Pushing the Limits of the Swampland Distance Conjecture

Memoria de Tesis Doctoral realizada por

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presentada ante el Departamento de Física Teórica
de la Universidad Autónoma de Madrid
para optar al Título de Doctor en Física Teórica

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Mayo de 2022

A ti, mamá.

Agradecimientos

Quiero empezar mostrando mi más sincero agradecimiento a mi director de tesis. Ángel, muchísimas gracias por *todo*. Gracias por guiarme por el a veces caótico camino de hacer un doctorado. Gracias por haberme enseñado tanto de tantas cosas. Por esas clases magistrales totalmente improvisadas en tu despacho, en las que nos dabas a tus estudiantes pequeñas píldoras que contenían muchísima sabiduría. Gracias también por esa pasión que tienes por la física. Creo que es muy importante mantenerla, y cuando la mía se venía abajo tú me transmitías la tuya. Gracias por apoyarme en cada idea que se me ocurría. Por intentar comprender a qué me refería, si tenía sentido o no, y por darme las referencias necesarias para aprender sobre ello. Gracias también por involucrarme en tus propias ideas, por esos correos en los que me preguntabas si me apetecía pensar en tal tema que te interesaba, siempre dándome la opción de decir que no. Por supuesto, todas eran ideas muy interesantes y yo acababa aprendiendo mucho trabajando en ellas. Por último, gracias por darme apoyo moral. Hacer un doctorado puede ser muy cargante psicológicamente, te enfrentas a muchos retos. Así que gracias por darme un empujón cuando lo necesitaba, o un tirón de orejas cuando me hacía falta. En fin, gracias por *todo*!

This gratitude extends to my academic siblings: Eduardo, Alessandro, Ginevra, Matilda (dude/dude's), Jesús and Roberta. Thanks for the great company during different parts of this journey. I have enjoyed a lot meeting you to discuss something, compute something, or just to figure out what was that something. Thanks for sharing your thoughts and doubts, I learnt a lot from them. Matilda, thanks for being so nice and funny. Also for our disagreements, in which we convince the other one at the same time, so at the end we are still disagreeing. All in all, I think we make a very nice academic family and hope we see each other around!

I would also like to thank the rest of the IFT string phenomenology group. I really enjoyed the journal clubs. I had so much fun that I would really get frustrated if I couldn't attend. Gracias a los seniors: Luís y Fernando, por compartir vuestra sabiduría y enseñarnos a ser críticos con los papers que se iban presentando en el journal club. Also the postdocs: Florent, Michael, Pierre, Thibaut and Mikel, for making the journal club possible (which is highly non-trivial) and for sharing your knowledge. To Florent, also thanks for our collaboration(s) and for the "Friday mandatory beers". Last but not least, thanks to the other students: Álvaro, Edu, Joan, David, Alberto, Gonzalo, Ignacio y Luca, for sharing their thoughts and doubts and making the journal clubs way more fun. Álvaro, gracias por ser tan majo y compartir con todos nosotros tu siempre magnífico punto de vista. Y sobre todo gracias por las charlas sobre la vida en la academia y más allá de ella, tu modo de verlo me ayudó y sigue ayudando mucho. Alberto, gracias por interesarte en comentar conmigo las preguntas que te rondan la cabeza, "de locos".

También me gustaría extender este agradecimiento al resto de miembros del IFT. Las pequeñas interacciones son las que hacen un día de trabajo más agradable. En este aspecto me gustaría destacar al personal de limpieza y de recepción, por saludarnos todas las mañanas con una sonrisa. También quiero dar gracias a las secretarias: Rebeca Alameda, Rebeca Bello, Laura, Isabel y Mónica, por ayudarnos tanto y en tantas ocasiones, y encima de buena gana. Por último, quiero agradecer a mis compañeros de MMA: Josemi, Daniel y David. Con vosotros la docencia ha sido mucho más divertida que solo dar clase.

Sin duda alguna, un aspecto destacable del doctorado ha sido la estancia en Harvard. For giving me the opportunity of living this amazing adventure, I want to thank Cumrun

Vafa. Aunque también han estado presentes en muchas otras etapas de mi doctorado, quiero agradecer aquí a Irene y a Miguel por adoptarme durante esta estancia. Por ser tan majos y darme tan buenos consejos. En general, gracias por ser una gran referencia para mí, no solo en como hacer física, sino también en como divertirse haciéndola. Como no podría ser de otra forma, estoy profundamente agradecido con mi familia de Boston: Alba, Alfonso, Fer, Helena, Iván, Max y Verónica. Me lo pasé en grande con vosotros, muchísimas gracias por todos los viajes, cenas y cervezas. Max y Fer, gracias por haber sido mis padres (evitándome dormir debajo de un puente). Los fines de semana no habrían sido los mismos sin ir a vuestra casa (aunque no me prepararais tuppers). Ich bedanke mich auch bei Max für die Unterhaltungen über Physik und das Leben als Physiker.

Bueno, no todo ha dado vueltas alrededor de la física. Así que ya es hora de salir un poco del ámbito académico (aunque solo un poco, porque aún quedan muchos físicos por mencionar). En el plano personal, esta época se ha visto marcada por un duro golpe que quiso coincidir con el principio del primer año. Por si fuera poco, también nos ha tocado vivir una pandemia que nos ha obligado a encerrarnos en casa durante mucho tiempo. Menos mal que yo pasé el confinamiento con mi maravilloso compañero de piso, Jrancho. Fran y Xurxo, muchísimas gracias por todo. Sobran y faltan las palabras, sinceramente. Es imposible hacer una lista de las cosas que os agradezco, así que mencionaré solo algunas. Graziâ Franxo, por aberme acompañao tôh êttô añô y por aberme sacao ziempre una carcahá (no puedê ebitâl-lo, erê ilarante). Grazas Xurxo, por acompañarme todos estes anos e por todo o apoio que me deches (vou botar de menos os teus abrazos, aínda que non o creas).

A pesar de la pandemia, hubo algo de tiempo para pasarlo bien en el IFT y por Madrid. Y yo he tenido la suerte de hacerlo con gente maravillosa. Primero una mención especial a mis compañeros de despacho: Llorenç, Walter y Xurxo (sí, otra vez). Llorenç, gràcies per ser una persona tan interessant. Dóna gust parlar amb tu de qualsevol tema. Gràcies també per tots els moments graciosos que ens has donat (tu saps a quals em referisc). Walter, gracias por ser tan majo y por soportarnos. Además, recuerda que una lata de cerveza tiene la cantidad de energía para ir a por un pinche doce pack completo como no a la verga vámonos (pico, ya). También quiero agradecer a Alejandra, Alejandro, Ángel, Arturo, Francesca, Gabriel, Judit, Mattia, Rosa y Sergio. Sergio, gracias por todas las historias tan graciosas que nos cuentas en los almuerzos. Judit, gràcies per mai haver menyspreat els meus problemes, encara que clarament foren drames del primer món.

Pero no he pasado todo mi tiempo en Madrid. De vez en cuando volvía a Málaga, donde quedaba con María y Rolando para no parar de hablarles de mi tesis y otros temas relacionados. Muchas gracias a ambos por ser tan buenos amigos. Aunque no he vuelto a ir mucho por Granada, uno no olvida la carrera ni la gente con la que la hizo. Además he tenido la suerte de que algunes sigan muy presentes durante el doctorado, así que muchas gracias Blanca, Olalla y Fran (sí, otra vez). Gracias Blanca por ser la mejor compañera de laboratorio del mundo, además de una persona maravillosa. Y gracias a Olalla por ser tan divertida, no hay manera de aburrirse contigo. También quiero hacer mención especial a Sonia. Gracias por hacerme compañía en los momentos más duros de mi vida y por seguir siendo una muy buena amiga.

Otro aspecto muy importante para mí ha sido el kárate. Ir a clase ha sido sin duda la mejor forma de desfogar y desconectar. Esto es aún mejor cuando compartes dojo con un montón de gente maravillosa. Así que muchas gracias a todos los karatekas del dojo Zentrum, él ambiente que propiciáis es excelente y me lo he pasado en grande. En especial

quiero mencionar al sensei, Luis. Gracias por ser tan amable y por cuidar tanto de todos tus alumnos. Hortaz gain, karate da Marta ezagutu neben lekua. Milesker dojo hau eta ez beste edozein guztiz ausaz aukeratziatik. Milesker zure laguntziatik eta neri hainbeste zainketiatik. Eta milesker udaberrixak askoz polittauak ittiatik

Por último quiero dar las gracias a las dos personas más importantes de mi vida. Papá y Tata, muchas gracias por estar ahí. Muchas gracias Papá por haberme inculcado el amor por el conocimiento y por haberme alentado siempre a seguir con mi carrera científica. Muchas gracias Tata por hacerme tanto de reir y por haberme dado tan buenos momentos. Gracias a ambos por insistir en llamarme aunque yo no pare de deciros "ando muy liado". Y, sobre todo, muchas gracias por preocuparos por mí más que nadie.

Esta tesis doctoral está basada en los siguientes artículos:

[1] *Transplanckian axion monodromy!?*,
G. Buratti, **J. Calderón** and A. M. Uranga,
JHEP **05** (2019) 176, [arXiv:1812.05016]

[2] *Tackling the SDC in AdS with CFTs*,
F. Baume and **J. Calderón Infante**,
JHEP **08** (2021) 057, [arXiv:2011.03583]

[3] *The Convex Hull Swampland Distance Conjecture and Bounds on Non-geodesics*,
J. Calderón-Infante, A. M. Uranga and I. Valenzuela,
JHEP **03** (2021) 299, [arXiv:2012.00034]

[4] *Dynamical Cobordism and Swampland Distance Conjectures*,
G. Buratti, **J. Calderón-Infante**, M. Delgado and A. M. Uranga,
JHEP **10** (2021) 037, [arXiv:2107.09098]

[5] *At the End of the World: Local Dynamical Cobordism*,
R. Angius, **J. Calderón-Infante**, M. Delgado, J. Huertas and A. M. Uranga,
Pendiente de publicación en JHEP, [arXiv:2203.11240]

Abstract

The goal of this thesis is to push the limits of the Swampland Distance Conjecture (SDC). This is one of the most relevant conjectures in the Swampland program, whose aim is to identify the criteria that an effective field theory must satisfy in order to be consistently coupled to quantum gravity. For this purpose, we study the SDC in two different contexts: Running solutions and AdS/CFT. After an introduction motivating the subject, we give a short review of the Swampland program. Special attention is given to the SDC and to the limitations that motivate this thesis.

We start by presenting a first example of running solution and its interplay with the SDC. It teaches us that the naive extension of the conjecture to this context fails. However, we argue that this is not a violation of the SDC in its usual formulation, but that it is protected in a non-trivial way. Motivated by these results, we propose that consistency of the SDC along the RG flow of the theory imposes constraints on the potentials that are attainable in quantum gravity. To characterize them, we consider the behaviour of the SDC along non-geodesic trajectories in field space. We first study some examples and then generalize the results by providing a geometric formulation of the SDC. It turns out to be equivalent to a convex hull condition, similar to the one appearing in a extension of the Weak Gravity Conjecture.

We then move to another type of running solutions, the so-called dynamical cobordisms. They receive this name for being the spacetime realization of cobordisms between different theories. Based on several examples in theories with dynamical tadpoles, we propose that the behaviour of scalars as one hits a cobordism wall allows for a distinction between two types of them: On one hand there are interpolating domain walls, in which scalars remain at finite field distance and after which spacetime continues. On the other hand, scalars explore infinite field distance as we approach a wall of nothing ending spacetime. For the latter case, we furthermore find some universal scaling relations between spacetime geometric quantities such as the distance to the wall or the scalar curvature and the field space distance. By performing a bottom-up effective field theory analysis of these kind of solutions, we moreover relate this universal scalings to the presence of exponential potentials at infinite field distance limits. These results suggest a relation between Cobordism, de Sitter (dS) and Distance conjectures.

Finally, we turn our attention to the SDC in the context of AdS/CFT. In all the examples we consider, infinite distance limits in the CFT turn out to be weak coupling points in which some sector decouples. The SDC tower of states from the CFT perspective is then formed by the higher-spin conserved currents that characterize these points. After reviewing the main entries of the AdS/CFT dictionary that are relevant for the conjecture, we warm up with the most well-known example of this duality in String Theory. We then perform some purely CFT analysis in the context of 4d $\mathcal{N} = 2$ theories. For those with Einstein gravity dual, we are able to show that the exponential decay rate of the tower of states is at least order one. In addition, we take a closer look to an specific example with known bulk dual in String Theory. It exhibits interesting features such as infinite field distance limits induced by quantum corrections, parametrically large exponential decay rate for the tower and intriguing candidates for the stringy object that give rise to it.

Resumen

El objetivo de esta tesis es empujar los límites de la Conjetura de la Distancia (SDC por sus siglas en inglés). Esta es una de las conjeturas más relevantes en el programa Ciénaga, que trata de encontrar criterios que una teoría de campos efectiva debe satisfacer para poder acoplarse a gravedad cuántica de manera consistente. Para ello, estudiamos la SDC en dos contextos: Soluciones dinámicas y AdS/CFT. Tras una introducción motivando el tema, daremos un corto repaso al programa Ciénaga. Prestaremos especial atención a la SDC y a las limitaciones que motivan esta tesis.

Empezamos presentando un primer ejemplo de solución dinámica y su relación con la SDC. Esta nos enseña que la extensión más directa de la conjetura a este contexto no funciona. Sin embargo, argumentamos que esto no conlleva una violación de la SDC, sino que esta se ve protegida de una forma no trivial. Con estos resultados como motivación, proponemos que el que la SDC sea consistente a lo largo del flujo de renormalización de la teoría impone ciertas restricciones sobre los potenciales posibles en teorías de gravedad cuántica. Para caracterizarlas, consideramos el comportamiento de la SDC a lo largo de trayectorias no geodésicas en espacio de campos. Primero estudiamos algunos ejemplos y luego generalizamos los resultados dando una formulación geométrica de la SDC. Esta resulta ser equivalente a una condición sobre la envolvente convexa similar a la que aparece en una extensión de la Conjetura de Gravedad Débil.

Tras esto pasamos a otro tipo de soluciones, los llamados cobordismos dinámicos. Este nombre se debe a que describen cobordismos entre distintas teorías de forma dinámica en el espacio-tiempo. Basándonos en ejemplos en teorías con renacuajos dinámicos, proponemos que el comportamiento de los escalares al alcanzar el muro de cobordismo permite distinguir entre dos tipos: Por un lado, encontramos muros de dominio en los cuales los escalares permanecen a distancia de campos finita y tras los cuales el espacio-tiempo continúa. Por otro lado, los escalares exploran distancia de campos infinita al acercarnos a un muro de la nada tras el cual deja de haber espacio-tiempo. Además, en el último caso encontramos ciertas relaciones universales entre cantidades geométricas espaciotemporales tales como la distancia al muro o la curvatura escalar y la distancia en espacio de campos. Tras realizar un análisis de este tipo de soluciones en teorías efectivas genéricas, también vinculamos estas relaciones universales a la presencia de potenciales exponenciales en límites distancia de campos infinita. Estos resultados sugieren una conexión entre las conjeturas Cobordismo, de Sitter (dS) y de la Distancia.

Por último, estudiamos la SDC en el contexto de AdS/CFT. En todos los ejemplos que consideramos, los límites a distancia infinita en la CFT resultan ser puntos de acople débil para algún subsector de la teoría. La torre de estados de la SDC desde la perspectiva de la CFT está formada por las corrientes conservadas de espín alto que caracterizan estos puntos. Tras repasar las entradas del diccionario AdS/CFT más relevantes para el estudio de la conjetura, la ponemos a prueba en el ejemplo más famoso de esta dualidad en Teoría de Cuerdas. Después llevamos a cabo un análisis centrado en la CFT, más concretamente en teorías en 4d y con $\mathcal{N} = 2$. Para aquellas con dual gravitacional Einstein, somos capaces de demostrar que la velocidad de caída exponencial de la torre de estados es como poco de orden uno. Además, estudiamos más a fondo un ejemplo específico cuyo dual gravitacional es conocido en Teoría de Cuerdas. Este muestra ciertas características interesantes, como por ejemplo distancias infinitas inducidas por correcciones cuánticas, velocidades de caída exponencial paramétricamente grandes para la torre de estados y candidatos intrigantes para el objeto cuerdoso del que surgen.

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Part I

Preliminaries

1
Introduction

Fundamental or basic research seeks to understand what happens around us. Thanks to it, today we enjoy deep knowledge about the laws of Nature. In the long run, this has led to enormous technological progress. Despite this, it is important to keep in mind that this research is not driven by economical profit, but rather by pure curiosity.

The Two Pillars of Modern Physics

Even having such a deep knowledge, we can not claim to understand everything at the fundamental level. Coming back to the end of the 19th century, one could have thought that this was the case thanks the huge successes in physics. But Nature is always more impressive than we expected. At that same time, the physicist William Thomson (Lord Kelvin) was mentioning a couple of 'clouds' that were challenging our understanding. Ultimately, these challenges were so great that required a paradigm shift and led to the two pillars of modern physics: Relativity and Quantum Mechanics. These very fascinating and counter-intuitive set of principles have granted us an even better understanding of the laws of Nature, from the very small to the very large scales.

The search for a relativistic and quantum mechanical framework led to the formulation of quantum field theory (QFT), that allows us to describe the (in our current understanding) most fundamental objects in Nature, particles. Within this framework the Standard Model of particle physics (SM) was born. This very successful model reproduces to an incredible precision all currently available experiments probing the smallest scales. It contains a set of particles as basic constituents. On one side we have matter particles, such as the electron. These are the building blocks forming all the matter that we see. On the other hand we have force mediators, responsible for transmitting the electromagnetic, the strong and the weak interactions. An example of such is the photon (the particle of light), mediator of the electromagnetic interactions. Finally we have the Higgs Boson, a very special particle that comes as a byproduct of the mechanism that generates for the rest of particles their masses. Its discovery at the LHC in 2012 [6, 7] represented the final piece of evidence for this model.

Despite its great success, we know that the Standard Model can not be the end of the story. Even ignoring its own theoretical issues, there is something that we experience in our everyday life that it is missing. Only three out of the four fundamental interactions that have been observed are described within this model. In fact, the one that is missing is the most relevant when going to the largest scales. It is the responsible for keeping us standing on the ground, or for making the apples fall from the trees to the floor. The Standard Model does not describe gravity.

Coming back to the one of the two pillars of modern physics, formulating a relativistic theory of gravity led to a very striking feature: Gravity is about the dynamics of spacetime itself. Such a theory was proposed by Albert Einstein in 1915 [8] under the name of General Relativity (GR). Among its successes, this theory is the basis of the ΛCDM model of cosmology, which describes most of the history of our Universe with astonishing precision. Furthermore, Einstein's theory predicts very fascinating objects and phenomena that after many years we have been able to observe. Such an example are gravitational waves, a form of radiation in which spacetime oscillates. They were famously observed by the Laser Interferometer Gravitational Wave Observatory collaboration (LIGO) in 2016 [9]. Another example is that of black holes. These amazing objects were found as the first solution to Einstein's equations and have played a central role in the study of the theory. Even though their existence had been confirmed in many different ways, a few years ago the Event Horizon Telescope collaboration (EHT) presented new direct evidence in the form of the first picture of a black hole [10]. Even more recently, a picture of Sagittarius A* (the black hole in the center of our own galaxy) has been achieved by the same collaboration [11].

Quantum Gravity, String Theory and the Swampland

Again, despite the great success of General Relativity, it can not be the end of the story. It is a relativistic theory of gravity, but it is not a quantum one. In fact, when regarded as a QFT it turns out to be what we call UV incomplete. This means that the theory breaks down when trying to describe very high energy processes, i.e., very small scales. It is then an effective field theory (EFT) that must be completed to something more fundamental, a theory of Quantum Gravity (QG). Even though a deep understanding of gravity at the quantum level is missing, we have some good insights coming from black hole physics. QFT arguments close to the horizon of a black hole suggest that it evaporates by releasing Hawking radiation [12, 13]. This in turn implies that these objects have an entropy controlled by the area of the horizon. This led to a very important insight into quantum gravity, the holographic principle. It states that, in a theory of quantum gravity, the number of fundamental degrees of freedom describing a system should grow with its area, and not with its volume as we are used to.

A theory describing all interactions in a single framework is said to be a theory of everything. Remarkably, we have a candidate: String Theory (ST). This very rich and yet poorly understood theory proposes a paradigm shift. The fundamental objects are no longer point-like particles but extended objects such as strings. It is thus not a QFT, but at low energies it reproduces the physics of such. The strings are so tiny that without enough energy we cannot resolve them as an extended object. Their vibrational modes are then seen as different point-like particles, and we describe the lightest ones within an effective field theory that would be ultimately completed to the full string theory.

Even though there is no experimental evidence for string theory and one may not expect to have it anytime soon, it is a very interesting theory. A good reason for this is that it is known to be a consistent theory of quantum gravity. First, it is known to be a UV complete quantum theory unlike General Relativity. Second, it always contains the graviton (particle mediator of gravity) as one of the light vibrational modes of the string. Therefore, the EFT that one obtains at low energies is coupled to gravity and is known to be completed to quantum gravity by full-fledged string theory. Regardless whether it describes our Universe or not, string theory is a fantastic arena to draw lessons about quantum gravity. Among its successes, it has been possible to reproduce the entropy of certain black holes by counting the microstates that form them in string theory [14]. Moreover, it has led to the first explicit realization of the holographic principle in the form of the AdS/CFT correspondence [15, 16]. This fascinating duality proposes that quantum gravity in AdS space in d dimensions is described by Conformal Field Theory (CFT) in $d-1$ dimensions, a theory without gravity. Even outside of string theory, this duality has motivated a lot of research and has applications in many different contexts.

A beautiful feature of string theory is that it fixes the number of spacetime dimensions as something fundamental. Unfortunately, every string theory we know predicts more dimensions than we have observed in our Universe. For instance, in the case of Superstring Theory we are forced to work with ten space-time dimensions. If this theory describes our Universe, six of them must be hidden to us in the form of compact dimensions. It turns out that this process of compactification from ten to four dimensions is highly non-unique, leaving us with a huge amount of possible 4d EFTs coming from string theory at low energies. The set of all these different effective field theories was dubbed as the string theory Landscape, and the large number of possibilities motivated the idea that 'everything is possible in string theory'. This means that any effective field theory can be found in the Landscape, this is, can be completed to a theory of quantum gravity. If this were true, then string theory (or in general quantum gravity) will not teach us anything about our Universe at low energies.

In contraposition to this way of thinking, the idea of the Swampland arises. Not everything is possible in string theory, there are EFTs that cannot be consistently completed to quantum gravity. These theories are said to belong to the Swampland [17], and it is the goal of the Swampland program to characterize the constraints that an effective field theory must satisfy so that it does (not) belong to the Swampland. Unfortunately, the lack of a framework describing quantum gravity in full glory makes it very difficult to identify these constraints. The best we can do now is to look for common patterns in EFTs that are known to be consistent with quantum gravity (for example coming from string theory) and promote them to conjectures.

These so-called Swampland conjectures are usually supported by a plethora of examples in string theory. Some others also enjoy evidence or even proofs in the context of AdS/CFT. As a complement to this top-down approach, it is of great interest to give bottom-up arguments for the conjectures. Despite their sometimes heuristic nature, they provide a good motivation based on physical principles and/or expectations about quantum gravity. For instance, basic principles of physics such as unitarity, causality and locality usually play a central role in these arguments. In addition, they also involve considerations that are more intrinsic to quantum gravity such as black hole evaporation or entropy bounds. Last but not least, there are known connections between different conjectures. Some of them can be derived from others (usually in restricted setups) or are shown to be merged in different contexts. This leaves us with a web of Swampland conjectures, rather than with a number of disconnected statements. This fact could be pointing to some underlying principle and gives support to the program as a whole.

The Swampland Distance Conjecture and its Limitations

Among these conjectures we find the Swampland Distance Conjecture (SDC), to which this thesis is mostly devoted. It is one of the most studied ones and plays a central role in the whole program, as it has been heavily tested in string theory and enjoys connections to most of the other conjectures. In what follows we describe its content without entering into technicalities. The idea is to give an intuition, leaving the precise formulation of the conjecture for later. We comment on a couple of limitations of this conjecture. In particular, the ones motivating the research presented in this thesis.

A very important concept in QFT is the vacuum or ground state of the theory. It is the state of minimum energy and things like particles can be viewed as states built over it. It can happen that this state is not unique. In fact, it there can be a set of continuously connected vacua. We call this the moduli space of the theory. Importantly, in the case of continuously connected vacua it is possible to define a notion of distance that measures how long is a trajectory between two of them.

The SDC uncovers an universal behaviour when exploring infinite distances in the moduli space of an EFT coupled to gravity. It asserts that the regime of validity in energy of the effective field theory description, beyond which it must be completed to quantum gravity, goes to zero when going to infinite distances in moduli space. In particular, it states that this happens exponentially fast in the moduli space distance. This is pretty remarkable, since there is no reason to believe that an EFT becomes less powerful when trying to describe more far away vacua.

This conjecture has very interesting phenomenological implications, since it limits the regime of validity of the effective field theory for describing processes in which different states are explored. However, it is not directly applicable to phenomenologically interesting scenarios for two reasons. First, the states that are explored in them are not the ones with minimum energy. What naturally happens is that states with less and less energy are explored as the theory tries to reach one of minimum energy. Furthermore, this happens dynamically as an observer moves in spacetime. On the other hand, the conjecture is stated as a property of the moduli space itself, without taking into account these dynamics. This is the first limitation of the SDC to which this thesis is dedicated. We push this limit of applicability by studying different types of dynamical configurations and their interplay with the SDC.

The SDC enjoys a huge amount of evidence, but lacks a fundamental reason explaining why it should be true. In addition, all this evidence comes from string theory. Since we have no proof that string theory is the only theory of quantum gravity, one could worry that the the conjecture is telling us something about string theory but not about any theory of quantum gravity. These two facts call for new perspectives on this conjecture. This is the second limitation of the SDC to which this thesis is dedicated. We give a new perspective by studying the SDC in the context of AdS/CFT. This opens up a new window for finding more evidence and understanding this conjecture.

Plan of the Thesis

This thesis is divided in six parts. Part I contains this introduction and some background material in the form of a brief review of the Swampland program in section 2. The main results are contained in parts II, III and IV. Some conclusions and outlook are presented in V. Additionally, part VI contains various appendices extending the material in the rest of the thesis.

Parts II and III are dedicated to the study of the SDC in dynamical configurations. We start part II by studying a first and specially interesting example in chapter 3. It will teach us that the naive extension of the SDC to dynamical settings does not work. Motivated by what we learn in chapter 3, we propose in chapter 4 that the SDC constrains the potentials that can appear in quantum gravity. We study them by considering the interplay between the SDC and non-geodesics (trajectories that do not minimize the distance). This will allows us to reformulate it in a way that strengthen its connection to the Weak Gravity Conjecture (WGC), another very important conjecture in the Swampland program.

Another type of dynamical configuration is studied in part III. They are particularly interesting because they represent a dynamical realization of cobordisms, the protagonist of a very profound Swampland conjecture called Cobordism Conjecture. We start in chapter 5 by studying the behaviour of the fields spaces distance in this kind of configurations. We argue that infinite field distance is related to a special type of them, the ones we call dynamical cobordisms to nothing. They are further studied in chapter 6. There we uncover a universal behaviour in a large number of examples as they explore infinite field distance, which calls for a relation to the SDC. Furthermore, we present a bottom-up effective field theory analysis that suggest a relation between this and asymptotically exponential potentials, also suggesting a link to the dS Conjecture.

