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Optimal Pollution Standards and Non-Compliance in a Dynamic Framework *

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Abstract

In this paper we present a Stackelberg differential game to study the dynamic interaction between a polluting firm and a regulator who sets pollution limits overtime. At each time, the firm settles emissions taking into account the fine for non-compliance, and balances current costs of investments in a capital stock which allows for future emission reductions. We show that the optimal effective pollution limit path, which is the pollution level above which the fine is truly imposed, decreases overtime, inducing a rise in capital stock and a decrease in both emissions and the level of non-compliance. If the effective pollution limit coincides with the pollution limit set by the regulator, we generally find a bounded value of the severity of the fine that maximizes social welfare. If the effective pollution limit is larger than the pollution limit set by the regulator due to fine discounts in exchange for firm's investment in capital, the effect of a more severe fine depends on the magnitude of this discount. In the limiting scenario with a sufficiently large severity of the fine, emissions coincide with the effective pollution limit and no penalties are levied, since the firm shows adequate adaptation progress through capital investment.

Key words: non-compliance; fines; pollution standards; dynamic regulation; Stackelberg differential games.

JEL Codes: C61, C73, K32, K42, L51, Q28.

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1 Introduction

The analysis of regulatory standard setting and compliance issues has been traditionally studied in static contexts, see the literature review in Section 2. The typical problem is very simple and can be summarized as follows. At one moment in time, a regulator sets a regulatory limit taking into consideration the possibility that firms rationally decide to exceed that limit. The latter is likely to happen if firms' expected costs of violating the limit (basically expected sanctions) are smaller than firms' compliance costs.

The static approach, which is very useful for many reasons, is of limited applicability when we seek to explain the progressive implementation of more stringent limits overtime, as well as the firms' progressive adaptation to harsher regulatory environments. However, this is a common characteristic of many regulations dealing with pollution. For example, the US National Pollutant Discharge Elimination System (NPDES) permit program under the Clean Water Act contains a number of programs and initiatives which require facilities which discharge pollutants into waters of the US to obtain a permit to release specific amounts of pollution.[1] Among others, the Multi-Sector General Permit for stormwater discharges associated with industrial activity and the Vessel General Permit for Vessel Discharges have been progressively tightened since their respective initial issuances.[2]

One of the reasons why regulations are gradually tightened overtime is that polluting facilities might have difficulties to comply even with very lax pollution limits in the starting periods.

[1]Consult the Environmental Protection Agency (EPA)'s webpage for additional information (http://cfpub.epa.gov/npdes/).

[2]The first one has been progressively tightened in 2000, 2008 and 2013, since the initial issuance on September 29, 1995 (http://cfpub.epa.gov/npdes/home.cfm?program_id=6 for details). The second one was issued initially in 2008, and then reissued in 2013 (http://cfpub.epa.gov/npdes/home.cfm?program_id=350).

Progressive adaptation to the new environmental requirements can help facilities to easily comply with much more tighter limits after some time. Since adaptation is typically costly, regulators might find it convenient to help facilities with low (or even absent) fines for non-compliance in case of adequate adaptation progress. An example in point is the EPA's Audit Policy, which lists some conditions for enforcement relief with the NPDES programs. In particular, fines for non-compliance can be reduced up to 100% of the nongravity-based part and up to 75% of the gravity-based component if firms promptly disclose and correct any discovered violations as a result of a self-audit procedure.[3] These practices generally involve large investment efforts in compliance-promoting activities, including information costs such as training personnel about regulatory requirements. Overtime, the likelihood of violations decreases because facilities are able to identify and correct problems before they become violations, see Stafford (2005).

In the present paper, we consider the dynamic setting of pollution standards in a context of imperfect compliance. To our knowledge, our study is a first attempt in the literature to address this issue (see Section 2, below). Within this context, we specifically aim to answer two research questions. The first question is whether pollution limits should be soft in the beginning and progressively tighter overtime. Generally, such soft regulatory introductions may obey political reasons since new regulations impose high cost burdens on the industry and may typically find harsh opposition. So, are there efficiency reasons as well to justify such progressive regulatory tightening? The second question is whether it would be socially beneficial that regulators offer penalty discounts in exchange for environmentally-friendly actions to help facilities to better adequate to more stringent regulations. Again, are there efficiency reasons to support such

[3]Incentives for Self-Policing: Discovery, Disclosure, Correction and Prevention of Violations − Final Policy Statement, 60 Fed. Reg. 66,706 (December 22, 1995), revised on April 11, 2000.

practices?

We model the dynamic interaction between a representative polluting firm and a regulator by means of a differential game played à la Stackelberg. Emissions are considered as an input to produce a consumption good. The firm invests to enlarge the capital stock, while the regulator sets the pollution standard overtime. At each point in time, the firm can either comply with the standard or pollute above the standard and then be subject to an expected fine. The firm then faces a double dilemma, it has to balance on the one hand, the productive gains from higher emissions with the associated increment in the fine for non-compliance; and on the other hand, the current cost of investment in a capital stock (which is a substitute for emissions in the production of output) with the future fine reductions from lower emissions. While the firm seeks to maximize discounted profits throughout an infinite time horizon, the regulator is concerned about social welfare, that is, firm's profits net of environmental damages and the administrative costs of imposing sanctions.

In the first part of the paper, we consider a base model where the expected fine is quadratic in the degree of non-compliance and penalty discounts are not allowed. Assuming that the initial capital stock is small, we analytically show that the optimal path for the pollution standard is decreasing overtime. Interestingly, the induced emissions and the induced degree of non-compliance are decreasing as well, while the capital stock increases over time. Note that the fact that a tighter standard induces a lower rate of non-compliance cannot be explained in a static setting (in there, a tighter standard always predicts a larger gap between emissions and the standard). However, in a dynamic context the firm can progressively adapt to a stricter standard by means of capital investment. Within this base model, we also perform a comparative statics exercise with respect to the severity of the fine (that is, for a given degree of non-compliance, we analyze variations in the resulting expected penalty). We find that the severity of the fine must be kept bounded in order to reach a maximum social welfare level. The reason for this result is that the regulator and the firm value non-compliance differently. If initially small, a more severe penalty induces the firm to exert additional compliance efforts, which reduces firm's profits while improving social welfare due to lower damages from pollution. However, when the severity of the fine is very large, the regulator considers that the firm is putting too much effort on compliance and too little on productive activities. An additional increase in the severity of the fine would lead the regulator to relax the standard to help the firm to comply. Firm's profits would increase and the resulting higher emissions would reduce social welfare.

In the second part of the paper, we extend our base model to explore the consequences of offering discounts in the fine for non-compliance in exchange for the firm's investments in capital. Technically, we do so by introducing the concept of *effective standard*, which is the pollution level above which the fine for non-compliance is truly imposed. This effective standard is laxer than the announced standard if the firm invests to increase the capital stock. The level of non-compliance is now a broader concept, which depends on the stock of capital, the level of emissions, the capital investment and the pollution limit. In this new scenario, we show similar patterns for the evolution of the relevant variables as those found in the base model, that is, decreasing standards, decreasing emissions, decreasing degree of non-compliance and increasing capital stock. Within this extended model, we perform several comparative statics exercises with respect to the severity of the fine and the amount of the fine discount. First, for a given severity of the fine, we interestingly find that offering a fine discount in exchange for firm's capital investment is socially desirable, and we indeed numerically find the value of the discount that maximizes social welfare. The existence of a bounded fine discount which attains a maximum social welfare again stems from the different valuation of non-compliance by firm and regulator. Initial increases in the fine discount induce firm's investments, reducing emissions and improving social welfare. However, when the fine discount is already too high, the cost of additional investments associated with increases in the fine discount is perceived by the regulator as excessively high. Thus, the regulator would relax the emission standard making compliance easier, although the associated rise in emissions would reduce social welfare. Second, for a given fine discount, the severity of the fine that maximizes social welfare need not be bounded as in the base model. When the fine discount is small enough, the severity of the fine that maximizes social welfare has a bounded value. However, for large fine discounts, the severity of the fine that maximizes social welfare is infinitely high. This limiting situation induces full compliance with the effective pollution limit, since the firm is threatened by a very large punishment if it pollutes above that limit. Emissions are still above the pollution limit imposed by the regulator, but due to fine discounts no penalties are levied. Indeed, the corresponding social welfare level obtained in this limiting situation is very close to the first-best level.

The remainder of the paper is organized as follows. In section 2, we review the related literature and stress our contribution. In section 3 we present the base model, where the fine depends only on the degree of non-compliance. In section 4, we present the characteristics of the optimal policy in the base model. In section 5, we extend the model to allow for discounts of the fine for non-compliance. We conclude in Section 6. All the proofs are in the Appendix.

2 Contribution to the literature

To our knowledge, and as pointed out in the introduction, the theoretical analysis of pollution standards and non-compliance issues has been only studied in static contexts, see for example Downing and Watson (1974), Harford (1978), Veljanovski (1984), Kambhu (1989), Jones (1989), Jones and Scotchmer (1990), Keeler (1995), or Arguedas (2008, 2013). These studies explore the relationship between regulatory stringency and both monitoring and enforcement strategies under alternative modeling structures in a one period model. Modelling alternatives include exogenous versus endogenous standard settings, alternative regulatory objectives, hierarchical versus single-layered governments, compliance-constrained versus imperfect-compliance-inducing policies, etc. The dynamic approach followed in this paper is then crucial to explain the progressive implementation of more stringent regulations, as well as firms' progressive adaptation to comply with tighter norms. Among the papers in the static context, the closest to ours is Arguedas (2013). There, penalty reductions in exchange for investment efforts by polluting firms are specifically modelled and questioned as appropriate from an optimal perspective. Arguedas (2013) finds three critical conditions for the social convenience of such reductions: (i) administrative costs of sanctioning, (ii) imperfect compliance, and (iii) fines progressive in the degree of non-compliance. In the present paper, we assume these three specific conditions in a dynamic framework. While Arguedas (2013) does not characterize the optimal fine discount, we are able to find it numerically.

