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Welfare Gain and the Demand for Annuities

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Abstract

This paper extends the annuity demand theory, giving new reasons for the small annuities demand. Regarding this problem, Yaari (1965) claims, under the condition that no one can die in debt, that a selfish consumer will fully annuitized her savings, insofar as annuity asset yield dominate conventional assets yield. However, we demonstrated mathematically that, in a standard life-cycle model, when borrowings are unconstrained and financial markets are complete, a selfish consumer may prefer not to annuitize her savings. In addition, we analyze the desire to purchase annuities according to the risk aversion coefficient and wealth composition.

Keywords: Annuities Demand, Complete Markets, Myopic-selfish Behavior, Life-cycle Model.

JEL: D11, D80, D91, G22, H55, J14, J17.

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1 INTRODUCTION

Upon retirement, individuals have to decide whether or not to finance their future consumption using annuities. Regarding this problem, conventional economic theory states that a selfish consumer will fully annuitize her savings, insofar as the yield of annuity assets dominates that of conventional assets. This result, obtained by Yaari (1965), was proved under the following conditions: i) the consumer's preferences are depicted by a constant relative risk aversion (CRRA) utility function, ii) the economic agent is a selfish consumer, iii) the yield of annuity assets dominates that of conventional assets, and iv) a negative asset position is forbidden when the consumer does not invest in annuities. Under these assumptions, however, there is no empirical research which supports Yaari's theorem. By contrast, many explanations have been proposed to give insights about the mismatch between the theoretical results and the small demand for annuities in the real world. For example, Bernheim (1991) suggests that consumers have altruistic feelings. Warshawsky (1988), Friedman and Warshawsky (1990) and Mitchell et al. (1999) report that the yield of private annuities does not exceed the market interest rate because of transaction costs. Kotlikoff and Spivak (1981) demonstrate that intra-family transfers may substitute annuities by more than 70 percent. Yagi and Nishigaki (1993) claim that it is optimal to allocate wealth not only in annuities, but also in conventional assets, when annuity payouts are constant. Even Brown and Warshawsky (2001) have pointed out other imperfections in the annuity market, such as the lack of protection against inflation, the irreversibility of annuitization and institutional barriers. However, research in the field is still fashionable, not only because the reduced demand for annuities continues to be puzzling for economists, but also because of the increasing concern in developed countries about the feasibility of the social security system.

Recently, Davidoff et al. (2005) have found that full annuitization is optimal under complete markets, under even less restrictive assumptions than those used by Yaari (1965), and also a large fraction of annuitized wealth remains optimal even with incomplete markets. Therefore, they conclude that the lack of the demand for annuities is caused by behavioral biases.

This paper, following Davidoff et al. (2005), contributes to annuity demand theory demonstrating that a selfish consumer may prefer not to annuitize her savings, whenever

a stream of future earnings is expected. This sort of behavior can be explained in two different ways. On the one hand, the result is obtained by weakening Yaari's fourth condition (see above) or, equivalently, by allowing the consumer to have a negative asset position upon death, but at longevity age.¹ On the other hand, an alternative explanation for the result is found by assuming an agent with shortsightedness, as is suggested by Davidoff et al. (2005). That is to say, the consumer is only worried about having positive wealth up to a certain date.²

Hence, we aim to give new insights on the small demand for annuities. To our knowledge, no one has mathematically proved that a selfish consumer under complete markets can be better off by not annuitizing her wealth. This unexpected result is only derived so long as there is a stream of future earnings. Thus, previous results under complete market settings are complementary to these, since researches³ have thus far assumed an initial wealth, instead of a stream of earnings, which has to be allocated throughout the agent's lifespan.

The importance of this approach to the annuity puzzle is twofold. Firstly, it gives some theoretical clues concerning the macroeconomic consequences of private pension plans held by baby boomers, specially defined contribution (DC) plans. Secondly, it can give a better understanding of consumer behavior. The former is necessary for policy makers and insurance companies in order to encourage the demand for annuities; since the alternative asset depletes wealth faster, even to the point of outliving resources. Therefore, given the concern about the future feasibility of the Social Security in the developed countries, it is convenient to study different alternatives⁴ which increase the demand for life annuity benefits by DC holders. This could contribute to a deeper knowledge on consumption-savings models. However, this point needs further research.

This paper proceeds as follows: section II reviews the most relevant literature about the demand for annuities. Section III describes the model that we use to determine optimal consumption and develops two subsections, which explain the reasons why the consumer may not decide to purchase annuities. The first subsection compares consumption trajec-

¹The final condition widely imposed on these models is zero wealth at longevity age.

²It is convenient to consider this date, in general, as her remaining expected life time, which is always less than the time necessary to reach longevity.

³This is not the case of Yaari (1965) and Hakansson (1969).