Finally, in IV we investigate the realization of the SDC in AdS/CFT. It contains section 7, which starts introducing the natural tools for studying the SDC within the AdS/CFT dictionary. As a warm up, we show how the SDC is satisfied in the most well-known example of this duality in string theory. We then turn to a purely CFT analysis that gives new evidence for this conjecture through the AdS/CFT dictionary. Finally, we come back to an specific example of AdS/CFT duality coming from string theory in which we can further extend the previous analysis. Furthermore, we find that this example shows several interesting behaviours from the string theory perspective. These include infinite field distance limits induced by quantum corrections, the SDC being satisfied more radically than in the usual examples and some intriguing candidates for the stringy object responsible for it.

2

Review of the Swampland Program

In this chapter we provide a review of some aspects of the Swampland program. The main focus will be in the Swampland Distance Conjecture, as it is the main protagonist of this thesis. This review is intended to be brief and focused on topics that will be relevant for the rest of the thesis. For this reason, some very important conjectures in the Swampland program will not appear here, but will be stated when necessary along the rest of the thesis. For more complete reviews and lecture notes on the Swampland program see [18–21]. Some of the material in this chapter already appeared in the author's publication [20].

We first start introducing some generalities about the Swampland program in Section 2.1. In Section 2.2 we introduce the Swampland Distance Conjecture. A couple of limitations of this conjecture are discussed, since filling this gap is the goal of the thesis. In sections 2.3 and 2.4 we introduce the No Global Symmetries and Weak Gravity Conjecture (WGC) respectively. In both cases we discuss their relations to the SDC, as well as some refinements and generalizations that will be relevant for the rest of the thesis.

2.1 Generalities of the Swampland Program

The punchline of the Swampland program is that not every EFT (weakly coupled to Einstein gravity) can be obtained as the low energy limit of a UV complete theory of quantum gravity. It is then said that an EFT that falls in this category belongs to the Swampland [17], and the goal of the program is to stablish the criteria that an EFT should satisfy for it to be embeddable in quantum gravity. This idea is depicted in figure 2.1.

The progress made so far is in the form of several conjectures presenting candidates for these criteria. The lack of a definitive quantum gravity framework makes it impossible to prove these conjectures in full glory, but it does not mean that we cannot make any progress. There are many ways in which these constraints can be studied, here we describe five of them:

- Gathering evidence: It is very important to gather evidence for the conjectures (or disprove them). This has to be done in models that are known to be consistent with quantum gravity. The landscape of string theory vacua is the perfect playground for this purpose. In fact, all the conjectures are supported with evidence coming from string theory when they are proposed. However, the lack of a non-perturbative definition of string theory makes it impossible to prove them in full generality even within this framework. Another complementary way of gathering evidence is through AdS/CFT. The caveat is that it can only be used for testing conjectures in AdS vacua. The advantage is that we do have full non-perturbative definition of what a CFT is.

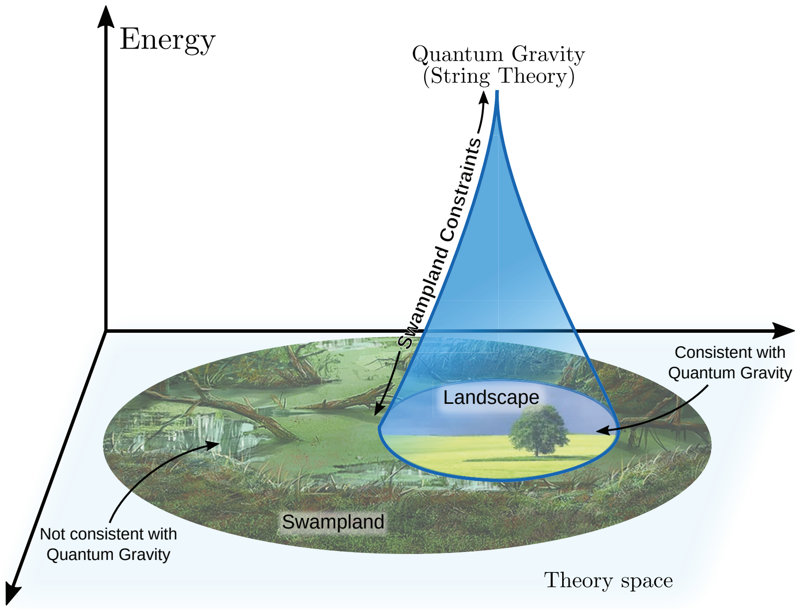

Figure 2.1: The Swampland and Landscape of EFTs. The space of consistent EFTs forms a cone because Swampland constraints become stronger at high energies. This figure is taken from [20].

This, together with the techniques that have been developed for studying them, allows us to attempt for fairly general derivations of certain conjectures within this framework (see e.g. [22–25]).

- Finding bottom-up rationales: Given that the evidence is gathered in restricted quantum gravity settings, one can worry about a "lamppost effect". This is, that the conjectures are not about quantum gravity, but only about string theory and/or AdS/CFT. For this reason, it is very appealing to look for model independent bottom-up rationales for the conjectures. Even though these are not proofs by any means, they provide a motivation based on physical principles and/or expectations about quantum gravity. Also related to this, maybe such a lamppost effect is not possible because string theory is the only theory of quantum gravity. This is called the String Universality Principle or String Lamppost Principle, and there has been progress in showing that under some restrictions in the dimensionality and number of supercharges of the theory, any EFT consistent with Swampland conjectures can be derived from string theory (see e.g. [26–32]).

- Self-consistency tests: There are operations that, given a consistent EFT should render another consistent one. The archetypical example is dimensional reduction. It is then interesting to consider the consistency of the conjectures under it. This has led to many interesting results like strong forms and refinements of the conjectures (see e.g. [33, 34]).

- Connections between conjectures: Usually some criteria are used to motivate others, or links between two conjectures are found. Thanks to this, today we have a web of Swampland conjectures rather than a series of disconnected statements. This means that evidence for some conjectures can be taken as evidence for others. In the same way, a counterexample for a single conjecture can shake the grounds of the whole program. But most importantly, these links motivate the existence of some underlying principle behind all these criteria, and gives credence to the program as a whole.

- Phenomenological implications: Finding the phenomenological implications of these criteria is a goal by itself. It could be that quantum gravity actually has some impact well below the Planck scale. This is of great interest since it can potentially lead to connections with experiments. From other perspective, this also serves as a test for the conjectures, as they could be disproved by experiments in a rather straightforward way. For a review focused on phenomenological aspects of the Swampland we refer to [21].

All these approaches (and any other that has not been mentioned here) are very interesting by themselves and help us in making progress towards the final goal of the Swampland program. Nevertheless, it is important to keep in mind that this is not a one way road map. Progress is made in a series of back and forward interactions between conjectures and any way that is available to learn about them. In fact, this makes it more exciting. As an example, a conjecture can be proposed based on relations to other conjectures and on certain amount of evidence, but it may lack a bottom-up rationale. Finding some exciting phenomenological implication motivates studying it. Finding new evidence and imposing self-consistency test may then lead to some refinement or strong version of the conjecture. After this, the relation to other conjectures and its phenomenological implications can be revisited. Moreover, one may find some bottom-up rationale based on the physical mechanisms that are relevant for the conjecture in several setups.

Thanks to all this, today we have a web of Swampland conjectures with a fair amount of proposed criteria and relations among them. Some conjectures are of course in firmer ground than others. Interestingly, it is usually the case that the more phenomenologically interesting a conjecture is, the less supported it is and viceversa. In figure 2.2, some of the most relevant criteria are presented in a map of conjectures. Among them, the No Global Symmetries, Weak Gravity and Distance conjectures are arguably the most supported ones. They are not only the very well established, but also the pillars of the whole program because of the strong relations that they enjoy among themselves and to other conjectures. They will be the main focus of this review, and other conjectures that are relevant for the rest of the thesis will be presented in relation to them.

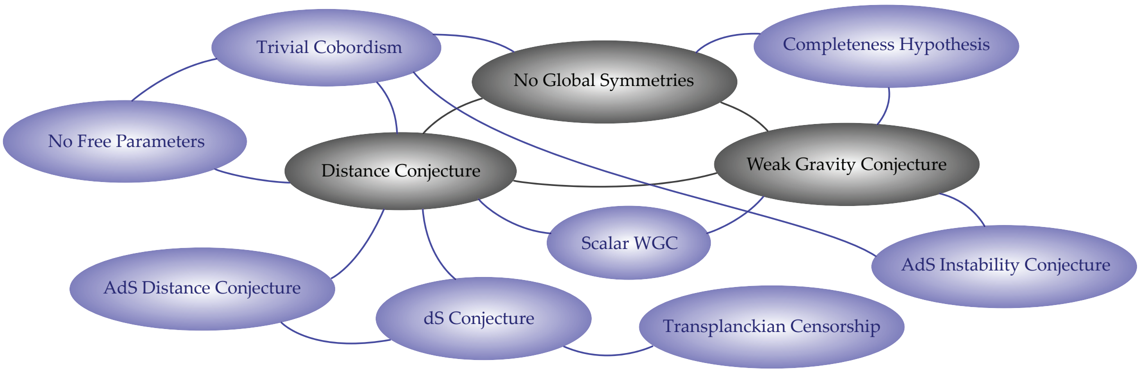

Figure 2.2: Map of the Swampland conjectures. The conjectures in black are at the core of the Swampland program, and they will be introduced in later sections. This figure is taken from [20].

2.2 The Swampland Distance Conjecture

We start with the conjecture to which this thesis is mainly devoted. In later sections we introduce other conjectures, always keeping in mind their relations to this one. Let us first give the statement, leaving the physical intuition for the discussion about evidence in string theory.

Consider a D-dimensional effective field theory coupled to Einstein gravity and with some moduli space $\mathcal{M}$ parametrized by massless scalar fields. The action includes the term

$$S \supset M_{Pl}^{D-2} \int d^D x \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j \right) , \tag{2.1}$$

where G_{ij} is identified as the metric in moduli space, measuring distances in Planck units.

The first statement of this conjecture is that the moduli space is non-compact. Starting from a point, there always exist another point at infinite geodesic moduli space distance. Note that a point is at infinite distance if every trajectory approaching the point has infinite length. The second and more important statement describes what happens if we try to approach some point at infinite moduli space distance:

Swampland Distance Conjecture: There is an infinite tower of states that becomes exponentially light at any infinite field distance limit as

$$\frac{M_{SDC}}{M_{Pl}} \sim e^{-\alpha D_\phi} \quad \text{with } D_\phi \to \infty, \tag{2.2}$$

in terms of the geodesic moduli space distance D_ϕ [35].

The exponential rate α, apart from being positive, is not specified by the conjecture. It is expected to be an $\mathcal{O}(1)$ constant, as otherwise it could spoil the exponential behaviour, but its origin is not known. Concrete lower bounds have been proposed in the literature [36–39].

The infinite tower of states signals the breakdown of the EFT, as it is impossible to have an effective field theory description weakly coupled to Einstein gravity with infinitely many light degrees of freedom. Hence, there is a QG cut-off associated to the infinite tower of states, which decreases exponentially in terms of the field distance,

$$\frac{\Lambda_{QG}}{M_{Pl}} \sim e^{-\alpha D_\phi} . \tag{2.3}$$

For simplicity, here we are taking this cut-off to coincide with the first state of the tower. An arguably more accurate way of defining it is via the species bound cut-off [40–43], whose exponential rate will differ from α by an order one factor depending on the space-time dimension and the details of the tower of states (see [44]).

The SDC enjoys large amounts of evidence in string theory, it has been heavily tested in the context of 4d $\mathcal{N}=2$ theories from Type II compactifications [45–49], 5d $\mathcal{N}=1$ from M-theory and its 6d F-theory uplifts [50–53] and 4d $\mathcal{N}=1$ from type II [39, 54, 55] and F-theory [56]. These studies not only give evidence for the SDC, but also uncover interesting connections to cutting-edge mathematics. They provide tools for describing the physics near to a plethora of infinite field distance limits beyond the one corresponding to weak coupling and large volume. As a consequence, we get a much deeper understanding of string theory compactifications.

To have a better intuition about the SDC, let us describe how it usually works in compactifications of string theory on Calabi-Yau (CY) manifolds without entering in much detail (for a detailed discussion see [20] and references therein). The moduli space is parametrized by scalars controlling the string coupling and the size and shape of the CY manifold. They control the mass of different towers of states in Planck units. For example: the string coupling controls the mass of the tower of string excitations, the size of some internal dimension controls the mass of the KK tower related to it, and the volume of some 2- or 3-cycle controls the mass of any extended object that may be wrapped on it. One or many of these towers become massless at any infinite moduli space distance limit. Their mass scale usually behaves as a power-law way in some coupling and/or size. It then happens that the moduli space line element behaves logarithmically with them, thus recovering the exponential law of the SDC.

The SDC is an example of conjecture that enjoys a refinement motivated by how it is realized in a plethora of examples that were studied to find evidence for it (see discussion in Section 2.1). Notice that the conjecture by itself does not say anything about the nature of the tower of states. The Emergent String Conjecture (ESC) fills this gap with the following statement:

Emergent String Conjecture: Any infinite distance limit is either a decompactification limit or a limit in which there is a weakly coupled string becoming tensionless [53].

Most importantly, this refinement makes more apparent the already present connection between the SDC and dualities. It can be interpreted as predicting the existence of a duality at every infinite distance limit such that the tower provides the new fundamental (weakly coupled) degrees of freedom of the dual description. Then the ESC further implies that this new weakly coupled description will involve a weakly coupled string and/or new spacetime dimensions.

To make this point clearer, let us give an archetypical example for each of these cases, namely the strong coupling limit of type II theories. The case of an emergent string is found in type IIB. At strong coupling the D1 is becoming tensionless exponentially with the field space distance. This signals a new duality frame in which it is the fundamental object. In this case one finds that it behaves exactly as the type IIB fundamental string, uncovering the famous type IIB S-duality. On the other hand, the case of a KK tower is found in type IIA. At strong coupling the tower of D0s become massless exponentially with the field space distance. Again, this signals a new frame in which these modes are fundamental. This dual frame is M-theory on S^1, and from this perspective the D0s are indeed the KK modes.

Despite the great amount of evidence gathered for this conjecture, the SDC still lacks a firm bottom-up rationale (see [57] for a proposal valid in limits with vanishing $U(1)$ gauge couplings). Combined with having only evidence for string theory, one may worry about a possible string lamppost effect as discussed in Section 2.1. For this reason, in Chapter 7 we discuss realizations of the SDC in AdS/CFT and provide evidence from a purely CFT perspective, a priori independent of string theory.

Another aspect of the SDC is its phenomenological implications. An immediate consequence is that effective field theories are only valid for finite scalar field variations. From (2.3) and taking into account that $\Lambda_0 \leq M_{Pl}$ one gets

$$D_\phi \leq \frac{1}{\alpha} \log \frac{M_{Pl}}{\Lambda_{QG}}, \tag{2.4}$$

which is telling us that the maximum field variation actually depends on the QG cut-off of the EFT. This means that the higher the energies involved in the process changing the vev of the scalar, the smaller is the maximum field distance that can be described within the effective field theory. This statement has direct implications for inflation, for which one would require the cut-off to be above the Hubble scale.

However, the SDC is stated for the moduli space parametrized by exactly massless scalars. In order to apply it to an inflationary scenario, it is necessary to understand what happens when a potential is added. Based on physical grounds, one would expect that the SDC should also apply to the valleys of the potential, i.e. to directions along which the potential may not be exactly flat but the relevant energies are smaller than a given cut-off. Furthermore, the SDC is an asymptotic statement as the field space distance goes to infinity. For phenomenological implications, it is needed to quantify how much field distance can be traversed before the exponential behaviour kicks in. Again based on physical grounds, one would expect that this happens after at most some order one distance (in Planck units). These expectations, together with the one that $\alpha \sim \mathcal{O}(1)$ stated before, are encapsulated in the refined SDC:

Refined SDC: The exponential behaviour with $\alpha \sim \mathcal{O}(1)$ should be manifest when $D_\phi \gtrsim \mathcal{O}(1)$. In addition, the conjecture should also hold for scalars with nearly flat potential [58].

The second statement of this refined version will be of special interest to us, since in chapter 4 we will argue that it leads to constraint on the potentials that are consistent with quantum gravity.

Even considering the Refined SDC, applications of the SDC to phenomenological scenarios require extending it to spacetime solutions in which scalars run, exploring large field space distances. A priori this is not trivial. For instance consider cosmological scenarios such as multi-field inflation. The field space trajectory that is explored as the cosmological time advances is not necessarily a geodesic, while the SDC as it is formulated only applies to geodesic field distance. This motivates the study of the SDC in solutions in which scalars run with some spacetime coordinate, dubbed in this thesis as running solutions. As advanced, this will be the focus of most of the thesis, including chapters 3, 5 and 6.

2.3 No Global Symmetries and Cobordism Conjectures

The No Global Symmetries Conjecture is probably the first Swampland conjecture ever made. In fact, it is hard to give the reference in which it was proposed because it has been a common lore since the early days of quantum gravity, well before the concept of Swampland was even introduced.

A global symmetry can be defined as a transformation described by a unitary local operator that commutes with the Hamiltonian and acts non-trivially on the Hilbert space of physical states.[1] The latter requires the existence of at least one charged local operator, so we say that the symmetry acts faithfully. The content of the conjecture is then that this should not be present in a complete theory of quantum gravity. This is:

No Global Symmetries: There are no global symmetries in quantum gravity (i.e. any symmetry is either broken or gauged).

Notice that in this form of the conjecture the symmetry may be broken at an arbitrarily high scale or gauged as weakly as desired. For this reason, it is not very useful for putting tight constraints on phenomenological models. Nevertheless, it has very profound theoretical implications. First, it is forbidding at the fundamental level one of the most important concepts in theoretical physics.[2] Moreover, some of the most important conjectures such as the WGC or SDC can be thought of as strong versions or consequences of the No Global Symmetries Conjecture.

Consider a theory with an $U(1)$ gauge symmetry. The limit of vanishing gauge coupling $g \to 0$ can be thought of as restoring the global symmetry. When one tries to engineer this limit in quantum gravity, it turns out to be at infinite field space distance. In some sense, it seems like quantum gravity is forbidding the restoration of a global symmetry by pushing it infinitely far away in the space of vacua. But this is not the end of the story, by virtue of the SDC an infinite tower of states become massless in this limit. From this perspective, the SDC is a mechanism that protects quantum gravity from having global symmetries. The more you restore it, the lower is the cutoff of your effective field theory. Notice however that this not a good bottom-up rationale for the SDC. The missing piece is having a good reason for the restoration of a global symmetry in all infinite field distance limits in quantum gravity. Despite this, it is a very interesting connection that gives some physical meaning to the SDC tower in this context.

The No Global Symmetries Conjecture enjoys great amounts of evidence in very different contexts. This ranges from a proof in perturbative string theory [60], non-perturbative checks in various string theory setups and even a proof in AdS/CFT under certain assumptions [22, 23]. Moreover, there is a bottom-up rationale based on black hole physics that motivates the conjecture. It can be summarized as follows: Hawking radiation is blind to any kind of global charge. This is, gravity does not distinguish between positive or negative global charge, and there is no interaction that does as in the case of gauge charges. Therefore, if a global symmetry is to be exactly conserved, the evaporation of a black hole with a certain amount of global charge has to lead to a planckian size remnant. Since this happens for any black hole with any arbitrary amount of global charge, we end up with a theory with infinite number of states below a certain energy scale. Even though there is not a rigorous argument that rules out this possibility, it certainly sounds problematic [61]. Moreover, it has been argued that this would go against holographic entropy bounds such as the Covariant Entropy Bound [62].

A natural extension of this conjecture also forbids the presence of p-form global symmetries. These are generalizations of the concept of global symmetry in which the charged operators are supported on p-dimensional manifolds. This generalized global symmetries

[1] Here we avoid entering into more formal and general definitions in terms of topological operators. For this we refer to [59].

[2] Of course, this does not imply that global symmetries are not useful and explain many things at low energies.

were introduced in [59], and we refer to it for a proper definition. Let us only mention that the $p = 0$ case recovers the usual picture of a symmetry acting on local charged operators. The main idea for extending the conjecture is that a p-form global symmetry leads to a regular one upon compactification on a p-dimensional torus. As the conjecture should hold upon compactification of the theory, one concludes that generalized global symmetries should also be absent in quantum gravity.

Let us introduce another very profound conjecture whose basis is the absence of global symmetries in quantum gravity, the Cobordism Conjecture. Two k-dimensional compact manifolds are said to be cobordant if their union is the boundary of another $k+1$-dimensional compact manifold, called cobordism manifold. The set of cobordant manifolds, together with the operation of disjoint union, form the cobordism group of k-dimensional manifolds. This notion can be refined by imposing some further structure in the two manifolds and the cobordism between them, leading to the cobordism group of k-dimensional manifolds with some structure s, denoted as Ω_k^s.

It was argued in [63] that the presence of a non-trivial cobordism group leads to a $(D-k-1)$-form global symmetry in the theory. Let us summarize the agument here: If a k-dimensional compact manifold is a consistent compactification of a D-dimensional theory, then one can use it to build $(D-k-1)$-dimensional gravitational solitons. A cobordism between two such compact manifolds can then be interpreted as a process in which a gravitational soliton turns into another. In this way, each cobordism class is translated to an invariant that cannot be changed under time evolution, i.e., a global charge. Following the logic that there cannot be p-form global symmetries in quantum gravity, we end up with the conjecture:

Cobordism Conjecture: All cobordism groups in a D-dimensional theory of quantum gravity must vanish

$$\Omega_k^{QG} = 0\,. \tag{2.5}$$

Otherwise they give rise to a $(D-k-1)$-form global symmetry with charges $[M] \in \Omega_k^{QG}$ [63].

Notice that here the superscript QG means the yet unknown structure that a background in quantum gravity has. In practice one has an effective field theory with some gauge and matter content, i.e. with some given structure. If it has some non-trivial cobordism group, the conjecture is used to predict a refinement of this structure that trivialises it. This in turn can be related to either breaking or gauging the cobordism charge. The latter is naturally related to having some condition (usually a Bianchi identity) forbidding any background with non-zero cobordism charge [64]. On the other hand, breaking it predicts the presence of defects that should trivialise it [63]. This, together with other Swampland constraints, has been used to test the string universality principle (see e.g. [30, 32, 65]).

The cobordism group encodes some notion of connectedness between manifolds. By considering compactifications of a D-dimensional theory on these k-dimensional manifolds, this also leads to a notion of connectedness between effective field theories. This is depicted in figure 2.3. The Cobordism Conjecture then encodes the expectation that all EFTs coming from quantum gravity are connected if one allows for arbitrarily high energy processes. Equivalently, the Cobordism Conjecture asserts that any effective field theory must admit the introduction of a boundary ending spacetime into nothing. This has been used in [66] to argue that there is no topological obstruction for bubbles of nothing (see [67]) in any effective field theory coming from quantum gravity.

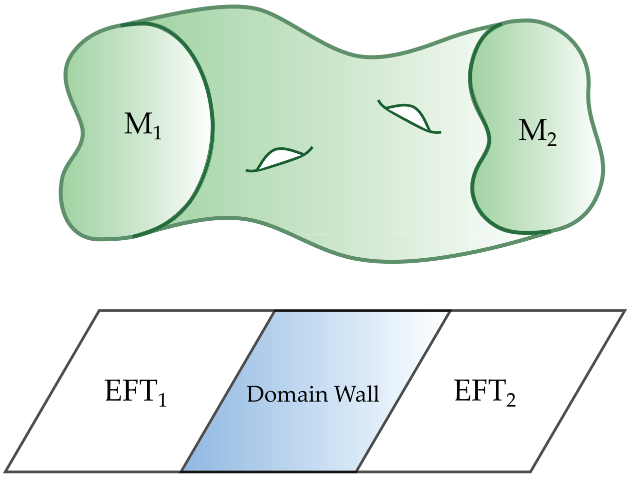

Figure 2.3: Cobordism between two manifolds. The EFTs arising from compactifications on them are then connected by a domain wall. This picture is taken from [20].

In chapters 5 and 6 we will study running solutions featuring cobordisms in a dynamical way. We will find some interesting connections between these dynamical cobordisms and field space distances. We will relate infinite field distance to cobordisms to nothing, suggesting some connection between the SDC and the Cobordism Conjecture.

2.4 The Weak Gravity Conjecture

The Weak Gravity Conjecture can be viewed as a strong version of the No Global Symmetries. One of the implications of the latter is that limits in which some gauge coupling goes to zero should be forbidden in some way. However, it does not impose any strict bound on the gauge coupling g. The WGC puts a constrain on the spectrum of charged objects with respect to any $U(1)$ gauge symmetry and to the cut-off of the EFT. The smaller is the gauge coupling, the stronger these constraints become. Thus, it realizes in a quantitative way the fact that $g \to 0$ limits should be obstructed in quantum gravity. In what follows we introduce the conjecture and some generalizations and refinements that will be relevant for the rest of the thesis. For an extensive review dedicated to this conjecture see [68].

The WGC has an electric and a magnetic version. The first one states:

Electric Weak Gravity Conjecture: Given a $U(1)$ gauge theory, weakly coupled to Einstein gravity, there exists an electrically charged state with

$$\frac{Q}{m} \geq \frac{\mathcal{Q}}{M}_{\text{extremal}} = \mathcal{O}(1) \tag{2.6}$$

in Planck units. Here $\mathcal{Q}$ and M are the charge and mass of an extremal black hole, and

$$Q = qg\,, \tag{2.7}$$

where q is the quantized charge of the state and g is the gauge coupling [69].

Unlike the SDC, this conjecture has no unknown parameters. The extremal charge to mass ratio is computed by looking for extremal black hole solutions in the theory. It is model dependent since, for example, it gets contributions coming from massless scalars controlling the kinetic term of the $U(1)$ (see e.g. [36,70]). This very precise formulation is due to the bottom-up rationale based in black hole physics that was used to derive it in the first place. It is obtained by imposing that extremal black holes should be able to decay. Otherwise they would lead to an infinite number of stable remnants, which is argued to be pathological as it was done for the motivation of the No Global Symmetries Conjecture. In fact, (2.6) is nothing but the kinematical condition for an extremal black hole to be able to emit the particle while remaining subextremal.

Another interpretation for this conjecture, and the one giving it its name, is that gravity should be the weakest force. Indeed, if the inequality (2.6) is satisfied then gravity is not stronger than the electric repulsion between a pair of the particles. This has been argued to be necessary to avoid the formation of an arbitrary number of bound states. We should note however that the precise bound obtained by this requirement is in general different from the one in (2.6). We would then distinguish between the WGC and the Repulsive Force Conjecture [70,71]. Interestingly, it has been argued that in asymptotic limits in field space these two actually coincide [36].

Now we turn to the magnetic version. It asserts that:

Magnetic Weak Gravity Conjecture: The EFT cut-off Λ is bounded from above by the gauge coupling as [69]

$$\Lambda \lesssim g M_{Pl}^{(D-2)/2} \,. \tag{2.8}$$

This is indeed obtained as the magnetic dual of the electric WGC, in the sense that the gravitational interaction should be weaker than the magnetic one. This requires the existence of a magnetic monopole whose mass is smaller than its charge in Planck units. Taking into account that the magnetic coupling is the inverse of the electric one and that the mass of the magnetic monopole is of the order of the cutoff divided by the gauge coupling squared one arrives to (2.8). Equivalently, it can also be obtained by imposing that the theory should have some monopole that is not a black hole. This is, that the radius of the monopole should be larger than its Schwarzschild radius.