In the dynamic context, the endogenous determination of the regulatory standards and the monitoring and enforcement issues have been considered separately in the literature.[4] On the

[4]In the case of tradable permits, only Innes (2003), Stranlund et al. (2005) and Lappi (2013) jointly consider enforcement issues and intertemporal permit trading, although neither of them jointly considers the endogeneity of the permits and the possibility of non-compliance with these permits overtime. Both Innes (2003) and Stranlund et al. (2005) constrain the analysis to enforcement policies that induce full compliance, such that the former assumes an enforcement strategy which consists only of a costly penalty for permit violations, while the second considers the reporting and monitoring functions of enforcement altogether. Lappi (2013) allows for the possibility of non-compliance, assuming that the auditing probability is subjective and, therefore, compliance decisions are made according to the firms' beliefs about that probability. However, emission permits are also exogenous in that setting.

one hand, all the papers that deal with standard setting assume perfect compliance and, therefore, abstract from modeling inspection frequencies and fines for non-compliance. On the other hand, all the papers that deal with dynamic enforcement do so assuming exogenous (or constant overtime) regulatory regimes. We now elaborate on these two related strands of the literature.

Regarding the literature that considers dynamic problems of regulatory standard setting assuming perfect compliance, Beavis and Dobbs (1986) is probably the first study of this kind. There, the environmental authority selects the pollution limit and the time at which that limit comes into force. Fines for non-compliance are not specifically modelled, since it is assumed that the firm complies with the standard once the regulatory period starts. The spirit of this study is similar to ours, and calls for the social convenience of polluters' gradual adaptation to new environmental regulations. The main difference is that we model a more realistic progressive tightening of the standard, which induces a decrease in the degree of non-compliance overtime. However, Beavis and Dobbs (1986) only consider a first period of no regulation where compliance is not an issue, and a second period of full compliance with the regulation. This difference allows us to explain the progressive decrease in non-compliance levels, even under more stringent pollution limits overtime, a result that cannot be explained in the context of Beavis and Dobbs (1986).

Some extensions of pollution standards setting in dynamic contexts include uncertainty about the exact pollution limit and the time at which the regulatory period starts (Hartl, 1992), persistent pollution (Conrad, 1992, Falk and Mendelsohn, 1993) or incentive-compatibility issues (Benford, 1998) among others, but the common characteristic of all these studies is that polluters are assumed to comply with pollution limits, and the enforcement aspects of the policy are absent.

Regarding the literature on dynamic enforcement, relevant references are those of Harrington (1988), Harford and Harrington (1991), Raymond (1999) and Friesen (2003). A common feature of all these studies is the specific modelling of monitoring decisions. There, budget constrained regulators distribute inspections in response to compliance information, thus prompting increased compliance. However, in all these studies firms are assumed to undertake discrete decisions whether to comply or to violate a given regulation, and then be moved to the bad group in the case of non-compliance. Therefore, the progressive tightening of the regulations overtime cannot be explained in these models either.

An extensive number of works in the literature suggests that fines for non-compliance should be bounded. Our paper also fits with this literature showing the conditions under which the fine which maximizes social welfare is bounded. Besides the above mentioned papers that include the possibility of targeting enforcement in dynamic settings, the literature gives alternative justifications for such a conclusion. Examples include self-reporting of emission levels (Livernois and Mckenna 1999), penalty evasion (Kambhu 1989), possible inverse relationships between fines and conviction probabilities (Andreoni 1991), marginal deterrence (Shavell 1992), specific versus general enforcement (Shavell 1991), hierarchical governments (Saha and Poole 2000; Decker 2007), regulatory dealing (Heyes and Rickman 1999), or others.

Finally, our paper is also related to the literature that analyzes the incentives set by different policy measures for investing in greener technologies, see Jaffe et al. (2003) or Requate (2005) for detailed overviews. The first works within this literature are those of Downing and White (1986) and Milliman and Prince (1989), who show that taxes and auctioned permits generally provide the largest innovation and adoption incentives. Later studies analyze the consequences of partial diffusion of new technologies (Requate and Unold, 2003), imperfect competition in the supply of abatement technology (Maia and Sinclair-Desgagné, 2005), uncertainty with regards to abatement costs or permit prices (Zhao, 2003 and Insley, 2003), uncertainty with regards to regime changes (Nishide and Nomi, 2009), international competition (Feenstra et al., 2001), induced technological progress (Krysiak, 2011), etc.

3 The base model

A representative firm produces a consumption good with capital and emissions as inputs.[5] Let $Y(t)$, $K(t)$, $E(t)$ respectively denote the levels of production, capital and emissions of the facility at time t. For mathematical convenience we assume that the production function is linear quadratic, as follows:

$$Y(K(t),E(t)) = K(t) + \sigma E(t) - \frac{[K(t)+\sigma E(t)]^2}{2}. \tag{1}$$

Note that this is a concave function with a constant marginal rate of technical substitution between emissions and capital given by $\sigma > 0$.

The flow of emissions is a decision variable of the firm, while capital is accumulated over time according to the investment decision, $I(t)$, such that

$$\dot{K}(t) = I(t) - \delta K(t), \quad K(0) = k_0, \tag{2}$$

where $\delta > 0$ is the rate of depreciation of capital and $k_0 \geq 0$ is the level of capital at $t = 0$. At each time t, investment costs are a quadratic function of the investment level, given by $C(I(t)) = cI^2(t)/2$, with $c > 0$.

From the point of view of the firm, capital accumulation is costly while emissions are free in

[5]Copeland and Taylor (1994) and some other papers cited therein also consider emissions as an input in the production process.

the absence of any regulation. However, instantaneous emissions cause environmental damages, given by the quadratic expression $D(E(t)) = d(E(t))^2/2$, with $d > 0$. [6]

We assume that an environmentally concerned regulator wants to make the firm (partially) responsible for the environmental damages caused. The regulator imposes an emission target or pollution limit which can vary across time, $L(t)$. At each time t, the firm faces a fine which depends on the level of non-compliance with the regulation, given by $N(E(t), L(t)) = E(t) - L(t)$, as long as $E(t) > L(t)$. We assume the fine to be quadratic in the degree of non-compliance, that is, $F(E\,(t)\,, L\,(t)) = f[E\,(t) - L\,(t)]^2/2$, or zero under compliance, where $f \geq d$ measures the severity of the fine.[7]

Note that the fine does depend on the degree of non-compliance, determined by the firm's decisions on emissions and the regulator's decision on the emission standard, all of them time-varying variables. Moreover the fine is also dependent on the severity of the fine, f, considered as a constant. The reason for this modelling is that in practice the general structure of the fine is less flexible than the level of the standard itself. Thus, we assume a time-invariant quadratic shape for the fine, although the specific level of the standard upon which the firm can be penalized is variable through time.

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[6]In this paper, we do not consider cumulative pollution effects. Cases we have in mind are those of degradable pollution, assimilative waste or flow pollutants, in which per-period environmental damages are mainly affected by per-period pollution. Probably, the clearest example of a flow pollutant is noise, but there are others, such as organic effluents associated with brewing or paper production (methane, carbon monoxide, mono-nitrogen oxides or sulfur dioxide, among others) or liquid waste loads on water bodies with large assimilative capacity. For example, cumulative effects are not considered in Beavis and Dobbs (1986) either.

[7]The assumption $f \geq d$ allows us to concentrate on the subset of most intuitive results. In particular, this assumption ensures a non-negative optimal standard. Also, in this paper we abstract from monitoring issues. One possibility is to assume that monitoring is perfect, such that the firm that pollutes above the effective standard is surely caught and fined. This occurs for example in cases where facilities are subject to continuous monitoring, that is, in cases where accurate monitoring devices (installed by the regulator or by the facilities themselves) measure noise levels, sulfur dioxide levels, waste load levels, etc. Another possibility is to assume that the firm is risk-neutral and the fine is actually an expected fine, such that parameter f includes an exogenous probability of being caught and fined. The convex specification for the expected fine may well reflect the fact that firms are more likely discovered and punished when the degree of non-compliance is large than when it is small.

Following the standard literature on pollution regulation, we consider a differential game played *à la Stackelberg*, where the regulator is the leader and the firm is the follower. A feedback Stackelberg equilibrium is the solution we look for. As it is usual for this type of differential games with an infinite time horizon, we assume that the agents (the firm and the regulator) employ stationary strategies, i.e, their strategies and value functions do not explicitly depend on time (see, for example, Dockner et al., 2000).

The firm, who acts as the Stackelberg follower, chooses the paths for emissions and investment in order to maximize the present value of profits over an infinite time horizon, taking into account the time evolution of the capital stock. Instantaneous profits are given by the income from production, minus the investment costs and the fine for non-compliance. Considering the consumption good as the *numéraire*, the dynamic maximization problem for the firm is the following:[8]

$$\max_{I,E} \int_0^\infty \left[Y(K,E) - C(I) - F(E,L)\right] e^{-\rho t} dt$$
$$\text{s.t.: } \dot{K} = I - \delta K, \quad K(0) = k_0, \tag{3}$$

where $\rho > 0$ is the discount factor.

The regulator acts as a Stackelberg leader and decides the optimal pollution limit path, $L(t)$, taking into account the firm's optimal response to the policy, $\hat{I}(K, L)$ and $\hat{E}(K, L)$ (i.e., the investment and emissions reaction functions that solve problem (3)). The regulator is concerned about the firm's profits, the environmental damages and the social cost of enforcing the pollution limit. We define the latter as a proportion $h \in (0, 1)$ of the fine for non-compliance. The

[8]The time argument is omitted here and henceforth when no confusion can arise. As a general principle, upper-case letters denote time-dependent (either state or control) variables, while lower-case letters denote time-independent parameters.

particular case where $h = 0$ means that the total amount of the penalty (if any) is socially redistributed lump-sum. However, the more general case where $h > 0$ reflects the existence of positive social costs associated with imposing penalties, such as administrative costs of court processes, citizens' discomfort with law transgressions, etc.[9] We assume that the sanctioning costs for the regulator are lower than the actual penalty for the firm, i.e. $h < 1$.

