.30}$$

The coefficients are determined by the intersection numbers $r_a^k = [\Pi_a]\cdot[\beta^k]$ and $s_{ak} = -[\Pi_a]\cdot[\alpha_k]$. The 4d fields of interest arise from the RR 3- and 5-forms

$$a_l = \int_{[\alpha_l]} C_3\,, \qquad B_2^k = \int_{[\beta^k]} C_5\,, \qquad \text{with } dB^k = -\delta^{kl}(*\,da_l)\,. \tag{2.31}$$

The duality follows from the 10d duality of C_3 and C_5. Notice that other components such as $\int_\beta C_3$ do not survive the orientifold projections since C_3 and C_5 are orientifold even and odd, respectively.

The surviving fields interact with the open strings through the CS action (2.20), that leads to the 4d terms

$$\begin{aligned} S_{aF} =&\ \tfrac{1}{2}\left(\textstyle\int_{D6_b} C_3\wedge \mathrm{tr}\,F_b^2 + \int_{D6_{b'}} C_3\wedge\mathrm{tr}\,F_b^2\right) &= r_b^k\int_{4d} a_k\,\mathrm{tr}\,F_b^2 \\ S_{BF} =&\ \tfrac{1}{2}\left(\textstyle\int_{D6_a} C_5\wedge\mathrm{tr}\,F_a - \int_{D6_{a'}} C_5\wedge\mathrm{tr}\,F_a\right) &= N_a s_{ak}\int_{4d} B_2^k\wedge F_{U(1)_a}\,. \end{aligned} \tag{2.32}$$

The factors of 1/2 arise because of the orientifold action, and the relative minus sign in the $B\wedge F$ term follows from $F_a = -F_{a'}$. The N_a coefficient is due to the $U(1)_a$ trace normalization. The first term is a coupling of the scalars a_l to the $SU(N_b)$ gauge bosons. The second couples the 2-forms B_2^k to the $U(1)_a$ groups. It can be rewritten in terms of the scalars as

$$S_{BF} = N_a\,s_{ak}\int_{4d} B_2^k\wedge\mathrm{tr}\,F_a \quad\to\quad -\tfrac{1}{2}\int_{4d} d^4x\,(N_a s_{ak}\,A_a + da_k)^2\,, \tag{2.33}$$

where the kinetic term for the scalars comes from the kinetic term for the 2-forms that we had omitted previously.

Now, from the above couplings, one can construct diagrams such as that in Fig. 2.1(b) which have the right structure to cancel the anomaly (2.29). Their amplitude is indeed proportional to

$$\Delta_{ab}^{GS} \sim -\tfrac{1}{2}N_a\,s_{ak}\,r_b^l\,\delta_l^k = -N_a\,[\Pi_a]\cdot[\Pi_b] + N_a\,[\Pi_{a'}]\cdot[\Pi_b] \tag{2.34}$$

which is what is precisely required to cancel the contribution from the triangle diagrams (2.29). We see that in fact, the anomaly vanishes. Nevertheless, we will often refer to these $U(1)$ symmetries as 'anomalous', with the implicit understanding that the GS cancellation mechanism is in force.

The actions (2.32) and specially the $B \wedge F$ term (2.33) have very important consequences that we summarize next:

- Notice that the scalar fields a_k are axionic in the sense described in section 2.2.3, i.e. they come from the integral of a p-form gauge field around an internal p-cycle. As we emphasized above, such scalars should satisfy a periodicity condition of the schematic form $a_k \sim a_k{+}1$, that is, they should parametrize circles. Happily, the actions (2.32) and (2.33) are consistent with these identifications. The scalars appear either derivatively or together with a term which is topological and quantized, the *instanton number* n of $SU(N_b)$ in this case. Therefore, the measure e^{iS} that enters path integrals is invariant under $S(a_k) \to S(a_k + 1) = S(a_k) + 2\pi n$.

- The form of the action (2.33) tells us that under a $U(1)_a$ gauge transformation, the scalar fields must transform as

$$A_a \to A_a + d\Lambda\,, \qquad a_k \to a_k - N_a\, s_{ak}\, \Lambda\,. \tag{2.35}$$

- This coupling also tells us that naively "anomalous" $U(1)$'s are massive. The axionic scalars that participate in the GS mechanism are eaten up by the gauge fields and disappear from the spectrum. The masses acquired by the gauge bosons are of order of the string scale, and hence, at low energies they appear as global symmetries of the effective action.

  Even $U(1)$ bosons whose triangle anomaly diagrams vanish without the need of the GS mechanism may get a mass if they have non-vanishing $B \wedge F$ couplings. The condition for a particular linear combination of $U(1)$'s, say $\sum c_a\, U(1)_a$, to remain massless is

$$\sum_a c_a\, N_a\, s_{ak} = 0 \qquad \text{for all } k. \tag{2.36}$$

  Of course, this condition is sufficient to guarantee that anomalous triangle diagrams vanish, but it is not necessary.

- Finally, let us anticipate that $B \wedge F$ couplings are related to Fayet-Illiopoulos (FI) terms of the effective action. As we saw in section (2.2.3), the scalars a_k belong to hypermultiplets of the $\mathcal{N} = 2$ theory obtained before the introduction of O-planes and D-branes. These multiplets decompose as two chiral superfields of $\mathcal{N} = 1$. Let us denote by Φ_k the chiral multiplet which contains a_k. Its scalar components is $\Phi_k = \varphi_k + i a_k$. Now, in order to preserve the $U(1)_a$ gauge symmetry (2.35), the Kähler potential, which has to be a real function of the chiral multiplets, has necessarily the structure

$$K(\Phi_k + \Phi_k^* + \sum_a N_a\, s_{ak} V_a)\,, \tag{2.37}$$

  where V_a is the $U(1)_a$ vector supermultiplet of $\mathcal{N} = 1$. The expansion of (2.37) in powers of V_a yields, at the linear level, a FI term controlled by

$$\xi_a = N_a\, s_{ak} \frac{\partial K(\Phi + \Phi^*)}{\partial \Phi}\Big|_{V=0} \tag{2.38}$$

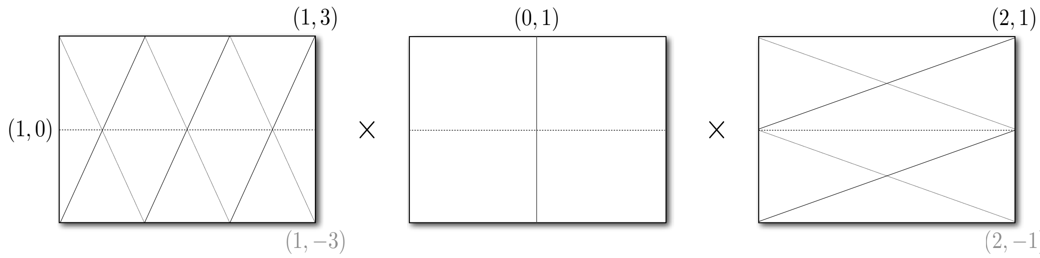


Figure 2.2: $\mathbf{T}^6$ orientifolded by $z_i \to \bar{z}_i$. The dashed lines represent O6-planes, the continuous lines represent the 3-cycle $(1,3) \times (0,1) \times (2,1)$, while the shaded ones are its orientifold image.

Thus, supersymmetry relates $B \wedge F$ and FI couplings.

Finally let us mention that gravitational anomalies are cancelled by similar Green-Schwarz mechanisms whose explicit discussion we skip.

2.4 A toroidal orientifold model

We now want to apply the tools presented in this section to the explicit construction of a model with a gauge group and spectrum corresponding to that of the Standard Model. Several such constructions have been obtained in e.g. [65–69].The setup we present in this section will be used in chapter 5 to exemplify the results of [49].

Generalities of toroidal compactifications

Consider type IIA theory compactified on a factorized 6-torus $\mathbf{T}^6 = \mathbf{T}_1^2 \times \mathbf{T_2}^2 \times \mathbf{T_3}^2$.[9] We need the compactification space to have a $\mathbb{Z}_2$-symmetry so that we can construct a consistent orientifold projection. The easiest way to achieve this is to consider rectangular tori parametrized by three complex coordinates $z_i = x_i + iy_i$ with the identifications

$$z_i \sim z_i + R_i^{(1)}\,, \qquad z_i \sim z_i + iR_i^{(2)}\,. \tag{2.39}$$

This is consistent with the projection $z_i \to \bar{z}_i$, which defines eight O6-planes that span the x_i directions and sit at locations $y_i = 0, \frac{1}{2}R_i^{(2)}$.

The 1-cycles of each torus are defined by the equations $n_i R_i^{(1)} y_i = m_i R_i^{(2)} x_i$. They are parametrized by two integers (n,m) that describe how many times the fundamental cycles $[a] = (1,0)$ and $[b] = (0,1)$ are wrapped. These models contain stacks of N_a D6-branes, that wrap 3-cycles which we take to be factorizable and characterized by the wrapping numbers (n_i^a, m_i^a), and orientifold images on cycles $(n_i^a, -m_i^a)$, see figure 2.2. The general formula for the topological intersection number of any two such 3-cycles is

$$I_{ab} = (n_1^a m_1^b - m_1^a n_1^b) \times (n_2^a m_2^b - m_2^a n_2^b) \times (n_3^a m_3^b - m_3^a n_3^b)\,. \tag{2.40}$$

[9]This space has trivial holonomy and hence preserves more supersymmetries than a CY manifold of strict $SU(3)$ holonomy. The reduction of supersymmetry is realized entirely by the introduction of intersecting branes and O-planes. In any case, the results of previous sections can be applied to this setup with little or no modification.

The RR tadpole cancellation conditions (2.25) read in this case[10]

$$\begin{aligned} \sum_a N_a n_1^a n_2^a n_3^a &= 16 \\ \sum_a N_a n_1^a m_2^a m_3^a &= 0 \qquad \text{and permutations of 1, 2, 3.} \end{aligned} \tag{2.41}$$

Standard Model-like $USp(2)$ setups

In this setup, we can construct a large class of models that realize the gauge group of the Standard Model and its low energy spectrum (plus right-handed neutrinos). We focus here on models first constructed in [41]. They contain four stacks of branes, denoted a(*baryonic*), b (*left*), c (*right*) and d (*leptonic*) for reasons that will become clear shortly. They have $N_a = 3$, $N_b = 2$ $N_c = N_d = 1$. The *left* stack b is taken to be orthogonal to the O6-planes, so that it is its own orientifold image and hosts a gauge group $USp(2) \equiv SU(2)$. It can be identified with the electroweak $SU(2)_L$ group, and the whole gauge symmetry of the configuration is $U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$. The spectrum of the Standard Model is reproduced if the stacks have the following intersection numbers

$$\begin{aligned} I_{ab} = \ 3 \quad &; \quad I_{ac} = I_{ac'} = \ -3 \\ I_{db} = \ -3 \quad &; \ \ I_{cd} = \ -3 \ ; \ I_{cd'} = \ 3 \ . \end{aligned} \tag{2.42}$$

The resulting chiral fermions are shown in table 2.1. The $U(1)$ charges clarify the names

Intersection	Matter fields	$SU(3)_a \times SU(2)_b$	Q_a	Q_c	Q_d	Y
(ab)	Q_L	$3(\mathbf{3},\mathbf{2})$	1	0	0	1/6
(ac)	U_R	$3(\bar{\mathbf{3}},\mathbf{1})$	-1	1	0	-2/3
(ac')	D_R	$3(\bar{\mathbf{3}},\mathbf{1})$	-1	-1	0	1/3
(bd)	L	$3(\mathbf{1},\mathbf{2})$	0	0	-1	-1/2
(cd)	E_R	$3(\mathbf{1},\mathbf{1})$	0	-1	1	1
(cd')	N_R	$3(\mathbf{1},\mathbf{1})$	0	1	1	0

Table 2.1: Standard Model spectrum and $U(1)$ charges in toroidal models realizing the intersection numbers (2.42).

given to the different stacks. The last column reproduces the hypercharges of the Standard Model fields. These are realized in our models by the $U(1)$ linear combination

$$Y = \tfrac{1}{6}(Q_a - 3Q_c + 3Q_d) \tag{2.43}$$

Since $U(1)_Y$ gauge bosons must remain massless in a phenomenologically viable model, we need to impose the constraints (2.36) that reads in this case

$$s_{ak} \ - \ s_{ck} \ + s_{dk} \ = \ 0 \quad \text{for all } k. \tag{2.44}$$

Recall that this condition implies that $U(1)_Y$ has no $B \wedge F$ couplings and therefore is massless, anomaly free (without the need of the GS mechanism) and furthermore has no FI term.

[10] Cancellation of discrete K-theory charges which we have not discussed requires also $\sum N_a m_1^a n_2^a n_3^a \in \mathbb{Z}_2$ and permutations [191].

N_i	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)
$N_a = 3$	$(1, 0)$	$(n_2^a, 1)$	$(3, m_3^a)$
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 1$	$(n_1^c, 1)$	$(1, 0)$	$(0, 1)$
$N_d = 1$	$(1, 0)$	$(n_2^d, -3)$	$(1, m_3^a)$

Table 2.2: D6-brane wrapping numbers giving rise to a SM spectrum.

Explicit realization

We have presented what could be called a *protomodel* for the achievement of a Standard Model-like theory. We must however find an explicit realization of this proposals, i.e. find particular wrapping numbers for the branes that realize the appropriate intersection numbers and potentially satisfy the RR tadpole condition, while retaining a massless hyperscharge. That is, they have to satisfy equations (2.42), (2.41) and (2.44). In table 2.2 we present a large class of models reproducing the correct intersections. They are parametrized by four integers n_1^c, n_2^a, n_2^d, and m_3^a. RR tadpole cancellation is not automatically satisfied, but can be implemented with the addition of $(9n_2^a + n_2^d - 16)$ D6-branes (or antibranes, depending on the sign) parallel to the orientifold plane. They have no intersection with the rest of the branes and do not modify the discussion in any significant way. Finally, the hypercharge will remain massless if

$$n_1^c \;=\; n_2^a m_3^a + n_2^d m_3^a \; . \tag{2.45}$$

Final remarks

- The (*proto*)models we have constructed are non-supersymmetric. Still, there are scalar fields at the intersections that play the role of squarks, sleptons and Higgs scalars. These may be massive, massless or even tachyonic depending on the complex structure of the toroidal compactification. There are regions in moduli space in which many of them become massless, indicating that (at least some of) the gauge branes preserve common supersymmetries. In these cases, however, the branes introduced to cancel RR tadpoles will spoil the supersymmetry.

  One has to make sure that at least there are no tachyonic fields that would render the model unstable. The ultimate stability of these models depends often on the introduction of additional ingredients, such as background fluxes, which we do not discuss. In any event, these simple models are useful to study general features that appear in more realistic setups as well.

- There are similar toroidal compactifications which do preserve supersymmetry and lead to the same spectra and gauge groups, see e.g. [70, 71]. As we have repeatedly emphasized, the preservation of supersymmetry is highly desirable string compactifications. These models involve slightly more complicated ingredients such as toroidal orbifolds. We will briefly discuss them in chapter 5.

- As we will see explicitly in chapter 5, the $U(1)$ gauge bosons orthogonal to the hypercharge get a Sückelberg mass from $B \wedge F$ couplings, becoming effectively global symmetries of the low energy theory. These massive $U(1)$'s would naively forbid certain couplings that are phenomenologically desirable. For example, neutrino Majorana masses seem to violate $U(1)_c$ and $U(1)_d$ in our explicit constructions. In other models the situation is even more dramatic since the couplings that are apparently forbidden include Yukawa couplings or μ terms for the Higgses. In the coming chapters we will study mechanisms (non-perturbative effects) by which these couplings can arise.

Chapter 3

Effective actions and their non-perturbative contributions

We have presented in chapter 2 general features of compactifications of type II theories. In particular, we have focused on the description of the massless low energy spectra that arises in CY and toroidal compactifications of type IIA theories in the presence of orientifold planes and intersecting D-branes. In this chapter we finish the review of material needed to present the results of this thesis. We describe general features of the effective actions of these setups and some of the methods available for their computations [20, 53, 55–57]. This introduces the crucial (although somewhat artificial) distinction between perturbative and non-perturbative terms of the action. The latter are the main focus of this work.

As in the previous chapter, we focus here on type IIA models and rely on mirror symmetry and T-duality to obtain similar results for type IIB when necessary (as in chapter 4).

3.1 The orientifolded closed string spectrum

The massless spectrum coming from open strings in orientifold type IIA models has been introduced in section 2.3.3. We also presented the closed string fields of CY compactifications in 2.2.3. There were geometric moduli, coming from the ten dimensional metric G_{MN}, that describe deformations of the complex structure and Kähler class of the CY. The former belong to $\mathcal{N} = 2$ hypermultiplets, while the latter are part of vector multiplets. Coming from the p-form 10d gauge fields B_2 and C_3 there were axionic scalars that pair up with geometric moduli. A final scalar ϕ came from the 10d dilaton. We now proceed to implement the orientifold projection on these fields and describe the $\mathcal{N} = 1$ chiral multiplets to which they belong.

Let us first introduce appropriate basis for the relevant cohomologies of the CY. At degree three we introduce a symplectic basis of harmonic forms $\{\alpha_k, \beta^k\}_{k=0,\ldots,h_{2,1}}$ which is actually dual to the homology basis introduced in section 2.3.4. The 3-forma α_k and β^k are even and odd, respectively, under the orientifold projection, and satisfy

$$\int_{\mathbf{X}_6} \alpha_k \wedge \beta^l = \delta_k^l \,. \tag{3.1}$$

Similarly, harmonic forms of even degree are classified according to their behavior under the orientifold. We define a basis of even (odd) $(1,1)$-forms $\{\omega_\alpha\}$ ($\{\omega_a\}$) with $\alpha = 1, \ldots, h^+_{1,1}$ ($\alpha = 1, \ldots, h^1_{1,1}$) and their dual $(2,2)$ forms $\{\tilde{\omega}^\alpha\}$ ($\{\tilde{\omega}^a\}$) such that

$$\int_{\mathbf{X}_6} \omega_\alpha \wedge \tilde{\omega}^\beta = \delta_a^b\,; \qquad \int_{\mathbf{X}_6} \omega_a \wedge \tilde{\omega}^b = \delta_a^b\,. \tag{3.2}$$

Now recall that, under the projection introduced by O6-planes, the Kähler and holomorphic forms of the CY behave as $\mathcal{R}J = -J$ and $\mathcal{R}\Omega_3 = \overline{\Omega}_3$. It can also be seen that the orientifold acts on the 10d p-form fields as

$$\mathcal{R}\phi = \phi\,, \qquad \mathcal{R}B_2 = -B_2\,, \qquad \mathcal{R}C_1 = -C_1\,, \qquad \mathcal{R}C_3 = C_3\,. \tag{3.3}$$

Counting the number of independent parameters and taking into account their parity, it is easy to see that Kähler moduli will pair up with B_2-descendant axions, while complex structure moduli will combine with axions that come from C_3. As shown in the bibliography, the actual way in which this is done is through the complexified Kähler and 3-forms

$$\begin{aligned} J_c &\equiv B_2 + iJ = -i\sum_{a=1}^{h^-_{1,1}} T_a\,\omega_a \\ \Omega_c &\equiv C_3 + i\,e^{-\phi}\,\mathrm{Re}\,(\Omega_3) = iS\alpha_0 + i\sum_{k'=1}^{h_{2,1}} U_{k'}\alpha_{k'}\,. \end{aligned} \tag{3.4}$$

The last terms represent expansions in terms of the cohomology of the CY. The coefficients are the scalar components of the $\mathcal{N}=1$ 4d effective theory. These are the Kähler moduli $\{T_a\}$ coming from vector multiplets of the original $\mathcal{N}=2$ theory, and the *axio-dilaton* S and complex structure moduli $\{U_k\}$ that were originally in hypermultiplets. Oftentimes we combine the axio-dilaton $S \equiv U_0$ with the complex structure moduli into a single set $\{U_k\}_{k=0,\ldots,h_{2,1}}$. We will denote with the same symbols the chiral multiplets and their scalar components. Vector multiplets (of $\mathcal{N}=1$) arise from the expansion of C_3 in a basis of even $(1,1)$ forms. They will not play an important role in our discussion.

Several crucial remarks are in order here

- The real part of the hypermultiplet (vector multiplet) scalars are geometric moduli that parametrize the volumes of cycles of odd (even) dimensions of the CY manifold.
- The imaginary components, on the other hand, are axionic, and hence subject to periodicity conditions as stressed in the previous chapter.
- The dilaton, that parametrizes the string coupling constant through $g_s = e^\phi$, appears only in the moduli of complex structure, i.e. those coming from $\mathcal{N}=2$ hypermultiplets.
- The fermionic components of the closed string chiral multiplets are not charged under the gauge theories living on the worldvolumes of D-branes. They are thus not very interesting for phenomenology. The scalars, however, play important roles in two ways: first, their vev's control the strengths of the couplings of matter fields; second, as we saw in eq. (2.35), the (axionic) imaginary parts of complex structure chiral fields do transform non-trivially under $U(1)$ transformations of the D6-brane gauge fields. They can hence be important in terms that involve charged matter fields.

We will develo these points extensively in the rest of this chapter. They are the main ideas that underlie the results of this work.

3.2 The effective action: perturbative vs. non-perturbative terms

We want to describe now the general form of the effective action that governs the dynamics of closed and open string fields. We also introduce in this section the important separation of perturbative and non-perturbative effects which is at the core of this work.

Open string vector multiplets are denoted V_A (with field strength chiral multiplets $(W_A)^\alpha$) while chiral matter multiplets charged under them (from D-brane intersections) are called C_I. Actions with $\mathcal{N}=1$ supersymmetry contain two types of terms. D-terms are real and their measure span the whole superspace, while F-terms are holomorphic functions of the chiral fields and their integrals span half of the superspace:

$$S_D \sim \int d^4x\, d^2\theta\, d^2\overline{\theta}\ K(\Phi+\Phi^*) \qquad\qquad S_F \sim \int d^4x\, d^2\theta\, F(\Phi)\ +\ \text{h.c.} \tag{3.5}$$

D-terms include the Kähler potential and FI terms for $U(1)$'s. F-terms include superpotentials, gauge kinetic functions and the less familiar higher F-terms. Our main interest is in F-terms for reasons that will become apparent soon.

Periodicity of the axions (the imaginary components of closed string chiral fields) restricts extraordinarily the possible forms of F-terms. We have already seen in section 2.3.4 (c.f. eqs. (2.32) and (2.33)) two possible ways in which axions can appear in the action: namely at the derivative level or as coefficients of some topological integral term (such as Chern classes). A third manner, still consistent with axion periodicity, is through periodic functions $f(a)=f(a+1)$, basically functions that can be expanded in terms of exponentials $f(a)=f(e^{2\pi i\,a})$. This condition is specially restrictive for F-terms, since holomorphy relates the couplings of the axions to those of their geometric moduli partners.

3.2.1 The superpotential

$$S_W = \int d^4x\, d^2\theta\, W(\Phi)\ +\ \text{h.c.} \tag{3.6}$$

Scalars do not arise with derivatives at the level of the superpotential, and there are no topological invariants constructed from chiral superfields. Hence the only options for axions to appear in a superpotential is through exponential functions. Together with the holomorphy condition, this implies that it can be written as[1]

$$\begin{aligned} W(T_a,U_k,C_I) = &\quad W_0^{\text{cl}}(e^{-T_a}) + W_{\text{n.p.}}^{\text{cl}}(e^{-T_a},e^{-U_k}) \\ &+\ W_0^{\text{mat}}(e^{-T_a},C_I) + W_{\text{n.p.}}^{\text{mat}}(e^{-T_a},e^{-U_k},C_I)\,. \end{aligned} \tag{3.7}$$

[1]There are several reasons for the negative signs in the exponentials: they imply that these interactions vanish, first, when α' and g_s vanish, and second, when the volumes of the cycles of the CY grow to infinity. Also, exponentials with the wrong sign would generically lead to scalar potentials that are not bounded from below and hence unstable.

The two lines correspond to terms that involve only closed string fields and those which include also matter (charged) fields, respectively.

In each line we have also distinguished, somewhat artificially, those terms $W_{\text{n.p.}}$ that include the complex structure moduli U_k (recall that we are including the axio-dilaton here) and those that do not, W_0. This seemingly arbitrary distinction is extremely important for us. Its motivation is that, as we saw in (3.4), the string coupling constant $g_s = e^{\phi}$ appears only in complex structure chiral multiplets. This implies that W_0^{cl} and W_0^{mat} are independent of g_s and hence arise at tree-level in a perturbative expansion. On the other hand, quantum corrections to these quantities arise from the terms $W_{\text{n.p.}}^{\text{cl}}$ and $W_{\text{n.p.}}^{\text{mat}}$, but the way in which g_s appears on them is as $e^{-U_k} \propto e^{-1/g_s}$. These type of terms are intrinsically non-perturbative, they have an essential singularity at $g_s = 0$ and do not appear at any order of perturbation theory. The final punchline is that the superpotential is exact at tree-level in perturbation theory, but can receive non-perturbative corrections.

The separation between perturbative and non-perturbative effects has some immediate consequences:

- **Neutral superpotential:** the superpotential for complex structure multiplets vanishes exactly in perturbation theory. Hence the only way to stabilize these moduli (a requirement for phenomenologically viable models) is through non-perturbative effects [43], at least in the absence of other ingredients such as background fluxes. The stabilization of Kähler moduli can in principle be achieved perturbatively, but non-perturbative effects can help in the task.

- **Charged superpotential:** an even more important consequence of non-perturbative effects is the generation of superpotential terms that effectively violate global $U(1)$ gauge symmetries [40–42]. Remember that these symmetries arise from the $U(N) = SU(N) \times U(1)$ gauge fields of D6-branes. When their field strength is coupled to a 2-form through a $q(B \wedge F)$ couplings, the gauge bosons acquire a Stückelberg mass and the symmetry become effectively global. Recall also, from eq. (2.35), that these couplings result in a non trivial transformation of the axion fields of hypermultiplets $U \to U + iq\Lambda$. This implies that e^{-U} behaves as a "composite" field of $U(1)$ charge $-q$. We can construct non-perturbative terms such as

$$Y_{IJK} \sim e^{-U} C_I C_J C_K \tag{3.8}$$

  that are $U(1)$ invariant if the charges of the matter fields sum up to q. Once we stabilize the complex structure moduli, these terms would lead to an effective violation of the global $U(1)$ symmetry. The particular case of (3.8) represents a Yukawa coupling. Notice that such terms cannot be generated perturbatively.

  We saw in the toroidal models of section 2.4 that there were indeed some phenomenologically desirable couplings such as Majorana masses for neutrinos that were apparently forbidden by $U(1)$. Other models contain among the "forbidden" couplings some of the Yukawas, or μ terms for the Higgses. Non-perturbatively, these couplings can arise naturally as we have described.

The results of chapter 5 will describe some aspects of non-perturbatively generated superpotential and their phenomenological properties in models of intersecting D6-branes.

3.2.2 Gauge kinetic functions

$$S_f = \int d^4x \, d^2\theta \, f(\Phi) \, \mathrm{tr}\,(W^\alpha W_\alpha) \; + \; \text{h.c.} \tag{3.9}$$

Similar arguments can be applied to constrain the form of holomorphic gauge kinetic functions $f_A(T_a, U_k)$.[2] In this case axionic fields can appear in two ways, through exponential functions, as in the superpotential, or together with the topological instanton number (or second Chern class) that couples to the imaginary part of the gauge kinetic function

$$S_f \sim \mathrm{tr} \int \left[(\mathrm{Re}\, f_A) \, F^A \wedge * F^A + (\mathrm{Im}\, f_A) \, F^A \wedge F^A \right] \, . \tag{3.10}$$

We saw in section 2.3.4, in (2.32), that at first order, these terms appear from the reduction of the CS coupling between the D6-brane and the RR 3-form

$$S_f^{(0)} = -r_A^k \int_{4d} \mathrm{Im}\,(U_k) \, \mathrm{tr}\,(F^A \wedge F^A) \, , \tag{3.11}$$

where $r_A^k = [\Pi_A] \cdot [\beta^k]$. Indeed, one could think of these terms as defining the periodicity of the axions. In the general case, when the coefficients r_A^k are mutually prime (i.e. g.c.d (r_A^k)), the periodicity is just $U_k \sim U_k + 1$. It is then easy to see that holomorphy and periodicity of axions implies a general form

$$f_A(T_a, U_k) = \sum_k -r_k^A \, U_k + f_A^{\text{1-loop}}(e^{-T_a}) + f_{\text{n.p.}}(e^{-T_a}, e^{-U_k}) \, . \tag{3.12}$$

We have again separated the perturbative corrections to (3.11), which involve only the Kähler multiplets and are independent of g_s, from the non-perturbative ones that involve the complex structure moduli.

The results presented in section 4.3 deal indeed with non-perturbative corrections to gauge kinetic function, although in type IIB theories in higher dimensions.

3.2.3 Higher F-terms

$$S_{\text{B.W}} = \int d^4x \, d^2\theta \, \omega_{\bar{i}_1 \ldots \bar{i}_n, \bar{j}_1 \ldots \bar{j}_n}(\Phi) \left(\overline{\mathcal{D}}^{\dot\alpha} \overline{\Phi}^{\bar{i}_1} \, \overline{\mathcal{D}}_{\dot\alpha} \overline{\Phi}^{\bar{j}_1} \right) \ldots \left(\overline{\mathcal{D}}^{\dot\alpha} \overline{\Phi}^{\bar{i}_n} \, \overline{\mathcal{D}}_{\dot\alpha} \overline{\Phi}^{\bar{j}_n} \right) \tag{3.13}$$

The final type of F-terms we will consider are the so-called multi-fermion, Beasley-Witten, or higher derivative F-terms [72, 73]. They are somewhat less known than superpotentials and gauge kinetic functions, so we pause a moment to describe their structure.

Higher F-terms are specified by a holomorphic function $\omega_{\bar{i}_1 \ldots \bar{i}_n, \bar{j}_1 \ldots \bar{j}_n}(\Phi)$ of the chiral fields $\Phi = \varphi + \theta^\alpha \psi_\alpha$ analogous to the superpotential, but also include $2n$ anti-chiral fields that appear through the derivative

$$\overline{\mathcal{D}}^{\dot\alpha} \overline{\Phi} = \overline{\psi}^{\dot\alpha} + \theta_\alpha (\sigma^\mu)^{\dot\alpha \alpha} \, \partial_\mu \overline{\varphi} \, . \tag{3.14}$$

These introduce either fermions or derivatives of scalar fields in the action, hence the names of these F-terms. At first sight, the couplings (3.13) do not seem supersymmetric, but a careful

[2] We do not consider possible terms involving charged matter fields C_I or mixed terms between different gauge groups. We also omit some factors of α' and 2π in our discussion.

analysis shows that they are, provided the functions ω satisfy certain constraints. First, they must be holomorphic functions of chiral fields. Second, they must be antisymmetric in the $\bar{i}$ and $\bar{j}$ indices separately, and symmetric under their exchange. This property hints towards the interpretation of these functions as follows. Using the Kähler metric $g_{i\bar{j}}$ on the moduli space $\mathcal{M}$ parametrized by the scalars φ, we can raise one of the two sets of indices, say the $\{\bar{j}\}$. Then, because of the antisymmetry of the indices in each set, we can interpret the function $\omega_{\bar{i}_1\ldots\bar{i}_n,\bar{j}_1\ldots\bar{j}_n} g^{j_1\bar{j}_1}\ldots g^{j_n\bar{j}_n}$ as a holomorphic section of $\overline{\Omega}^n_{\mathcal{M}}\otimes\wedge^n T\mathcal{M}$, i.e. the space of $(0,n)$-forms with values in the antisymmetrized tensor product of n tangent bundles of $\mathcal{M}$. Now, the final condition on these functions is that these sections represent non-trivial cohomology classes of $\mathcal{H}^n_{\bar{\partial}}(\mathcal{M},\wedge^n T\mathcal{M})$, i.e. upon the raising of one set of indices, ω must be closed, but non-exact under $\bar{\partial}$. Closedness implies that (3.13) is supersymmetric (the integrand is chiral), while non-exactness implies that it cannot be written as a D-term globally in moduli space [72, 73].

For $n = 1$, Beasley-Witten terms represent deformations of the complex structure of the moduli space $\mathcal{M}$, which can be either the Kähler or the complex structure moduli space of the CY compactification manifold. This interpretation is quite intuitive since $\omega_{\bar{i}}^{\,j}$ belongs to the cohomology $\mathcal{H}^1_{\bar{\partial}}(\mathcal{M},T\mathcal{M})$ and this is precisely the space in which, for a Kähler manifold, complex structures and their deformations live (c.f. section 2.2.2). Terms with $n > 1$ are less studied and in principle less interesting since they involve more derivatives or more fermions, and are hence more suppressed at low energies.

Now, since the functions ω are holomorphic, the way in which closed string chiral multiplets can enter in Beasley-Witten terms is quite constrained by periodicity of axions. Again, they can only appear exponentiated[3]

$$\omega(T_a,U_k) = \omega_0(e^{-T_i}) + \omega_{\text{n.p.}}(e^{-T_i},e^{-U_k})\,, \tag{3.15}$$

with the by now familiar distinction between perturbative and non-perturbative terms. Notice however, that these ω functions accompany terms which involve derivatives of chiral fields. Nothing prevents axions from appearing in derivative form in the actions. In particular, higher F-terms which involve $2k$ derivatives of complex structure moduli would imply k inverse powers of the the string coupling constant, g_s^{-2k}. Such a dependence on the dilaton is however not expected to arise in string theory for $k > 1$. From a worldsheet perspective, the coupling constant dependence of a term computed from a worldsheet with Euler number $\chi(\Sigma)$ is $g_s^{-2+\chi(\Sigma)}$. Since the Euler number is non-negative, only factors of g_s^n are expected with $n \geq -2$. We conclude that no more than two derivatives of complex structure moduli are expected to appear in higher F-terms.

In section 4.2 we will study non-perturbative corrections to the hypermultiplet moduli space metric of type IIB CY compactifications. These are $n = 1$ Beasley-Witten terms with two derivatives of Kähler moduli, which are the ones that contain the dilaton on the type IIB side, as dictated by mirror symmetry.

[3]As for the gauge kinetic functions, we are ignoring here a possible dependence on charged matter fields C_I.

3.2.4 D-terms

Non-holomorphic terms are under less control, and much more difficult to compute than holomorphic ones, since nothing prevents them to get corrections at any order in perturbation theory. The real part of the chiral fields, i.e. the dilaton and the geometric Kähler and complex structure moduli, can in principle appear in them in an arbitrary way. In this work we are interested in non-perturbative effects, and these are effectively only relevant when they do not have to compete with perturbative terms (at least for $g_s \to 0$, the realm in which we can actually do computations). Also, the computation of non-perturbative effects in string theory relies heavily on the holomorphic properties of F-terms, and is under control only for them. Hence, D-terms will not be relevant for us.

Let us just comment here that there are Kähler potentials for each type of closed moduli, and they are decoupled even at the non-perturbative level. These functions define the geometries of the moduli spaces (although their complex structure can be deformed by Beasley Witten terms as explained above). As we mentioned in section 2.2.3, for $\mathcal{N}=2$ CY compactifications, the vector multiplet moduli space (Kähler parameters in type IIA) is of special Kähler type, while the hypermultiplet moduli space (complex structure in type IIA) is quaternionic Kähler. After reduction of supersymmetry to $\mathcal{N}=1$, no general constraint remains on the moduli geometries other than Kählerity.

3.2.5 Summary

In this section we have seen how periodicity of axions, together with holomorphy, highly constrains the possible dependence of F-terms on closed string fields. Of special importance is the dependence on the dilaton (and hence on g_s) which for type IIA string theory belongs to the complex structure moduli.

We saw, in particular, that g_s can only appear non-perturbatively in the superpotential, i.e. the tree level result is exact in perturbation theory, and is only corrected by non-perturbative effects. Similarly, in gauge kinetic functions, complex structure moduli only appear at tree level, with a linear dependence, and perturbative corrections are independent of g_s and only arise at one loop. Further corrections only appear non-perturbatively. We have also seen how holomorphy constrains corrections to higher F-terms, although their g_s dependence is slightly more complicated and depends on the particular term of interest.

These crucial results are known as non-renormalization theorems . They are general features of supersymmetric field theories, and can be deduced in pure field theoretical terms without the need to refer to string theory or to axions. For example, one can compute the effective superpotential through supersymmetric Feynman diagrams and see that they systematically vanish for loop-diagrams of any order. For us, the theorems are just a natural consequence of the axionic periodicity properties and the holomorphy of F-terms. We did not even need to rely on perturbation theory to derive them. Furthermore, the decoupling of complex structure and Kähler moduli in F-terms is an artifact of perturbation theory and should have no special meaning in non-perturbative terms. The decoupling of the Kähler potentials, on the other hand, is a non-perturbative result which implies in particular that the hypermultiplet moduli space receives no quantum corrections at all.

The basic tool that we have at hand for practical computations in physics is perturbation theory. This flaw is specially acute in string theory since we do not even have a complete non-perturbative definition of the theory. The formulation we use, namely worldsheet string theory, is intrinsically perturbative and relies on an expansion in g_s (the genus expansion). Hence, our results are usually valid only for small coupling constant, and are not reliable outside this realm.[4]

This means that, although quite artificial for arbitrary g_s, the distinction between perturbative and non-perturbative effects is indeed very important at a practical level. As mentioned above, since non-perturbative effects are much more suppressed than perturbative ones, they are irrelevant when they have to compete with each other. On the other hand, we have seen a few examples where there are terms of the action that only arise non-perturbatively. These are always F-terms since they are the ones subject to non-renormalization theorems. In particular, neutral superpotentials for complex structure moduli arise only at the non-perturbative level, and charged superpotentials are also exclusively non-perturbative because of global "anomalous" $U(1)$'s. Non-perturbative contributions to other F-terms may also be quite important since they are the only corrections and, even though small, could change qualitative features of the action.

As a consequence, non-perturbative effects are interesting both for phenomenological reasons, since they can drastically change the behavior of string theory compactifications at low energies; and for purely theoretical reasons, as sources of information not contained in the worldsheet formulation of string theory, and hence crucial in the aim of finding a complete definition of the theory (such as string field theory).

As a final comment, let us at last motivate the name axion that we have been using for the imaginary part of our closed string chiral fields. A first reason appeared already in the previous chapter, where we saw that these fields can couple to the second Chern class of open string gauge fields, just like the familiar QCD axion does. The second one is a consequence of perturbation theory. Except when they appear as coefficients of some topological instanton number, our periodic fields can only appear with derivatives or through exponential functions, which are intrinsically non-perturbative for complex structure fields, as we have emphasized. This means that the periodicity of these fields gets enhanced to a Peccei-Quinn shift symmetry at the perturbative level

$$U(x) \longrightarrow U(x) + c\,, \qquad \forall c \in \mathbb{R}\,, \qquad \text{(perturbatively)}. \tag{3.16}$$

This is usually stated by saying that these fields are axions whose $\mathbb{R}$ shift symmetries are broken to $\mathbb{Z}$ by non-perturbative effects. From our perspective, this symmetry $\mathbb{R}$ was never there. It is only a consequence of neglecting e^{-1/g_s} effects and should play no role in a non-perturbative description of the theory.

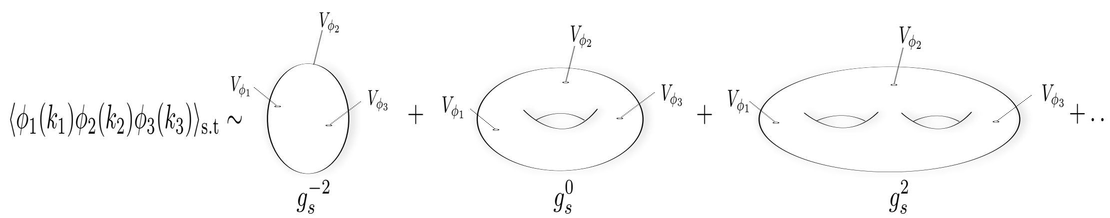
3.3 Perturbative string theory

The next question to face is naturally what are the specific functions that enter the effective actions. As we just stressed, the terms that can be obtained from first principles in the

[4] With the notable exception of results protected by symmetries, specially supersymmetry. This is most welcome since we have implicitly been using perturbative string theory to obtain the BPS spectra at low energies. The results presented so far are robust even away of the small coupling region since they are protected by supersymmetry.

worldsheet formulation of string theory are the perturbative ones. We describe some aspects of their computation in this section, although our main interest in this work is in the more involved non-perturbative couplings.

In perturbative string theory (in asymptotically flat spacetime), physical amplitudes involving a set of spacetime fields $\{\phi_i\}$ are obtained through the calculation of 2d CFT correlators with insertions of worldsheet vertex operators associated to the fields $\{V_{\phi_i}\}$. The explicit computation of a given correlator is generally performed through a perturbative expansion in the coupling constant of the 2d theory, namely α'.[5] The physical amplitude is then obtained as a sum of the 2d correlators on worldsheets Σ with different topologies but the same vertex insertions. Each correlator is weighted by a factor of $g_s^{-2+\chi(\Sigma)}$, where $\chi(\Sigma)$ denotes the Euler number of the worldsheet. Hence, the amplitude is given by a perturbative power series in α' and g_s. This is schematically represented as



Once an amplitude of interest has been computed, one can infer an effective spacetime action from which it could have been derived. This is the effective action we are interested in. In particular, this method could be applied to compute the spacetime actions for type IIA superstrings in ten flat dimensions that we presented with almost no motivation in (2.2) for closed strings, and in (2.19) and (2.20) for open ones. These actions are the first order approximation, both in g_s and α' to the complete result. Higher order corrections could be obtained by more precise (in α') CFT computations on worldsheets with higher genus (or Euler number).

We are mostly interested, however, in the computation of effective actions of the much more complicated scenarios of toroidal or CY compactifications with D6-branes and O6-planes. The worldsheet theories are much more involved in these backgrounds, and the computations of 2d correlators become intractable in many cases. An alternative way to proceed is to perform a KK reduction and an orientifold projection on the original 10d actions.[6] This method was applied in section 2.3.4 to obtain the effective couplings (2.32) involved in the GS anomaly cancellation mechanism.

The procedure, although useful in many cases, only shifts the difficulties to the computation of quantum corrections to the action in ten flat dimensions and its reduction to four in the compactification background of interest. Of course, these steps are extremely complicated in most cases. Furthermore, the KK reduction is likely to miss many ingredients of the full 4d effective action. Since it is a purely field theoretical tool, aspects related to the extended nature of the strings (and hence parametrized by α') such as winding modes, may be easily overlooked with this method.

[5] Throughout this work we have obviated the presence of α' in most equations. Since it is a parameter with spacetime dimensions, its presence can be easily inferred by dimensional analysis.

[6] To obtain the effective action at low energies one would also need to apply the renormalization group equations to the resulting couplings.

Notice finally that, due to the way in which it is computed (or defined), the spacetime effective action is intrinsically perturbative in g_s. Non-perturbative effects in α' can be relatively easily included by computing instanton corrections in the worldsheet correlators, the so-called worldsheet instantons. These are well-defined since we have a 2d action, namely the Polyakov action whose coupling constant is α', from which they can be derived in the usual field theoretical way. On the contrary, the worldsheet formulation of string theory does not provide us with an spacetime action (this is in fact what we are trying to find), whose coupling constant should be g_s, from which non-perturbative effects could be derived. Several relatively successful attempts have been performed to define such non-perturbative completions of string theory, and are the area of study of string field theory. These are extremely complicated and under poor control even in flat backgrounds, and are not yet ripe for applications to superstring compactifications.