The WGC is probably one of the best studied conjectures in the Swampland programs. It enjoys evidence coming from a variety of different approaches. These include: Higher derivative corrections to black holes and positivity bounds coming from unitarity and causality (see e.g. [72–76]), relations to weak cosmic censorship [77], AdS/CFT (see e.g. [24,25,78–80]) and, of course, string theory compactifications (see [19] for a summary and references therein).

As said before, the two versions of the WGC obstruct in some sense the $g \to 0$ limit in quantum gravity. However, they are not sufficiently strong to put a QG cutoff in the sense that no EFT (weakly coupled to Einstein gravity) will be valid above that energy scale. This was the case for the SDC because it predicts an infinite tower of states. In the case of the electric WGC one only needs to include a charged particle, something that of course can be done withing effective field theory. For the magnetic version, the cutoff is related to the fact that one cannot describe the dynamics of magnetic monopoles in a $U(1)$ theory. However, beyond that cutoff one could realize that the $U(1)$ comes from the spontaneous symmetry breaking of a gauge symmetry. In that case there is an effective field theory description which is valid above the magnetic WGC cutoff.

Nevertheless, there are stronger versions of the WGC that are powerful enough to set a QG cutoff. These are the sublattice [24, 25, 33] and tower [75] versions. They impose that there should be an infinite tower of states satisfying the electric WGC. In the case of the sublattice version, it is further imposed that they should fill a sublattice of the charge lattice. Let us mention that there is a potential counterexample to the Sublattice WGC, but further analysis would be needed to confirm it [51]. These two versions also enjoy a profound connection to the SDC. It has been confirmed in many setups that they are essentially the two sides of the same coin. This happens because some gauge coupling g goes to zero at infinite distance in field space, and the same tower is the responsible for satisfying both the SDC and Tower/Sublattice WGC [36, 81]. Interestingly, in this case one can fix the parameter α in the SDC in terms of the massless scalars contribution to the charge to mass ratio of extremal black holes appearing in the WGC.

There are also a couple of very natural generalizations to the WGC. The first one extends it to theories with multiple $U(1)$ gauge symmetries. It comes from the same physical principle as the WGC using black holes physics. Imposing that extremal black holes with several $U(1)$ charges should be able to decay one obtains:

Convex Hull WGC: For multiple $U(1)$ gauge fields, the WGC is satisfied if the convex hull of the charge to mass ratio $\vec{z} = \vec{Q}/m$ of all the states contains the extremal region [82].

The extemal region is obtained by looking for multi-charged extremal black hole solutions in the theory and can take diverse shapes (see e.g. [36]). Interestingly, the Convex Hull WGC is stronger than just imposing that the WGC should hold for each $U(1)$ separately. For example, having a extremal state for each $U(1)$ would suffice for satisfying the latter but would clearly violate the former.

In chapter 4 we will formulate a Convex Hull version for the SDC, similar in spirit to the Scalar WGC proposed in [71]. This formulation of the SDC strengthen the connection with the WGC that was mentioned before.

The second generalization extends the WGC to p-form gauge symmetries. This is precisely what is obtained by gauging the p-form global symmetries that were mentioned in section 2.3. The charged objects are in general extended, in particular their worldvolume is $(p+1)$-dimensional. For example, a string can be charged under a 1-form gauge symmetry, whose gauge field is a 2-form. Then, the WGC asserts that there should exist some charged state whose tension to charge ratio is smaller than the one corresponding to extremal p-branes in the theory.

For the case $p = -1$ we have the so-called Axion WGC. In this case we have 0-form gauge fields, this is, axions. The charged objects are instantons and the analogous to the mass and gauge coupling are the action S and the inverse of the axion decay constant f respectively. The conjecture then implies that:

Axion WGC: In the presence of an axion, there should be an instanton with charge q satisfying

$$S \lesssim q\frac{M_{Pl}}{f}\,. \tag{2.9}$$

Notice that we have replaced the strict inequality in terms of the extremal charge to mass ratio of the black object by an inequality up to an order one factor. This is due to the lack of a well-defined notion of extremality for instantons (see [33, 83, 84] for a couple of proposals). It also looses the bottom-up rationale in comparison with the usual WGC. Despite this, it has been tested in several scenarios (see e.g. [85–96]).

An interesting aspect of the Axion WGC is its implications for models of natural inflation. In this scenarios the inflaton is an axion rolling down a non-perturbative potential generated by instantons. This potential is computed in the diluted gas approximation, which requires that $S \gtrsim 1$ for all instantons in order to keep control of the instanton expansion. A property of these models is that the maximum displacement during inflation is bounded by the periodicity of the potential, given by the axion decay constant f. Putting these two bounds together with the Axion WGC we recover a bound on the maximum field excursion during inflation of the form

$$D_\phi \lesssim f \lesssim M_{Pl}\,. \tag{2.10}$$

This means that if the instanton giving the leading contribution to the potential is the one satisfying the Axion WGC the field excursion cannot be transplanckian. Notice that we have assumed this instanton to have unit charge, otherwise the field range available for inflation would be even smaller. In conclusion, the Axion WGC disfavours large field natural inflation scenarios. A loophole to this argument should be noted: It could happen that the (unit charge) instanton giving the leading contribution to the potential is not the one satisfying the Axion WGC, but it is another one with charge q. In this case the bound is relaxed by a factor of q. This shows the relevance of finding how large can be the charge of the WGC conjecture satisfying state in a theory.

An alternative to natural inflation that also involves an axion as the inflaton is the so-called axion monodromy inflation [97], see also [98–105]. In this models, the axion periodicity is unfolded in the potential so that it does not limit the maximum field excursion. The periodicity of the axion is still respected by having a multi-branched potential. In fact, fixing a branch of the potential can be understood as the spontaneous symmetry breaking of the 0-form gauge symmetry corresponding to the axion periodicity.

There are also partial studies trying to rule out transplanckian excursions in axion monodromy models by invoking the backreaction on the scalar kinetic terms reducing the effectively traversed distance [106].[3] There are also discussions ruling out particular models using 10d lifts [108, 109] or other mechanisms [110]. In Chapter 3 we show that this is not conclusive by presenting a fully backreacted transplanckian axion monodromy model and its interplay with the SDC when considered in running solutions.

[3] For other discussions of backreaction related to flattening of the potential, see [107].

Part II

Transplanckian Field Ranges and Non-geodesics

3
Transplanckian Axion Monodromy

In this chapter we present a first type of running solution exploring infinite field distance and its interplay with the SDC and other Swampland conjectures. Not only the former constrain the possibilities to attain field ranges larger than the Planck scale. As reviewed in section 2.4, some mechanisms constraining transplanckian excursions in natural inflation and axion monodromy models have been considered in the literature. These results would seem to motivate that transplanckian field ranges are not physically attainable in Quantum Gravity. If correct, this statement would have profound implications for certain phenomenological applications, like the construction of inflation models with sizable gravitational wave backgrounds (which for single-field inflation are directly related to the distance traversed by the inflaton).

In what follows we prove that this is in fact incorrect, and that transplanckian field excursions are physically realized in string theory. Indeed, the running solution that we consider is a completely explicit example of axion monodromy model exploring infinite field distance, with full backreaction taken into account in terms of the complete 10d supergravity solution. The complete background turns out to be given by a simple and well-known warped throat, the Klebanov-Strassler throat [111, 112], when regarded as a flux compactification on a Sasaki-Einstein manifold $\mathbf{X}_5$, with a 5d axion rolling in the radial direction of a (locally) AdS_5 spacetime.

The dynamics of the transplanckian axion can be described within an effective field theory, which we discuss explicitly based on a consistent truncation provided in [111]. This, together with the full 10d solution, allows for a discussion of the validity of effective actions for the transplanckian excursion. We show that the configuration is free from oftentimes feared problems: no pathology arises neither when the axion winds its period a large number of times, and no infinite tower of states becomes exponentially light as we explore infinite distance in field space. As a consequence, this represents a counterexample to the naive extension of the SDC to running solutions stating that the exponential behaviour of the tower should be dynamically realised in all of them. As we will see, the falloff of the tower is delayed by having a strongly non-geodesic trajectory in field space. In turn, this will be tightly related to the transplanckian axion, since it is the responsible of both delaying the tower and having a non-geodesic trajectory.

Freund-Rubin vacua such as $AdS_5 \times \mathbf{X}_5$ with 5-form flux on $\mathbf{X}_5$ are often described as not yielding good effective field theories, since the compactification radius is comparable to the AdS radius. However, we are not interested in describing an effective field theory which describes the stabilization of the compactification breathing mode, which cannot be decoupled (in the Wilsonian sense) from the KK tower of states. We are interested in the effective dynamics of a parametrically less massive field giving the running in field space.

Our effective theory is suitable for that purpose, and can be regarded as describing the low energy dynamics of a scalar in a gravitational background which is fixed at higher scales, save for backreaction effects which are duly included in the effective field theory description.

We focus on 5d models because the kinds of Klebanov-Strassler throats we need (either for the conifold or for generalizations) have been most studied in this setup. On the other hand, there are less studied but completely analogous throats based on locally $AdS_4 \times \mathbf{X}_7$ configurations in M-theory, which we also discuss and lead to 4d transplanckian axion monodromy configurations in precisely the same fashion as the 5d models.

We work in configurations with negative vacuum energy. This is not an obstruction from the fundamental viewpoint of establishing the existence of transplanckian field excursions in string theory. On the other hand, it does not yield realistic models for inflation. Related to this, our configurations have scalars depending on spatial directions, rather than time-dependent ones. In fact, formally the sign flip required to switch from space to time dependent scalar profiles correlates with the sign flip for the vacuum energy. This suggests a tantalizing link between positive cosmological constant and time dependent background, which in the present context is reminiscent of the dS/CFT correspondence [113].

The rest of the chapter is organized as follows. In Section 3.1 we describe the KS solutions from the perspective of producing 5d axion monodromy models, focusing on the conifold example. In section 3.1.1 we describe the 5d compactification on $\mathbf{X}_5$ with no 3-form fluxes, leading to the AdS_5 vacuum. In section 3.1.2 we describe the KS solution [112] (actually, its KT asymptotic form [111]) and in section 3.1.3 we establish that it describes an axion monodromy solution in which the field range traversed is arbitrarily large, in particular transplanckian. In section 3.1.4 we relate hypothetical backgrounds with finite axion field ranges with duality walls in the UV of the holographically dual field theories, which have so far not been shown to admit a gravitational description. In Section 3.2 we turn to the effective field theory description. In section 3.2.1 we review the effective field theory in [111] for the axion and compactification moduli. In section 3.2.2 we obtain an effective action at energies hierarchically below the KK scale, which actually encodes the axion dynamics and its backreaction effects. In section 3.3 we discuss 4d configurations from M-theory compactifications, with exactly the same axion monodromy physics as the previous 5d examples. Finally, we give a summary of the chapter in section 3.4. Appendix A discusses a dual Hanany-Witten configuration of D4- and NS5-branes useful to illustrate the absence of pathologies as the axion winds around its period.

The content of this chapter already appeared in the author's publication [1]. Here it has beeen adapted with slight modifications for a better fit in the context of this thesis.

3.1 Warped throats and transplanckian axion monodromy

In the following we review the Klebanov-Strassler (KS) throat [112]. We intentionally emphasize its structure as a 5d compactification in which the introduction of the RR 3-form flux yields a 5d axion monodromy model, for which the KS throat is an explicit fully backreacted solution. We then show that the axion roll in this configuration is transplanckian. Actually, for this purpose it suffices to focus on the region far from the tip of the throat, so we use the simpler expressions of the Klebanov-Tseylin (KT) throat [111], supplemented with the boundary conditions derived from the KS smoothing of its naked singularity. For the latter reason, we still refer to the configuration as KS throat.

3.1.1 The 5d theory

Consider as starting point the type IIB Freund-Rubin $\mathrm{AdS}_5 \times T^{1,1}$ background

$$ds^2 \,=\, R^2\,\frac{dr^2}{r^2} \,+\, \frac{r^2}{R^2}\,\eta_{\mu\nu}dx^\mu dx^\nu \,+\, R^2\,ds^2_{T^{1,1}} \tag{3.1}$$

with

$$R^4 \,=\, 4\pi(\alpha')^2 g_s N \tag{3.2}$$

and with N units of RR 5-form flux through $T^{1,1}$. The type IIB complex coupling is constant, and we will keep it set at $\tau = i/g_s$ (introduction of non-trivial constant C_0 is straightforward via minor changes in the fluxes below).

This is the near horizon limit of a set of N D3-branes at a conifold singularity [114]. The line element $ds^2_{T^{1,1}}$ corresponds to a (unit volume) 5d horizon $T^{1,1}$, which is an $\mathbf{S}^1$ bundle over $\mathbf{P_1} \times \mathbf{P_1}$ with first Chern classes $(1,1)$, hence the name. Topologically, it is an $\mathbf{S}^2 \times \mathbf{S}^3$. Denoting by σ_2 and σ_2' the volume forms of the two $\mathbf{P_1}$'s, we have a harmonic 2-form $\omega_2 = \sigma_2 - \sigma_2'$ and its (dual in $T^{1,1}$) harmonic 3-form ω_3. They are Poincaré duals of the 3- and 2-spheres, and $\omega_2 \wedge \omega_3$ is the volume form on $T^{1,1}$.

On top of the complex dilaton, the resulting effective 5d theory has a massless axion, given by the period of the NSNS 2-form over $\mathbf{S}^2 \subset T^{1,1}$

$$\oint\!\!\!\int_{\mathbf{S}^2} B_2 = \phi \quad \text{namely} \quad B_2 \,=\, \phi\,\omega_2. \tag{3.3}$$

The periodicity $\phi \sim \phi + 1$ is set by the exponential of the action of a fundamental string wrapped on the $\mathbf{S}^2$. Above the scale of massless fields, there is the scale $1/R$. This is the scale of KK modes, but also the scale of stabilization of the breathing mode of $T^{1,1}$. It is possible to write an effective action for this dynamical mode[1]; in this action, the potential is minimized at the value (3.2), and with a negative potential energy cosmological constant, such that the maximally symmetric solution is the AdS_5 space in (3.1). For a simplified discussion in the completely analogous case of $\mathrm{AdS}_5 \times \mathbf{S}^5$, see [115]; we will discuss such effective actions in a more general context later on.

The above background is a particular case of the general class of $\mathrm{AdS}_5 \times \mathbf{X}_5$ vacua, where $\mathbf{X}_5$ is a Sasaki-Einstein variety. These are gravitational duals to systems of D3-branes at singularities, and have been intensely explored in the literature. Large classes of these models admit also the introdution of 3-form fluxes to be described below, and thus lead to axion monodromy models. To emphasize this direct generalization, we will oftentimes write $\mathbf{X}_5$ instead of $T^{1,1}$.

[1] Since this scale is not hierarchically lower than the KK masses, this effective action should be interpreted as arising from a consistent truncation, rather than a Wilsonian one.

3.1.2 The KS solution

Once we have described the compactification to 5d, we would like to describe the introduction of a RR flux on $\mathbf{S}^3 \subset T^{1,1}$

$$\frac{1}{(2\pi)^2\alpha'} \oint_{\mathbf{S}^3} F_3 = M. \tag{3.4}$$

Our key observation is that the resulting 5d theory is an axion monodromy model for ϕ. This simply follows because the self-dual 5-form field strength

$$\tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \tag{3.5}$$

satisfies the modified Bianchi identity

$$d * \tilde{F}_5 = d\tilde{F}_5 = H_3 \wedge F_3. \tag{3.6}$$

From the KK perspective the flux (3.4) induces a 5d topological coupling

$$\int_{10d} F_3 \wedge B_2 \wedge F_5 \quad \longrightarrow M \oint_{5d} \phi \, F_5. \tag{3.7}$$

As already noted in [103, 116] [2], this is a 5d version of the Dvali-Kaloper-Sorbo term [118, 119] associated to a monodromy for the axion. Clearly, as ϕ winds around its basic period, there is a corresponding increase for the flux of $\tilde{F}_5$ through $T^{1,1}$ (and, by self-duality, through the non-compact 5d space), as follows,

$$N = \oint_{T^{1,1}} \tilde{F}_5 = N_0 + M\phi. \tag{3.8}$$

In the following we take the reference value N_0 to be reabsorbed into a redefinition of ϕ.

The presence of a scalar potential of the axion monodromy kind, arising from the reduction of the 10d $|\tilde{F}_5|^2$ terms, will be manifest in the 5d effective action discussed in Section 5.3. We are interested in the behaviour of this theory as the value of ϕ changes over a large range. Clearly, the presence of this potential term implies that moving the scalar vev adiabatically away from the minimum leads to off-shell configurations, for which the computation of the backreaction is not clearly defined. A natural solution is to instead consider configurations in which the scalar ϕ is allowed to roll, so that the spacetime dependent background allows to remain on-shell[3]. The KS solution is precisely an explicit 10d solution of this rolling configuration in which the axion ϕ is allowed to roll along one of the *spatial* directions. (As discussed in the introduction, the realization of time dependent roll suggests an interesting interplay with the question of realizing de Sitter vacua). We now review the 10d KS solution (actually, its KT limit with KS boundary conditions) from this perspective.

The KS throat describes a configuration in which the axion has a dependence on the radial direction. Concretely, ϕ is a harmonic form in the radial direction in the underlying AdS_5, hence

$$\Delta\phi = 0 \quad \rightarrow \quad \phi \sim M \log r. \tag{3.9}$$

[2]While finishing this paper, we noticed the recent [117], which involves a similar structure of flux and axion, albeit in a different approach to axion monodromy.

[3]This is in fact a natural viewpoint in inflationary axion monodromy models, in which the interesting solutions correspond to physical time-dependent rolls of the scalar down its potential.

This corresponds to the fact that the combination $G_3 = F_3 - \frac{i}{g_3} H_3$ is imaginary self-dual, and in fact (2,1) i.e. supersymmetry preserving, when regarded as a flux in the conifold CY threefold $\mathbf{X}_6$, i.e. when combining the radial coordinate r with the angular manifold $T^{1,1}$. The metric then simply corresponds to a warped version of $M_4 \times \mathbf{X}_6$ of the general class in [120,121]

$$ds_{10}^2 = h^{-1/2}(r) dx_n dx_n + h^{1/2}(r)(dr^2 + r^2 ds_{T^{1,1}}^2) \tag{3.10}$$

with

$$h(r) = \frac{1}{4r^4} M^2 \log \frac{r}{r_*} \tag{3.11}$$

with r_* some reference value. In short, the metric is of the form (3.1) with the radius (3.2) including a radial dependence

$$N \sim M^2 \log r, \tag{3.12}$$

which follows from (3.8). As explained, this is the KT solution, which has a naked singularity at $r \to 0$. The KS solution provides a smoothing of this based on the deformed conifold[4]. In fact we will be interested in the region of large r, and how it extends to infinity, so the KT solution suffices.

The above solution describes precisely all the effects of the backreaction for arbitrarily large values of the axion and number of windings along its period. As one moves towards large r, the axion is climbing up its potential and inducing larger flux N due to the monodromy. The flux and stored energy backreact on the stabilization of the breathing mode of the compactification space, whose minimum tracks the value of ϕ from (3.2), (3.8) and (3.12)

$$R^4 \sim g_s M \phi \sim g_s M^2 \log r. \tag{3.13}$$

The non-compact geometry is locally AdS$_5$ with varying radius R. Hence, there is also a backreaction in the vacuum energy, with runs towards less negative values as

$$V_0 \sim (\log r)^{-1}. \tag{3.14}$$

The slow growth of the vacuum energy can be regarded as a flattening of the potential, albeit different from the polynomial ones in [107].

From the holographic perspective, each winding of ϕ on its period corresponds to a cycle in the cascade of Seiberg dualities, in which, as one moves to the UV (larger r), the effective number of colors increases by (actually twice) a factor M

$$SU(N_0) \times SU(N_0 + M) \to SU(N_0 + 2M) \times SU(N_0 + M) \to \\ \to SU(N_0 + 2M) \times SU(N_0 + 3M). \tag{3.15}$$

[4]When regarded from the 5d perspective, this implies that the direction r "ends" at a finite distance. Of course this is not relevant for the discussion below, which only deals with the large r regime. Moreover, even if one would be interested in having a radial dimension with no end, it is straightforward to modify (3.11) or even its full KS version, e.g. by introducing a large number P of additional explicit D3-branes, producing an AdS$_5$ at the bottom of the KS throat, effectively removing the endpoint for r. This corresponds to the mesonic branches of the cascade [122].

Although we will not exploit this holographic picture (as the supergravity solution speaks for itself), we will use it in Appendix A to explain why no disaster arises when the axion rolls around its period[5]. In particular there are no states becoming massless or light as one crosses the "zero" value, an effect often feared to play a lethal role for the discussion of monodromy dynamics in effective field theory. The fact that this effect is absent in our model supports the expectation that it is not a generic problem of axion monodromy models (but rather, either of particular models realizing the idea, or of partial analysis of those models without full inclusion of backreaction).

3.1.3 Transplanckian axion field range

Let us use the above solution to quickly show that the 5d field ϕ traverses a transplanckian distance in field space. A more systematic discussion is presented in Section 5.3.

The distance traversed by ϕ from a reference point r_0 to infinity is given by

$$D_\phi \,=\, \int_{r_0}^{\infty} \left(G_{\phi\phi}\, \frac{d\phi}{dr}\, \frac{d\phi}{dr} \right)^{\frac{1}{2}} dr \,=\, \int_{r_0}^{\infty} \left(G_{\phi\phi} \right)^{\frac{1}{2}} \frac{d\phi}{dr}\, dr, \tag{3.16}$$

where $G_{\phi\phi}$ is the metric in field space, which is determined by the 5d kinetic term for ϕ, in the 5d Einstein frame

$$S_5 \,=\, \frac{1}{2\kappa_5^2} \int d^5x \, \sqrt{-g_5} \left(R_5 \,-\, G_{\phi\phi}\, \partial_m \phi\, \partial_n \phi\, g^{mn} \right). \tag{3.17}$$

Since the compactification volume varies, certain care is required. We must define a fixed reference radius R determining the 5d Planck scale, and introduce a 5d dynamical breathing mode $\tilde{R}$ encoding any variation (see [115] for a similar parametrization). Hence, focusing just on the parametric dependence, we write

$$V_{\mathbf{X}_5} \,=\, R^5\, \tilde{R}^5, \tag{3.18}$$

$$ds^2 \,=\, g^{(5)}_{mn}\, dx^m\, dx^n \,+\, (R\tilde{R})^2\, (g_{\mathbf{X}_5})_{ij}\, dy^i\, dy^j. \tag{3.19}$$

We now focus on the reduction on $\mathbf{X}_5$ of the 10d action for the metric and kinetic term of B_2. In the 10d Einstein frame we have

$$S_{10d} \,=\, \frac{1}{2\kappa_{10}^2} \int d^{10}x\, \sqrt{-g_{10}} \left(R_{10} \,-\, \frac{1}{12\, g_s}\, H_{MNP}\, H^{MNP} \right). \tag{3.20}$$

As explained, the reference value R fixes the 5d Planck scale

$$\frac{R^5}{2\kappa_{10}^2} = \frac{1}{2\kappa_5^2} \tag{3.21}$$

and the factor $\tilde{R}^5$ is reabsorbed by rescaling the 5d metric to the 5d Einstein frame

$$(g_5)_{mn} \,\to\, \tilde{R}^{-\frac{10}{3}}\, (g_5)_{mn}. \tag{3.22}$$

[5]See [99] for some discussion of periodic effects in axion monodromy.

We follow the effect of this rescaling in the kinetic term of the component of B_2 given by (3.3). The dependence on $\tilde{R}$ is as follows:

$$\int d^{10}x\,\sqrt{-g_{10}}\,g^{mn}g^{ik}g^{jl}\partial_m B_{ij}\partial_n B_{kl} \xrightarrow{\text{compact.}} \tag{3.23}$$
$$\xrightarrow{\text{compact.}} \int d^5x\,\sqrt{-g_5}\,(R\tilde{R})\,(g_5)^{mn}\,\partial_m\phi\,\partial_n\phi \xrightarrow{\text{Einstein}} \int d^5x\,\sqrt{-g_5}\,(R\tilde{R}^{-4})\,(g_5)^{mn}\,\partial_m\phi\,\partial_n\phi\,.$$

Hence, we have $\tilde{R}^4 \sim M^2 \log r$ and thus

$$G_{\phi\phi} \sim (\,M^2\,\log r\,)^{-1}. \tag{3.24}$$

We have $\phi \sim M \log r$, hence the distance (3.16) is

$$D_\phi \;=\; \int G_{\phi\phi}^{\frac{1}{2}}\,\frac{d\phi}{dr}\,dr \;\sim\; \int dr\,(M^2\,\log r)^{-\frac{1}{2}}\,M\,\frac{dr}{r} \;=\; \int \frac{ds}{s^{\frac{1}{2}}} \tag{3.25}$$

for $s = \log r$. This becomes arbitrarily large for large r, showing that the 5d scalar ϕ rolls through a transplanckian distance in field space.

The 10d backreacted solution for this transplanckian axion monodromy configuration allows to address many of the objections to transplanckian field excursions in string theory or quantum gravity, and study how the present models avoid those potential pitfalls. As many of these are related to the regimes of validity of effective field theories for the axion dynamics, we postpone their discussion until section 5.3.

The above AdS_5 vacua admit generalizations associated to D3-branes at more general CY threefold singularities, which have been extensively studied in the toric case. The dual backgrounds correspond to type IIB Freund-Rubin $\text{AdS}_5 \times \mathbf{X}_5$, where $\mathbf{X}_5$ is the 5d horizon of the 6d CY cone. The construction of KT backgrounds by introducing (possibly a richer set of) 3-form fluxes is a straightforward extension of our above discussion (see for instance [123] for complex cones over del Pezzo surfaces), so there is a large class of constructions leading to transplanckian axion monodromy. Being more careful, we should make clear that only CY singularities admitting complex deformations can complete their KT throats into smooth supersymmetric KS-like throats [124]; other choices admit no supersymmetric KS completion [123, 125, 126], and actually lead to runaway instabilities [123, 127], a fact which has recently motivated the "local AdS - Weak Gravity Conjecture" [116], generalizing the "AdS-WGC" in [128]. However, even with the restriction to CY singularities admitting complex deformations, there is an enormous class of such explicit constructions (built with standard toolkits, see e.g. [129]), and thus leading to transplanckian axion monodromy.

3.1.4 Duality walls

The fact that the axion traverses an arbitrarily large distance in field space as one moves to larger distances in r is intimately related to the RG flow structure in the holographic field theory. As mentioned in section 3.1.2, the axion winding around its period corresponds to completing a cycle in the Seiberg duality cascade of the $SU(N) \times SU(N+M)$ field theory. The steps in the energy scale in each duality cycle relate to the radial distance required for the scalar to wind around its period. The infinite range in energy as one moves up to the UV in the field theory provides an infinite range in radial distance on the gravity side, which allows for an arbitrarily large axion field range with finite gradient energy density.

Hence, the nice properties of the holographic field theory RG flow relates to the fact that the gravity side is described by a supergravity background.