The dynamic maximization problem for the regulator is then the following:

$$\max_L \int_0^\infty \Big[Y(K, \hat{E}(K,L)) - C(\hat{I}(K,L)) - D(\hat{E}(K,L)) - hF(\hat{E}(K,L), L)\Big] e^{-\rho t} dt$$
$$\text{s.t.: } \dot{K} = \hat{I}(K,L) - \delta K, \quad K(0) = k_0. \tag{4}$$

Once the optimal pollution limit path is determined, the corresponding optimal capital investment, emissions and stock of capital paths can be obtained. All the characteristics of the solution are analyzed in the following section.

4 The optimal solution in the base model

In this section, we characterize the optimal policy that solves problem (4) and the corresponding optimal paths for emissions, investments and the capital stock. The regulator fixes the pollution limit, L, knowing the best-reaction functions of the firm. Interestingly, the firm's best-reaction function for investment in the base model is independent on the pollution limit set by the regulator, see (25) in the Appendix. Therefore, the accumulation of capital is unaffected by the decision taken by the regulator, who in consequence behaves as a static player.

[9]We assume that the administrative costs of imposing sanctions are increasing in the level of non-compliance. Sanctioning costs may increase with this level (and eventually, with the level of the fine) since individuals can strongly resist to the imposition of larger fines, engage in avoidance activities,etc., see Polinsky and Shavell (1992). Stranlund (2007), Arguedas (2008) or, more recently, Arguedas (2013), also consider sanctioning costs dependent on the level of the fines.

The following proposition presents the characteristics of the optimal policy in the base model (all the proofs are in the Appendix).

Proposition 1 *In the base model, the optimal pollution limit is given by:*[10]

$$L^{*b}(K) = L_0^{*b}(1-K), \qquad L_0^{*b} = \frac{\sigma(f + h\sigma^2 - d)}{\Psi} > 0, \tag{5}$$

where $\Psi = f(d+\sigma^2) + h\sigma^4$; *the induced optimal emissions and capital investment strategies are:*

$$E^{*b}(K) = E_0^{*b}(1-K), \qquad E_0^{*b} = \frac{\sigma(f + h\sigma^2)}{\Psi} > 0, \tag{6}$$

$$I^{*b}(K) = I_0^{*b} + I_1^{*b}K, \qquad I_0^{*b} = \frac{b_F^b}{c}, \quad I_1^{*b} = \frac{a_F^b}{c}, \tag{7}$$

where

$$a_F^b = \frac{c(\rho+2\delta) - \sqrt{\Delta^b}}{2} < 0, \quad b_F^b = \frac{2cd^2 f(f+\sigma^2)}{(c\rho + \sqrt{\Delta^b})\Psi^2} > 0, \tag{8}$$

$$\Delta^b = c^2(\rho+2\delta)^2 + \frac{4cd^2 f(f+\sigma^2)}{\Psi^2}; \tag{9}$$

and the optimal capital stock evolves as:

$$K^b(t) = (k_0 - \bar{K}^b)e^{\theta^b t} + \bar{K}^b, \tag{10}$$

where the steady-state value of the capital stock and the speed of convergence towards this value are respectively given by

$$\bar{K}^b = \frac{d^2 f(f+\sigma^2)}{d^2 f(f+\sigma^2) + c\delta(\delta+\rho)\Psi^2} \in (0,1), \quad \theta^b = \frac{c\rho - \sqrt{\Delta^b}}{2c} < 0. \tag{11}$$

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[10]Superscript b stands for base model.

The optimal path for the pollution limit, given by expression (5), is inversely related to the evolution of the stock of capital, which is characterized in (10). If initially the stock of capital is small ($k_0 < \bar{K}^b$), then the stock of capital increases towards its long-run value from below. Therefore, the regulator fixes a lax emission limit at the beginning, and this limit becomes more stringent over time as the capital grows. The pollution standard converges towards its steady-state value, $\bar{L}^b$, which is given by the following expression:

$$\bar{L}^b = \frac{\sigma(f + h\sigma^2 - d)\left(1 - \bar{K}^b\right)}{\Psi} = \frac{c\delta(\delta+\rho)\sigma(f + h\sigma^2 - d)\Psi}{d^2 f(f+\sigma^2) + c\delta(\delta+\rho)\Psi^2}. \tag{12}$$

From expression (5), and since $K^b(t) \le \bar{K}^b < 1$, we can infer that the standard is always positive (recall that we have assumed that $f \ge d$).

Correspondingly, both the optimal emissions and capital investment paths characterized in (6) and (7) are also inversely related to the evolution of the capital stock. Therefore, provided $k_0 < \bar{K}^b$, both the optimal emissions and capital investment paths decrease to their steady-state values, $\bar{E}^b$ and $\bar{I}^b$, which are the following:

$$\bar{E}^b = \frac{\sigma(f + h\sigma^2)\left(1 - \bar{K}^b\right)}{\Psi} = \frac{c\delta(\delta+\rho)\sigma(f + h\sigma^2)\Psi}{d^2 f(f+\sigma^2) + c\delta(\delta+\rho)\Psi^2}, \tag{13}$$

$$\bar{I}^b = \delta\bar{K}^b = \frac{\delta d^2 f(f+\sigma^2)}{d^2 f(f+\sigma^2) + c\delta(\delta+\rho)\Psi^2}. \tag{14}$$

Interestingly, the optimal pollution limit decreases over time, as well as the associated degree of non-compliance. The reason is that the firm progressively adapts to more stringent standards through capital investment and, therefore, the induced emissions are closer to the required limits, even if those limits have become more stringent. This can be easily seen substracting (5) from (6) , which results in:

$$E^{*b}(K) - L^{*b}(K) = (1-K)\frac{\sigma d}{f(d+\sigma^2) + h\sigma^4}. \tag{15}$$

The degree of non-compliance in the long run converges to a positive value, given by the combination of expressions (12) and (13), as follows:

$$\bar{E}^b - \bar{L}^b = \frac{c\delta(\delta+\rho)\sigma d\Psi}{d^2 f(f+\sigma^2) + c\delta(\delta+\rho)\Psi^2}. \tag{16}$$

It is now interesting to compare the optimal emissions, investment and capital accumulation time-paths under the optimal policy characterized in Proposition 1 with the corresponding paths under no regulation and the first-best scenario. In the scenario of no regulation, the firm does not have to comply with any pollution limit. Therefore, the maximization problem of the firm in this case simply reads:

$$\begin{aligned} &\max_{I,E} \int_0^\infty \left[Y(K,E) - C(I)\right] e^{-\rho t} dt \\ &\text{s.t.: } \dot{K} = I - \delta K, \quad K(0) = k_0, \end{aligned} \tag{17}$$

and the solution is denoted as $(E^{\text{NR}}, I^{\text{NR}}, K^{\text{NR}})$, where the superscript stands for the no-regulation scenario, and is given in (27) in the Appendix. Under no regulation, the firm does not have any incentives to invest in capital. Then, the capital stock depreciates until zero and emissions increase towards the long-run value $1/\sigma$ (the inverse of the marginal rate of substitution between emissions and capital).

On the other hand, the first-best solution arises when the firm fully accounts for the environmental damages in its optimization problem. This solution is defined by the optimal investment and emission paths that maximize the difference between the value of production and the sum of investment costs and environmental damages, as follows:

$$\begin{aligned} &\max_{I,E} \int_0^\infty \left[Y(K,E) - C(I) - D(E)\right] e^{-\rho t} dt \\ &\text{s.t.: } \dot{K} = I - \delta K, \quad K(0) = k_0, \end{aligned} \tag{18}$$

and the solution is denoted as $(E^{\text{FB}}, I^{\text{FB}}, K^{\text{FB}})$, where the superscript stands for the first-best scenario, and is given in (28) and (29) in the Appendix. In the first-best solution, if the initial stock of capital is small the investment effort is initially strong and, as the capital increases towards its long-run value, investments are reduced to the amount strictly necessary to replace the depleted capital. Correspondingly, emissions decrease to the steady-state value.

The following proposition shows that the optimal emissions and capital accumulation time-paths associated with the optimal policy characterized in Proposition 1 evolve between the paths in the two extreme scenarios presented in (17) and (18), for all t,[11] while it is not necessarily true that capital investment levels under the optimal policy are lower than those under the first-best scenario for all t.[12]

Proposition 2 *Assuming $f \geq d$, the steady-state values and optimal paths for capital stock, investment and emissions induced by the optimal policy in the base model are related to the respective steady-state values and optimal paths under no regulation and the first-best outcome as follows:*

(i) $\bar{K}^{FB} > \bar{K}^b > 0$, $\Delta^{FB} > \Delta^b > 0$, *and* $0 > -\delta > \theta^b > \theta^{FB}$, *then* $K^{FB}(t) > K^b(t) > K^{NR}(t)$ *for all* $t > 0$.

(ii) $\bar{I}^{FB} > \bar{I}^b > 0$, *although it is not necessarily true that* $I^{FB}(t) > I^b(t)$ *for all* $t \geq 0$. *In the particular case where* $k_0 = 0$, $I^{FB}(0) > I^b(0) > 0$.

[11] $K^i(t), E^i(t), I^i(t)$ refer to the optimal capital stock, emissions and investment time-paths for the different scenarios, $i \in \{NR, FB, b\}$.

[12] In the particular case with $h = 0$, the regulator could induce the firm to act as in the first-best scenario, if the severity of the fine were $f = d$, which then implies $L^{*b}(K) = 0$ at any time. Note that under this strict liability specification, the firm's maximization problem is identical to the first-best maximization problem. This particular case is out of the scope of this paper.

(iii) $\bar{E}^{FB} < \bar{E}^{b} < 1/\sigma$ *and* $E^{FB}(t) < E^{b}(t) < E^{NR}(t)$ *for all* $t > 0$.

We are now interested in performing a comparative statics exercise about the effect of the severity of the fine, f, on the optimal policy characterized in Proposition 1. In particular, we want to analyze the consequences on the long-run values of emissions, the capital stock, the pollution limit and the degree of non-compliance as a consequence of a rise in f. Remember that the severity of the fine is not a regulator's decision variable, but a given parameter which determines the final fine the firm has to face in case of non-compliance, as well as the social costs born by the regulator. In consequence, the outcome of the optimal policy characterized in Proposition 1 is clearly influenced by this parameter.