Yukawa couplings and worldsheet instantons

In order to exemplify the worldsheet computation of perturbative terms (in a g_s expansion) of the effective action, we discuss now Yukawa couplings in type IIA intersecting D6-brane models [70, 74, 75]. This particular example is very illustrative for us, since it is non-perturbative in α', and the calculations bear strong similarities with effects that are non-perturbative in g_s.

In the language of (3.7), perturbative Yukawa couplings belong to the matter part of the superpotential $W_0^{\text{mat}}(e^{-T_a}, C_I)$. They have the particular form

$$W_Y = Y_{ijk}(e^{-T_a})\, H_i\, Q_{Lj}\, q_{Rk}\,. \tag{3.17}$$

Notice that the Kähler moduli T_a, as defined in (3.4), are proportional to the volume of 2-cycles of the CY manifold. These areas are naturally measured in units of α', and hence the moduli scale secretly as $T_i \propto (\alpha')^{-1}$. This immediately implies that the Yukawa functions $Y_{ijk}(e^{-T_a})$ we are looking for are non-perturbative in α'. Now, how do these effects appear in actual computations?

As mentioned above, from the 2d CFT perspective, non-perturbative α' effects arise in the usual field theoretical way, from topologically non-trivial solutions to the classical equations of motion. Notice that the Polyakov action for the string (or the Nambu-Goto action) is just the volume of the worldsheet. The equations of motion tell us that the area of the string should be minimized. Topologically non-trivial configurations in which the area of the string is minimized are obviously those in which the (euclidean) worldsheet Σ wraps non-trivial cycles in the homology of the 10d space.

This result can be also intuitively understood from the 4d spacetime point of view. A worldsheet that wraps entirely a cycle of the compactification space would look in 4d as localized in spacetime. Such type of localized configuration of fields in spacetime are the usual form in which instantons appear in field theory, and are weighted non-perturbatively in the corresponding coupling constant.

We reach the conclusion that we have to compute 2d CFT correlators in which the string wraps an internal cycle. To obtain a Yukawa coupling (3.17), we have to introduce also vertex operators for the corresponding matter fields H_i, Q_{Lj}, and q_{Rk}. Since these are chiral open string fields that live at the intersections of gauge D-branes, we must consider worldsheets with boundaries on the corresponding branes. Furthermore, only diagrams with disk topology can contribute. Other topologies would introduce powers of the dilaton, and hence of complex structure chiral fields, which are forbidden by periodicity of the axions. The diagrams of interests are represented in figure 3.1.

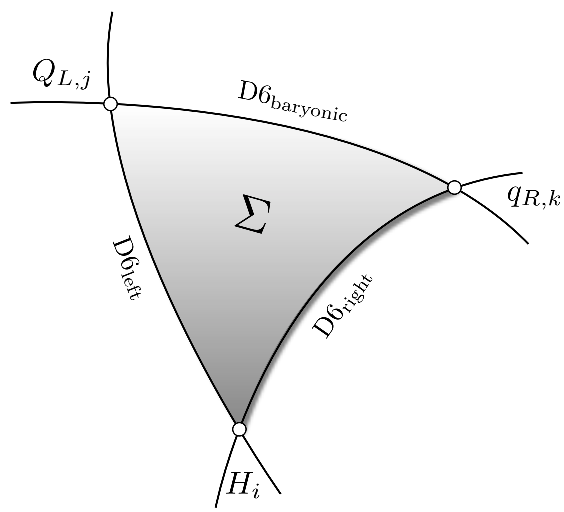


Figure 3.1: Worldsheet instanton contributing to the Yukawa coupling (3.17).

A final property, which will be further developed in the next section in relation to D-brane instantons, is that only configurations that preserve some supersymmetry can contribute to the superpotential. The superpotential is an F-term which is only integrated over half of the superspace, and these type of terms are always associated to BPS states of the theory. Instantons that break all supersymmetries can only contribute to D-terms of the effective action. We saw in section 2.3.2 that supersymmetric 3-cycles are the special Lagrangian cycles. The analogous statement for even dimensional cycles is that supersymmetric cycles are those defined by holomorphic equations. These are obviously called holomorphic cycles.

The result is that Yukawa couplings in type IIA intersecting brane models take the form

$$Y_{ijk} = h_{\text{qu}} \sum_{\Sigma = \mathcal{D}_{ijk}} e^{-\frac{\text{Vol}_\Sigma(T_a)}{\alpha'}} \ . \tag{3.18}$$

Here we have reintroduced the dependence on α' to emphasize the non-perturbative character of the coupling. The sum runs over all appropriate cycles: holomorphic 2-cycles with disk topologies that join the intersections where the matter fields H_i, Q_{Lj}, and q_{Rk} live. The universal coefficient h_{qu} stands for one-loop quantum fluctuations of the worldsheet around the minimal area classical solution. We will not need the details of this term here. Finally, the exponential factor is just the classical Polyakov action of the instanton, namely, the complexified area of the worldsheet. We know that this area is a Kähler modulus of the CY, and always appears as a complex scalar, together with the corresponding axionic field.

3.4 D-brane instantons

We have strongly emphasized in the previous sections the importance of non-perturbative terms of the effective action of string compactifications, both for theoretical and phenomenological reasons. Since we cannot derive them from first principles, we must rely on indirect ways to infer their presence and form. Ideally, we would like to have different methods of computation of these effects to check against each other, and to serve as a validation of the calculations themselves. In this section we will introduce the standard D-brane instanton calculus which is the most systematic tool available so far, see [20, 76] for reviews. In the next chapter, we will develop a different computational method that, in some cases, is more powerful and easy to use. The agreement between both approaches is a strong check of their validity, and part of the evidence that non-perturbative effects should indeed be a part of a complete formulation of string theory.

The question that we face is how to introduce effects that are non-perturbative in g_s, and how to compute their contributions to the 4d effective actions. The answer is quite obvious in the light of the previous discussions, and specially of the description of worldsheet instanton effects.

We need to consider elements that are intrinsically non-perturbative, and do not appear in the worldsheet expansion of perturbative string theory. The natural objects to study are D-branes, whose tensions scale as $1/g_s$, as we have seen in section 2.3.1. So far we have only considered D-branes whose worldvolumes span the non-compact Minkowski dimensions. As we know, these lead to ground states of the theory with gauge groups living on the brane. Other configurations one could study are classified according to the number of non-compact dimensions which they span.

- **Domain walls**: consider a D6-brane that wraps a 4-cycle of the CY manifold, or equivalently a D4-brane wrapping a 2-cycle, or a D2-brane localized in the CY. Just like gauge D-branes, these setups also represent vacua of the theory. From the 4d point of view, these would look like objects that span two spatial dimensions plus time, i.e. domain walls. Phenomenologically, these are not very interesting configurations since they break 4d Poincare invariance and their existence is quite constrained by cosmological measurements.

- **Cosmic strings**: a very similar situation arises when one considers D4-branes wrapping 3-cycles of the internal space. These are vacua that, from the 4d perspective, include objects that extend to infinity on one spatial direction plus time, i.e. cosmic strings. Again, they are not Poincare invariant and cosmological observations severely constrain their possible existence in our Universe.

- **D-brane particles**: a more interesting setup arises when one considers objects that look point-like in space and propagate in time, i.e. solitons. These could arise from D6-branes wrapping the entire CY, from D4-branes wrapping 4-cycles, D2-branes wrapping 2-cycles or from D0-branes localized in the internal space. If the wrapped cycles are supersymmetric, the objects are BPS and the particles are protected by supersymmetry and hence stable. On the other hand, their mass is roughly the volume of the wrapped cycles as measured in units of the string scale, and are therefore very massive. These states decouple in general in the low energy regime, although there are regions in Kähler moduli space in which some cycles become massless and have a strong influence in the effective theory. A prominent example in which they play a crucial role is the resolution of the conifold singularity by Strominger in [83].

In compactifications with O-planes, they are less relevant, since they generically do not survive the orientifold projection. This agrees nicely with the fact that there exist no BPS particle representation of the $\mathcal{N} = 1$ supersymmetry algebra.

Notice finally, that the actions of these particles depend on the volume of even dimensional cycles, which are measured by Kähler moduli. This is a further hint that these are not the non-perturbative objects that we are looking for, which should depend on complex structure moduli, i.e. the volume of odd-dimensional cycles.

- **D-brane instantons**: consider a D2-brane with euclidean worldvolume (sometimes denoted E2-brane), which is entirely wrapped on a 3-cycle of the CY. From the 4d perspective, this is an object that is localized both in space and in time, i.e. it looks like an instanton. The situation is very similar both to the familiar instantons of gauge theories, and to the worldsheet instantons described in the previous section. In analogy with them, the contributions to the effective action are expected to be proportional to $e^{-S_{\rm cl}}$, where $S_{\rm cl}$ is the classical action of the instanton. Now the actions of the instantons are given by the DBI and CS actions of section 2.3.1. Classically, they just yield the volumes of the 3-cycles wrapped by the instantons, and these are precisely expressed in terms of complex structure moduli. Therefore, these are the objects that we were looking for, they have the right dependence on closed string moduli and on g_s to make non-perturbative contributions to the effective actions.

In the remaining of this work we discuss general features of D-brane instantons and their computation, and some of their consequences for the phenomenology of string compactifications.

3.4.1 D-brane instanton calculus

Consider an euclidean E2-brane $\mathcal{E}$ whose worldvolume is wrapped entirely over a 3-cycle $\Pi_{\mathcal{E}}$. In order to survive the projection, the cycle must be orientifold invariant, or else we can consider combinations with image cycles $\Pi_{\mathcal{E}} + \Pi_{\mathcal{E}'}$. Now, the (complexified) volume of the cycle, i.e. its classical action, will be controlled by a complex structure modulus $U_{\mathcal{E}}$ that can be expressed linearly in terms of the basis of moduli U_k we introduced in section 3.1. In analogy with gauge field and worldsheet instantons, we expect the contribution of $\mathcal{E}$ to the 4d effective action to take the form

$$S_{4d}^{\mathcal{E}} = \int_{4d} d^4x\, d^2\theta\, h(e^{-T_a})\, \mathcal{O}(\Phi_1, \ldots, \Phi_n)\, e^{-U_{\mathcal{E}}} \tag{3.19}$$

We are focusing here on F-terms which can arise, as we will see, when $\Pi_{\mathcal{E}}$ is a supersymmetric cycle, i.e. special Lagrangian. The chiral operator $\mathcal{O}(\Phi)$ depends on spacetime matter chiral fields, and determines whether the F-term is a superpotential, a gauge kinetic term or a higher derivative F-term. The function h includes a one-loop determinant contribution and can depend on Kähler moduli in an appropriate way.

A systematic way to obtain contributions such as (3.19) in a semiclassical approximation would be to compute worldsheet diagrams in the background given by the instanton, including vertex operators corresponding to the chiral fields $\mathcal{O}(\Phi)$, see fig. 3.2. One should then sum up (or integrate) the result over all possible instanton configuration, which involves in particular a crucial integration over zero modes of the instanton. One would obtain in this way non-perturbative contributions to a physical amplitude, from which the effective action could be inferred. There may be important subtleties in this process that can be better understood from a slightly different perspective.

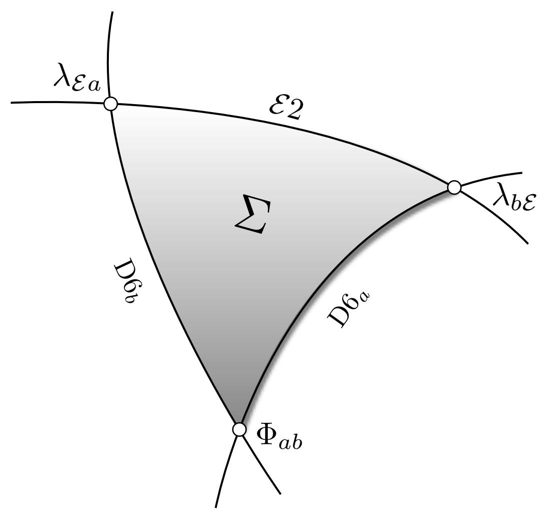


Figure 3.2: The figure can be interpreted in two ways. First, as computing a non-perturbative physical amplitude involving the spacetime chiral field Φ_{ab}. The open string fields λ are then non-physical boundary condition changing operators. Second, as an interaction between the instanton zero-modes λ and the matter field Φ. This term enters the zero-mode part of the worldvolume action of the instanton $S_{\mathcal{E}}^{(z.m)}$ in (3.21).

Taking the analogy with field theory instantons seriously, the contribution to the effective action of a particular E2-instanton configuration should be just given by $e^{-S_{\mathcal{E}}}$, the exponential of the (zero-dimensional effective) action of the instanton. One should again sum up (integrate) the contributions from all possible instanton configurations. In the semiclassical approximation, these include quantum fluctuations around the instanton background. As we know, fluctuations of a D-brane instanton are parametrized open strings which have at least one extreme attached to the E2-brane. These will include both massless and massive fields from the E2 worldvolume perspective. Let us collectively denote the former by $\mathcal{M}_0$ (the set of zero modes) and the latter by $\mathcal{M}_{\text{n.z.}}$ (non-zero modes). The contribution to the physical effective action should be

$$S_{4d}^{\mathcal{E}} = \sum_{\text{inst}} \int d\mathcal{M}_0 \, d\mathcal{M}_{\text{n.z.}} \, e^{-S_{\mathcal{E}}} \,. \tag{3.20}$$

Notice that the integral is just the partition function on the worldvolume of the E2-brane. The sum is over instantonic configurations which are not continuously connected to one another, e.g. E2-branes wrapping different homology cycles.

Now, how does this expression relates to (3.19)? First let us express the (zero-dimensional) instanton action as a sum of three terms

$$S_{\mathcal{E}} = S_{\mathcal{E}}^{(\text{clas})} + S_{\mathcal{E}}^{(\text{n.z})} + S_{\mathcal{E}}^{(\text{z.m})} \tag{3.21}$$

Here $S_{\mathcal{E}}^{\text{clas}}$ is the classical action, which is proportional to the complexified volume $U_{\mathcal{E}}$ and independent of the open string fields. $S_{\mathcal{E}}^{\text{n.z}}$ is the action for the non-zero modes, and includes at least a mass term for these fields. Finally, $S_{\mathcal{E}}^{\text{z.m}}$ corresponds to terms that include the zero-modes.

We can try to perform the integration (3.20) over the worldvolume fields of the instanton. Of crucial importance is the integration over the fermions, since they can easily lead to a zero result due to the standard rules of grassmannian integration $\int d\psi = 0$ and $\int d\psi\, \psi = 1$. The fermions must necessarily appear in the integrand if we want to obtain a non-zero contribution, i.e. they have to be *saturated*.

The integration over fermionic non-zero modes can always be saturated using the mass terms of the instanton action $S_{\mathcal{E}}^{\text{n.z}}$. Together with the bosonic massive fields, the $d\mathcal{M}_{\text{n.z.}}$ integration yields a prefactor (one-loop determinant) which is not very important to our discussion, and is indeed quite hard to obtain in practical situations.

The crucial step is the integration over the zero-modes in $\mathcal{M}_0$, specially the fermionic ones. The particular spectrum of massless fields on the worldvolume depends on the specific configuration of the instanton, and we will discuss it in some detail in the next section. There is, however, a universal sector common to all instantons which plays a special role. The bosonic part of this universal sector contain the four fields x^μ that parametrize the position of the instanton in 4d spacetime. These are goldstone bosons of the Poincare symmetry that is broken by the instanton. Furthermore, since the instanton also breaks (at least some of) the supersymmetries of the 4d theory, there will appear goldstino zero-modes in the worldvolume theory.

Generically, an instanton will break all of the $\mathcal{N} = 1$ supersymmetries and will yield four fermion zero-modes θ^α and $\bar{\theta}^{\dot{\alpha}}$. This would lead to the generation of a 4d D-term of the effective action. If, however, the E2-brane wraps a special Lagrangian 3-cycle which preserves the same supersymmetries as the O-planes and the D6-branes, the instanton will be BPS and preserve half of the supersymmetry. In this case there will only be two goldstinos θ^α and integration over the universal zero modes will lead precisely the familiar F-term structure

$$S_{4d}^{\mathcal{E}} = \int_{4d} d^4x\, d^2\theta\, A_{\text{n.z.}} \int d\mathcal{M}'\, e^{-U_{\mathcal{E}} - S_{\mathcal{E}}^{(\text{z.m})}} \tag{3.22}$$

Here $A_{\text{n.z.}}$ is the factor coming from the integration over non-zero modes. The remaining integration is only over zero modes which we have separated into the universal sector $\{x, \theta\}$ and the rest $\mathcal{M}'$. It is clear that, if the instanton is to yield a non-vanishing F-term contribution, all fermionic zero modes other than θ^α must be saturated with interactions from $S_{\mathcal{E}}^{(\text{z.m})}$.

Notice that (3.22) is now very close to our original guess (3.19). The only step that separates both expressions is the integration over non-universal zero modes. Indeed, in a configuration in which $\mathcal{M}'$ is empty, both equations are equivalent for $h = A_{\text{n.z.}}$ and $\mathcal{O}(\Phi) = 1$. This simplest case corresponds to the generation of a neutral superpotential for complex structure moduli, which is already very interesting for applications to moduli stabilization.

3.4.2 Instanton zero-modes

We proceed now to study in detail the general instanton configurations and their (fermionic) zero modes. The analysis bears strong similarities with that of the spectrum of gauge D6-branes in section 2.3.3, with the important different that the instanton is localized in the non-compact dimensions.

Universal zero-modes $(x, \theta, \overline{\tau})$

This sector was introduced in the last paragraph and arises from open strings with both ends attached to the instanton E2-brane, i.e. the $\mathcal{E}-\mathcal{E}$ sector. As mentioned before, it contains the four goldstone bosons x^μ associated to the breaking of Poincare invariance. Furthermore it contains goldstinos from the supersymmetries broken by the instanton.

Let us assume that the 3-cycle $\Pi_\mathcal{E}$ is supersymmetric with respect to the orientifold. This means that a putative D6-brane that would wrap the special lagrangian cycle $\Pi_\mathcal{E}$ would break the $\mathcal{N}=2$ supersymmetry preserved by the CY to the same $\mathcal{N}=1$ subalgebra remaining after the orientifold projection, while it would break the orthogonal complement. We call their respective generators $(\mathcal{Q}^\alpha, \overline{\mathcal{Q}}^{\dot\alpha})$ and $(\mathcal{Q}'^\alpha, \overline{\mathcal{Q}}'^{\dot\alpha})$. Now, due to its localization in the four extended dimensions, the instanton $\mathcal{E}$ does not preserve the physical supersymmetry $(\mathcal{Q}^\alpha, \overline{\mathcal{Q}}^{\dot\alpha})$, but instead preserves the one generated by $(\mathcal{Q}'^\alpha, \overline{\mathcal{Q}}^{\dot\alpha})$. Hence, before the orientifold projection, the instanton contains the goldstinos θ^α and $\overline{\tau}^{\dot\alpha}$, associated to the broken $(\mathcal{Q}^\alpha, \overline{\mathcal{Q}}'^{\dot\alpha})$.

As we have previously seen, the breaking $\mathcal{N}=2 \to \mathcal{N}=1$ occurs at the cycles wrapped by the O6-planes and D6-branes. Away from this loci, an instanton would locally feel the original $\mathcal{N}=2$ supersymmetry, and hence contain the four goldstinos θ^α and $\overline{\tau}^{\dot\alpha}$. In this case, a mechanism to saturate the extra $\overline{\tau}^{\dot\alpha}$ zero-modes is needed if the instanton is to yield a non-vanishing contribution to the effective action. We will discuss this configuration briefly in the next section.

A simpler situation arises if the instanton feels locally already only the $\mathcal{N}=1$ preserved by the orientifold. This happens if $\mathcal{E}$ wraps either the same cycle as a stack $\mathcal{D}$ of D6-branes, or a cycle that is invariant under the orientifold projection. The first setup $\Pi_\mathcal{E} \in \Pi_\mathcal{D}$ corresponds to the stringy description of a usual gauge instanton of the Chan-Paton gauge theory that lives on $\mathcal{D}$. The extra goldstinos $\overline{\tau}^{\dot\alpha}$, as well as other fermionic zero-modes that arise from open strings stretching between $\mathcal{E}$ and $\mathcal{D}$, can be saturated against each other from interactions that appear in $S_\mathcal{E}^{(\mathrm{z.m})}$. We will not analyze this case in detail.

The second setup, in which the instanton cycle is its own orientifold image $\Pi_\mathcal{E} = \Pi_{\mathcal{E}'}$ is also very interesting. Consider a stack of N E2 branes wrapping such a cycle. One has to distinguish the two possible ways in which the orientifold projection may act on the Chan Paton factors of open strings in the $\mathcal{E}-\mathcal{E}$ sector [77–80]:

- $USp(N)$ instanton: if the original $U(N)$ gauge group is projected down to an $USp(N)$ subgroup, one gets $\frac{1}{2}N(N-1)$ zero modes θ^α and $\frac{1}{2}N(N+1)$ zero-modes $\overline{\tau}^{\dot\alpha}$. Notice that for this case one needs an even number of branes and, even for the minimal situation $N=2$, one gets too many fermion zero-modes to generate an F-term.

- $O(N)$ instanton: here one can also have an odd number of $\mathcal{E}$ branes. One obtains $\frac{1}{2}N(N+1)$ zero-modes θ^α and $\frac{1}{2}N(N-1)$ zero-modes $\overline{\tau}^{\dot{\alpha}}$.

It is clear that a single $O(1)$ instanton has the right structure to generate a neutral superpotential for the complex structure moduli

$$S_{4d}^{\mathcal{E}} = \int_{4d} d^4x\, d^2\theta\, A_{\text{n.z.}} e^{-U_{\mathcal{E}}} \,. \tag{3.23}$$

Unless otherwise stated, we will only consider cycles wrapped by single instanton branes, either $O(1)$ or $U(1)$ instantons depending on whether $\Pi_{\mathcal{E}} = \Pi_{\mathcal{E}'}$ or not.

Deformation zero-modes (c, $\overline{\chi}^{\dot{\alpha}}$, γ^α)

Just like in gauge D6-branes, there can be further massless fields in the $\mathcal{E}-\mathcal{E}$ sector, parametrizing possible deformations of the 3-cycle wrapped by the instanton. As we commented in section 2.3.2, they are counted by the first Betti numbers of the wrapped cycle $b_1(\Pi_{\mathcal{E}})_\pm$. These are the number of non-trivial homology 1-cycles that are even and odd, respectively, under the orientifold projection. The precise spectrum is however not the same as that of a hypothetical D6-brane wrapped on $\Pi_{\mathcal{E}}$ due to the localization of the instanton in the non-compact directions. The result is that, in the instanton worldvolume, there will live $b_1(\Pi_{\mathcal{E}})_-$ chiral fermionic zero-modes γ^α, and $b_1(\Pi_{\mathcal{E}})_+$ pairs of bosonic and anti-chiral fermionic zero modes (c, $\overline{\chi}^{\dot{\alpha}}$).

Again, in order to generate a contribution to the effective action, one must get rid of the extra fermionic zero-modes, either by saturation from interactions in $S_{\mathcal{E}}^{(\text{z.m})}$, or by some other mechanism like the inclusion of fluxes. We will only be interested in situations in which these deformation zero-modes are simply absent. Cycles that satisfy $b_1(\Pi) = 0$ are called *rigid*.

Charged zero-modes (λ)

If a $U(1)$ instanton $\mathcal{E}$ wraps a rigid cycle which intersects stacks $\mathcal{D}_a$ of N_a D6-branes, there are generically zero-modes from the $\mathcal{E}-\mathcal{D}$ sector that are charged under the spacetime gauge groups [40–42]. We only consider the case in which the gauge branes are not invariant under the orientifold projection, i.e. $\Pi_{\mathcal{D}} \neq \Pi_{\mathcal{D}'}$, so that the gauge groups are $U(N_a) = SU(N_a) \times U(1)_a$. The situation is again similar but not equal to that of intersecting D6-branes, which was briefly discussed in 2.3.2. In this case the zero-modes consist purely of fields with a single grassmannian degree of freedom [81]. Their net multiplicity is given by intersection numbers: there will be fields $\lambda_{a\mathcal{E}}$ counted by $I_{\mathcal{D}_a\mathcal{E}} = [\Pi_{\mathcal{D}_a}] \cdot [\Pi_{\mathcal{E}}]$, and fields $\lambda_{a\mathcal{E}'}$ counted by $I_{\mathcal{D}_a\mathcal{E}'} = [\Pi_{\mathcal{D}_a}] \cdot [\Pi_{\mathcal{E}'}]$. All of them transform under the fundamental representation $\square_a$ of $SU(N_a)$.[7] If any of the intersection numbers is negative, there will arise instead fields $\lambda_{\mathcal{E}a}$ and/or $\lambda_{\mathcal{E}'a}$ in the antifundamental $\overline{\square}_a$. Finally, if the instanton is $O(1)$, the same arguments apply by simply setting $\mathcal{E}' = \mathcal{E}$.

[7] Recall our convention, that fields that transform under the fundamental $\square_a$ (antifundamental $\overline{\square}_a$) carry charge +1 (-1) under the corresponding abelian $U(1)_a$ factors.

Multi-instanton zero-modes (μ^α, $\bar{\mu}^{\dot{\alpha}}$, m)

A final set of zero modes can arise if various instantonic E2-branes intersect each other [82]. Their multiplicity is counted by the same intersection numbers as in the case of gauge D6-branes. A typical case occurs when $\mathcal{E}$ intersects its orientifold image $\mathcal{E}'$. We can borrow the results of section 2.3.3, and eq. (2.26). There will be chiral fermions μ^α counted by $\frac{1}{2}(I_{\mathcal{E},\mathcal{E}'} - I_{\mathcal{E},O6})$ and with $U(1)_\mathcal{E}$ charge +2; and pairs of bosons and anti-chiral fermions $(m, \bar{\mu}^{\dot{\alpha}})$ counted by $\frac{1}{2}(I_{\mathcal{E},\mathcal{E}'} + I_{\mathcal{E},O6})$ with $U(1)_\mathcal{E}$ charges (2,-2). Negative multiplicities would simply indicate a positive number of fields with the opposite charges.

3.4.3 Non-perturbative superpotentials

We will study in the following some mechanisms by which zero-modes other than the universal θ^α needed to generate an F-term can be saturated. We will see that the integration over these fields leads to contributions to different terms of the effective action. Let us begin with a discussion of superpotentials.

Lifting of zero modes

As we saw above, the configuration with less amount of zero-modes is a single $O(1)$ instanton wrapping a rigid cycle. In this case there are no extra zero-modes and the instanton contributes automatically to a neutral superpotential (3.23). This result can have important phenomenological consequences, since in the absence of other ingredients such as background fluxes, it is the only way to generate a superpotential for the complex structure moduli, and hence their only stabilization mechanism [43].

An even more interesting case is that of a rigid $O(1)$ instanton that intersects gauge D6-branes, and hence contains charged zero-modes [40–42]. If we want to generate a contribution to the effective action, we must make sure that all these extra fermions can be saturated in eq. (3.22) from interactions that appear in $S_\mathcal{E}^{(\text{z.m})}$. In fact, we have already seen a way in which such an interaction can arise. Figure 3.2 is nothing else than the contribution to the E2-instanton action from a worldsheet instanton that involves two charged zero-modes and a chiral matter field Φ_{ab}

$$S_\mathcal{E}^{(\text{z.m})} = h\, e^{-\text{Vol}_\Sigma(T_a)} \lambda_{\mathcal{E}a} \Phi_{ab} \lambda_{b\mathcal{E}} \,, \tag{3.24}$$

where contraction of gauge indices is understood. Here h is a constant not relevant for us, and the exponent is the classical action of the worldsheet instanton, the complexified area of the worldsheet (recall the discussion of worldsheet instanton in 3.3 and specially eqs. (3.17) and (3.18)). Integration over the zero-modes $\lambda_{\mathcal{E}a}$ and $\lambda_{b\mathcal{E}}$ introduces a term $h\, e^{-\text{Vol}_\Sigma(T_a)} \Phi_{ab}$ in the effective action contribution.

A rigid $O(1)$ instanton for which all the charged zero-modes can be saturated by interactions similar to (3.24), will lead to a spacetime effective superpotential

$$S_{4d}^\mathcal{E} = h \int_{4d} d^4x\, d^2\theta\, f(e^{-T_a})\, \Phi_{a_1 b_1} \cdot \ldots \cdot \Phi_{a_n b_n}\, e^{-U_\mathcal{E}} \,, \tag{3.25}$$

which is precisely of the expected form (3.19).

Violation of global $U(1)$'s

At this stage, the $U(1)_a$ factors and their possible "anomalies" enter the discussion in an extremely interesting manner. Notice that the chiral fields Φ that enter the superpotential are charged under these groups, as e.g. dictated by invariance of (3.24). Hence, saturation of a zero-mode with charge $+1$ under $U(1)_a$ introduces a matter field with charge -1 in the superpotential. The net charge of the zero modes is just $N_a I_{a\mathcal{E}}$, and hence the total $U(1)_a$ charge of the chiral fields in the superpotential is $-N_a I_{a\mathcal{E}}$. At low energies, once the moduli have been stabilized and the $U(1)$ bosons have acquired a Stückelberg mass, the couplings (3.25) violate these global symmetries.

One could equivalently say that the zero mode measure $d\mathcal{M}_0$ in (3.20) carries this charge. This is just a well-known statement for gauge field theory instantons, it is the fact that instanton backgrounds "violate" $U(1)$ with a chiral anomaly.

This discussion resembles very much our study of the GS mechanism in section 2.3.4. We showed there that a $U(1)_a$ can be apparently anomalous, if their gauge bosons couple to a RR 2-form $B_2^k = \int_{\beta_k} C_5$ which are dual to the axions $a^k = \int_{\alpha^k} C_3$. The coupling occurred through a term

$$S_{BF} = q_a^k \, B_2^k \wedge F_{U(1)_a} \, , \qquad \text{where} \quad q_a^k = N_a [\alpha^k] \cdot [\Pi_a] \, , \tag{3.26}$$

so that the GS mechanism cancels the anomaly. This coupling also implied a non-trivial gauge transformation of the axion a^k, and hence of the complex structure modulus to which it belongs

$$U^k = -\frac{1}{g_s} \mathrm{Vol}(\alpha^k) - i \, a^k \quad \longrightarrow \quad U^k + i \, q_a^k \, \Lambda \, . \tag{3.27}$$

This means that a term e^{-U^k} behaves as a field with $U(1)_a$ charge $-q_a^k$. This term is precisely the classical action of an instanton wrapped along the cycle α^k, and $-q_a^k$ coincides precisely with the charge of its zero-modes $-q_a^k = N_a I_{a\mathcal{E}}$. It is then obvious that the charge contained in the matter fields in the superpotential (3.25) cancels precisely that of the classical action of the instanton $e^{-U_{\mathcal{E}}}$.

Computation of the net number of charged zero modes or equivalently the $U(1)$ charges of and instanton are useful tools in considering what type of terms and instanton can contribute to. Running the argument around, one can determine what type of instantons can contribute to a given term by determining its anomalous $U(1)$ charges.

Some phenomenological applications

As we have emphasized, non-perturbative charged superpotentials are specially important since they are forbidden at the perturbative level. Furthermore, the suppression factor $e^{-\mathrm{Vol}_{\mathcal{E}}/g_s}$ by which they are weighted could help in understanding some of the hierarchies of the Standard Model. It is of crucial importance that this coefficient is in general independent of the Yang-Mills coupling constants of the spacetime gauge theory $g_a^2 = \frac{g_s}{\mathrm{Vol}_{\mathcal{D}_a}}$ (c.f. eq. (3.12)). The E2 instanton classical action can then be expressed as

$$S_{\mathcal{E}}^{(\mathrm{clas})} = \frac{1}{g_a^2} \frac{\mathrm{Vol}_{\mathcal{E}}}{\mathrm{Vol}_{\mathcal{D}_a}} \, . \tag{3.28}$$

We see that the instanton suppression factor coincides with the usual one of gauge field theory instantons only if the E2-brane $\mathcal{E}$ wraps the same cycle as the D6-brane $\mathcal{D}_a$. This is in perfect agreement with our previous discussion, $\mathcal{E} = \mathcal{D}$ corresponds precisely to the stringy description of gauge instantons. On the other hand, if the instanton wraps a different cycle, the ratios of volumes can generate a new hierarchy which, in specific circumstances, can have the precise magnitude to generate non-perturbative couplings with the desired strength. Instantons whose worldvolume does not coincide with that of a gauge D-brane do not have an analog in pure gauge theory, and are hence called *stringy*.

This mechanism of generating hierarchies is most welcome. Instantons could lead to superpotentials that range from neutral superpotential for complex structure moduli, linear terms for matter fields that can generate supersymmetry breaking à la Polonyi, Majorana masses for right-handed neutrinos, the μ-parameter of supersymmetric extensions of the Standard Model, Yukawa coupling for certain families, Weinberg operators for neutrino masses, etc.

Many string models that come close to the Standard Model lack several of these couplings at the perturbative level due to global $U(1)$ symmetries. Indeed we found such a situation in the toroidal example of section 2.4, where we saw that neutrino Majorana masses were indeed forbidden by $U(1)_c$ and $U(1)_d$. Non-perturbative effects can generate such couplings, and furthermore generate desired hierarchies if the volumes of the CY have the appropriate size.

Unfortunately, there could also be non-perturbative contributions to some couplings which are not welcome from a phenomenological perspective. Most prominently, couplings violating baryon and/or lepton number which may lead to too fast proton decay generically plague supersymmetric and GUT extensions of the Standard Model and their stringy realizations. In string model building, baryon and lepton number often appear as gauge symmetries that have effectively become global by the GS mechanism, and are hence respected at the perturbative level. If we want these models to be viable, we must provide some mechanism which ensures that proton stability is also respected by non-perturbative effects, at least to a cote consistent with present experiments (mean-life $\tau_p \gtrsim 10^{32}$ years). In chapter 5 will present such a mechanism and discuss its implementation in specific (MS)SM-like constructions.

3.4.4 Corrections to the gauge kinetic function

We have seen how instantons with only the universal θ^α and possibly charged zero-modes lead to contributions to the superpotential of the effective action. If we want to study corrections to the effective action, we must consider instantons with some additional structure.

It turns out that the appropriate configuration is that of an $O(1)$ instanton without charged zero modes, but with an extra deformation zero-mode γ^α coming from a non-zero Betti number $b_1(\Pi_{\mathcal{E}})_-$. It can be shown, by considering a diagram such as that in figure 3.3, that the worldvolume action of such an instanton receives a one-loop contribution in which these modes interact with space time gauge bosons F_a. The structure is precisely the one needed to generate a contribution to the gauge kinetic function upon saturation of the zero-modes γ^α.

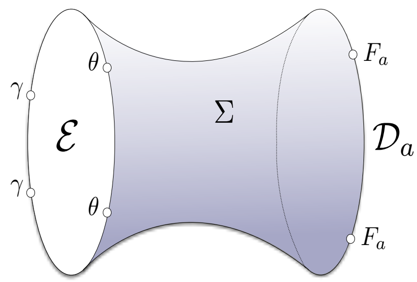


Figure 3.3: Diagram generating an interaction for deformation zero modes γ^α on the worldvolume of an instanton $\mathcal{E}$ that contributes to the 4d gauge kinetic function.

3.4.5 Higher F-terms

The final type of F-terms, namely Beasley-Witten F-terms, can be generated by BPS-instanton whose zero-modes comprise n extra anti-chiral Weyl spinors, denoted collectively as $\overline{\mu}_{\bar{i}}^{\dot{\alpha}}$, $\bar{i} = 1, \ldots, n$, which couple to anti-chiral spacetime (physical) fields $\overline{\Phi}^{\bar{i}}$ through an interaction in the instanton worldvolume

$$S_{\mathcal{E}}^{(\mathrm{z.m})} \sim (\overline{\mu}_{\bar{i}})^{\dot{\alpha}}\, \overline{\mathcal{D}}_{\dot{\alpha}} \overline{\Phi}^{\bar{i}} . \tag{3.29}$$

The type of fields $\overline{\Phi}^{\bar{i}}$ that enter the effective action will depend on the type of zero-modes $\overline{\mu}_{\bar{i}}^{\dot{\alpha}}$ involved in the interaction (3.29). They will be Kähler moduli if the zero-modes ore of the kind $(c,\, \overline{\chi}^{\dot{\alpha}})$ that parametrize deformations of the instanton cycle (and are counted by $b_1(\Pi_{\mathcal{E}})_+$); while they will correspond to complex structure moduli if the extra zero-modes are of the universal type $\overline{\tau}^{\dot{\alpha}}$ that appear in $U(1)$ instantons.

3.4.6 Non-perturbative D-terms

Let us finally comment that, although less relevant, non-perturbative contributions to D-terms can in principle be generated by non-supersymmetric instantons. These have at least eight goldstino zero modes from the broken supersymmetries $(\theta^\alpha,\, \overline{\theta}^{\dot{\alpha}})$ and $(\tau^\alpha,\, \overline{\tau}^{\dot{\alpha}})$. If some mechanism is found to saturate the extra τ fields, and in the absence of other zero-modes, the instanton has the right structure to generate a D-term. Explicit computations are usually complicated and in general poorly understood, so we omit their discussion here.

3.4.7 Multiple instantons effects

We have focused so far in configurations involving single instantons, whose properties are relatively well understood. In a given compactification, however, one would have to take into account the contributions from arbitrary configurations, in which several instantons may appear simultaneously on different internal cycles, possibly with multiple wrapping and intersections with each other. Such situations are, of course, much more difficult to handle.

At first sight, a multi-instanton configuration has too many fermionic zero-modes to produce any non-zero contribution to the effective action. These come from the zero-modes that the individual instantonic branes have by themselves, and from zero-modes living in their intersections. As we describe next, there are at least two situations in which these zero-modes can be saturated against each other to yield a non-trivial contribution to F-terms.

Walls of BPS stability

In type II compactifications (or orientifolds thereof) on CY threefolds, the spectrum of BPS D-branes can jump across (or at) real codimension one walls in moduli space. For the case of instantonic branes, what on one side of the wall appears as a single instanton may become on the other side (or at the wall) a multi-instanton configuration. It is convenient to distinguish, following [84, 85], the concepts of threshold stability and of genuine marginal stability.

- **Wall of marginal stability**

  At a wall of marginal stability, a BPS D-brane on one side of the wall splits into two D-branes at the wall, which misalign their BPS phases on the other side of the wall, thus making the overall object non-BPS.

  In the case of D-brane instantons in type IIA models, the real parameter ξ parametrizing the direction transverse to the wall is a complex structure modulus, and couples as a Fayet-Illiopoulos (FI) term to the D-brane worldvolume theory. This allows for a microscopic description of the D-brane system at the wall, where the BPS D-brane is split into components [86]. For simplicity, consider a 4d $\mathcal{N} = 2$ compactification, so that the worldvolume theory on the BPS D-brane system is the dimensional reduction to zero dimensions of a 4d theory with four supercharges. For 4d $\mathcal{N} = 1$ type II orientifolds, suitable orientifold projections of the forthcoming worldvolume descriptions would lead to similar wall structures.

  Consider for concreteness the split of a single instanton into two components. A typical wall of marginal stability is described by a worldvolume theory with gauge group $U(1)$ (the relative $U(1)$ of the two branes) and a charged chiral multiplet ϕ, with charge normalized to $+1$, with no superpotential, and with D-term potential

$$V_D^{\mathcal{E}} = (\,|\phi|^2 + \xi(U)\,)^2 \,. \tag{3.30}$$

  For $\xi(U) = 0$, we have $\phi = 0$, and the relative $U(1)$ and supersymmetry are unbroken, corresponding to a BPS system of two D-branes. For $\xi(U) < 0$, a vev for ϕ restores supersymmetry but breaks the $U(1)$, corresponding to a single bound BPS D-brane. For $\xi(U) > 0$, there is a non-supersymmetric minimum at $\phi = 0$, with unbroken $U(1)$, corresponding to a non-BPS system of two D-branes. There is a manifest discontinuity in the spectrum of BPS D-branes in the system. Similar walls exist with additional number of chiral multiplets, as long as all carry equal sign charges. Hence a condition to have marginal stability walls is that sub-objects have non-zero intersection number, which counts the (net) multiplicity of such fields.

- **Wall of threshold stability**

  At a wall of threshold stability, the BPS D-brane splits at the wall, but the decay products recombine back into a BPS state at the other side of the wall.

A typical wall of threshold stability is described by a theory with gauge group $U(1)$ and two chiral multiplets ϕ_1, ϕ_2 with opposite charges ± 1. Hence the D-term potential on the worldvolume reads

$$V_D^{\mathcal{E}} = (\,|\phi_1|^2 - |\phi_2|^2 + \xi(U)\,)^2 \tag{3.31}$$

There may potentially exist a superpotential or not, depending on the model, but we need not consider either possibility at the moment. For $\xi(U) = 0$ we have a BPS system of two D-branes, while for $\xi(U) \neq 0$ the D-brane forms a single BPS bound state, regardless of the sign of $\xi(U)$. The spectrum of BPS D-branes is potentially changed only at the wall (but remains unchanged away from it). Hence typically walls of threshold stability arise when the sub-objects have zero intersection number, leading to zero net chirality for the modes of open strings stretched between them.

Consider the computation of non-perturbative effects from D-brane instantons in a string compactification. The statement that non-perturbative F-terms arise from BPS D-brane instantons may suggest that they are discontinuous at real codimension one walls in moduli space, in plain contradiction with supersymmetry of the 4d effective action, which requires nice holomorphic dependence on the moduli.

This puzzle was raised in [44] and addressed in several examples [44, 45] for 4d $\mathcal{N} = 1$ theories (see also [87]). The main results for D-brane instantons that contribute to the non-perturbative superpotential are

- BPS D-brane instantons that contribute to the superpotential must have exactly two fermion zero modes. Thus they cannot become non-BPS, since they do not have enough fermion zero modes for the four required Goldstinos. Hence such instantons cannot cross walls of genuine marginal stability.

- BPS D-brane instantons that contribute to the superpotential can cross walls of threshold stability. The decay products at the wall conspire to produce a superpotential contribution in a multi-instanton process, restoring holomorphic dependence of the superpotential on the moduli (and rendering it essentially independent of $\xi(U)$).

BPS D-brane instantons not contributing to the superpotential, but to higher F-terms, can cross walls of marginal stability and become non-BPS. The non-perturbative amplitude is in a non-trivial class of the Beasley-Witten cohomology [72, 73], so that locally they can be written as D-terms, but not globally due to an obstruction localized on the BPS locus.

A prominent example of higher F-term correction in the $\mathcal{N} = 1$ setup is provided by corrections to moduli space metrics in $\mathcal{N} = 2$ theories. A beautiful systematic understanding of marginal wall crossing for BPS particles in 4d quantum field theories was provided in [88] in terms of continuity of non-perturbative effects of BPS instantons in their 3d compactification. The result provides a physical interpretation of the wall crossing formula in [102]. A generalization for string compactifications is expected to hold, despite technical difficulties in making it completely precise.

