

On-line characterization of transient neuronal activity

David Arroyo^{1,†}, Pablo Chamorro¹, José María Amigó²,
Francisco B. Rodríguez¹ and Pablo Varona¹

¹ *Grupo de Neurocomputación Biológica, Escuela Politécnica Superior,
Universidad Autónoma de Madrid, 28049 Madrid, Spain*

² *Centro de Investigación Operativa, Universidad Miguel Hernández, Avda. de la
Universidad s/n, 03202 Elche, Spain*

Abstract. Characterization and control of nonlinear and non-stationary processes is an active topic in the field of the applied theory of dynamical systems. In this context classical control techniques cannot be applied straightforward, and thus observation and actuation should be properly incorporated into a real-time feedback (or closed-loop) methodology. One of the possible application scenarios of this methodology is depicted by neural activity. In this work we analyze the problem related to the first component of the real-time closed-loop technology for the case of neural activity. This being the case, we discuss different methods to classify dynamics and to detect events in an automatic and fast way.

Keywords: time series, order patterns, event characterization, entropy, time-frequency representation

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† **Corresponding author:** david.arroyo@uam.es

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1. Introduction

The implementation of a real-time closed-loop activity dependent technology for neuroscience studies, cannot be achieved without an adequate (by means of precision and computation-time demand) procedure to codify and classify the interplay of processes occurring in multiple temporal and spatial scales, even when some of the state variables are not accessible. If the core of our work is drawn by transient characteristics of signals, methods defined only by frequency analysis cannot be applied. Certainly, model based procedures and time-frequency analysis (using, for example, the Hilbert and/or wavelet transforms) of time series should be used to categorized non-stationary behaviour. Nevertheless, those methods are very dependant on the proper selection of a meaningful set of parameters and, consequently, can lead to an extra overload of the event characterization procedure. On the other hand, ergodic theory underlines the possibility of studying dynamical systems by coarse-grained versions of the associated orbits or trajectories [1]. As a matter of fact, if one assigns a partition to the state space of a dynamical system, then its dynamics can be interpreted using the framework of information theory. Nevertheless, the discretization of orbits cannot be done arbitrarily, since the underlying partition must be at least a generating one. The determination of generating partitions is a very complex (and open) problem, and thus the segmentation of the state space is usually done by approximating the generating partition [2]. The translation of time series into ordinal patterns is a way to get such approximation [3]. In this work, and from the perspective of information theory, we study the ordinal patterns of time series obtained from both neural models and recordings from living neurons. Different measures and analysis are calculated and compared to those associated to time-frequency methods, taking into account the requirements and inherent limitations of *online* event detection and classification.

2. Instantaneous frequency estimation: Intrinsic Mode Functions (IMFs)

According to Bedrosian's theorem the Hilbert transform of the product of a lowpass (narrowband) and a high pass signal is given by the product of the lowpass signal and the Hilbert transform of the high pass signal. In [4] the Empirical Mode Decomposition (EMD) is presented as way to extract the IMFs of a given signal. EMD approximates the upper and lower envelopes of a signal by interpolating its local extrema using spline functions. Once the envelopes have been obtained, the mean value of them is computed and the difference between the original signal and the mean value is interpreted as an

IMF. This sifting process is applied on the subsequent IMFs until an stoppage criterion is satisfied. For each IMF, instantaneous frequency is given by the derivative of the phase of the associated analytical signal [5, Chapter 10]. The plot of the instantaneous frequency of IMFs defines the Hilbert spectrum of a signal (see Fig. 1(b)).

3. Wavelet-based entropy measures

The multiresolution analysis proposed by Mallat in [6] leads to the decomposition of a signal into a set of levels of description, which makes possible to compute the *Wavelet Entropy* (WE)[7]. Let us consider $\{\psi_{j,k}(t)\}$ being a family of orthonormal functions in $L^2(\mathbb{R})$, a sequence S of length M , and the associated wavelet coefficients $C_j(k) = \langle S, \psi_{j,k} \rangle$. The average energy of a detail level j is

$$E_j = \frac{1}{N_j} \sum_k |C_j(k)|^2, \quad (1)$$

where N_j is the number of wavelet coefficient for scale j . The total energy is calculated as

$$E_{tot} = \|S\|^2 = \sum_{j<0} \sum_k |C_j(k)|^2 = \sum_{j<0} E_j, \quad (2)$$

and the *Relative Wavelet Energy* (RWE) is

$$p_j = \frac{E_j}{E_{tot}} \quad (3)$$

for $j = 1, 2, \dots, \log_2(M)$. Therefore, entropy can be determined assuming p_j as a probability distribution function.