Starting at a low level of f, an increase in f results in a greater capital stock, lower emissions and a better compliance, while the effect on the pollution limit is ambiguous. However, if the social cost per unit of the fine is not too large, these effects are reversed at certain values of f, except for the degree of non-compliance, which decreases monotonically with f. In particular, increases in f above a certain value $\hat{f}^{b}_{L}$, result in a loosening of the pollution limit. Thus, from this value on the severity of the fine and the pollution limit are substitute instruments in the regulatory problem. The following proposition summarizes this interesting finding.

Proposition 3 *The effect of a more severe penalty, characterized by a higher value of parameter* f*, over steady-state values of the more relevant variables of the model can be summarized as:*[13]

i) If $h < h_{\max} \equiv \frac{d+\sigma^2}{2\sigma^2}$*, then:*

$$(\bar{K}^{b})_{f}, |\theta^{b}|_{f}, (b^{b}_{F})_{f}, (-a^{b}_{F})_{f} \begin{cases} > 0 & \text{if} \quad f \in [0, \hat{f}^{b}_{K}), \\ < 0 & \text{if} \quad f > \hat{f}^{b}_{K}, \end{cases} \qquad (\bar{E}^{b})_{f} \begin{cases} < 0 & \text{if} \quad f \in [0, \hat{f}^{b}_{E}), \\ > 0 & \text{if} \quad f > \hat{f}^{b}_{E}, \end{cases}$$

[13]The subscript f denotes partial derivative with respect to f.

$$0 < \hat{f}_K^b < \hat{f}_E^b \quad \textit{if} \quad h > 0; \quad \textit{and } \hat{f}_K^b = \hat{f}_E^b = 0 \quad \textit{if} \quad h = 0.$$

ii) If $\frac{2d}{2d+\sigma^2}\frac{d+\sigma^2}{2\sigma^2} \equiv h_{\min} < h < h_{\max} \equiv \frac{d+\sigma^2}{2\sigma^2}$*, then:*

There exists a $\hat{f}_L^b \geq 0$ *such that* $(\bar{L}^b)_f > 0$ *for any* $f > \hat{f}_L^b$.

Furthermore, $\max\left\{d, \hat{f}_L^b\right\} < \hat{f}_K^b < \hat{f}_E^b$.

iii)

$$(\bar{E}^b - \bar{L}^b)_f < 0, \textit{ for any } h > 0,\ f \geq 0.$$

The analytical results in Proposition 3 can be explained with the help of Figure 1. This figure illustrates the relationship between the severity of the fine (i.e., the value of f, in the horizontal axes) and the long-run values of the capital stock, emissions, the pollution limit and the degree of non-compliance (in the vertical axes), for $h \in (h_{\min}, h_{\max})$.[14] For low values of f, an increase in this parameter is associated with a higher capital and lower emissions in the long run. Moreover, the speed of convergence of these variables towards their steady-state values is also increased. However, the effect of a more severe penalty on these variables is reversed for a certain value of f. Thus, further increases in f above $\hat{f}_K^b$ will decrease the capital stock, and above $\hat{f}_E^b$ will increase the emissions in the long run, see Figure 1 (left). Correspondingly, as shown in Figure 1 (right), for this example[15] an increase of f below $\hat{f}_L^b$ tightens the emission limit, although in any case an increase in the severity of the fine above $\hat{f}_L^b$ will lead to a laxer

[14]For the numerical illustration the parameters' values are: $c = d = 1,\ \rho = 0.05,\ \delta = 0.15,\ \sigma = 2,\ k_0 = 0,\ h = 0.3$. For this specification $h \in (h_{\min}, h_{\max}) = (0.2083, 0.625)$. Capital and emissions are measured respectively on the left and right vertical axes of Figure 1 (left), while the pollution limit and the degree of non-compliance are measured, respectively, on the left and right vertical axes of Figure 1 (right).

[15]For $f \in [0, \hat{f}_L^b)$, and for the vast majority of the values for h, $\bar{L}^b$ decreases monotonously with f. However, for h close to $h_{\min}$, $\bar{L}^b$ either always increases or it increases within a first period and decreases henceforth.

emission limit in the long run.[16] Thus, for values of the fine above $\hat{f}_L^b$, the pollution limit and the severity of the fine are substitutes, while in this example they are complementary instruments for values of f below this bound. Finally, the degree of non-compliance decreases monotonically with the severity of the fine.

It is interesting to note that the effect of a higher f on the evolution of the variables over time is the same as that at the steady state, with some exceptions: the effect of a higher f on emissions at a particular time t is ambiguous in the interval $\left(\hat{f}_K^b, \hat{f}_E^b\right)$; the corresponding effect on the optimal pollution limit at time t is ambiguous in the interval $\left(\hat{f}_L^b, \hat{f}_K^b\right)$; and, finally, the effect of a higher f on the degree of non-compliance at time t is ambiguous for $f > \hat{f}_K^b$.

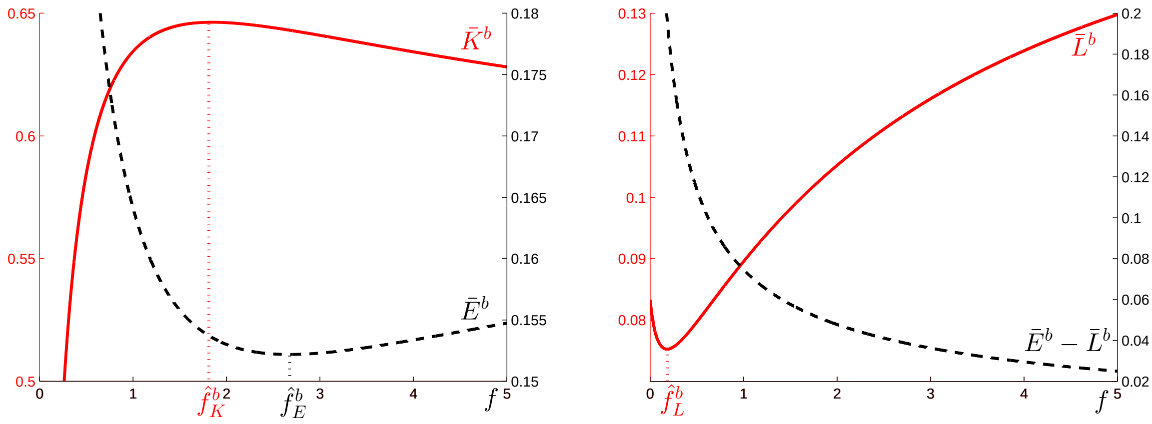


Figure 1: $\bar{K}^b$, $\bar{E}^b$ w.r.t. f (left); $\bar{L}^b$, $\bar{E}^b - \bar{L}^b$ w.r.t. f (right)

The next question is to analyze the effect of a more severe fine on the accumulated profits of the firm and, more importantly, on the accumulated social welfare. Due to its analytical complexity, we rely on a numerical simulation, which is robust to changes in the parameters' values in footnote 14. Figure 2 shows the relationship between the value function of the firm, $V_{\mathrm{F}}^b(k_0)$ and the social welfare, $V_{\mathrm{R}}^b(k_0)$ with respect to the severity of the fine, f.

[16] In the example, $\hat{f}_L^b < d$, although for greater values of h (for example $h = 0.5$), $\hat{f}_L^b > d$, and $\bar{L}^b$ would be U-shaped within the interval we are interested in, that is, $f \in (d, \infty)$.

We first observe that social welfare is improved and firm's profits lessens when moving from no regulation ($f = 0$) to a regulatory scenario in which the firm is marginally penalized when polluting above the pollution standard specified by the regulator. But the effect of successive increments in the severity of the fine is not monotonous. Firm's profits reach a minimum at $\hat{f}_K^b$, which coincides with the value of f for which the firm's capital investment is maximum. Focusing on social welfare, we also observe that it improves with the severity of the fine up to a maximum, $\hat{f}^b$, which lays not only to the right of $\hat{f}_K^b$, but it is also greater than $\hat{f}_E^b$.[17]

Our interpretation of this non-monotonic effect of the severity of the fine f on firm's profits and social welfare is centered on the different valuation of non-compliance by the firm and the regulator. Since $h \in (0, 1)$, a more severe penalty affects the firm strongly than the regulator. In consequence, when the severity of the fine is low, a more severe penalty will push the firm to better comply with the environmental standard, increasing capital investments and reducing emissions. This will reduce firm's profits, but will increase society's welfare, which bears a lower damage from pollution. Nevertheless, from the viewpoint of the regulator (less concerned on compliance than the firm), an excessively severe penalty might eventually lead the firm to focus too much on compliance and to little on production. Hence, the regulator would decide a laxer standard, making it easier for the firm to comply with higher emissions and production. Higher production will increase firm's welfare, but the damage linked with higher emissions will lead social welfare down.

To conclude this section, Table 1 summarizes the effects of increases in f on the long-run values of the relevant variables of the model (the capital stock, the emissions, the pollution

[17]The parameter values presented in footnote 14 result in $\hat{f}_L^b = 0.18223 < \hat{f}_K^b = 1.84615 < \hat{f}_E^b = 2.6723 < \hat{f}^b = 2.73391$. The numerical result $\hat{f}^b > \hat{f}_K^b$ is robust to parameter changes, as long as $h \in (h_{\min}, h_{\max})$.

