arXiv:0902.0713v1 [q-fin.ST] 4 Feb 2009

Correction to "Leverage and volatility feedback effects in high-frequency data" [J. Financial Econometrics 4 (2006) 353–384] *

Amparo Baíllo†

Universidad Autónoma de Madrid (Spain)

Abstract

Bollerslev *et al.* (2006) study the cross-covariances for squared returns under the Heston (1993) stochastic volatility model. In order to obtain these cross-covariances the authors use an incorrect expression for the distribution of the squared returns. Here we will obtain the correct distribution of the squared returns and check that, under this new distribution, the result in Appendix A.2 in Bollerslev *et al.* (2006) still holds.

*Research partially supported by Spanish grant MTM2007-66632.

†Address for correspondence: Departamento de Matemáticas, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid (Spain). Phone: +34 914977641. e-mail: amparo.baillo@uam.es

1. Correction to "Leverage and volatility feedback effects in high-frequency data"

Bollerslev *et al.* (2006) study the cross-covariances for squared returns under the Heston (1993) stochastic volatility model

$$\begin{aligned} dp_t &= (\mu + cV_t)\,dt + \sqrt{V_t}\,dB_t \\ dV_t &= \kappa(\theta - V_t)\,dt + \sigma\sqrt{V_t}\,dW_t, \end{aligned} \tag{1}$$

where B_t and W_t are correlated Brownian motions with $\mathrm{corr}(dB_t, dW_t) = \rho$. For simplicity, it is assumed that $\mu = c = 0$. If the continuously compounded returns from time t to time $t+\Delta$ are defined as $R_{t,t+\Delta} = p_{t+\Delta} - p_t = \int_t^{t+\Delta} \sqrt{V_u}\,dB_u$, then in Appendix A.2 of Bollerslev *et al.* (2006) it is proved that, for $n = 0, 1, 2, \ldots$,

$$\mathrm{cov}(R^2_{t+(n-1)\Delta,t+n\Delta}, R_{t-\Delta,t}) = (1 - \kappa a_\Delta)^{n-1} \rho\, \sigma\, \theta\, a^2_\Delta, \tag{2}$$

where $a_\Delta = (1 - e^{-\kappa\Delta})/\kappa$.

In order to obtain (2) these authors use the following distribution of the squared returns

$$R^2_{t,t+\Delta} = 2\int_t^{t+\Delta} R_{u,u+\Delta}\,\sqrt{V_u}\,dB_u + \int_t^{t+\Delta} V_u\,du. \tag{3}$$

Observe, however, that $R^2_{t,t+\Delta} = (p_{t+\Delta} - p_t)^2$ cannot depend on the values of $R_{u,u+\Delta}$ with $u \in [t, t+\Delta]$, that is, on returns which are posterior to $t+\Delta$. Here we will obtain the correct expression for the distribution of the squared returns and check that, under this new distribution, result (2) in Bollerslev *et al.* (2006) still holds.

In order to obtain the distribution of the squared returns observe that $R^2_{t,t+\Delta} = p^2_{t+\Delta} + p^2_t - 2\,p_t\,p_{t+\Delta}$. By Itô's Lemma we have that

$$p^2_{t+\Delta} = p^2_t + 2\int_t^{t+\Delta} p_u\,\sqrt{V_u}\,dB_u + \int_t^{t+\Delta} V_u\,du. \tag{4}$$

Using (1) and (4) we have that

$$R^2_{t,t+\Delta} = 2\int_t^{t+\Delta} R_{t,u}\,\sqrt{V_u}\,dB_u + \int_t^{t+\Delta} V_u\,du. \tag{5}$$

The only difference between (3) and (5) is the first integral. This is why a large part of the proof of (2) in Bollerslev *et al.* (2006) remains valid. More concretely, it is still true that

$$\mathrm{cov}(R^2_{t+(n-1)\Delta,t+n\Delta}, R_{t-\Delta,t}) = E\left(\int_{t+(n-1)\Delta}^{t+n\Delta} V_u\,du, \int_{t-\Delta}^{t} \sqrt{V_u}\,dB_u\right).$$

To show this it may easily be checked that $E\left(\int_{t+(n-1)\Delta}^{t+n\Delta} R_{t+(n-1)\Delta,u}\sqrt{V_u}\,dB_u \mid F_{t+(n-1)\Delta}\right) = 0.$

References

Bollerslev, T., Litvinova, J. and Tauchen, G. (2006). Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics*, 4, 353–384.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The Review of Financial Studies*, 6, 327–343.