In contrast with this picture, it is interesting to point out that a different kind of RG flow behaviour of duality cascades has been contemplated, purely from the field theory perspective. These are known as duality walls, and correspond to duality cascade RG flows in which, as one moves to the UV, the energy steps in each duality cycle decrease; more concretely, the number of duality cycles in a given energy slice increases as one moves up to the UV, in such a way that there is a limiting energy, at which the number of cycles per energy interval diverges. Such RG flows have been introduced in [130], and proposed to relate to quiver gauge theories of D-branes at singularities in e.g. [131–133]. However, there is no concrete string theory D-brane realization of such RG flows. In particular, systematic searches for gravity backgrounds dual to gauge theories with duality walls have produced no such results [123].

The absence of such backgrounds, at least in the context of supergravity, has an interesting implication for our perspective on field ranges in axion monodromy models. Gravitational solutions dual to duality walls would require an axion winding around its period an infinite number of times in a finite range in the radial distance. This is compatible with finite gradient energy densities only if the kinetic term of the axion varies so as to render finite the traversed distance in field space. This kind of behaviour would produce axion monodromy models where superplanckian field ranges cannot be attained. Hence, the absence of supergravity backgrounds of this kind is a signal that superplanckian axion monodromy models are actually generic in the present setup, whereas those with limiting field ranges are exotic, if at all existent.

3.2 Effective field theory analysis

In the previous section we have shown a fully backreacted explicit 10d solution for axion monodromy models with arbitrarily large field ranges. In this section we bring the discussion to the context of the 5d effective field theory, where much of the discussion of swampland conjectures is carried out.

3.2.1 Effective field theory for axion and breathing mode

From the 10d solution it is clear that the relevant dynamics in 5d involves the axion ϕ and the breathing mode of $\mathbf{X}_5 = T^{1,1}$, coupled to 5d gravity. It is interesting to device an effective field theory describing the dynamics for these degrees of freedom in the KS solution[6]. This provides a concrete context in which to test the regime of validity of the effective field theory to describe transplanckian axion monodromy, or to test other swampland conjectures.

The 5d effective field theory can be obtained starting from the 10d type IIB effective action, and using a suitable ansatz for the compactification, which allows for general dynamics for the relevant 5d fields. This strategy was in fact put forward in [111] to produce the 5d action we are interested in. We review the key ingredients relevant for our purposes, and adapted to our present notation.

[6]Inclusion of the dilaton is discussed in section 3.2.3

We consider the metric ansatz

$$ds_{10}^2 = L^2\big(e^{-5q}\, ds_5^2 + e^{3q} ds_{T^{1,1}}^2\big) \tag{3.26}$$

Here q is a 5d field encoding the breathing mode of $T^{1,1}$. Also, ds_5^2 is the line element in the 5d non-compact spacetime, defined in the 5d Einstein frame thanks to the prefactor e^{-5q}. The explicit L scales out the line elements to geometries of unit radius.

There are M units of F_3 flux over the $\mathbf{S}^3 \in T^{1,1}$ and there is a 5d axion defined by (3.3). The modified Bianchi identity (3.6) implies that the flux of $\tilde{F}_5$ over $T^{1,1}$ is given by (3.8).

The 5d effective action for the 5d scalars ϕ and q, collectively denoted by φ^a, is given by

$$S_5 = -\frac{2}{\kappa_5^2}\int d^5x\,\sqrt{-g_5}\left[\frac{1}{4}R_5 - \frac{1}{2}G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b - V(\varphi)\right], \tag{3.27}$$

with the kinetic terms and potential given by

$$G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b \;=\; 15(\partial q)^2 + \frac{1}{4}g_s^{-1}e^{-6q}(\partial\phi)^2\;, \tag{3.28}$$

$$V(\varphi) \;=\; -5e^{-8q} + \frac{1}{8}M^2 g_s\, e^{-14q} + \frac{1}{8}(N_0 + M\phi)^2 e^{-20q}. \tag{3.29}$$

The different terms in the potential have a clear interpretation. The first negative contribution corresponds to the curvature of the compactification space $T^{1,1}$, the second is the contribution from the M units of F_3 flux on the $\mathbf{S}^3$, and the third corresponds to the contribution from the 5-form flux over $T^{1,1}$, and has the typical axion monodromy structure. We note that, despite the bare quadratic dependence, the backreaction of ϕ on the geometry will produce a different functional dependence of the potential energy at the minimum, as shown below. Also, as already explained, the above action should be regarded as a consistent truncation in supergravity, so we will take special care to discuss the role of other physical degrees of freedom, like KK modes.

Since the above effective theory is general, it should reproduce the basic AdS$_5$ background for $M = 0$. The potential becomes

$$V(\varphi) \;=\; -5e^{-8q} + \frac{1}{8}N_0^2 e^{-20q}. \tag{3.30}$$

The potential has a minimum at

$$e^{6q} \;=\; \frac{N_0}{4} \tag{3.31}$$

with negative potential energy at the minimum

$$V_0 \;=\; -3\,e^{-8q}. \tag{3.32}$$

Comparing (3.26) with the standard expression for AdS$_5 \times T^{1,1}$ metric (3.1), we recover the scaling of the $T^{1,1}$ radius R with N_0

$$R^2 \sim e^{3q} \;\rightarrow\; R^4 \sim N_0 \tag{3.33}$$

with other factors reabsorbed in L in (3.26). Taking the value for V_0 (3.32) and removing a factor of e^{-5q} to change to the 10d frame, we recover the same scaling for the radius of the AdS_5 vacuum.

The KS throat (actually its asymptotic KT form) is a solution of the above effective action. Following [111], we take the following ansatz for the metric

$$ds_{10}^2 \,=\, s^{-1/2}(r)\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu \,+\, h^{1/2}(r)\,(dr^2 + r^2 ds_{T^{1,1}}^2). \tag{3.34}$$

In terms of (3.26), this corresponds to

$$e^{3q} = r^2\,h^{1/2}(r) \quad , \quad ds_5^2 \,=\, e^{5q}\,[\,s^{-1/2}(r)\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu \,+\, h^{1/2}(r)\,dr^2\,]. \tag{3.35}$$

The effective theory admits a solution where

$$\phi \,=\, M\,\log r \quad , \quad s(r) \,=\, h(r) \,=\, \frac{1}{4r^4}M^2 \log\frac{r}{r_*}, \tag{3.36}$$

with r_* some reference value. This is just the throat solution discussed in Section 3.1.2.

The effective action can be exploited to recover the result of the transplanckian field range covered by the axion. Since the 5d effective action is already in the 5d Einstein frame, we can read out and evaluate the kinetic term for ϕ in (3.28)

$$G_{\phi\phi} \,\sim\, e^{-6q} \,=\, [\,r^4\,h(r)\,]^{-1} \,\sim\, (\,M^2\,\log r\,)^{-1}. \tag{3.37}$$

We thus recover, in a more precise setting, the result (3.24), and thus the corresponding unbounded (and hence transplanckian) field range.

3.2.2 The axion effective field theory

As explained, the above action should be regarded as a consistent truncation in supergravity, but not as a Wilsonian effective action. In other words, at the scale $1/R$ at which the stabilization of the breathing mode occurs, there are many other modes, corresponding to KK excitations of the 10d fields in $\mathbf{X}_5$ which are not included in the action. Note that this scale goes as $1/R \sim (\log r)^{-1/4}$. On the other hand, the effective dynamics for the axion occurs at far lower scales, set by $\partial\phi = 1/r$. Similarly, the scale of the backreaction on the compactification radius or the vacuum energy is measured by their derivatives with respect to r, which are similarly suppresed by $1/r$ (or even with additional inverse powers of $\log r$). It is therefore interesting to construct an effective field theory including just the axion and intended to describe its dynamics at those scales (hence, including the backreaction on the volume and vacuum energy).

For this, we minimize the scalar potential for q keeping ϕ fixed. This gives the condition

$$\frac{5}{2}\,(N_0 + M\phi)^2\,x^2 \,+\, \frac{7}{4}\,g_s M^2\,x \,-\, 40 \,=\, 0 \quad , \quad \text{with } x = e^{-6q}. \tag{3.38}$$

Rather than solving the above exactly, since we are focusing on the large r regime, where ϕ is large and x is comparably small, we drop the subleading second term, and obtain

$$e^{6q} \,=\, \frac{1}{4}(N_0 \,+\, M\phi). \tag{3.39}$$

This reproduces the result of the KS solution that $e^{6q} \sim M^2 \log r$ for $\phi \sim M \log r$, so we are capturing the relevant physics.

We should replace that value in the potential. Again restricting to large r, we drop the second term in (3.29) and obtain

$$V = -e^{-8q} \left[5 - \frac{1}{8} (N_0 + M\phi)^2 e^{-12q} \right]. \tag{3.40}$$

This has the same structure as (3.30) with the replacement $N_0 \to N_0 + M\phi$. The potential should be regarded as a function of ϕ only, by simply replacing (3.39) in this expression. It is therefore clear that considering a profile $\phi = M \log r$ leads to the appropriate change in the vacuum energy, so that the backreaction of the axion monodromy is duly included.

The complete axion action should include its kinetic term, obtained from that in (3.28) by using (3.39). We recover a kinetic term

$$\sim (N_0 + M\phi)^{-1} (\partial\phi)^2, \tag{3.41}$$

which again reproduces the familiar result about the transplanckian distance traveled in the rolling solution considered.

This effective action suffices to describe the dynamics of the transplanckian axion monodromy, so it is a well-defined setup to test/propose swampland conjectures on effective actions. For instance, one natural idea is to consider if there is an analog of the swampland distance conjecture, and there is a tower of states becoming exponentially light as the axion travels at arbitrarily large distances. This is not the case, as follows. The invariant distance in axion field space goes (for large ϕ) as $d \sim \phi^{1/2}$; on the other hand, the masses of KK modes (which are the primary suspects for fields becoming light at large ϕ, since R increases), scale as $m_{\rm KK} \sim e^{-4q} \sim (\phi)^{-2/3}$, hence $m_{\rm KK} \sim d^{-4/3}$ and there is no tower of exponentially light states. This is compatible with the swampland distance conjecture, if interpreted as applying to field ranges approaching points at infinite distance in moduli space [35, 45]. It is also compatible with the oftentimes used version for transplanckian geodesic distances, since in the next section we will show that our axion travel does not follow a geodesic. However the model provides a beautiful way in which a fully backreacted monodromic axion can travel arbitrarily large distance in field space without triggering the appearance of exponentially light states.

There are other interesting questions that can be addressed in the present setup, such as the application of swampland constraints on the scalar potential, or the realization of the weak gravity conjecture in the present setup, etc. Since the underlying model is a string theory compactification on a smooth geometry with fluxes, we expect no new surprises or novel mechanisms related to these other swampland conjectures.

3.2.3 Inclusion of the dilaton

As announced, in this section we show that the underlying reason for the compatibility of the transplanckian axion monodromy model with the swampland distance conjectures is that the axion does not follow a geodesic in the moduli of light fields. The crucial ingredients to understand this are the spacetime dependence of the axion, and the inclusion of the dilaton in the moduli space.

The original KT 5d effective action [111] includes further fields beyond those included in the earlier discussion. Indeed, it contains fields $\varphi^a = q, f, \Phi, \phi$, where f describes a possible asymmetric volume for the $\mathbf{S}^2$ and $\mathbf{S}^3$ of $T^{1,1}$, and Φ is the dilaton. The 5d action for these fields has the structure (3.27) with

$$\begin{aligned} G_{ab}(\varphi) &= \mathrm{diag}\left(15, 10, \frac{1}{4}, \frac{1}{4}e^{-\Phi-4f-6q}\right) \\ V(\varphi) &= e^{-8q}\left(e^{-12f} - 6e^{-2f}\right) + \frac{1}{8}M^2 e^{\Phi+4f-14q} + \frac{1}{8}(N_0 + M\phi)^2 e^{-20q}. \end{aligned} \tag{3.42}$$

The pure AdS$\times T^{1,1}$ solution for $M = 0$ shows that in this action the breathing mode q and asymmetric mode f are heavy modes, while the axion ϕ and dilaton Φ remain as light fields. Morally, we should thus consider the later as parametrizing a moduli space at scales hierarchycally below the KK scale, with a potential induced by the introduction of non-zero M. This is manifest because the terms including M in the potential are subdominant with respect to the first, M-independent, one.

This allows to integrate out q and f. We may minimize the leading potential for f, and set $f = 0$ (as implicit in the previous section). For the minimization of q, we proceed as in the previous section and recover (3.39).

Note that, in the resulting theory for the axion and the dilaton, there is a non-trivial potential for the dilaton. This is however compatible with its constant value in the axion monodromy solution in an interesting way: the spacetime dependence of the axion has a non-trivial backreaction in the dilaton, through the dilaton dependence of the axion kinetic term, which induces an effective potential for the dilaton balancing the original one and allowing for a constant dilaton solution. Quantitatively, the equation of motion for a general field in the presence of a spacetime-dependent axion background reads

$$\frac{1}{\sqrt{g}}\partial_\nu\left(\sqrt{g}g^{\mu\nu}G_{ac}\partial_\mu\varphi^a\right) = \frac{1}{2}\frac{\partial G_{\phi\phi}}{\partial\varphi^c}(\partial\phi)^2 + \frac{\partial V}{\partial\varphi^c}. \tag{3.43}$$

For the dilaton, the condition to allow for a constant dilaton $e^\Phi = g_s$ is the vanishing of the right-hand side, which is proportional to

$$-e^{-6q-\Phi}(\partial\phi)^2 + e^{-14q+\Phi}M^2. \tag{3.44}$$

This indeed vanishes in the KT solution, allowing for a constant dilaton. As anticipated, the spacetime dependence of the axion exerts a force on the dilaton keeping it constant on the slope of its bare potential.

The scale of this effect is set by the gradient of the axion $\partial\phi$, which is hierarchycally below the KK scale. This implies that the corresponding backreaction effect for the other fields q and f is negligible, and can be ignored when they are integrated out, as implicit in our above discussion. It also implies that it is not appropriate, in a Wilsonian sense, to integrate out the dilaton dynamics, as it occurs at the scale relevant for axion dynamics.

This last observation raises an important point. In checking the interplay of our axion monodromy model with the swampland distance conjectures, the moduli space on which distances should be discussed is that spanned by the axion and the dilaton, as their potential on this moduli space is hierarchically below the KK scale cutoff. As we have shown, in this moduli space the KT solulion describes an axion monodromy model traversing transplanckian (and actually arbitrarily long) distances without encountering infinite towers of light states. However, as we now argue, this does not contradicts swampland distance conjectures, since the trajectory does not correspond to a geodesic in the axion-dilaton moduli space.

After replacement of q and f by their values at the minimum of their potentials, the kinetic term for ϕ, Φ reads

$$\mathcal{L}_{kin} = \frac{1}{8}(\partial\Phi)^2 + \left(\frac{e^{-\Phi}}{2\,(N_0 + M\phi)} + \frac{5M^2}{24\,(N_0 + M\phi)^2} \right) (\partial\phi)^2. \tag{3.45}$$

At large ϕ we can neglect the subleading second term in the kinetic term of ϕ and get

$$\mathcal{L}_{kin} = \frac{1}{8}(\partial\Phi)^2 + \frac{e^{-\Phi}}{2\,(N_0 + M\phi)}(\partial\phi)^2. \tag{3.46}$$

To look at the geodesics of this theory it is convenient to change variables

$$\begin{aligned} x &= \tfrac{4}{M}\sqrt{N_0 + M\phi}, \\ y &= 2e^{\Phi/2} = 2\sqrt{g_s}. \end{aligned} \tag{3.47}$$

This leads to

$$\mathcal{L}_{kin} = \frac{1}{2y^2}\left[(\partial x)^2 + (\partial y)^2 \right], \tag{3.48}$$

which is the metric of the hyperbolic plane. Geodesics of this space, considering y the vertical axis, are vertical lines or half-circles centered in the horizontal axis. On the other hand, the KT solution corresponds to horizontal lines at different constant values of the dilaton.

3.3 The 4d case

The above discussion has been carried out in the 5d context because, being holographically dual to 4d gauge theories, these are the best studied warped throats. However, there are well studied supergravity solutions of the form $\mathrm{AdS}_4 \times \mathbf{X}_7$, and supergravity solutions of the KT kind when the horizon variety $\mathbf{X}_7$ admits the introduction of fluxes [134]. In the following we review these backgrounds and show that they realize in 4d the same kind of transplanckian axion monodromy as the 5d configurations described above.

The starting point is the $\mathrm{AdS}_4 \times \mathbf{X}_7$ background, which can be regarded as arising from the near-horizon limit of a stack of N coincident M2-branes [15]

$$ds^2 \,=\, h(r)^{\frac{2}{3}}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu \,+\, h(r)^{\frac{1}{3}}\,(\, dr^2 \,+\, r^2\, ds^2_{\mathbf{X}_7}\,), \tag{3.49}$$

where now Greek indices label non-compact coordinates spanning, together with r, the 4d spacetime. The harmonic function is

$$h(r) \,=\, \frac{2^5\, \pi^2\, N\, \ell_p^6}{r^6}. \tag{3.50}$$

Namely, we have

$$ds^2 \,=\, \frac{R^4}{r^4}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu \,+\, R^2\, \frac{dr^2}{r^2} \,+\, R^2\, ds^2_{\mathbf{X}_7}\,, \tag{3.51}$$

where

$$R^6 = 2^5 \pi^2 N \ell_p^6. \tag{3.52}$$

There are N units of flux of the 7-form field strength F_7 (dual to the 4-form field strength F_4) through $\mathbf{X}_7$.

Consider an $\mathbf{X}_7$ with a non-trivial 4-cycle[7], on which we turn on M units of 4-form field strength flux F_4. Taking the dual 3-cycle Π_3 in $\mathbf{X}_7$, there is a 4d axion

$$\phi = \int_{\Pi_3} C_3. \tag{3.53}$$

This axion is monodromic, as follows from the reduction of the 11d Chern-Simons coupling

$$\int_{11d} F_4 \wedge F_4 \wedge C_3 \to \int_{4d} M \phi F_4. \tag{3.54}$$

The monodromy implies that the value of N varies with ϕ as

$$N = N_0 + M \phi, \tag{3.55}$$

with N_0 a reference value, which we take zero in what follows.

This leads to a 4d analog of the KT throat found in [134] and given by a flux background

$$F_4 = d^3x \wedge dh^{-1} + M *_7 \omega_3 - M \frac{dr}{r} \wedge \omega_3. \tag{3.56}$$

Here ω_3 is the Poincare dual to the 4-cycle in $\mathbf{X}_7$, so the second term corresponds to the F_4 flux through the 4-cycle. The third term corresponds to a rolling scalar profile $d\phi = dr/r$, hence

$$\phi \sim M \log r. \tag{3.57}$$

Hence we have the axion rolling logarithmically up its monodromic potential, exactly as in the the 5d KS solutions discussed above. The first term correspond to the dual of the flux of F_7 through $\mathbf{X}_7$, which varies with the radial coordinate due to the axion monodromy.

The harmonic function $h(r)$ is

$$h(r) = M^2 \left(\frac{\log r}{6r^6} + \frac{1}{36r^6} \right) \tag{3.58}$$

(up to some ρ/r^6 factor, which defines a reference value which we take to be zero). It also determines the metric by replacement in (3.49).

The solution, just like in the 5d KT example, has a naked singularity at $r = 0$, which is presumably smoothed out at least for certain geometries $\mathbf{X}_7$, although no analog of the full KS solution has been found. It would be interesting to develop the dictionary

[7] Such horizons can be obtained for instance by taking the near horizon limit of M2-branes at toric $CY_3 \times \mathbf{C}$ (leading to 3d $\mathcal{N} = 1$ theories), where the CY_3 admits a complex deformation corresponding to the size of a 3-cycle. The horizon $\mathbf{X}_7$ then contains (an $\mathbf{S}^1$ worth of) such 3-cycle, and hence its dual 4-cycle.

of fractional M2-brane theories and their gravity duals further to gain insight into such smoothings. This however lies beyond the scope of the present paper.

It is straightforward to compute the 4d kinetic term of the axion ϕ as in the simplified 5d calculation in section 3.1.3. Specifically, the Einstein-Hilbert and 3-form kinetic term in the 11d action read

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g_{11}} \left(\mathcal{R}_{11} + \frac{1}{2} |F_4|^2 \right). \tag{3.59}$$

Define the volume of $\mathbf{X}_7 = (R\tilde{R})^7$, where R defines the backgound value and $\tilde{R}$ its breathing mode. The KK reduction to 4d contains the terms

$$S_4 = \frac{1}{2\kappa_4^2} \int d^{11}x \sqrt{-g_4} \left(\tilde{R}^7 \mathcal{R}_4 + c \tilde{R} g^{mn} \partial_m \partial_n \phi \right). \tag{3.60}$$

Here we have introduced

$$\kappa_4^2 = \frac{\kappa_{11}^2}{R^7}. \tag{3.61}$$

Also, the factor $\tilde{R}$ in the axion kinetic term arises from an $\tilde{R}^7$ from the compactification volume and a factor $\tilde{R}^{-6}$ from three inverse metrics of $\mathbf{X}_7$ required for the contractions of $|\omega_3|^3$. Finally c is a constant that depends on geometrical properties of the cycles in $\mathbf{X}_7$.

Going to the 4d Einstein frame we have

$$S_4 = \frac{1}{2\kappa_4^2} \int d^{11}x \sqrt{-g_4} \left(\mathcal{R}_4 + c \tilde{R}^{-6} g^{mn} \partial_m \partial_n \phi \right). \tag{3.62}$$

So the kinetic term for the axion gives

$$G_{\phi\phi} \sim \tilde{R}^{-6} \sim (M^2 \log r)^{-1}. \tag{3.63}$$

This is exactly as in the 5d example, and again leads to arbitrarily large, in particular transplanckian, field ranges traversed by the axion roll.

3.4 Summary

In this chapter we have considered a first example of running solution in String Theory and its interplay with the SDC. This is a particularly interesting case since it goes against the naive extension of the SDC to running solutions. This is, there is no tower of states becoming light exponentially with the field space distance travelled by the scalars in the running solution. As we have explained, the KK modes do become light but as a power law in the field space distance. We showed however that this is not against the usual formulation of the SDC in section 3.2.3. The key point is that field excursion of the scalar in the running solution is a highly non-geodesic trajectory in the field space of the EFT. In this way, even though the SDC in its usual formulation in terms of the geodesic field distance is satisfied, the falloff of the tower is delayed in the running solution. For this argument it is crucial that the dilaton has to be included in the EFT describing the field space trajectory of the running solution in section 3.2.2.

The complete 10d background is the well-known Klebanov-Strassler throat [111,112] and was reviewed in 3.1. When regarded as a flux compactification on a Sasaki-Einstein manifold $\mathbf{X}_5$, the 10d uplift encodes the full backreaction of the field dynamics on the geometry and other scalar fields.

The 5d effective field theory was described in Section 5.3. The kind of flux compactifications discussed in 3.2.1 are usually argued not to yield good effective field theories, but rather only consistent truncations. The reason is the coincidence of the stabilization scale of the breathing mode, the AdS scale and the KK scale. However, we argue in 3.2.2 that the dynamics of the running solution lie well-below these scales. We use this build an a priori well-behaved EFT in the wilsonian sense describing these dynamics.

4

The Convex Hull Swampland Distance Conjecture and Bounds on Non-geodesics

An important point in the discussion of SDC is that it should apply to *adiabatic motion* in moduli space. Morally, this amounts to varying the scalar values as vevs, i.e. with no spacetime variation. In fact, as shown Chapter 3, backgrounds with spacetime varying scalars can lead to transplanckian motion without encountering exponentially falling towers of states.[1] A useful way to understand this point is that adiabatic motion corresponds to moving along geodesics in moduli space, while spacetime dependence introduces extra forces in moduli space motion, leading to non-geodesic trajectories. This would seem to imply that the SDC, in the adiabatic sense, should apply only to geodesic trajectories in moduli space.

On the other hand, although the SDC and its variants are most precisely stated and studied for exactly massless moduli, on physical grounds they should be expected to hold in the presence of scalar potentials, as long as the relevant masses and energies remain smaller than the cutoff, i.e. a pseudomoduli space. From this perspective, consider a theory with a moduli space $\mathcal{M}$ parametrized by a set of scalars ϕ^i, such that the SDC is satisfied. If a potential $V(\phi)$ is now introduced, the motion of scalars is restricted to the valleys of this potential, which can either correspond to left-over massless moduli, or to directions along which the potential may not be exactly flat but the relevant energies are smaller than a given cutoff Λ. Let us denote this (pseudo)moduli space $\overline{\mathcal{M}}$. At energies below Λ, we may integrate out the heavy directions of $\mathcal{M}$ and obtain and effective theory for the light scalars φ^a parametrizing $\overline{\mathcal{M}}$.

Now this leads to the following conundrum. In the effective theory below Λ, one can study the SDC by considering geodesic trajectories in the moduli space $\overline{\mathcal{M}}$. On the other hand, the trajectory can be regarded as uplifted to a trajectory in $\mathcal{M}$, so that distances along it can be computed as in the parent theory.[2] But these trajectories in general do not correspond to geodesics in $\mathcal{M}$, and could in principle violate the SDC, *even if the SDC is obeyed for geodesics in* $\mathcal{M}$!

A most relevant aspect of this apparent puzzle is that, if actually realized, the SDC would cease to make sense as a swampland constraint. Given an effective theory violating the SDC in its moduli space, one could always argue that this corresponds to the theory

[1] For spacetime dependence and transplanckian scalar travel, see also [58, 135, 136].

[2] An important point is that the kinetic terms on the effective theory in $\overline{\mathcal{M}}$ are affected by the integration of the heavy modes; this is captured by the statement that in the effective theory below Λ, the metric on $\overline{\mathcal{M}}$ is the induced metric from the embedding of $\overline{\mathcal{M}} \subset \mathcal{M}$. This is equivalent to the statement that the distance is obtained from the trajectory when embedded in $\mathcal{M}$.

on $\overline{\mathcal{M}}$, and that above certain scale Λ the theory is completed to a larger moduli space $\mathcal{M}$ which obeys it, and which can in principle be completed into a quantum gravity theory. In fact, there is no reason why this cannot occur in a nested manner with several effective theories scalating up to some higher energy scale at which finally the SDC is fulfilled. In other words, since the notion of moduli scape in the presence of multi-scale potentials is a scale-dependent notion, the SDC constraint would only apply in a certain energy regime, but then, *which* energy regime?

We have guided the reader through this argument to make our main point manifest. We propose that the above situation cannot occur in a theory of quantum gravity, and that the SDC must apply at any energy scale, namely, in any of the effective theories valid at any intermediate energy scale. This has the following profound implication: since arbitrary scalar potentials in a moduli space $\mathcal{M}$ can easily lead to subspaces $\overline{\mathcal{M}}$ violating the SDC, the validity of the SDC at all scales in quantum gravity theories constitutes a non-trivial constraint on consistent potentials in quantum gravity theories.

Notice that the discussion in chapter 3 is a very non-trivial example of these ideas. The 5d theory presented in section 3.2.1 indeed satisfies the SDC for any geodesic. Due to the presence of the potential we were able in section 3.2.2 to integrate out a combination of the breathing mode and the axion. The resulting light mode is the one describing the transplanckian excursion in the spacetime varying solution that avoids the exponentially falling tower of states. However, as argued in section 3.2.3, the effective theory at that energy scale has to include the dilaton. Crucially, it was found that the light mode parametrizes a non-geodesic trajectory in this two-dimensional pseudomoduli space. All in all, the SDC is respected for geodesics of the pseudomoduli space at any energy scale and only in truly spacetime varying solutions the exponentially falling tower can be avoided.