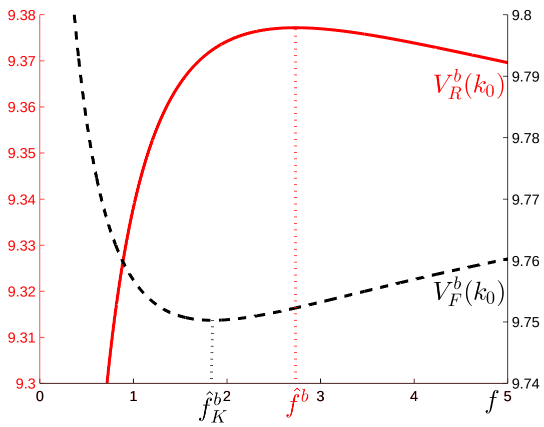


Figure 2: $V_{\text{R}}^b(k_0),\ V_{\text{F}}^b(k_0)$ w.r.t. f

$f < \hat{f}_L^b$	$\hat{f}_L^b < f < \hat{f}_K^b$	$\hat{f}_K^b < f < \hat{f}_E^b$	$\hat{f}_E^b < f < \hat{f}^b$	$f > \hat{f}^b$
$\bar{K}^b \uparrow$	$\bar{K}^b \uparrow$	$\bar{K}^b \downarrow$	$\bar{K}^b \downarrow$	$\bar{K}^b \downarrow$
$\bar{E}^b \downarrow$	$\bar{E}^b \downarrow$	$\bar{E}^b \downarrow$	$\bar{E}^b \uparrow$	$\bar{E}^b \uparrow$
$\bar{L}^b \downarrow$	$\bar{L}^b \uparrow$	$\bar{L}^b \uparrow$	$\bar{L}^b \uparrow$	$\bar{L}^b \uparrow$
$\bar{E}^b - \bar{L}^b \downarrow$	$\bar{E}^b - \bar{L}^b \downarrow$	$\bar{E}^b - \bar{L}^b \downarrow$	$\bar{E}^b - \bar{L}^b \downarrow$	$\bar{E}^b - \bar{L}^b \downarrow$
$V_{\text{F}}^b(k_0) \downarrow$	$V_{\text{F}}^b(k_0) \downarrow$	$V_{\text{F}}^b(k_0) \uparrow$	$V_{\text{F}}^b(k_0) \uparrow$	$V_{\text{F}}^b(k_0) \uparrow$
$V_{\text{R}}^b(k_0) \uparrow$	$V_{\text{R}}^b(k_0) \uparrow$	$V_{\text{R}}^b(k_0) \uparrow$	$V_{\text{R}}^b(k_0) \uparrow$	$V_{\text{R}}^b(k_0) \downarrow$

Table 1: Effect of a more severe penalty for different ranges of the penalty f

limit, the degree of non-compliance) as well as on the optimal firm's profits and social welfare, for the values of the per-unit social cost of imposing fines such that $h_{\min} < h < h_{\max}$.[18]

5 The extended model: allowing for fine discounts

In this section, we analyze the optimal policy if the firm is not penalized for polluting above the emission limit set by the regulator as long as it invests to increase the stock of capital. We broaden the concept of non-compliance, as follows:

$$N(K(t), E(t), I(t), L(t)) = E(t) - [L(t) + \beta \dot{K}(t)] = E(t) - [L(t) + \beta(I(t) - \delta K(t))], \quad (19)$$

[18]If the social cost satisfies $h \in (0, h_{\min})$, then $\hat{f}_K^b < d$ and $\hat{f}^b$ can also be smaller than d.

where $\beta \geq 0$, and the equation (2) has been considered.

Note that $\beta = 0$ corresponds to the base case analyzed in the previous section. Now, $\beta > 0$ allows the regulator to be more flexible with respect to the imposition of the fine and take into account the investment efforts of the firm. In this case, the *effective* standard is $L(t) + \beta \dot{K}(t)$. Therefore, a laxer effective limit is imposed if the firm invests to increase the capital stock. The level of non-compliance now depends on the stock of capital, the level of emissions, the capital investment and the pollution limit, i.e., $N(K, E, I, L)$. Then again, the fine is quadratic in the degree of non-compliance, that is, $F(K, E, I, L) = f[E - (L + \beta(I - \delta K))]^2/2$, or zero under compliance. The fine is now a function of the gap between emissions and the effective emission limit, $L + \beta(I - \delta K)$. Considering this broader concept of the fine in the dynamic maximization problem for the firm described in (3), the best-response functions of the firm are:[19]

$$\hat{E}(K, L) = \underline{E} + E^{\mathrm{L}} L + E^{\mathrm{K}} K, \qquad \hat{I}(K, L) = \underline{I} + I^{\mathrm{L}} L + I^{\mathrm{K}} K. \tag{20}$$

The main difference with the base model where $\beta = 0$ (see (25) in the Appendix) lies in the fact that $I^{\mathrm{L}} < I^{\mathrm{L}b} = 0$. Now, the pollution standard chosen by the regulator does not only affect the emissions of the firm, but it also has an effect on its investment decisions. A tighter pollution limit (a lower L) induces a rise in the investment effort of the firm, and correspondingly, a higher pollution limit discourages investment affecting the accumulation of capital. In consequence, the regulator who fixes the optimal time path for the emission limit knowing the best-response functions chosen by the firm in (20), is no longer a static player as in the case with $\beta = 0$.

The following proposition presents the characteristics of the optimal policy in the extended model.

[19]See the proof of Proposition 4 in the Appendix for the detailed expressions.

Proposition 4 *In the extended model where fine discounts are allowed (i.e. $\beta > 0$), the optimal strategies for the pollution limit, emissions and investment in capital are given by:*

$$L^*(K) = L_0^* + L_1^* K, \qquad E^*(K) = E_0^* + E_1^* K, \qquad I^*(K) = I_0^* + I_1^* K, \tag{21}$$

with

$$V_F(K) = a_F \frac{K^2}{2} + b_F K + c_F, \quad V_R(K) = a_R \frac{K^2}{2} + b_R K + c_R, \tag{22}$$

being the value function of the firm and the regulator, respectively, and

$$\begin{aligned}
&L_0^* = (cd + (b_R - b_F)\beta\sigma^3)\frac{\sigma\Phi - 2h\sigma^3\Omega}{\Phi^2}, \quad L_1^* = -(cd + (a_F - a_R)\beta\sigma^3)\frac{\sigma\Phi - 2h\sigma^3\Omega}{\Phi^2},\\
&E_0^* = \frac{f^2\beta\sigma^3(b_F - b_R) + \Phi - cdf^2}{\sigma\Phi}, \quad E_1^* = -\frac{f^2\beta\sigma^3(a_R - a_F) + \Phi - cdf^2}{\sigma\Phi}.\\
&I_0^* = \frac{b_F}{c} + \frac{f^2\beta\sigma[cd + (b_R - b_F)\beta\sigma^3]}{c\Phi}, \quad I_1^* = \frac{a_F}{c} - \frac{f^2\beta\sigma[cd + (a_F - a_R)\beta\sigma^3]}{c\Phi}.\\
&\Omega = cf + (c + f\beta^2)\sigma^2 > 0, \qquad \Phi = c\Psi + f^2\beta^2\sigma^4 > 0.
\end{aligned}$$

The optimal time-path of the capital stock reads:

$$K(t) = (k_0 - \bar{K})e^{\theta t} + \bar{K}, \tag{23}$$

where

$$\begin{aligned}
\bar{K} &= \frac{f^2\beta^2\sigma^4 b_R + f^2\beta\sigma cd + b_F c\Psi}{f^2\beta^2\sigma^4(a_F - a_R) + f^2\beta\sigma cd + \Phi(c\delta - a_F)},\\
\theta &= \frac{f^2\beta^2\sigma^4(a_R - c\delta) - f^2\beta\sigma cd + (a_F - c\delta)c\Psi}{c\Phi}.
\end{aligned}$$

In contrast with the results presented in the previous section, now we cannot provide general conclusions about the evolution of the different variables through time. However, for the parameters' values presented in footnote 14, the capital stock converges towards a positive long-run value from below, with a speed of convergence given by $|\theta|$. The temporal evolution of emissions and the pollution limit are equivalent to those without fine discounts. The effective pollution standard is initially laxer, and it becomes tighter over time as the capital stock grows. Nevertheless, although the effective pollution limit becomes more stringent, capital growth allows the firm to reduce the degree of non-compliance overtime. These results are robust to changes in parameter's values.

In what follows, we are interested in performing two exercises. The first is similar to the one we performed in the previous section: When fine discounts are allowed, is there a bounded value of the severity of the fine for which social welfare is maximum? The second one is idiosyncratic of this section: Does the existence of fine discounts associated with investment in capital give room for social welfare improvements?

We then perform a comparative static analysis on parameter f, which measures the severity of the fine, and parameter β, which defines to what extent the emission limit can be raised with the capital investment of the firm. This analysis is carried out numerically, and results are robust to changes in the parameters' values unless said otherwise.

We first start with the comparative statics on the severity of the fine (that is, parameter f). Let us recall that in the base model, ($\beta = 0$), where the pollution limit was independent of the investment decisions, the effect of a more severe penalty was, in general, non-monotone (see Proposition 2). In particular, when the fine was low, a rise in f increased social welfare but the reverse occurred when the fine was large (see Figure 2). In consequence, we could compute the magnitude of the fine, $\hat{f}^b$, which resulted in the highest social welfare level.

The following claim[20] characterizes the effect of a change in the severity of the fine on social

[20]From now on since our results have been generated numerically, but cannot be proven analytically, we state them as claims rather than propositions.

welfare when fine discounts are allowed, i.e. assuming $\beta > 0$.

Claim 1 *For a given fine discount $\beta > 0$,*

i) If β is small, then there exists $\hat{f} \in (\hat{f}^b, \infty)$ such that:

$$(V_R(k_0))_f \begin{cases} > 0 & \textit{if} \quad f \in [0, \hat{f}), \\ < 0 & \textit{if} \quad f > \hat{f}. \end{cases}$$

Furthermore, $\hat{f}$ increases with β.

ii) If β is large enough, then $(V_R(k_0))_f > 0$ for all $f > 0$.

iii) $\hat{f} > \hat{f}^b$ with $\lim_{\beta \to 0} \hat{f} = \hat{f}^b$ and $\lim_{\beta \to 0} V_R(k_0) = V_R^b(k_0)$.

When the allowed fine discount is not too large (part i) of the claim), initial rises in the severity of the fine increase social welfare, up to a certain value, from which further increases result in social losses.Then, there exists a finite value $\hat{f}$ at which the society attains the highest social welfare level. Also, the level of the fine which maximizes social welfare is more severe the larger the discount, i.e., $\hat{f}$ increases with β. Moreover, when the fine discount is very small (β approaches zero), $\hat{f}$ approaches $\hat{f}^b$ and social welfare matches its value when fine discounting were not allowed (part iii) of the claim).