⁴Brown and Warshawsky (2001) suggest several alternatives concerning this point.

tories under different investment possibilities. The second subsection is devoted to the analysis of the welfare gain from not purchasing annuities when a stream of future earnings is introduced into the model. Some tables containing the Annuity Equivalent Wealth (AEW) are presented. Section V concludes by pointing out the importance of the findings and suggesting directions for future research.

2 OVERVIEW OF LIFE ANNUITIES DEMAND

Annuity demand theory has experienced significant development since Yaari (1965)'s seminal paper, in which he proved that under the life-cycle hypothesis with an uncertain lifetime, the optimal behavior of a selfish individual is to hold all assets in the form of annuities. This statement is of extraordinary importance because, if confirmed, it supplies economists with an important tool that enables both a deeper knowledge of the consumer's behavior, and also it enables the implementation of more efficient public policies; e.g. do people save because of a concern for other people, or simply out of concern for themselves? Or, what is the best way to finance retirees income? Unfortunately, there is no empirical testing that confirms Yaari's result completely. In contrast, there seem to be several circumstances in which it becomes unattractive to purchase annuities.

The desire to leave a bequest at the time of death, according to the literature, is one of the most cited reasons for not fully annuitizing. However, and although Yaari (1965) already contemplates this alternative, there does not exist a consensus regarding the importance of the bequest motive on the consumer's behavior; and thus, whether or not the bequest motive can be used to justify the small demand for annuities. According to this literature, there are researchers, on the one hand, such as Abel (1985) or Gokhale et al. (2001) who prove that accidental, or involuntary, bequests⁵ are sufficiently rich to explain household inherited-wealth, reported by Kotlikoff and Summer (1981);⁶ or Hurd (1989) in which it is determined that most bequests are accidental and desired bequests are small on average. Others like Brown (1999) point out that decisions about whether or not to annuitize DC plans are not affected by the bequest motive. But, on the other hand, Bernheim (1991) suggests that the existence of a strong bequest motive explains why people

⁵Wealth not consumed by a selfish individual at her death.

⁶They stated that roughly eighty percent of U.S. wealth was inherited.

maintain a positive fraction of resources in bequeathable forms.

Another well known reason is the low return on private annuities. Of course it can not be guaranteed that a selfish consumer would prefer to purchase annuities when their yield does not exceed the market interest rate. Nevertheless, this is usually due to the price of fair annuities being increased by transaction costs, or loads,⁷ which diminish the annuity yield below the risk free interest rate. For instance, Warshawsky (1988) shows for the United States that Government Bond yields exceeded annuity yields during the period 1919-1967. In contrast, Friedman and Warshawsky (1990) obtain the opposite result, using the same methodology, from 1968 to 1983. Finally, a more recent analysis, see Mitchell et al. (1999), has reported that the internal rates of return for SPIAs⁸ available in 1995 were between 1 and 2 percent below the market returns.

In addition to these justifications, earlier researches have pointed out other imperfections in the annuity markets, as additional motives which decrease annuities demand. Brown and Warshawsky (2001)⁹ suggest, among other possibilities, the following imperfections: i) the lack of protection against inflation, ii) the irreversibility of annuitization, and iii) institutional barriers and legal issues. Others have shown that constant payouts, offered by the private annuity system, is another motive for not fully annuitizing. This leads Yagi and Nishigaki (1993) to claim that it is optimal to allocate wealth both to annuities and to conventional assets, when annuity payouts are constant throughout the retirement period. Recently, Davidoff et al. (2005) have proved, under less restrictive assumptions than those used by Yaari (1965), that positive annuitization remains optimal even with incomplete markets. As a consequence, they suggest either psychological or behavioral biases as possible explanations for the limited annuity demand.

Finally, private annuities have almost perfect substitute assets, such as family transfers and Social Security benefits, that obviously affect their demand. The former may substitute annuities by more than 70 percent, even when markets are complete. This is so, as was shown by Kotlikoff and Spivak (1981), because family members possess a level of information of each other that reduces problems like moral hazard, adverse se-

⁷Annuity loads can be decomposed into the following three components: a) reserves, which are affected by the adverse selection problem, b) administrative costs and c) commissions and profits.

⁸Single-Premium Immediate Annuities (SPIA).

⁹Read this paper for a comprehensive extent literature review concerning these imperfections.

lection and deception, which nevertheless do affect insurance companies. According to economic theory, insurance companies should substitute dollar for dollar for private annuities. Therefore, coupling this statement with the fact that Social Security benefits, in the developed countries, are the major flow of income for retirees, we can expect that private annuities should have a small demand. However, even with the existence of the social security system, Auerbach (1987) obtains the result that households do not significantly offset social security insurance by reducing their purchase of private life insurance. Consequently, we can conclude that, although several reasons have been suggested, the demand for annuities is still a puzzle for economists.