The realization of the SDC in moduli spaces of light fields in the presence of potentials[3] has been explored in diverse examples in flux compactifications in string theory e.g. [106, 139, 140]. These top-down approaches are valuable, yet very model dependent. In this paper we instead initiate a model-independent bottom-up approach, closer to the spirit of the swampland program. Our strategy is instead to characterize the non-geodesic trajectories which are nevertheless 'sufficiently geodesic' to allow the realization of the SDC. The approach is very model independent, since it only involves geometrical properties of the moduli space, and some information about the towers hiding at its asymptotic regions. Characterization of the non-geodesicity allowed by the SDC for trajectories in a moduli space, leads in interesting examples to explicit bounds. These bounds can be subsequently tested against concrete models, and are interestingly saturated in string theory flux compactification in the asymptotic limits [141].

The model-independent approach allows us to devise an illuminating rephrasing of the SDC in terms of a Convex Hull condition, similar in spirit to that arising in the context of the Weak Gravity Conjecture [82], or its scalar WGC (SWGC) extensions [71, 142–146]. In particular, we characterize SDC towers by a scalar charge to mass ratio as in the SWGC; this controls the exponential decay rate along asymptotic trajectories characterized in terms of their asymptotic unit tangent vectors. Conversely, by considering the space of such vectors for all possible trajectories, we define an 'extremal region' by the set of charge to mass ratios ensuring a fixed minimum decay rate along any possible trajectory. Although

[3]In the original work [35] (see also [137, 138]), it was already noted that the SDC also applies to the subspaces parametrizing minimum loci for the potential for a given effective cutoff scale, and that this can imply powerful constraints on the potentials consistent with QG.

in the original formulation of the SDC [35], the decay rate is an undetermined $\mathcal{O}(1)$ factor, concrete lower bounds have been proposed in [36–39, 45, 147]. This allows to express the SDC in a given physical system as the condition that the convex hull of the scalar mass to charge ratios of its towers contains the extremal region. If the convex hull condition is not satisfied for arbitrary trajectories, one can use the convex hull to recover the above mentioned bounds on the non-geodesicity of the trajectories. Alternatively, it can also be used to predict the existence of new towers.

The fact that the scalar charge to mass ratio in our Convex Hull SDC agrees with the SWGC is a tantalizing hint, although the physical requirement of 'extremality' in both situations does not seem to be necessarily identical. It would be interesting to explore the relationship between the WGC states and the SDC towers in this Convex Hull context, possibly along the lines of [36, 81].

The chapter is organized as follows. In Section 4.1 we consider the example of non-geodesic trajectories in a moduli space given by one hyperbolic plane (section 4.1.1) or products thereof (section 4.1.2), and derive bounds on the non-geodesicity of trajectories obeying the SDC. The analysis of the multi-moduli cases motivates Section 4.2, where we frame the multi-axion examples in a general reformulation of the SDC (section 4.2). This allows us in Section 4.3 to formulate our Convex Hull SDC (section 4.3.1), and recover and vastly generalize results of the previous sections, as we show in several explicit examples (section 4.3.2). In Section 4.4 we revisit the results about asymptotic flux compactifications in [141], and show that they realize the critical behaviours of non-geodesicity. Section 4.5 contains a summary of the chapter.

This chapter is based on the author's publication [3] with slight modifications for a better fit in the context of this thesis.

4.1 Non-geodesic bounds in the hyperbolic plane

We focus our analysis on trajectories approaching points at infinity in moduli space, in the spirit of the SDC, since the interesting physics occurs in the asymptotic region near infinity. Also, it often corresponds to weakly coupled regimes, where effective actions and scalar potentials can be reliably computed. Moreover, fairly general moduli spaces simplify in the asymptotic regime, so that very simple moduli space geometries are useful templates for the asymptotics of general moduli spaces.

In this section, we discuss moduli spaces given by a hyperbolic plane, or products thereof. Despite their apparent simplicity, they are key to describing moduli spaces of general CY compactifications near their boundaries at infinity [45, 46, 141], to the extent of encoding much of the dynamics of these models [148]. Moreover, they allow for explicit computations which will be useful to motivate our generalizations in later sections.

4.1.1 One hyperbolic plane

Consider a 4d effective theory with two real moduli s and ϕ with kinetic terms

$$\frac{n^2}{s^2}\left(\partial_\mu s\,\partial^\mu s + \partial_\mu \phi\,\partial^\mu \phi\right), \tag{4.1}$$

where n is a free parameter. In other words, the moduli space $\mathcal{M}$ is given by the upper half-plane with metric

$$dD_\phi^2 = \frac{n^2}{s^2}\left(ds^2 + d\phi^2\right) \tag{4.2}$$

We note that n determines the Ricci scalar curvature

$$R = -\frac{2}{n^2}. \tag{4.3}$$

This geometry is ubiquitous in string theory, with ϕ corresponding to some periodic axion and s its 'saxion' partner (although we do not assume susy, we stick to this name). For instance, the type IIB complex coupling in 10d, the 4d axio-dilaton in string compactifications, and the Kähler and complex structure moduli of 2-tori in toroidal (and orbifold and orientifolds thereof) compactifications.

In many of these, the $SL(2,\mathbf{R})$ symmetry of the above geometry lead to an exact infinity discrete $SL(2,\mathbf{Z})$ duality symmetry. However, we work in a more general perspective, so that our analysis is valid in the absence of this symmetry. On one hand, in many compactifications, we would like to regard the above metric as a good approximation to the moduli space (or suitable subspaces thereof) of CY compactifications, in the large s asymptotic region; hence the region near $s = 0$ is not relevant to this physics context, and the duality $s \to 0$ and $s \to \infty$ is a mere artifact. Second, the discrete axion periodicity (which would be present even near $s \to \infty$) is in general spontaneously broken in the presence of potentials of axion monodromy[4] kind [97, 119], a generic situation in flux compactifications [101, 102]. Hence we consider ϕ to take real values, with no identification whatsoever.

We consider that the SDC is satisfied on this moduli space $\mathcal{M}$, namely there exists a tower of states with mass scale[5]

$$M \sim s^{-a},\ a > 0. \tag{4.4}$$

If s parametrizes the vertical axis, all geodesics in this space are either vertical lines or half-circles with centers on the $s = 0$ line. Thus, the only geodesics approaching $s \to \infty$ are vertical lines with $\phi = \text{const}$. For these geodesics, the distance behaves as

$$D_\phi \sim n \log s \tag{4.5}$$

(with n taken positive herefrom). The mass scale of the tower reads

$$M \sim \exp\left(-\frac{a}{n}D_\phi\right) \sim \exp\left(-\alpha D_\phi\right) \tag{4.6}$$

thus leading to the SDC with decay rate $\alpha = \frac{a}{n}$. This is indeed $\mathcal{O}(1)$ in many realizations in string theory.

[4]More precisely, the discrete periodicity is preserved due to the multiple-branched structure of the potential. However, it is spontaneously broken when the problem under study (e.g. adiabatic motion in moduli space) is restricted to a single branch.

[5]The fact that the overall scale is independent of ϕ does not imply that the masses of individual states in the tower can not depend on ϕ. Indeed, a typical structure is given by $M_n = M|n+\phi|$, with n labeling states in the tower and M depending only on s, as required for our analysis. Here the ϕ-dependence is determined by the fact [45, 47] that the different states in the tower are generated by monodromy in ϕ, i.e. $\phi \to \phi + 1$ is equivalent to $n \to n + 1$. More general axion dependences in the tower scale will be easily included in the analysis in Section 4.3.

Let us now consider a general trajectory approaching $s \to \infty$ in this moduli space. For reasonable trajectories, we can can use s to parametrize it,[6] so that the curve is defined by the expression

$$\phi = f(s) \tag{4.7}$$

for some function f that we assume sufficiently smooth. This is a template to describe the moduli space of an effective theory in which there is partial moduli stabilization, and the light direction can be parametrized by s.

Recalling footnote 2, note that the distance in this effective theory is not measured by just the metric component g_{ss}, but rather by the effective metric obtained upon replacing the s-dependent value of ϕ in the underlying metric. This is equivalent to measuring distance along the trajectory with the ambient space metric (4.2) in $\mathcal{M}$. This yields

$$dD_\phi = \frac{n}{s}\sqrt{1 + f'(s)^2}\, ds. \tag{4.8}$$

We can now classify general trajectories in three different kinds, according to the asymptotic behaviour of $f'(s)$ in the $s \to \infty$ limit:

- The *Asymptotically Geodesic* case:

  This corresponds to $f'(s) \to 0$, and we have

  $$dD_\phi = n\frac{ds}{s}. \tag{4.9}$$

  Trajectories of this class approach a geodesic when $s \to \infty$. Therefore the SDC is automatically satisfied with decay rate

  $$\alpha_{\text{geod.}} = \frac{a}{n}. \tag{4.10}$$

- The *Critical* case:

  This corresponds to $f'(s) \to \beta = $ const. and we have

  $$dD_\phi = \sqrt{1 + \beta^2}\, n\frac{ds}{s}. \tag{4.11}$$

  The tower of states has mass scale

  $$M \sim \exp\left(-\frac{\alpha}{\sqrt{1+\beta^2}} D_\phi\right) \tag{4.12}$$

  which is consistent with the SDC, but modifies the scale of the exponential. We can define a factor

  $$\nu \equiv \frac{\alpha_{\text{geod}}}{\alpha_{\text{non-geod}}} \tag{4.13}$$

  that measures such a modification, so that $M \sim \exp(-\frac{\alpha_{\text{geod}}}{\nu} D_\phi)$. In this case we have

  $$\nu_{\text{crit.}} = \sqrt{1+\beta^2} \;\to\; \alpha_{\text{crit}} = \frac{a}{n\sqrt{1+\beta^2}}. \tag{4.14}$$

  Hence, $\nu \to \infty$ corresponds to a violation of the SDC in a non-geodesic trajectory.

[6] Actually, requiring that the trajectory eventually goes to $s \to \infty$ makes this parametrization always valid for sufficiently large s.

- The *Swampy* case:

  This corresponds to $f'(s) \to \infty$, and we have

$$dD_\phi = n \frac{f'(s)}{s} ds\,. \tag{4.15}$$

  Here we can evaluate the behaviour of the tower by computing

$$\frac{d\log M}{dD_\phi} = \frac{d\log M}{ds}\frac{ds}{dD_\phi} = -\frac{a}{n}\left(f'(s)\right)^{-1} \to 0. \tag{4.16}$$

  This violates the SDC since the tower mass scale is no longer falling exponentially with the distance.

We note that the critical case corresponds to the maximum deviation from a geodesic still consistent with the SDC. It therefore provides a non-trivial bound that any light direction in the moduli space (after partial moduli stabilization by some scalar potential) must obey. Notice that this is the class of curves in which the saxion varies linearly with the axion ($s \sim a$). This will be contrasted with explicit string models of flux compactifications in Section 4.4, where we show that this class of models saturates the bound.

It is interesting to explore the characterization of these classes in terms of a geometrical quantity of the trajectories. A natural scalar measure of non-geodesicity is the modulus $|\Omega|$ of the proper acceleration[7]

$$\Omega^i = T^j \nabla_j T^i\,, \tag{4.17}$$

where T is the normalized tangent vector of the trajectory and ∇ is de covariant derivative in moduli space. In our case we have

$$|\Omega|^2 = \frac{\left(f'(s) + f'(s)^3 - s f''(s)\right)^2}{n^2\left(1+f'(s)^2\right)^3}. \tag{4.18}$$

We can now characterize the three classes of paths above in terms of the asymptotic behaviour for $|\Omega|$ as follows. The *Asymptotically Geodesic* case corresponds to $f'(s) \to 0$ which translates to $|\Omega|^2 \to 0$. The proper acceleration vanishes asymptotically, since the trajectory approaches a geodesic. Contrary, for the *Swampy* case, one has $f'(s) \to \infty$ leading[8] to $|\Omega|^2 \to \frac{1}{n^2}$. Thus, the proper acceleration at $s \to \infty$ attains a maximal value, and signals a hard violation to the SDC. Finally, the *Critical* case corresponds to

$$f'(s) \to \beta = \text{const.} \implies |\Omega|^2 \to \frac{1}{n^2}\frac{\beta^2}{1+\beta^2}\,. \tag{4.19}$$

The change in the parameter of the exponential defined in (4.13) can be written as

$$\nu = \frac{1}{\sqrt{1 - n^2|\Omega|^2}}. \tag{4.20}$$

[7] Actually, since our trajectories are not worldlines, we should use the term "extrinsic curvature", but we stick to the kinematical language.

[8] One also needs that $\frac{sf''}{f'^3} \to 0$. This is satisfied for all functions of the form $f(s) = s^n, (\log s)^n, e^{s^n}$.

We can thus recast the criterion that the trajectory respects the SDC as a bound on the non-geodesicity:

$$|\Omega|^2 < \frac{|R|}{2} = \frac{1}{n^2} \tag{4.21}$$

In the next subsection, we will generalize these bounds to higher dimensional moduli spaces in which there is more than one hyperbolic plane. We will see that they cannot be stated simply in terms of the modulus of the proper acceleration, as the direction will also matter. In other words, the bounds will vary depending on the type of trajectory/ the growth sector to which the trajectory belongs.

4.1.2 Product of hyperbolic planes

We now consider a moduli space given by a product of hyperbolic planes. For simplicity, we consider the case of two, which suffices to illustrate the point. The metric is

$$dD_\phi^2 = \frac{n^2}{s^2}\left(ds^2 + d\phi^2\right) + \frac{m^2}{u^2}\left(du^2 + d\psi^2\right). \tag{4.22}$$

As in Section 4.1.1, we focus in paths approaching the infinite distance regime $s, u \to \infty$. Note that this setup (4.22) nicely models the asymptotic behaviour of CY moduli spaces, cf. Section 4.4, thus our discussion is of direct relevance to the study of non-geodesic paths in CY moduli space.

In order to satisfy the SDC for geodesics, it is enough to introduce a tower of states for each of the hyperbolic planes, with mass scales

$$M_s \sim s^{-a}, \qquad M_u \sim u^{-b}, \qquad a, b > 0. \tag{4.23}$$

It is clear that these towers enforce the SDC for geodesic paths contained in a single hyperbolic plane. Moreover, one can show that they also suffice to recover the SDC for more general geodesics.[9]

Let us consider non-geodesic paths approaching infinity in different ways, some of which correspond to different growth sectors, in the terminology of [46]. For instance, we can consider paths that only move on one of the hyperbolic planes, namely approaching $s \to \infty$ while keeping u, ϕ fixed, or alternatively, approaching $u \to \infty$ with s, ϕ fixed. In that case, the situation is equivalent to the one discussed in section 4.1.1 and we can simply borrow the classification of asymptotically geodesic, critical and swampy trajectories. We could also apply the criterion (4.21) on the proper acceleration $|\Omega|^2$, although this would yield two *different* bounds for the two different curvature paratemeters n and m. In other words, there is no single discriminating criterion on $|\Omega|$ which applies to both kinds of paths.

On the other hand, we may consider a path

$$\psi = f(s) \quad \text{with } \phi, u \text{ const.} \tag{4.24}$$

so that we move along a trajectory involving the axion and saxion of different hyperbolic planes. The distance along this curve is given by

$$dD_\phi = \sqrt{\frac{n^2}{s^2} + \frac{m^2}{u^2} f'(s)^2}\, ds = \frac{n}{s}\sqrt{1 + \frac{m^2}{n^2}\frac{s^2}{u^2} f'(s)^2}\, ds\,. \tag{4.25}$$

[9] Physical realizations like CY compactification may produce additional towers. This is ignored in this section for simplicity, but is included in the general analysis in later sections.

In the second equality we recognize the two terms in the square root to be the geodesic and the non-geodesic contribution to the field distance.

As done in section 4.1.1, we can classify the trajectories in the same three different families, depending on the relevance of the non-geodesic contribution in the asymptotic limit $s \to \infty$. We thus see that the critical case is:

$$\frac{m^2}{n^2}\frac{s^2}{u^2}f'(s)^2 \to \text{const.}\,, \tag{4.26}$$

which means that $sf' \to \gamma = \text{const.}$, implying the critical behaviour

$$f(s) \to \gamma \log s\,. \tag{4.27}$$

If $f(s)$ grows slower or faster than this critical case one gets the asymptotically geodesic and the swampy case respectively.

The SDC will only be satisfied for asymptotically geodesic or critical trajectories. Only for those paths, the towers of states in (4.23) will still exhibit the exponential behaviour in terms of the distance along the path. Recall that the critical case is the maximum deviation from a geodesic still consistent with the exponential behaviour required by the SDC, although the factor in the exponential changes. For a tower with $M_s \sim s^{-a}$ one finds

$$M_s \sim \exp\left(-\frac{\alpha}{\nu}\right)\,, \tag{4.28}$$

with $\alpha = \frac{a}{n}$ being the geodesic decay rate, and the factor of violation of the SDC given by

$$\nu = \sqrt{1 + \frac{m^2}{n^2}\frac{\gamma^2}{u^2}}\,. \tag{4.29}$$

In comparison with the case of dual axion-saxion pair, we find that this factor varies when choosing different $u = \text{const.}$ planes.

This also produces a direct relation, albeit a different one, between ν and the modulus of the proper acceleration. Indeed we find

$$\nu = \frac{1}{\sqrt{1 - m|\Omega|}}\,. \tag{4.30}$$

Hence the existence of two axionic directions on which the path can wind lead to different relations between ν and $|\Omega|$. An implication is that, taking the $\nu \to \infty$ limit, they lead to different discriminating criteria. The critical values correspond to $|\Omega| \to \frac{1}{n}$ and $|\Omega| \to \frac{1}{m}$, when moving along ϕ or ψ respectively.

The difficulty in finding a criterion based solely on the modulus of the proper acceleration is that $|\Omega|$ misses the information about the direction in moduli space. And trying to include this information more explicitly may quickly run into nasty and unphysical dependences on the coordinates chosen in moduli space. In the next section we provide a concrete description which avoids these pitfalls, and yet allows to provide a purely geometrical reformulation of the SDC in general moduli spaces. We will then translate this general criterion into a Convex Hull SDC condition in section 4.3.

4.2 A Geometric formulation of the SDC

Let us recap our approach in general language. Consider some theory satisfying the SDC, i.e. containing towers of states decaying exponentially for every geodesic approaching an infinite distance limit of the moduli space $\mathcal{M}$. When adding a scalar potential lifting some of the directions, we will be left with a new moduli space $\overline{\mathcal{M}}$ whose geodesic trajectories might lift to non-geodesic trajectories in $\mathcal{M}$. To satisfy the SDC in the new IR theory, we need that the level of non-geodesicity of these trajectories is small enough to still allow for exponentially falling towers. Given the field metric of $\mathcal{M}$ and the towers of states, we can always identify the non-geodesic trajectories that are consistent with satisfying the SDC in the IR theory, where the limiting cases are dubbed critical paths. This was done in Section 4.1 for the case of products of hyperbolic planes, obtaining specific bounds for the critical paths. However, this procedure requires specific information about the geometry of the moduli space, so it needs to be worked out case by case.

In this section, we are going to take a step back and reformulate the SDC in a language that will allow us to generalise the results of section 4.1 for general moduli spaces. This will be later translated into a Convex Hull condition in analogy to the WGC in section 4.3. The strategy is to provide a geometric description of the criteria for trajectories to fulfill or violate the SDC. We will keep the nomenclature introduced in section 4.1 to distinguish between the different types of trajectories, namely:

- Asymptotically geodesic paths: they approach a geodesic in the asymptotic limit, so the exponential rate is that of the geodesic.
- Critical paths: non-geodesics that still marginally allow for the exponential decay of the tower, although the exponential rate differs from the geodesic one.
- Swampy paths: they highly deviate from geodesics so the tower does no longer fall exponentially.

4.2.1 Geometric formulation

The general formulation of the SDC establishes that, for any geodesic in moduli space in an infinite distance limit there exists a tower of states with mass scale

$$M = \exp\left(-\alpha\, D_\phi\right) \tag{4.31}$$

with positive $\alpha > 0$. Stronger versions of the conjecture further require $\alpha > \mathcal{O}(1)$, motivated by string theory setups. Let us now focus in the mildest version and simply require $\alpha > 0$, leaving the study of the implications of $\alpha > \mathcal{O}(1)$ for the next subsection.

Consider a trajectory γ approaching an infinite distance point in moduli space. We start by rewriting the exponential decay rate α of the tower mass scale as

$$\alpha(D_\phi) = -\frac{d\log M}{dD_\phi} = -T^i\,\partial_i \log M\,, \tag{4.32}$$

where T is the normalized tangent vector of γ, and M is implicitly evaluated along it.

This rewriting shows that the only information about γ relevant for the SDC is the limiting tangent vector when approaching the infinite distance point. This agrees with our observation that the modulus of the proper acceleration (4.17) is not the right quantity to discriminate the behaviour of asymptotic trajectories.

An important observation is that the set of allowed asymptotic tangent vectors T near a point at infinity is in general restricted, in particular for asymptotically geodesic trajectories.[10] For instance, in the hyperbolic plane case, any curve approaching $s \to \infty$ with bounded ϕ must have $\dot{\phi} \to 0$; hence only the asymptotic tangent vector in the s direction is allowed. On the other hand, relaxing the asymptotic geodesicity requirement allows to explore more general vectors T, as in the case of critical or swampy trajectories. To reflect this fact, we define the subspace $\mathbb{G}$ as that spanned by asymptotic tangent vectors of asymptotically geodesic trajectories.[11]

Let us now consider the implications of the SDC (in its milder version) for such asymptotically geodesic trajectories. We note that what appears in (4.32) is the scalar product between the (limit) tangent vector and the gradient of $\log M$. This implies that a single tower of states along an asymptotically geodesic direction suffices to satisfy the SDC for any other direction, except for the orthogonal ones. Thus, the minimal requirement of the SDC is that there exist as many towers as orthogonal limit tangent vectors in $\mathbb{G}$.

Actually, in general, there may exist other towers of states beyond the above minimal set. Hence it is convenient to consider a new subspace, denoted by $\mathbb{M}$, spanned by the gradient vectors of (log of) the scale M, for all existing towers of states. Note that in many string theory realizations the directions associated to such towers are "dense"; for instance, in a KK compactification on $\mathbf{S}^1 \times \mathbf{S}^1$ near the decompactification limit $R_1, R_2 \to \infty$, there are towers of KK states with masses

$$M^2 = \left(\frac{n_1}{R_1}\right)^2 + \left(\frac{n_2}{R_2}\right)^2 . \tag{4.33}$$

Hence, for any rational direction of γ i.e. $R \equiv R_1/q_1 = R_2/q_2$, there is a tower of states with mass $M \sim n/R$ by taking the states $n_1 = nq_1$, $n_2 = nq_2$.

In terms of these spaces, the mildest version of the SDC (with $\alpha > 0$) can be expressed as:

For any vector in $\mathbb{G}$ (i.e. any asymptotically geodesic tangent vector), there must be at least one non-orthogonal vector in $\mathbb{M}$ (i.e. a suitable tower of states becoming massless).

Equivalently, the projection of $\mathbb{M}$ onto $\mathbb{G}$ should completely fill the latter:

$$\mathcal{P}_{\mathbb{G}}\mathbb{M} = \mathbb{G} . \tag{4.34}$$

Incidentally, this implies that their dimensions satisfy $\dim \mathbb{M} \geq \dim \mathbb{G}$.

This geometric formulation of the SDC resembles a kind of Completeness Hypothesis where the towers of states play the role of the charge spectra in gauge field theories. Analogously, here the role of the charge space is played by the space of asymptotic tangent vectors of asymptotically geodesic trajectories. Stronger conditions - similar to the WGC - will appear when further requiring the towers to satisfy a lower bound for the exponential

[10] Notice that for this it is crucial that the point at infinity is singular. For a regular point, space is locally flat and thus any tangent vector corresponds to a geodesic passing through it.

[11] Strictly speaking, we are interested in the subset containing all asymptotically geodesic vectors, which may not necessarily form a vector subspace. However, being it the case in all the examples at hand motivated by string theory, we will treat $\mathbb{G}$ as a well-defined vector subspace.

rate $\alpha \geq \alpha_0$, with α_0 some order one constant. This mild formulation, though, already allows us to extract interesting conclusions. The first observation is that a single tower of states might not suffice to satisfy the SDC whenever the space $\mathbb{G}$ is spanned by more than one tangent vector while $\mathbb{M}$ remains one-dimensional. This can occur e.g. in higher dimensional moduli spaces in which the tower misses to depend on at least one of the saxions. A second observation is that, by satisfying the above criterion, we are actually fulfilling the SDC along a more general set of trajectories beyond geodesics. We will characterize this set of trajectories in the next subsection.

4.2.2 Non-geodesic bounds

We can now turn to characterizing which trajectories could satisfy or violate the SDC. Indeed, the presence of a single tower makes the mild version of the SDC satisfied for all its non-orthogonal directions, and not only geodesics. Taking into account all possible towers, this defines a subset $\mathcal{T}_{SDC}$ composed by all the directions that satisfy the SDC. In this way, the SDC can be reformulated as imposing that this subset must contain all the asymptotically geodesic directions,

$$\mathbb{G} \subset \mathcal{T}_{SDC}\,. \tag{4.35}$$

Clearly, swampy trajectories violating the SDC will therefore correspond to those not belonging to $\mathcal{T}_{SDC}$. Notice that, for this mild version, $\mathcal{T}_{SDC}$ is just the whole set of directions in moduli space minus the orthogonal complementary of the subspace $\mathbb{M}$. Hence, critical trajectories also belong to $\mathcal{T}_{SDC}$ in this mild formulation.

Let us now turn to the stronger version of the SDC, in which we impose a lower bound for the exponential rate $\alpha \geq \alpha_0$ with α_0 some $\mathcal{O}(1)$ constant. It is reasonable to assume that α cannot take arbitrarily small values as otherwise it would violate the exponential behaviour required by the SDC. Moreover, all string theory examples studied so far have $\mathcal{O}(10^{-1}–10)$, and precise bounds have been given in the context of towers of BPS particles in Calabi-Yau compactifications [36,45]. There, one finds that $\alpha \geq \frac{1}{\sqrt{2n}}$ for a CY_n, implying $\alpha \geq \frac{1}{\sqrt{6}}$ for a CY_3 Type II compactification to four dimensions. A lower bound has also been motivated by using the Transplanckian Censorship Conjecture [37,38,147] or by identifying infinite distance limits with RG flow endpoints of BPS strings in 4d $\mathcal{N}=1$ EFTs [39]. Here, we will not commit to any of these specific values for α_0 although it would be extremely interesting to get a better understanding of this.

We can now extend the last formulation in (4.35) to include the lower bound on α by finding an appropriate definition of $\mathcal{T}_{SDC}$. To do this, let us recast the scalar product in (4.32) in terms of the angle θ between the (limit) tangent vector and the gradient of $\log M$, and get

$$\alpha = -|\partial \log M| \cos\theta\,. \tag{4.36}$$

Here we see that a single tower will make the SDC with $\alpha \geq \alpha_0$ satisfied for any direction such that

$$\cos\theta \leq -\frac{\alpha_0}{|\partial \log M|}\,. \tag{4.37}$$

This is, each tower will then come with an associate cone of directions satisfying the SDC, $\mathcal{C}_M(\alpha_0)$, and defined by (4.37). Therefore, $\mathcal{T}_{SDC}$ will be formed by the union of the associate cones of all the towers of states (see figure 4.1), this is,

$$\mathcal{T}_{SDC} = \bigcup \mathcal{C}_{M_i}(\alpha_0)\,. \tag{4.38}$$

With this definition, the SDC with $\alpha \geq \alpha_0$ reduces again to (4.35). Notice that not all critical trajectories will satisfy (4.37), so only a subset of them will belong to $\mathcal{T}_{SDC}$, whose definition now depends on α_0. We will translate this condition into a convex hull condition in Section 4.3, which provides an equivalent but simplified and more elegant formulation of the above criterion.