When the firm is allowed to discount a large part of its investments (part ii) of the claim), social welfare increases monotonously with the severity of the fine. In this case, from a social perspective the fine should be set as large as possible.[21]

[21] We cannot characterize analytically the value of β above which an infinite punishment for non-compliance would be socially desirable. In the numerical simulation it holds that this threshold (0.217) is smaller than the marginal rate of technical substitution $1/\sigma = 0.5$ between capital and emissions.

Next, we perform a comparative statics exercise on the fine discount β, taking the severity of the fine, f, as given. We ask whether fine discounts are socially desirable. The result is presented next.

Claim 2 *For a given severity of the fine $f > 0$, there always exists a bounded fine discount $\hat{\beta} \in (0, \infty)$ such that*

$$(V_R(k_0))_\beta \begin{cases} > 0 & \text{if} \quad \beta \in [0, \hat{\beta}), \\ < 0 & \text{if} \quad \beta > \hat{\beta}. \end{cases}$$

Furthermore, there exist $\hat{\beta}_F, \hat{\beta}_E$ and $\hat{\beta}_L$ with $0 < \hat{\beta}_F < \hat{\beta}_L < \hat{\beta} < \hat{\beta}_E$ such that

$$(V_F(k_0))_\beta \begin{cases} < 0 & \text{if} \quad \beta \in [0, \hat{\beta}_F), \\ > 0 & \text{if} \quad \beta > \hat{\beta}_F, \end{cases}$$

$$(\bar{E})_\beta \begin{cases} < 0 & \text{if} \quad \beta \in [0, \hat{\beta}_E), \\ > 0 & \text{if} \quad \beta > \hat{\beta}_E, \end{cases} \qquad (\bar{L})_\beta \begin{cases} < 0 & \text{if} \quad \beta \in [0, \hat{\beta}_L), \\ > 0 & \text{if} \quad \beta > \hat{\beta}_L, \end{cases}$$

$$\left(\bar{K}\right)_\beta > 0, \quad \left(N(\bar{K}, \bar{E}, \bar{I}, \bar{L})\right)_\beta < 0 \quad \forall \beta > 0.$$

The previous claim suggests that switching from the base scenario where fine discounts are not allowed to the extended model in which fine discounts are allowed results in a social welfare improvement as long as β is small enough (within 0 and $\hat{\beta}$). However, when the allowed fine discount is sufficiently large (larger than $\hat{\beta}$), further discounts reduce social welfare. Thus, for a given f, society would attain a maximum welfare level at $\hat{\beta}$.

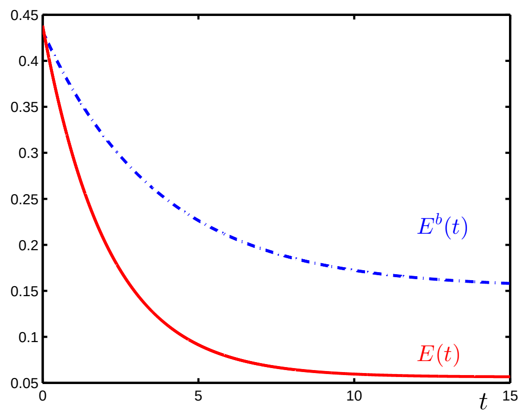
The explanation for this result is the following. Starting from the base scenario without fine discounts, we analyze successive increases in the fine discount, β. Due to the greater incentive to invest in capital, the stock of capital at the steady state increases. A greater capital stock allows the firm to produce with lower emissions and the regulator to implement a more stringent pollution standard at the steady state. Still the degree of non-compliance decreases.

The firm is pushed to invest but since the regulator has tightened the standard, the associated investment costs exceed the gains from a better compliance, hence firm's profits are reduced. Correspondingly, the social welfare is increased due to a lower environmental damage together with a lower social cost from non-compliance. Because the social costs associated with non-compliance are softer than the penalty paid by the firm ($h < 1$), the incentive to rise investment linked to a rise in the fine discount is differently perceived by the firm and the regulator. Thus, when the fine discount is already too high, the firm reaction to further increases is still a rise in investments in order to reduce the fine for non-compliance. However, for the regulator, the reduction in the social costs associated with non-compliance, the increase in production and the lower environmental damage might not be enough to counterbalance the rise in the investment costs. Thus, instead of tightening the standard, the regulator would relax the standard making it easier for the firm to comply and therefore, alleviating the incentive to invest in capital. A softer emission standard increases firm's profits, but the slight increase in emissions worsens social welfare.

By comparing the corresponding long-run values under $\beta = 0$ and $\beta = \hat{\beta}$ (where the social optimum is maximum), we observe that long-run emissions, the effective pollution limit and the degree of non-compliance are lower under $\beta = \hat{\beta}$. In fact, these results (and also the fact that the capital stock is larger under $\beta = \hat{\beta}$) also hold for any time period, as shown in Figures 3 and 4.[22]

In Claim 1 we have analyzed the effect of the magnitude of the fine assuming that the allowed discount is fixed (which may or may not be monotone). Equivalently, Claim 2 describes the effect of the fine discount given the magnitude of the fine. Our next question is whether a

[22] We have considered the same parameter values as in footnote 14, and also $f = 2 > d = 1$.



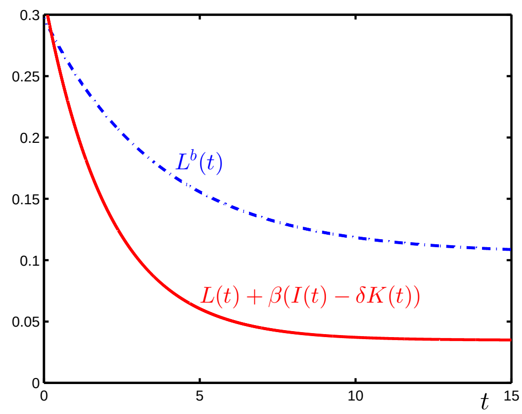


Figure 3: Emissions (left); Effective emission limit (right)

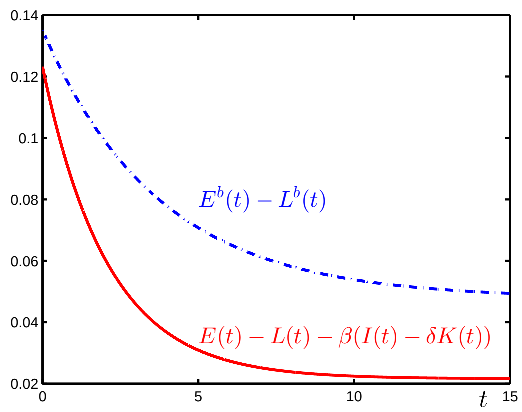


Figure 4: Degree of non-compliance

combination of parameters values (f, β) exists, for which the solution of the Stackelberg game reaches its maximum.

Figure 5 depicts the social welfare level when the initial capital stock is zero as a function of the fine discount, β, for different values of the magnitude of the fine, f. For $f = 1$, the dashed line reaches its maximum at $\hat{\beta} \backsimeq 1.1232$, with $V_{\text{R}}(k_0) = 9.5482$. Increasing the magnitude of the fine to $f = 2$ leads to a new and higher maximum of social welfare at $\hat{\beta} \backsimeq .807$, with $V_{\text{R}}(k_0) = 9.55416$. Further increases in the magnitude of the fine are associated with new $\hat{\beta}$ at which the social welfare is increased. This process continues indefinitely.[23]

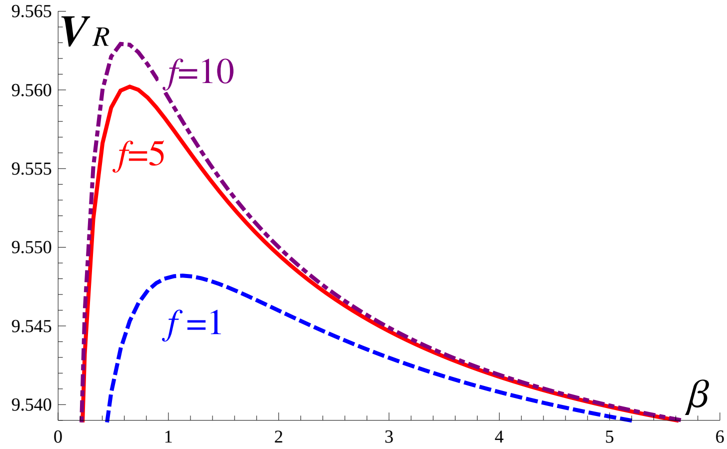


Figure 5: $V_{\text{R}}(k_0)$ w.r.t. β for different values of f.

This new finding is then summarized in the following:

Claim 3 *There is no pair of parameters* $(f, \beta) \in (0, \infty) \times (0, \infty)$ *for which* $V_{\text{R}}(k_0)$ *reaches a maximum.*

In consequence, the best option for the society would be to penalize as much as possible, and for this magnitude of the fine select the fine discount β which maximizes social welfare. This limiting case when f tends to infinity is equivalent to a new scenario where *full compliance* with the effective pollution limit is assumed (and therefore, no penalty is levied). In this context, the firm is allowed to exceed the pollution standard set by the regulator in its net investment times the fine discount $\hat{\beta}$. Once the pollution standard is chosen, investment must guarantee full compliance with the effective pollution limit, $\tilde{I}(E, L, K) = \delta K + (E - L)/\hat{\beta}$. The firm

[23] We have numerically computed a rise in the maximum value of $V_{\text{R}}(k_0)$ when switching from $f = 10^{12}$ to $f = 10^{13}$. Interestingly, the value of β that maximizes social welfare when the severity of the fine tends to infinity converges to a finite value, $\hat{\beta}$.

maximization problem then reads:

$$\max_{E} \int_0^\infty \left[Y(K,E) - C(\tilde{I}(E,K,L)) \right] e^{-\rho t} dt$$
$$\text{s.t.: } \dot{K} = \tilde{I}(E,L,K) - \delta K, \quad K(0) = k_0.$$

Knowing the best-reaction function of the firm, $\hat{E}(L,K)$, the regulator solves the following dynamic optimization problem:

$$\max_{L} \int_0^\infty \left[Y(K,\hat{E}(K,L)) - C(\tilde{I}(\hat{E}(K,L),K,L)) - D(\hat{E}(K,L)) \right] e^{-\rho t} dt$$
$$\text{s.t.: } \dot{K} = \tilde{I}(\hat{E}(K,L),K,L) - \delta K, \; K(0) = k_0.$$

Denoting the social welfare in this full compliance scenario as $V_{\text{R}}^{\text{FC}}(K)$, for the parameters' values considered we obtain the following ranking of social welfare levels:

$$V^{\text{NR}}(k_0) = 7.5 < V_{\text{R}}^{b}(k_0) = 9.3772 \,(\text{for } \hat{f}^b) < V_{\text{R}}^{\text{FC}}(k_0) = 9.56632 \,(\text{for } \hat{\beta}) < V^{\text{FB}}(k_0) = 9.56702.$$

If we compute the distance to the social welfare in the first-best scenario, we observe that the scenario with no fine discounts ($V_{\text{R}}^{b}(k_0)$) distances 1.98%, while the scenario with discounts and full compliance ($V_{\text{R}}^{\text{FC}}(k_0)$) distances 0.0073%. Therefore, this limiting scenario results in a social welfare level roughly equal to the first-best level.