Poly-instantons

A different kind of multi-instanton effect was introduced in [46]. In (3.21) we considered the different terms that contribute to the worldvolume action of an instanton $S_{\mathcal{E}}$, and split them into a classical term $S_{\mathcal{E}}^{(\text{clas})}$, a one-loop contribution involving massive modes $S_{\mathcal{E}}^{(\text{n.z})}$, and an interaction term for the zero-modes $S_{\mathcal{E}}^{(\text{z.m})}$. We have implicitly only considered perturbative contributions to these terms so far. However, it is intuitively clear that the worldvolume action of an instanton, just like that of a gauge D-brane, should also receive non-perturbative corrections. In fact, it was shown in [46] that the perturbative instanton action $S_{\mathcal{E}}$ coincides term by term with the gauge kinetic function of a putative D6-brane that would wrap the same cycle as $\mathcal{E}$. As we know, this gauge kinetic functions receives non-perturbative corrections, and there is in principle no reason why we should not include them in the instanton action too.

Let us consider the effect of a two instanton process, in which we interpret instanton $\mathcal{E}_2$ as correcting the worldvolume action of instanton $\mathcal{E}_1$. The contriburion to the effective 4d physical action would then be a generalization of (3.20):

$$S_{4d}^{\mathcal{E}} = \int d\mathcal{M}_1 \, e^{-S_{\mathcal{E}_1}^{\text{pert}} + \int d\mathcal{M}_2 \, e^{-S_{\mathcal{E}_2}}} , \tag{3.32}$$

where $\mathcal{M}$ includes all (zero and non-zero) modes of the corresponding instanton, and $S_{\mathcal{E}_1}^{\text{pert}}$ includes only perturbative terms. The integration over the zero modes of $\mathcal{E}_2$ includes an integration over the relative positions of the two instantons in the four non-compact directions. Thus, poly-instanton effects are not restricted to coincident instantons, as opposed to multi-instantons that come from decay products of a stability wall.

Of course, the action of the second instanton would itself be corrected by other instantons, including $\mathcal{E}_1$. The complete contribution would be a power-tower of instantons, with terms of the schematic form

$$\Delta S_{4d} \sim e^{-S_1 + e^{-S_2 + e^{-S_3 + \dots}}} . \tag{3.33}$$

It is also illustrative to expand one of the exponentials in (3.32) to obtain

$$\Delta S_{4d} \sim e^{-S_1 + e^{-S_2}} = \sum_{n=0}^{\infty} \frac{1}{n!} \, e^{-S_1} (e^{-S_2})^n \tag{3.34}$$

Microscopically, the n^{th} term corresponds to a poly-instanton process with one $\mathcal{E}_1$ instanton and n independent $\mathcal{E}_2$ instantons.

The existence of poly-instanton effects is puzzling in the following way. Consider the computation of worldsheet instanton effects in a $SO(32)$ heterotic compactification. As was mentioned before, worldsheet instantons are better defined than D-brane instantons. The corrections can be seen to include a usual sum over single instantons multiply covering curves of the internal manifold. The model has a proposed S-dual Type I (an orientifolded version of type IIB theory) description, for which the poly-instanton calculus can be applied. The duality maps the heterotic sum over worldsheet instanton, to contributions from single E1-instantons on the Type I side. Now, E1 contributions are expected also to arise from poly-instanton configurations, just like the E2 instantons in type IIA theory we have described, but these have no analogues in the heterotic side.

This clash has led to claims that poly-instanton effects violate the duality between heterotic and type I theory, or that heterotic theory may be missing some ingredients that include poly-instantons. In section 4.3 we propose a solution of this puzzle and reconcile both theories from an alternative computation of non-perturbative effects.

Notice, moreover, that poly-instanton effects have been recently applied in phenomenological models in the context of moduli stabilization, as in [90, 91]. Therefore the proper understanding of these configurations is important beyond the formal level.

Chapter 4

D-brane instanton effects from D-brane particle loops

We begin now the presentation of the original results of this thesis. In this chapter we study a computational tool for non-perturbative effects which does not rely on the instanton calculus outlined in section 3.4. The method provides a powerful mechanism to compute and resum contributions from different instanton configurations in a simple manner. It is based on the well known duality between instantons in d dimensions and solitons in $d-1$ which arises upon compactification on a circle. Being obtained from configurations that contain solitonic particles rather than instantons (which are comparatively less understood), our results have clear and in many cases illuminating interpretations, which are obscure in computations based on instanton calculus. The situations in which our perspective is most advantageous always involve multiple instantons, in particular decay products of BPS wall-crossing and poly-instantons, that were both introduced in 3.4.7.

We first present in section 4.1 a general qualitative description of the method. We then analyze its explicit application to two particular compactifications of type IIB strings, namely, CY compactifications to 4d in section 4.2, and orientifold compactifications to 8d in section 4.3. The setups were studied in [47] and [48], respectively.

4.1 The c-map and non perturbative effects

In section 3.4 we discussed in some detail the physics of D-brane instantons in type IIA theories. We focused there on non-perturbative contributions from individual configurations involving single instantons, except in 3.4.7. In that section, we saw that multi-instanton effects have a crucial importance in the understanding of formal properties of non-perturbative effects and dualities, but are usually quite involved and hard to handle.

We would very much like to have a tool which permits the computation of complete non-perturbative terms of the effective action, or at least, that resums the contribution of large sets of instantonic configurations at once. This complete contributions should contain in many cases terms that arise from multi-instantons, either decay products of BPS stability walls or poly-instantons.

In this section we develop such a tool from ideas that were originally presented in [99,100]. Our setups will be based in this chapter in type IIB theories, although we will invoke often T-duality and hence type IIA, together with the lift of the latter to M-theory. When we need them for the IIB theory, we will present results dual to those described in chapters 2 and 3 for type IIA. Furthermore, we will focus on compactifications that preserve at least eight supercharges, such as CY compactifications to 4d which lead to $\mathcal{N} = 2$ theories.

Consider the computation of an Ep-instanton effect in a compactification of type IIB string theory to d dimensions. That is, we study the theory on a background given by $\mathbb{R}^d \times \mathbf{X}^{10-d}$, with $\mathbf{X}$ a small compact space.[1] Assume that we want to compute the contributions from the instanton $\mathcal{E}$ to a term $\mathcal{O}(\phi^i)$ in the action that involve the product of n elementary fields ϕ^i. As discussed previously, such a term could be inferred from a physical amplitude involving the fields as external legs, computed in the background of the instanton.

Let us instead compactify one of the non-compact dimensions on a circle $\mathbf{S}^1$ to $(d-1)$ dimensions. An Ep-instanton in d dimensions would still be a D-brane instanton of the $(d-1)$ dimensional theory. If we perform T-duality on the circle $\mathbf{S}^1 \to \tilde{\mathbf{S}}^1$ we obtain type IIA theory compactified on $\mathbb{R}^{d-1} \times \tilde{\mathbf{S}}^1 \times \mathbf{X}^{10-d}$. On this side of the duality, the instanton is an E$(p+1)$-brane that wraps $[\Pi_{\mathcal{E}}] \times \tilde{\mathbf{S}}^1$, and can contribute to the dual amplitude $\mathcal{O}(\tilde{\phi}^i)$. From the $(d-1)$-perspective this still looks like an instanton, but we could also consider the d dimensional point of view of type IIA in which our E-brane would look like a D-brane particle whose worldline runs along $\tilde{\mathbf{S}}^1$. The chain of compactifications and T-duality is known as the c-map, and is represented schematically by

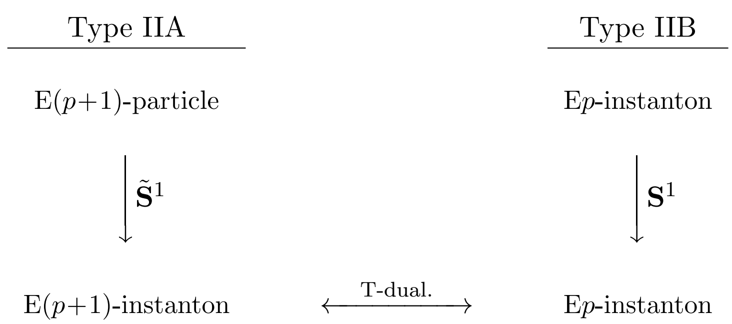


Figure 4.1: The c-map.

From the type IIA d dimensional perspective, the computation of interest would be no longer an instanton one, but rather a process of integrating out the massive solitonic E-brane field. This type of computation is of course much simpler and well-understood than the one involving instantons. If no subtleties arise in the steps connecting type IIA in d dimensions to some other point of the chain of dualities in figure 4.1, the results can be directly interpreted there as non-perturbative instanton contributions. Notice furthermore, that the momentum along the circle on the particle interpretation is mapped upon T-duality to the instanton number, i.e. the wrapping number of the instanton. Hence a simple summation over KK

[1]We consider the background to have euclidean signature, as in any standard computation of instanton effects.

states yields the resummation of multiple contributions on the dual side. This is one of the advantages of the method.

Generalities of one-loop amplitudes

We have reached the conclusion that if we want to compute E-brane instanton effects on a type II theory in d or $d-1$ dimensions, we can equivalently integrate out massive E-brane particles on a dual theory on $\mathbb{R}^{d-1} \times \mathbf{S}^1$, with the massive particles running along the circle.

The explicit calculation can be performed in a worldline formulation of quantum field theory, à la Schwinger, see e.g. [89]. If we want to compute the contribution of an E-brane particle with mass μ to an amplitude $\mathcal{A}_\phi$ involving k elementary fields ϕ^i, we need to evaluate the one-loop integral

$$\mathcal{A}_\phi = \int_0^\infty \frac{dt}{t} \sum_\ell \int d^{d-1}\mathbf{p}\; e^{-t\left(\mathbf{p}^2 + \frac{(\ell-c)^2}{R^2} + \mu^2\right)} \left[\prod_{r=1}^k \int_0^t d\tau_r V_{\phi^i}(\tau_r)\right] . \tag{4.1}$$

Here t parametrizes the coordinate on the worldline of the particle, and $\mathbf{p}$ and ℓ are its momenta along the non-compact d–1 and compact $\mathbf{S}^1$ directions, respectively. R is the radius of the compactification circle, and c represents the Wilson lines along it to which the E-brane particle couples. The last factor in brackets contains the vertex operator corresponding to the external lines for the fields ϕ^i. The integration is over the possible position of each insertion along the worldline.

In this chapter we will be concerned with amplitudes in which all external fields are equal. Correspondingly, there will be a symmetrization coefficient $1/k!$ in (4.1). For BPS protected amplitudes, the vertex operator part can simply be replaced by a factor of t^k and insertions encoding the coupling to the external fields whose precise form we do not need for the moment. Omitting these insertions, the amplitude takes the form

$$\mathcal{A}_{\phi^i} = \frac{1}{k!\,\pi^{(d-1)/2}} \sum_\ell \int d^{d-1}\mathbf{p} \int_0^\infty \frac{dt}{t}\; t^k\; e^{-t\left(\mathbf{p}^2 + \frac{(\ell_9-c)^2}{R^2} + \mu^2\right)} . \tag{4.2}$$

Integrating over continuous momenta and performing a Poisson resummation over the discrete one, we get an equivalent expression in terms of windings w

$$\mathcal{A}^{(d/2-k)}_{\phi^i} = \frac{\sqrt{\pi}\,R}{k!} \sum_w e^{-2\pi i w c} \int \frac{dt}{t}\; t^{k-d/2}\; e^{-\frac{\pi^2 R^2 w^2}{t} - t\mu^2} , \tag{4.3}$$

where we have indicated on $\mathcal{A}$ that the amplitude depends on the number $(d/2-k)$. For non-zero winding $w \neq 0$, we can use the integral representation of modified Bessel functions

$$\int_0^\infty \frac{dt}{t} t^{-s} e^{-\frac{A}{t} - tB} = 2\left|\frac{B}{A}\right|^{s/2} K_s\left(2\sqrt{|AB|}\right) \tag{4.4}$$

to rewrite the amplitude as

$$\mathcal{A}^{(d/2-k)}_{\phi^i} = \frac{2\sqrt{\pi}\,R}{k!} \sum_{w\neq 0} e^{-2\pi i w c} \left(\frac{\mu}{\pi R|w|}\right)^{d/2-k} K_{d/2-k}\left(2\mu\pi R|w|\right) . \tag{4.5}$$

It is interesting to point out that using the differentiation formula for Bessels functions one can write

$$\mathcal{A}_{d,k}(\mu) \;=\; \left(-\frac{1}{\mu}\frac{d}{d\mu}\right)^k \mathcal{A}_{d,k=0}(\mu) \tag{4.6}$$

namely the k-leg amplitude can be obtained as the k^{th} derivative of the vacuum amplitude. In section 4.3.2 the amplitudes with external legs will indeed be generated from a Schwinger-like vacuum amplitude in a general background, by differentiation with respect to background vector multiplets, on which the BPS masses depend.

The lift to M-theory

It is often useful to perform the above loop integrals not in the weak-coupling limit of the Type IIA theory compactified on a circle, but rather in its lift to M-theory compactified on a two-torus $\mathbf{T^2}$, with volume V_2 and complex structure $\tau = \tau_1 + i\tau_2$.[2] The generalizations of eqs. (4.1)-(4.5) to this setup are quite straightforward: one must only take into account that the BPS particles have KK momenta $\{\ell_I\}_{I=1,2}$ along both directions of the $\mathbf{T^2}$. The amplitude reads

$$\mathcal{A}_{\phi^i} = \frac{1}{k!\,\pi^{(d-1)/2}} \int d^{d-1}\mathbf{p} \int_0^\infty \frac{dt}{t}\, t^k \sum_{\{\ell_I\}} e^{-t\left(\mathbf{p}^2 + G^{IJ}(\ell_I+c_I)\,(\ell_J+c_J)+\mu^2\right)} . \tag{4.7}$$

where G^{IJ} is the inverse metric on the torus: $G^{IJ}\ell_I\ell_J = (V_2\tau_2)^{-1}|\tau\ell_1 - \ell_2|^2$. Integrating over the non-compact momenta $\mathbf{p}$ and performing a Poisson resummation over the compact ones $\{\ell_I\}$ we obtain an expression analogous to (4.3):

$$\mathcal{A}_{\phi^i} = \frac{\pi V_2}{k!} \sum_{\{w^I\}} e^{2\,\pi\, i\, w^I\, c_I} \int_0^\infty \frac{dt}{t}\, t^{k-(d+1)/2}\, e^{-\pi^2\, G_{IJ}\, w^I\, w^J/t - t\,\mu^2} . \tag{4.8}$$

The integral can be explicitly evaluated in terms of Bessel functions with a result

$$\mathcal{A}_{\phi^i} = \frac{2\pi V_2}{k!} \sum_{\{w^I\}'} e^{2\,\pi\, i\, w^I\, c_I} \left|\frac{\mu^2}{\pi^2 G_{IJ} w^I w^J}\right|^{\frac{d-2k+1}{4}} K_{\frac{d-2k+1}{2}}\left(2\mu\pi\sqrt{G_{IJ}w^I w^J}\right) , \tag{4.9}$$

where the sum runs only over values with $(w^1, w^2) \neq (0,0)$. Notice that the M-theory picture has a manifest $SL(2,\mathbb{Z})$ invariance with respect to the $\mathbf{T^2}$. This will translate to the $SL(2,\mathbb{Z})$ invariance of the non-perturbative effects on the IIB side.

All the formulas presented so far are the contributions of a single BPS particle of mass μ. To obtain the full amplitude, we should resum the contribution from all such particles. As we see, the 'only' information we need to compute the instanton contributions is the spectrum of BPS particles with the masses and quantum numbers (or their couplings to the external fields ϕ). We can obtain this information in several particular setups, and the above simple formulas yield the desired non-perturbative contributions to the spacetime amplitudes and couplings.

[2] Henceforth we define fields in such a way that the imaginary part corresponds to geometric moduli, while the real part corresponds (for Kähler moduli) to Wilson line moduli. The convention is opposite to that in chapters 2 and 3.

4.2 D1/D(-1) corrections to the hypermultiplet moduli space

In this section we would like to apply the c-map to the computation of non-perturbative corrections to the hypermultiplet moduli space of type II compactifications. Starting in IIA (resp. IIB) string theory compactified on a CY threefold X, one can compute the quantum corrections to the vector-multiplet moduli space $\mathcal{V}_A$ (resp. $\mathcal{V}_B$) by computing one-loop diagrams of BPS particles arising from D-branes on holomorphic (resp. special lagrangian) cycles. Upon $\mathbf{S^1}$ compactification to 3d, diagrams involving D-brane particles running along the $\mathbf{S^1}$ can be regarded as instantons correcting the 3d vector multiplets. These can actually be dualized to 3d hypermultiplets parametrizing a quaternionic Kahler space $\hat{\mathcal{V}}_A$ (resp. $\hat{\mathcal{V}}_B$), fibered over the 4d vector moduli space with fibers given by the electric and magnetic Wilson lines along $\mathbf{S^1}$. Upon T-duality and decompatification, $\hat{\mathcal{V}}_A$ (resp. $\hat{\mathcal{V}}_B$) become the 4d hypermultiplet moduli space of the T-dual IIB theory (resp. IIA). The D-brane particles map to 4d instantons from D-branes on holomorphic (resp. special lagrangian) cycles.

In this section we apply the c-map to relate loops of D2/D0-brane particles to D1/D(-1)-brane instanton corrections to hypermultiplet moduli space of type IIB compactifications. Higher derivative corrections to hypermultiplets are not well understood in general (see [114] for a partial analysis of higher derivative corrections), so most analysis focus on the computation of hypermultiplet moduli space metrics [101, 105]. We will also focus in this case in the explicit computation in this section.

4.2.1 The one-loop computations

The c-map was first applied to the case of the conifold in [99] and [100]. Their method can be easily generalized, and applied to the computation of type IIA or type IIB D-brane instantons in mutually local sectors. In this section we elaborate on the computation of type IIB D1/D(-1)-brane instanton effects by computing a one-loop diagram of D2/D0-brane particle states in type IIA compactified to 3d. The relevant formulas were presented in section 4.1.

Actually, this computation can be lifted to M-theory. Namely, we consider M-theory compactified on $\mathbf{X}_6$, and subsequently compactified to 3d on $\mathbf{T^2}$. The amplitude of interest is a one loop diagram of 5d BPS states, with momentum along the $\mathbf{T^2}$. Such BPS particles are supersymmetric gravitons, and wrapped M2-branes, with multiplicities counted by the Gopakumar-Vafa (GV) invariants.[3] Recalling the general results on instanton effects of section 3.2.3, we see that in order to contribute to the moduli space metric, the cycles wrapped by the branes must be rigid, i.e. must have genus zero. Hence corrections to the hypermultiplet moduli space from wrapped M2-branes are related to the genus zero GV invariants.

The type IIB non-perturbative brane instanton effects are recovered by taking the limit of zero area of the $\mathbf{T^2}$, keeping its complex structure τ constant (which becomes the complex string coupling in the IIB side). The M-theory computation reproduces a sum over (p, q) string instanton effects, or equivalently over general D1/D(-1)-brane instantons, including D1-branes with non-trivial worldvolume fluxes. Rather than taking the intermediate step of the type IIA limit (which would correspond to a particular limit of weak coupling $\tau \to i\infty$) we work at general τ and recover the full result.

[3] GV invariants and their relation to topological strings are discussed in section 4.2.2. For the moment we can think of them as quantities that parametrize the multiplicities of BPS M2-brane particles.

General expressions for these non-perturbative effects in 4d IIB models on $\mathbf{X}_6$ have been obtained in [107, 108], by starting with worldsheet instantons and imposing $SL(2,\mathbb{Z})$ invariance. The M-theory perspective provides a simple derivation of the result in a manifestly $SL(2,\mathbb{Z})$ invariant formulation; it also explains the observation that the result depends on $\mathbf{X}_6$ only through its Euler characteristic and the genus 0 GV invariants.

The graviton piece

Let us first describe the contribution arising from 5d gravitons. These are 11d gravitons, truncated to their zero modes in $\mathbf{X}_6$, and running with arbitrary momenta in the $\mathbf{T^2}$ (and the non-compact 3d). The computation can be regarded as a truncation (to the zero mode sector in $\mathbf{X}_6$) of the computation in [118] of D(-1)-brane instanton contributions to a certain R^4 term for IIB on $\mathbf{S^1}$ from graviton one-loop diagram in M-theory on $\mathbf{T^2}$. Indeed, the reduction of R^4 terms on a CY $\mathbf{X}_6$ provide corrections to the hypermultiplet moduli space metric, see e.g. [106]. Morally, the R^4 term with three curvatures insertions along $\mathbf{X}_6$ (producing a numerical factor given by the Euler characteristic) leads to a correction to the 3d Einstein term, which can be recast as a correction to the hypermultiplet metric.

The 5d graviton BPS states are obtained by considering the quantum ground states of gravitons in $\mathbf{X}_6$. Accounting for boson-fermion cancellations, their index is given by the Euler characteristic of $\mathbf{X}_6$, $\chi_E(\mathbf{X}_6) = 2\,(h^{1,1} - h^{2,1})$. The correction to the hypermultiplet moduli space metric is obtained from a 1-loop diagram of these states, with one insertion of the curvature tensor. We can borrow the results of section 4.1 setting $k = 1$ and $d = 4$. The one-loop graviton amplitude (4.7) with one graviton insertion reads

$$\mathcal{A}_g \;=\; -\frac{\chi_E(\mathbf{X}_6)}{\pi^{3/2}} \int d^3p \int_0^\infty dt \,\sum_{\{\ell_I\}} e^{-t\,(\mathbf{p}^2 + G^{IJ}\,\ell_I\,\ell_J)}\,, \tag{4.10}$$

where G^{IJ} denotes the inverse metric on the $\mathbf{T^2}$. Integrating over 3d momenta $\mathbf{p}$, and performing a Poisson resummation on the discrete momenta ℓ_I, we have

$$\mathcal{A}_g = -\chi_E(\mathbf{X}_6)\,\pi\,V_2 \sum_{\{w^I\}} \int_0^\infty ds\, s^{1/2}\, e^{-s\,\pi^2\,G_{IJ}\,w^I\,w^J}\,, \tag{4.11}$$

where V_2 is the $\mathbf{T^2}$ area. For a $\mathbf{T^2}$, $G_{IJ}\,w^I\,w^J = |w_1 + \tau\,w_2|^2\,V_2/\tau_2$ with $\tau = \tau_1 + i\tau_2$ the $\mathbf{T^2}$ complex structure. Leaving out the $\vec{w} = \vec{0}$ term and performing the integral over s, we obtain

$$\mathcal{A}_g = -\frac{\chi_E(\mathbf{X}_6)}{2\,\pi^{3/2}\sqrt{V_2}} \sum_{\{w^I\}'} \frac{\tau_2^{3/2}}{|\,w_1 + \tau\,w_2\,|^3} \;\equiv\; -\frac{\chi_E(\mathbf{X}_6)}{2\,\pi^{3/2}V_2^{1/2}}\,E_{1/2}(\tau)\,. \tag{4.12}$$

where the sum runs over values $(w_1, w_2) \neq (0,0)$, and $E_n(x)$ is the non-holomorphic Eisenstein series of order n.

The M2-brane piece

We now compute the one-loop diagram with one external graviton leg and with the particle in the loop given by an M2-brane wrapped on a genus zero holomorphic 2-cycle $C \in H_2(\mathbf{X}_6,\mathbb{Z})$

of $\mathbf{X}_6$. The effective particle has a five-dimensional mass $m_C = \mathrm{Vol}(C) = \int_C J = t_C \sqrt{V_2/\tau_2}$, where J is the Kähler form,[4] and its worldline action has a coupling to a gauge field $A_\mu = \int_C C^{(3)}$. The one-loop amplitude is thus given by (4.9) that reads in this case

$$\mathcal{A}_{\mathrm{M2}} = \frac{1}{2\,\pi^{3/2}\sqrt{V_2}} \sum_{(w_1,w_2)\neq(0,0)} \frac{\tau_2^{3/2}}{|w_1+\tau w_2|^3} \left(1+2\,\pi\,|w_1+\tau w_2|\,t_C\right) e^{-2\,\pi\,(|w_1+\tau w_2|\,t_C \,-\, i\,w^I\,A_I)}\,. \tag{4.13}$$

The calculation performed above was for a membrane wrapped on a fixed, rational (genus zero) holomorphic curve in a class $[C] \in H_2(\mathbf{X}_6,\mathbb{Z})$. The full amplitude is a sum over all possible curve classes. Let $[\gamma_a]$ denote a basis of $H_2(\mathbf{X}_6,\mathbb{Z})$, and expand any curve $[C] = k_a[\gamma_a]$. The number of BPS M2-branes wrapped on a genus zero representative of the class $[C]$ is given by the genus zero GV invariant $n^{(0)}_{\mathbf{k}}$. The total correction to the hypermultiplet moduli space metric from such M2-branes is proportional to

$$\mathcal{A}_{\mathrm{M2\ total}} = \frac{1}{2\,\pi^{3/2}\sqrt{V_2}} \sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+\tau n|^{3/2}} \left(1+2\,\pi\,|m+\tau n|\,k_a\,t^a\right) e^{-S_{m,n}} \tag{4.14}$$

where we have introduced

$$S_{m,n} \;=\; 2\,\pi\,(\,|m+\tau n|\,k_a\,t^a \;-\; i\,m\,b^a \;-\; i\,n\,c^a\,)\,, \tag{4.15}$$

the action of an euclidean M2-brane particle wrapped on the (m,n) 1-cycle on $\mathbf{T^2}$. For later convenience, we have named our Wilson lines as follows:

$$\int_{[\gamma_a]\times S^1_{i=1,2}} C^{(3)} = (b^a, c^a)\,, \tag{4.16}$$

where the two components represent Wilson lines along the first and the second circle of the $\mathbf{T^2}$, respectively.

By shrinking one of the torus 1-cycles one can recover an interpretation in type IIA on $\mathbf{X}_6 \times \mathbf{S^1}$. From this perspective, the above amplitude contains in particular the contributions from worldsheet instantons (M2-branes wrapping the M-theory circle) and from D2/D0-brane bound states (from M2-branes wrapped on the M-theory circle, and carrying momentum along it). In fact, by taking such a IIA limit, we can bring the formula (4.14) to a form that is reminiscent of the Gopakumar-Vafa formula in [93]. By taking, for instance, $\tau_2 \to \infty$, and dropping the sum over n, we see that the second term in the expansion becomes

$$\frac{1}{2} \sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(n)\neq 0} \frac{1}{n^2}\, e^{-2\,\pi\,(\,k_a\,t^a \,-\, i\,c^a\,)\,|n|} \tag{4.17}$$

$$= \sum_{\mathbf{k}} n^{(0)}_{k_a}\, \mathrm{Li}_2(e^{-2\,\pi\,(\,k_a\,t^a \,-\, i\,c^a\,)})\,, \tag{4.18}$$

where we have introduced the *dilogarithm* $\mathrm{Li}_2(x) = \sum_{n=1}^\infty x^n/n^2$. The appearance of such dilogarithms is reminiscent of the structure in [102]. In section 4.2.4 we will consider the role of dilogarithms in this connection.

[4] We have normalized here the Kähler parameters t^a such that $\int_{[\gamma_a]} J = t^a\,\sqrt{V_2/\tau_2}$.

The type IIB instanton interpretation

The above results can be related to type IIB 4d brane instantons by shrinking the $\mathbf{T^2}$ keeping the complex structure τ fixed. This process maps the manifest geometric $SL(2,\mathbb{Z})$ invariance of M-theory to the S-duality group of IIB. In order to do so, it is convenient to notice that the above structures clearly reproduce the type IIB result in [107,108], which we review here for completeness. In the off-shell $\mathcal{N}=2$ formalism of these references, the hypermultiplet metric is encoded in a single function, the tensor potential, which contains a classical piece $\chi_{\rm cl.}$, plus contributions related to D(-1)-brane and (p,q) 1-brane instantons

$$\chi = \chi_{\rm cl.} + \chi_{\rm D(-1)} + \chi_{\rm 1-brane} \tag{4.19}$$

with

$$\begin{aligned}
\chi_{\rm cl} &= 4\, r^0\, \tau_2^2\, \frac{1}{3!}\, \kappa_{abc}\, t^a\, t^b\, t^c\ , \\
\chi_{\rm D(-1)} &= \frac{r^0 \tau_2^{1/2}}{2(2\pi)^3}\, \chi_E(\mathbf{X}_6) \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m\tau+n|^3}\ , \\
\chi_{\rm 1-brane} &= -\frac{r^0 \tau_2^{1/2}}{(2\pi)^3} \sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m\tau+n|^3} \left(1+2\pi|m\tau+n|\, k_a t^a\right) e^{-S_{m,n}}\ .
\end{aligned} \tag{4.20}$$

Here κ_{abc} are the classical triple intersection form on the CY, and r^0 is a scalar in the tensor multiplet to render the tensor potential of appropriate degree, see [107,108] for notation and further details. The $n^{(0)}_{\mathbf{k}}$ denote the genus zero GV invariants. Also,

$$S_{m,n} = 2\pi k_a \left(|m\tau+n|\, t^a - \ i m\, c^a - \ i n\, b^a \right)\ , \tag{4.21}$$

is the tension of a type IIB (p,q) string wrapped $s=\gcd(m,n)$ times around a holomorphic cycle, with $(p,q)=(m,n)/s$, and agrees with the action (4.15) of an M2-brane wrapped on the 2-cycle on $\mathbf{X}_6$ times a 1-cycle on $\mathbf{T^2}$. The axion scalars b^a, c^a are integrals of the IIB NSNS and RR 2-forms over the 2-cycle γ_a correspond to the 3-form integrals in the M-theory picture.

As mentioned, the M-theory derivation of these known results explains naturally their structure, in particular the $SL(2,\mathbb{Z})$ invariance, and the apperance of the topological invariants $\chi_E(\mathbf{X}_6)$ and $n^{(0)}_{\mathbf{k}}$.

4.2.2 Relation to topological strings and BPS wall crossing

The method of computation of instanton effects through the c-map presented here is not only a powerful computational tool, but provides also a deep insight into non-perturbative aspects of string theory. We will argue in the following, that it naturally establishes a connection between instanton effects and topological strings, and hence provides an elegant explanation for the continuity of such effects across lines of BPS stability.

Topological strings are extremely interesting theories lying in the border of physics and mathematics. They could be regarded as simplified versions of string theory, simple enough as to admit deep understanding and mathematical manageability, yet rich enough as to provide a lot of interesting results both for string theories and pure mathematics. Unfortunately, topological strings comprise a vast subject which requires deep mathematical results that cannot be appropriately covered in this work. We refer the reader to the many reviews that have been written on the subject. Physicist readers may [29–31] specially useful. In this section we restrict to pointing out the relation of our computations to the Gopakumar-Vafa (GV) interpretation of the topological A-model.

There may seem to be an immediate problem with the idea that the topological string computes non-perturbative D-brane instanton effects, because it depends on the 'wrong' moduli. Namely, one might have guessed that e.g. the A-model could describe D-brane instantons effects from topological A-branes, but the latter actually depend on complex structure moduli, which are visible only in the B-model! Equivalently, the A-model is usually related to the vector multiplets of the physical type IIA string theory, while non-perturbative D-brane instantons correct the hypermultiplet moduli space.

The c-map provides the solution to this apparent problem. Since it includes a T-duality, the c-map swaps type IIA and IIB theories, and the roles of vector and hypermultiplets. This provides a physical interpretation of the proposed S-duality of topological strings [104], by which the A-model computes non-perturbative effects of the B-model, and viceversa. As we discuss now, the connection between topological strings and physical non-perturbative effects can be explicitly understood by relating the computation carried out in section 4.2.1 to the GV calculation of the A-model topological partition function.

The Gopakumar-Vafa interpretation

In the GV interpretation [93–95] the A-model on $\mathbf{X}_6$ relates to a one-loop diagram in M-theory on $\mathbf{X}_6 \times \mathbf{S^1}$, involving BPS particles from M2-branes wrapped on holomorphic cycles on $\mathbf{X}_6$, running with momentum along $\mathbf{S^1}$. In the type IIA picture, one recovers a Schwinger diagram involving D2/D0-branes coupled to a graviphoton field strength background, and including non-perturbative effects from D2/D0-particle pair production. Using the c-map (further circle compactification to 3d and T-duality) these determine non-perturbative effects from D1/D(-1)-brane instantons in the T-dual type IIB on $\mathbf{X}_6$.

The GV amplitude in M-theory on $\mathbf{X}_6 \times S^1$ is determined by the GV invariants $n^r_{\mathbf{k}}$, which provide the multiplicity of M2-branes wrapped on a holomorphic cycle in the class $[\gamma] = k_i[\gamma_i]$, with $SU(2)_L$ spin content $[(\frac{1}{2})+2(0)]\otimes[(\frac{1}{2})+2(0)]^r$. Here $[\gamma_i]$ are a basis of $H_2(\mathbf{X}_6,\mathbb{Z})$. Then, the topological string partition function can be written as

$$F_{\text{top.}} = \sum_{r,\mathbf{k},p>0} \frac{n^r_{\mathbf{k}}}{p} \left(2\sin\frac{p\lambda}{2}\right)^{2r-2} \exp\left[\,-2\pi\, p\, k_i t_i\right]. \tag{4.22}$$

This partition function is derived by computing the one-loop Schwinger effect mentioned before directly in M-theory, in the presence of a constant, self-dual graviphoton background F. The computation is identical in spirit to those carried out in section 4.2.1. The perturbative part of this diagram reproduces the perturbative corrections to the vector multiplets of type IIA on $\mathbf{X}_6$ computed by the topological string, which in the presence of the graviphoton background read

$$S_{R^2,4d} = \int d^4x \sum_g F_g(t_i)\,\lambda^{2g-2} R_+^2 = \int d^4x\, F_{\text{top}}(\lambda, t_i)\, R_+^2\,, \tag{4.23}$$

where the combination $g_s F = \lambda$ plays the role of topological string coupling. In addition, the GV formulation includes non-perturbative information from the D2/D0-brane particles. Through the c-map, the M-theory computation describes corrections from worldsheet and D1/D(-1)-brane instantons to the hypermultiplet moduli space of type IIB on $\mathbf{X}_6$.

In the above discussion, different values of r determine different kinds of corrections. It is interesting to keep track of the label r in this process and translate it to D-brane instanton language, both from the spacetime and brane worldvolume viewpoints. From the spacetime viewpoint, in M-theory on $\mathbf{X}_6$, the label r defines the spin content of the 5d M2-brane particle multiplet. In the spacetime Schwinger computation, a particle with spin r introduces at least r powers of the graviphoton field strength F, i.e. leads to a $(2r+2)$-derivative correction to vector multiplets. Via the c-map, it leads to a $(2r+2)$ derivative correction to hypermultiplets. From the brane worldvolume viewpoint, in M-theory on $\mathbf{X}_6$, the label r determines the number of fermion zero modes in the superparticle Quantum Mechanics (whose quantization leads to the spacetime spin content). From this viewpoint, the F^{2r} correction in the Schwinger computation, arises from the coupling of the graviphoton field strength to the worldline fermion zero modes, and saturation of the latter in the path integral. Mapping the BPS particles to D-brane instantons in the T-dual type IIB, r determines the number of fermion zero modes of the D-brane instanton (besides the four universal goldstinos, which relate to the center of mass half-hypermultiplet). From familiar D-brane instanton physics, it leads to a $(2r+2)$-derivative correction to the hypermultiplets, in agreement with the spacetime picture above.

As mentioned before, not much is known about higher-derivative F-terms for hypermultiplets, so we primarily focus on corrections to the hypermultiplet metric. These arise from the $r=0$ sector, namely D-brane instantons with four fermion zero modes, or D-brane particles in hypermultiplets. Geometrically, these correspond to M2-branes on $\mathbf{S^2}$ 2-cycles, with multiplicity counted by the genus 0 GV invariants $n_{\mathbf{k}}^{(0)}$. In the IIA picture they become D2-branes on $\mathbf{S^2}$'s with induced D0-brane charge, and in the IIB picture D1-brane instantons on $\mathbf{S^2}$'s with induced D(-1)-brane charge.

Topological strings and wall-crossing

The proposed relation between non-perturbative effects and topological strings has important consequences. Most prominently, they underlie the continuous behavior of instanton effects at walls of BPS stability, which was discussed in 3.4.7. We focus on walls of threshold stability, the ones that affect instantons with mutually local charges such as D1/D(-1), and are the only relevant ones for $\mathcal{N}=1$ superpotentials.[5].

The c-map procedure generates non-perturbative D-brane instanton corrections manifestly continuous throughout moduli space. This follows from the fact that GV BPS index is constant and has no wall crossing. This property, was already emphasized in [95], and is familiar and extensively used in the literature on BPS states.

The continuity of BPS indices counting particle degeneracies is in contrast with the analog for BPS instantons. Even when the *classical* BPS D-brane particle splits, there is a *quantum* BPS bound state at threshold which keeps the index unchanged. For D-brane *instantons*, the

[5]The above system allows nonetheless for a non-trivial discussion of marginal stability walls, as argued in an explicit example in [47].

splitting at the threshold wall implies the real disappearance of the single D-brane instanton contribution to the 4d non-perturbative effective action. Restoration of the continuity requires the existence of microscopically non-trivial multi-instanton processes.

These results can be rephrased as the statement that non-perturbative terms are insensitive to the stability conditions of the underlying BPS objects. More abstractly, they would be defined at the level of the category of holomorphic D-branes [92]. These branes are precisely the type of extended objects that enter topological string theories.

4.2.3 $\mathcal{N}=1$ superpotentials and their wall crossing

We have gained a good understanding of threshold wall crossing for D-brane instantons in 4d $\mathcal{N}=2$ theories. It would be interesting to extend this understanding to 4d $\mathcal{N}=1$ theories, where such transitions are particularly important, as the only wall crossing phenomenon for D-brane instantons contributing to the superpotential. In this section we describe mechanisms for reduction of supersymmetry, and their interplay with the T-duality between particles and instantons. The latter is shown to still underly the continuity of non-perturbative effects across stability walls.

Introduction of fluxes

A standard mechanism to reduce the amount of supersymmetry is the introduction of closed string fluxes [115–117] (already exploited for a different purpose in the context of matrix model / gauge theory duality [96–98]). In this section we compute non-perturbative superpotentials from D-brane instanton sums in 4d $\mathcal{N}=1$ flux compactifications, and their threshold wall crossing, by relating them to the underlying 4d $\mathcal{N}=2$ theories. The analysis ignores the possible presence of other ingredients breaking $\mathcal{N}=2$ supersymmetry, like orientifold planes, to be discussed later.

The effect of fluxes on D-brane instantons has been actively investigated. In particular fluxes can turn D-brane instantons with 4 fermion zero modes into D-brane instantons with 2 fermion zero modes, allowing them to contribute to the non-perturbative superpotential. This has been established from the coupling of the D-brane instanton fermion zero modes to the flux, computed using D-brane action techniques [123–126], or CFT correlators [127, 128]. Unfortunately these microscopic techniques must be applied to individual instantons, and cannot take advantage of the powerful resummation techniques of the underlying $\mathcal{N}=2$ theory. The latter is however fully exploited in the macroscopic effective field theory technique proposed in [103]. Starting with the effective 4d $\mathcal{N}=2$ theory of the flux-less compactification, including non-perturbative effects from resummed D-brane instantons, the effect of fluxes is simply described by the introduction of the flux superpotential [129]. The non- perturbative superpotential is automatically reproduced by the evaluation of Feynman diagrams involving the spacetime interactions from fluxes and instantons.

More quantitatively, the effective action in $\mathcal{N}=1$ terms can be written

$$\int d^2\theta\, d^2\overline{\theta}\, K(Z,\overline{Z}) + \int d^2\theta\, W_{\rm flux}(Z) \tag{4.24}$$

where Z denotes the $\mathcal{N}=2$ hypermultiplet moduli written in $\mathcal{N}=1$ terms, and $K(Z,\overline{Z})$ encodes the moduli space metric. The $\mathcal{N}=1$ D-term can be written as an F-term as

$$\int d^2\theta\, d^2\overline{\theta}\, K(Z,\overline{Z}) \simeq \int d^2\theta\, \frac{\partial^2 K}{\partial \overline{Z}^2}\, \overline{DZ}\, \overline{DZ} \tag{4.25}$$

where $\overline{DZ}$ is an anti-chiral multiplet whose lowest component is the 4d fermion in the Z. Integrating out the moduli, the non-perturbative superpotential in the flux compactification is [103]

$$W_{\text{non-pert.}} = \frac{\partial^2 W_{\text{flux}}}{\partial Z^2}\, \frac{\partial^2 K}{\partial \overline{Z}^2} \tag{4.26}$$

The net effect is therefore to turn the $\mathcal{N}=2$ F-term into an $\mathcal{N}=1$ superpotential term.

This can be applied to the introduction of fluxes in the 4d $\mathcal{N}=2$ compactifications of section 4.2.1. For example, consider the concrete example of type IIB on the resolved conifold, or rather the equivalent mirror picture of type IIA on the deformed conifold. The flux configuration we consider is M units of NSNS 3-form flux through the $\mathbf{S^3}$ A-cycle, and $-K$ units through the (suitably regularized) non-compact dual B-cycle, plus possibly RR fluxes along non-compact cycles to preserve $\mathcal{N}=1$ supersymmetry. Using the periods of Ω, the flux superpotential is given by

$$W_{\text{flux}} \simeq \int_X H_3 \wedge \left(\frac{1}{g_s}\,\mathrm{Re}\,\Omega + iC_3\right) = \frac{1}{2\pi i} M\, Z\, \ln Z \,-\, K\, Z \tag{4.27}$$

where $Z = |z|/g_s + ix$, with z the complex structure modulus and x the RR 3-form along $\mathbf{S^3}$. The model is mirror to the IIB conifold model in [116]. The modulus Z is stabilized at $Z_0 = \exp\left(-2\pi\, K/M\right)$.

The D2-brane instanton corrections to the moduli space metric were computed in [99] (and have been derived directly from (4.21) in [109]). The relevant component for our computation is

$$\begin{aligned} K_{\bar{Z}\bar{Z}} &= \sum_{m\neq 0} C_m(z)\, \exp\left[-2\pi\left(\frac{|mz|}{g_s} - i\, mx\right)\right] \\ \text{with} \qquad C_m(z) &= \sum_{n=0}^{\infty} \frac{\Gamma(\frac{1}{2}+n)}{2\sqrt{\pi}\, n!\, \Gamma(\frac{1}{2}-n)} \left(\frac{g_s}{4\pi\, |mz|}\right)^{n+\frac{1}{2}} \end{aligned} \tag{4.28}$$

Using (4.26), the $\mathcal{N}=1$ non-perturbative superpotential reads

$$W_{\text{n.p.}} \simeq \frac{M}{Z_0} \sum_{m\neq 0} C_m(z_0)\, e^{-2\pi\, m Z_0} \tag{4.29}$$

where $z_0 = \mathrm{Re} Z_0$. Notice that the moduli in the prefactor should be considered as fixed at their vevs at the minimum, as the above superpotential is valid at scales below the moduli stabilization scale.

Concerning our interest in threshold wall crossing, the close relation between D-brane instantons in the parent $\mathcal{N}=2$ model and its $\mathcal{N}=1$ flux descendant makes it clear that the continuity of the superpotential across threshold walls is automatically encoded in the continuity of the GV invariants in the $\mathcal{N} = 2$ theory.