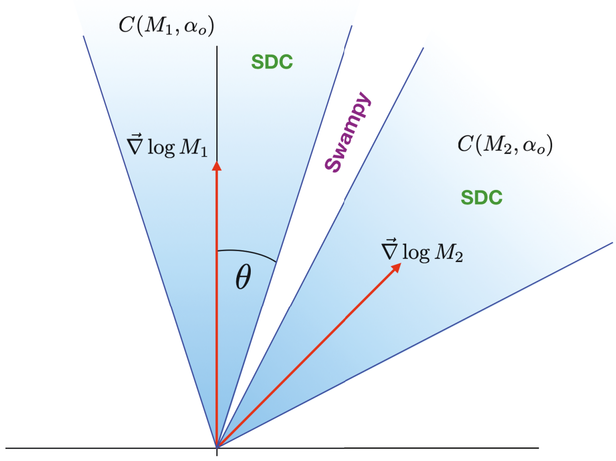

Figure 4.1: Pictorial representation of the subset of directions $\mathcal{T}_{SDC}$. There are two towers of states and their associated cones are represented. Every direction outside both of these cones does violate the SDC for $\alpha \geq \alpha_0$.

In general, determining $\mathcal{T}_{SDC}$ is not only associated to the non-geodesicity of the trajectory but requires full information about the tower of states. However, it becomes a purely geometric condition in the particular case that

$$\mathbb{M} = \mathbb{G} \,. \tag{4.39}$$

In this situation, the realization of the SDC is such that the non-geodesicity of trajectories is directly related to the slow-down of the exponential falloff of state masses along them. It is in this case when the moduli space geometry completely determines the structure of the towers near infinity, as indeed occurred in earlier examples, and in most string theory examples. Hence, it is a natural framework to discriminate the asymptotically geodesic, critical and swampy trajectories, generalizing our discussion in Section 4.1.

Our approach allows to go even further, and also obtain the modification factor ν in the exponential decay rate for the non-geodesic cases, as follows. The exponential rate in (4.32) can be written as

$$\alpha = -(\mathcal{P}_{\mathbb{M}} T)^i \partial_i \log M = -(\mathcal{P}_{\mathbb{G}} T)^i \partial_i \log M \,. \tag{4.40}$$

where we have used (4.39) in the last step. Notice that $\mathcal{P}_{\mathbb{M}} T$ is nothing else than $\cos(\theta)$ defined in (4.36). For a unit vector in $\mathbb{G}$, the result of this expression is the non-vanishing exponential decay rate required to fulfil the SDC along geodesics, i.e.

$$\alpha = |\mathcal{P}_{\mathbb{G}} T| \, \alpha_{\text{geod}}. \tag{4.41}$$

Hence, the factor ν in (4.13) is given by

$$\nu = |\mathcal{P}_{\mathbb{G}} T|^{-1} = (1 - |\mathcal{P}_{\mathbb{G}^\perp} T|^2)^{-1/2}. \tag{4.42}$$

It is straightforward to apply these concepts to recover the results for the hyperbolic space in Section 4.1.1. As can be readily checked from (4.4), the relevant subspaces are spanned by ∂_s,

$$\mathbb{G} = \mathbb{M} = \langle\, \partial_s \,\rangle\,. \tag{4.43}$$

For trajectories $\phi = f(s)$ we have the tangent vector

$$T = \frac{s}{n\sqrt{1 + f'(s)^2}} \left(\partial_s + f'(s)\,\partial_\phi\right). \tag{4.44}$$

Non-geodesic trajectories are those with a nontrivial ∂_ϕ component in their limit tangent vector. Finally, the criterion depending on the modulus of the proper acceleration (4.20) is recovered from (4.42) by checking, in the limit $s \to \infty$, the relation

$$\mathcal{P}_{\mathbb{G}^\perp} T = n|\Omega|. \tag{4.45}$$

One can similarly recover the results for products of hyperbolic spaces from these considerations, as the interested reader in encouraged to check. We instead move on to provide an even more intuitive formulation of these criteria in terms of a Convex Hull condition similar to that used for WGC.

4.3 The Convex Hull SDC

In this Section we formulate the SDC in terms of a Convex Hull condition in the space of asymptotic trajectories. This will let us to easily recover our earlier results about different classes of asymptotic trajectories in a pictorial way which is more familiar in the Swampland program. Moreover, it will also allow us to generalize the story for any combination of towers of states and any asymptotic structure of the field space.

4.3.1 General formulation

Consider a trajectory γ approaching an infinite distance point in moduli space and $\vec{T}$ its normalised tangent vector. As in section 4.2, we denote by $\mathbb{G}$ the subspace spanned only by asymptotically geodesic vectors, i.e. that approach a geodesic trajectory at infinite distance. We have seen that requiring the existence of an infinite tower of states becoming light along any of these asymptotically geodesic trajectories, actually allows for satisfying the SDC along a more general set of trajectories characterised by vectors in $\mathcal{T}_{SDC} \supset \mathbb{G}$. This larger space allows for a certain level of non-geodesicity, including critical paths but excluding swampy trajectories, according to the nomenclature summarised at the beginning of section 4.2. If we require a stronger version of the SDC in which the exponential rate in (4.32) satisfies a lower bound $\alpha \geq \alpha_0$, only a subset of the critical paths will be included in $\mathcal{T}_{SDC}$. In other words, the SDC with $\alpha \geq \alpha_0$ is equivalent to requiring that, for any direction in $\mathbb{G}$, there must exist a tower of states such that the gradient vector of $\log M$ projected onto that direction is sufficiently large. Our goal now is to translate this statement into a convex hull condition.

The key observation is that there is a formal analogy with WGC quantities. The gradient of M can be regarded as the scalar charge of the tower under the moduli. We can also think of $\mathbb{G}$ as the vector space of possible 'charge' directions. Hence, the previous criterion can be rephrased as requiring that *for every charge direction, there must exist a charged infinite tower of states satisfying* $\alpha(D_\phi) \geq \alpha_0$ *asymptotically*, where α_0 is a fixed contant (of order 1) which quantifies the criterion of fast enough decay to satisfy the SDC.

The SDC conditions can thus be formulated in analogy with the scalar version of the WGC [71]. In particular for a tower with (scalar dependent) mass scale M, we can define a scalar charge to mass ratio

$$\vec{z} = -g^{-\frac{1}{2}} \vec{\nabla} \log M \,, \tag{4.46}$$

where $g^{1/2}$ is a matrix whose square is the field metric (more precisely, introducing the n-vein $e_i^a e_j^b \delta_{ab} = g_{ij}$, we have $z^a = -e_i^a g^{ij} \partial_j \log M$). The inclusion of the metric absorbs a piece in (4.32), such that scalar products become cartesian in the following.

The scalar WGC requires the existence of at least one state satisfying $|\vec{z}| \geq \mathcal{O}(1)$, such that the gravitational force acts weaker than the scalar force [71] (see [146] for a different motivation of this proposal). Hence, the order one factor is typically fixed such that states saturating the scalar WGC should feel no force. Unlike with the usual WGC, the order one factor is not associated to extremality of black holes but, for convenience, we will keep the terminology *extremal* to refer to those states saturating the bound. At first glance, it seems that the scalar WGC is different to the SDC, as for the latter what matters is not the modulus of the scalar charge to mass ratio but the projection over a trajectory. However, we will se that the SDC can actually be understood as a Convex hull Scalar WGC in which the extremal states are instead identified as those decaying exponentially with a minimum rate α_0.

Consider a vector space of dimension equal to the number of scalars under consideration, and a general unit vector $\vec{n}$ therein to parametrize the asymptotic behaviour of a general trajectory. This is related to the earlier vector $\vec{T}$ by $n^a = e_i^a T^i$, and is unit norm with respect to the Cartesian dot product. We define the *extremal states* as those with a scalar charge to mass ratio vectors $\vec{z}$ satisfyng

$$\vec{n} \cdot \vec{z} = \alpha_0 \tag{4.47}$$

for some fixed $\alpha_0 > 0$ determining the lower bound for the SDC exponent. From (4.32), we see that for fixed $\vec{n}$, this corresponds to the full set of towers with exponential rate α_0 along the asymptotic trajectory defined by $\vec{n}$. It corresponds to a hyperplane orthogonal to $\vec{n}$, at a distance α_0 from the origin. Scanning over all possible unit vectors,[12] we define the *extremal region* as the enveloping hypersurface defined by the set of all such hyperplanes. It corresponds to a sphere or radius α_0, see figure 4.2.

By allowing the sphere in Figure 4.2 to take any radius, we recover the mildest version of the SDC in which α is an undetermined positive constant, $\alpha > 0$. However, it seems reasonable to consider a finite radius for the extremal ball which cannot be taken parametrically small, as that would spoil the exponential behaviour and violate the SDC. Determining how small α can get is one of the biggest open questions of the SDC and, as explained above, specific lower bounds have been proposed in the literature [36–39,45,147].

[12]Note that we allow for both positive and negative values of all components of $\vec{n}$. This is unphysical for the scalars becoming large in trajectories going off to points at infinity. However, we allow for this possibility at the formal level, to produce a simpler formulation of the convex hull condition, out of which the physical constraints follow from simple restriction of the allowed trajectories.

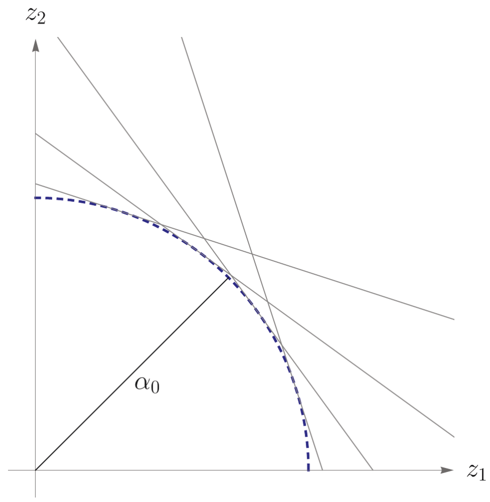

Figure 4.2: The extremal region as envelope of hyperplanes.

One possibility motivated by the key role of the scalar charge to mass ratio above is that α_0 can indeed be determined by using the scalar WGC or some sort of no-force requirement, as it was probably envisioned in [71]. This would be very interesting as it might be used to provide a bottom-up rationale for the SDC.

It is now straightforward to define the SDC in terms of a convex hull[13] condition:

Convex Hull SDC: In a theory with a set of towers corresponding to scalar charge to mass ratios $\vec{z}_i$, the requirement that the SDC is satisfied (with at least decay rate α_0) by any trajectory is exactly the condition that the convex hull of the vectors $\vec{z}_i$ contains the above defined extremal region, namely the unit ball of radius α_0.

Alternatively, it is possible that the the SDC convex hull condition is not satisfied, so the SDC does not hold (with decay rate α_0) for all trajectories, but it still applies to some trajectories. In this situation we can put bounds on the trajectories not to become swampy, constraining the level of non-geodesicity allowed such that the SDC is satisfied. This situation naturally occurs when we start with a UV theory satisfying the SDC and then add a scalar potential lifting some directions, so we are left with a IR moduli space whose geodesics might lift to non-geodesics from the UV perspective. In this case, we can use the convex hull SDC in the UV theory to constrain the allowed set of non-geodesic trajectories that would still allow us to comply with the SDC in the IR. The advantage of this formulation is that it also allows us to incorporate the possibility that new towers of states appear in the IR theory when adding the scalar potential. Hence, the Convex Hull SDC can be used to constrain either:

- the spectra of the theory, by requiring as many towers as needed to satisfy the convex hull condition,
- or the possible trajectories along which the SDC can be satisfied for a fixed set of towers and, therefore, the scalar potentials consistent with quantum gravity.

[13]To achieve a full convex hull, we formally use the method of images and also include the mirror vectors along the negative directions mentioned in footnote 12.

This latter option is not possible in the usual WGC, as the charge lattice is typically a fixed input of the theory.[14] However, it is very natural in the context of the SDC, as the allowed set of trajectories consistent with quantum gravity is still an open question, as it depends in turn on what scalar potentials can be realised in quantum gravity.

4.3.2 Examples

In this Section we illustrate these ideas with examples, reproducing and generalizing the results in previous sections.

4.3.2.1 The hyperbolic plane complex scalar revisited

Consider the case of the hyperbolic plane in section 4.1.1. Using the metric (4.2) the charge to mass ratio vector for a tower with mass scale M is

$$\vec{z} = -\frac{s}{n} \left(\partial_\phi \log M, \partial_s \log M\right) . \tag{4.48}$$

Asymptotically geodesic trajectories have a tangent vector which approaches $\vec{n} = (0, 1)$ asymptotically. Critical trajectories are parametrised by (4.7) with constant $f' = \beta$, so the unit vector is

$$\vec{n} = \frac{1}{\sqrt{1+\beta^2}} (\beta, 1) . \tag{4.49}$$

For these trajectories the vector $\vec{n}$ is constant so that (4.47) corresponds to the equation of a straight line in the plane (z_1, z_2). Different trajectories with different values of f' will give rise to different straight lines, e.g. horizontal lines correspond to a purely saxionic trajectory, and bigger β leads to bigger slopes.

For the particular case of a single tower $M \sim s^{-a}$, c.f. (4.4), we have a single point (and its image) at $\vec{z} = (0, \pm a/n)$. Clearly, the convex hull of these two points does not contain the ball of radius α_0, hence it does not satisfy the SDC for any trajectory. The SDC is satisfied only in the purely saxionic (geodesic) direction if $a/n > \alpha_0$, or trajectories close enough to it. We can then use the convex hull condition to put a bound of how much a trajectory can deviate from the geodesic saxionic trajectory. For this porpuse, we just need to compute the angle $\cos\theta = 1/\sqrt{1+\beta_{\text{max}}^2}$ at which a tangent trajectory to the ball passes by the point $\vec{z} = (0, \pm a/n)$ (see Figure 4.3). This occurs for a trajectory with

$$\beta_{\text{max}} = (\cos\theta)^{-2} - 1 = \left(\frac{a}{n\alpha_0}\right)^2 - 1 \tag{4.50}$$

Hence, critical trajectories with $\beta \leq \beta_{\text{max}}$ will satisfy the SDC with a exponential rate given by

$$\alpha_{\text{crit.}} = \frac{a}{n\sqrt{1+\beta^2}} , \tag{4.51}$$

recovering the result (4.14) in section 4.1.1.

[14] Actually, the charge lattice can vary after higgsing a gauge group. If the higgsing is too large, this can lead to a violation of the WGC in the IR, even if it was originally satisfied in the UV. This is a known loophole of the WGC [149]. From our perspective, by analogy with the SDC, the resolution is that the the amount of higgsing should be restricted in quantum gravity, so the WGC could also be used to constrain the allowed IR charge lattices.

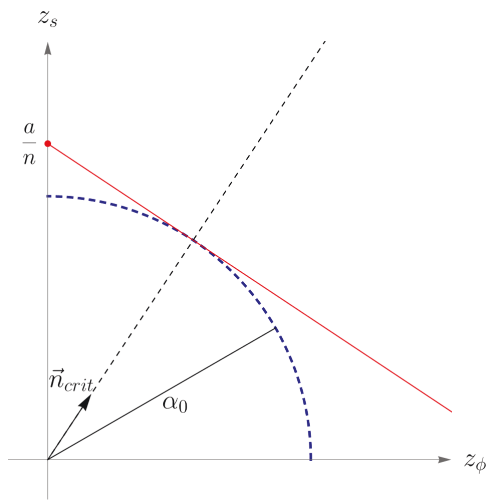

Figure 4.3: The bound on almost saxionic trajectories.

Alternatively, if one is interested in enforcing the SDC for any trajectory, we have to introduce more towers, such that the convex hull of their $\vec{z}$'s encloses the extremal region. In Figure 4.4 we depict situations with different towers and fulfilling, or not, the SDC for any trajectory. It is instructive to compare with the criterion in section 4.2.1 in terms of the cones comprising $\mathcal{T}_{SDC}$, see Figure 4.5.

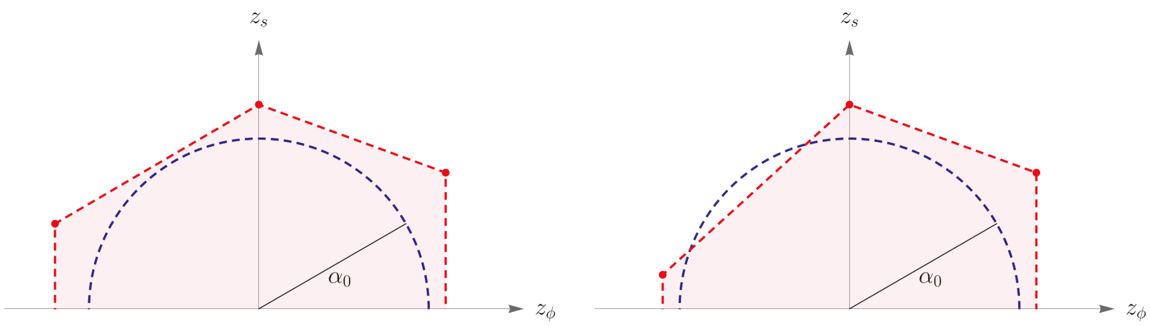

Figure 4.4: The convex hull satisfied or not.

4.3.2.2 Two saxions

Let us consider now a theory with two saxion-like real scalars, namely with a metric

$$dD_\phi^2 = \frac{n_1^2}{s_1^2}\, ds_1^2 + \frac{n_2^2}{s_2^2}\, ds_2^2\,. \tag{4.52}$$

This can be considered as template for the situation with two complex scalars, with hyperbolic space metric (4.22), if we restrict to trajectories not involving the corresponding axions.

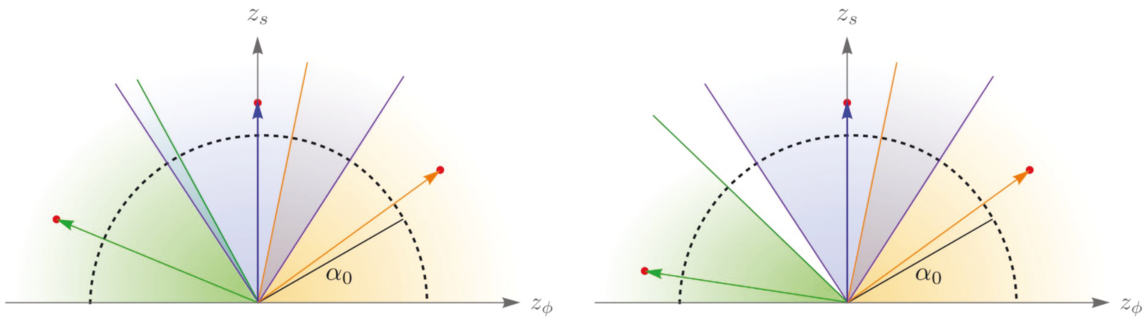

Figure 4.5: Same setups as those shown in Figure 4.4 from the perspective of the subset $\mathcal{T}_{SDC}$.

The scalar charge to mass ratio for a general tower with mass scale $M(s_1, s_2)$ is

$$\vec{z} = -\Big(\frac{s_1}{n_1}\,\partial_{s_1}\,\log M\,,\,\frac{s_2}{n_2}\,\partial_{s_2}\,\log M\,\Big)\,. \tag{4.53}$$

A typical situation is to have two towers, each ensuring the SDC along its corresponding saxionic direction

$$M_1 \sim s_1^{-a_1} \quad , \quad M_2 \sim s_2^{-a_2}\,. \tag{4.54}$$

This corresponds to the values $\vec{z}_1 = (a_1/n_1, 0)$ and $\vec{z}_2 = (0, a_2/n_2)$ respectively. In Figure

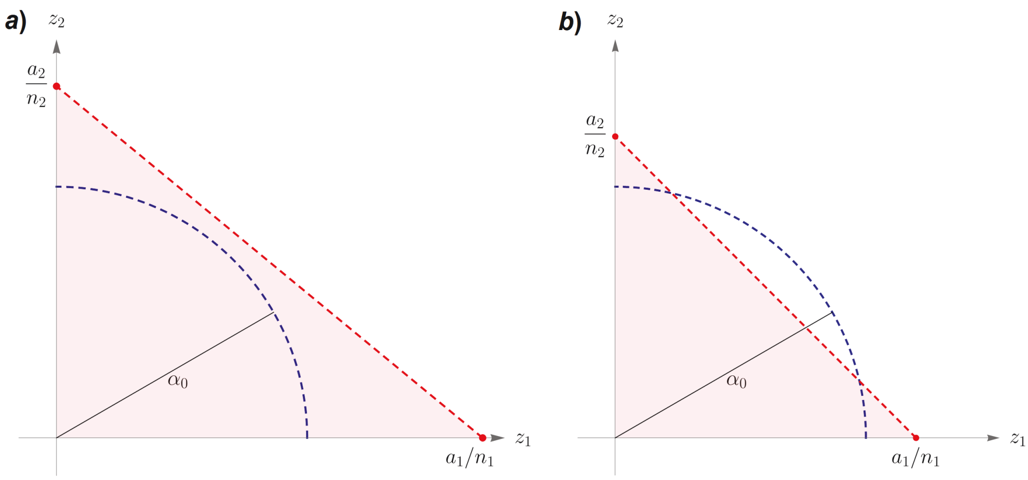

Figure 4.6: The convex hull satisfied or not for two saxions, depending on the specific values of a_i, n_i. For simplicity, we only show the positive quadrant.

4.6 we depict some examples of the corresponding convex hull conditions. Note that even if the SDC is satisfied along each saxion direction individually, it may fail along some other mixed trajectories, see Figure 4.6b. This is reminiscent of similar behaviours in the WGC, see e.g. [89]. The condition that the SDC is satisfied (with decay rate α_0) for any trajectory is straightforward to get from the geometric figure:

$$\frac{a_1 a_2}{n_1 n_2}\left[\left(\frac{a_1}{n_1}\right)^2 + \left(\frac{a_2}{n_2}\right)^2\right]^{-\frac{1}{2}} > \alpha_0\,. \tag{4.55}$$

It is interesting to compare this with the case in which the states are, or are not, mutually BPS. For instance, if we consider that the two towers of states are mutually BPS and can form threshold bound states, we expect there are towers with mass scales

$$M \,=\, q_1\, M_1 \,+\, q_2\, M_2\,. \tag{4.56}$$

For these, the scalar charge to mass ratio is given by

$$\vec{z}_{q_1,q_2} \,=\, \left(\frac{a_1}{n_1}\,\frac{q_1}{M}\, s_1^{-a_1}, \frac{a_2}{n_2}\,\frac{q_2}{M}\, s_2^{-a_2} \right) \tag{4.57}$$

Denoting its two components $\vec{z} = (z_1, z_2)$, they all lie in the hyperplane

$$\frac{n_1}{a_1}\, z_1 \,+\, \frac{n_2}{a_2}\, z_2 \,=\, 1\,, \tag{4.58}$$

which is the line joining the two towers, namely the red dashed line in Figure 4.6. Hence, mutually BPS states do not need to comply with our definition of extremal in the context of the SDC convex hull. This can be important in the case in which the towers correspond to excitation modes of mutually BPS strings, as e.g. in [39]. It also contrasts with the usual WGC, where only BPS states are expected to have a *gauge* charge to mass ratio equal to an extremal black hole. It would be interesting to clarify the interplay of the two notions of extremality to possibly improve on the SDC convex hull formulation.

On the other hand, if we consider towers of states which are not mutually BPS, they might form an ellipse in the scalar charge to mass ratio plane, satisfying the SDC convex hull condition more easily. This can play an important role when checking the SDC in the context of BPS towers of particles in CY compactifications and patching the results from different growth sectors.[15]

4.3.2.3 Decoupled Saxion-Axion

In this section we consider trajectories involving a saxionic scalar s, and an axionic scalar ψ corresponding to a *different* saxion u, namely the metric reads

$$dD_\phi^2 \,=\, \frac{n^2}{s^2}\, ds^2 \,+\, \frac{m^2}{u^2}\, d\psi^2\,. \tag{4.59}$$

Clearly the prefactor of the second term can be removed by redefining ψ, but we prefer to keep it. This allows an easier interpretation of the results as a subsector in a model with two hyperbolic plane complex scalars, c.f.(4.22).

The scalar charge to mass ratio has the form

$$\vec{z} \,=\, -\left(\frac{u}{m}\, \partial_\psi \log M, \frac{s}{n}\, \partial_s \log M \right) \tag{4.60}$$

Asymptotically, the trajectory can be parametrized with s, so it reads $\psi = f(s)$. The corresponding unit vector is

$$\vec{n} \,=\, \frac{1}{\sqrt{\frac{m^2}{u^2} f'^2 + \frac{n^2}{s^2}}} \left(\frac{m}{u} f', \frac{n}{s} \right) = \frac{1}{\sqrt{\frac{m^2}{n^2} \frac{s^2}{u^2} f'^2 + 1}} \left(\frac{m}{n}\frac{s}{u} f', 1 \right), \tag{4.61}$$

[15]Each growth sector corresponds to an specific ordering of the saxions, regarding which one grows faster when approaching the infinite distance loci. Typically, the tests of the SDC in this context focus on each growth sector independently [36,46].

where the last equality is just a convenient rewriting. As derived in section 4.1.2, critical trajectories in this subsector (with u constant) obey $sf' \to \gamma = \text{const.}$, yielding $f(s) \to \gamma \log s$. For these trajectories, the unit vector reads

$$\vec{n} = \frac{1}{\sqrt{1+\beta^2}}(\beta, 1)\,, \tag{4.62}$$

where we have defined $\beta = \frac{m}{nu}\gamma$. If $f(s)$ grows faster, we recover $\vec{n} = (1,0)$ (swampy paths), while if it grows slower we obtain $\vec{n} = (0,1)$ (asymptotically geodesic paths). As in the previous examples, (4.62) scans over different directions in the 2d plane of $\vec{z}$, covering all possible critical trajectories.

It is straightforward to consider different possible towers and analyze whether the Convex Hull SDC is satisfied, or else, which bounds it sets on the parameters of the model and the allowed trajectories. For instance, since (4.62) is formally like (4.49), if we consider a single tower with scaling $M \sim s^{-a}$, we obtain a critical value of the decay rate along the trajectory (4.51). In other words,

$$\gamma = \frac{nu}{m}\sqrt{\left(\frac{a^2}{n^2\alpha_{\text{crit}}^2} - 1\right.}\,. \tag{4.63}$$

from which we can extract the value of α_{crit} in terms of γ. Only along trajectories with $\gamma \leq \gamma(\alpha_{\text{crit}} = \alpha_0)$ the SDC is satisfied. This defines the maximal amount of excitation the axion ψ can have not to spoil a given exponential decay rate α_{crit} along the trajectory.