3 THE MODEL

The aim of this section is to analyze consumption trajectories and utility levels under an uncertain lifetime. The analysis will help to understand both the psychological biases, and the economic circumstances, under which individuals are not willing to purchase annuities. Moreover, the paper will show the main consequences derived from this irrational behavior.

We can assume under uncertain lifetime and complete markets that consumption can be financed, for the sake of simplicity, by two alternative assets; e.g. conventional assets and annuities. On one hand, conventional assets yield a safe interest rate r and have the important property that the investment is bequeathable. On the other hand, annuities are an actuarial contract between the consumer and an insurance company. This contract consists of paying, at the beginning of the period, a single premium to the insurance company in exchange for a lottery under which if the consumer survives at the end of the period, she will receive the safe interest rate r plus a risk premium μ contingent on her mortality risk; but, if she does not survive at the end of the period, she will not receive anything. Thus, while conventional assets are a bequeathable investment, annuities are not.

The consumption allocation process differs according to how important is the desire to leave bequests. However, we will follow the theoretical stream that assumes the consumer to be a selfish individual. Taking this into account, Yaari (1965) claims that the consumer

will fully annuitize her savings. This theorem was proved under the condition of having a positive asset position throughout the lifespan. But, if the condition is relaxed, instead of financing consumption through annuities, the consumer may achieve a higher utility level by borrowing money and increasing present consumption. This result, although mathematically proved in the following pages, needs a further explanation.

The assumptions used by Yaari (1965) are rather close to reality. Therefore, it does not seem useful to weaken the borrowing constraint, unless we are suspicious that the consumer makes decisions subject to some irrationality. Following Davidoff et al. (2005), we propose that the economic agent behaves myopically and so, once the consumer has retired, she is only worried about having positive wealth during her remaining expected life, and not up to longevity. The main consequence derived from this behavior is a smaller demand for annuities, as well implying that consumers are more likely to outlive their resources.

We use the following additional necessary assumptions, in order to purchase actuarially fair life-annuities: i) the only source of uncertainty is the time of death. Longevity T is known in advance and is unalterable. ii) The probability of death Ω^{10} is known and exogenous, and is also common to every consumer. iii) The mature asset yields a known and safe interest rate. Nevertheless, and although these three conditions eliminate both the adverse selection problem (i.e. there are no healthier consumers who are more willing to purchase annuities than other healthier ones, at a given premium) and the ruin¹¹ problem, because the probability of death is known, there are two remaining loads which decrease the annuities internal rate of return. Consequently, if we assume that annuity yields at least dominate maturity yields, then we need to include that iv) there are no family agreements which permit transfers (see Kotlikoff and Spivak, 1981), and finally v) annuity markets are complete and their administrative costs are negligible.

Unfortunately, we have to realize that all of these conditions are not realistic. Nonetheless, they are necessary not only to analyze the myopic behavior, because of the possibility that the agent is not concerned about her future wealth as of a certain age; but also to calculate, on the one hand, the level of utility achieved by the consumer when her wealth is

¹⁰See Definition 1 located in the appendix, page 21.

¹¹The possibility that an insurance company will not have enough reserves, in order to satisfy the contracts, because of unexpected circumstances.

fully annuitized and, on the other hand, the level of utility achieved without purchasing annuities. The standard comparison between these two utility levels is labeled “Annuity Equivalent Wealth”¹² (AEW), and it will help later on to determine which asset is preferred by the consumer.

We shall proceed by introducing the mathematical problem faced by our consumer. The expected utility function U , which we assume to be similar for every consumer at a given age, will first be explained and, subsequently, two budget constraints will be presented. Each constraint will differ according to the asset selected to finance future consumption.

Firstly, the individual at age x , as a selfish consumer, is depicted by the following expected utility function, that she maximizes by the selection of a consumption plan,

$$U(x) = \int_x^T \frac{\Omega(s)}{\Omega(x)} \beta(s-x) u(c(s,x)) ds \text{ for all } x \in [0, T], \quad (1)$$

where in terms of Fisher-Yaari-Bommier¹³ (FYB), we henceforth denote: $\frac{\Omega(s)}{\Omega(x)}$ as the (Fisher) rational discount function, $\beta(s-x)$ as the (Yaari) subjective discount function, and $\frac{\Omega(s)}{\Omega(x)}\beta(s-x)$ as the (Bommier) overall discount function. They measure the value at age x associated with a unit of consumption at age s . In particular, $\frac{\Omega(s)}{\Omega(x)}$ is the probability that an individual of age x will be alive at age s , while $\beta(s-x)$ is the widely used time discount factor from age x to age s , or $e^{-\delta(s-x)}$, $\forall \delta \geq 0$. Hence, both $\Omega(\cdot)$ and $\beta(\cdot)$ are positive real functions valued less than or equal to one. On the other hand, $u(\cdot)$ is assumed to be a CRRA utility function of a risk averse consumer ($u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma > 0$), like the one used by Yaari (1965). And, $c(s,x)$ represents the rate of expenditure¹⁴ on consumption at age s , of an x year-old consumer.