Remarkably the present setup provides the full non-perturbative superpotential arising from an infinite sum over multiwrapped D2-brane instantons. To our knowledge this is the first time such contributions to the superpotential can be successfully resummed (see [130] for partial results in this direction, and [46, 131, 132] for multi-wrapped instanton contributions to other quantities, motivated by heterotic-type I duality). We hope this kind of result to have an interesting impact on model building applications of D-brane instantons.

Introduction of gauge D-branes

We have seen that multiwrapped instanton contributions are quite generic in $\mathcal{N} = 2$ and $\mathcal{N} = 1$ non-perturbative effects. This may seem to lead to a potential puzzle in a different 4d $\mathcal{N} = 1$ context, in which the reduction from $\mathcal{N} = 2$ is obtained by introducing 4d spacefilling D-branes, leading to 4d $\mathcal{N} = 1$ gauge sectors. In such models, certain D-brane instantons (wrapped on the same internal cycles as the gauge D-branes) admit the interpretation of gauge theory instantons. In many examples, the non-perturbative effects in 4d $\mathcal{N} = 1$ gauge theories are ensured, by R-symmetry and holomorphy, to arise only at the 1-instanton level. In these models, such macroscopic considerations forbid the contribution from multiwrapped instantons. In this section we take a small detour to understand microscopically the absence of multiwrapped instantons, emphasizing the key differences with previous systems where they are present.

We focus on a prototypical example where the non-perturbative effect is an ADS superpotential for $N_f = N_c - 1$ SQCD [133]. Its derivation from D-brane instanton physics has been discussed e.g. in [135], however assuming (rather than deriving) the absence of multiwrapped instanton contributions. For our purposes, it will be enough to consider the $N_c = 1$ analog of the above ADS superpotential, generated by a D-brane instanton on top of a single 4d gauge D-brane, see [134] for a detailed analysis.

For concreteness we consider type IIA on a deformed conifold geometry X, with a D6-brane wrapped on $\mathbf{S^3}$, whose non-perturbative effects can be computed from a T-dual type IIB on $X \times \mathbf{S^1}$, with a D5-brane on $\mathbf{S^3}$ and located at a point in $\mathbf{S^1}$. Namely the sum over D2-brane instantons on $\mathbf{S^3}$ maps to a one-loop diagram of D3-brane particles running in a loop with momentum on $\mathbf{S^1}$. As in previous systems, the D3-brane particle momentum maps to the T-dual D2-brane instanton number. We need to show that due to the presence of the D5-brane, the 4d $\mathcal{N} = 1$ superpotential arises only from D3-brane particles with zero momentum on $\mathbf{S^1}$. This follows from looking at the fermion zero modes on the D3-brane particle worldline, which can be obtained from the open string spectrum. In the D3-D3 open string sector there are four fermion zero modes θ^α, $\overline{\tau}_{\dot\alpha}$, and in the D3-D5 sector there are one complex bosonic mode $b_{\dot\alpha}$ and two fermionic zero modes $\beta_{\dot\alpha}$, $\overline{\beta}_{\dot\alpha}$. D3-brane particles with zero momentum on $\mathbf{S^1}$ have a quantum wavefunction which is constant on $\mathbf{S^1}$, and interact with the localized D5-brane. This is reflected by a non-vanishing worldline coupling involving the D3-D5 zero modes of the form

$$S_{\text{D3 ferm.}} = \overline{\tau}^{\dot\alpha} \, (\, b_{\dot\alpha}\overline{\beta} \, + \, \overline{b}_{\dot\alpha} \, \beta \,) \tag{4.30}$$

This lifts all fermion modes except the two fermion zero modes θ^α, and allows the D-branes to contribute to the superpotential. For D3-branes with non-zero momentum, their interaction with the localized D5-brane is weighted by the average of their wavefunctions at the D5-brane location, namely

$$\int dx_0 \, e^{ik(x-x_0)} \, = \, 0 \tag{4.31}$$

The D3-D5 couplings are absent and the D3-branes have too many fermion zero modes to contribute to the superpotential. An identical argument can be run in the more general case of configurations realizing $N_f = N_c - 1$ SQCD, with N_c D5-branes on $\mathbf{S^3}$, and N_f D5-branes on a dual 3-cycle. Hence we recover a result consistent with the expectation from gauge theory instanton physics.

Introduction of orientifold planes

We can consider reducing to 4d $\mathcal{N} = 1$ by an orientifold quotient. The main effect of such orientifolds is that they can project out certain fermion zero modes of D-brane instantons, allowing them to contribute to the superpotential. For instance for D1-brane instantons, this would correspond to wrapping them on holomorphic curves of RP_2 topology. Clearly a systematic computation of such 4d $\mathcal{N} = 1$ superpotentials would require a formulation of topological string in orientifold models (dubbed real topological string). Recent progress in this field (see e.g. [138–142]) holds the promise of such applications, although the connection of the real topological strings to the physical orientifold theories is not fully understood. However, the existence of a GV interpretation for real topological string amplitudes almost guarantees the applicability of some $\mathcal{N} = 2$ lessons to $\mathcal{N} = 1$. The particular lesson of relating D-brane instantons to D-brane particle loops by T-duality, and threshold bound states to multi-instanton processes, survives in $\mathcal{N} = 1$ orientifold theories, and explain the continuity of non-perturbative $\mathcal{N} = 1$ superpotentials through threshold walls.[6] This should be related to the continuity of unoriented GV invariants for the real topological string.

4.2.4 More general D-branes charges

The computation of non-perturbative effects from D-brane instantons with general charges is an important question, with much recent progress, see [110–113], yet with open questions. In this section we consider the extent to which topological strings, can describe effects from general D-brane instantons, beyond the D1/D(-1) sector. We suggest some interesting connections, supporting the idea that the topological string underlies the continuity of non-perturbative effects across general lines of marginal stability (and thus also the wall crossing formula in [102]).

D6-brane charge in topological strings

We have repeatedly mentioned that the topological string only includes effects from the sector of D2/D0-brane charge. However there is a trick [120,121] that allows to consider more general configurations describing bound states of one D6-brane with induced D2/D0-brane charges, as we review following [121].

[6] For a particular example of such a situation see [47].

The basic idea is to consider a new kind of state in M-theory on $X \times \mathbf{S^1} \times \tilde{\mathbf{S}}^{\mathbf{1}}$, given by a KK monopole, described as a Taub-NUT (TN) geometry K_4 with $\mathbf{R^3}$ base filling the (euclidianized) 3d Minkowski directions, and circle fiber along $\tilde{\mathbf{S}}^1$. In the presence of such object, we have M-theory on $X \times \mathbf{S}^1 \times K_4$, where K_4 asymptotes to $\tilde{S}^1 \times M_3$ but is topologically $\mathbf{R^4}$. One considers this geometry, in the presence of a second-quantized gas of spinning particles from M2-branes wrapped on holomorphic cycles of X. The relation comes from two possible type IIA reductions of this configuration.

• First, by shrinking $\mathbf{S^1}$, one recovers type IIA theory on $X \times K_4$, which topologically is $X \times \mathbf{R^4}$. The M2-branes become D2-branes, with momentum along $\mathbf{S^1}$ becoming D0-brane charge, and with momentum along $\tilde{\mathbf{S}}^1$ giving angular momentum of this D2/D0-brane particle. The result is a partition function of a second quantized system of D2/D0-brane particles, namely

$$Z'_{\text{top.}} = \exp\, F'_{\text{top}} \tag{4.32}$$

where, using the GV interpretation, F'_{top} is the topological string free energy without the contribution from the constant maps. Note that this quantity is determined in terms of the GV invariants.

• Reducing instead along $\tilde{\mathbf{S}}^1$, one obtains type IIA on $X \times \mathbf{S^1}$. The TN geometry becomes a D6-brane wrapped on $X \times \mathbf{S^1}$, bound to a set of D2-branes (from the M2-branes) and D0-branes (from momentum along $\mathbf{S^1}$) [7]. This yields the partition function of the world-volume theory on a D6-brane wrapped on X, bound with D2 on the class $\mathbf{k} \in H_2(X,\mathbb{Z})$ and q_0 units of D0 charge, normalized to the partition function of a pure D6-brane with no induced charges, essentially [143]

$$Z'_{\text{top.}} = \sum_{\mathbf{k},q_0} N_{\mathbf{k},q_0}\, e^{\lambda(\,q_0 + t_a k_a\,)} \tag{4.33}$$

The multiplicities $N_{\mathbf{k},q_0}$ count the BPS groundstates of the D6/D2/D0 system and suffer wall crossing. In a particular chamber in moduli space they are given by the mathematical Donaldson-Thomas (DT) invariants [122].

The above argument is usually interpreted as relating the topological string on X with the number of BPS groundstates of the D6/D2/D0-theory on X. In such terms, the relation can only hold in a particular chamber in moduli space, since the index of BPS D6/D2/D0-particle suffers wall crossing while the GV invariants do not. On this chamber, which we refer to as topological chamber, the equality between (4.32) and (4.33) provides a rigorous relation between the mathematical DT and the GV invariants, which we exploit below. Nevertheless, in the following sections we argue the existence of a universal relation, valid throughout moduli space, and deeply related to the interpretation of the topological string as computing non-perturbative D-brane instanton effects, which are continuous upon wall crossing.

D5/D1/D(-1)-instanton wall crossing and topological strings

The key observation is that the M-theory 9-11 flip actually relates the GV topological string with a D6/D2/D0-brane on $X \times \mathbf{S^1}$, namely with a *3d compactification of the system of 4d*

[7] Inclusion of D4-brane charge can be attempted by using monodromies of the NSNS B-field, at least modulo wall crossing phenomena in this process. Given these subtleties, we focus our discussion on subsets of states with no D4-brane charge.

D6/D2/D0-brane particles. Moreover the D6-brane worldvolume theory is second-quantized in the argument, suggesting that we actually describe it as a first-quantized BPS D6/D2/D0-brane particle running along the $\mathbf{S^1}$ in a one-loop Schwinger diagram. Using our by now familiar c-map, the 9-11 flip of the GV topological string is actually computing non-perturbative D5/D1/D(-1)-brane instanton effects in a T-dual 4d type IIB on X.

This observation has several important (and closely related) implications regarding wall crossing:

• Since D5/D1/D(-1)-brane instanton effects in IIB are computed by the GV topological string (via the 9-11 flip), they are continuous throughout moduli space. Namely are continuous across walls of marginal stability of the underlying D-brane instantons.

• The index of BPS D6/D2/D0-brane 4d particles jumps across walls of marginal stability, but their non-perturbative effects upon $\mathbf{S^1}$ compactification to 3d must be continuous (being determined by GV invariants). This is possible only due to the mechanism in [88] and the wall crossing formulas of [102]. The connection between D-brane instanton effects and the topological string connection implies these relations, in that it leads to manifestly continuous non-perturbative effects.

• Beyond the well-known relation between GV invariants and the index of 4d BPS D6/D2/D0-particle states *in the topological chamber* (where the latter are given by DT invariants), we claim that there is a precise relation between GV invariants and the index of 4d BPS D6/D2/D0-particle states *throughout moduli space*. In any given chamber, the 3d non-perturbative effects from the 4d particles on $\mathbf{S^1}$ must reproduce the GV topological string partition function as described in the 9-11 flip argument. Conversely, the D6/D2/D0 indices in the different chambers could be generated by expanding the GV partition function in different basis of instantons (corresponding to those descending from the different sets of BPS particles in the 4d lift).

It would be very interesting to work this out explicitly in examples where the DT invariants are known throughout moduli space, e.g. [136, 137].

Computation of D5/D1/D(-1)-instanton effects

In this section we describe more quantitatively the non-perturbative D5/D1/D(-1)-brane instanton corrections to the IIB hypermultiplet metric. The preliminary conclusion is that the physical interpretation of $Z'_{\rm top}$ in terms of D5/D1/D(-1)-instantons is limited by the present ability to describe quaternionic Kahler metrics, and allows the inclusion of non-perturbative effects in a certain linear approximation. In the topological string chamber, where the BPS multiplicites are the mathematical DT invariants, the linear approximation corresponds to truncation onto single-instanton processes. As one moves to other chambers, however, the linear approximation contains information from certain multi-instanton processes as well.

A second limitation concerns the sector of charge under consideration. The topological string allows to describe arbitrary bound states of D2/D0-branes with one D6-brane, but it does not allow the inclusion of anti-D2 brane states. Therefore the results in this section are restricted to (potentially multi-) instanton processes with total charge corresponding to one D6-brane and to positive D2/D0-brane charge. A more complete description thus remains as an open question.

The discussion of non-perturbative effects from D-brane instantons with mutually non-local charges has been considered in [112,113]. Since the instantons break too many isometries of hypermultiplet moduli space, the description of the metric requires a twistor formalism, which provides the basis of our coming discussion. Sketchily (see references for details), the construction of the quaternionic Kahler metric on the hypermultiplet moduli space $\mathcal{M}$ is encoded in a contact structure on its twistor space $\mathcal{Z}$, which is a $\mathbf{P^1}$ fibration over $\mathcal{M}$. The twistor space can be covered by open sets, which are locally flat when expressed in the so-called Darboux coordinates. The contact structure specifies the set of transition functions (contact transformations) between Darboux coordinates in different patches. All the information is encoded in a set of holomorphic functions, subject to some contraints. For cases with enough isometries, one of these functions called the contact potential becomes the $\mathcal{N} = 2$ tensor potential mentioned in section 4.2.1.

The perturbative moduli space metric is recovered from a twistor space with three patches. The correction to the metric from brane instantons can be implemented by the introduction of additional patches in the twistor space, with the corresponding contact transformations. In the linear approximation, where instanton effects are considered a small deformation of the structure of the perturbative theory, the information is encoded in a single holomorphic transition function, which for D-brane instantons with mutually local charges is of the form [112]

$$G_{\rm local} \;=\; \frac{1}{(2\pi)^2}\sum_{k_\Lambda} n_{k_\Lambda}\,{\rm Li}_2\left(e^{-2\pi i\, k_\Lambda \xi^\Lambda}\right) \tag{4.34}$$

where k_Λ denote the instanton integer charges, and ξ^Λ are the moduli, so the exponent reproduces the instanton central charge.

A particular case of the above formula has already appeared in (4.17), as the computation of the D2/D0-brane one-loop diagram, or D1/D(-1)-instanton non-perturbative effects. There, k_0 is the D1-brane worldvolume flux quantum, other k_a label the 2-cycle homology class, the multiplicities n_{k_Λ} correspond to the corresponding genus zero GV invariants, and the dilogarithm sums over wrapping numbers of such D1-branes.

In the present situation, the topological string (via the 9-11 flip) dictates a simple generalization of this computation, namely a one-loop diagram of D6/D2/D0-brane particles. These are simply added in the topological string description, so the result is

$$G_{\rm D5/D1/D(-1)} \;=\; \frac{1}{(2\pi)^2}\sum_{\mathbf{k},q_0} N_{\mathbf{k},q_0}\,{\rm Li}_2\,(e^{\,l^\Lambda \rho_\Lambda \,-\, k_\Lambda \xi^\Lambda}) \tag{4.35}$$

where the central charges of the different D-branes are displayed by introducing a symplectic basis of electric and magnetic charges (k_Λ, l^Λ), and moduli $(\rho_\Lambda, \xi^\Lambda)$. This is precisely of the form considered in [112] for the corrections from D-brane instantons with mutually non-local charges, in the linear approximation mentioned above.

In the context [112], there was no clear interpretation for the D-brane multiplicities. In our setup however, the multiplicities $N_{\mathbf{k},q_0}$ have a clear interpretation as the 4d D6/D2/D0-particle indices in the topological chamber, namely the mathematical DT-invariants. Given the derivation of the above result from the 9-11 flip in M-theory, the sum runs over all possible D2/D0-brane states, without inclusion of antibranes (as these are the states that can be bound to a D6-brane in the topological chamber).

Our statement is that the above expression provides the correct non-perturbative effects even if one moves to other chambers, even if some of these D6/D2/D0-brane states disappear from the BPS spectrum. In order to illustrate the point, consider a situation where a state γ disappears through a primitive wall crossing $\gamma \to \gamma_1 + \gamma_2$. Restricting to the relevant sector, on the stable side the contribution to (4.35) is

$$N_{\gamma_1}\,\mathrm{Li}_2(e^{-Z_{\gamma_1}})\,+\,N_{\gamma_2}\,\mathrm{Li}_2(e^{-Z_{\gamma_2}})\,+\,N_{\gamma}\,\mathrm{Li}_2(e^{-Z_{\gamma}}) \tag{4.36}$$

On the other side, the first two terms are still generated by γ_1 and γ_2. The last term cannot be generated by γ since it is unstable. However it is simple to argue that it is generated by a 2-instanton process involving both γ_1 and γ_2. This follows from the considering the linear approximation to the wall crossing formula [102], as we explain.

Following the discussion in [112,113] (inspired in [88]), the correction to the hypermultiplet moduli space metric from a D-brane instanton with charge γ and BPS phase θ_γ is described as a change in the contact structure of the twistor space $\mathcal{Z}$. Denoting z a complex coordinate on the $\mathbf{P^1}$ fiber of $\mathcal{Z}$, the instanton introduces new patches, elongated along the real lines (rays) $\ell_\pm$ where the phase $\arg z = \pm\theta_\gamma$. The contact structure is encoded in a transition function across $\ell_\pm$, which is a symplectomorphism U_γ

$$U_\gamma\,=\,\exp\,\mathrm{Li}_2\left(\,e_\gamma\right)\,\to\,\exp\,\mathrm{Li}_2\left(\,e^{-2\pi\,Z_\gamma}\right) \tag{4.37}$$

where the arrow indicates how the symplectomorphism is represented as a transition function as the exponential of the dilogarithm in (4.35). For mutually non-local D-brane instanton charges, such symplectomorphisms do not commute, and form an algebra [102]

$$[e_{\gamma_1},e_{\gamma_2}]\,=\,(-1)^{\langle\gamma_1,\gamma_2\rangle}\,\langle\gamma_1,\gamma_2\rangle\,e_{\gamma_1+\gamma_2} \tag{4.38}$$

The contribution from summing over all instantons is associated to taking the products of such symplectomorphisms, ordered according to the value of the BPS phase θ_γ (e.g. counter-clockwise around the origin $z=0$). As one moves in moduli space, the BPS phases change and become aligned at walls or marginal stability, beyond which the ordering of the aligned BPS states changes. The continuity of the moduli space metric requires that the spectrum of BPS states jumps, such that the products over the BPS spectra is unchanged,

$$\prod_{\gamma^+} U_{\gamma^+}^{\Omega^+(\gamma^+)}\,=\,\prod_{\gamma^-} U_{\gamma^-}^{\Omega^-(\gamma^-)} \tag{4.39}$$

where the products run through BPS states with charges $\gamma^\pm$ on the sides of a wall, and are ordered according to their BPS phase, and Ω denotes the BPS multiplicities in the corresponding region.

Going back to our primitive crossing example, the wall crossing formula is

$$U_{\gamma_1}^{N_{\gamma_1}}\,U_{\gamma}^{N_{\gamma}}\,U_{\gamma_2}^{N_{\gamma_2}}\,=\,U_{\gamma_2}^{N_{\gamma_2}}\,U_{\gamma_1}^{N_{\gamma_1}} \tag{4.40}$$

The linear approximation to the right hand side directly reproduces (4.36). The left hand side reproduces it upon use of the Baker-Hausdorff-Campbell formula, and linearization, with

$$N_\gamma\,=\,(-1)^{\langle\gamma_1,\gamma_2\rangle}\,\langle\gamma_1,\gamma_2\rangle \tag{4.41}$$

namely, the primitive wall crossing formula in [119]. The last contribution in (4.36) thus arises as a linear piece from the combination of the two instantons γ_1, γ_2, as announced.

A similar application of the general wall crossing formula implies that the topological string answer (4.35) is robust under wall crossing. This nicely dovetails the fact that the result can be unambiguously expressed in terms of GV invariants (via their connection with DT invariants). The amusing part is that the way in which different terms in (4.35) are generated changes from chamber to chamber, and in particular can involve multi-instanton effects. Our point, however, is that the topological string is clever enough to ignore all microscopic complications and provide an answer automatically valid thoroughout moduli space, at least for instanton processes in the charge sector under consideration. Although the equality (4.39) is already known, we claim that, at least at the linear level, it is a consequence of the fact that (4.35), (which computes the U's), is computed by the topological string, which is manifestly robust across walls, as opposed to just some function of the GV invariants that might otherwise change in form from chamber to chamber in the moduli space. It would be interesting to extend this picture to other charge sectors, with multiple D6-branes, or including anti-D2-brane charges.

4.2.5 Final remarks

In this section we have discussed the connection between the GV interpretation of the topological string and 4d non-perturbative effects from D-brane instantons. This relation underlies the continuity of non-perturbative effects through walls of BPS stability. The connection is realized via compactification to 3d, and hence fits nicely with the physical interpretation of the wall crossing formula in [102]. Along the way, we have made contact with the literature on computation of D-brane instantons effects in 4d $\mathcal{N} = 2$ theories, and reproduced some of those results in a computationally powerful way. In addition, we have also discussed mechanisms to reduce supersymmetry to $\mathcal{N} = 1$ and studied its wall crossing. Explicit computations in the $\mathcal{N} = 1$ case, and specially in the presence of D-branes and O-planes, are under poor control. The generalization of the c-map method to this case is one of the main open directions left for the future.

In the following section, however, we study a higher dimensional setup (and hence with higher supersymmetry), in which the c-map can be explicitly analyzed, even in the presence of gauge D-branes and O-planes.

4.3 D-instanton corrections in 8d type IIB orientifolds

In this section, we would like to apply the c-map to the computation of instanton effects in a situation that include gauge D-branes and O-planes. This is of course a very important issue since in the end, our main target are orientifold compactifications, which naturally include gauge branes. As we have seen, however, theories with only four supercharges are more difficult to handle in general, and the c-map method is not yet ripe for application in such setups. In this section we focus on a higher dimensional theory which, while including open strings and orientifold planes, preserves more supersymmetry and is hence more manageable.

4.3.1 The type I' computation

The background

The background we consider is 9d type I' theory [148], namely type IIA on $\mathbf{S^1}$ modded out by the orientifold action $\Omega R(-1)^{F_L}$, where R flips the circle coordinate and F_L is the left-moving fermion number. There are two O8-planes and 32 D8-branes to cancel the RR tadpole. We work in the covering space picture.

Throughout the section we focus on the configuration with 16 D8-branes on top of each O8-plane, hence the 9d gauge symmetry is $SO(16)^2$. There is local RR tadpole cancellation and constant dilaton profile along the circle. We will compactify this theory on a further circle and perform a T-duality along it. Following the c-map chain of figure 4.1, we can compute instanton corrections in the dual type IIB model in 8d by computing loops of particles in the type I' setup.

Therefore, the spectrum of 9d BPS one-particle states, together with their quantum numbers, will be important for us: there are perturbative open string BPS states stretching among the D8-branes, hence carrying integer winding charge for states in the $(\mathbf{120},\mathbf{1})+(\mathbf{1},\mathbf{120})$, and with half-integer winding for states in the $(\mathbf{16},\mathbf{16})$. In addition, there are non-perturbative BPS particle states arising from D0-brane bound states. For D0-brane bound states in the bulk, there is a single BPS state with $2n$ units of D0-brane charge (as counted in the covering space), and neutral under the gauge symmetry. For D0-branes on top of each O8-plane, there is a single BPS state with charge $2n$, transforming in the $\mathbf{120}$ of the corresponding $SO(16)$, and a single BPS state with charge $2n+1$ in the $\mathbf{128}$. These BPS multiplicities can be simply understood by regarding these states as KK momentum modes of the gravity multiplet and the E_8 vector multiplets in the lift to Horava-Witten theory. The different transformation properties of the boundary D0-brane states is due to the momentum shift from the Wilson lines in the eleventh direction, breaking $E_8 \to SO(16)$, see e.g. [149]. More general type I' configurations and their BPS spectrum have been considered in [150, 151], whose results will be useful in some generalizations.

In this section we consider the compactification of this 9d model to 8d on a further $\mathbf{S^1}$, along which we may have general $SO(16)^2$ Wilson lines. This model is related by T-dualtity to the type IIB orientifold in [144], where towers of D(-1)-brane instantons generate contributions to couplings F^4 and R^4. In a dual heterotic model, these are generated as one-loop threshold corrections. These corrections have been studied from different viewpoints, see e.g. [152–159]. Our purpose is to determine these couplings in the type I' picture where they can be easily described in terms of a one-loop diagram of 9d BPS particles, possibly winding around the $\mathbf{S^1}$ compactification circle to 8d. Perturbative BPS open strings and non-perturbative D0-brane BPS bound states will produce contributions T-dual to perturbative and D(-1)-instanton contributions in the type IIB orientifold.

The non-perturbative contributions

The computations in this subsection follow [147]. We are interested in computing the one-loop diagram of a 9d BPS particle on $M_8 \times \mathbf{S^1_9}$, with four external insertions of gauge fields strengths (or curvatures). For concreteness we focus on D0-branes, although the result is more general, as we will see later in this section. The amplitude has the general structure (4.1). The Wilson line $\tilde{c} = c + A$ along $\mathbf{S^1_9}$, has a piece c from a gauge boson in the gravity multiplet (i.e. the RR 1-form for BPS D0-branes) and a piece A encoding possible Wilson lines of vector multiplet gauge bosons (i.e. the D8-brane $SO(16)$ Wilson lines for D0-branes). The external vertex operators V_F can simply be replaced by a factor of t^4 and insertions encoding the coupling to the external gauge bosons or curvatures. Focusing on the former case, we obtain an insertion of $\mathrm{tr}_{\mathbf{R}} F^4$ for a BPS particle in a representation $\mathbf{R}$ of the gauge group. Corrections involving curvatures are discussed at the end of this s.

The relevant formulas for the one-loop computations were obtained in section 4.1, and they can be applied by setting $d = 9$ and $k = 4$ to our case. We will focus in this section on the case $w_9 \neq 0$ (states with $w_9 = 0$ lead to a tree-level contribution which will be considered later in this section). We can then use the expression (4.5) in terms of Bessel functions. For our case of interest $d/2 - k = 1/2$, the saddle point approximation is exact, $K_{1/2}(x) = \sqrt{\frac{\pi}{2x}} e^{-x}$, and we get

$$\mathcal{A}_{(1/2)} \;=\; \frac{1}{4!} \sum_{w_9 \neq 0} \frac{1}{|w_9|}\, e^{-2\pi R_9 |w_9 \mu| - 2\pi i w_9 \tilde{c}}\;. \tag{4.42}$$

For a bound state of n D0-branes, $\mu = n/g_s$ and $c = nc_0$, with c_0 the RR 1-form Wilson line. Defining $q = e^{2\pi i \tau}$ with $\tau = i\frac{R_9}{g_s} + c_0$, and reintroducing the external field strengths F to recover the contribution to the effective action, rather than the amplitude, we obtain

$$\Delta S^{D0} = \frac{1}{4!} \sum_{w_9 > 0} \frac{1}{w_9}\, q^{n\, w_9}\, e^{2\pi i\, w_9 A}\, \mathrm{tr}_{\mathbf{R}} F^4 \;+ \text{c.c.} \tag{4.43}$$

This expression already suggests an instanton expansion. Notice that the prefactor is simply constant, as expected for the high supersymmetry of the system.

This basic expression allows for the computation of instanton corrections to 8d theories obtained upon further circle compactification to 8d, possibly with Wilson lines. In the following we work out several examples of gauge and gravitational corrections. In the latter case, suitable curvature traces replace $\mathrm{tr}\, F^4$ in the expression.

The $SO(16)$ model

Consider the simplest situation where there are no Wilson lines on the circle $\mathbf{S^1_9}$, so that the 8d gauge group is $SO(16)^2$. The non-perturbative gauge threshold corrections are easily computed from the D0-brane one-loop diagrams, using the BPS bound state spectrum information: For each boundary there exists a D0-brane BPS bound state of mass $|2n|$, in the representation **120** of the corresponding $SO(16)$, and one state of mass $|2n-1|$ in the **128**, for each non-zero integer n. Focusing on a single $SO(16)$, the contribution is

$$
\begin{aligned}
\Delta S^{D0}_{SO(16)} &= \frac{2}{4!}\sum_{w_9,n>0}\frac{1}{w_9}\,q^{w_9(2n)}\,\mathrm{tr}_{\mathbf{120}}F^4 + \frac{2}{4!}\sum_{w_9,n}\frac{1}{w_9}\,q^{w_9(2n-1)}\,\mathrm{tr}_{\mathbf{128}}F^4 + \text{c.c.} \\
&= \frac{1}{3}\,\mathrm{tr}\,F^4\sum_k\sum_{\ell|k}\frac{1}{\ell}\left[2q^{2k}-q^k\right] + \frac{1}{8}\,(\mathrm{tr}\,F^2)^2\sum_k\sum_{\ell|k}\frac{1}{\ell}\left[-q^{2k}+2q^k\right] + \text{c.c.}
\end{aligned}
\tag{4.44}
$$

where in the last equality we have used (4.101) to rewrite the expression in terms of traces in the vector representation. This result agrees with the result in [147] for heterotic strings on $\mathbf{T}^2$.

The $SO(8)^2$ model

Let us now consider turning on $\mathbb{Z}_2$ valued Wilson lines which break each 9d $SO(16)$ factor to $SO(8)^2$. This model was discussed in [144]. The 9d BPS states in $SO(16)$ representations pick up different phases according to their behaviour under the decomposition

$$
\begin{aligned}
&\mathbf{120} \to (\mathbf{28},\mathbf{1})_+ + (\mathbf{1},\mathbf{28})_+ + (\mathbf{8_v},\mathbf{8_v})_- \\
&\mathbf{128} \to (\mathbf{8_s},\mathbf{8_s})_+ + (\mathbf{8_c},\mathbf{8_c})_-
\end{aligned}
\tag{4.45}
$$

where the subindex $\pm$ corresponds to having $e^{2\pi i\,A} = \pm 1$.

We focus on F^4 correction associated to only one of the $SO(8)$ factors, in which case only states charged under this factor contribute. The result is (we will drop the + c.c. term in the following)

$$
\begin{aligned}
\Delta S^{D0}_{SO(8)} = \frac{2}{4!}\Big[&\sum_{n,w_9}\frac{1}{w_9}\,q^{2n\,w_9}\,\mathrm{tr}_{\mathbf{28}}\,F^4 + 8\sum_{n,w_9}\frac{1}{w_9}\,q^{2n\,w_9}\,(-1)^{w_9}\,\mathrm{tr}_{\mathbf{8_v}}\,F^4 \\
&+ 8\sum_{n,w_9}\frac{1}{w_9}\,q^{(2n-1)\,w_9}\,\mathrm{tr}_{\mathbf{8_s}}\,F^4 + 8\sum_{n,w_9}\frac{1}{w_9}\,q^{(2n-1)\,w_9}\,(-1)^{w_9}\,\mathrm{tr}_{\mathbf{8_c}}\,F^4\Big]\,.
\end{aligned}
\tag{4.46}
$$

Using the trace identities (4.102), we obtain

$$
\begin{aligned}
\Delta S^{D0}_{SO(8)} &= \frac{1}{3}\,\mathrm{tr}\,F^4 \sum_{w_9,n}\left[\frac{1}{2w_9}\,q^{(2n)\,2w_9} - \frac{1}{2w_9-1}\,q^{(2n)\,(2w_9-1)} - \frac{1}{2w_9}\,q^{(2n-1)\,2w_9}\right] \\
&\quad + \frac{1}{8}\,(\mathrm{tr}\,F^2)^2 \sum_{n,w_9}\left[\frac{1}{w_9}\,q^{2n\,w_9} + 2\times\frac{1}{2w_9}\,q^{(2n-1)\,2w_9}\right] \\
&\quad - 8\,\mathrm{Pf}\,F \sum_{n,w_9}\frac{1}{2w_9-1}\,q^{(2n-1)(2w_9-1)} \\
&= \frac{1}{2}\,\mathrm{tr}\,F^4 \sum_k\sum_{\ell|k}\frac{1}{\ell}\left[q^{4k} - q^{2k}\right] - \frac{1}{8}\,(\mathrm{tr}\,F^2)^2 \sum_k\sum_{\ell|k}\frac{1}{\ell}\left[q^{4k} - 2q^{2k}\right] \\
&\quad - 8\,\mathrm{Pf}\,F \sum_k\sum_{\ell|2k-1}\frac{1}{\ell}\,q^{2k-1}\,. \qquad (4.47)
\end{aligned}
$$

This agrees with the correction in [144], up to an overall minus sign.

Note that in principle the D0-branes could seem to generate mixed corrections $\mathrm{tr}\,F_1^2\,\mathrm{tr}\,F_2^2$ for two $SO(8)$ factors from the same boundary. However these vanish due to a cancellation between D0-branes in different representations. Explicitly,

$$
\begin{aligned}
&\sum_{n,w_9}\frac{1}{w_9}\,q^{2n\,w_9}\,(-1)^{w_9}\,\mathrm{tr}_{\mathbf{8_v}}\,F_1^2\,\mathrm{tr}_{\mathbf{8_v}}\,F_2^2 + \sum_{n,w_9}\frac{1}{w_9}\,q^{(2n-1)\,w_9}\,\mathrm{tr}_{\mathbf{8_s}}\,F_1^2\,\mathrm{tr}_{\mathbf{8_s}}\,F_2^2 \\
&+ \sum_{n,w_9}\frac{1}{w_9}\,q^{(2n-1)\,w_9}\,(-1)^{w_9}\,\mathrm{tr}_{\mathbf{8_c}}\,F_1^2\,\mathrm{tr}_{\mathbf{8_c}}\,F_2^2 \qquad (4.48) \\
= \;& \mathrm{tr}\,F_1^2\,\mathrm{tr}\,F_2^2\left[\sum_{n,w_9}\frac{1}{2w_9}\,q^{2n\,2w_9} - \sum_{n,w_9}\frac{1}{2w_9-1}\,q^{2n\,(2w_9-1)} + 2\times\frac{1}{2w_9}\,q^{(2n-1)\,2w_9}\right] = 0
\end{aligned}
$$

where we have used (4.102) and the cancellation follows after some simple manipulations.

Perturbative contributions

Perturbative corrections to the gauge couplings we are computing arise in two different ways. First of all, bound states of D0 branes with zero winding in the circle $\mathbf{S_9}$ yield a tree-level contribution which can be easily obtained from (4.3) by setting $w_9 = 0$:

$$
\begin{aligned}
\Delta S^{\mathrm{Pert}} &= \frac{\sqrt{\pi}\,R}{4!}\sum_{n\in\mathbb{Z}}\int\frac{dt}{t}\,t^{-\frac{1}{2}}\left[e^{-t\frac{(2n)^2}{g_s^2}}\,\mathrm{tr}_{\mathbf{120}}F^4 + e^{-t\frac{(2n-1)^2}{g_s^2}}\,\mathrm{tr}_{\mathbf{128}}F^4\right] \qquad (4.49) \\
&= \frac{4\tau_2}{4!\,\pi}\sum_{w>0}\left[\frac{1}{w^2}\mathrm{tr}_{\mathbf{120}}F^4 + \frac{(-1)^w}{w^2}\mathrm{tr}_{\mathbf{128}}F^4\right] = \frac{\pi\,\tau_2}{3\cdot 4!}\left[2\,\mathrm{tr}_{\mathbf{120}}F^4 - \mathrm{tr}_{\mathbf{128}}F^4\right]\,.
\end{aligned}
$$

Using the trace identities of appendix 4.A, we can calculate these contributions for the $SO(16)^2$ and the $SO(8)^4$ models:

$$
\begin{aligned}
\Delta S^{\mathrm{Pert}}_{SO(16)} &= \frac{\tau_2\,\pi}{6}\,\mathrm{tr}\,F^4_{SO(16)} \\
\Delta S^{\mathrm{Pert}}_{SO(8)} &= \frac{\tau_2\,\pi}{6}\,\mathrm{tr}\,F^4_{SO(8)}\,. \qquad (4.50)
\end{aligned}
$$

Again, these terms agree with the heterotic results in the literature.

The spectrum of BPS particles in the 9d type I' theory also contains perturbative states, corresponding to open strings winding in the interval $\mathbf{S^1_{10}}/\mathbb{Z}_\mathbf{2}$. Since the BPS condition forbids any oscillation excitation, they are simply the groundstates of open strings stretching between the D8-branes. There are states starting and ending on the same $SO(16)$ stack, therefore transforming in the corresponding **120** and labeled by an integer winding w_{10}, and states stretching between the two $SO(16)$ stacks, thus in the $(\mathbf{16},\mathbf{16})$ and labeled by a half-integer winding $w_{10}-1/2$. In the $\mathbf{S^1_9}$ compactification to 8d, one-loop contributions from these states can be analyzed using formulas similar to the above, by simply taking into account their different 9d masses. The results correspond to perturbative one-loop F^4 terms.

The starting point is the analog of the amplitude (4.1), for perturbative states:

$$\begin{aligned}
\mathcal{A}^{\rm Pert} &= \frac{1}{\pi^8 4!}\sum_{\substack{\ell_9\in\mathbf{Z}\\ w_{10}\in\frac{\mathbf{Z}}{2}}}\int d^8\mathbf{p}\int_0^\infty \frac{dt}{t}\, t^4\, e^{-t\left(\mathbf{p}^2+\frac{(\ell_9-\tilde b)^2}{R_9^2}+w_{10}^2R_{10}^2\right)} \\
&= \frac{\sqrt{\pi}R_9}{4!}\sum_{\substack{w_9\in\mathbf{Z}\\ w_{10}\in\frac{\mathbf{Z}}{2}}}\int_0^\infty \frac{dt}{t}\, t^{-1/2}\, e^{-\frac{\pi^2R_9^2w_9^2}{t}}\, e^{-tw_{10}^2R_{10}^2}\, e^{-2\pi i w_9\tilde b}\ , \qquad (4.51)
\end{aligned}$$

where we have integrated over continuous momenta and performed a Poisson resummation over the discrete momentum ℓ_9. Here, $\tilde b$ contains the NSNS 2-form and the information about the Wilson line along $\mathbf{S}^1_9$, in complete analogy with $\tilde c$ for the non-perturbative contributions: $\tilde b = b + A = w_{10}b_0 + A$.

Let us first consider the contributions from states with non-zero winding in the interval $\mathbf{S^1_{10}}/\mathbb{Z}_\mathbf{2}$, but with $w_9=0$, the analog of (4.49):

$$\begin{aligned}
\Delta S^{\rm F1}_{w_9=0,w_{10}\neq 0} &= \frac{\sqrt{\pi}R_9}{4!}\sum_{w_{10}\in\mathbf{Z}}\int_0^\infty \frac{dt}{t}\, t^{-1/2}\, e^{-tw_{10}^2R_{10}^2}\ \mathrm{tr}_{\,\mathbf{120}}F^4 \\
&\quad +\frac{\sqrt{\pi}R_9}{4!}\sum_{w_{10}\in\mathbf{Z}}\int_0^\infty \frac{dt}{t}\, t^{-1/2}\, e^{-t(w_{10}-1/2)^2R_{10}^2}\ \mathrm{tr}_{\,(\mathbf{16},\mathbf{16})}F^4 \\
&= \frac{2U_2}{\pi 4!}\sum_{\ell_{10}>0}\frac{1}{\ell_{10}^2}\ \mathrm{tr}_{\,\mathbf{120}}F^4 + \frac{2U_2}{\pi 4!}\sum_{\ell_{10}>0}\frac{(-1)^{\ell_{10}}}{\ell_{10}^2}\ \mathrm{tr}_{\,(\mathbf{16},\mathbf{16})}F^4 \\
&= \frac{\pi U_2}{3\cdot 4!}\ \mathrm{tr}_{\,\mathbf{120}}F^4 - \frac{\pi U_2}{6\cdot 4!}\ \mathrm{tr}_{\,(\mathbf{16},\mathbf{16})}F^4 \qquad (4.52)
\end{aligned}$$

where we performed a Poisson resummation and omitted the (divergent) term $\ell_{10}=0$. Here, $U = U_1 + i\,U_2 \equiv b_0 + iR_9R_{10}$ is the volume modulus of the compactification torus (in the covering space) which upon T-duality becomes the complex structure modulus of type IIB.

States with both winding numbers w_9 and w_{10} different from zero contribute to the amplitude in a way equivalent to stacks of D0 branes with non-zero winding number along

$\mathbf{S^1_9}$ (see eq.(4.43)). The result is

$$\begin{aligned}\Delta S^{\rm F1}_{w_9\neq 0, w_{10}\neq 0} &= \frac{2}{4!}\sum_{w_9,w_{10}>0}\frac{1}{w_9}\,q'^{\,w_9\,w_{10}}\,e^{-2\pi i w_9 A}\,{\rm tr}_{\,\mathbf{120}}F^4 \,+\, {\rm c.c}\\ &+\frac{2}{4!}\sum_{w_9,w_{10}>0}\frac{1}{w_9}\,q'^{\,w_9\,(w_{10}-\frac{1}{2})}\,e^{-2\pi i w_9 A}\,{\rm tr}_{\,(\mathbf{16},\mathbf{16})}F^4 + {\rm c.c.}\,, \end{aligned} \quad (4.53)$$

where we have defined $q' = e^{2\pi i U}$.