Interpreting the result as applied to subsector of a two complex scalar model, a trajectory deviating from a geodesic single saxionic direction by exciting the axion of the second complex scalar preserves the SDC if the axion grows with at most the log dependence $\psi = f(s) \to \gamma \log s$ and γ above. We leave to the interested reader the discussion of further possibilities of tower distributions and the corresponding bounds.

Combining the results of sections 4.3.2.1, 4.3.2.2 and 4.3.2.3, we complete the analysis of a two complex dimensional moduli space given by a product of hyperbolic planes. This can be trivially generalised to products of more than two hyperbolic planes. As explained in section 4.4, they are good templates of the asymptotic geometry realised at the infinite distance limits of Calabi-Yau compactifications.

4.4 Constraints on the potential and asymptotic flux compactifications

Throughout this paper, we have argued that consistency of the SDC at any energy scale put constraints on the set of nearly-flat field trajectories allowed by quantum gravity. This is because the moduli space of a theory, and consequently the identification of geodesic paths, varies when going to the IR and integrating out heavy scalar degrees of freedom. But by placing bounds on the trajectories we are actually constraining the scalar potentials consistent with quantum gravity! In this section, we give some first steps translating our bounds to the potential and comparing with previous literature on the asymptotic behaviour of scalar potentials in string theory.

A natural setup in which to apply our above strategy is string theory flux compactifications. These are most often described by starting with a flux-less compactification, with a moduli space on which a potential is subsequently introduced by means of a flux superpotential. The resulting theory may maintain a moduli space of smaller dimension, if moduli stabilization is only partial, or the resulting potential may admit valleys which can be discussed as pseudomoduli. From our vantange point we are thus led to propose that the most general flux compactification must necessarily lead to potentials such that the resulting (pseudo)moduli space still satisfies the SDC. In particular, this implies that geodesics in this (pseudo)moduli space must belong to $\mathcal{T}_{SDC}$ defined in (4.38), and it should be impossible to get a valley along a highly turning trajectory which is not in $\mathcal{T}_{SDC}$. We will see below that that in a fairly general class of models, the flux potentials precisely yield nearly-flat trajectories which are critical according to the definition at the beginning of section 4.2. In other words, the valleys of the potential have the maximum level of non-geodesicity (from the perspective of the original UV moduli space) that it is allowed to satisfy the SDC in the IR.

The asymptotic behaviour of the potential have been considered in quite some detail in CY flux compactifications in [141]. The setup is compactifications of M-theory on Calabi-Yau fourfolds with G_4 fluxes [120, 150], for which the mathematical machinery of asymptotic Hodge theory allows to study the asymptotic form of the flux potential near any infinite distance limit in complex structure moduli space. By taking the F-theory limit, one recovers a 4d $\mathcal{N} = 1$ theory with a flux-induced scalar potential. This allows us to study, not only the more familiar infinite distance limits in perturbative Type IIB/A, but also other types of limits for finite g_s. In the following, we summarize the results of [141] that are relevant to our discussion, in order to reinterpret them from the new perspective advocated in this paper.

All infinite distance limits in complex structure moduli space of Calabi-Yau can be described as the loci of $\hat{n}$ intersecting complex divisors. In an appropriate parametrization, these are described by

$$t^j = \phi^j + is^j \to i\infty\,, \qquad j = 0, \ldots, \hat{n}\,, \tag{4.64}$$

while all the other coordinates remain finite. Taking ϕ^j and s^j to be the axion and saxion of complex scalars, the above limits correspont to sending to infinity some of the saxion vevs. Using the Nilpotent Orbit Theorem [151], one can show (see e.g. [45, 46]) that the Kähler potential takes the following form in the asymptotic limit,

$$K = -\log(p_d(s^j) + \mathcal{O}(e^{2\pi i t^j})) \tag{4.65}$$

where $p_d(s^j)$ is a polynomial of degree d on the saxions, and d characterizes the type of singular limit.[16] More concretely, d is associated to the properties of a monodromy transformation encoding the action of the axionic discrete shift symmetry in the limit. For single moduli limits, i.e. $j = 1$, the field metric exhibits the hyperbolic behaviour studied in Section 4.1.1:

$$dD_\phi^2 = \frac{n^2}{s^2}\left[(ds)^2 + (d\phi)^2\right] + d\Delta_{finite}^2 \tag{4.66}$$

with $n = d/4$ and $d\Delta_{finite}$ only depending on the moduli that are not taken to the asymptotic limit. The same behaviour occurs if we restrict to paths in some *growth sector* in multi-moduli limits. This amounts to approaching the infinite distance limit in such a way

[16] Although this also holds for singular loci at finite distance in moduli space, we restrict the discussion to infinite distance regimes, so $d \neq 0$.

that some axion vevs are much bigger than others. Namely, choosing as suitable ordering, we have $s^1 \gg s^2$, $s^2 \gg s^3$, ... and so on. In this so-called *strict asymptotic regime* we can neglect polynomial terms of the form s^j/s^{j+1}. The leading term of the Kähler potential can then be factorized,[17] yielding

$$dD_\phi^2 = \sum_i \left(\frac{n_i^2}{(s^i)^2}\left[(ds^i)^2 + (d\phi^i)^2\right]\right) + \dots . \tag{4.67}$$

to leading order in the asymptotic limit. Namely, each complex modulus whose saxion is taken to the asymptotic limit parametrizes a hyperbolic plane, c.f. Section 4.1.2. Hereby our motivation to use hyperbolic metrics as toy models to illustrate our proposal and results in this paper.

Interestingly, not only the field metic, but also the flux-induced scalar potential is highly constrained in the infinite distance limits. In particular, it is possible to build an adapted basis of 4-cycles for each growth sector such that the corresponding cohomology group is divided in orthogonal subspaces under the Hodge norm in the strict asymptotic regime [141]. This induces a split of the G_4 flux in different components G_4^ℓ, such that the scalar potential behaves as $V \simeq \sum_\ell \left(||G_4^\ell||^2\right.$. Here $||G_4^\ell||^2$ denotes the Hodge norm of each flux component, whose moduli dependence can be completely determined using the discrete data characterizing the singular limit. Amusingly, the moduli dependence is such that the potential behaves as an homogeneous function[18] to leading order in the large field limit,

$$V(\lambda s^j, \lambda\phi^j) \simeq \lambda^m V(s^j, \phi^j) \tag{4.68}$$

This was exploited in [141] to consider the question of the backreaction on the saxions due to the motion of the axion away from its minimum e.g. along an inflationary valley in an axion monodromy scenario [101, 102] (see [97, 98, 119] for early axion monodromy models unrelated to flux compactifications).

As explained in [141], the resulting backreaction for potentials satisfying (4.68) is of the form

$$s \sim \beta\phi \tag{4.69}$$

with β a flux-independent parameter. The above relation holds for each individual hyperbolic plane independently; namely, a trajectory in which the axions are excited away from their minima necessarily requires the saxions to have a backreaction linear in the corresponding axions. This implies that the valleys of the potential at the asymptotic regimes occur along (4.69), so that e.g. highly turning axionic trajectories are not realised. Therefore, a tower of states decaying exponentially in the saxionic field, will also decay exponentially in terms of the axion, eventually signaling the EFT breakdown for large field variations.

This linear backreaction, and their correlation to the SDC, had previously been noted in certain models of Type II flux compactifications [106], see also [139, 140, 155]. It is also

[17] For each saxion s^j, it is possible to define some integer d_i characterizing the singularity. If all $d_i \neq 0$, then one simply has $n_i = d_i/4$. However, the factorization of K breaks down when some $d_i = 0$ and a more detailed analysis is required. We refer the reader interested in the details of these degenerate cases to [152, 153].

[18] There were a few exceptions in [141] in which the potential was not homogeneous to leading order (see also [154]). However, they are not relevant for our analysis as they do not allow for parametrically large axionic field variations in a controlled regime with $s \gg 1$.

highly correlated to the difficulties for obatining mass hierarchies in these flux compactifications, as studied in [110, 156]. The analysis in [141] supports that this behaviour is universal in flux Calabi-Yau compactifications, as it is tied to asymptotic properties of the moduli space inherited from Hodge Theory. But is it a general feature of potentials consistent with quantum gravity?

We are now ready to reinterpret (4.69) from a new perspective and provide an answer to the above question in view of the results of our paper. Noticing that the asymptotic moduli space metric is that of a hyperbolic plane, the linear result (4.69) corresponds to the critical case of non-geodesic trajectories in Section 4.1.1. Namely, it corresponds to traveling along a trajectory which is as non-geodesic as possible in a way compatible with the distance conjecture. It is very exciting that string theory flux compactification thus saturate the non-geodesicity bound of the hyperbolic plane. It also implies that the flux potentials are consistent with the SDC being satisfied at any energy scale, providing evidence for our proposal.

Clearly, other asymptotic metrics could lead to different parametrizations of the critical paths. But the conclusion of our work is equivalent: the potential should be such that it only generates (pseudo)moduli spaces that ensure consistency of the SDC along the RG flow. This has interesting implications for single field inflation, including axion monodromy models. It would be interesting to turn the question around, and determine from a bottom-up perspective what is the more general form of the potential that generates nearly-flat trajectories corresponding to critical paths. In other words, such that the lightest field is associated to a field direction that coincides with a critical trajectory. One could then try to compare these general bounds on the potentials coming from the Convex Hull SDC with other swampland conjectures constraining the asymptotic form of the potential as the de Sitter conjecture [157].

Before closing this section, we would like to point out that a counterexample to the linear backreaction above was presented in [1] by considering field variations with a spatial dependence. There, it was shown that the stabilization of the breathing mode is such that the resulting light mode avoids the KK tower to fall exponentially when approaching infinite distance. However, this does not contradict our proposal, since this dangerous direction is not a geodesic from the perspective of the low energy pseudomoduli space. In other words, it belonged to the subspace $\mathbb{G}^{\perp}$ of the low energy pseudomoduli space, and thus the SDC was still satisfied in the IR.

4.5 Summary

In this chapter we have discussed the interpretation of the Swampland Distance Conjecture in effective theories with scalar potentials leading to valleys of light fields. We have argued that the SDC is meaningful as a swampland constraint only if it applies at any scale, and that this poses non-trivial constraints of the potentials. We have approached the problem of characterizing these contraints by first studying the structure of non-geodesic trajectories near points at infinity in moduli spaces, and characterizing the constraints implied by the SDC. The analysis is carried out in hyperbolic spaces or products thereof, which provide a good template of general CY moduli spaces near infinite distance loci. We have shown that the critical behaviour of maximal non-geodesicity compatible with the SDC corresponds to axion variations with a linear backreaction on their corresponding saxions. We have argued that this agrees with the structure of flux compactifications near infinite distance loci. This suggests that string theory flux potentials are the most generic ones compatible with the SDC.

We have also reformulated the SDC in terms of a Convex Hull condition, in which scalar charge to mass ratio of SDC towers determine the exponential falloff α along asymptotic trajectories. The SDC is satisfied with an exponential rate lower bounded by α_0 if the convex hull of the scalar charge to mass ratio of the towers includes the ball of radius α_0. This allowed a very intuitive pictorial rederivation of the above mentioned results. For a given set of towers, it can be used to determine the set of trajectories consistent with the SDC, recovering the critical behaviour of maximum non-geodesicty above. Conversely, it can be used to argue for the existence of more than one tower in higher dimensional spaces.

Part III

Dynamical Cobordisms and Distance Conjectures

5

Field Space Distances in Dynamical Cobordisms

In this chapter we turn our attention to another type of running solutions and the field distance that they explore. Their study was pioneered in [158], where they received the name *dynamical cobordisms*. The reason is that they describe how cobordisms between different theories happen dynamically, i.e., in a spacetime-varying solution to the equations of motion.

As reviewed in section 2.3, the Cobordism Conjecture [63] states that any configuration in a consistent theory of quantum gravity should admit, at the topological level, the introduction of a boundary ending spacetime into nothing. Accordingly, we will refer to such boundaries as *walls of nothing*. Equivalently, it implies that any two consistent theories of quantum gravity must admit, at the topological level, an interpolating configuration connecting them, as a generalized domain wall separating the two theories. We will refer to such configurations as *interpolating domain walls*. The Cobordism Conjecture is topological in nature, and it assures the existence of walls of nothing and interpolating domain walls. However it is important to endow them with dynamics to see how they are actually realized in spacetime. This further motivates to study these dynamical cobordisms as running solutions.

An exploration of the Cobordism Conjecture beyond the topological level was undertaken in [158] via the study of spacetime varying solutions to the equations of motion in theories with dynamical tadpoles, namely, potentials which do not have a minimum and thus do not admit maximally symmetric solutions (see [159–162] for early work and [163–166] for related recent developments, and [167, 168] for a complementary approach to cobordism solutions). In the solutions in [158], which we refer to as Dynamical Cobordisms, the fields run along a spatial coordinate until the solution hits a singularity at finite distance in spacetime, which (once resolved in the full UV theory) ends spacetime.

In [158], these solutions were studied in theories with tadpoles for dynamical fields (dubbed *dynamical tadpoles*, as opposed to topological tadpoles, such as RR tadpoles, which lead to topological consistency conditions on the configuration[1]). These are ubiquitous in the presence of scalar potentials, and in particular in non-supersymmetric string models. In theories with dynamical tadpoles the solutions to the equations of motion vary over the non-compact spacetime dimensions[2]. Based on the behaviour of large classes of string models, it was proposed in [158] that such spacetime-dependent running solutions must hit singularity at a finite distance Δ in spacetime (as measured in the corresponding

[1]Note however that dynamical tadpoles were recently argued in [169] to relate to violation of swampland constraints of quantum gravity theories.

[2]see [159–162] for early work and [163–166] for related recent developments, and [167, 168] for a complementary approach to cobordism solutions

Einstein frame metric), scaling as $\Delta^{-n} \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. Furthermore, it was argued that this singularity is resolved in the full UV description to the wall of nothing of the Cobordism Conjecture. On the other hand, interpolating cobordism walls connecting different theories were not discussed. We will address them in this chapter.

In the presence of a dynamical tadpole, the scalars are forced to vary over spacetime in this running solution. It is then a perfect arena for our purpose of studying field excursions in dynamical setups. As the main result of this chapter, we argue that, when a running solution in theories with dynamical tadpoles hits a wall, the sharp distinction between interpolating domain walls and walls of nothing is determined by the behaviour of scalar fields as one reaches the wall via a remarkable correspondence:

- When scalars remain at finite distance points in field space as one hits the wall, it corresponds to an interpolating domain wall, and the solution continues across it in spacetime (with jumps in quantities as determined by the wall properties);
- On the other hand, when the scalars run off to infinity as one reaches the wall (recall, at a finite distance in spacetime), it corresponds to a wall of nothing, capping off spacetime beyond it.

We also argue that scalars reaching singular points at finite distance in moduli space upon hitting the wall still define interpolating domain walls, rather than walls of nothing; hence, walls of nothing are not a consequence of general singularities in moduli space, but actually to those at infinity in moduli space. This suggests that, in the context of dynamical solutions[3], the walls of nothing of the Cobordism Conjecture are closely related to the Swampland Distance Conjecture. Following this logic, and motivated by the Distant Axionic String Conjecture [55], we propose the Cobordism Distance Conjecture. It states that any infinite field distance limit in QG can be explored in a dynamical cobordism configuration, as we approach a wall of nothing. In addition,

We illustrate these ideas in several large classes of string theory models, including massive IIA, and M-theory on CY threefolds. Moreover, we also argue that our framework encompasses the recent discussion of EFT string solutions in 4d $\mathcal{N}=1$ theories in [55] (see also [39]), where saxion moduli were shown to attain infinity in moduli space at the core of strings magnetically charged under the corresponding axion moduli. We show that EFT string solutions are the cobordism walls of nothing of $\mathbf{S}^1$ compactifications of the 4d $\mathcal{N}=1$ theory with certain axion fluxes on the $\mathbf{S}^1$.

The chapter is organized as follows. In Section 5.1 we present the main ideas in the explicit setup of running solutions in massive IIA theory, and their interplay with type I' solutions [170]. In Section 5.2 we carry out a similar discussion for M-theory on CY threefolds with G_4 flux (in Section 5.2.1) and their relation to strongly coupled heterotic strings [171]. In Section 5.2.2 we use it to discuss domain walls across singularities at finite distance in moduli space, following [172]. In Section 5.3 we discuss the $\mathbf{S}^1$ compactification of general 4d $\mathcal{N}=1$ theories. In Section 5.3.1 we introduce dynamical tadpoles from axion fluxes, whose running solutions hit walls of nothing at which saxions run off to infinity. In Section 5.3.2 we relate the discussion to the EFT strings of [55]. In Section 5.4 we discuss the moduli space distances in walls of nothing and interpolating walls in 4d $\mathcal{N}=1$ theories

[3] Note that, in setups with no dynamical tadpole, one can still have e.g. cobordism walls of nothing without scalars running off to infinity: for instance, 11d M-theory, which does not even have scalars, admits walls of nothing defined by Horava-Witten boundaries; similar considerations may apply to potential theories with no moduli (or with all moduli stabilized at high enough scale).

with non-trivial superpotentials of the kind arising in flux compactifications. In Section 5.5 we discuss our proposal in non-supersymmetric string theories, in particular the 10d $USp(32)$ string. In Section 5.6 we offer a summary of the chapter. Appendix B provides some observations on cobordism walls in holographic throats.

This chapter contains part of the results already published in the author's paper [4]. The rest of results in this paper are left for the next chapter.

5.1 Cobordism walls in massive IIA theory

Walls of nothing and infinite moduli space distance

In this section we consider different kinds of cobordism walls in massive IIA theory [173], extending the analysis in [158]. The Einstein frame 10d effective action for the relevant fields is

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_E}\,\{\,[\,R-\frac{1}{2}(\partial\phi)^2\,]-\frac{1}{2}e^{\frac{5}{2}\phi}F_0^2-\frac{1}{2}e^{\frac{1}{2}\phi}(F_4)^2\,\}\,, \tag{5.1}$$

where the Romans mass parameter is denoted by F_0 to suggest it is a 0-form field strength flux. This theory is supersymmetric, but has a dilaton tadpole

$$\mathcal{T} \sim e^{\frac{5}{2}\phi}F_0^2\,, \tag{5.2}$$

so the theory does not admit 10d maximally symmetric solutions. The solutions with maximal (super)symmetry are 1/2 BPS configurations with the dilaton depending on one coordinate x^9, closely related to that in [174]. In conventions closer to [170], the Einstein frame metric and dilaton are

$$(G_E)_{MN} = Z(x^9)^{\frac{1}{12}}\,\eta_{MN}\,,\quad e^{\phi} = Z(x^9)^{-\frac{5}{6}}\,,\quad \text{with}\;\; Z(x^9)\sim -F_0\,x^9\,, \tag{5.3}$$

where we have set some integration constant to zero. The solution hits a singularity at $x^9=0$. The spacetime distance from a general position x^9 to the singularity is [158]

$$\Delta = \int_{x^9}^{0} Z(x^9)^{\frac{1}{24}}\,dx^9 \sim Z(x^9)^{\frac{25}{24}}\,F_0^{-1} \sim F_0^{-1}e^{-\frac{5}{4}\phi} \sim \mathcal{T}^{-\frac{1}{2}}\,, \tag{5.4}$$

in agreement with the scaling relation $\Delta^{-2}\sim\mathcal{T}$, that was dubbed Finite Distance lesson in [158]. Following the Dynamical Cobordism proposal therein, the singularity is resolved in string theory into a cobordism wall of nothing, defined by an O8-plane (possibly dressed with D8-branes to match the F_0 flux to be absorbed)[4], ending the direction x^9 as a boundary.

We now notice that, since $Z\to 0$ implies $\phi\to\infty$ as $x^9\to 0$, the dilaton runs off to infinity in moduli space as one hits the wall, as befits a wall of nothing from our discussion in the introduction.

According to the SDC, there is an infinite tower of states becoming massless in this limit. It signals a breakdown of the effective field theory near the wall of nothing. This fits nicely with our observation that the wall can only be microscopically defined in the

[4] This imposes a swampland bound on the possible values of F_0 that are consistent in string theory.

UV complete theory, and works as a boundary condition defect at the level of the effective theory.

It is a natural question to ask whether this tower of states becoming light is apparent in the description of the wall of nothing in the UV complete theory. From the effective field theory perspective, the SDC tower corresponds to D0-branes which end up triggering the decompactification of the M-theory eleventh dimension. However, in the UV complete description, there are a finite number of extra massless states, responsible for the enhancement of the perturbative open string gauge group to the exceptional symmetries which are known to arise from the heterotic dual theory [170] (see also [175]). On the other hand, there is no signal of an infinite tower of states becoming massless simultaneously. The appearance of the SDC in this context has thus different implications in the EFT and UV complete picture.

Interpolating domain walls

There is a well known generalization of the above solutions, which involves the inclusion of D8-branes acting as interpolating domain walls across which F_0 jumps by one unit. The general solution of this kind is provided by (5.3) with a piecewise constant F_0 and a piecewise continuous function Z [170].

The D8-brane domain walls are thus (a very simple realization of) cobordism domain walls interpolating between different Romans IIA theories (differing just in their mass parameter). The point we would like to emphasize is that, since Z remains finite across them, the dilaton remains at finite distance in moduli space, as befits interpolating domain walls from our discussion in the introduction.

5.2 Cobordism walls in M-theory on CY3

In this section we recall results from the literature on the strong coupling limit of the heterotic string, also known as heterotic M-theory [171,176–178] (see [179,180] for review and additional references). They provide straightforward realizations of the different kinds of cobordism walls in M-theory compactifications on CY threefolds. The discussion generalizes that in [158], and allows to study the behaviour at singular points at finite distance in moduli space, in particular flops at conifold points.

5.2.1 M-theory on CY3 with G_4 flux

We consider M-theory on a CY threefold $\mathbf{X}$, with G_4 field strength fluxes on 4-cycles. For later convenience, we follow the presentation in [172]. We introduce dual basis of 2- and 4-cycles $C^i \in H_2(\mathbf{X})$ and $D_i \in H_4(\mathbf{X})$, and define

$$\int_{D_i} G_4 = a_i \quad , \quad \oint\!\!\oint_{C^i} C_6 = \tilde{\lambda}^i \, . \tag{5.5}$$

We also denote by b_i the 5d vector multiplet of real Kähler moduli, with the usual Kähler metric and the 5d $\mathcal{N}=1$ prepotential

$$G_{ij} = -\frac{1}{2}\frac{\partial^2}{\partial b_i \partial b_j}\ln \mathcal{K} \, , \qquad \mathcal{K} \equiv \frac{1}{3!} d_{ijk} b^i b^j b^k \, , \tag{5.6}$$

with d_{ijk} being the triple intersection numbers of $\mathbf{X}$. We have the familiar constraint $\mathcal{K} = 1$ removing the overall modulus V, which lies in a hypermultiplet.

The 5d effective action for these fields is

$$\begin{aligned} S_5 &= -\frac{M_{p,11}^9}{2} L^6 \left[\int_{M_5} \sqrt{-g_5} \Big(R + G_{ij}(b) \partial_M b^i \partial^M b^j + \frac{1}{2V^2} \partial_M V \partial^M V + \lambda(\mathcal{K} - 1) \Big) \right. \\ &\qquad \left. + \frac{1}{4V^2} G^{ij}(b) a_i \wedge \star a_j + d\tilde{\lambda}^i \wedge a_i \right] - \sum_{n=0}^{N+1} \alpha_i^{(n)} \int_{M_4^{(n)}} \Big(\tilde{\lambda}^i + \frac{b^i}{V} \sqrt{g_4} \Big) \ . \end{aligned} \tag{5.7}$$

Here λ is a Lagrange multiplier, and L the reference length scale of the Calabi-Yau. With hindsight, we include 4d localized terms which correspond to different walls in the theory, with induced 4d metric g_4.

The G_4 fluxes a_i induce dynamical tadpoles for the overall volume and the Kähler moduli b_i. There are 1/2 supersymmetric solutions running in one spacetime coordinate, denoted by y, with the structure

$$\begin{aligned} ds_5^2 &= e^{2A} ds_4^2 + e^{8A} dy^2 \ , \\ V &= e^{6A} \quad , \quad b^i = e^{-A} f^i \ , \\ e^{3A} &= \Big(\frac{1}{3!} d_{ijk} f^i f^j f^k \Big) \ , \\ (d\tilde{\lambda}^i)_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma} e^{-10A} \left(-\partial_{11} b^i + 2 b^i \partial_{11} A \right) . \end{aligned} \tag{5.8}$$

The whole solution is determined by a set of one-dimensional harmonic functions. They are given in terms of the local values of the G_4 fluxes,

$$d_{ijk} f^j f^k = H_i \ , \qquad H_i = a_i y + c_i \ . \tag{5.9}$$

Here the c_i are integration constants set to have continuity of the H_i, and hence of the f_i, across the different interpolating domain walls in the system, which produce jumps as follows. Microscopically, the interpolating domain walls correspond to M5-branes wrapped on 2-cycles $[C] = \sum n_i C^i$, leading to jumps in the fluxes that in units of M5-brane charge are given by

$$\Delta a_i = n_i \ . \tag{5.10}$$

Hence, interpolating domain walls maintain the theory at finite distance in moduli space. This is not the case for cobordism walls of nothing, which arise when $e^A \to 0$ (related to having a singular behaviour for the metric), and hence $V \to 0$, which sits at infinity in moduli space.[5] This regime was already discussed (in the simpler setup of K3 compactifications) in [158], where the cobordism domain was argued to be given by a Horava-Witten boundary (dressed with suitable gauge bundle degrees of freedom, as required to absorb the local remaining G_4 flux), in agreement with the strong coupling singularity discussed in [171]. The wall appears at a finite spacetime distance Δ following the scaling $\Delta^{-2} \sim \mathcal{T}$ in [158].

[5] In fact, in general CYs this limit is not under perturbative control due to instanton corrections. Here we ignore this with the intention of exemplifying a possible behaviour. It would be certainly interesting to include those corrections to this setting.

5.2.2 Traveling across finite distance singularities in moduli space

The setup of M-theory on a CY3 $\mathbf{X}$ allows to address the question of whether walls of nothing could arise at finite distance in moduli space, if the scalars hit a singular point in moduli space. This is actually not the case, as can be explicitly shown by following the analysis in [172] for flop transitions.

Specifically, they considered the flop transition between two Calabi-Yau manifolds with $(h_{1,1}, h_{2,1}) = (3, 243)$, in the setup of a CY3 compactification of the Horava-Witten theory, namely with two boundaries restricting the coordinate y to an interval. In our more general setup, one may just focus on the dynamics in the bulk near the flop transition as one moves along y. Hence we are free to locate the flop transition point at $y = 0$.

In terms of the Kähler moduli $t^i = V^{\frac{1}{3}} b_i$ of $\mathbf{X}$, and changing to a more convenient basis

$$t^1 = U \quad , \quad t^2 = T - \frac{1}{2}U - W \quad , \quad t^3 \;=\; W - U \; , \tag{5.11}$$

and similar (proper transforms under the flop) for $\tilde{\mathbf{X}}$, the Kähler cones of $\mathbf{X}$ and $\tilde{\mathbf{X}}$ are defined by the regions

$$\mathbf{X}: \qquad W > U > 0 \; , \; T > \frac{1}{2}U + W \; , \tag{5.12}$$

$$\tilde{\mathbf{X}}: \qquad U > W > 0 \; , \; T > \frac{3}{2}U \, . \tag{5.13}$$

This shows that the flop curve is C_3, and the area is $W - U$, changing sign across the flop.