This utility function has the important feature that, a single monetary unit of consumption delivers a greater utility as the consumer ages. This is due to property six of the rational discount function (see Definition 1, pag. 21). However, nowadays there exists a controversy about what type of rational discount function should be used. According to this, there are three main theoretical streams. The first supports the use of the common

¹²See Brown (2003), Brown and Warshawsky (2001), Mitchell et al. (1999), Friedman and Warshawsky (1990), among others.

¹³See Bommier (2001), Fisher (1977).

¹⁴We assume the rate of expenditure on consumption is a smooth function, $c \in \mathcal{C}^\infty([0, T] \times [0, T])$.

life tables, Hurd (1989), and Hurd and McGarry (1995). Another supports the necessity of continuously correcting the survival probability due to time misperceptions, specially close to young and old ages, Hamermesh (1985), Hurd and McGarry (2002) or Gan et al. (2003). And others like Bommier (2001, 2003a,b) state that more accurate results can be obtained using hyperbolic discounting, because individuals make inconsistent decisions. This paper, however, will not enter into the discussion and so, we will plug the common life tables, as proxy values, into the rational discount function.

Secondly, the constraint faced by the consumer depends on whether she purchases annuities or not. Thus, we introduce two alternative budget constraints.

$$k(x) + \int_x^T \frac{R(s)}{R(x)} (w(s) - c(s, x)) ds = 0, \quad (2)$$

and

$$k(x) + \int_x^T \frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)} (w(s) - c(s, x)) ds = 0. \quad (3)$$

(2) and (3) are respectively the budget constrained when consumption is financed by investing in conventional assets, and when consumption is financed by annuities. $k(x)$ is initial wealth at age x , and $w(s)$ is income at age s . There is no restriction about the source of income; so, $w(\cdot)$ may be either Social Security benefits when the agent is retired, or a salary if she continues working, or the sum of many different periodical earnings. On the other hand, $\frac{R(s)}{R(x)}$ is the financial present value at age x , of a monetary unit received at age s , and equivalently, $\frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)}$ is the actuarial present value; that is,

$$\frac{R(s)}{R(x)} = e^{-\int_x^s r(j) dj},$$

and

$$\frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)} = e^{-\int_x^s (r(j) + \mu(j)) dj},$$

so that $r(j)$ is the safe interest rate at age j yielded by the conventional asset, and $r(j) + \mu(j)$ is the actuarially fair interest perceived in the case of being alive at the end of period j . Neither (2) and (3) constrain wealth to be nonnegative along the lifespan. Therefore, the consumer may be in debt at any time, although both (2) and (3) implicitly assume that wealth at longevity should be zero.¹⁵

¹⁵This is a necessary condition which represents the consumer's selfish behavior into the budget constraint.

3.1 Consumption Trajectories

In this model, our individual must decide whether to invest in annuities or not. Her decisions are subject to an irrational behavior known as myopia which, in this case, means that she is not worried about having a negative asset position from a certain date.¹⁶ Moreover, for the sake of reality we assume that financial markets do not allow individuals to die in debt, and so we can expect that individuals will consume their income in each period. That is, we will follow Yaari (1965)'s problem only when the consumer reaches an asset position of zero. We suggest this behavior as a possible explanation for the small demand for annuities because it is usual to see how retirees, with Social Security benefits, prefer not to annuitize their DC plans, in order to enjoy their wealth at the beginning of their retirement, due to the higher probability of becoming unhealthy over the remaining expected lifetime. Nonetheless, the decision to purchase annuities depends on many more aspects which will be explained later on.

Firstly, according to this model the individual's welfare is based on her consumption trajectories. Therefore, we need to know not only how consumption increases over time, but also what the initial consumption is. This fact was studied previously by Barro and Friedman (1977), Davies (1981), Levhari and Mirman (1977), among others, who compare consumption trajectories when the consumer purchases annuities, with those trajectories when she does not. Optimal consumption growth, both in the case of investing in conventional assets, and in the case of purchasing annuities, are standard problems in optimal control theory. The results are:

$$\frac{\frac{\partial}{\partial s} c(s, x)}{c(s, x)} = \frac{r(s) - \mu(s) - \delta}{\gamma}, \quad (4)$$

and

$$\frac{\frac{\partial}{\partial s} \hat{c}(s, x)}{\hat{c}(s, x)} = \frac{r(s) - \delta}{\gamma}, \forall s \in [x, T]. \quad (5)$$

Hereinafter, in order to distinguish the two alternative investments, we denote by \hat{c} the consumption trajectory when annuities are purchased.