6 Conclusions

Our paper constitutes a first attempt in the literature to address the determination of optimal pollution standards overtime in a context of imperfect compliance. Considering a representative firm which uses emissions as an input to produce a consumption good and invests to increase a capital stock, we have assumed that the firm can decide to exceed the pollution limit set by the regulator at any time and then be subject to a fine if it finds it profitable to do so. In the base model, we have considered a fine quadratic in the degree of non-compliance, which is the difference between firm's emissions and the pollution limit set by the regulator. In the extended model, we have introduced the concept of effective pollution limit, and we have redefined the fine as being quadratic in the difference between emissions and the effective pollution limit, which is laxer than the pollution limit set by the regulator if the firm invests to increase the capital stock.

We have shown that the optimal path for the effective pollution limit decreases overtime, resulting in a decrease in emissions and an increase in the stock of capital. Interestingly, while the firm faces a more stringent pollution limit overtime, however the degree of non-compliance decreases overtime. This result is valid in both the base and the extended models. While this result cannot be found in static models of regulatory enforcement typically considered in the literature (in those settings, a tighter standard always results in a larger gap between emissions and the standard), however in a dynamic context the firm can progressively adapt to more stringent regulations by means of capital investment. In fact, a fine dependent on emissions in excess of the effective pollution limit helps the firm to decrease the level of non-compliance even more, since the firm is induced to invest in capital accumulation even more than without discounts.

We have analyzed the question of how severe the penalty for non-compliance should be. To that aim, we have performed a comparative statics analysis regarding the severity of the fine. In the base model where the fine depends on emissions in excess of the pollution limit set by the regulator, we have generally found non-monotone relationships between the main variables and the severity of the fine, *ceteris paribus*. In particular, in general the optimal pollution limit decreases with the severity of the fine for low values of the fine, but it always increases for large values of the fine. This means that the pollution limit and the fine act as substitute instruments provided the level of the fine exceeds a certain threshold, whereas they might be substitute or complementary instruments when the level of the fine is below this threshold. The induced emissions and capital stock are, respectively, U-shaped and inverse U-shaped in the severity of the fine and, finally, the degree of non-compliance decreases with the severity of the fine. All this results in a first decreasing and then increasing relationship between firm's profits and the severity of the fine, and also in a first increasing and then decreasing relationship between social welfare and the severity of the fine. The latter implies that there always exists a bounded value of the severity of the fine that maximizes social welfare.

In the extended model where fine discounts are allowed, we are able to affirmatively answer our second research question: *Should the fine be reduced in exchange for firm's capital investments?* Moreover, for a given severity of the fine, we can always find the finite value of the fine discount at which social welfare attains its maximum. The first question on the severity of the fine can also be studied in the extended model. We have numerically found the same sort of non-monotonicity only when the fine discount is small. If, to the contrary, the fine discount is large, social welfare increases monotonically with the severity of the fine. Finally, if we compute the fine discount which maximizes social welfare for successive values of the severity of the fine, we observe that the value of social welfare increases with the severity of the fine at the social maximizing fine discount. A limiting scenario with a sufficiently large severity of the fine would be equivalent to an alternative scenario of full compliance with the effective pollution limit, since the firm is threatened by a very large punishment if it pollutes above that limit. Emissions are still above the pollution limit imposed by the regulator, but no penalties are levied. This scenario roughly mimics the first-best scenario in terms of social welfare levels.

Several extensions of our model are possible, such as including monitoring issues, cumulative pollution effects, or imperfect information on the part of the regulator and/or the polluting firm. All these issues deserve further research.

7 Appendix

Proof of Proposition 1.

We start with the maximization problem for the firm, presented in (3). In view of the linear-quadratic structure of the problem we conjecture a quadratic value function for the firm in K, denoted by $V_{\text{F}}^{b}(K)$:

$$V_{\text{F}}^{b}(K) = \frac{a_{\text{F}}^{b}}{2}K^2 + b_{\text{F}}^{b}K + c_{\text{F}}^{b}. \tag{24}$$

The Hamilton-Jacobi-Bellman equation for the firm's problem is:

$$\rho V_{\text{F}}^{b}(K) = \max_{I,E}\left\{K + \sigma E - \frac{(K+\sigma E)^2}{2} - c\frac{I^2}{2} - f\frac{(E-L)^2}{2} + (V_{\text{F}}^{b})'(K)(I - \delta K)\right\}.$$

The best-response functions for the firm are given by:

$$\hat{E}^b(K,L) = \underline{E}^b + E^{\text{L}b}L + E^{\text{K}b}K, \qquad \hat{I}^b(K,L) = \hat{I}^b(K) = \underline{I}^b + I^{\text{L}b}L + I^{\text{K}b}K, \tag{25}$$

$$\underline{E}^b = \frac{\sigma}{\sigma^2 + f},\ \ E^{\text{L}b} = \frac{f}{\sigma}\underline{E}^b,\ \ E^{\text{K}b} = -\underline{E}^b, \qquad \underline{I}^b = \frac{b_{\text{F}}^b}{c},\ \ I^{\text{L}b} = 0,\ \ I^{\text{K}b} = \frac{a_{\text{F}}^b}{c}.$$

Now, the regulator fixes the pollution limit, L, knowing the best-reaction functions of the firm presented in (25). Since $I^{\text{L}b} = 0$ the regulator, behaves as a static player whose maximization problem reads:

$$\max_L\left\{Y(K,\hat{E}^b(K,L)) - C(\hat{I}^b(K,L)) - D(\hat{E}^b(K,L)) - hf\frac{[\hat{E}^b(K,L) - L]^2}{2}\right\} =$$

$$\max_L\left\{(K+\sigma\hat{E}^b(K,L)) - \frac{(K+\sigma\hat{E}^b(K,L))^2}{2} - c\frac{\hat{I}^b(K)^2}{2} - d\frac{\hat{E}^b(K,L)^2}{2} - hf\frac{[\hat{E}^b(K,L)-L]^2}{2}\right\}.$$

The optimal strategy for the regulator is to set an emission limit which satisfies:

$$Y_E(K, \hat{E}^b(K,L)) = D'(\hat{E}^b(K,L)) + hf[\hat{E}^b(K,L) - L]\frac{E^{\text{L}b} - 1}{E^{\text{L}b}}.$$

From this condition, the optimal emission limit is the affine function of the stock of capital, $L^{*b}(K)$, given in (5). As long as $h < 1$, it follows that $\left(L_0^{*b}\right)_f > 0$.

From the optimal emission limit in (5) and the best-response functions of the firm, the optimal emissions and investment are given by the expressions in (6) and (7). And it is easy to see that $\left(E_0^{*b}\right)_f < 0$ for all f. Further, the coefficients which define the firm's value function, $V_{\text{F}}^b(K)$, defined in (24) can now be computed and are given by (8) and:

$$c_{\text{F}}^b = \frac{(b_{\text{F}}^b)^2 + c}{2c\rho} - \frac{d^2 f(f+\sigma^2)}{2\rho\Psi^2} > 0. \tag{26}$$

Once the optimal investment strategy is known, integrating the differential equation which defines the capital stock dynamics, the optimal time path of the capital stock is given by (10).

Proof of Proposition 2

We first characterize the optimal paths for the case of no regulation and the first-best scenario, and then we compare these two cases with the optimal equilibrium characterized in Proposition 1.

First, with no regulator the Hamilton-Jacobi-Bellman equation for problem (17) is:

$$\rho V_{\text{F}}^{\text{NR}}(K) = \max_{I,E}\left\{K + \sigma E - \frac{(K+\sigma E)^2}{2} - c\frac{I^2}{2} + (V_{\text{F}}^{\text{NR}})'(K)(I - \delta K)\right\}.$$

We again conjecture a quadratic value function in K, denoted by $V_{\text{F}}^{\text{NR}}(K)$:

$$V_{\text{F}}^{\text{NR}}(K) = \frac{a_{\text{F}}^{\text{NR}}}{2}K^2 + b_{\text{F}}^{\text{NR}}K + c_{\text{F}}^{\text{NR}},$$

and $a_{\rm F}^{\rm NR}, b_{\rm F}^{\rm NR}, c_{\rm F}^{\rm NR}$ are unknowns to be determined. The only feasible solution for these unknowns associated with a convergent time path of the capital stock is: $a_{\rm F}^{\rm NR} = b_{\rm F}^{\rm NR} = 0, c_{\rm F}^{\rm NR} = 1/(2\rho)$. Therefore, the solution under no regulation is the following:

$$I^{\rm NR} = 0, \qquad E^{\rm NR}(K) = \frac{1}{\sigma}(1-K), \quad \text{and} \quad K^{\rm NR}(t) = k_0 e^{-\delta t}, \tag{27}$$

where the optimal time path of capital is obtained by solving (2) once the optimal investment has been replaced.

In the particular case $k_0 = 0$, then $K(t) = 0$, $E(t) = 1/\sigma$ for any $t \geq 0$ and the social welfare is $V^{\rm NR}(0) = 1/(2\rho) - d/(2\rho\sigma^2)$.