Finally there are contributions from states in the **120** with zero winding in the interval, corresponding to the massless gauge bosons. These states can be considered from eq.(4.51) by taking $w_{10} = 0$ and $w_9 \neq 0$, or equivalently, from eq.(4.1) by taking $\mu = 0$:

$$\begin{aligned}\Delta S^{\rm F1}_{w_9\neq 0, w_{10}=0} &= \frac{\sqrt{\pi}R_9}{4!}\sum_{w_9\in\mathbf{Z}}\int_0^\infty \frac{dt}{t}\, t^{-1/2}\, e^{-2\pi i w_9 A - \frac{\pi^2 R_9^2 w_9^2}{t}}\ {\rm tr}_{\,\mathbf{120}}F^4\\ &= \frac{2}{4!}\sum_{w_9>0}\frac{e^{-2\pi i A w_9}}{w_9}\ {\rm tr}_{\,\mathbf{120}}F^4. \end{aligned}\quad (4.54)$$

Let us consider the $SO(16)^2$ theory by setting $A = 0$ in the above equations. In this case the contribution in (4.54) is divergent and the reason is because it arises from massless states running in the loop. This divergence may however be regularized[8] using the prescription described in [164]. For our case, this prescription boils down to simply adding a term [118], $\log(\tau_2 U_2/\Lambda^2)$, which is consistent with modular invariance and where Λ^2 is chosen such that it cancels the logarithmic divergence in the sum. By using this procedure we obtain

$$\Delta S^{\rm F1}_{w_9\neq 0, w_{10}=0} \to -\frac{1}{4!}\,\log(\tau_2 U_2)\,{\rm tr}_{\,\mathbf{120}}F^4. \quad (4.55)$$

The total contribution from open strings in this model is the sum of the terms (4.52), (4.53) and (4.55). Taking into account the trace identities of appendix 4.A we obtain

$$\begin{aligned}\Delta S^{\rm F1}_{SO(16)} &= \frac{-3}{4!}({\rm tr}\,F^2)^2\log(\tau_2 U_2|\eta(U)|^4)\\ &+\frac{8}{4!}{\rm tr}\,F^4\left[-\log(\tau_2 U_2) + 2\sum_{w_9,w_{10}>0}(q'^{w_9 w_{10}} + 2\,q'^{w_9(w_{10}-\frac{1}{2})} + {\rm c.c.})\right]\end{aligned}\quad (4.56)$$

For the $SO(8)^8$ case we simply have to take into account the effect of the Wilson lines on each state and the breaking of the representations

$$\begin{aligned}\mathbf{120} &\to (\mathbf{28},\mathbf{1})_+ + (\mathbf{1},\mathbf{28})_+ + (\mathbf{8_v},\mathbf{8_v})_- \quad (4.57)\\ (\mathbf{16};\mathbf{16}) &\to (\mathbf{8_v},\mathbf{1};\mathbf{8_v},\mathbf{1})_+ + (\mathbf{8_v},\mathbf{1};\mathbf{1},\mathbf{8_v})_- + (\mathbf{1},\mathbf{8_v};\mathbf{8_v},\mathbf{1})_- + (\mathbf{1},\mathbf{8_v};\mathbf{1},\mathbf{8_v})_+\end{aligned}$$

Note that in this case (4.54) is only divergent for states in the $(\mathbf{28},\mathbf{1})_+$ and not for those in the $(\mathbf{8_v},\mathbf{8_v})_-$ for which $e^{2\pi i A} = -1$. For the latter, the sum in (4.54) yields just a

[8] For further discussions on these issues, see section 4.3.3.

moduli independent contribution which we will not consider. It turns out that the sum of the contributions to $\mathrm{tr}\, F^4$ from the $(\mathbf{8_v}, \mathbf{8_v})_-$ states with non-zero winding cancel the contributions from the $(\mathbf{8_v}, \mathbf{1}; \mathbf{8_v}, \mathbf{1})_+$ and $(\mathbf{8_v}, \mathbf{1}; \mathbf{1}, \mathbf{8_v})_-$ with non-zero winding. Explicitly,

$$\sum_{w_9,w_{10}} \frac{1}{w_9}\, q'^{\,w_9\, w_{10}}\, (-1)^{w_9} + \sum_{w_9,w10} \frac{1}{w_9}\, q'^{\,(w_{10}-\frac{1}{2})\, w_9} + \sum_{w_9,w_{10}} \frac{1}{w_9}\, q'^{\,(w_{10}-\frac{1}{2})\, w_9}\, (-1)^{w_9} = 0 \quad (4.58)$$

where the cancellation is (for no obvious physical reason) analogous to that in (4.48), as can be easily checked by trading $q' \to q^2$, $w \to n$.

Finally, the contributions to $\mathrm{tr}\, F^4$ from states with zero winding in $\mathbf{S^1_9}$ (eq.(4.52)) also vanishes and therefore we conclude that this term does not receive (moduli dependent) contributions from fundamental strings (in the $SO(8)^4$ model). Open strings in the **28** do however have a non-vanishing contribution to the $(\mathrm{tr}\, F^2)^2$ term. By collecting all the pieces we obtain,

$$\begin{aligned} \Delta S^{\mathrm{F1}}_{SO(8)} &= \frac{1}{4!}\left[\frac{\pi U_2}{3} - \log(\tau_2 U_2) + 2\sum_{w,m} \frac{1}{m}\left[q'^{\,wm} + \mathrm{c.c.}\right]\right] \mathrm{tr}_{\,\mathbf{28}} F^4 \\ &= -\frac{3}{4!} \log\left(\tau_2 U_2 |\eta(U)|^4\right) (\mathrm{tr}\, F^2)^2 \end{aligned} \quad (4.59)$$

which agrees with the heterotic result in [144]. This result is also recovered in the type IIB model in [144], even though it is there treated as a local model. The agreement follows because of the cancellation (4.58) for contributions from open strings charged under different gauge factors.

There are also additional perturbative contributions to mixed terms like $\mathrm{tr}\, F_i^2 \mathrm{tr}\, F_j^2$, which can be easily computed. Skipping the details, we get that

$$\begin{aligned} \Delta S^{\mathrm{Mixed}}_{SO(8)} &= \frac{1}{4!}\sum_{w,m} \frac{1}{m}\, q'^{wm}\, (-1)^m \left[\mathrm{tr}\, F_1^2\, \mathrm{tr}\, F_2^2 \,+\, \mathrm{tr}\, F_3^2\, \mathrm{tr}\, F_4^2\right] \\ &+ \frac{1}{4!}\sum_{w,m} \frac{1}{m}\, q'^{(w-\frac{1}{2})\, m} \left[\mathrm{tr}\, F_1^2\, \mathrm{tr}\, F_3^2 \,+\, \mathrm{tr}\, F_2^2\, \mathrm{tr}\, F_4^2\right] \\ &+ \frac{1}{4!}\sum_{w,m} \frac{1}{m}\, q'^{(w-\frac{1}{2})\, m}\, (-1)^m \left[\mathrm{tr}\, F_1^2\, \mathrm{tr}\, F_4^2 \,+\, \mathrm{tr}\, F_2^2\, \mathrm{tr}\, F_3^2\right] \end{aligned} \quad (4.60)$$

These corrections have not been computed before for the heterotic or the type IIB dual.

It is interesting that both perturbative and non-perturbative contributions can be discussed on an equal footing in the language of one-loop diagrams of BPS particles. Notice that this relates the matching of the 8d corrections in dual pictures to the matching of the BPS spectra of the 9d theories. Therefore the matching of our results (and those in [144]) simply follows from heterotic-type I' duality in 9d, which is well understood at the BPS level [151]. This viewpoint will be useful in section 4.3.4.

Gravitational couplings

In this section we compute gravitational and mixed corrections from one-loop diagrams of BPS particles. For concreteness we focus on non-perturbative D0-brane states, although clearly the perturbative terms can be obtained similarly.

Gravitational R^4 or mixed R^2F^2 corrections can be obtained by replacing the $\mathrm{tr}_{\mathbf{R}}F^4$ insertion in the loop diagram by suitable couplings to the external source of field strength or curvature. This can most easily be done by computing in a background source, and subsequently picking the corresponding term in the Taylor expansion in the background. The computation in a background is exactly as the one used in the computation of anomaly polynomials, hence we may borrow the results of contributions from different kinds of fields, which we gather in Appendix 4.B. Note that for previously computed pure gauge corrections, the $\mathrm{tr}_{\mathbf{R}}F^4$ term can be recovered from the corresponding term in the expansion of the Chern character $\mathrm{tr}_{\mathbf{R}}e^F$.

Let us compute the non-perturbative $\mathrm{tr}\,R^4$ correction in the $SO(16)^2$ model. The D0-branes in the boundary are in vector multiplets, and contain states of spin 1/2 contributing to gravitational couplings via the A-roof polynomial (4.105). Taking into account their $SO(16)^2$ multipliticies, they contribute in the following way,

$$\Delta S^{R^4}_{s=\frac{1}{2}} = \frac{1}{(4\pi)^4}\frac{1}{360}\,\mathrm{tr}\,R^4 \sum_{n,m}\frac{1}{m}\left[240\,q^{2n\,m} + 256\,q^{(2n-1)m}\right] . \tag{4.61}$$

There are also contributions from bound states of $2n$ D0-branes in the bulk (as counted in the covering space). They contain massive spin 3/2 states, which from (4.108) contribute as

$$\Delta S^{R^4}_{s=\frac{3}{2}} = \frac{1}{(4\pi)^4}\frac{248}{360}\,\mathrm{tr}\,R^4 \sum_{n,m}\frac{1}{m}\,q^{m(2n)} . \tag{4.62}$$

The total contribution is thus

$$\Delta S^{R^4}_{SO(16)} = \frac{1}{(4\pi)^4}\frac{1}{360}\,\mathrm{tr}\,R^4 \sum_{k}\sum_{\ell|k}\frac{1}{\ell}\left[232\,q^{2k} + 256\,q^{k}\right] . \tag{4.63}$$

This agrees with the heterotic string result in [147].

Let us also compute the full non-perturbative gravitational and mixed corrections in the $SO(8)^4$ model. Focusing on mixed $\mathrm{tr}\,R^2\mathrm{tr}\,F^2$ corrections for a fixed $SO(8)$ factor, there are contributions only from boundary D0-branes charged under it. Using the relevant term in (4.107), the contribution is

$$\begin{aligned}\Delta S^{R^2F^2}_{SO(8)} &= -\frac{1}{6}\frac{1}{(4\pi)^4}\,\mathrm{tr}\,R^2\Bigg(\mathrm{tr}_{\mathbf{28}}F^2\sum_{n,m}\frac{1}{m}q^{2n\,m} + 8\,\mathrm{tr}_{\mathbf{8_v}}F^2\sum_{n,m}\frac{1}{m}q^{2n\,m}(-1)^m \\ &\qquad + 8\,\mathrm{tr}_{\mathbf{8_s}}F^2\sum_{n,m}\frac{1}{m}q^{(2n-1)\,m} + 8\,\mathrm{tr}_{\mathbf{8_c}}F^2\sum_{n,m}\frac{1}{m}q^{(2n-1)\,m}(-1)^m\Bigg) \\ &= -\frac{1}{(4\pi)^4}\,\mathrm{tr}\,R^2\,\mathrm{tr}\,F^2\sum_{k}\sum_{\ell|k}\frac{1}{\ell}\,q^{2k} .\end{aligned} \tag{4.64}$$

For purely gravitational couplings, there are contributions from all D0-brane states, both from those bound to the D8-branes and from those in the bulk. Taking into account their multiplicites, and their spin components, the contribution is

$$\begin{aligned}\Delta S_{SO(8)}^{R^4} &= \frac{1}{(4\pi)^4}\left(\frac{1}{360}\,\mathrm{tr}\,R^4+\frac{1}{288}\,(\mathrm{tr}\,R^2)^2\right)\times\\ &\quad\times\Big(\sum_{n,m}\frac{1}{m}\,q^{2n\,m}\,[\,112\,+\,128\,(-1)^m]+\sum_{n,m}\frac{1}{m}\,q^{(2n-1)m}\,[\,128\,+\,128(-1)^m]\Big)\\ &\quad+\frac{1}{(4\pi)^4}\left(\frac{248}{360}\,\mathrm{tr}\,R^4+\frac{56}{288}\,(\mathrm{tr}\,R^2)^2\right)\sum_{n,m}\frac{1}{m}\,q^{2n\,m}\\ &= \frac{1}{(4\pi)^4}\left(\mathrm{tr}\,R^4+\frac{7}{12}\,(\mathrm{tr}\,R^2)^2\right)\sum_k\sum_{\ell|k}\frac{1}{\ell}\,q^{2k} \qquad (4.65)\end{aligned}$$

The results (4.64) and (4.65) agree with the heterotic and type IIB computations in [144], up to rescalings of traces and field strengths (which do not modify the agreement for the pure gauge correction).

4.3.2 More general configurations and the prepotential

General Wilson lines and D8-brane positions

An interesting generalization corresponds to considering the type IIB orientifold with D7-branes at more general positions in the transverse 2-plane, as encoded in the vevs of complex scalars in vector multiplets. In the type I' picture, a real component of these scalars corresponds to turning on more general D8-brane Wilson lines along the $\mathbf{S}^1_9$ wrapped by the D0-branes. These degrees of freedom are complexified by considering general positions of the D8-branes, away from the O8-plane. In this section we discuss these generalizations, which turn out to be very simple in our language.

Let us start by considering the configuration with 16 D8-branes on top of each O8-plane, with general Wilson lines along the $\mathbf{S}^1_9$. We focus on non-perturbative gauge corrections from D0-branes, which are generated only for gauge bosons arising from a single $SO(16)$. The gauge group is generically broken to $U(1)^8$, and we denote by ϕ_i and F_i the Wilson line and field strength for the i^{th} $U(1)$ factor. The $SO(16)$ weight vector Λ of a D0-brane state encodes its $U(1)^8$ charges, so its contribution follows from (4.43), namely

$$\Delta S_{F^4}^{D0} = \frac{1}{4!}\sum_{i,j,k,l}\sum_m \frac{1}{m}\,q^{n\,m}\,e^{2\pi i\,m\,\Lambda_i\cdot\phi_i}\,\Lambda_i\,\Lambda_j\,\Lambda_k\,\Lambda_l\,F_iF_jF_kF_l \qquad (4.66)$$

It is straightforward to realize that the above results for $SO(8)$ are a particular case.

It is also easy to include general D8-brane positions away from the $SO(16)^2$ point. The basic observation [150,151] is that as a D8-brane is moved away from the O8-plane, the BPS D0-branes stuck on it grow fundamental strings joining them and the dislocated D8-brane, in a way dictated by charge conservation. This string creation effect is dual to the Hanany-Witten brane creation effect [165], and is responsible for an increase in the mass of the 9d BPS state. More explicitly, denoting φ_i the i^{th} D8-brane position, the mass of a bound state of n D0-branes with $SO(16)$ weight Λ is shifted as $n\to n+\Lambda_i\varphi_i$. Therefore its contribution to the F^4 couplings (allowing simultaneously for general Wilson lines) can be expressed as

$$\Delta S_{F^4}^{D0} = \frac{1}{4!}\sum_{i,j,k,l}\sum_m \frac{1}{m}\,e^{2\pi i\,\tau\,m\,n}\,e^{2\pi i\,m\,\Lambda_i\cdot\Phi_i}\,\Lambda_i\,\Lambda_j\,\Lambda_k\,\Lambda_l\,F_iF_jF_kF_l \qquad (4.67)$$

where the complex scalars $\Phi = \phi + \tau\varphi$ correspond the complexification of D8-brane positions and $\mathbf{S}^1_9$ Wilson lines. This complexification will be manifestly geometric in the M-theory perspective in section 4.3.3. The above expressions can be encoded in a generating functional [146], the 8d prepotential $F(\Phi)$, by promoting the complex scalars to supermultiplets

$$\Phi_i \;=\; \Phi_i \,+\, \theta\gamma^{\mu\nu}\,\theta\, F_{i,\mu\nu} \tag{4.68}$$

and writing, modulo constants,

$$\int d^8\theta\, F(\Phi) \;=\; \int d^8\theta \sum_m \frac{1}{m^5}\, e^{2\pi i\,\tau n}\, e^{2\pi i\, m\,\Lambda_i\cdot\Phi_i} \;\;=\; \int d^8\theta \sum_m \frac{1}{m^5}\, e^{2\pi i\,\tau n}\, \mathrm{tr}_{\,\mathbf{R}} e^{2\pi i m\Phi} \tag{4.69}$$

where Φ is the background for a supermultiplet in the representation $\mathbf{R}$. This can be regarded as the computation of a Schwinger one-loop vacuum diagram in the presence of a background for the superfields (4.68). Schematically it is the trace of the operator $e^{2\pi i\Phi}$ over the one-particle BPS spectrum of the spacetime theory

$$F(\Phi) \;=\; \mathrm{tr}_{\,\mathcal{H}}\, e^{2\pi i\Phi}\;. \tag{4.70}$$

The diverse F^4 terms are recovered from the fourth derivative w.r.t the background fields, as discussed in section 4.1.

General gravitational terms

The idea can be generalized to include curvature terms, using ideas from section (4.3.1). We describe a curvature tensor background as in the computation of anomaly polynomials (see e.g. [163])

$$\frac{R}{2\pi} \;=\; \mathrm{diag}\,(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)\otimes\begin{pmatrix} & 1\\ -1 & \end{pmatrix} \tag{4.71}$$

with Pontryagin classes given by

$$\frac{1}{(2\pi)^2}\mathrm{tr}\,R^2 \;=\; \sum_{i<j}\epsilon_i^2\epsilon_j^2 \quad,\quad \frac{1}{(2\pi)^4}\mathrm{tr}\,R^4 \;=\; \sum_{i<j<k<l}\epsilon_i^2\epsilon_j^2\epsilon_k^2\epsilon_l^2. \tag{4.72}$$

Following [146] we simply promote the backgrounds ϵ_ℓ to supermultiplets W_ℓ similar to (4.68)

$$\epsilon_\ell \;\to\; W_\ell \;=\; G_\ell \,+\, R^\ell_{\mu\nu}\,\theta\gamma^{\mu\nu}\theta \tag{4.73}$$

with G_i and $R^i_{\mu\nu}$ the graviphoton and curvatures associated to the i^{th} Cartan generator in the Lorentz group.

The prepotential is given by a trace over the one-particle BPS spectrum of the anomaly polynomial operator, regarded as a function of supermultiplets. For instace, for boundary D0-branes,

$$F(\Phi,W) \;=\; \mathrm{tr}_{\,\mathcal{H}}\, e^{2\pi i\Phi}\,\hat{A}(W) \tag{4.74}$$

and all gauge and gravitational corrections are obtained from $\int d^8\theta\, F(\Phi_i,W_\ell)$.

4.3.3 The M-theory point of view

The non-perturbative contributions are due to D0-branes. These states admit a simple interpretation in the Horava-Witten M-theory lift, as momentum modes of the E_8 vector multiplets on the boundaries (as used to derive the type I' D0-brane bound state multiplicities). The non-perturbative contribution should therefore admit a simple description as a one-loop diagram of massless E_8 fields in the 10d boundary compactified on $\mathbf{T^2}$ down to 8d. This description makes the modular properties of the result manifest. It also allows for generalizations of the computation in models with general Wilson lines, not necessarily related to perturbative type I'.

The one-loop diagram

Consider Horava-Witten theory on a 2-torus, i.e. M-theory on $\mathbf{R}^8 \times \mathbf{S}^1_{(9)} \times (\mathbf{S}^1_{(10)}/\mathbf{Z}_2) \times \mathbf{S}^1_{(11)}$. We will calculate the 1-loop amplitude of massless E_8 gauge bosons with 4 external insertions of gauge field strengths. We do not consider in this section external insertions of curvature tensors, for which we should also include 1-loop diagrams of bulk gravitons. The generalization is straightforward. The general formulas to be used here were reviewed in section 4.1.

We study the massless E_8 gauge bosons which live at 10-dimensional boundaries of the $\mathbf{S^1_{(10)}}/\mathbf{Z}_2$ interval. Since these particles are stuck at the boundaries we only need to sum over the KK-momenta they carry in the $\mathbf{S^1_{(9)}} \times \mathbf{S^1_{(11)}} = \mathbf{T^2_{(9,11)}}$ directions,

$$
\begin{aligned}
\mathcal{A}^{E_8} &= \frac{1}{4!}\int_0^\infty \frac{dt}{t} t^4 \sum_{\ell_I} \int d^8\mathbf{p}\; e^{-\pi t\left(\mathbf{p}^2 + G^{IJ}\widetilde{\ell}_I\widetilde{\ell}_J\right)} \\
&= \frac{1}{4!}\int_0^\infty \frac{dt}{t} \sum_{\ell_9,\ell_{11}} e^{-\pi t \frac{1}{V_2\tau_2}|\widetilde{\ell}_9 - \tau\widetilde{\ell}_{11}|^2}
\end{aligned}
\tag{4.75}
$$

where we have denoted,

$$
\widetilde{\ell}_I = \begin{pmatrix} \widetilde{\ell}_9 \\ \widetilde{\ell}_{11} \end{pmatrix} = \begin{pmatrix} \ell_9 - \mathbf{\Lambda}\cdot\mathbf{A}_9 \\ \ell_{11} - \mathbf{\Lambda}\cdot\mathbf{A}_{11} \end{pmatrix} \tag{4.76}
$$

where $\mathbf{\Lambda}$ denotes the weight vectors of the adjoint ($\mathbf{248}$) of E_8 and $\mathbf{A}_I$ denotes the Wilson lines along the $I = 9, 11$ directions of the $\mathbf{T^2_{(9,11)}}$. The action of the modular group is manifest in this expression, so the invariance group of the result is the subgroup of $SL(2,\mathbb{Z})$ preserving the Wilson line structure. This will be discussed shortly.

Performing a Poisson resummation on the KK momenta ℓ_9, we get a sum over winding numbers w_9,

$$
\mathcal{A}^{E_8} = \frac{\sqrt{\pi}}{4!}\int_0^\infty \frac{dt}{t} t^{-1/2} \sum_{w_9,\ell_{11}} e^{-\frac{\pi^2 w_9^2}{t} - \tau_2^2\widetilde{\ell}_{11}^2 t}\; e^{2\pi i w_9 \widetilde{\ell}_{11}\tau_1} e^{2\pi i w_9 \mathbf{\Lambda}\cdot\mathbf{A}_9}\;. \tag{4.77}
$$

In what follows we consider the Wilson lines $\mathbf{A}_{11}$ to implement the breaking $E_8 \to SO(16)$, so as to connect with the type I' description in previous sections. In the above expression, it is convenient to split off the contribution from $w_9 = 0$, which corresponds to the tree level term from the type I' perspective. After Poisson resummation on ℓ_{11} and integration over t it becomes

$$\mathcal{A}^{E_8}_{(w_9=0)} = \frac{\tau_2}{4!\pi}\sum_{w_{11}\neq 0}\frac{1}{w_{11}^2}e^{2\pi i w_{11}\mathbf{\Lambda}\cdot\mathbf{A}_{11}} \tag{4.78}$$

where we exclude the divergent $w_{11} = 0$ term (which is absent in a global tadpole free theory). Splitting $\mathbf{248}\to\mathbf{120}+\mathbf{128}$, reintroducing the external F insertions and summing over weights we obtain

$$\begin{aligned}\Delta S^{E_8}_{w_9=0} &= \frac{2\tau_2}{4!\pi}\left[\sum_{w_{11}>0}\frac{1}{w_{11}^2}\,\mathrm{tr}_{\,\mathbf{120}}F^4 + \sum_{w_{11}>0}\frac{(-1)^{w_{11}}}{w_{11}^2}\,\mathrm{tr}_{\,\mathbf{128}}F^4\right]\\ &= \frac{4\pi}{4!}\tau_2\,\mathrm{tr}\,F^4\end{aligned} \tag{4.79}$$

where in the last line we have used $\sum_{w_{11}>0}\frac{1}{w_{11}^2} = \pi^2/6$ and $\sum_{w_{11}>0}\frac{(-1)^{w_{11}}}{w_{11}^2} = -\pi^2/12$, and the trace identities (4.101). Although the result is independent of the Wilson line $\mathbf{A}_9$, it is understood that if $SO(16)$ is broken into several factors, each receives a contribution of this form. This reproduces the tree level F^4 coupling for D7-branes, as announced.

The non-zero w_9 contribution in (4.77) becomes

$$\mathcal{A}^{E_8}_{(w_9\neq 0)} = \frac{1}{4!}\sum_{\substack{w_9\neq 0\\ \ell_{11}\in\mathbf{Z}}}\frac{1}{|w_9|}e^{-2\pi\tau_2|w_9(\ell_{11}-\mathbf{\Lambda}\cdot\mathbf{A}_{11})|}\;e^{2\pi i\tau_1 w_9(\ell_{11}-\mathbf{\Lambda}\cdot\mathbf{A}_{11})}\;e^{2\pi i w_9\mathbf{\Lambda}\cdot\mathbf{A}_9}\;. \tag{4.80}$$

Splitting $\mathbf{248}\to\mathbf{120}+\mathbf{128}$, corresponding to $\mathbf{\Lambda}\cdot\mathbf{A}_{11}$ being in $\mathbb{Z}$ or $\mathbb{Z}+\frac{1}{2}$, and with suitable relabelings of ℓ_{11}, the contribution can be recast as

$$\begin{aligned}\Delta S^{E_8}_{\rm non-pert} &= \frac{2}{4!}\sum_{\substack{w_9>0\\ \ell_{11}>0}}\frac{1}{w_9}\,q^{w_9\ell_{11}}\;\mathrm{tr}_{\,\mathbf{120}}\left(F^4\,e^{2\pi i w_9\mathbf{\Lambda}\cdot\mathbf{A}_9}\right) + \mathrm{c.c.}\\ &\quad+\frac{2}{4!}\sum_{\substack{w_9>0\\ \ell_{11}>0}}\frac{1}{w_9}\,q^{w_9(\ell_{11}-1/2)}\;\mathrm{tr}_{\,\mathbf{128}}\left(F^4\,e^{2\pi i w_9\mathbf{\Lambda}\cdot\mathbf{A}_9}\right) + \mathrm{c.c.}\end{aligned} \tag{4.81}$$

where we have reintroduced the F^4 insertion. The factor of 2 arises from summing over negative w_9. We have also removed the contribution from $\ell_{11} = 0$, which corresponds to the massless $SO(16)$ gauge bosons. It can be considered in the perturbative type I' sector, together with the contributions from winding open strings. The latter should arise from wrapped M2-branes in the Horava-Witten theory, whose contribution cannot be reliably computed in M-theory.

The above amplitude, as already indicated in the subindex, reproduces the D0-brane contributions in type I' in section 4.3.1. The precise match requires redefining $q\to q^2$ in (4.81), in order to change the unit of D0-brane charge from the quotient space convention (implicit in the Horava-Witten picture) to the covering space (used in the type I' picture in previuos sections). The above expression reproduces the type I' result for general Wilson lines $\mathbf{A}_9$. A slight generalization, allowing for deviations of $\mathbf{A}_{11}$ from the $SO(16)$ case can easily be shown to reproduce the type I' contribution for general D8-brane positions in the interval. Hence the complexification of D8-brane positions and Wilson lines is naturally geometrized in terms of complex Wilson lines in the M-theory setup.

The $SO(8)^4$ modular group

From the M-theory perspective, the modular group $SL(2,\mathbb{Z})$ of type IIB has a natural geometrical interpretation, and the invariance subgroup of our setups is that preserved by the Wilson lines $\mathbf{A}_9$, and $\mathbf{A}_{11}$. In this subsection we explicitly compute this invariance subgroup for the $SO(8)^4$ model and recover results from the literature. In order to do so, it is convenient to use dimensional regularization to parametrize the infrared divergencies caused by massless states running in the loop. This allows us to derive an alternative expression of the D0-brane 1-loop amplitudes (4.75) in terms of non-holomorphic Eisenstein series, which make the modular properties of the model manifest. Let us compute these gauge amplitudes in $8+2\epsilon$ dimensions and perform a Poisson resummation over both KK-momenta ℓ_I, instead of just ℓ_9:

$$\begin{aligned}\mathcal{A}^{E_8} &= \frac{1}{4!}\,V_2^\epsilon\int_0^\infty\frac{dt}{t}t^4\sum_{\ell_I}\int d^{8+\epsilon}\mathbf{p}\;e^{-\pi t\left(\mathbf{p}^2+G^{IJ}\tilde{\ell}_I\tilde{\ell}_J\right)}\\ &= \frac{1}{4!}\,V_2^\epsilon\int_0^\infty\frac{dt}{t^{1+\epsilon}}\sum_{\ell_9,\ell_{11}}\;e^{-\pi t\frac{1}{V_2\tau_2}|\tilde{\ell}_9-\tau\tilde{\ell}_{11}|^2}\\ &= \frac{\Gamma(1+\epsilon)}{4!\,\pi^{1+\epsilon}}\sum_{(w_9,w_{11})}\frac{\tau_2^{1+\epsilon}}{|w_9+\tau w_{11}|^{2+2\epsilon}}\,e^{2\pi i(w_9\mathbf{\Lambda}\cdot\mathbf{A_9}+w_{11}\mathbf{\Lambda}\cdot\mathbf{A_{11}})}.\end{aligned}\qquad(4.82)$$

where it is here and in the following implied that the term $(w_9,w_{11})=(0,0)$ is excluded in the sum. The prefactor V_2^ϵ involving the volume of the torus $\mathbf{T^2_{(9,11)}}$ has been consistently included for dimensional reasons. For $\mathbf{\Lambda}\cdot\mathbf{A_9}=\mathbf{\Lambda}\cdot\mathbf{A_{11}}=0$ the expression in (4.82) is the non-holomorphic Eisenstein series of order $1+\epsilon$, $E_{1+\epsilon}(\tau)$, which has a pole in ϵ. The physical reason for this divergence is in our case that there are massless states running in the loop.

To analyze this expression for the $SO(8)^4$ model we just need to consider the corresponding Wilson lines and their effect in the **248** states (4.45). The corresponding contributions are

$$\begin{aligned}(\mathbf{28},\mathbf{1}):\ \Delta S^{E_8}_{(\mathbf{28},\mathbf{1})} &= \frac{1}{4!\,\pi}\sum_{(w_9,w_{11})}\frac{\tau_2^{1+\epsilon}}{|w_9+\tau w_{11}|^{2+2\epsilon}}\,\mathrm{Tr}_{(\mathbf{28},\mathbf{1})}F^4_{SO(8)}=\frac{3}{4!\pi}\,E_{1+\epsilon}(\tau)\,(\mathrm{tr}\,F^2)^2\\ (\mathbf{8_v},\mathbf{8_v}):\ \Delta S^{E_8}_{(\mathbf{8_v},\mathbf{8_v})} &= \frac{1}{4!\pi}\sum_{(w_9,w_{11})}\frac{(-1)^{w_9}\tau_2^{1+\epsilon}}{|w_9+\tau w_{11}|^{2+2\epsilon}}\,\mathrm{Tr}_{(\mathbf{8_v},\mathbf{8_v})}F^4\\ &= \frac{8}{4!\pi}\,[E_{1+\epsilon}(\tau/2)-E_{1+\epsilon}(\tau)]\,\mathrm{tr}\,F^4\\ (\mathbf{8_s},\mathbf{8_s}):\ \Delta S^{E_8}_{(\mathbf{8_s},\mathbf{8_s})} &= \frac{1}{4!\,\pi}\sum_{(w_9,w_{11})}\frac{(-1)^{w_{11}}\tau_2^{1+\epsilon}}{|w_9+\tau w_{11}|^{2+2\epsilon}}\,\mathrm{Tr}_{(\mathbf{8_s},\mathbf{8_s})}F^4\\ &= \frac{1}{4!\pi}\,[E_{1+\epsilon}(2\,\tau)-E_{1+\epsilon}(\tau)]\,(-4\mathrm{tr}\,F^4+3(\mathrm{tr}\,F^2)^2-96\mathrm{Pf}F)\\ (\mathbf{8_c},\mathbf{8_c}):\ \Delta S^{E_8}_{(\mathbf{8_c},\mathbf{8_c})} &= \frac{1}{4!\,\pi}\sum_{(w_9,w_{11})}\frac{(-1)^{w_9+w_{11}}\tau_2^{1+\epsilon}}{|w_9+\tau w_{11}|^{2+2\epsilon}}\,\mathrm{Tr}_{(\mathbf{8_c},\mathbf{8_c})}F^4\\ &= \frac{1}{4!\pi}[2E_{1+\epsilon}(\tau)-E_{1+\epsilon}(2\,\tau)-E_{1+\epsilon}(\tau/2)]\,(-4\mathrm{tr}\,F^4+3(\mathrm{tr}\,F^2)^2+96\mathrm{Pf}F)\end{aligned}$$

where, for convenience, we have included in the definition of the non-holomorphic Eisenstein series, a factor of $\frac{\gamma(1+\epsilon)}{\pi^\epsilon}$:

$$E_{1+\epsilon} = \frac{\Gamma(1+\epsilon)}{\pi^\epsilon} \sum_{(w_9,w_{11})} \frac{\tau_2^{1+\epsilon}}{|w_9 + \tau w_{11}|^{2+2\epsilon}} \, . \tag{4.83}$$

In order to relate these expressions to the heterotic results in [144] we need to make the change of variables $\tau \to -1/(2\tau)$. Collecting the terms, we obtain the following total contribution,

$$\begin{aligned}
\Delta S^{E_8} \;&=\; \frac{12}{\pi\, 4!}\,[E_{1+\epsilon}(4\tau) - E_{1+\epsilon}(2\,\tau)]\mathrm{tr}F^4 + \frac{3}{\pi\, 4!}\,[2\,E_{1+\epsilon}(2\,\tau) - E_{1+\epsilon}(4\tau)](\mathrm{tr}F^2)^2 \\
&\quad + \frac{96}{\pi\, 4!}\,[3\,E_{1+\epsilon}(2\,\tau) - 2\,E_{1+\epsilon}(\tau) - E_{1+\epsilon}(4\tau)]\,\mathrm{Pf}F \\
&=\; \frac{12}{\pi\, 4!}\,[E_{1+\epsilon}(4\tau) - E_{1+\epsilon}(2\,\tau)]\mathrm{tr}F^4 + \frac{3}{\pi\, 4!}\,[2\,E_{1+\epsilon}(2\,\tau) - E_{1+\epsilon}(4\tau)](\mathrm{tr}F^2)^2 \\
&\quad + \frac{96}{\pi\, 4!}\,[E_{1+\epsilon}(\tau + \frac{1}{2}) - E_{1+\epsilon}(\tau)]\,\mathrm{Pf}F.
\end{aligned} \tag{4.84}$$

This expression can be compared to the heterotic results in [144] with the aid of the first Kronecker limit formula, which, with our conventions reads

$$E_{1+\epsilon}(\tau) = \frac{\pi}{\epsilon} + \pi\left[\gamma_E - \log 4\pi - \log(\tau_2|\eta(\tau)|^4)\right] + O(\epsilon), \tag{4.85}$$

where γ_E is the Euler-Mascheroni constant. (4.84) coincides with the heterotic results if we simply drop the divergent term in (4.85), i.e. if we substitute every Eisenstein series of order $1+\epsilon$ in (4.84) by a renormalized Eisenstein series of order 1 defined by[9]:

$$\hat{E}_1(\tau) \equiv \lim_{\epsilon\to 0}\left[E_{1+\epsilon}(\tau) - \frac{\pi}{\epsilon} - \pi(\gamma_E - \log 4\pi)\right] \tag{4.86}$$

This renormalization procedure is manifestly consistent with modular invariance. Note also that in practice the renormalization is only necessary for the $(\mathrm{tr}F^2)^2$ term in (4.84), which is the only divergent one, in agreement with the results of section 4.3.1.

The alternative expression (4.84) is useful to determine the modular invariance group of the effective action. It is simply made up of $SL(2,\mathbb{Z})$ transformations

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{4.87}$$

which leave the renormalized Eisenstein functions $\hat{E}_1(\tau)$, $\hat{E}_1(2\tau)$ and $\hat{E}_1(4\tau)$ invariant. Note that the modular properties of renormalized and unrenormalized Eisenstein functions are the same so we will work in the following with the latter. Since $E_1(\tau)$ is modular invariant, and the invariance group of $E_1(4\tau)$ is a subgroup of the invariance group of $E_1(2\tau)$, we only need to look at the function $E_1(4\tau)$, which transforms as

$$\begin{aligned}
E_1(4\,\tau) \to &\sum_{(w_9,w_{11})} \frac{1}{|c\tau+d|^2}\frac{4\,\tau_2}{|w_9 + \frac{a\tau+b}{c\tau+d}\,4w_{11}|^2} \\
&\sum_{(w_9,w_{11})} \frac{4\,\tau_2}{|(dw_9 + 4bw_{11}) + (\frac{c}{4}w_9 + aw_{11})\,4\,\tau|^2}.
\end{aligned} \tag{4.88}$$

[9]For further details on this renormalization and its relation to other regularization schemes see [160], [161].

Clearly, this function is only invariant if $c = 4n$ ($n \in \mathbb{Z}$), i.e. under transformations of the form

$$\begin{pmatrix} a & b \\ 4n & d \end{pmatrix} \in SL(2,\mathbb{Z}) \ . \tag{4.89}$$

Hence we have recovered the known result that the effective action is invariant under the subgroup $\Gamma_0(4) \in SL(2,\mathbb{Z})$.

4.3.4 Polyinstanton effects

We have seen that the F^4 and R^4 terms of the 8d theory can be obtained as a one-loop computation, in terms of the spectrum of 9d BPS one-particle states in vector (short) multiplets, in an 8d analog of [162]. In the type I' model, such BPS states are fundamental strings or D0-branes, leading to perturbative or non-perturbative corrections, with the latter reproducing elegantly the D(-1)-brane instanton sums of the T-dual type IIB orientifold. In the heterotic dual, the 9d BPS particles are winding and momentum states of fundamental strings, and the one-loop diagram reproduces the worldsheet instanton contributions computed in the literature. The one-loop description therefore shows that the agreement of the 8d corrections in type II-heterotic duals follows from the agreement of the 9d spectrum of one-particle BPS states in heterotic-type I' duality, which has been extensively studied in [151].

Polyinstantons in 8d

It is worthwhile to note that the contribution from a single 9d BPS D0-brane can correspond to a multi-instanton contribution on the type IIB side. This is particularly manifest for BPS D0-brane bound states of k elementary D0-branes. This is the 8d analog of a similar phenomenon in the 4d $\mathcal{N} = 2, 1$ context. The fact that multiple instantons can conspire to contribute to the non-perturbative $\mathcal{N} = 1$ superpotential (or $\mathcal{N} = 2$ hypermultiplet metric) [44, 45] was analyzed and interpreted in section 4.2 as the fact that in the T-dual theory the corresponding BPS particles form a bound state at threshold.

In the 4d setup [46] considered a different kind of multiple instanton effect, dubbed polyinstanton, which also has an 8d analog in our setup. The polyinstantons in [46] were claimed to violate heterotic-type I duality. In this section we address this puzzle for 8d polyinstantons, shedding light from a new perspective, valid also in the 4d setup. The bottom line is that polyinstanton processes can be interpreted as reducible Feynman diagrams which do not contribute to the microscopic 1PI effective action.

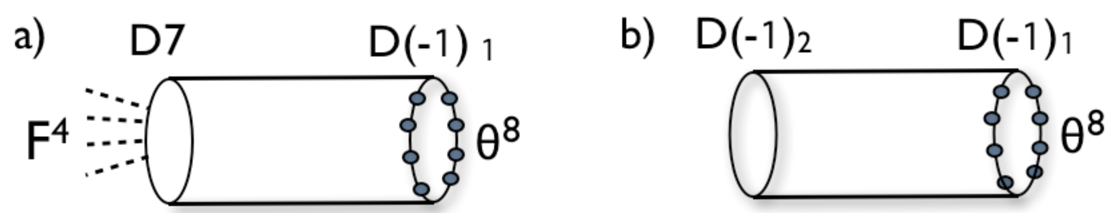


Figure 4.2: a) D(-1)-brane instanton correction to the F^4 coupling on a D7-brane. b) A related diagram describes a D(-1)-brane instanton correction to the action of a second D(-1)-brane instanton.

Let us start by introducing the 8d polyinstanton corrections to e.g. F^4, in complete analogy with the F^2 corrections [46]. The microscopic diagram leading to a D7-brane F^4 correction from a D(-1)-brane instanton includes a cylinder diagram with a boundary on the D7-brane (with 4 fields strength insertions) and a boundary on the D(-1) (with 8 fermion zero mode insertions saturating the instanton Goldstinos), see figure 4.2a. As shown in figure 4.2b, there is a similar diagram, with the D7-brane replaced by a second D(-1)-brane instanton, and with no insertions on the correspoding boundary. Labeling the two instantons $1, 2$ to avoid confusion, this diagram represents the correction from D$(-1)_1$ to the action of D$(-1)_2$. Considering now an F^4 term induced by D$(-1)_2$, the inclusion of this correction would naively lead to a contribution to the 8d effective action schematically of the form

$$\int d^8x \,\mathrm{tr}\, F^4 \, e^{-(S_2+e^{-S_1})} = \int d^8x \,\mathrm{tr}\, F^4 \sum_{n=0}^{\infty} \frac{1}{n!}\, e^{-S_2}\, (e^{-S_1})^n \tag{4.90}$$

Microscopically, the n^{th} term corresponds to a polyinstanton process with one D$(-1)_2$ instanton and n independent D$(-1)_1$ instantons. The zero modes of the latter are saturated through D$(-1)_1$-D$(-1)_2$ cyclinders as in Figure 4.2b. The result involves an integration over the relative positions of the instantons in the 8d space, just like in the 4d case [46]. The contribution is therefore in principle not localized on coincident instantons, as opposed to the multiinstantons studied in [44]. In particular, since the polyinstantons in general sit at different locations in the internal space, the saturation of fermion zero modes can take place independently of the distances among instantons in 8d.

Polyinstantons and heterotic-type II orientifold duality

It is straightfoward to use the F^4 results in previous sections to compute these effects exactly (i.e. by summing over multiple instantons of each kind), in particular for D(-1)-brane instantons sitting on top of D7-branes. However, a general analysis, together with the type I' interpretation in terms of the 9d one-particle BPS spectrum, suffices to make the clash with the heterotic result manifest, and to suggest its resolution.

The 8d corrections arising from standard D(-1)-brane instantons correspond under T-duality to one-loop diagrams of 9d BPS D0-brane one-particle states. These are directly translated to one-loop diagrams of 9d BPS states in the heterotic dual, reproducing the genus one worldsheet instanton contributions. This contribution in principle includes certain D-brane multi-instantons, namely those T-dual to 9d particles which form BPS bound states at threshold, and whose hallmark is that their contribution is localized on configurations of coincident instantons.

Polyinstanton processes however involve instantons whose T-dual particles do not combine into 9d one-particle BPS bound states. This is manifest as in general the individual instantons sit at different points in the internal space, and this separation can persist in the type I' dual, e.g. when they map to D0-branes on different $SO(16)$ boundaries. Therefore they are manifestly not included in the one-loop diagram of one-particle BPS states, and hence in the heterotic genus one worldsheet contribution.

There is a clear way out of this potential clash with duality. The heterotic genus one worldsheet diagram (and so the type I' one-loop diagram) computes the one-loop correction to the 1PI action. Namely it includes the effects of massless states (and is hence non-holomorphic) but does not include reducible contributions. These can be later generated by computing tree level diagrams using the effective vertices of the 1PI action. We will now argue that D-brane polyinstanton effects actually correspond to such reducible diagrams, and hence do not contribute to the 1PI action, restoring agreement between type II orientifolds and their heterotic duals.

Polyinstantons in spacetime as reducible diagrams

In type IIB the polyinstantons correspond to individual D(-1)-instantons joined by cylinder diagrams. As the instantons are in general located at (possibly widely) different locations in 8d space, it is natural to interpret the cylinders as a tree level closed string exchange, and the corresponding processes as reducible. Thus polyinstantons do not induce new terms in the microscopic 1PI action, but are rather generated by Wick contractions of other elementary effective vertices in the 1PI action. The picture is particularly clear in the type I' model, where the polyinstanton is given by a Feynman diagram with a loop of BPS particles with four field strength insertions, joined by closed string propagators to other loops of BPS particles (which can be subsequently joined to other propagators and loops), see Figure 4.3.

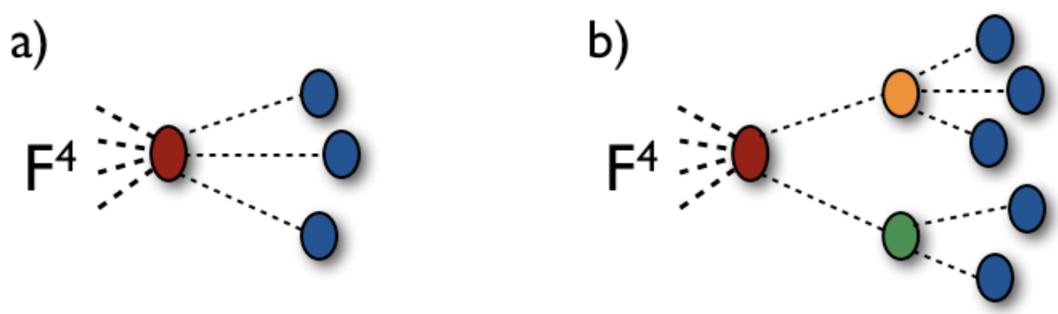


Figure 4.3: Polyinstanton processes as reducible spacetime Feynman diagrams. Colored blobs denote elementary instanton interactions in the 1PI action, joined by propagating closed string modes. Summing over polyinstanton processes like a), with arbitrary numbers of blue blobs, reproduces an effective exponential correction to the action of the red blob instanton. Figure b) shows richer polyinstanton processes, involving the elementary interactions described by (4.91) and (4.93).