So, (4) represents the way in which consumption increases over time when an x year old consumer invests in conventional assets, while (5) does so when an x year old con-

¹⁶This statement is equivalent to the claim that the consumer assumes complete markets in which loans are allowed.

sumer invests in annuities. Note from (4) and (5) that consumption with an annuitized wealth grows faster. Concretely, over the lifespan, (4) decreases markedly when the consumer approaches longevity, while (5) leads to a continuously increasing consumption up to longevity, so long as $r(s) > \delta$ for all $s \in [x, T)$.

Secondly, if our aim is to understand why an individual at age x decides to allocate her wealth to conventional assets, instead of annuities, it is a necessary, but not sufficient, condition that at least her $c(x, x)$ should be greater than her $\hat{c}(x, x)$. According to this, we present in Table 1 the main factors that determine which initial consumption is greater:

Table 1: DETERMINATION OF THE VALUE

$\frac{c(x,x)}{\hat{c}(x,x)}$		
γ	$w(s) > 0^\dagger$	$w(s) = 0^\ddagger$
$(0, 1)$	> 1	> 1
$\{1\}$	> 1	$= 1$
$(1, \rightarrow)$	unknown result	< 1

[†] For at least one $s \in [x, T)$.

[‡] For all $s \in [x, T)$.

To understand the meaning of Table 1 it is convenient to realize that the introduction of annuities in an uncertain lifetime model produces both a substitution effect and an income effect on initial consumption. This is so, because annuities offer a greater yield than conventional assets. In relation to this fact, Levhari and Mirman (1977) shows that when $w(s) = 0$ for all $s \in [x, T)$, life-span uncertainty affects optimal consumption in two opposing ways. On the one hand, the individual substitutes more future consumption for present consumption than under certainty. On the other hand, uncertain lifetime decreases (resp. increases) consumption because of having the possibility of a longer (resp. shorter) life. Moreover, the risk aversion coefficient γ together with a stream of future earnings plays a fundamental role in the determination of the value $\frac{c(x,x)}{\hat{c}(x,x)}, \forall x \in [0, T)$ (see Table 1). In particular, the risk aversion coefficient γ is necessary to calculate the present value of a future consumption. For example, whenever γ equals 1, a marginal

unit of future consumption financed either by conventional assets, or by annuities, has the same marginal present value. Consequently, both initial consumptions will be alike. This fact shows up in column 3 (no future earnings) of Table 1. By contrast, assuming that $\gamma \in (0, 1)$ (resp. $(1, \rightarrow)$), the future consumption financed by conventional assets is less (resp. more) preferred than the future consumption financed by annuities. Hence, because of the substitution effect $c(x, x)$ should be greater (resp. lower) than $\hat{c}(x, x)$. In the second term, the stream of future earnings also modifies the optimal initial consumption. Thus, the present value of a stream of future earnings under actuarially fair insurance is lower than under conventional assets. Then, future earnings have a negative effect on $\hat{c}(x, x)$ in comparison with $c(x, x)$.

In short, when γ and the stream of future earnings are taken into account simultaneously, we can note (see columns 2 and 3) that $c(x, x)$ is greater than $\hat{c}(x, x)$ for a wider range of γ values. Concretely, the greater are future earnings with respect to initial capital $k(x)$, the wider is the range of γ values for which $c(x, x)$ is greater than $\hat{c}(x, x)$. So, assuming that γ belongs to the open range $(1, \rightarrow)$,¹⁷ there exists a slight difference between considering an initial wealth, or a stream of future earnings. Nevertheless, this fact does not mean that the demand for annuities depends on the risk aversion coefficient. By contrast, we should take into account the fact that, when comparing $c(x, x)$ with $\hat{c}(x, x)$, there are two opposing effects. The less risk averse the consumer is, $\hat{c}(x, x)$ becomes lower with respect to $c(x, x)$ but, simultaneously, (5) becomes greater than (4), and vice versa. Therefore, the risk aversion coefficient leads to opposing results.

We find, however, that the difference between this result and those obtained previously, lies in the introduction of a stream of future earnings under complete markets. Therefore, it becomes interesting to analyze what the present value of future earnings is, with respect to initial wealth. For instance, in the appendix it is proved that, it is less (resp. more) likely that a retiree will decide to annuitize her DC plan when she has a much greater (resp. lower) accumulated capital in Social Security benefits. The implications of this statement are: i) individuals with resources in the form of capital (resp. non capital income) are more (resp. less) willing to purchase annuities; ii) Social Security

¹⁷Previous studies suggest that γ is typically above unity. For instance, Davies (1981) suggests with a similar model a value of 4 as the best guess.

benefits do not offset annuitized private pension plans completely, as Auerbach (1987) claims. In fact, there exists a threshold that is positively related with the present value of future earnings, above which the consumer will decide to annuitize her initial wealth. iii) individuals who choose not to purchase annuities outlive their initial wealth faster. Hence, their $c(x, x)$ will be greater than those individuals who have annuitized their wealth (or $\hat{c}(x, x)$), because they anticipate consumption. But they will consume exactly their income in each period from the date that they have depleted their wealth. A representation of such consumption trajectories is provided in Figure 1.