4. Ordinal analysis of non-stationary behavior

Order patterns represent a way to estimate generating partitions and to identify changes in dynamics [8]. Given a closed interval $I \subset \mathbb{R}$ and a map $f : I \rightarrow I$, the *orbit* of (the initial condition) $x \in I$ is defined as the set $\mathcal{O}_f(x) = \{f^n(x) : n \in \mathbb{N}_0\}$, where $\mathbb{N}_0 = \{0\} \cup \mathbb{N} = \{0, 1, \dots\}$, $f^0(x) = x$ and $f^n(x) = f(f^{n-1}(x))$. Orbits are used to define *order L-patterns* (or order patterns of length L), which are permutations of the elements $\{0, 1, \dots, L-1\}$, $L \geq 2$. We write $\pi = [\pi_0, \pi_1, \dots, \pi_{L-1}]$ for the permutation $0 \mapsto \pi_0, \dots, L-1 \mapsto \pi_{L-1}$. The point $x \in I$ is said to define (or realize) the order L -pattern $\pi = \pi(x) = [\pi_0, \pi_1, \dots, \pi_{L-1}]$ if

$$f^{\pi_0}(x) < f^{\pi_1}(x) < \dots < f^{\pi_{L-1}}(x). \quad (4)$$

Alternatively, x is said to be of type π . The set of all possible order patterns of length L is denoted by \mathcal{S}_L , and the set P_π is defined as

$$P_\pi = \{x \in I : x \text{ is of type } \pi\}, \quad (5)$$

where $\pi \in \mathcal{S}_L$. Taking into account Birkhoff's ergodic theorem [9, p.34], if f is ergodic with respect to the invariant measure μ , then the orbit of $x \in I$ visits the set P_π with relative frequency $\mu(P_\pi)$, for almost all x with respect to μ . Accordingly, the so-called *permutation entropy* can be computed from the relative frequencies of the different order patterns as

$$H_L = - \sum_{\pi \in \mathcal{S}_L} \mu(P_\pi) \log \mu(P_\pi). \quad (6)$$

From a practical point of view, $\mu(\cdot)$ can be estimated from the histogram of the order patterns associated to a given orbit, and a sliding window can be applied in order to detect non-stationarity.

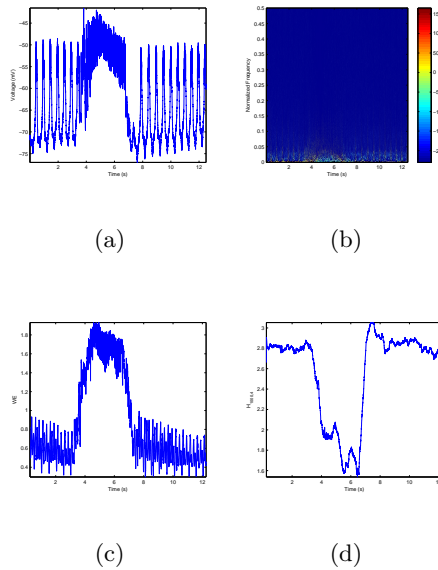


Figure 1: Experimental analysis of the change of the overall spike frequency of pyloric neurons due to cardiac sac activity. (a) Effect of cardiac sac activity on pyloric network output. (b) Associated Hilbert spectrum. (b) Analysis through the Wavelet entropy. (c) Permutation entropy of pyloric network output.

5. Results and discussion

The inclusion of any strategy to detect and control dynamics in real-time resorts to precise, automatic and, not less important, fast procedures. Time-frequency methods are very useful to identify non-stationarities [10], but their inner characteristics requires to examine thoroughly the type of signal to study in order to select the proper parameters (interpolation procedure, mother wavelet and central frequency, etc.) [11]. In the case of EMD, the physical meaning of the IMFs is not always clear and could determine misleading results [10]. Furthermore, Hilbert spectrum allows the identification of changes in dynamics only after a convenience postprocessing (wich erodes automatic event detection). In order to illustrate the virtues and limitations of the methods here explained, we have studied the dynamics change of cardiac sac activity [12] (see Fig. 1). According to our experiments, modifications in dynamics can be detected by any of the described methods. However, configuration and implementation of permutation entropy is less complex and adequate for real-time closed-loop applications.

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