The exponential combinatorics of polyinstantons in (4.90) is simply the combinatorics of spacetime Feynman diagrams with two basic kinds of interaction diagrams, see Figure 4.3a. For massless closed string states, these can be explicitly obtained in the factorization limit. One basic interaction vertex corresponds to an instanton coupling to F^4 with emission of n massless closed string states. It simply follows from expanding the F^4 instanton corrections in the fluctuations of the dynamical modulus controling the instanton action, in our case τ. Expanding it into a vev plus fluctuation, $\tau_0 + \tau$, each F^4 instanton correction produces terms of the following form,

$$q^N \,\mathrm{tr}\, F^4 \;\to\; \sum_{n=0}^{\infty} \frac{(2\pi i N)^n}{n!}\, q_0^N \,\mathrm{tr}\, F^4\, \tau^n \ . \tag{4.91}$$

The second kind of vertex is the emission of a massless closed string from an instanton. Although unfamiliar, this contribution indeed exists, as follows. The fluctuation τ is a complex scalar belonging to a multiplet whose on-shell structure has the form

$$\Phi_\tau \,=\, \tau \,+\, \ldots + \theta^8\,\partial^4\overline{\tau}\,. \tag{4.92}$$

This follows from the orientifold truncation of the chiral on-shell superfield Φ, satisfying $\bar{D}\Phi = 0$, $D^4\Phi = \bar{D}^4\bar{\Phi} = 0$, in [166]. This gives the supersymmetric completion of the instanton action, and the last term corresponds to an interaction saturating all the instanton goldstino zero modes with one insertion of $\overline{\tau}$. Therefore one generates the couplings

$$\int d^8x\, d^8\theta\, e^{2\pi i N(\tau_0+\Phi_\tau)} \;\to\; \int d^8x\; q_0^N\; \partial^4\overline{\tau} \,+\, \cdots\,. \tag{4.93}$$

The term we required has been separated out explicitly. The other terms can be used to construct more involved diagrams, as in Figure 4.3b.

As we will argue in the next section, the interpretation of polyinstantons as reducible diagrams remains valid in 4d with minor modifications, due only to the lower supersymmetry which leads to a reduced number of instanton fermion zero modes. The relevant cylinder diagram stretching between the different instantons saturates 4 fermion zero modes on a boundary, while the other remains empty. The cylinder can be regarded as a tree exchange of closed strings, and in the factorization limit the two basic interactions are analogous to (4.91), (4.93) with simple modifications: reduction $F^4 \to F^2$ and $\theta^8\partial^4\bar{\tau} \to \theta^4\partial^2\bar{\tau}$. Hence the ideas apply to the 4d $\mathcal{N}=2$ K3 compactifications in section 4.3.5, and also to the 4d $\mathcal{N}=1$ orbifolds thereof considered in [46], see also [132].

The fact that polyinstantons contribute to Wilsonian actions is consistent with many of the physical arguments in [46]. For instance, consider a gauge sector on a stack of D-branes whose gauge kinetic function receives non-perturbative corrections from other instantons. If the gauge sector develops a gaugino condensate, its scale is determined by the full gauge kinetic function (including its exponential corrections).

Effective 1PI and Wilsonian actions

In the previous discussion, the 1PI action in principle contains couplings to massive closed string fields. It does not integrate out reducible diagrams with massive particle exchange, therefore it should be considered a microscopic 1PI action, rather than an effective 1PI action. However, it is very convenient for the comparison of one-loop corrections between orientifolds and heterotic.

This microscopic 1PI action is not direcly related to the Wilsonian action, therefore there is no conflict with the existence of local polyinstanton terms in the latter, which are absent in the former. However, it is customary to define an *effective* 1PI, integrating out reducible diagrams with exchange of massive particles (but not reducible diagrams with exchange of light particles). The local terms in this action directly relate to the Wilsonian action, and do contain polyinstanton terms.

It is worthwhile to mention that our BPS particle viewpoint provides an efficient resummation tool for these effects. For instance, consider the one-instanton contribution to $\mathrm{tr}\, F^4$

terms, with general structure

$$\mathrm{tr}\, F^4 (\sum_k N_k q(\tau)^k) \tag{4.94}$$

This implies a modification of τ, equivalently of the instanton action to

$$\tau' = \tau + \sum_k N_k q(\tau)^k \tag{4.95}$$

So the $\mathrm{tr}\, F^4$ correction including polyinstantons (those with only one level of branches form the 'main' instanton) is obtained by replacing $\tau \to \tau'$ in (4.94), namely the coefficient of the $\mathrm{tr}\, F^4$

$$\sum_k N_k q(\tau')^k = \sum_k N_k \exp[2\pi i k (\sum_r N_r q^r)] = \sum_k N_k q(\tau)^k + \sum_{k,r} N_k N_r q^k q^r + \ldots \tag{4.96}$$

Further iterations would produce contributions from polyinstantons with more levels of branches from the main instanton.

In this picture the polyinstantons on the orientifold side arise from higher loop diagrams. These corrections (or their 4d $\mathcal{N}=2$ version) do not violate non-renormalization theorems, since these are loop diagrams in the 9d theory. Given the one-to-one map between heterotic/orientifold 9d BPS states, we expect a similar higher-loop interpretation of the above terms in the heterotic picture.

4.3.5 Compactification

In this section we consider compactification of the 8d theory on K3, leading to models with 4d $\mathcal{N}=2$ supersymmetry. From the type II perspective, the models can be regarded as an orientifold of a K3$\times\mathbf{T}^2$ compactification. Cancellation of RR tadpoles requires the gauge D-branes to carry a non-trivial gauge bundle with instanton number 24 (assuming no spacetime filling D3-branes in the IIB model, or D4-branes in the type I'). For concreteness, we focus on models where the internal bundle can be embedded in $SO(16)\times SO(16)$, so that in the type I' picture the D8-branes on each boundary carry instanton numbers $(12+n, 12-n)$. Focusing on a given $SO(16)$, the K3 gauge bundle with structure group K breaks the gauge group to the commutant H. This can be further broken to its Cartan subalgebra by turning on generic D-brane Wilson lines or positions on $\mathbf{T}^2$. These compactifications are very similar (and often dual) to $E_8 \times E_8$ models on K3$\times\mathbf{T}^2$ studied in e.g. [167–174]. The 4d theory contains hypermultiplets describing the K3 moduli and the compactification bundle moduli, and vector multiplets describing the gauge D-brane Wilson lines and/or positions on $\mathbf{T}^2$. There is a further vector multiplet containing the dilaton, arising from the $\mathbf{T}^2$ compactification of a 6d tensor multiplet. In certain orientifolds of K3 orbifolds there are additional vector multiplets arising from 6d tensor multiplets, determined by the orientifold projection on twisted sectors [175]. Although such models do not admit a perturbative heterotic dual, the non-perturbative corrections to their effective action can be studied from the type II orientifold side with our present techniques.

Due to the reduction of the supersymmetry, the non-perturbative corrections from BPS instantons with minimal number of fermion zero modes correspond to terms $\mathrm{tr}\, F^2$, namely contributions to the vector multiplet prepotential, or to gravitational corrections $\mathrm{tr}\, R^2$, which can be included in a generalized prepotential. D-brane instantons with additional neutral fermion zero modes may contribute to higher F-terms, but we rather focus on the minimal case. Due to the familiar $\mathcal{N} = 2$ decoupling theorems, the corrections are independent of the hypermultiplets, so they are insensistive to the K3 geometry or the bundle moduli. The resulting effective action is therefore fixed in terms of the topological properties of the model, namely the instanton number n.

In the type IIB picture, there are perturbative contributions to the prepotential, as well as non-perturbative contributions arising from D(-1)-brane instantons and euclidean D3-branes wrapped on K3. In the type I' picture these contributions can be computed as a one-loop diagram of 5d BPS particles. These correspond to either perturbative 9d states (open strings winding along the interval) in quantum groundstates on K3, 9d D0-branes (in the bulk or at the boundaries, and labeled by the D0-brane charge) in quantum groundstates on K3, or genuinely 5d particles arising from D4-branes wrapped on K3. Although all these particles are on equal footing at the level of the 5d BPS spectrum, the BPS multiplicities of the D4-brane particle states and their quantum numbers under the unbroken gauge groups are not known, and we skip their discussion. In what follows we focus on the D0-brane particles, with the discussion of the perturbative states being similar. We also consider trivial Wilson lines, whose further inclusion is straightforward.

The D(-1)-brane instanton contribution is T-dual to a one-loop diagram of 5d particles corresponding to the quantum groundstates of D0-branes on K3, with zero momentum on the interval. The 5d BPS degeneracies are determined by the cohomology of the D0-brane quantum mechanics problem on K3. Focusing on corrections to the gauge kinetic function of the 4d gauge group H arising from a 9d $SO(16)$ factor, there are non-perturbative contributions from the D0-branes at the corresponding boundary. These transform in the representations **120** or **128**, which we denote generically R_{16}, so their dynamics is coupled to the K3 gauge bundle. Using the decomposition

$$\begin{aligned} SO(16) &\;\to\; K \times H \\ R_{16} &\;\to\; \sum_i (\, R_{K,i}, R_{H,i}) \end{aligned} \tag{4.97}$$

a 9d D0-brane state in the $R_{SO(16)}$ produces a number n_{R_H} 5d D0-brane BPS states in the representation R_H, given by the index of the relevant Dirac operator

$$\begin{aligned} n_{R_H} &= \sum_i \int_{\mathrm{K3}} \mathrm{Ch}_{R_{K,i}}(F)\, \hat{\mathrm{A}}(R) \\ &= \sum_i \int_{\mathrm{K3}} \left(\mathrm{tr}_{R_{K,i}} F^2 + \dim R_{K,i}\, \mathrm{tr}\, R^2 \right) \end{aligned} \tag{4.98}$$

The contribution from these 5d particles to the gauge kinetic function of H can be obtained from the general expression (4.2) for $d = 5$, $k = 2$. Since $d/2 - k = 1/2$, it can be recast in the form (4.42), formally identical (and for good reasons as we will see) to F^4 corrections from 9d particles. It is thus given by

$$\Delta S_{4d} = \sum_m \frac{1}{m} \sum_i q^{n_{R_{16}}\, m}\, n_{R_H}\ \mathrm{tr}_{\,R_H} F^2 \tag{4.99}$$

Here $n_{R_{16}} = 2n, 2n+1$ is the D0-brane charge of a 9d state in the $R_{16} = \mathbf{120}, \mathbf{128}$ of $SO(16)$, respectively. The inclusion of general Wilson lines and brane positions is straightforward, following section 4.3.2. Note that an expression similar to (4.99) is valid for the contribution of other 5d BPS states, like perturbative states or D4-brane particles, by simply replacing the corresponding instanton exponential weight q, and using the appropriate BPS multiplicities n_{R_H}. The latter are straightforward to obtain for perturbative states, but are not known for D4-brane particles.

For 5d D0-branes, the expression (4.99) can be recast as

$$\begin{aligned} \Delta S_{4d}^{D0} &= \sum_m \frac{1}{m} \sum_i q^{n_{R_{16}}\, m} \sum_i \int_{\mathrm{K3}} \left(\mathrm{tr}_{R_{K,i}} F^2 + \dim R_{K,i}\, \mathrm{tr}\, R^2 \right) \int_{4d} \mathrm{tr}_{\,R_H} F^2 \\ &= \sum_m \frac{1}{m}\, q^{n_{R_{16}}\, m} \int_{\mathrm{K3}\times M_4} \left(\mathrm{tr}_{\,R_{16}} F^4 + \mathrm{tr}_{\,R_{16}} F^2\, \mathrm{tr}\, R^2 \right) \\ &= \sum_m \frac{1}{m}\, q^{n_{R_{16}}\, m} \int_{\mathrm{K3}\times M_4} \mathrm{Ch}_{R_{16}}(F)\, \hat{\mathrm{A}}(R) \end{aligned} \tag{4.100}$$

where in the last equality we have included the purely gravitational terms, which are computed similarly, including the contribution from 9d bulk D0-brane states. Eq. (4.100) shows that the contribution from 5d D0-brane states can be obtained by simple dimensional reduction on K3 of the F^4 corrections in the 8d theory. This follows from the fact that the contributing 5d particles are just groundstates of the 9d particles, so no information is lost in the dimensional reduction truncation. From the type IIB viewpoint, the D(−1) instantons of the 4d theory are essentially those of the 8d theory, with 8 fermion zero modes lifted by the interaction with the curvature and gauge bundle on K3. Careful saturation of the latter for the diverse amplitudes should reproduce the prefactor corresponding to the index of the Dirac operator discussed above, which in this language should be regarded as the Euler characteristic of the relevant instanton moduli space. It is interesting that the type I' picture provides an alternative, and very transparent, interpretation of this factor.

The above argument can be repeated for the perturbative type I' contribution. Hence the 8d prepotential directly produces a large part of the 4d $\mathcal{N} = 2$ prepotential (as applied in certain local orbifolds in [146]) and many of the properties of the latter are inherited to the former. This applies in particular to our interpretation of the polyinstanton processes in section 4.3.4. There is furthermore no obstruction to applying further freely acting orbifold quotients which reduce the supersymmetry to 4d $\mathcal{N} = 1$, maintaining the basic properties of the instantons and polyinstantons, as in [46].

4.A Trace structures

This appendix provides some trace identities used in the main text. Most can be extracted from [176], except the pfaffian contribution for $SO(8)$, which we have directly computed.

$$\begin{aligned}
&\mathbf{SO(16)}\\
&\mathrm{tr}_{\,\mathbf{120}} F^4_{SO(16)} = 8\mathrm{tr}\, F^4_{SO(16)} + 3(\mathrm{tr}\, F^2_{SO(16)})^2\\
&\mathrm{tr}_{\,\mathbf{128}} F^4_{SO(16)} = -8\mathrm{tr}\, F^4_{SO(16)} + 6(\mathrm{tr}\, F^2_{SO(16)})^2
\end{aligned} \tag{4.101}$$

$$\begin{aligned}
&\mathbf{SO(8)}\\
&\mathrm{tr}_{\,\mathbf{8_s}} F^2_{SO(8)} \;=\; \mathrm{tr}_{\,\mathbf{8_c}} F^2_{SO(8)} \;=\; \mathrm{tr}\, F^2\\
&\mathrm{tr}_{\,\mathbf{28}} F^4_{SO(8)} \;=\; 3\,(\mathrm{tr}\, F^2_{SO(8)})^2\\
&\mathrm{tr}_{\,\mathbf{8_s}} F^4_{SO(8)} \;=\; -\frac{1}{2}\,\mathrm{tr}\, F^4_{SO(8)} \;+\; \frac{3}{8}\,(\mathrm{tr}\, F^2_{SO(8)})^2 \;-\; 12\mathrm{Pf}\, F\\
&\mathrm{tr}_{\,\mathbf{8_c}} F^4_{SO(8)} \;=\; -\frac{1}{2}\,\mathrm{tr}\, F^4_{SO(8)} \;+\; \frac{3}{8}\,(\mathrm{tr}\, F^2_{SO(8)})^2 \;+\; 12\mathrm{Pf}\, F
\end{aligned} \tag{4.102}$$

where

$$\mathrm{Pf}\, F = \frac{1}{2^8}\, t_8^{\mu_1\ldots\mu_8}\, \varepsilon_{a_1\ldots a_8}\, F^{a_1a_2}_{\mu_1\mu_2}\cdots F^{a_7a_8}_{\mu_7\mu_8} \tag{4.103}$$

where t_8 is the Lorentz $SO(8)$ antisymmetric invariant tensor.

4.B Characteristic classes

The computation of an amplitude with external gauge field strength or curvature insertions can be performed by computing in the presence of a general gauge or curvature background and selecting the appropriate term in a power expansion. The computation in general backgrounds is a standard tool in the computation of anomalies, from which we can borrow the results, see e.g. [163]. In those conventions, the Chern character is given by

$$\mathrm{ch}(F) = \mathrm{tr}_{\,\mathbf{R}}\left[\exp\left(\frac{iF}{2\pi}\right)\right] = r + \frac{i}{2\pi}\mathrm{tr}_{\,\mathbf{R}} F - \frac{2}{(4\pi)^2}\mathrm{tr}_{\,\mathbf{R}} F^2 - \frac{i}{6(2\pi)^3}\mathrm{tr}_{\,\mathbf{R}} F^3 + \frac{2}{3(4\pi)^4}\mathrm{tr}_{\,\mathbf{R}} F^4 + \cdots \tag{4.104}$$

where $r = \mathrm{tr}_{\,\mathbf{R}}\mathbf{1}$ denotes the dimension of the representation $\mathbf{R}$ of the gauge group. The A-roof genus (relevant for spin 1/2 particles) is

$$\widehat{A}(R) = 1 + \frac{1}{12(4\pi)^2}\mathrm{tr}\, R^2 + \frac{1}{(4\pi)^4}\left[\frac{1}{288}\left(\mathrm{tr}\, R^2\right)^2 + \frac{1}{360}\mathrm{tr}\, R^4\right] + \cdots \tag{4.105}$$

while the polynomial relevant for spin 3/2 particles is given by

$$\mathrm{tr}\left[\exp\left(\frac{R}{2\pi}\right)\right] = k + \frac{1}{2\pi}\mathrm{tr}\, R + \frac{2}{(4\pi)^2}\mathrm{tr}\, R^2 + \frac{1}{6(2\pi)^3}\mathrm{tr}\, R^3 + \frac{2}{3(4\pi)^4}\mathrm{tr}\, R^4 + \cdots \tag{4.106}$$

where $k = \mathrm{tr}\,\mathbf{1}$. Since our D0-particles are in the $\mathbf{8_v} + \mathbf{8_s}$ of the $SO(8)$ Lorentz group, $k = 8$.

In the Horava-Witten picture we get spin 1/2 states from the boundaries and spin 3/2 states from the bulk. The boundary states carry one spinor index and one gauge index (in representation $\mathbf{R}$) and thus the contribution from the spin 1/2 states is obtained from

$$\begin{aligned}\widehat{A}(R)\ \mathrm{ch}(F) &= r - \frac{2}{(4\pi)^2}\mathrm{tr}_{\mathbf{R}}F^2 + \frac{r}{12(4\pi)^2}\mathrm{tr}\,R^2 \\ &\quad + \frac{1}{(4\pi)^4}\left[\frac{2}{3}\mathrm{tr}_{\mathbf{R}}F^4 - \frac{1}{6}\mathrm{tr}\,R^2\mathrm{tr}_{\mathbf{R}}F^2 + \frac{r}{288}\left(\mathrm{tr}\,R^2\right)^2 + \frac{r}{360}\mathrm{tr}\,R^4\right] + \cdots \end{aligned} \tag{4.107}$$

The contribution from the gravitino and its spin 1/2 partner is obtained from

$$\widehat{A}(R)\ \mathrm{tr}\left[\exp\left(\frac{R}{2\pi}\right)\right] = 8 + \frac{17}{6(4\pi)^2}\mathrm{tr}\,R^2 + \frac{1}{(4\pi)^4}\left[\frac{56}{288}\left(\mathrm{tr}\,R^2\right)^2 + \frac{248}{360}\mathrm{tr}\,R^4\right] + \cdots \tag{4.108}$$

Note that the spin 3/2 states (bulk) can only give a contribution corresponding to the $2n$-tower since the $(2n-1)$-tower will always have one (unpaired) D0-particle stuck at the O8-plane.

Chapter 5

Discrete gauge symmetries in D-brane models

We turn now to a somewhat more phenomenological aspect of non-perturbative effects of string compactifications. We describe here how, in some situations, discrete gauge subgroups of gauge $U(1)$'s with a Stückelberg mass survive even at the non-perturbative level, thus preventing some couplings to appear in the effective action.

The subject is extremely important for string theory model building. First, as we discussed extensively in chapter 3, instantons effectively break "global" $U(1)$ symmetries of the low energy theory, and are responsible for the generation of couplings that are perturbatively forbidden. These terms are most welcome since they may be needed for phenomenological purposes. Some examples include Majorana masses for neutrinos, μ terms in supersymmetric models, or perturbatively forbidden Yukawa couplings. If the breakdown of the symmetry is not complete, but a discrete remnant, say $\mathbb{Z}_n \subset U(1)$, survives even non-perturbatively, some of the couplings may be impossible to generate. In some cases, these symmetries will render apparently appealing models unviable for phenomenological purposes.

On the other hand, discrete symmetries may work in our favor, since they can forbid unwanted terms in the Lagrangian. In fact, they are a common tool at the purely field theoretical level, in the study of extensions of the Standard Model. They have been invoked for example in order to guarantee the absence of flavour changing neutral currents (FCNC) in two-Higgs models or in flavour models of fermion masses. In the context of the MSSM, discrete symmetries seem unavoidable in order to explain the observed baryon stability. Although indeed such symmetries do their phenomenological job, their fundamental origin is quite obscure.

If string theory is to describe the observed physics, a natural question is whether discrete gauge symmetries arise in string compactifications. Unlike in field theory, in string theory constructions we are not free to impose any symmetry, rather one should determine whether they are present or not in each given model.

In this chapter we study systematically the appearance of discrete gauge symmetries in type IIA models of intersecting D6-branes. We will hence make extensive use of the results of chapters 2 and 3. Our discussion is however expected to be quite general and apply to other constructions like type IIB models with magnetized D-branes, D-branes at singularities, etc, as expected from mirror symmetry. Also, they should admit a lift to M-theory on G_2 manifolds, along the lines of [190]. We will make some comments about discrete gauge symmetries in the context of F-theory in section 5.3

5.1 Discrete gauge symmetries and D-branes

A simple way to obtain a $\mathbb{Z}_N$ symmetry would be to break spontaneously some non-anomalous $U(1)$ symmetry by giving a vev to a field ϕ with charge N. Any term of the Lagrangian would be of the schematic form

$$\mathcal{L} \sim \langle\phi\rangle^k \, \mathcal{O}(\Psi)\,, \tag{5.1}$$

where $\mathcal{O}$ is a field-dependent operator, necessarily of $U(1)$ charge $-kN$. It is clear that the $\mathbb{Z}_N \in U(1)$ group consisting of transformations $\psi_q \to e^{\frac{2\pi\, i\, q}{N}}\psi_q$ would still be a symmetry of the lagrangian, even after the stabilization of ϕ. This mechanism works as long as there exist some other field in the spectrum with a charge that is mutually prime with N. Otherwise one could redefine the charges of the fields as $q \to \frac{q}{\text{g.c.d.}\{q_i\}}$ and still preserve charge integrality. In the following, we assume that the charges of every $U(1)$ are normalized to g.c.d.$\{q_i\} = 1$.

In string models one could in principle consider D- and F-flat directions in the scalar potential in which an appropriate scalar with charge N gets a vev (see e.g. ref. [189] for an attempt in this direction in the heterotic setup). However this mechanism is very much dependent on the existence and dynamical preference for a particular choice of flat direction. The assessment of the existence of the symmetry thus requires a delicate analysis of this point. We would rather like to know whether there is a mechanism within string theory by which interesting discrete gauge symmetries survive in a natural way, without tuning scalar vevs to that purpose.

5.1.1 Discrete gauge symmetries from $B \wedge F$ couplings

In this section we describe the appearance of discrete gauge symmetries in intersecting D6-brane models from the analysis of their $B \wedge F$ couplings. The mechanism by which discrete gauge remnants of $U(1)$ gauge symmetries with Stückelberg masses is reminiscent of a field theory argument given by Banks and Seiberg in [181].

We described in detail the setups of interest in chapter 2. Let us recall that the coupling that generates the masses of the $U(1)$ gauge bosons is of the form (2.32)

$$S_{BF} = \sum_{k=0}^{h_{2,1}} N_a \, s_{ak} \int_{4d} B_2^k \wedge F_{U(1)_a}\,, \tag{5.2}$$

where, in the language of section 2.3.4, $s_{ak} = [\alpha_k]\cdot[\Pi_a]$. This coupling had several important consequences that were extensively discussed in the review chapters. In particular, they were responsible for the GS mechanism, and the non trivial transformation properties (2.35) of the axions a_k dual to B_2^k

$$A_a \to A_a + d\Lambda_a\,, \qquad a_k \to a_k - N_a\, s_{ak}\, \Lambda_a\,. \tag{5.3}$$

This transformation translates to the complex structure chiral fields U_k, whose imaginary parts are just the axions. The transformation (5.3) implies in fact that the instanton measure $e^{-U_{\mathcal{E}}}$ behaves as a charged collective field, i.e. $e^{-U_{\mathcal{E}}} \to e^{q\Lambda_a}\, e^{-U_{\mathcal{E}}}$, with the charge q given by

$$q = N_a\,[\Pi_{\mathcal{E}}]\cdot[\Pi_a] = r_{\mathcal{E}}^k\, N_a\, s_{ak}\,, \tag{5.4}$$

where we have expanded the cycle wrapped by the instanton as $[\Pi_{\mathcal{E}}] = r_{\mathcal{E}}^k\,[\alpha_k]$.

Now, stabilization of the complex structure moduli leads to the breakdown of $U(1)$ groups through instanton effects because of this charge. It is clear then that if all instantons carry a charge which is a multiple of some integer n, the perturbatively exact $U(1)$ will be broken only to a discrete subgroup $\mathbb{Z}_n$. From (5.4) we can see that the necessary and sufficient condition to preserve a $\mathbb{Z}_n \subset U(1)_a$ is that[1]

$$N_a\,[\alpha_k]\cdot[\Pi_a] = N_a\, s_{ak} = 0 \mod n\,, \tag{5.5}$$

for all $k = 0,\ldots,h_{2,1}(\mathbf{X}_6)$. The mechanism is a non-perturbative and topological (and hence robust) version of the one discussed around eq. (5.1).

Notice that condition (5.5) is simply the statement that all of the $B\wedge F$ coupling or the gauge brane have coefficients which are multiple of k. In general, the factor of N_a implies the appearance of a $\mathbb{Z}_{N_a}$ discrete gauge symmetry. This corresponds to the general fact that the actual gauge group on a stack of N D-branes is $[SU(N)\times U(1)]/\mathbb{Z}_N$, with the $\mathbb{Z}_N$ corresponding to the center of $SU(N)$, i.e. the N-ality. Namely, the group element $\mathrm{diag}(\alpha,\ldots,\alpha)$ with $\alpha = e^{2\pi i/N}$ can be regarded as belonging to $SU(N)$ or to the diagonal $U(1)$; the quotient by $\mathbb{Z}_N$ implies that the two possibilities should be regarded as completely equivalent. The charges of fields under this $\mathbb{Z}_N$ are given by their N-ality, and so this $\mathbb{Z}_N$ does not imply any selection rule beyond $SU(N)$ gauge invariance; hence, it is not very interesting by itself.

In general, we may be interested in discrete subgroups of $U(1)$ linear combinations of the form

$$Q \;=\; \sum_a c_a\, Q_a \tag{5.6}$$

In order to properly identify the discrete gauge symmetry from the $B\wedge F$ coupling, we fix the normalization such that $c_a \in \mathbb{Z}$, and $\gcd(c_a) = 1$. The $B\wedge F$ couplings read

$$S_{BF} \;=\; \big(\textstyle\sum_a c_a N_a\, s_{ak}\big)\, B^k \wedge F \tag{5.7}$$

where F is the field strength associated to the Q generator. So there is a $\mathbb{Z}_n$ gauge symmetry if the quantities $(\sum_a c_a N_a s_{ak})$ are multiples of n, for all k, i.e. if

$$\sum_a c_a N_a [\Pi_a]\cdot[\alpha_k] \;=\; 0 \mod n, \quad \forall k\,. \tag{5.8}$$

In our normalization, fields in the fundamental of $SU(N_a)$ have $U(1)_a$ charges $q_a = 1$, while fields in the two-index symmetric or antisymmetric tensor representation have $q_a = 2$ (and the opposite charges for the conjugate representations). For a field with charges q_a under the $U(1)_a$, its charge under the $\mathbb{Z}_n$ symmetry is $\sum_a c_a\, q_a \mod n$.

[1] Notice that we have to require that all instanton effects, either supersymmetric or not, preserve the discrete symmetry. Therefore, we have to consider any possible values of $r_{\mathcal{E}}^k$.

5.1.2 Discrete anomaly cancellation

There are diverse arguments strongly suggesting that global symmetries, either continuous or discrete, are expected to be broken by quantum gravitational effects, and hence cannot exist in any consistent quantum theory including gravity (see [177–179] for early viewpoints, and e.g. [180, 181] and references therein, for more recent discussions). This suggests that discrete symmetries should have a gauge nature so that they are respected by such corrections [182–184]. Similarly to continuous gauge symmetries, discrete gauge symmetries should respect certain anomaly cancellation conditions, which strongly restrict the possibilities in specific theories [185].

The fact that all D-brane instantons (including gauge instantons) preserve these $\mathbb{Z}_n$ symmetries suggests that they are anomaly-free (even if the corresponding $U(1)$'s are anomalous). It is worthwhile to verify this directly, using the conditions in [185].

Let us recall a few results from section 2.3. The RR-tadpole cancellation condition (2.25) reads

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4\,[\Pi_{O6}] = 0\,. \tag{5.9}$$

As discussed in section 2.3.3, The chiral part of the spectrum of these models is

$$\begin{array}{ll} \text{Gauge group} & \prod_a U(N_a) \\ \text{Ch. fermions} & \sum_{ab} I_{ab}\,(\Box_a,\overline{\Box}_b) \,+\, \sum_{ab'} I_{ab'}\,(\Box_a,\Box_b) \,+ \\ & +\,\sum_a \left(\frac{I_{aa'}+I_{a,O6}}{2}\, \begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a \,+\, \frac{I_{aa'}-I_{a,O6}}{2}\, \Box\!\Box_a \right)\,. \end{array} \tag{5.10}$$

In our normalization, fields in the fundamental of $SU(N_a)$ have $U(1)_a$ charges $q_a = 1$, while fields in the two-index symmetric or antisymmetric tensor representation have $q_a = 2$ (and the opposite charges for the conjugate representations). For a field with charges q_a under the $U(1)_a$, its charge under the $\mathbb{Z}_n$ symmetry is $\sum_a c_a q_a$ mod n. Together with condition (5.8), these results are enough to check that discrete gauge anomalies indeed cancel in our setups.

The mixed $\mathbb{Z}_n - SU(N_b)^2$ anomaly is

$$\begin{aligned} &\sum_a c_a\, N_a\, \frac{1}{2}(I_{ab} + I_{ab'}) = \frac{1}{2}\sum_a c_a\, N_a\,[\Pi_a]\cdot([\Pi_b] + [\Pi_{b'}]) = \\ &= \frac{1}{2}\sum_k 2r_b^k \sum_a c_a\, N_a\,[\Pi_a]\cdot[\alpha_k] \end{aligned} \tag{5.11}$$

where the factor of $\frac{1}{2}$ arises from the $SU(N_b)$ quadratic Casimir in the fundamental. Using (5.8), the above expression is of the form $\frac{1}{2}n$ times an integer, as required by anomaly cancellation.

For the mixed $\mathbb{Z}_n$-gravitational anomaly, we have

$$\sum_a c_a \left\{ \sum_b N_a N_b\, I_{ab} + \sum_{b'\neq a'} N_a N_b\, I_{ab'} + 2\, \frac{I_{aa'}+I_{a,\mathrm{O6}}}{2}\, \frac{N_a(N_a-1)}{2} + \right.$$
$$\left. + 2\, \frac{I_{aa'}-I_{a,\mathrm{O6}}}{2}\, \frac{N_a(N_a+1)}{2} \right\} = \tag{5.12}$$
$$= \sum_a c_a \left(\sum_b N_a N_b\, I_{ab} + \sum_{b'} N_a N_b I_{ab'} - N_a I_{a,\mathrm{O6}} \right) = 3 \sum_a c_a N_a\, I_{a,\mathrm{O6}}$$

where in the last line, the sum in b' includes a', and in the last equality we use the tadpole condition (5.9). Since $[\Pi_{\mathrm{O6}}]$ must be an integer linear combination of the 3-cycles $[\alpha_k]$, the condition (5.8) ensures that the last expression is a multiple of n, as required by anomaly cancellation.

The cancellation of mixed anomalies with other $U(1)$'s proceeds in an analogous fashion. The cubic $\mathbb{Z}_N^3$ anomalies on the other hand cancel as an automatic consequence of the $U(1)^3$ anomaly cancellation in this setting.

It is interesting to compare the situation with discrete gauge symmetries in heterotic compactification, studied mostly in the context of toroidal orbifolds. Discrete gauge symmetries arise from continuous $U(1)$'s broken by vevs of dynamical fields with charge n. If the $U(1)$ is the (unique) anomalous one, it is possible to generate anomalous discrete gauge symmetries [187], with anomaly canceled by the Green-Schwarz mechanism, as for the parent $U(1)$. The physical interest of this situation leans on the fact that instantons violating the discrete symmetry are necessarily very much suppressed, since their strength is controlled by SM gauge couplings. In our present D-brane constructions, we are implicitly focusing on symmetries preserved by *any* instanton, which are hence non-anomalous. Still, there are situations in which it may be physically meaningful to relax this requirement and consider anomalous discrete symmetries. For instance, in models where a subset of instantons have large strength (e.g. to generate neutrino masses or Yukawa couplings), while the remaining are hierarchically suppressed in comparison. Then, discrete symmetries respected by the former and violated by the latter could be anomalous, and behave similarly to the above mentioned heterotic ones.

5.2 Discrete gauge symmetries and Standard Model brane constructions

We now turn to the study of discrete gauge symmetries in brane constructions of phenomenological interest. We first review the classification of discrete gauge symmetries of the MSSM in [186], and later study its implementation in various proposed D-brane realizations of MSSM-like models.

5.2.1 Discrete gauge symmetries in the MSSM

A crucial difference between the non-SUSY SM and the MSSM is that in the latter the most general dimension four effective Lagrangian respects neither baryon- nor lepton-number conservation. The most general superpotential consistent with gauge invariance and leading to dimension four operators has the structure

$$\begin{aligned} W_{\rm MSSM} &= Y_U^{ij}\, Q_iU_jH_u \,+\, Y_D^{ij}\, Q_iD_jH_d \,+\, Y_L^{ij}\, L_iE_jH_d \,+\, \mu\, H_uH_d \,+ \\ &+\, \lambda^{ijk}\, U_iD_jD_k \,+\, \lambda^{ijk'}\, Q_iD_jL_k \,+\, \lambda^{ijk''}\, L_iL_jE_k \,+\, \mu_R^i\, L_iH_u\,. \end{aligned} \tag{5.13}$$

Here $(Q, U, D, L, E, N, H_u, H_d)$ are the MSSM quark, lepton and Higgs superfields in standard notation. The first line contains the usual Yukawa couplings and the μ-term, and respects baryon and lepton number; the UDD terms in the second line violate baryon-number in one unit, whereas the rest violate lepton-number also in one unit. If all the terms in the second line were present and unsuppressed, the proton would decay with a lifetime of a few minutes. The simplest solution to avoid this problem is to assume some discrete symmetry, like e.g. R-parity or baryon triality B_3, forbidding all or some of the couplings in the second line.

In [186] the possible $\mathbb{Z}_N$ generation independent discrete symmetries of the MSSM were classified in terms of the three generators R, L, A given in table 5.1. Defining

$$R_N \;=\; e^{i\,2\pi R/N} \;\;,\;\; L_N \;=\; e^{i\,2\pi L/N} \;\;,\;\; A_N \;=\; e^{i\,2\pi A/N} \;\;, \tag{5.14}$$

a $\mathbb{Z}_N$ gauge symmetry generator may be written as

$$g_N \;=\; R_N^m \times A_N^n \times L_N^p \quad,\quad m,n,p = 0,1,..,N-1\;. \tag{5.15}$$

This is the most general $\mathbb{Z}_N$ symmetry allowing for the presence of all standard Yukawas QUH_u, QDH_d, LEH_d (and also LH_uN_R in the presence of right-handed neutrinos). Note that one can obtain further but equivalent discrete symmetries by multiplying by some power of a discrete subgroup of the hypercharge generator $e^{i\,2\pi(6Y)/N}$, where we use $6Y$ to make hypercharges integer.

	Q	U	D	L	E	N_R	H_u	H_d
R	0	-1	1	0	1	-1	1	-1
L	0	0	0	-1	1	1	0	0
A	0	0	-1	-1	0	1	0	1

Table 5.1: Generation independent generators of discrete $\mathbb{Z}_N$ gauge symmetries in the MSSM.

As discussed in [185,186], the mixed $\mathbb{Z}_N \times SU(3)^2$, $\mathbb{Z}_N \times SU(2)^2$ and mixed gravitational anomaly constraints yield

$$nN_g \;=\; 0 \;\;,\;\; \text{mod } N \tag{5.16}$$

$$(n+p)N_g \;-\; nN_D \;=\; 0 \;\;,\;\; \text{mod } N \tag{5.17}$$

$$-N_g(5n+p-m) \;+\; 2nN_D \;=\; \eta\,\frac{N}{2} \;\;,\;\; \text{mod } N \tag{5.18}$$

where N_g, N_D are the number of generations and Higgs sets respectively and $\eta = 0, 1$ for N =odd, even [2].

[2] In the presence of N_g right-handed neutrinos, which is the generic case in brane models, the mixed gravitational anomaly gets simplified to $-4nN_g + 2nN_D = (\eta/2)N$ mod N.

As discussed in section 5.1.2, only discrete *gauge* symmetries are expected to exist in consistent theories including gravity. Therefore, it is a relevant question to assess the conditions for the above symmetries to be discrete gauge symmetries. A necessary condition is anomaly cancellation. The R_2 symmetry corresponds to the usual R-parity and it is anomaly free (in fact all R_N are anomaly free for any N in the presence of right-handed neutrinos). In addition, for the $N_g = 3$ physical case, there are three anomaly free $\mathbb{Z}_3$'s: L_3, R_3L_3 and $R_3L_3^2$, as the reader may easily check using (5.18). The symmetry $B_3 = R_3L_3$ was introduced in [186] and is usually called *baryon triality*; it allows for dimension 4 operators violating lepton number, but not violating baryon number, so the proton is sufficiently stable. There are also additional $\mathbb{Z}_9$ and $\mathbb{Z}_{18}$ anomaly free discrete symmetries [188] which involve the A_N generators. However, imposing also the purely Abelian cubic condition of [185] and absence of massive fractionally charged states singles out R-parity R_2 and baryon triality B_3.

The phenomenologically interesting couplings allowed or forbidden by these discrete symmetries are displayed in table 5.2. The $\mathbb{Z}_6$ obtained by multiplying R_2 and B_3 is usually called *hexality* [188] and forbids all dangerous couplings but allows for a μ-term and the Weinberg operator LLH_uH_u (and hence left-handed and right-handed neutrino Majorana masses).

	H_uH_d	UDD	QDL	LLE	LH_u	LLH_uH_u	$QQQL$	$UUDE$
R_2		x	x	x	x			
$B_3 = R_3L_3$		x					x	x
L_3			x	x	x	x	x	x
$R_3L_3^2$		x	x	x	x		x	x
$R_2 \times R_3L_3$		x	x	x	x		x	x

Table 5.2: Operators forbidden by the anomaly-free $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries.

This ends our review of anomaly free discrete $\mathbb{Z}_N$ gauge symmetries in the MSSM.

5.2.2 Discrete gauge symmetries in SM-like brane models

We turn now to the appearance of discrete gauge symmetries in explicit SM-like brane models. As in section 2.4, in our examples we will concentrate in toroidal type IIA orientifolds (or orbifolds thereof) with intersecting D6-branes, although from the context it transpires that much of the analysis holds in more general orientifolds; for instance, in the large class of Gepner MSSM-like orientifold models constructed in [194–196]. Similar results also hold in other MSSM-like constructions as well, like type IIB orientifolds with magnetized D-branes, related to IIA models by mirror symmetry (T-duality in the toroidal setup), or in heterotic compactifications with $U(1)$ bundles [199,200]. Similar analysis can in principle be carried out in other setups, like D3/D7-branes at singularities, although the presence of extra multiplets beyond the MSSM ones in these models makes the analysis more model-dependent.

As we saw in the particular example analyzed in section 2.4, much of the analysis of $U(1)$ symmetries of (MS)SM-like orientifolds can be characterized in terms of 'protomodels', i.e. the gauge groups on the relevant sets of D6-branes, and the intersection numbers pattern required to reproduce the chiral matter content. These structures can subsequently be implemented in different compactifications, based on geometric spaces (toroidal or not), or non-geometric

CFT setups. Results based on the protomodel structure are largely independent on their specific realization.

There are two large classes of SM-like orientifolds (toroidal or not), depending on whether the electroweak $SU(2)_L$ group is realized from a $Sp(2)$ group or from a $U(2)$. We analyze only the former, since the latter require the introduction of tilted tori and a generalization of our formalism which is not very illuminating. The general discussion can be found in [49].

The $USp(2)$ brane models

These type of constructions were described in some detail in section 2.4. We make extensive use of the results obtained there, which we review here for clarity.

In this class of models there are four stacks of D-branes, denoted a (*baryonic*), b (*left*), c (*right*) and d (*leptonic*). They have $N_a = 3$, $N_b = 2$, $N_c = N_d = 1$, but the stack b is taken coincident with its orientifold image, so that the initial gauge group is $U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$. The chiral fermion content reproduces the SM quarks and leptons if the D6-brane intersection numbers are given by

$$\begin{array}{rcl} I_{ab} = \ 3 & ; & I_{ac} = I_{ac'} = \ -3 \\ I_{db} = \ -3 & ; & I_{cd} = \ -3 \ ; \ I_{cd'} = \ 3 \ . \end{array} \tag{5.19}$$

with the remaining intersections vanishing. As usual, negative intersection numbers denote positive multiplicities of the conjugate representation. The spectrum of chiral fermions was shown in table 2.1, and corresponds to three SM quark-lepton generations. In addition there are three right-handed neutrinos N_R, whose presence is generic in this kind of constructions. Higgs multiplets arise from the intersections I_{bc} and $I_{bc'}$.

One linear combination of the three $U(1)$'s, i.e.

$$Y \ = \ \frac{1}{6}\,(Q_a \ - \ 3Q_c \ + \ 3Q_d) \ , \tag{5.20}$$

corresponds to the hypercharge generator; it is anomaly free, and should be required to be massless, namely its $B \wedge F$ couplings should vanish. In the language of previous sections, we have

$$s_{ak} \ - \ s_{ck} \ + s_{dk} \ = \ 0 \quad \text{for all } k. \tag{5.21}$$

where we have accounted for a factor of $N_a = 3$ in the s_{ak} term, and have recalled that $N_c = N_d = 1$. Another one, $(3Q_a - Q_d)$ is anomalous (with anomaly canceled by the GS mechanism) and becomes massive as usual. The remaining orthogonal linear combination Y' is anomaly free and will become massive or not depending on the structure of the couplings of the $U(1)$'s to the RR 2-forms in explicit realization of the 'protomodel'. Note that one can identify the generators of the previous section as $R = -Q_c$, $L = Q_d$ and $Q_a = 3B$, with B the baryon number. There is no analogue of the A generator in this class of models due to the absence of a $U(1)_b$ associated to the electroweak group.