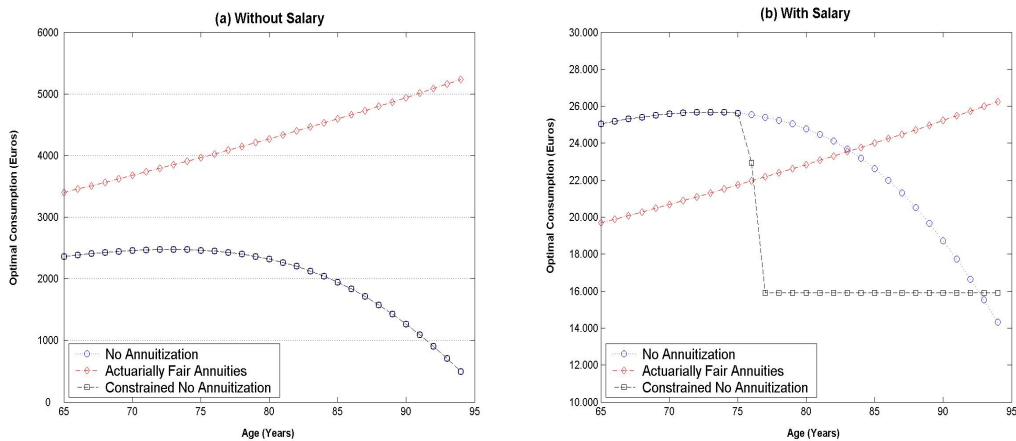


Figure 1: Selected consumption trajectories, $r = .02$, $\delta = 0$, $\gamma = 2$, $w = 16.000$, $k(65) = 100.000$

These two cases represent the consumption trajectories when the only source of income is initial wealth (Figure 1.a) and, when besides the initial wealth, a periodic salary is introduced (Figure 1.b). The consumption trajectories are the cases shown in columns 2 and 3, row 3 of Table 1. Specifically, blue circles (\circ) plot the consumption path of a consumer who is able to borrow money and who does not purchase annuities. Red diamonds (\diamond) plot the consumption of an individual who purchases actuarially fair annuities. And, black boxes (\square) plot the consumption trajectory when the myopic consumer does not purchase annuities and financial markets do not allow individuals to die in debt.

Note from Figure 1. that consumption under annuitized wealth, $\hat{c}(s, x)$ is not only always greater than $c(s, x)$ for all $s \in [x, T)$, but also it increases from year to year; while non-annuitized consumption decreases in Figure 1 from age 75. On the other hand, con-

trary to case (a), we are able to see from case (b) that non-annuitized initial consumption is greater than annuitized initial consumption. Therefore, see Figure 1.b, non-annuitized consumption is greater than optimal annuitized consumption up to age 76.

In conclusion, whenever γ belongs to the interval $(0, 1)$, the early consumption financed by conventional assets is greater than the early consumption financed by annuities. Furthermore, $c(x, x)$ can exceed $\hat{c}(x, x)$ for γ -values that are greater than, although close to, 1 and the greater is the stream of future earnings. These facts, nevertheless, do not necessarily imply that specific values of γ lead to conventional assets being more preferred than annuities.

3.2 Welfare Gains

It is pointed out by Yaari (1965) and proved by Hakansson (1969) that a selfish consumer will fully annuitize her savings, so long as she cannot die in debt. Subsequently, Kotlikoff and Summer (1981), Hurd (1989), Mitchell et al. (1999) and Brown and Warschawsky (2001) show that a selfish consumer will fully annuitize her initial wealth, even with complete markets. Recently, Davidoff et al. (2005) has extended the annuity demand theory by presenting less restrictive conditions under complete markets, under which we still get full annuitization. However, it has not been proved mathematically that a selfish consumer may prefer not to annuitize her savings under complete markets, when both an initial wealth, and a stream of future earnings are taken into account simultaneously.

We shall proceed by explaining the circumstances under which individuals are not willing to purchase annuities. The explanation follows two steps. Firstly, we show, with the help of Table 2 below, that the risk aversion coefficient does not explain the demand for annuities, at least for the most common range of γ -values used by economists. Secondly, we state **Proposition 1**, which is the main finding of the paper, and we shall outline its consequences upon the consumption allocation process.