The first-best solution to problem (18) can be found by solving the following Hamilton-Jacobi-Bellman equation:

$$\rho V^{\rm FB}(K) = \max_{I,E}\left\{(K+\sigma E) - \frac{(K+\sigma E)^2}{2} - c\frac{I^2}{2} - d\frac{E^2}{2} + (V^{\rm FB})'(K)(I-\delta K)\right\},$$

where $V^{\rm FB}(K)$ denotes the value function of the problem. Following the same procedure as for solving problem (17), we then easily obtain the following result:

$$I^{\rm FB}(K) = \frac{b^{\rm FB}}{c} + \frac{a^{\rm FB}}{c}K, \qquad E^{\rm FB}(K) = \frac{\sigma}{d+\sigma^2}(1-K), \tag{28}$$

where $V^{\rm FB}(K) = a^{\rm FB}K^2/2 + b^{\rm FB}K + c^{\rm FB}$, with

$$a^{\rm FB} = \frac{c(\rho+2\delta) - \sqrt{\Delta^{\rm FB}}}{2} < 0, \qquad b^{\rm FB} = \frac{2cd}{(d+\sigma^2)\left[c\rho + \sqrt{\Delta^{\rm FB}}\right]} > 0,$$
$$c^{\rm FB} = \frac{(b^{\rm FB})^2}{2c\rho} + \frac{\sigma^2}{2\rho(d+\sigma^2)} > 0, \quad \Delta^{\rm FB} = c^2(\rho+2\delta)^2 + \frac{4cd}{d+\sigma^2},$$

and the optimal time-path of the capital stock is:

$$K^{\rm FB}(t) = (k_0 - \bar{K}^{\rm FB})e^{\theta^{\rm FB}t} + \bar{K}^{\rm FB}, \tag{29}$$

$$\bar{K}^{\text{FB}} = \frac{d}{d + c(d+\sigma^2)\delta(\delta+\rho)} > 0, \quad \theta^{\text{FB}} = \frac{c\rho - \sqrt{\Delta^{\text{FB}}}}{2c} < 0,$$

where $\bar{K}^{\text{FB}}$ is the long-run value of the capital stock, and $|\theta^{\text{FB}}|$ is the speed of convergence towards this value.

The corresponding long-run values of emissions and investment read:

$$\bar{E}^{\text{FB}} = \frac{\sigma}{d+\sigma^2}\left(1 - \bar{K}^{\text{FB}}\right) = \frac{c\delta(\delta+\rho)\sigma}{d + c(d+\sigma^2)\delta(\delta+\rho)}, \qquad \bar{I}^{\text{FB}} = \delta\bar{K}^{\text{FB}} = \frac{d\delta}{d + c(d+\sigma^2)\delta(\delta+\rho)}.$$

Finally, we compare the optimal paths under no regulation, the first best and the base model.

(i) Since $f \geq d$, it follows that $\bar{K}^{\text{FB}} > \bar{K}^b$ and both are positive. Moreover, under this condition it is also true that $\Delta^{\text{FB}} > \Delta^b > 0$, hence inequality $\theta^{\text{FB}} < \theta^b$ follows. It is immediate to verify that $\theta^b < -\delta$.

(ii) $\bar{I}^{\text{FB}} > \bar{I}^b > 0$ follows straightforwardly from part (i). Since $f \geq d$, then $a^{\text{FB}} < a^b_{\text{F}} < 0$ immediately follows from $\Delta^{\text{FB}} > \Delta^b$. Moreover, proving $b^{\text{FB}} > b^b_{\text{F}}$ is equivalent to prove:

$$\frac{\Delta^{\text{FB}} - c^2(\rho+2\delta)^2}{2cd(c\rho + \sqrt{\Delta^{\text{FB}}})} > \frac{\Delta^b - c^2(\rho+2\delta)^2}{2cd(c\rho + \sqrt{\Delta^b})}.$$

Provided that $f(x) = (x - c^2(\rho+2\delta)^2)/(c\rho + \sqrt{x})$ is an increasing function and $\Delta^{\text{FB}} > \Delta^b$, $b^{\text{FB}} > b^b_{\text{F}}$ follows. Thus, $I^{\text{FB}}(0) = b^{\text{FB}}/c > b^b_{\text{F}}/c = I^{*b}(0) > 0$. However, nothing can be said of the comparison between $I^{*b}(K)$ and $I^{\text{FB}}(K)$.

(iii) The results immediately follow from part (i), the expressions of $E^{\text{NR}}(K)$, $E^{\text{FB}}(K)$, $E^{*b}(K)$, and inequalities:

$$\frac{1}{\sigma} > \frac{\sigma(f + h\sigma^2)}{\Psi} > \frac{\sigma}{d+\sigma^2}.$$

Proof of Proposition 3

i) From (8), (26) and (11) it follows that sign$|\theta^b|_f$ =sign$(\bar{K}^b)_f$ =sign$(b^b_{\text{F}})_f = -$sign$(a^b_{\text{F}})_f$ is given by:

$$\text{sign}(h\sigma^2(2f+\sigma^2)-(d+\sigma^2)f) = \text{sign}(2h\sigma^2(f+\sigma^2)-\Psi). \quad (30)$$

Here and henceforth we will be assuming $d+\sigma^2(1-2h)>0$, i.e. $h<(d+\sigma^2)/(2\sigma^2)=h_{\max}$. Under this condition, the sign in (30) is positive for $f\in[d,\hat{f}^b_K)$, and negative for $f>\hat{f}^b_K$, with:[24]

$$\hat{f}^b_K = \frac{h\sigma^4}{d+\sigma^2(1-2h)}.$$

The effect on emissions is obtained from (13),

$$(\bar{E}^b)_f = -\frac{dh\sigma^3}{\Psi^2}\left(1-\bar{K}^b\right) - \frac{\sigma(f+h\sigma^2)}{\Psi}(\bar{K}^b)_f.$$

In consequence, if $(\bar{K}^b)_f>0$ then $(\bar{E}^b)_f<0$, implying $\hat{f}^b_E>\hat{f}^b_K>0$ for all $h>0$, and $\hat{f}^b_E=\hat{f}^b_K=0$ for $h=0$. To prove that $\hat{f}^b_E$ is finite it is sufficient to see than $(\bar{E}^b)_f$ is positive for sufficiently large values of f: $\lim_{f\to\infty}(\bar{E}^b)_f=\infty$.

ii) The effect of f on the emission limits can be computed from (12):

$$(\bar{L}^b)_f = \frac{d\sigma(d+(1-h)\sigma^2)}{\Psi^2}\left(1-\bar{K}^b\right) - \frac{\sigma(-d+f+h\sigma^2)}{\Psi}(\bar{K}^b)_f, \quad (31)$$

where $1-\bar{K}^b$ can be written as a function of $(\bar{K}^b)_f$, and then:

$$(\bar{L}^b)_f = \left\{\frac{[d+(1-h)\sigma^2][d^2f(f+\sigma^2)+c\delta(\delta+\rho)\Psi^2]}{\Psi d\sigma(2h\sigma^2(f+\sigma^2)-\Psi)} - \frac{\sigma(f+h\sigma^2-d)}{\Psi}\right\}(\bar{K}^b)_f.$$

We know that $(\bar{K}^b)_f<0$ if and only if $f>\hat{f}^b_K$, i.e. $2h\sigma^2(f+\sigma^2)-\Psi<0$. Then, a sufficient condition for the expression in brackets to be negative is $f+h\sigma^2-d>0$. In

[24] If $h>(d+\sigma^2)/(2\sigma^2)=h_{\max}$ the sign in (30) would be positive for all $f\geq 0$.

consequence, if $\hat{f}_K^b > d$, then $(\bar{K}^b)_f < 0$ implies $(\bar{L}^b)_f > 0$ and therefore $\hat{f}_L^b < \hat{f}_K^b$. It is easy to prove that

$$\hat{f}_K^b > d \Leftrightarrow h > \frac{2d}{2d+\sigma^2}\frac{d+\sigma^2}{2\sigma^2} \equiv h_{min}.$$

Thus $(\bar{L}^b)_f > 0$ for all $f > \hat{f}_L^b$. To study the sign of $(\bar{L}^b)_f$ for $f \in [0, \hat{f}_L^b)$, expression (31) can be re-written as an expression which sign is given by a second-order convex polynomial in f. Depending on whether this polynomial has no real roots or it has two roots with different or equal sign, the different behaviours in footnote 15 appear.

iii) The derivative $(\bar{E}^b - \bar{L}^b)_f$ can be computed and proved negative for any positive h.

Proof of Proposition 4

From the dynamic maximization problem for the firm described in (3), and assuming a quadratic value function for the firm, $V_{\text{F}}(K)$ in (22), the best-response functions of the firm are given by (20) where

$$E^{\text{L}} = \frac{cf}{\Omega}, \quad \underline{E} = \frac{\sigma}{f}E^{\text{L}} + \frac{f\beta(b_{\text{F}}+\beta\sigma)}{\Omega}, \quad E^{\text{K}} = \frac{E^{\text{L}}-1}{\sigma} + \frac{f\beta(a_{\text{F}}-c\delta)}{\Omega},$$
$$I^{\text{L}} = -\frac{f\beta\sigma^2}{\Omega}, \quad \underline{I} = -\frac{I^{\text{L}}}{\sigma} + \frac{b_{\text{F}}(f+\sigma^2)}{\Omega}, \quad I^{\text{K}} = \frac{I^{\text{L}}(1-\beta\delta\sigma)}{\sigma} + \frac{a_{\text{F}}(f+\sigma^2)}{\Omega},$$

with $\Omega = cf + (c + f\beta^2)\sigma^2$, $E^{\text{L}} \in (0,1)$, and $I^{\text{L}} < 0$.

Assuming a quadratic value function for the regulator, $V_{\text{R}}(K)$ in (22), the optimal emission standard, $L^*(K)$ in (21) is obtained by solving the maximization problem of the regulator in (4), taking into account the best-response functions in (20). The optimal emissions, $E^*(K)$, and the optimal investment, $I^*(K)$, in (21) immediately follow from these best-response functions once $L^*(K)$ is known.

From the optimal investment decision in (21) and the capital stock dynamics in (2), the optimal time path of the capital stock can be written as in (23).

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