Depending on the structure of the $B \wedge F$ couplings in the model, it is possible to realize the following discrete symmetries:

- **R_N-symmetries**

  Since $R = -Q_c$, a R_N symmetry will appear if $s_{ck} \in N\mathbb{Z}$ for all k in the model. In particular standard R-parity will appear if $s_{ck} \in 2\mathbb{Z}$ for all k.

- **L_N symmetries**

  Again, since $L = Q_d$ in the brane notation, a L_N symmetry will appear if $s_{dk} \in N\mathbb{Z}$ for all k in the model.

- **Baryon triality**

  One can study the realization of combinations like $B_3 = R_3 L_3$. Using the above results, this requires the condition $s_{ck} + s_{dk} \in 3\mathbb{Z}$, for all k. Now from (5.21) this is equivalent to the condition $s_{ak} \in 3\mathbb{Z}$ for all k. An equivalent derivation is that B_3 can be related to baryon number B by

  $$B_3 = 2Y/3 - B/3 \tag{5.22}$$

  In any SM-like D-brane model, baryon number is realized as $U(1)_a$, and hence B_3 arises from its $\mathbb{Z}_9$ subgroup. Due to the additional multiplicity of $N_a = 3$, this only requires $s_{ak} \in 3\mathbb{Z}$ for all k in the model.

- Other combinations may be studied analogously.

We already presented an explicit class of torioidal realizations of this 'protomodel' in section 2.4. We generalize the construction to include an arbitrary number N_g of fermion quark/lepton generations [41].The set of SM branes and their wrapping numbers is shown in table 5.3. Here n_a^2, m_a^3, n_c^1, n_d^2, m_d^3 are integers. The branes b are mapped to themselves

N_i	(n^1, m^1)	(n^2, m^2)	(n^3, m^3)
$N_a = 3$	$(1, 0)$	$(n_a^2, 1)$	(N_g, m_a^3)
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 1$	$(n_c^1, 1)$	$(1, 0)$	$(0, 1)$
$N_d = 1$	$(1, 0)$	$(n_d^2, -N_g)$	$(1, m_d^3)$

Table 5.3: D6-brane wrapping numbers giving rise to a SM spectrum.

under the orientifold action, so that the corresponding gauge group is $USp(2)$, identified with $SU(2)_L$. It is easy to check that indeed these wrapping numbers give rise to the chiral spectrum of a SM with N_g quark/lepton generations, as in table 2.1. The hypercharge remains massless as long as

$$n_c^1 \ = \ n_a^2 m_a^3 + n_d^2 m_d^3 \ . \tag{5.23}$$

The other two linear combinations are generically massive. RR tadpoles cancel in this model if

$$3m_a^3 \ = \ N_g m_d^3 \ \ . \tag{5.24}$$

In addition one should add $(3n_a^2 N_g + n_d^2 - 16)$ D6-branes (or antibranes, depending on the sign) along the orientifold plane. They have no intersection with the rest of the branes and do not modify the discussion in any way.

In this model, the non-vanishing $B \wedge F$ couplings from (5.2) are

$$\begin{aligned} F^a &\wedge \; 3(N_g B_2^2 \; + \; n_a^2 m_a^3 B_2^3) \\ F^c &\wedge \; n_c^1 B_2^3 \\ F^d &\wedge \; (-N_g B_2^2 + n_d^2 m_d^3 B_2^3)\,, \end{aligned} \tag{5.25}$$

where we have denoted B_2^k, $k = 0, 1, 2, 3$ the RR 2-forms $B_2^k = \int_{\beta^k} C_5$ associated to the basis of odd-cycles

$$\begin{aligned} [\beta^0] &= [(0,1)]_1\,[(0,1)]_2\,[(0,1)]_3\,, \qquad [\beta^1] = [(0,1)]_1\,[(1,0)]_2\,[(1,0)]_3\,, \\ [\beta^2] &= [(1,0)]_1\,[(0,1)]_2\,[(1,0)]_3\,, \qquad [\beta^3] = [(1,0)]_1\,[(1,0)]_2\,[(0,1)]_3\,. \end{aligned} \tag{5.26}$$

It is easy to see that the structure of $B \wedge F$ couplings (5.25) naturally contains some of the discrete gauge symmetries discussed above:

i) Baryon triality is quite generic. Indeed, the $\mathbb{Z}_9$ required for matter parity appears automatically for the physical case $N_g = 3$ as long as $n_a^2 m_a^3$ is multiple of 3. More generally, a $\mathbb{Z}_{N_g}$ discrete baryon symmetry will be present if $n_a^2 m_a^3$ is multiple of N_g.

ii) Since $R = -Q_c$, the R_N discrete symmetries (including R-parity) are naturally generated with $N = n_c^1$.

iii) Similarly, since $L = Q_d$, a L_{N_g} discrete symmetry appears whenever $n_d^2 m_d^3$ is a multiple of N_g.

iv) The symmetry $R_3 L_3^2$ is a $\mathbb{Z}_3$ subgroup of the $U(1)$ generated by $Q_c + Q_d$, hence it is realized as a discrete gauge symmetry whenever $n_c^1 + n_d^2 m_d^3 = 3$. This is still compatible with (5.23); for instance $n_c^1 = 1$, $n_d^2 m_d^3 = 2$, $n_a^2 m_a^3 = -1$.

Note that some of these symmetries may be realized simultaneously, thereby generating a larger discrete gauge symmetry group. For instance, *hexality*, being a product of R_2 and B_3 will appear for $n_c^1 = 2$ and $n_a^2 m_a^3$ a multiple of 3. These conditions are still compatible with (5.23).

The above class of examples is non-SUSY, still there are scalars at the intersection (not all massless) which play the role of squarks, sleptons and Higgs scalars, so that it makes sense the study of the couplings forbidden or allowed by discrete $\mathbb{Z}_N$ symmetries. Also, as already emphasized, it is a useful illustration of patterns which may arise in SUSY realizations in other setups richer than toroidal orientifolds.

On the other hand, there are also supersymmetric toroidal orbifold models with electroweak symmetry realized as $SU(2)_L = USp(2)$, and reproducing an MSSM-like matter content. Consider the MSSM-like models in [70], realized in an orientifold of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ as in [71]. The wrapping numbers (n_a^i, m_a^i) of the different MSSM D6$_a$-branes on the different 2-tori are shown in table 5.4 (ignoring the additional branes required for RR tadpole cancellation), and the resulting spectrum and charge assignments are shown in table 5.5. This corresponds to the intersection numbers (5.19) with a trivial relabeling $d \leftrightarrow d'$. Note that the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold truncates the gauge group on $2N_a$ D6$_a$-branes to $U(N_a)$

It is easy to check, using that the $B \wedge F$ couplings are

$$\begin{aligned} F^a &\wedge \; 3N_g\,(\,B_2^2 \; - \; B_2^3\,) \\ F^d &\wedge \; N_g\,(\,B_2^2 \; - \; B_2^3\,)\,. \end{aligned} \tag{5.27}$$

N_α	(n^1, m^1)	(n^2, m^2)	(n^3, m^3)
$N_a = 6$	$(1, 0)$	$(N_g, 1)$	$(N_g, -1)$
$N_b = 4$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 4$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_d = 2$	$(1, 0)$	$(N_g, 1)$	$(N_g, -1)$

Table 5.4: D-brane wrapping numbers giving rise to an $SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L}$ extension of the MSSM with N_g quark-lepton generations. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold truncates the gauge group on $2N_a$ D6$_a$-branes to $U(N_a)$.

Sector	Matter fields	$SU(3) \times SU(2)_L \times SU(2)_R$	Q_a	Q_d	Q_{B-L}
(ab)	Q_L	$3(\mathbf{3}, \mathbf{2}, \mathbf{1})$	1	0	1/3
(ac)	Q_R	$3(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2})$	-1	0	-1/3
(db)	L_L	$3(\mathbf{1}, \mathbf{2}, \mathbf{1})$	0	-1	-1
(dc)	L_R	$3(\mathbf{1}, \mathbf{1}, \mathbf{2})$	0	1	1
(bc)	H	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	0	0	0

Table 5.5: Left-Right MSSM spectrum and $U(1)$ charges obtained from table 5.4, for the particular choice $N_g = 3$. The $B - L$ generator is defined as $Q_{B-L} = \frac{1}{3}Q_a + Q_d$.

In this model $U(1)_{B-L}$ remains as a continuous gauge symmetry, generated by $Q_a/3 + Q_d$. Using a hypercharge shift, this implies that Q_c has no $B \wedge F$ couplings. Hence it does not make much sense to discuss discrete R_N symmetries which are contained in a continuous symmetry.

On the other hand the realization of B_3 as a discrete gauge symmetry is automatic for the physical case with $N_g = 3$. On top of a nice and simple realization of baryon triality in an explicit MSSM-like D-brane model, this example shows an interesting link between this symmetry and the number of generations. Note that alternatively, since $U(1)_{B-L}$ is a gauge symmetry of the massless spectrum, $R_3 L_3^2$ also remains as a discrete symmetry.

5.2.3 Discrete gauge symmetries and $SU(5)$ unification

It is interesting to explore whether discrete gauge symmetries also appear in models with a unified gauge symmetry like $SU(5)$. It is possible to construct type II orientifolds with a $SU(5)$ gauge group and appropriate matter and SM Higgs multiples. However, in these models the Yukawa couplings $10 \times 10 \times 5_H$ can only appear at the non-perturbative level, since they violate the $U(1) \subset U(5)$ symmetry, which is perturbatively exact. Such couplings are on the other hand easy to obtain in the context of F-theory GUT's, see section 5.3. In the framework of type II orientifolds, they can be generated by D-brane instanton effects, see [192, 193] for further discussion. The fact that certain instantons must play an important role in this class of models gives an added interest to the question of whether certain phenomenologically undesirable operators are protected against analogous non-perturbative effects; discrete gauge symmetries are the perfect tool to enforce such property.

For instance, a potential problem of generic $SU(5)$ unification models is the presence of dimension 4 couplings $10\cdot\bar{5}\cdot\bar{5}$, which contain UDD, DQL and LLE couplings, giving rise to fast proton decay. Other potentially dangerous dimension 5 couplings are $10\cdot 10\cdot 10\cdot \bar{5}$ which contain the operators $QQQL$ and $UUDE$ which may also give rise to too fast proton decay. We would like to see whether discrete symmetries forbidding these couplings are generated in brane models.

A large set of Gepner model $SU(5)$ orientifolds models was studied in [196–198]. We will restrict, however, to a study of the $U(1)$ symmetries in the simplest intersecting D-brane setting which may contain $SU(5)$ unification as described in e.g. [192]. Consider a stack 1 of five D6-branes with gauge group $U(5)_1$ intersecting a single D6-brane 2 with gauge group $U(1)_2$. The minimal structure of D6-brane intersections required to get a $SU(5)$ GUT is as in table 5.6. The subindices show the $U(1)_1\times U(1)_2$ charges, and asterisks denote orientifold

Intersection	I_{ab}	$U(5)_1\times U(1)_2$
$11'$	3	$\mathbf{10}_{(2,0)}$
12	3	$\bar{\mathbf{5}}_{(-1.1)}$
$22'$	3	$\mathbf{1}_{(0,-2)}$
$12'$	1	$\mathbf{5}^H_{(1,1)}+\bar{\mathbf{5}}^H_{(-1,-1)}$

Table 5.6: Configuration of intersecting D6-branes realizing an $SU(5)$ GUT.

image D6-branes. The $\mathbf{10}$'s and $\bar{\mathbf{5}}$'s arise from the $11'$ and $1'2'$ intersections, with $I_{11'}=3$, $I_{12}=-3$, respectively, whereas the Higgs fields reside at $12'$ intersections. The D-type Yukawas $\mathbf{10}_{(2,0)}\cdot\bar{\mathbf{5}}_{(-1,1)}\cdot\bar{\mathbf{5}}^H_{(-1,-1)}$ are allowed by the $U(1)$ symmetries, whereas the U-type coupling $\mathbf{10}_{(2,0)}\cdot\mathbf{10}_{(2,0)}\cdot\mathbf{5}^H_{(1,1)}$ is forbidden.

Since neither L nor B generators commute with $SU(5)$, it is not possible to generate symmetries like baryon triality or lepton triality as discrete symmetries of the $SU(5)$ model. However it is easy to obtain R-parity or some $\mathbb{Z}_N$ generalization, as discrete subgroups of the generator

$$Q_X \;=\; \frac{1}{2}(Q_1 \;-\; 5Q_2) \;=\; 5(B-L) \;-\; 4Y\ . \tag{5.28}$$

This is the familiar $U(1)$ in the branching $SO(10)\to SU(5)\times U(1)$, under which the $\mathbf{16}$ and the $\mathbf{10}$ decompose as

$$\begin{aligned}\mathbf{16} \quad &\to \quad \mathbf{10}_1+\bar{\mathbf{5}}_{-3}+\mathbf{1}_5\\ \mathbf{10} \quad &\to \quad \mathbf{5}_2+\bar{\mathbf{5}}_{-2}\end{aligned} \tag{5.29}$$

Therefore if $\frac{1}{2}(s_{1k}-5s_{2k})\in N\mathbb{Z}$, then the $B\wedge F$ couplings imply that a $\mathbb{Z}_N$ subgroup of $U(1)_X$ survives as a discrete gauge symmetry. This suffices to forbid the L- and B-violating couplings in $10\cdot\bar{5}\cdot\bar{5}$. For $N=2$ one recovers the usual R-parity, since Q_X is mod 2 equal to $B-L$ (up to hypercharge shift), whereas for e.g. $N=4$ one recovers a $\mathbb{Z}_4$ symmetry first suggested by Krauss and Wilczek [183]. These generalizations of R-parity have however the shortcoming of forbidding neutrino Majorana masses.

A clarification is in order here. The above linear combination has non-integer coefficients, contrary to our normalization (5.6). Actually this follows because any SM field arises from a string with both endpoints on the branes 1 or 2, so its charge under $Q_1 - 5Q_2$ is even. The factor of 1/2 in (5.28) brings back the normalization to minimum unit charge. Note however that other possible (potentially massive) states in the full theory, arising from strings stretching between the SM and hidden branes, would have fractional charge assignments under X. Namely, taking into account all fields in the string model, we should normalize the combination as $Q_1 - 5Q_2$, according to (5.6). However, a $\mathbb{Z}_{2N}$ subgroup acts only as a $\mathbb{Z}_N$ symmetry in the SM fields, identified with the generator (5.28).

In principle there could also be discrete symmetries coming from the orthogonal $U(1)$ symmetry

$$Q_Z \;=\; \frac{1}{2}(\,5Q_1\,+\,Q_2\,) \tag{5.30}$$

under which the fields have charges 10_5, $\bar{5}_{-2}$, 1_1, 5^H_3, $\bar{5}^H_{-3}$. A $\mathbb{Z}_2$ subgroup of Q_Z would allow for neutrino Majorana masses but would forbid the instanton generation of U-quark Yukawas, since all fields would be odd except for $\bar{5}$.

Thus within this type of brane configurations R-parity (or $\mathbb{Z}_N$ generalizations) may in principle appear as a discrete gauge symmetry. However additional discrete symmetries will typically forbid either the generation of U-quark Yukawas or neutrino Majorana masses or both. It seems also difficult to forbid dim=5 couplings $10 \times 10 \times 10 \times \bar{5}$ without forbidding at the same time U-quark Yukawa couplings. It would be interesting to see whether these conclusions based on the simplest D-brane configuration remain true in more general cases.

5.3 Discrete gauge symmetries in local F-theory GUTs

F-theory can be regarded as a non-perturbative generalization of type IIB compactifications with D7-branes. In the same spirit, local F-theory GUTs can be regarded as a non-perturbative generalization of type IIB models with GUT theories localized on stacks of D7-branes. However, a key difference in both situation is the status of $U(1)$ symmetries (and so, for instance, the presence or not of certain couplings, like the up-type Yukawa in $SU(5)$ theories). Since $U(1)$ symmetries are so intimately linked with discrete gauge symmetries, it is worthwhile to explore the extension to the realm of F-theory of our earlier description of discrete gauge symmetries in D-brane models.

Unfortunately, a review of F-theory and all the necessary ingredients that we use in this section would take up to much space of this thesis. Since F-theory is not directly related to the rest of our work, we omit such a discussion and refer the reader to reviews such as [201, 202]

The physics of $U(1)$ gauge theories in F-theory is in general poorly understood in compact examples. We therefore focus on local F-theory GUT models, which have been extensively studied (see e.g. [203–206]). The starting point is provided by F-theory 7-branes wrapped on a local 4-cycle S in the base of the elliptically fibered CY fourfold, leading to an $SU(5)$ GUT theory (with no overall $U(1)$ factor). There are other 7-branes on other 4-cycles S_a (which are non-compact in the local description) which intersect S along complex curves Σ_a (matter curves). These intersections support charged matter, in representations of the gauge factors of both 7-branes dictated by the enhanced symmetry at the intersection locus. In particular, local enhancements to $SU(6)$ lead to fields in the 5, $\bar{5}$, and local enhancements to $SO(10)$ lead to fields in the 10, $\overline{10}$. In addition, local rank-2 enhancements at points of S, due to intersections of several matter curves, correspond to Yukawa couplings among the fields supported on the latter. In this local picture, the $U(1)_a$'s supported by the non-compact 7-branes on S_a are global symmetries, which may or may not survive in a full fledged compactification, due to global geometrical effects. Even if they survive these effects, and seemingly manifest as 4d $U(1)$ gauge symmetries, they may acquire Stückelberg masses by their $B \wedge F$ couplings. We are thus interested in determining *necessary* conditions (which are not sufficient due to this global sensitivity) for $\mathbb{Z}_n$ subgroups of these $U(1)$'s to survive as discrete gauge symmetries of the model.

There are certainly many possibilities in F-theory model building. For concreteness we will focus on a particular class of models, in which there is a good control of the $U(1)_a$ charges of the different $SU(5)$ representations in the different curves; the basic ideas concerning the $B \wedge F$ couplings in F-theory however hold more generally. The models we focus on have an underlying E_8 structure globally on the 4-cycle S, in the sense that the pattern of matter curves and Yukawa points is determined by an unfolding of E_8 into $SU(5)$, according to

$$\begin{array}{lcl} E_8 & \to & SU(5)_{\rm GUT} \times SU(5)_\perp \\ 248 & \to & (24,1) + (10,5) + (5,\overline{10}) + (\overline{10},\bar{5}) + (\bar{5},10) + (1,24) \end{array} \tag{5.31}$$

where $SU(5)_\perp$ is actually split to $U(1)^4$, but is useful as shorthand for the corresponding charges. For the $SU(5)_{\rm GUT}$ 5's and 10's, and singlets, these are specifically given by

$$\begin{array}{cc} SU(5)_{\rm GUT} & U(1)^4 \\ 10 & (\underline{4,-1,-1,-1,-1}) \\ 5 & (\underline{3,3,-2,-2,-2}) \\ 1 & (\underline{1,-1,0,0,0}) \end{array} \tag{5.32}$$

where underlining means permutation of entries; also, conjugate $SU(5)_{\rm GUT}$ representations have opposite $U(1)^4$ charges. Note that we have represented charges with respect to five $U(1)$'s with generators Q_a $a = 1, \ldots, 5$ but constrained by $\sum_a Q_a = 0$ (corresponding to Cartan generators of $SU(5)_\perp$).

This class of models has been extensively discussed in e.g. [207–221]. The global E_8 structure throughout S allows the use of the so-called spectral cover construction, to encode most of the relevant information about the local geometry around S (sometimes referred to as semi-local model), including the 7-brane worldvolume gauge fluxes. Roughly speaking, the system is a configuration of F-theory 7-branes leading to an E_8 gauge theory on S, deformed by vevs (rather, backgrounds varying along S) of scalars in the adjoint of $SU(5)_\perp$, thus leaving only $SU(5)$ as the unbroken group. This point-dependent $SU(5)_\perp$ matrix can be diagonalized, in terms of five (point-dependent) eigenvalues ϕ_i (with $\sum_i \phi_i = 0$), leading to a 5-fold co vering of S, known as spectral cover. In general, the scalar profiles can have poles (it is formally a meromorphic Higgs bundle), so that the extra 7-branes go off to infinity and are non-compact in the semi-local model. Also, the 5-fold cover is in general branched, meaning that some of the $U(1)$'s are related by monodromies, subgroups of S_5 (the group of permutations of 5 elements), that describe the reshuffling of sheets of the cover as one loops around in S.

This description is fleshed out by describing the semilocal geometry of the elliptic fibration in the Tate form, describing the unfolding of E_8 into $SU(5)$:

$$x^3 - y^2 + xyz\, b_5 w + x^2 z^2\, b_4 w^2 + yz^3\, b_3 w^3 + xz^4\, b_2 w^4 + z^6\, b_0 w^5 = 0 \tag{5.33}$$

where $[z, x, y]$ are homogeneous coordinates in $\mathbb{P}_{[1,2,3]}$, parametrizing the elliptic fiber, w is a coordinate transverse to S, and b_i are functions (actually, sections of suitable line bundles) over S.

The 4-cycle S corresponds to the locus $w = 0$, where the above equation can be shown to describe a degeneration of the elliptic fiber leading to an $SU(5)$ gauge symmetry. The information on the extra 7-branes is encoded in the b_n, and is nicely captured by the $SU(5)$ spectral cover $\mathcal{C}_5$, a 5-sheeted branched cover of S living in an auxiliary non-CY threefold X; the latter is defined as a $\mathbb{P}_1$ bundle over S, $\mathbb{P}(\mathcal{O}_S \oplus K_S)$, where $\mathcal{O}_S$ and K_S are the trivial and canonical line bundles over S, respectively. The spectral cover is defined by the equation

$$b_0\, s^5 + b_2\, s^3 + b_3\, s^2 + b_4\, s + b_5 = 0 \tag{5.34}$$

where s is an affine coordinate in the $\mathbb{P}_1$ fiber in X, so S is defined by $s = 0$. The b_n are symmetric monomials in some variables ϕ_i, regarded as the Higgs vevs, with $b_1 = 0$ due to tracelessness of the $SU(5)_\perp$ generators. The spectral cover $\mathcal{C}_5$ contains the information about the matter curves, for instance the **10** matter curves are associated to its intersection with S; this is the locus $b_5 = 0$, which can be shown to correspond in (5.33) to a locus of enhanced $SO(10)$ symmetry. The **5** curves arise from an associated spectral cover $\mathcal{C}_{10}$, describing the representation of the Higgs field in the **10** of $SU(5)_\perp$. There are several techniques to compute the location of the different matter curves, and their homology classes, for which we refer the reader to the references.

The spectral cover is particularly useful to characterize the 7-brane worldvolume $U(1)_a$ fluxes, required to obtain chiral matter from the 6d multiplets localized on the matter curves. This is done in terms of a suitable line bundle $\mathcal{N}_5$ over the spectral cover $\mathcal{C}_5$. Once projected down to S, this defines an $SU(5)_\perp$ bundle V over S, which can be regarded as fully responsible for the breaking of the underlying E_8 symmetry to its commutant $SU(5)_{\rm GUT}$. In addition, the line bundle can include components corresponding to hypercharge flux F_Y, in order to break $SU(5)_{\rm GUT}$ to the SM group. As emphasized in [205] (see [222] for an earlier realization in a different context), masslessness of the hypercharge gauge boson requires the 2-form F_Y to be non-trivial on S, but trivial in the global geometry. Since this prevents F_Y to have $B \wedge F$ couplings to bulk 2-forms, its introduction is irrelevant for the purpose of studying discrete gauge symmetries, and we ignore it in the following.

To our knowledge, the computation of $B \wedge F$ couplings for the $U(1)^4$ factors in F-theory has not been carried out in detail in the literature in the spectral cover language. However, they are easily guessed to arise from a Chern-Simons (CS) coupling on the F-theory 7-brane worldvolume

$$\int_{S_a \times M_4} C_4 \wedge F_a \wedge F_a \tag{5.35}$$

This can be regarded as a simple generalization of the CS couplings on D7-branes. More rigorously, it can be easily derived from the dual picture of M-theory on a CY fourfold. The degenerations of the elliptic fiber on top of the 7_a-branes support harmonic 2-forms ω_a, normalized to $\int_{\rm ALE}\omega_a\wedge\omega_b=\delta_{ab}$, where ALE stands for the local ALE geometry transverse to the degeneration locus. The component of the M-theory 3-form C_3 along ω_a becomes the 7_a worldvolume gauge field, so its field strength $G_4=dC_3$ has a component

$$G_4 \;=\; \sum_a \omega_a\wedge F_a \tag{5.36}$$

The 11d effective action of M-theory has a Chern-Simons coupling

$$\int_{11d} C_3\wedge G_4\wedge G_4 \tag{5.37}$$

Upon replacing (5.36), and noticing that the M-theory C_3 maps to the F-theory C_4 under duality, we recover (5.35).

The coupling (5.35) produces the relevant $B\wedge F$ couplings for $U(1)_a$, as follows. We introduce two basis of dual 2-cycles $\{\alpha_k\}$, $\{\beta^k\}$ in S, with $\alpha_k\cdot\beta^l=\delta_k^l$. As suggested by the notation, they play a role analogous to the 3-cycles in previous sections. Some of these cycles may be trivial in the global geometry, so in what follows we implicitly restrict the range of k to globally non-trivial classes[3]. We define the 4d 2-forms

$$B^k \;=\; \int_{\beta^k} C_4 \tag{5.38}$$

In addition, we expand the magnetic flux of $U(1)_a$ as

$$F_a \;=\sum_k s_{ak}\beta^k \quad , \quad \text{namely } s_{ak}=\textstyle\int_{\alpha_k}F_a \tag{5.39}$$

Reduction of the coupling (5.35) leads to the $B\wedge F$ terms

$$\sum_a s_{ak}B^k\wedge F_a \tag{5.40}$$

Therefore, for a linear combination $Q=\sum_a c_aQ_a$ to leave a $\mathbb{Z}_n$ discrete gauge symmetry, the necessary condition is

$$\sum_a c_a\, s_{ak} \;=\; 0 \text{ mod } n \text{ for all } k \text{ (with } \beta^k \text{ non-trivial in global geometry)} \tag{5.41}$$

This agrees with the D-brane condition below (5.7) with $N_a=1$, as is the case here. Also, it corresponds to the $B\wedge F$ couplings in compactifications of the heterotic string with $U(1)$ bundles [223, 224].

The above condition is necessary, but not sufficient, for several reasons: First, the $U(1)$ may actually be broken by global effects, as mentioned. Even semi-locally, there are in general monodromies [207], which eliminate some of the relative $U(1)$'s (e.g. Q_1-Q_2 for a $\mathbb{Z}_2$ monodromy). For instance, a generic spectral cover $\mathcal{C}_5$ (5.34) is irreducible, so there are $\mathbb{Z}_5$ or S_5 monodromies that mix all sheets in the spectral cover, and leave no $U(1)$ symmetry whatsoever (since $SU(5)_\perp$ has no overall $U(1)$ factor). In order to lead to non-trivial $U(1)$

[3] In other words, what counts is the class of $[F]$ in the cohomology of the threefold, rather than that of the 4-cycle.

symmetries, the spectral cover must be split, with two or more disconnected components (and in fact the split should extend even globally), as we consider in upcoming examples. Note that, even if there is such a $U(1)$ symmetry, the global geometry may contain additional 2-forms, not present in the local model, coupling to the $U(1)$ with $B \wedge F$ couplings not satisfying the condition (5.41).

An important ingredient about discrete gauge symmetries from $B \wedge F$ couplings is their anomaly cancellation. As suggested from our discussion in section 5.1.2, this leans on the structure of corresponding mixed $U(1)$ anomalies, and their cancellation by a Green-Schwarz mechanism. The latter has not been worked out in the F-theory context, but we may adopt a safe attitude and focus on $U(1)$ factors which are anomaly free. For $SU(5)$ theories, there is one family-independent $U(1)$ factor, already appeared in section 5.2.3. It is the generator Q_X, arising in the decomposition of $SO(10) \to SU(5)_{\rm GUT} \times U(1)_X$. F-theory models where this $U(1)_X$ remains as the only remnant of the original $U(1)^4$ are based on an $S(U(4) \times U(1))$ spectral cover, rather than an $SU(5)$ one. The spectral cover factorizes in two reducible pieces $\mathcal{C}_4$, $\mathcal{C}_1$, with (5.34) now having an structure

$$(c_0\, s^4\, c_1\, s^3 \,+\, c_2\, s^2 \,+\, c_3\, s \,+\, c_4\,)\, (\, d_0\, s \,+\, d_1\,) \,=\, 0 \tag{5.42}$$

with $b_1 = c_0 d_1 + c_1 d_0 = 0$. This means that four sheets of the spectral cover mix among themselves, while the last remains factorized. The construction of $S(U(4) \times U(1))$ spectral covers is a generalization of that of $SU(5)$ spectral covers, carried out in [212]. The introduction of the 7-brane worldvolume fluxes is carried out in terms of two line bundles $\mathcal{N}_4$, $\mathcal{N}_1$ over $\mathcal{C}_4$, $\mathcal{C}_1$, which project onto S as $U(4)$ and $U(1)$ bundles V_4, L, respectively. Their first Chern classes are integer cohomology classes in S, and are constrained by

$$c_1(V_4) \,+\, c_1(L) \,=\, 0 \tag{5.43}$$

so the construction actually defines an $S(U(4) \times U(1))$ bundle, with a commutant $SU(5) \times U(1)_X$ in E_8.

This defines the 4d gauge group (ignoring hypercharge flux), before accounting for the $B \wedge F$ couplings of $U(1)_X$. These are controlled by $c_1(L)$, i.e. the cohomology class of $[F_X]$, considered as a class in the global geometry (rather than just in S). In order to show that they can indeed lead to interesting discrete gauge symmetries, we consider two explicit examples of compact models, in [212,215], leading to 3-generation $SU(5)$ GUTs (with hypercharge flux breaking to the SM), and for which the $S(U(4) \times U(1))$ structure holds even globally.

The global example in [212], is based on a base B_3 obtained from the Fano threefold $\mathbb{P}_4[4]$ (i.e. the subspace of $\mathbb{P}_4$ defined by a homogeneous equation of degree 4) by a geometric transition introducing a dP_7 del Pezzo 4-cycle. The basic 2-cycle classes are H and X (related to the hyperplane class in $\mathbb{P}_4$ and the exceptional divisor dP_7 itself), and the Kähler cone is spanned by H and $H + X$. A detailed construction of the elliptic fibration, and the worldvolume fluxes, led to the construction of a 3-generation F-theory $SU(5)$ GUT (broken to the SM by suitable hypercharge flux), with an additional $U(1)$ 4d gauge symmetry. The $B \wedge F$ couplings can be derived from the Fayet-Illiopoulos terms in eq. (138) in that reference, and read

$$(\, -12 B_1 \,+\, 8\, B_2\,) \wedge F \tag{5.44}$$

where $B_1 = \int_H C_4$, $B_2 = \int_{H-X} C_4$. There is therefore a $\mathbb{Z}_4$ discrete gauge symmetry, which corresponds to the generalized R-parity in [183]. The model in [215] has a more involved structure, but similar qualitative features. From the FI terms in eq. (5.6) in that reference, the $B \wedge F$ couplings have a structure

$$(\,6B_1\, -\, 12\,B_2\, +12B_3) \wedge F \tag{5.45}$$

so there is a $\mathbb{Z}_6$ discrete gauge symmetry of the generalized R-parity type. Beyond these concrete examples, there seems to be no fundamental obstruction to realizing a genuinely $\mathbb{Z}_2$ R-parity in other examples constructed using similar techniques. We hope this analysis suffices to show the appearance of discrete gauge symmetries in F-theory, and leave a more systematic understanding for future work.

Chapter 6

Conclusions and future directions

As we have seen in the review chapters 2 and 3, non-perturbative effects are a key part of string theory, both at formal and phenomenological levels. Obtaining the full non-perturbative effective action and understanding its general properties is hence an extremely important goal of string theory.

In this thesis we have discussed a number of aspects of non-perturbative effects in type II superstring theory compactifications. Rather than the relatively well known properties of single D-brane instanton configurations, we have studied here features of the resummed contributions of multiple instantons.

- In chapter 4 we have presented a powerful tool which can be used to compute and resum non-perturbative effects from certain sectors of D-brane instantons. We have recovered results from the literature that were inferred either by dualities with heterotic strings and the imposition of modular invariance [107, 108] or by direct computations by applying involved localization techniques to the moduli space of multi-instantons [144, 145]. Our work provides an extremely simple derivation of known results which moreover makes modular properties manifest from a nice geometric perspective. In addition, the method provides great insight into the usually obscure physics of multi-instanton configurations.

  In section 4.2 we have studied corrections to the hypermultiplet moduli space of type IIB CY compactifications. The computations carried out there reveal a tight relation between non-perturbative effects and topological string theories. As we argued, such connection underlies elegantly the continuity of holomorphic F-terms across lines of BPS stability. What on the instanton literature was interpreted as a conspiracy of multi-instanton zero-modes to saturate against each other [44,45], is just a simple consequence of the well-known continuity of indices counting BPS particle degeneracies. Although the setup was mostly restricted to $\mathcal{N} = 2$ CY compactifications and instantons in sectors of mutually local charges, prospects for the reduction of supersymmetry to $\mathcal{N} = 1$ and more general instantons were also discussed.

  In section 4.3 we have studied corrections to quartic gauge and curvature couplings in 8d type I' models, recovering and generalizing results from the heterotic and type IIB sides. This provides an interesting generalization of the analysis in section 4.2, to models containing orientifold planes. We have shown again that the spectrum of 9d

BPS particles codifies important information concerning the nature of multiple D-brane instanton effects on the type IIB side. Namely, loops of 9d bound states are mapped to multi-instanton effects of the 1PI effective action, in which several instantons conspire to cancel their additional zero modes and contribute to BPS protected quantities, similar to the effects in [44, 45]. On the other hand, processes involving multi-particle states in 9d correspond to polyinstanton effects. It is satisfactory that the type I' picture encodes the two subtly different situations in a simple way.

- In chapter 5 we have discussed the natural appearance of discrete gauge symmetries in large classes of string vacua, concretely those based on D-branes in type II orientifolds, and local 7-brane systems in F-theory GUTs. They have a number of novelties as compared with earlier studies of discrete gauge symmetries in string theory, which were mainly based on heterotic string compactifications. The main advantage of the present setup is that the discrete symmetries are manifest in the model, without resorting to the rather model dependent choices of flat directions required in the heterotic setup. Also, the discrete gauge symmetries are, by construction, anomaly free and are respected by non-perturbative instanton effects.

  We have shown how semi-realistic (MS)SM type II orientifold constructions naturally bring in discrete gauge symmetries which are $\mathbb{Z}_N$ subgroups of continuous $U(1)$ symmetries in the models. Specifically, they correspond to discrete subgroups of baryon and lepton number $U(1)$ symmetries (modulo discrete hypercharge rotations). The list of discrete symmetries arising is very limited and corresponds to the anomaly free classification of discrete gauge symmetries in [186]: generalized R-parities $R_N \subset U(1)_{B-L}$, lepton triality L_3, baryon triality $B_3 = R_3 L_3$, and the combination $R_3 L_3^2$.

  Note that all these symmetries forbid baryon decay through dimension four operators. Among these symmetries only R-parity and baryon triality (or hexality, which is the product of both) allow for neutrino Majorana masses and hence are phenomenologically preferred. In addition only baryon triality (or hexality) forbid dimension 5 B/L-violating operators. It is worth to notice that if SUSY is found with L-violating QDL couplings at LHC, it would be evidence for baryon triality and non-unification since we have shown that $SU(5)$ unification may only be consistent with R_N discrete symmetries like R-parity.

  Given their important role in models with underlying $SU(5)$GUTs, we have further studied the realization of R_N discrete symmetries in F-theory models with split $S(U(4)\times U(1))$ spectral cover construction.

  In our opinion the source of discrete gauge symmetries described in this chapter provide us with the best available understanding of proton stability in the MSSM.

Our work is expected to admit extensions in several directions. Regarding the computation and interpretation of multi-instantons in chapter 4, it would worth studying the application of the c-map to the calculation of instanton effects in more general setups. From a formal perspective, the inclusion of general sectors of mutually non-local charges and their connection to topological strings would be extremely interesting. This would require a more detailed study of the relation with the microscopic wall crossing formulation in terms of symplectomorphism in the twistor space $\mathcal{Z}$. On a more abstract line, one would like to understand the relation of universal quantities (in the sense of being insensitive to wall crossing), like the hypermultiplet space metric, with the universal category of holomorphic branes. It could be useful for this purposes to find an explicit relation between the global perspective provided by the c-map and the direct localization techniques of [144, 145].

From a phenomenological perspective, it would be extremely interesting to develop additional tools to reduce supersymmetry to $\mathcal{N} = 1$ while keeping the fantastic computational power of $\mathcal{N} = 2$. We suspect that the T-dual viewpoint in terms of a one-loop diagram of BPS particles could provide such a powerful framework. This would be an important step towards the systematic computation of non-perturbative superpotentials in 4d theories with four supercharges. The required computation of the relevant BPS multiplicities would most probably be related to the real topological string [138–142] and their generalized GV invariants.

The discussion of discrete gauge symmetries in chapter 5 was restricted to intersecting brane models in type IIA theories. Analogous arguments should in principle be applicable to mor general brane setups such as type IIB models with magnetized D-branes, D-branes at singularities, etc. The generalization to M-theory compactifications on G_2 manifolds and the relation to torsion cycles should work along the lines of [190]. A more challenging task would be to implement systematically the discussion of $U(1)$ symmetries and their discrete subgroups in general F-theory models. The difficulty lies in the fact that, as we saw in section 5.3, global aspects of F-theory compactifications are usually under poor control.

A different generalization of the results of chapter 5, namely the appearance of non-abelian discrete symmetries in D-brane models and their possible phenomenological applications, is under study [225].

Finally, on a more ambitious and long-termed level, it would be to interesting to understand and study the expected connections between instanton effects in the worldsheet formulation of String Theory and in the intrinsically non-perturbative formulation of String Field Theory. The former has been the subject of the works presented in this thesis. Some aspects of the latter have been studied by the author in [25, 26].

Conclusiones y direcciones futuras

(Spanish translation of chapter 6)

Como hemos visto en los capítulos de resumen 2 y 3, los efectos no-perturbativos son una pieza clave de la teoría de cuerdas, tanto a nivel formal como fenomenolígico. La obtención de la acción efectiva completa y no-perturbativa, y la comprensión de sus propiedades generales es por tanto un objetivo extremadamente importante de la teoría de cuerdas.

En esta tesis hemos discutido ciertos aspectos de los efectos no-perturbativos de compactificaciones de cuerdas tipo II. Más que las propiedades relativamente bien conocidas de configuraciones instantónicas aisladas, hemos estudiado aquí características de las contribuciones resumadas de instantones múltiples.

- En el capítulo 4 hemos presentado un potente método aplicable al cálculo y resumación de efectos no-perturbativos de ciertos sectores de D-branas instantónicas. Hemos recuperado resultados de la literatura que habían sido obtenidos bien a través de dualidades y la imposición de invariancia modular [107, 108], o bien mediante cálculos directos basados en complicadas técnicas de localización en el espacio de módulos de multi-instantones [144, 145]. Nuestro trabajo aporta una derivación extremadamente simple de resultados conocidos que, más aún, hace las propiedades modulares manifiestas desde una perspectiva geométrica.

  En la sección 4.2 hemos estudiado correcciones al espacio de módulos de hipermultipletes de compactificaciones CY del tipo II. Los cálculos llevados a cabo revelan una estrecha relación entre los efectos no-perturbativos y las teorías topológicas de cuerdas. Como argumentamos, dicha conexión subyace elegantemente la continuidad de los términos-F holomorfos a través de lineas de estabilidad BPS. Lo que en la literatura de instantones es interpretado como una conspiración de modos cero de multi-instanton para saturarse [44, 45], es una simple consecuencia de la bien conocida continuidad de los índices que cuentan las degeneraciones de partículas BPS. A pesar de que nuestras configuraciones están restringidas mayormente a compactificaciones CY con supersimetría $\mathcal{N} = 2$ y a sectores de instantón con cargas mutuamente locales, las perspectivas para la reducción a supersimetría $\mathcal{N} = 1$ y a cargas más generals también han sido discutidas.

  En la sección 4.3 hemos estudiado correcciones a acoplos cuárticos gauge y de curvatura en modelos tipo I' en 8d, recuperando y generalizando resultados de teorías heteróticas y tipo IIB. Esto proporciona una generalización del análisis de la sección 4.2 a modelos que contienen planos orientifold. Hemos mostrado de nuevo que el espectro de estados de partículas BPS en 9d codifica información importante concerniente a la naturaleza de effectos de multi-instantón el una teoría dual tipo IIB. Más explícitamente, lazos de estado ligados en 9d corresponden a efectos de multi-instanton de la teoría efectiva

1PI, en la que varios instantones conspirean para cancelar sus modos cero adicionales, contribuyendo así a cantidades BPS protegidas (de manera similar a [44,45]). Por otro lado, procesos que involucran estados de multi-partículas en 9d corresponden a efectos de poli-instantón. Es satisfactorio observar que la imagen tipo I' codifica ambos tipos de procesos sutilmente distintos de forma sencilla.

- En el capítulo 5 hemos discutido la aparición natural de simetrías gauge discretas en amplias clases de vacíos de cuerdas, concretamente en aquéllos basados en orientifolds tipo II con D-branas gauge, y en modelos locales de 7-branas en teorías F de gran unificación. En comparación con estudios previos de simetrías discretas en teoría de cuerdas (principalmente basados en modelos heteróticos), nuestros resultados presentan una serie de importantes novedades. La principal ventaja del marco actual es que las simetrías discretas son manifiestas en el modelo, sin necesidad de recurrir a la elección de direcciónes planas de las teorías heteróticas. Además, las simetrías gauge discretas que presentamos son, por construcción, libres de anomalías y respetadas por efectos de instantón.

  Hemos mostrado como modelos parecidos el ME(MS) en orientifolds tipo II traen consigo naturalmente simetrías discretas gauge que son subgrupos $\mathbb{Z}_N$ de simetrías continuas $U(1)$ de los modelos. Específicamente, corresponden a los subgrupos de los $U(1)$ de número bariónico y leptónico (modulo rotaciones discretas de hipercarga). La lista de simetrías discretas que aparecen es muy limitada, y corresponde a la clasificación de simetrías discretas gauge libres de anomalías del [186]: paridades-R generalizadas, $R_N \subset U(1)_{B-L}$, trialidad leptónica L_3, trialidad bariónica $B_3 = R_3 L_3$ y la combinación $R_3 L_3^2$.

  Nótese que todas estas simetrías prohiben la desintegración de bariones mediada por operadores de dimensión 4. De entre estas simetrías, tan sólo paridad-R y la trialidad bariónica (o 'hexalidad', que es el producto de ambas) permiten masas de Majorana para los neutrinos, y son por tanto preferentes fenomenológicamente. Más aún, solamente trialidad bariónica (o hexalidad) prohiben acoplos de dimensión 5 que violan B/L. Merece la pena destacar que si se encuentra supersimetría con acoplos QDL violadores de L en el LHC, sería una indicación de trialidad bariónica y de no-unificación, ya que hemos visto que unificación $SU(5)$ sólo puede ser consistente con paridades R generalizadas.