We assume that a consumer decides to purchase annuities, so long as this asset leads to a welfare gain with respect to the alternative investment. In general, a common and easy way to prove that a welfare gain exists is by means of the annuity equivalent wealth (AEW), which is the proportion of annuitized wealth, that is necessary to achieve the maximum utility level when the consumer has no access to the annuity market. In other

words, AEW offers an intuition about whether or not the indifference curve under annuitized wealth is greater. Concretely, if this proportion is greater (resp. less) than one, annuities (resp. conventional assets) cause a welfare gain to the consumer. Table 2 reports the AEW values associated with the problem depicted in Figure 1, when γ equals 2. Nevertheless, we have introduced two additional cases in order to analyze the role of the risk aversion coefficient.

Table 2: ANNUITY EQUIVALENT WEALTH (AEW) VALUES

Age	$w(s) > 0^\dagger$			$w(s) = 0^\ddagger$		
	$\gamma = .75$	$\gamma = 2$	$\gamma = 5$	$\gamma = .75$	$\gamma = 2$	$\gamma = 5$
65	0.8982	0.9653	1.0389	1.4302	1.5342	1.6472
70	0.8011	0.9151	1.0440	1.4963	1.6412	1.7912
75	0.6993	0.8165	1.0492	1.5743	1.7740	1.9751
80	0.6472	0.7535	1.0547	1.6644	1.9369	2.2070
85	0.5940	0.7181	1.0612	1.7643	2.1317	2.4925
90	0.5450	0.6871	1.0719	1.8692	2.3540	2.8257
95	0.5107	0.6712	1.0943	1.9708	2.5860	3.1626

[†] For at least one $s \in [x, T)$, and [‡] for all $s \in [x, T)$.

Note: In this table, the mortality hazard rate is assumed to follow the Gompertz's Law ($\mu(x) = \alpha e^{\lambda x}$), where α is equal to $\exp(-11.74311)$ and $\lambda = 0.106402$, for all x between 65 and 110 years old. Further, it has been used the same values from Figure 1, in order to illustrate which consumption yields a greater utility in each case.

The values shown in each column correspond to the AEW of our myopic consumer, who starts at the age of 65 with an initial wealth of 100.000 euros and, for the case of receiving a periodical endowment, we assume an annual salary of 16.000 euros. Moreover, the values reported in Table 2 take into account the wealth already consumed. That is, the consumer's wealth decreases from year to year. This is the reason why the individual reinforces her decision to either finance the consumption through annuities, or to finance her consumption through conventional assets. So, once the consumer has invested in a specific asset, we may expect that she will hold the investment.

Now, if we pay attention to columns 3 and 6 of Table 2, we can see the AEW associated with Figure 1 (specifically, consumption trajectories plotted with black boxes). Table 2 (column 6) confirms that the consumer achieves a higher utility level by annuitizing her wealth, see Figure 1.a; while, in the case of Figure 1.b, the consumer prefers to increase her early consumption by investing in conventional assets, see Table 2 (column 3). However, the latter consumption trajectory depletes wealth faster and, as a consequence, the individual only can consume her income as of age 77.

It has been pointed out that the decision as to whether to annuitize or not does not depend on the risk aversion coefficient, Levhari and Mirman (1977). This result also shows up in Table 2. Note that the AEW values do not change drastically when the consumer's risk aversion, at the age of 65, varies from 0.75 to 2. In fact, given the resources assumed for the consumer, conventional assets yield greater welfare than annuities for γ values up to 3 (whenever there is a stream of future endowments). Thereinafter, the consumer will prefer to annuitize her wealth; e.g. $\gamma = 5$ as is shown in Table 2. Hence, the risk aversion coefficient does not explain the demand for annuities, because there exists a γ value, for which the consumer changes the investment in conventional assets to annuities, that i) depends on **Proposition 1** and ii) its value is, in general, outside the common range of γ values obtained by economists.¹⁸

Proposition 1 *The decision of whether or not to purchase an asset contingent upon her death, when the individual is selfish and faces an uncertain lifespan, depends on the relationship between the present value of future earnings and the initial wealth.*

Proposition 1 is principally of interest for understanding the small demand for annuities. Being specific, annuities are less (resp. more) preferred than conventional assets, the greater (resp. lower) is the present value of future earnings with respect to the initial wealth. Note that we are considering a selfish individual. So, it is not a necessary condition that the consumer has altruistic feelings, or a bequests motive, in order not to purchase annuities. Furthermore, **Proposition 1** assumes that not only does the yield of annuities dominate that of conventional assets, but also annuities are actuarially fair. Therefore, we can extract from Table 2 the following behavior: i) our myopic consumer

¹⁸The mathematical proof is found in the appendix, page ??.

may not purchase actuarially fair annuities, when there is a stream of future earnings; but ii) when the consumer is willing to purchase assets contingent on her death, the insurance company can supply unfair annuities. Table 3 below reports the maximum loading that the insurer can charge to the consumer. Finally, iii) when initial wealth is the only source of income, we find that the consumer purchases annuities, even with unfair premiums (although the yield of annuities must dominate that of conventional assets).

Table 3: MAXIMUM LOADS[†] ($\hat{\theta}$)

$k(65)$	$\hat{\theta}$		
	$\gamma = .75$	$\gamma = 2$	$\gamma = 5$
100.000	*	*	.4000
150.000	*	.0950	.6850
200.000	*	.3150	.8500
250.000	.0550	.4825	.9600

* Conventional assets yield a greater welfare.

Note: $w(s) = 16.000$ for all $s \in [x, T)$. The formula used to calculate the maximum load is as follows:

$$r(s) + (1 - \hat{\theta}) \cdot \mu(s).$$

Table 3 illustrates how a consumer demands a greater yield, the lower is the initial wealth with respect to the present value of future earnings (look at each column from the bottom to the top). Another important finding shows up when the consumer does not have earnings during future periods. In this particular case, our individual usually decides to annuitize.

In short, whenever DC-plan holders behave as our myopic consumer, we can state that Social Security benefits, or existing annuities, produce a crowding out effect on brand new annuities. However, this crowding out is not always dollar for dollar. In particular, the decision to annuitize new wealth depends on both future income, and on psychological aspects (e.g. risk aversion and subjective time discount). Hence, DC-plan holders with high Social Security benefits, or existing annuity payouts, will not annuitize their plans, unless the capital compounded in their DC pension plans is thought to be high enough. So, they choose a financial payout, which allows the possibility of leaving bequests.

4 CONCLUSIONS

This paper contributes to the theory of the demand for annuities, providing new insights on the annuity puzzle. We show that annuities may not cause a welfare gain relative to conventional assets. Consequently, the consumer is not willing to purchase annuities and so, she is more exposed to the risk of outliving her resources at the end of her life. This situation can be explained in two different ways. On the one hand, by relaxing the borrowing constraint condition imposed in Yaari (1965). On the other hand, by following Davidoff et al. (2005), and assuming that the economic agent is only concerned with having positive wealth during her remaining expected life, instead of doing so up to longevity. So, once one of these approaches is taken into account, we have found, firstly, that the risk aversion coefficient does not explain the demand for annuities and, secondly, that the decision as to whether or not to purchase an asset that is contingent upon her death, when the individual is selfish and faces an uncertain lifespan, depends on the relationship between the present value of future earnings and initial wealth.

The theoretical finding has important implications for the social security system, for private pension plans and for insurance companies. On the one hand, for example, Social Security does not offset, at the age of retirement, a dollar in public annuities by reducing a dollar in the private market. In particular, the crowding out effect is lower the greater is the wealth that is accumulated in the private market. On the other hand, we have shown how those economic agents, who finance their consumption through annuities, are willing to purchase actuarially unfair premiums, even to the point of demanding annuities with half of the risk premium contingent upon her death.

Finally, the conclusions obtained in this paper suggest extensions of the analysis, to include different annuities, such as SPIAs or constant annuity payouts, as well as to study the effects that fiscal policies can produce upon the demand for annuities.

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Appendix

In this appendix we shall present the main properties of the rational discount function or mortality hazard rate. Each property is essential both to determine the optimal allocation and to calculate the actuarial fair annuity. Subsequently, the proof of proposition 1 will be sketched.

Definition 1 Let $\Omega \in \mathcal{C}^2([0, T])$ denote the rational discount function, which means the probability that the consumer will be at age x . Ω has the following properties:

1. $\Omega(0) = 1$.
2. $\lim_{x \rightarrow T} \Omega(x) = 0$.
3. $0 < \Omega(x) \leq 1$.
4. $\Omega(s) < \Omega(x) \Leftrightarrow s > x$.
5. $-\frac{\partial}{\partial x} \frac{\Omega(x)}{\Omega(x)} = \mu(x) > 0$.

where $\mu \in \mathcal{C}^\infty([0, T])$ is the instantaneous mortality rate, also known by demographers as “mortality hazard rate”, or by actuaries as “force of mortality”.

6. The mortality hazard rate μ is an increasing function on age¹⁹

$$\frac{\partial}{\partial x} \mu(x) \geq 0, \forall x \in [0, T).$$

7. In particular, the probability of being alive at age x is given by the following mapping:

$$\begin{aligned} \Omega : [0, T) &\rightarrow (0, 1] \\ x &\mapsto \Omega(x) = e^{-\int_0^x \mu(s) ds} \end{aligned}$$

¹⁹In general, this is not true at young ages, although it is assumed for the sake of simplicity.