  Dada su importancia en modelos de unificación $SU(5)$, hemos estudiado la realización de simetrías discretas R_N en modelos de teoría F con construcciones de simetría partida $S(U(4) \times U(1))$.

  En nuestra opinión, la fuente de simetrías discretas descrita en el capítulo 5 aporta la mejor explicación de la estabilidad del protón en modelos MEMS hasta la fecha.

Es de esperar que nuestro trabajo admita extensiones en diversas direcciones. Respecto al cálculo e interpretación de multi-instantones en el capítulo 4, merecería la pena estudiar la aplicación el mapa-c al cómputo de efectos de instanton en situaciónes más generales. Desde una perspectiva formal, la inclusión de sectores generales de cargas mutuamente no-locales, así como su conexión con las cuerdas topólogicas sería extremadamente interesante. Ésto requeriría de un estudio más detallado de la relación con la formulación microscópica de los cruces de muros de estabilidad en términos de simplectomorfismos en el espacio de twistores $\mathcal{Z}$. En una línea más abstracta, sería deseable entender la relación de cantidades universales (en el sentido de insensibilidad respecto al cruce de muros), como la métrica del espacio de hipermultipletes, con la categoría universal de branas holomorfas. Para este propósito podría ser útil encontrar una relación explícita entre la perspectiva global aportada por el mapa-c, y las técnicas locales directas de localización de [144, 145].

Desde un punto de vista fenomenológico, sería extremadamente interesante desarrollar herramientas adicionales de reducción de simetría al caso $\mathcal{N} = 1$, manteniendo al mismo tiempo el fantástico poder computacional de $\mathcal{N} = 2$. Sospechamos que el punto de vista T-dual en términos de diagramas a un lazo con partículas BPS podría generar dicho marco de trabajo. Esto sería un importante paso hacia la importante computación sistemática de superpotenciales no-perturbativos en cuatro dimensiones con cuatro supercargas. El cálculo requerido de las multiplicidades BPS relevantes estaría muy problemente relacionado con la cuerda topólogica real y sus invariantes GV generalizados [138–142].

La implementación explícita de simetrías discretas gauge del capítulo 5 ha estado restringida a modelos de branas intersecantes en teorías tipo IIA. Argumentos análogos deberían ser aplicables en principio a situaciones de branas gauge más generales, tales como modelos tipo IIB con D-branas magnetizadas, modelos de branas en singularidades, etc. La generalización a compactificaciones de teoría M en variedades G_2 y su relación con ciclos de torsión deberán funcionar siguiendo las líneas de [190]. Una misión más desafiante sería la implementación sistemática de la discusión de simetrías $U(1)$ y sus subgrupos discretos en modelos generales de teorías F. La dificultad estriba en el hecho de que, como hemos visto en la sección 5.3, los aspectos globales de compactificaciones de teoría F no están generalmente bajo control.

Una generalización diferente de los resultados del capítulo 5, la aparición de simetrías discretas no-abelianas en modelos de D-branas y sus posibles aplicaciones fenomenológicas, está siendo estudiada [225].

Finalmente, a un nivel más ambicioso y a largo plazo, sería interesante entender las esperadas conexiones entre los efectos de instantón en la formulación perturbativa de la teoría de cuerdas y en la formulación intrínsicamente no-perturbativa de la teoría de campos de cuerdas. La primera ha sido el objeto de estudio de los trabajos presentados en esta tesis. Algunos aspectos de la segunda han sido estudiados por el autor en [25, 26].

Bibliography

[1] G. Aad *et al.* [ATLAS Collaboration], "Combined search for the Standard Model Higgs boson using up to 4.9 fb-1 of pp collision data at sqrt(s) = 7 TeV with the ATLAS detector at the LHC," Phys. Lett. B **710** (2012) 49 [arXiv:1202.1408 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], "Combined results of searches for the standard model Higgs boson in pp collisions at sqrt(s) = 7 TeV," [arXiv:1202.1488 [hep-ex]].

[3] S. Weinberg, "The Quantum theory of fields. Vol. 1: Foundations," Cambridge, UK: Univ. Pr. (1995) 609 p

[4] S. Weinberg, "The quantum theory of fields. Vol. 2: Modern applications," Cambridge, UK: Univ. Pr. (1996) 489 p

[5] J. F. Donoghue, E. Golowich and B. R. Holstein, "Dynamics of the standard model," Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **2**, 1 (1992).

[6] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [hep-th/9711200].

[7] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].

[8] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].

[9] G. 't Hooft, "Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking," NATO Adv. Study Inst. Ser. B Phys. **59**, 135 (1980).

[10] S. Weinberg, "Anthropic Bound on the Cosmological Constant," Phys. Rev. Lett. **59**, 2607 (1987).

[11] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," Phys. Rev. Lett. **38**, 1440 (1977).

[12] R. D. Peccei and H. R. Quinn, "Constraints Imposed by CP Conservation in the Presence of Instantons," Phys. Rev. D **16** (1977) 1791.

[13] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 1: Introduction," Cambridge, Uk: Univ. Pr. (1987) 469 P. (Cambridge Monographs On Mathematical Physics)

[14] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies And Phenomenology," Cambridge, Uk: Univ. Pr. (1987) 596 P. (Cambridge Monographs On Mathematical Physics)

[15] J. Polchinski, "String theory. Vol. 1: An introduction to the bosonic string," Cambridge, UK: Univ. Pr. (1998) 402 p

[16] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," Cambridge, UK: Univ. Pr. (1998) 531 p

[17] B. Zwiebach, "A first course in string theory," Cambridge, UK: Univ. Pr. (2009) 673 p

[18] K. Becker, M. Becker and J. H. Schwarz, "String theory and M-theory: A modern introduction," Cambridge, UK: Cambridge Univ. Pr. (2007) 739 p

[19] E. Kiritsis, "String theory in a nutshell," Princeton University Press, 2007

[20] L. E. Ibanez and A. M. Uranga, "String theory and particle physics: An introduction to string phenomenology," Cambridge, UK: Univ. Pr. (2012) 673 p

[21] E. Witten, "Noncommutative Geometry and String Field Theory," Nucl. Phys. B **268**, 253 (1986).

[22] E. Witten, "On background independent open string field theory," Phys. Rev. D **46**, 5467 (1992) [hep-th/9208027].

[23] B. Zwiebach, "Closed string field theory: An Introduction," [hep-th/9305026].

[24] W. Taylor and B. Zwiebach, "D-branes, tachyons, and string field theory," [hep-th/0311017].

[25] M. Kiermaier, Y. Okawa and P. Soler, "Solutions from boundary condition changing operators in open string field theory," JHEP **1103**, 122 (2011) [arXiv:1009.6185 [hep-th]].

[26] P. Soler, "Constructing open string field theory solutions from boundary condition changing operators," Prog. Theor. Phys. Suppl. **188**, 138 (2011).

[27] J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," Phys. Rev. Lett. **75**, 4724 (1995) [hep-th/9510017].

[28] G. 't Hooft, "Dimensional reduction in quantum gravity," [gr-qc/9310026].

[29] K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil and E. Zaslow, "Mirror symmetry," (Clay mathematics monographs. 1)

[30] A. Neitzke and C. Vafa, "Topological strings and their physical applications," [hep-th/0410178].

[31] M. Vonk, "A Mini-course on topological strings," [hep-th/0504147].

[32] N. Seiberg and E. Witten, "Electric - magnetic duality, monopole condensation, and confinement in N=2 supersymmetric Yang-Mills theory," Nucl. Phys. B **426**, 19 (1994) [Erratum-ibid. B **430**, 485 (1994)] [hep-th/9407087].

[33] N. Seiberg and E. Witten, "Monopoles, duality and chiral symmetry breaking in N=2 supersymmetric QCD," Nucl. Phys. B **431**, 484 (1994) [hep-th/9408099].

[34] M. R. Douglas and S. Kachru, "Flux compactification," Rev. Mod. Phys. **79**, 733 (2007) [hep-th/0610102].

[35] K. Becker, M. Becker and A. Strominger, "Five-branes, membranes and nonperturbative string theory," Nucl. Phys. B **456**, 130 (1995) [hep-th/9507158].

[36] E. Witten, "Nonperturbative superpotentials in string theory," Nucl. Phys. B **474**, 343 (1996) [hep-th/9604030].

[37] J. A. Harvey and G. W. Moore, "Superpotentials and membrane instantons," [hep-th/9907026].

[38] E. Witten, "World-sheet corrections via D-instantons," JHEP **0002** (2000) 030 [arXiv:hep-th/9907041].

[39] M. A. Shifman, (ed.), "Instantons in gauge theories," Singapore, Singapore: World Scientific (1994) 488 p

[40] R. Blumenhagen, M. Cvetic and T. Weigand, "Spacetime instanton corrections in 4D string vacua: The Seesaw mechanism for D-Brane models," Nucl. Phys. B **771**, 113 (2007) [hep-th/0609191].

[41] L. E. Ibanez and A. M. Uranga, "Neutrino Majorana Masses from String Theory Instanton Effects," JHEP **0703**, 052 (2007) [hep-th/0609213].

[42] B. Florea, S. Kachru, J. McGreevy and N. Saulina, "Stringy Instantons and Quiver Gauge Theories," JHEP **0705**, 024 (2007) [hep-th/0610003].

[43] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [hep-th/0301240].

[44] I. Garcia-Etxebarria and A. M. Uranga, "Non-perturbative superpotentials across lines of marginal stability," JHEP **0801** (2008) 033 [arXiv:0711.1430 [hep-th]].

[45] I. Garcia-Etxebarria, F. Marchesano and A. M. Uranga, "Non-perturbative F-terms across lines of BPS stability," JHEP **0807** (2008) 028 [arXiv:0805.0713 [hep-th]].

[46] R. Blumenhagen and M. Schmidt-Sommerfeld, "Power Towers of String Instantons for N=1 Vacua," JHEP **0807**, 027 (2008) [arXiv:0803.1562 [hep-th]].

[47] A. Collinucci, P. Soler and A. M. Uranga, "Non-perturbative effects and wall-crossing from topological strings," JHEP **0911** (2009) 025 [arXiv:0904.1133 [hep-th]].

[48] C. Petersson, P. Soler and A. M. Uranga, "D-instanton and polyinstanton effects from type I' D0-brane loops," JHEP **1006** (2010) 089 [arXiv:1001.3390 [hep-th]].

[49] M. Berasaluce-Gonzalez, L. E. Ibanez, P. Soler and A. M. Uranga, "Discrete gauge symmetries in D-brane models," JHEP **1112** (2011) 113 [arXiv:1106.4169 [hep-th]].

[50] P. Candelas, "Lectures On Complex Manifolds," In *Trieste 1987, Proceedings, Superstrings '87* 1-88.

[51] T. Hubsch, "Calabi-Yau manifolds: A Bestiary for physicists," Singapore, Singapore: World Scientific (1992) 362 p, 2nd ed. (1994) 374 p.

[52] B. R. Greene, "String theory on Calabi-Yau manifolds," [hep-th/9702155].

[53] F. G. Marchesano Buznego, "Intersecting D-brane models," Ph.D. thesis. [hep-th/0307252].

[54]

[55] R. Blumenhagen, M. Cvetic, P. Langacker, G. Shiu, "Toward realistic intersecting D-brane models," Ann. Rev. Nucl. Part. Sci. **55** (2005) 71-139. [hep-th/0502005];

[56] R. Blumenhagen, B. Kors, D. Lust, S. Stieberger, "Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes," Phys. Rept. **445** (2007) 1-193. [hep-th/0610327];

[57] F. Marchesano, "Progress in D-brane model building," Fortsch. Phys. **55** (2007) 491-518. [hep-th/0702094 [HEP-TH]];

[58] M. B. Green and J. H. Schwarz, "Infinity Cancellations in SO(32) Superstring Theory," Phys. Lett. B **151** (1985) 21.

[59] M. B. Green and J. H. Schwarz, "The Hexagon Gauge Anomaly in Type I Superstring Theory," Nucl. Phys. B **255** (1985) 93.

[60] J. Wess and J. Bagger, "Supersymmetry and supergravity," Princeton, USA: Univ. Pr. (1992) 259 p

[61] S. Weinberg, "The quantum theory of fields. Vol. 3: Supersymmetry," Cambridge, UK: Univ. Pr. (2000) 419 p

[62] F. Quevedo, S. Krippendorf and O. Schlotterer, "Cambridge Lectures on Supersymmetry and Extra Dimensions," arXiv:1011.1491 [hep-th].

[63] M. Berkooz, M. R. Douglas and R. G. Leigh, "Branes intersecting at angles," Nucl. Phys. B **480**, 265 (1996) [hep-th/9606139].

[64] H. Ooguri, Y. Oz and Z. Yin, "D-branes on Calabi-Yau spaces and their mirrors," Nucl. Phys. B **477**, 407 (1996) [hep-th/9606112].

[65] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, "Noncommutative compactifications of type I strings on tori with magnetic background flux," JHEP **0010**, 006 (2000) [hep-th/0007024].

[66] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadan, A. M. Uranga, "D = 4 chiral string compactifications from intersecting branes," J. Math. Phys. **42** (2001) 3103-3126. [hep-th/0011073].

[67] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadan, A. M. Uranga, "Intersecting brane worlds," JHEP **0102** (2001) 047. [hep-ph/0011132].

[68] L. E. Ibáñez, F. Marchesano, R. Rabadan, "Getting just the standard model at intersecting branes," JHEP **0111** (2001) 002. [hep-th/0105155].

[69] D. Cremades, L. E. Ibáñez, F. Marchesano, "Intersecting brane models of particle physics and the Higgs mechanism," JHEP **0207** (2002) 022. [hep-th/0203160].

[70] D. Cremades, L. E. Ibáñez, F. Marchesano, "Yukawa couplings in intersecting D-brane models," JHEP **0307** (2003) 038. [hep-th/0302105]

[71] F. Marchesano, G. Shiu, "MSSM vacua from flux compactifications," Phys. Rev. **D71** (2005) 011701. [hep-th/0408059].

[72] C. Beasley and E. Witten, "New instanton effects in supersymmetric QCD," JHEP **0501** (2005) 056 [arXiv:hep-th/0409149].

[73] C. Beasley and E. Witten, "New instanton effects in string theory," JHEP **0602** (2006) 060 [arXiv:hep-th/0512039].

[74] M. Cvetic and I. Papadimitriou, "Conformal field theory couplings for intersecting D-branes on orientifolds," Phys. Rev. D **68**, 046001 (2003) [Erratum-ibid. D **70**, 029903 (2004)] [hep-th/0303083].

[75] S. A. Abel and A. W. Owen, "Interactions in intersecting brane models," Nucl. Phys. B **663**, 197 (2003) [hep-th/0303124].

[76] R. Blumenhagen, M. Cvetic, S. Kachru and T. Weigand, "D-Brane Instantons in Type II Orientifolds," Ann. Rev. Nucl. Part. Sci. **59**, 269 (2009) [arXiv:0902.3251 [hep-th]].

[77] R. Argurio, M. Bertolini, S. Franco and S. Kachru, "Meta-stable vacua and D-branes at the conifold," JHEP **0706**, 017 (2007) [AIP Conf. Proc. **1031**, 94 (2008)] [hep-th/0703236].

[78] R. Argurio, M. Bertolini, G. Ferretti, A. Lerda and C. Petersson, "Stringy instantons at orbifold singularities," JHEP **0706** (2007) 067 [arXiv:0704.0262 [hep-th]].

[79] M. Bianchi, F. Fucito and J. F. Morales, "D-brane instantons on the T**6 / Z(3) orientifold," JHEP **0707**, 038 (2007) [arXiv:0704.0784 [hep-th]].

[80] L. E. Ibáñez, A. N. Schellekens, A. M. Uranga, "Instanton Induced Neutrino Majorana Masses in CFT Orientifolds with MSSM-like spectra," JHEP **0706** (2007) 011. [arXiv:0704.1079 [hep-th]]

[81] M. Cvetic, R. Richter, 2, T. Weigand, "Computation of D-brane instanton induced superpotential couplings: Majorana masses from string theory," Phys. Rev. **D76** (2007) 086002.

[82] R. Blumenhagen, M. Cvetic, R. Richter and T. Weigand, "Lifting D-Instanton Zero Modes by Recombination and Background Fluxes," arXiv:0708.0403 [hep-th].

[83] A. Strominger, "Massless black holes and conifolds in string theory," Nucl. Phys. B **451** (1995) 96 [arXiv:hep-th/9504090].

[84] J. de Boer, F. Denef, S. El-Showk, I. Messamah and D. Van den Bleeken, "Black hole bound states in $AdS_3 \times S^2$," JHEP **0811** (2008) 050 [arXiv:0802.2257 [hep-th]].

[85] J. de Boer, S. El-Showk, I. Messamah and D. Van den Bleeken, "Quantizing N=2 Multicenter Solutions," JHEP **0905**, 002 (2009) [arXiv:0807.4556 [hep-th]].

[86] S. Kachru and J. McGreevy, "Supersymmetric three-cycles and (super)symmetry breaking," Phys. Rev. D **61** (2000) 026001 [arXiv:hep-th/9908135].

[87] M. Cvetic, R. Richter, 2 and T. Weigand, "(Non-)BPS bound states and D-brane instantons," JHEP **0807**, 012 (2008) [arXiv:0803.2513 [hep-th]].

[88] D. Gaiotto, G. W. Moore and A. Neitzke, "Four-dimensional wall-crossing via three-dimensional field theory," Commun. Math. Phys. **299**, 163 (2010) [arXiv:0807.4723 [hep-th]].

[89] C. Itzykson and J. B. Zuber, "Quantum Field Theory," *New York, Usa: Mcgraw-hill (1980) 705 P.(International Series In Pure and Applied Physics)*

[90] M. Cicoli, C. P. Burgess and F. Quevedo, "Anisotropic Modulus Stabilisation: Strings at LHC Scales with Micron-sized Extra Dimensions," JHEP **1110**, 119 (2011) [arXiv:1105.2107 [hep-th]].

[91] M. Cicoli, F. G. Pedro and G. Tasinato, "Poly-instanton Inflation," JCAP **1112**, 022 (2011) [arXiv:1110.6182 [hep-th]].

[92] M. R. Douglas, "D-branes, categories and N = 1 supersymmetry," J. Math. Phys. **42** (2001) 2818 [arXiv:hep-th/0011017].

[93] R. Gopakumar and C. Vafa, "Topological gravity as large N topological gauge theory," Adv. Theor. Math. Phys. **2** (1998) 413 [arXiv:hep-th/9802016].

[94] R. Gopakumar and C. Vafa, "M-theory and topological strings I," arXiv:hep-th/9809187.

[95] R. Gopakumar and C. Vafa, "M-theory and topological strings II," arXiv:hep-th/9812127.

[96] R. Dijkgraaf and C. Vafa, "Matrix models, topological strings, and supersymmetric gauge theories," Nucl. Phys. B **644** (2002) 3 [arXiv:hep-th/0206255].

[97] R. Dijkgraaf and C. Vafa, "On geometry and matrix models," Nucl. Phys. B **644** (2002) 21 [arXiv:hep-th/0207106].

[98] R. Dijkgraaf and C. Vafa, "A perturbative window into non-perturbative physics," arXiv:hep-th/0208048.

[99] H. Ooguri and C. Vafa, "Summing up D-instantons," Phys. Rev. Lett. **77** (1996) 3296 [arXiv:hep-th/9608079].

[100] N. Seiberg and S. H. Shenker, "Hypermultiplet moduli space and string compactification to three dimensions," Phys. Lett. B **388** (1996) 521 [arXiv:hep-th/9608086].

[101] M. Rocek, C. Vafa and S. Vandoren, "Hypermultiplets and topological strings," JHEP **0602** (2006) 062 [arXiv:hep-th/0512206].

[102] Maxim Kontsevich, Yan Soibelman, "Stability structures, motivic Donaldson-Thomas invariants and cluster transformations" arXiv:0811.2435

[103] A. M. Uranga, "D-brane instantons and the effective field theory of flux compactifications," JHEP **0901** (2009) 048 [arXiv:0808.2918 [hep-th]].

[104] N. Nekrasov, H. Ooguri and C. Vafa, "S-duality and topological strings," JHEP **0410** (2004) 009 [arXiv:hep-th/0403167].

[105] D. Robles-Llana, F. Saueressig and S. Vandoren, "String loop corrected hypermultiplet moduli spaces," JHEP **0603** (2006) 081 [arXiv:hep-th/0602164].

[106] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, "R**4 couplings in M- and type II theories on Calabi-Yau spaces," Nucl. Phys. B **507** (1997) 571 [arXiv:hep-th/9707013].

[107] D. Robles-Llana, M. Rocek, F. Saueressig, U. Theis and S. Vandoren, "Nonperturbative corrections to 4D string theory effective actions from SL(2,Z) duality and supersymmetry," Phys. Rev. Lett. **98** (2007) 211602 [arXiv:hep-th/0612027].

[108] D. Robles-Llana, F. Saueressig, U. Theis and S. Vandoren, "Membrane instantons from mirror symmetry," Commun. Num. Theor. Phys. **1** (2007) 681 [arXiv:0707.0838 [hep-th]].

[109] F. Saueressig and S. Vandoren, "Conifold singularities, resumming instantons and non-perturbative mirror symmetry," JHEP **0707** (2007) 018 [arXiv:0704.2229 [hep-th]].

[110] S. Alexandrov, B. Pioline, F. Saueressig and S. Vandoren, "Linear perturbations of Hyperkahler metrics," Lett. Math. Phys. **87**, 225 (2009) [arXiv:0806.4620 [hep-th]].

[111] S. Alexandrov, B. Pioline, F. Saueressig and S. Vandoren, "Linear perturbations of quaternionic metrics," Commun. Math. Phys. **296**, 353 (2010) [arXiv:0810.1675 [hep-th]].

[112] S. Alexandrov, B. Pioline, F. Saueressig and S. Vandoren, "D-instantons and twistors," JHEP **0903**, 044 (2009) [arXiv:0812.4219 [hep-th]].

[113] S. Alexandrov, "D-instantons and twistors: Some exact results," J. Phys. A A **42**, 335402 (2009) [arXiv:0902.2761 [hep-th]].

[114] Y. Michel and B. Pioline, "Higher Derivative Corrections, Dimensional Reduction and Ehlers Duality," JHEP **0709** (2007) 103 [arXiv:0706.1769 [hep-th]].

[115] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908** (1999) 023 [arXiv:hep-th/9908088].

[116] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66** (2002) 106006 [arXiv:hep-th/0105097].

[117] K. Becker and M. Becker, "Supersymmetry breaking, M-theory and fluxes," JHEP **0107** (2001) 038 [arXiv:hep-th/0107044].

[118] M. B. Green, M. Gutperle and P. Vanhove, "One loop in eleven dimensions," Phys. Lett. B **409** (1997) 177 [arXiv:hep-th/9706175].

[119] F. Denef and G. W. Moore, "Split states, entropy enigmas, holes and halos," JHEP **1111**, 129 (2011) [hep-th/0702146 [HEP-TH]].

[120] D. Gaiotto, A. Strominger and X. Yin, "New Connections Between 4D and 5D Black Holes," JHEP **0602** (2006) 024 [arXiv:hep-th/0503217].

[121] R. Dijkgraaf, C. Vafa and E. Verlinde, "M-theory and a topological string duality," arXiv:hep-th/0602087.

[122] S. K. Donaldson and R. P. Thomas, "Gauge theory in higher dimensions," *Prepared for Conference on Geometric Issues in Foundations of Science in honor of Sir Roger Penrose's 65th Birthday, Oxford, England, 25-29 Jun 1996*

[123] E. Bergshoeff, R. Kallosh, A. K. Kashani-Poor, D. Sorokin and A. Tomasiello, "An index for the Dirac operator on D3 branes with background fluxes," JHEP **0510** (2005) 102 [arXiv:hep-th/0507069].

[124] P. K. Tripathy and S. P. Trivedi, "D3 brane action and fermion zero modes in presence of background flux," JHEP **0506** (2005) 066 [arXiv:hep-th/0503072].

[125] R. Kallosh, A. K. Kashani-Poor and A. Tomasiello, "Counting fermionic zero modes on M5 with fluxes," JHEP **0506** (2005) 069 [arXiv:hep-th/0503138].

[126] J. Park, "D3 instantons in Calabi-Yau orientifolds with(out) fluxes," Eur. Phys. J. C **67**, 263 (2010) [hep-th/0507091].

[127] M. Billo', L. Ferro, M. Frau, F. Fucito, A. Lerda and J. F. Morales, "Flux interactions on D-branes and instantons," JHEP **0810**, 112 (2008) arXiv:0807.1666 [hep-th].

[128] M. Billo', L. Ferro, M. Frau, F. Fucito, A. Lerda and J. F. Morales, "Non-perturbative effective interactions from fluxes," JHEP **0812**, 102 (2008) arXiv:0807.4098 [hep-th].

[129] S. Gukov, C. Vafa and E. Witten, "CFT's from Calabi-Yau four-folds," Nucl. Phys. B **584** (2000) 69 [Erratum-ibid. B **608** (2001) 477] [arXiv:hep-th/9906070].

[130] T. W. Grimm, "Non-Perturbative Corrections and Modularity in N=1 Type IIB Compactifications," JHEP **0710** (2007) 004 [arXiv:0705.3253 [hep-th]].

[131] P. G. Camara, E. Dudas, T. Maillard and G. Pradisi, "String instantons, fluxes and moduli stabilization," Nucl. Phys. B **795** (2008) 453 [arXiv:0710.3080 [hep-th]].

[132] P. G. Camara and E. Dudas, "Multi-instanton and string loop corrections in toroidal orbifold models," JHEP **0808** (2008) 069 [arXiv:0806.3102 [hep-th]].

[133] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Supersymmetric QCD," Nucl. Phys. B **241**, 493 (1984).

[134] C. Petersson, "Superpotentials From Stringy Instantons Without Orientifolds," JHEP **0805** (2008) 078 [arXiv:0711.1837 [hep-th]].

[135] N. Akerblom, R. Blumenhagen, D. Lust, E. Plauschinn and M. Schmidt-Sommerfeld, "Non-perturbative SQCD Superpotentials from String Instantons," JHEP **0704** (2007) 076 [arXiv:hep-th/0612132].

[136] D. L. Jafferis and G. W. Moore, "Wall crossing in local Calabi Yau manifolds," arXiv:0810.4909 [hep-th].

[137] W. y. Chuang and D. L. Jafferis, "Wall Crossing of BPS States on the Conifold from Seiberg Duality and Pyramid Partitions," Commun. Math. Phys. **292**, 285 (2009) arXiv:0810.5072 [hep-th].

[138] B. S. Acharya, M. Aganagic, K. Hori and C. Vafa, "Orientifolds, mirror symmetry and superpotentials," arXiv:hep-th/0202208.

[139] J. Walcher, "Evidence for Tadpole Cancellation in the Topological String," arXiv:0712.2775 [hep-th].

[140] D. Krefl and J. Walcher, "Real Mirror Symmetry for One-parameter Hypersurfaces," JHEP **0809** (2008) 031 [arXiv:0805.0792 [hep-th]].

[141] I. Brunner and M. Herbst, "Orientifolds and D-branes in N=2 gauged linear sigma models," arXiv:0812.2880 [hep-th].

[142] D. Krefl and J. Walcher, "The Real Topological String on a local Calabi-Yau," arXiv:0902.0616 [hep-th].

[143] D. Maulik, N. Nekrasov, A. Okounkov, R. Pandharipande, "Gromov-Witten theory and Donaldson-Thomas theory I," arXiv:math/0312059.

[144] M. Billo, L. Ferro, M. Frau, L. Gallot, A. Lerda and I. Pesando, "Exotic instanton counting and heterotic/type I' duality," JHEP **0907** (2009) 092 [arXiv:0905.4586 [hep-th]].

[145] M. Billo, M. Frau, F. Fucito, A. Lerda, J. F. Morales and R. Poghossian, JHEP **1005**, 107 (2010) [arXiv:1002.4322 [hep-th]].

[146] F. Fucito, J. F. Morales and R. Poghossian, "Exotic prepotentials from D(-1)D7 dynamics," JHEP **0910**, 041 (2009) arXiv:0906.3802 [hep-th].

[147] M. Gutperle, "A note on heterotic/type I' duality and D0 brane quantum mechanics," JHEP **9905** (1999) 007 [arXiv:hep-th/9903010].

[148] J. Polchinski and E. Witten, "Evidence for Heterotic - Type I String Duality," Nucl. Phys. B **460** (1996) 525 [arXiv:hep-th/9510169].

[149] S. Kachru and E. Silverstein, "On gauge bosons in the matrix model approach to M theory," Phys. Lett. B **396** (1997) 70 [arXiv:hep-th/9612162].

[150] O. Bergman, M. R. Gaberdiel and G. Lifschytz, "Branes, orientifolds and the creation of elementary strings," Nucl. Phys. B **509** (1998) 194 [arXiv:hep-th/9705130].

[151] O. Bergman, M. R. Gaberdiel and G. Lifschytz, "String creation and heterotic-type I' duality," Nucl. Phys. B **524** (1998) 524 [arXiv:hep-th/9711098].

[152] C. Bachas, C. Fabre, E. Kiritsis, N. A. Obers and P. Vanhove, "Heterotic/type-I duality and D-brane instantons," Nucl. Phys. B **509** (1998) 33 [arXiv:hep-th/9707126].

[153] E. Kiritsis and N. A. Obers, "Heterotic/type-I duality in $D < 10$ dimensions, threshold corrections and D-instantons," JHEP **9710** (1997) 004 [arXiv:hep-th/9709058].

[154] E. Kiritsis, "Duality and instantons in string theory," In *Trieste 1999, Superstrings and related matters* 127-205 [arXiv:hep-th/9906018].

[155] W. Lerche and S. Stieberger, "Prepotential, mirror map and F-theory on K3," Adv. Theor. Math. Phys. **2** (1998) 1105 [Erratum-ibid. **3** (1999) 1199] [arXiv:hep-th/9804176].

[156] W. Lerche, S. Stieberger and N. P. Warner, "Quartic gauge couplings from K3 geometry," Adv. Theor. Math. Phys. **3** (1999) 1575 [arXiv:hep-th/9811228].

[157] W. Lerche and S. Stieberger, "On the anomalous and global interactions of Kodaira 7-planes," Fortsch. Phys. **48** (2000) 155 [arXiv:hep-th/9903232].

[158] K. Foerger and S. Stieberger, "Higher derivative couplings and heterotic-type I duality in eight dimensions," Nucl. Phys. B **559** (1999) 277 [arXiv:hep-th/9901020].

[159] E. Gava, A. Hammou, J. F. Morales and K. S. Narain, "On the perturbative corrections around D-string instantons," JHEP **9903** (1999) 023 [arXiv:hep-th/9902202].

[160] D. M. Ghilencea and S. Groot Nibbelink, "String threshold corrections from field theory," Nucl. Phys. B **641** (2002) 35 [arXiv:hep-th/0204094].

[161] E. Kiritsis and B. Pioline, "On R**4 threshold corrections in type IIB string theory and (p,q) string instantons," Nucl. Phys. B **508**, 509 (1997) [arXiv:hep-th/9707018].

[162] J. A. Harvey and G. W. Moore, "Algebras, BPS States, and Strings," Nucl. Phys. B **463** (1996) 315 [arXiv:hep-th/9510182].

[163] L. Alvarez-Gaume, "An Introduction To Anomalies," HUTP-85/A092.

[164] L. J. Dixon, V. Kaplunovsky and J. Louis, "Moduli dependence of string loop corrections to gauge coupling constants," Nucl. Phys. B **355** (1991) 649.

[165] A. Hanany and E. Witten, "Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics," Nucl. Phys. B **492** (1997) 152 [arXiv:hep-th/9611230].

[166] M. B. Green and S. Sethi, "Supersymmetry constraints on type IIB supergravity," Phys. Rev. D **59** (1999) 046006 [arXiv:hep-th/9808061].

[167] G. Aldazabal, A. Font, L. E. Ibanez and F. Quevedo, "Chains of N=2, D=4 heterotic/type II duals," Nucl. Phys. B **461** (1996) 85 [arXiv:hep-th/9510093].

[168] T. Kawai, "$N = 2$ heterotic string threshold correction, $K3$ surface and generalized Kac-Moody superalgebra," Phys. Lett. B **372** (1996) 59 [arXiv:hep-th/9512046].

[169] N. Seiberg and E. Witten, “Comments on String Dynamics in Six Dimensions,” Nucl. Phys. B **471** (1996) 121 [arXiv:hep-th/9603003].

[170] G. Aldazabal, A. Font, L. E. Ibanez and A. M. Uranga, “New branches of string compactifications and their F-theory duals,” Nucl. Phys. B **492** (1997) 119 [arXiv:hep-th/9607121].

[171] M. Henningson and G. W. Moore, “Threshold corrections in K(3) x T(2) heterotic string compactifications,” Nucl. Phys. B **482** (1996) 187 [arXiv:hep-th/9608145].

[172] G. Lopes Cardoso, G. Curio and D. Lust, “Perturbative couplings and modular forms in N = 2 string models with a Wilson line,” Nucl. Phys. B **491** (1997) 147 [arXiv:hep-th/9608154].

[173] S. Stieberger, “(0,2) heterotic gauge couplings and their M-theory origin,” Nucl. Phys. B **541** (1999) 109 [arXiv:hep-th/9807124].

[174] M. Weiss, “Topological Amplitudes in Heterotic Strings with Wilson Lines,” JHEP **0708**, 024 (2007) [arXiv:0705.3112 [hep-th]].

[175] J. Polchinski, “Tensors from K3 orientifolds,” Phys. Rev. D **55** (1997) 6423 [arXiv:hep-th/9606165].

[176] J. Erler, “Anomaly Cancellation In Six-Dimensions,” J. Math. Phys. **35** (1994) 1819 [arXiv:hep-th/9304104].

[177] T. Banks, L. J. Dixon, “Constraints on String Vacua with Space-Time Supersymmetry,” Nucl. Phys. **B307** (1988) 93-108.

[178] L. F. Abbott, M. B. Wise, “Wormholes And Global Symmetries,” Nucl. Phys. **B325** (1989) 687.

[179] S. R. Coleman, K. -M. Lee, “Wormholes Made Without Massless Matter Fields,” Nucl. Phys. **B329** (1990) 387.

[180] R. Kallosh, A. D. Linde, D. A. Linde, L. Susskind, “Gravity and global symmetries,” Phys. Rev. **D52** (1995) 912-935. [hep-th/9502069].

[181] T. Banks, N. Seiberg, “Symmetries and Strings in Field Theory and Gravity,” Phys. Rev. **D83** (2011) 084019. [arXiv:1011.5120 [hep-th]].

[182] M. G. Alford, F. Wilczek, “Aharonov-Bohm Interaction of Cosmic Strings with Matter,” Phys. Rev. Lett. **62** (1989) 1071;

[183] L. M. Krauss, F. Wilczek, “Discrete Gauge Symmetry in Continuum Theories,” Phys. Rev. Lett. **62** (1989) 1221;

[184] J. Preskill, L. M. Krauss, “Local Discrete Symmetry And Quantum Mechanical Hair,” Nucl. Phys. **B341** (1990) 50-100.

[185] L. E. Ibáñez, G. G. Ross, “Discrete gauge symmetry anomalies,” Phys. Lett. **B260** (1991) 291-295.

[186] L. E. Ibáñez, G. G. Ross, "Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model," Nucl. Phys. **B368** (1992) 3-37.

[187] L. E. Ibáñez, "More about discrete gauge anomalies," Nucl. Phys. B **398** (1993) 301 [arXiv:hep-ph/9210211].

[188] H. K. Dreiner, C. Luhn, M. Thormeier, "What is the discrete gauge symmetry of the MSSM?," Phys. Rev. **D73** (2006) 075007. [hep-ph/0512163].

[189] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, "The Heterotic Road to the MSSM with R parity," Phys. Rev. D **77** (2008) 046013 [arXiv:0708.2691 [hep-th]].

[190] P. G. Camara, L. E. Ibanez and F. Marchesano, "RR photons," JHEP **1109**, 110 (2011) [arXiv:1106.0060 [hep-th]].

[191] A. M. Uranga, "D-brane probes, RR tadpole cancellation and K theory charge," Nucl. Phys. **B598** (2001) 225-246. [hep-th/0011048].

[192] R. Blumenhagen, M. Cvetic, D. Lust, R. Richter, 2, T. Weigand, "Non-perturbative Yukawa Couplings from String Instantons," Phys. Rev. Lett. **100** (2008) 061602. [arXiv:0707.1871 [hep-th]].

[193] L. E. Ibáñez, R. Richter, 2, "Stringy Instantons and Yukawa Couplings in MSSM-like Orientifold Models," JHEP **0903** (2009) 090. [arXiv:0811.1583 [hep-th]].

[194] T. P. T. Dijkstra, L. R. Huiszoon, A. N. Schellekens, "Supersymmetric standard model spectra from RCFT orientifolds," Nucl. Phys. **B710** (2005) 3-57. [hep-th/0411129].

[195] T. P. T. Dijkstra, L. R. Huiszoon, A. N. Schellekens, "Chiral supersymmetric standard model spectra from orientifolds of Gepner models," Phys. Lett. **B609** (2005) 408-417. [hep-th/0403196].

[196] P. Anastasopoulos, T. P. T. Dijkstra, E. Kiritsis, A. N. Schellekens, "Orientifolds, hypercharge embeddings and the Standard Model," Nucl. Phys. **B759** (2006) 83-146. [hep-th/0605226].

[197] E. Kiritsis, M. Lennek, B. Schellekens, "SU(5) orientifolds, Yukawa couplings, Stringy Instantons and Proton Decay," Nucl. Phys. **B829**, 298-324 (2010). [arXiv:0909.0271 [hep-th]].

[198] P. Anastasopoulos, G. K. Leontaris, R. Richter, A. N. Schellekens, "SU(5) D-brane realizations, Yukawa couplings and proton stability," JHEP **1012**, 011 (2010). [arXiv:1010.5188 [hep-th]].

[199] R. Blumenhagen, S. Moster, T. Weigand, "Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds," Nucl. Phys. **B751** (2006) 186-221. [hep-th/0603015].

[200] R. Blumenhagen, S. Moster, R. Reinbacher, T. Weigand, "Massless Spectra of Three Generation U(N) Heterotic String Vacua," JHEP **0705** (2007) 041. [hep-th/0612039].

[201] J. J. Heckman, "Particle Physics Implications of F-theory," arXiv:1001.0577 [hep-th].

[202] T. Weigand, "Lectures on F-theory compactifications and model building," Class. Quant. Grav. **27** (2010) 214004. [arXiv:1009.3497 [hep-th]].

[203] R. Donagi, M. Wijnholt, "Model Building with F-Theory," [arXiv:0802.2969 [hep-th]].

[204] C. Beasley, J. J. Heckman, C. Vafa, "GUTs and Exceptional Branes in F-theory - I," JHEP **0901**, 058 (2009). [arXiv:0802.3391 [hep-th]].

[205] C. Beasley, J. J. Heckman, C. Vafa, "GUTs and Exceptional Branes in F-theory - II: Experimental Predictions," JHEP **0901**, 059 (2009). [arXiv:0806.0102 [hep-th]].

[206] R. Donagi, M. Wijnholt, "Breaking GUT Groups in F-Theory," [arXiv:0808.2223 [hep-th]].

[207] H. Hayashi, T. Kawano, R. Tatar, T. Watari, "Codimension-3 Singularities and Yukawa Couplings in F-theory," Nucl. Phys. **B823**, 47-115 (2009). [arXiv:0901.4941 [hep-th]].

[208] R. Donagi, M. Wijnholt, "Higgs Bundles and UV Completion in F-Theory," [arXiv:0904.1218 [hep-th]].

[209] J. Marsano, N. Saulina, S. Schafer-Nameki, "F-theory Compactifications for Supersymmetric GUTs," JHEP **0908** (2009) 030. [arXiv:0904.3932 [hep-th]]

[210] J. J. Heckman, A. Tavanfar, C. Vafa, "The Point of E(8) in F-theory GUTs," JHEP **1008** (2010) 040. [arXiv:0906.0581 [hep-th]].

[211] J. Marsano, N. Saulina, S. Schafer-Nameki, "Monodromies, Fluxes, and Compact Three-Generation F-theory GUTs," JHEP **0908**, 046 (2009). [arXiv:0906.4672 [hep-th]].

[212] R. Blumenhagen, T. W. Grimm, B. Jurke, T. Weigand, "Global F-theory GUTs," Nucl. Phys. **B829** (2010) 325-369. [arXiv:0908.1784 [hep-th]]

[213] J. Marsano, N. Saulina, S. Schafer-Nameki, "Compact F-theory GUTs with U(1) (PQ)," JHEP **1004** (2010) 095. [arXiv:0912.0272 [hep-th]].

[214] E. Dudas, E. Palti, "Froggatt-Nielsen models from E(8) in F-theory GUTs," JHEP **1001** (2010) 127. [arXiv:0912.0853 [hep-th]].

[215] T. W. Grimm, S. Krause, T. Weigand, "F-Theory GUT Vacua on Compact Calabi-Yau Fourfolds," JHEP **1007** (2010) 037. [arXiv:0912.3524 [hep-th]].

[216] H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, "More on Dimension-4 Proton Decay Problem in F-theory – Spectral Surface, Discriminant Locus and Monodromy," Nucl. Phys. **B840** (2010) 304-348. [arXiv:1004.3870 [hep-th]].

[217] T. W. Grimm, T. Weigand, "On Abelian Gauge Symmetries and Proton Decay in Global F-theory GUTs," Phys. Rev. **D82** (2010) 086009. [arXiv:1006.0226 [hep-th]].

[218] J. Marsano, N. Saulina, S. Schafer-Nameki, "A Note on G-Fluxes for F-theory Model Building," JHEP **1011** (2010) 088. [arXiv:1006.0483 [hep-th]].

[219] E. Dudas, E. Palti, "On hypercharge flux and exotics in F-theory GUTs," JHEP **1009** (2010) 013. [arXiv:1007.1297 [hep-ph]].

[220] C. Ludeling, H. P. Nilles, C. C. Stephan, "The Potential Fate of Local Model Building," Phys. Rev. **D83** (2011) 086008. [arXiv:1101.3346 [hep-th]].

[221] M. J. Dolan, J. Marsano, N. Saulina, S. Schafer-Nameki, "F-theory GUTs with U(1) Symmetries: Generalities and Survey," [arXiv:1102.0290 [hep-th]].

[222] M. Buican, D. Malyshev, D. R. Morrison, H. Verlinde, M. Wijnholt, "D-branes at Singularities, Compactification, and Hypercharge," JHEP **0701** (2007) 107. [hep-th/0610007].

[223] R. Blumenhagen, G. Honecker, T. Weigand, "Loop-corrected compactifications of the heterotic string with line bundles," JHEP **0506** (2005) 020. [hep-th/0504232].

[224] R. Blumenhagen, G. Honecker, T. Weigand, "Supersymmetric (non-)Abelian bundles in the Type I and SO(32) heterotic string," JHEP **0508** (2005) 009. [hep-th/0507041].

[225] M. Berasaluce-Gonzalez, P. G. Cámara, F. Marchesano, A. M. Uranga, "Work in progress".