Near the flop point $y = 0$, the harmonic functions for the two CYs $\mathbf{X}$ and $\tilde{\mathbf{X}}$ have the form

$$\begin{array}{rcl} \mathbf{X} \text{ at } y \leq 0 & & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ H_T = -18y + k_T & , & \tilde{H}_T = 18y + k_T \; , \\ H_U = -25y + k_0 & , & \tilde{H}_U = 24y + k_0 \; , \\ H_W = 6y + k_0 & , & \tilde{H}_W = -5y + k_0 \; . \end{array} \tag{5.14}$$

Hence

$$\begin{array}{rcl} \mathbf{X} \text{ at } y \leq 0 & & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ H_{W-U} = 31y & , & \tilde{H}_{W-U} = -29y \; . \end{array} \tag{5.15}$$

Even though the flop point is a singularity in moduli space, and despite the sign flip for $W-U$, the harmonic functions are continuous and the solution remains at finite distance in moduli space. This agrees with the picture that it corresponds to an interpolating domain wall. In fact, as discussed in [172], the discontinuity in their slopes (and the related change in the G_4 fluxes) makes the flop point highly analogous to the above described interpolating domain walls associated to M5-branes.

The above example illustrates a further important aspect. It provides an explicit domain wall intepolating between two different (yet cobordant) topologies. It would be extremely interesting to extend this kind of analysis to other topology changing transitions, such as conifold transitions[6] [182]. This would allow for a further leap for the dynamical

[6]For a proposal to realize conifold transitions dynamically in a time-dependent background, see [181].

cobordism proposal, given that moduli spaces of all CY threefolds are expected to be connected by this kind of transitions [183].

We have thus established that physics at finite distance in moduli space gives rise to interpolating domain walls, rather than walls of nothing, even at singular points in moduli space. The implication is that the physics of walls of nothing is closely related to the behaviour near infinity in moduli space and hence to the SDC. In the following section we explore further instances of this correspondence in general 4d $\mathcal{N}=1$ theories.

5.3 $\mathbf{S}^1$ compactification of 4d $\mathcal{N}=1$ theories and EFT strings

In this section we study a systematic way to explore infinity in moduli space in general 4d $\mathcal{N}=1$ theories. This arises in a multitude of string theory constructions, ranging from heterotic CY compactifications to type II orientifolds on CY spaces [184]. Our key tool is an $\mathbf{S}^1$ compactification to 3d with certain axion fluxes. We will show that the procedure secretly matches the construction of EFT strings in [55] (see also [39]). Actually, this correspondence was the original motivation for this paper.

5.3.1 Cobordism walls in 4d $\mathcal{N}=1$ theories on a circle

We want to consider general 4d $\mathcal{N}=1$ theories near infinity in moduli space. According to [45,47,141], the moduli space in this asymptotic regime is well approximated by a set of axion-saxion complex fields, with metric given by hyperbolic planes. We start discussing the single-field case, and sketch its multi-field generalization at the end of this section.

Consider a 4d $\mathcal{N}=1$ theory with complex modulus $S=s+ia$, where a is an axion of unit periodicity and s its saxionic partner. We take a Kähler potential

$$K=-\frac{2}{n^2}\log(S+\bar{S})\ . \tag{5.16}$$

The 4d effective action is

$$\begin{aligned} S&=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-\frac{n^{-2}}{s^2}\left[(\partial s)^2+(\partial a)^2\right]\right\}, \\ &=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-(\partial\phi)^2-e^{-2n\phi}\,(\partial a)^2\right\}, \end{aligned} \tag{5.17}$$

where in the last equation we have defined $\phi=\frac{1}{n}\log ns$.

We now perform an $\mathbf{S}^1$ compactification to 3d with the following ansatz for the metric[7] and the scalars

$$\begin{aligned} ds_4^2&=e^{-\sqrt{2}\sigma}ds_3^2+e^{\sqrt{2}\sigma}R_0^2d\theta^2\,, \\ \phi&=\phi(x^\mu)\,,\qquad a=\frac{\theta}{2\pi}q+a(x^\mu)\,, \end{aligned} \tag{5.18}$$

where x^μ denote the 3d coordinates and $\theta\sim\theta+2\pi$ is a periodic coordinate. Regarding the axion as a 0-form gauge field, the ansatz for a introduces q units of its field strength

[7] We omit the KK $U(1)$ because it will not be active in our discussion.

flux (we dub it axion flux) on the S^1. We allowed for a general saxion profile to account for its backreaction, as we see next.

The dimensional reduction of the action (5.17) gives (see e.g. [185])

$$S_3 = \frac{M_{P,3}}{2} \int d^3x \sqrt{-g_3} \left\{ R_3 - G_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b - V(\varphi) \right\} , \tag{5.19}$$

where

$$G_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b = (\partial\sigma)^2 + (\partial\phi)^2 + e^{-2n\phi} (\partial a)^2 , \tag{5.20a}$$

$$V(\varphi) = e^{-2\sqrt{2}\sigma - 2n\phi} \left(\frac{q}{2\pi R_0} \right)^2 , \tag{5.20b}$$

and $M_{P,3} = 2\pi R_0 M_{P,4}^2$ is the 3d Planck mass.

The last term in the 3d action corresponds to a dynamical tadpole for a linear combination of the saxion and the radion, induced by the axion flux. We thus look for running solutions of the 3d equations of motion. We focus on solutions with constant axion in 3d $a(x^\mu) = 0$, for which the equations of motion read

$$\frac{1}{\sqrt{-g_3}} \partial_\nu \left(\sqrt{-g_3} g^{\mu\nu} \partial_\mu \sigma \right) = -\sqrt{2}\, e^{-2\sqrt{2}\sigma - 2n\phi} \left(\frac{q}{2\pi R_0} \right)^2 , \tag{5.21a}$$

$$\frac{1}{\sqrt{-g_3}} \partial_\nu \left(\sqrt{-g_3} g^{\mu\nu} \partial_\mu \phi \right) = -n\, e^{-2\sqrt{2}\sigma - 2n\phi} \left(\frac{q}{2\pi R_0} \right)^2 . \tag{5.21b}$$

We consider solutions in which the fields run with one of the coordinates x^3 (which with hindsight we denote by $r \equiv x^3$). We focus on a particular 3d axion-saxion ansatz

$$s(r) = s_0 - \frac{q}{2\pi} \log \frac{r}{r_0} \quad , \quad a(r) = a_0 . \tag{5.22}$$

for which the radion can be solved as

$$\sqrt{2}\sigma = \frac{2}{n}(\phi - \phi_0) + 2 \log \frac{r}{R_0} = \frac{2}{n^2} \log \left(1 - \frac{q}{2\pi s_0} \log \frac{r}{r_0} \right) + 2 \log \frac{r}{R_0} . \tag{5.23}$$

This, together with (5.22), provides the scalar profiles solving the dynamical tadpole. The motivation for this particular solution is that it preserves 1/2 supersymmetry, as we discuss in the next section in the context of its relation with the 4d string solutions in [55].

Note that as $r \to 0$, the radion blows up as $\sigma \to -\infty$, implying that the $\mathbf{S}^1$ shrinks to zero size, and the metric becomes singular. As one hits this singularity, the saxion goes to infinity, so we face a wall at which the scalars run off to infinity in moduli space. According to our arguments, it must correspond to a cobordism wall of nothing, capping off spacetime so that the $r < 0$ region is absent; hence the suggestive notation to regard this coordinate as a radial one, an interpretation which will become more clear in the following section. The finite distance Δ to the wall can be shown to obey the scaling $\Delta^{-2} \sim \mathcal{T}$ introduced in [158].

Note that the asymptotic regime near infinity in moduli space $s \gg 1$ corresponds to the regime

$$r \ll r_0 e^{\frac{2\pi}{q}(s_0 - 1)} . \tag{5.24}$$

Hence the exploration of the SDC's implications requires zooming into the region close to the wall of nothing.

Let us emphasize that the microscopic structure of the wall of nothing cannot be determined purely in terms of the effective field theory, and should be regarded as provided by its UV completion[8].

Multi-field generalization

Let us end this section by mentioning that the above simple model admits a straightforward generalization to several axion-saxion moduli a^i, s^i. One simply introduces a vector of axion fluxes q^i and generalizes the above running solution to

$$a^i = a_0^i + \frac{\theta}{2\pi} q^i \quad , \quad s^i(r) = s_0^i - \frac{q^i}{2\pi} \log \frac{r}{r_0} \,. \tag{5.25}$$

The corresponding backreaction on σ is

$$\sqrt{2}\,\sigma = -K(r) + K_0 + 2 \log \frac{r}{R_0} \,. \tag{5.26}$$

We leave this as an exercise for the reader, since the eventual result is more easily recovered by relating our system to the 4d string-like solutions in [55], to which we now turn.

5.3.2 Comparison with EFT strings

The ansatz (5.22) is motivated by the relation of our setup with the string-like solutions to 4d $\mathcal{N}=1$ theories discussed in [55], which we discuss next. This dictionary implies that those results can be regarded as encompassed by our general understanding of cobordism walls of nothing and the SDC.

In a 4d perspective, (5.22) corresponds to a holomorphic profile $z = re^{i\theta}$

$$S \;=\; S_0 + \frac{q}{2\pi} \log \frac{z}{z_0} \,. \tag{5.27}$$

The axion flux in (5.18) implies that there is a monodromy $a \to a + q$ around the origin $z = 0$. Hence, the configuration describes a BPS string with q units of axion charge. The solution for the metric can easily be matched with that in [55]. The 4d metric takes the form

$$ds_4^2 = -dt^2 + dx^2 + e^{2Z} dz d\bar{z} \,, \tag{5.28}$$

with the warp factor

$$2Z = -K + K_0 = \frac{2}{n^2} \log \frac{s}{s_0} \,. \tag{5.29}$$

This matches the 3d metric (5.28) by writing

$$ds_3^2 = e^{\sqrt{2}\,\sigma} \left(-dt^2 + dx^2\right) \Big(+ e^{2Z + \sqrt{2}\,\sigma} dr^2 \,. \tag{5.30}$$

and (5.23) ensures the matching of the S^1 radion with the 4d angular coordinate range.

$$\int_0^{2\pi} d\theta e^{\sigma/\sqrt{2}} R_0 = \int_0^{2\pi} d\theta e^{Z} r \,. \tag{5.31}$$

[8] In particular, possible constraints on q could arise from global consistency of the backreaction.

Hence, in 4d $\mathcal{N}=1$ theories there is a clear dictionary between running solutions in $\mathbf{S}^1$ compactifications with axion fluxes and EFT string solutions. The compactification circle maps to the angle around the string; the axion fluxes map to string charges; the coordinate in which fields run (semi-infinite, due to the wall of nothing) maps to the radial coordinate away from the string; the saxion running due to the axion flux induced dynamical tadpole maps to the string backreaction on the saxion, i.e. the string RG flow; the scalars running off to infinity in moduli space as one hits the wall of nothing map to the scalars running off to infinity in moduli space as one reaches the string core. Note that the fact that the wall of nothing is not describable within the effective theory maps to the criterion for an EFT string, i.e. it is regarded as a UV-given defect providing boundary conditions for the effective field theory fields.

This dictionary allows to extend the interesting conclusions in [55] to our context. For instance, the Distant Axionic String Conjecture in [55] proposes that every infinite field distance limit of a 4d $\mathcal{N}=1$ effective theory consistent with quantum gravity can be realized as an RG flow UV endpoint of an EFT string. We can thus map it into the proposal that every infinite field distance limit of a 4d $\mathcal{N}=1$ effective theory consistent with quantum gravity can be realised as the running into a cobordism wall of nothing in some axion fluxed $\mathbf{S}^1$ compactification to 3d. It is thus natural to extend this idea to a general conjecture

Cobordism Distance Conjecture: *Every infinite field distance limit of a effective theory consistent with quantum gravity can be realized as the running into a cobordism wall of nothing in (possibly a suitable compactification of) the theory.*

The examples in the previous sections provide additional evidence for this general form of the conjecture, beyond the above 4d $\mathcal{N}=1$ context.

5.4 4d $\mathcal{N}=1$ theories with flux-induced superpotentials

In the previous section we discussed cobordism walls in compactifications of 4d $\mathcal{N}=1$ theories on $\mathbf{S}^1$ with axion fluxes. Actually, it is also possible to study running solutions and walls in these theories without any compactification. This requires additional ingredients to introduce the dynamical tadpoles triggering the running. Happily, there is a ubiquitous mechanism, via the introduction of non-trivial superpotentials, such as those arising in flux compactifications. We discuss those vacua and their corresponding walls in this section. The discussion largely uses the solutions constructed in [186], whose notation we largely follow.

Let us consider a theory with a single axion-saxion complex modulus $\Phi = a + iv$. The 4d effective action, in Planck units, is

$$S = -\int d^4x\,\sqrt{-g}\left[\,\frac{1}{2}R + \frac{|\partial\Phi|^2}{4(\mathrm{Im}\,\Phi)^2} + V(\Phi,\overline{\Phi})\right] \tag{5.32}$$

with the $\mathcal{N}=1$ scalar potential

$$V(\Phi,\overline{\Phi}) \,=\, e^K\,(\,K^{\Phi\overline{\Phi}}\,|D_\Phi W|^2\, - 3|W|^2\,)\,. \tag{5.33}$$

We focus on theories of the kind considered in [186], where the superpotential is induced from a set of fluxes m^I, e_I, with $I=0,1$, and is given by

$$W \,=\, e_I f^I(\Phi) - m^I \mathcal{G}_I(\Phi) \tag{5.34}$$

for f^I, $\mathcal{G}_I$ some holomorphic functions whose detailed structure we do not need to specify.

In general, these fluxes induce a dynamical tadpole for Φ, unless it happens to sit at the minimum of the potential. The results in [186] allow to build 1/2 BPS running solutions depending on one space coordinate y with

$$ds^2 \,=\, e^{2Z(y)} dx_\mu dx^\mu \,+\, dy^2 \,. \tag{5.35}$$

For the profile of the scalar, the solution has constant axion a, but varying saxion. Defining the 'central charge' $\mathcal{Z} = e^{\mathcal{K}/2}\, W$ and $\mathcal{Z}_*$ its value at the minimum of the potential (and similarly for other quantities), the profile for the scalar v is

$$v(y) \,=\, v_* \,\coth^2\big(\frac{1}{2}|\mathcal{Z}_*|\, y\,\big) \tag{5.36}$$

Note that in [186] this solution was built as 'the left hand side' of an interpolating domain wall solution (more about it later), but we consider it as the full solution in our setup. Note also that we have shifted the origin in y with respect to the choice in [186].

The backreaction of the scalar profile on the metric is described by

$$Z(y) \,= d \,+\, e^{-\frac{1}{2}\hat{\mathcal{K}}_0}\Big[\,\log(-\sinh(\frac{1}{2}|\mathcal{Z}_*|\, y)) + \log\cosh(\frac{1}{2}|\mathcal{Z}_*|y)\,\Big] \tag{5.37}$$

where d is just an integration constant and $\hat{\mathcal{K}}_0$ is an additive constant in the Kähler potential.

The solution exhibits a singularity at $y = 0$, which (since the metric along y is flat) is at finite distance in spacetime from other points. On the other hand it is easy to see that the scalar v runs off to infinity as we hit the wall, since

$$v(y) \,\to\, 4\, v_*\, |\mathcal{Z}_*|^{-2}\, y^{-2} \quad \text{as } y \to 0\,. \tag{5.38}$$

This all fits very nicely with our picture that the solution is describing a cobordism wall of nothing, and that the solution for $y > 0$ is unphysical and not realized. This provides an effective theory description of the cobordism defects for general 4d $\mathcal{N} = 1$ theories, in a dynamical framework. It would be interesting to find explicit microscopic realizations of this setup.

Let us conclude this section by mentioning that it is possible to patch together several solutions of the above kind and build cobordism domain walls interpolating between different flux vacua. In particular in [186] the solution provided 'the left hand side' of one such interpolating domain wall solution whose 'right hand side' was glued before reaching (in our choice of origin) $y = 0$, hence before encountering the wall of nothing. The particular solution on the right hand side was chosen to sit at the minimum of the corresponding potential, for which there is no tadpole and thus the functions D and v are simply set to constants, fixed to guarantee continuity. Consequently, the solutions remain at finite distance in moduli space, in agreement with our picture for interpolating domain walls. In some sense, the flux changing membrane is absorbing the tadpole, thus avoiding the appearance of the wall of nothing. We refer the reader to [186] (see also [39]) for a detailed discussion.

5.5 Walls in 10d non-supersymmetric strings

The above examples all correspond to supersymmetric solutions, and even the resulting running solutions preserve some supersymmetry. This is appropriate to establish our key results, but we would like to illustrate that they are not restricted to supersymmetric setups. In order to illustrate that these ideas can apply more generally, and can serve as useful tools for the study of non-supersymmetric theories, we present a quick discussion of the 10d non-supersymmetric $USp(32)$ theory [187], building on the solution constructed in [159] and revised in [158][9].

The 10d (Einstein frame) action reads

$$S_E = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\,[\,R - \frac{1}{2}(\partial\phi)^2\,] - T_9^E\int d^{10}x\sqrt{-G}\,64\,e^{\frac{3\phi}{2}}\,, \tag{5.39}$$

where T_9^E is the (anti)D9-brane tension. The theory has a dynamical dilaton tadpole $\mathcal{T}\sim T_9^E g_s^{3/2}$, and does not admit maximally symmetric solutions. The running solution in [159] preserves 9d Poincaré invariance, and reads

$$\begin{aligned} \phi &= \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\log|\sqrt{\alpha_E}y| + \phi_0\,, \\ ds_E^2 &= |\sqrt{\alpha_E}y|^{\frac{1}{9}}\,e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-\frac{3\phi_0}{2}}e^{-\frac{9\alpha_E y^2}{8}}\,dy^2\,, \end{aligned} \tag{5.40}$$

where $\alpha_E = 64k^2T_9$. There are two singularities, at $y=0$ and $y\to\infty$, which despite appearances are located at finite spacetime distance, satisfying the scaling $\Delta^{-2}\sim\mathcal{T}$ introduced in [158]. In this case, there is no known microscopic description for the underlying cobordism defect, but we can still consider the effective theory solution to study its properties as we hit the walls. For instance, given the profile for the scalar in (5.40), it is easy to check that it goes to infinite field distance as any of the two singularities at $y=0,\infty$ are approached. This is in agreement with their interpretation as cobordism walls of nothing.[10]

5.6 Summary

Let us finally summarize the results of this chapter. We have considered running solutions solving the equations of motion of theories with tadpoles for dynamical fields. These configurations were shown to lead to cobordism walls of nothing at finite distance in spacetime [158], in a dynamical realization of the Cobordism Conjecture. We have also discussed interpolating domain walls across which we change to a different (but cobordant) theory/vacuum. We have shown that the key criterion distinguishing both kinds of walls is related to distance in field space: walls of nothing are characterized by the scalars attaining infinite distance in field space, while interpolating domain walls remain at finite distance.

Hence, cobordism walls of nothing provide excellent probes of the structure of the effective theory near infinite distance points, and in particular the Swampland Distance Conjectures. This viewpoint encompasses and generalizes that advocated for EFT strings

[9] For other references related to dynamical tadpoles in non-supersymmetric theories, see [160–165, 188].

[10] The interpretation of the $y\to 0$ singularity as a wall of nothing was deemed unconventional, since it would arise at weak coupling. It is interesting that we get additional support for this interpretation from the moduli space distance behaviour.

in 4d $\mathcal{N} = 1$ theories in [55]. Motivated by this, we propose the Cobordism Distance Conjecture, stating that any infinite field distance limit in QG can be explored dynamically as a wall of nothing is approached.

We have illustrated the key ideas in several large classes of string models, most often in supersymmetric setups (yet with nontrivial scalar potentials to produce the dynamical tadpole triggering the running); however, we emphasize that we expect similar behaviours in non-supersymmetric theories, as we have shown explicitly for the 10d non-susy $USp(32)$ theory.

6

Dynamical Cobordisms to Nothing and Swampland Conjectures

In the previous chapter we have studied the field excursion in solutions describing dynamical cobordisms triggered by dynamical tadpoles. For the case of dynamical cobordisms to nothing, one of the main results was that the solution actually explores infinite field distance as the wall is approached. This feature motivated some interplay with the Distance conjecture. The aim of this chapter is to make this connection more quantitative. The strategy is to look for universal behaviours of the solution as infinite distance in field space is explored. Indeed, the SDC proposes an universal behaviour between the mass scale of some towers of states and the field space distance in this limit. Therefore, it is natural to expect some universal relation between the field space distance and spacetime geometric quantities in the dynamical cobordism solution as one approaches the wall.

For this purpose we want to always keep the effective field theory perspective, from which the solution exhibits a singularity. These singularities are resolved in the full UV description, in terms of the corresponding cobordism configuration. In string theory examples, the latter often admits a tractable microscopic description involving geometries closing-off spacetime[1], possibly dressed with defects, as explained in the previous chapter. In this spirit, they were dubbed 'cobordism defects' or 'walls of nothing'. In this chapter we will mainly focus on the effective field theory description, where they remain as singular sources. To keep this in mind, we will refer to them as End-of-The-World (ETW) branes[2].

A common feature of these solutions is that the infinite field distance is explored in a finite spacetime distance as the singularity is approached. Therefore, two spacetime geometric quantities that are natural to consider for our purposes are the spacetime curvature R and the spacetime distance Δ to the singularity. The main result of this chapter is that they are indeed related to the field space distance D_ϕ by interesting scaling laws, namely (in Planck units)

$$\Delta \sim e^{-\frac{1}{2}\delta D_\phi}, \quad |R| \sim e^{\delta D_\phi}\,. \tag{6.1}$$

We will argue that this scaling relations (6.1) are universal by inspecting several explicit examples. This suggests that a simple universal local description near the ETW branes should be possible in the effective theory. We will also provide this local description by studying Dynamical Cobordisms near walls at which the scalars run off to infinite field space distance. In this local description, the solutions simplify dramatically and are

[1] Spacetimes with boundaries have also been considered in the holographic setup, see [189–195] for some recent approaches.

[2] This follows the nomenclature in some of the references in footnote 1.

characterized in terms of a critical exponent δ, which controls the asymptotic profiles of fields and the scaling relations (6.1) in a very direct way. The analysis does not rely on supersymmetry and can be applied to non-supersymmetric setups.

This provides a powerful universal framework to describe ETW branes within effective field theory. For pedagogical reasons, we first present this analysis and the use it to illustrate the many examples in which the scaling relations are found. We exploit it to describe Dynamical Cobordisms in several 10d string theories, including non-supersymmetric cases. We also use it to characterize warped throats [111,112] as Dynamical Cobordisms. We moreover show that the familiar 10d Dp-brane supergravity solutions can be regarded as Dynamical Cobordisms of sphere compactifications with flux, and are described by our local analysis with the D-branes playing the role of ETW branes. Finally, we argue that 4d small black hole solutions (see [196,197] for some reviews), including those of the recent work [57], can be similarly regarded as Dynamical Cobordisms of $\mathbf{S}^2$ compactifications with flux, with the small black hole core playing the role of ETW brane.

The chapter is organized as follows. In Section 6.1 we present the general formalism for the local description of Dynamical Cobordisms. In section 6.1.1 we present the general equations of motion, and in section 6.1.2 we apply them to describe the local dynamics near ETW branes, and derive the universal scaling relations. In Section 6.2 we apply the local description to several 10d examples, including massive IIA theory in section 6.2.1 and the non-supersymmetric $USp(32)$ theory of [187] in section 6.2.2. In Section 6.3 we interpret D-brane supergravity solutions as Dynamical Cobordisms (section 6.3.1) and express them as ETW branes in the local description (section 6.3.2). Similar ideas are applied in section 6.3.3 to the EFT string in 4d $\mathcal{N}=1$ theories in [55], and in section 6.3.4 to the Klebanov-Strassler warped throat [111,112]. In Section 6.4 we discuss small black holes as Dynamical Cobordisms. In section 6.4.1 we warm up by expressing the supergravity solution of D2/D6-branes on $\mathbf{T}^4$ as a Dynamical Cobordism, and in section 6.4.2 we relate it to small black holes via a further $\mathbf{T}^2$ compactification. In section 6.4.3 we consider more general small black holes, such as those in [57], and derive scaling relations despite the absence of a proper Einstein frame in 2d. In Section 6.5 we discuss the interplay of Swampland constraints with the results of our local description for the behaviour of several quantities near infinity in field space. In section 6.5.1 we consider the Distance Conjecture, the de Sitter conjecture and the Transplanckian Censorship Conjecture. In section 6.5.2 we discuss potentially large backreaction effects when the UV description of the ETW branes involve a large number of degrees of freedom, suggesting mechanisms to generate non-trivial minima near infinity in field space. In section 6.6 we offer some final thoughts. In Appendix C we generalize the ansatz in the main text to allow for non-zero constant curvature in the ETW brane worldvolume directions (section 3.1), and apply it to describe Witten's bubble of nothing as a 4d Dynamical Cobordism and provide its local description (section 3.2). In Appendix D we discuss subleading corrections to the local description, specially relevant in cases where the leading contributions vanish.

6.1 Local Dynamical Cobordisms

In this section we formulate our local effective description near End of The World (ETW) branes, in terms of gravity coupled to a scalar field. We would like to emphasize that we consider a general scalar potential, but remarkably derive non-trivial results for its asymptotic behaviour near infinity in field space. The key input is just that the dynamics should allow for the scalar to go off to infinity in field space in a finite spacetime distance.

Interestingly, the scalar potential generically behaves as an exponential near infinity in moduli/field space, suggesting a first-principles derivation of the 'empirical' evidence for such exponential potentials, coming from string theory and other swampland considerations. In particular, exponential potentials and constraints on them have been discussed in [167, 168], for the restricted case of bubbles of nothing (i.e. UV completed to a purely geometrical higher dimensional configuration, à la [67]). In contrast, our analysis holds for fully general ETW branes (and hence, allows for more general potentials, including cases without this asymptotic growth).

We focus on the case of a single scalar; however, our discussion also applies to setups with several scalars, by simply combining them into one effective scalar encapsulating the dynamics of the solutions (as illustrated in several of our examples in later sections).

6.1.1 General ansatz

Consider d-dimensional Einstein gravity coupled to a real scalar[3] field with a potential,

$$S = \int d^d x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) , \tag{6.2}$$

where we are taking $M_{Pl} = 1$ units. We focus on $d > 2$, and deal with the $d = 2$ case in some explicit examples in section 6.4.

ETW branes define boundaries of the d-dimensional theory, hence they are described as real codimension 1 solutions. We take the ansatz

$$\begin{aligned} ds^2 &= e^{-2\sigma(y)} ds^2_{d-1} + dy^2 \, , \\ \phi &= \phi(y) \, , \end{aligned} \tag{6.3}$$

where y parametrizes the coordinate transverse to the ETW brane.

We consider flat metric in the $(d-1)$-dimensional slices. The corresponding analysis for general non-zero constant curvature, carried out in the same spirit and leading to essentially similar results, is presented in Appendix C.

The equations of motion are

$$\phi'' - (d-1)\sigma'\phi' - \partial_\phi V = 0 \, , \tag{6.4}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V - \frac{1}{2}\phi'^2 = 0 \, , \tag{6.5}$$

$$(d-2)\sigma'' - \phi'^2 = 0 \, , \tag{6.6}$$

where prime denotes derivative with respect to y. The first one is the equation of motion for the scalar; for the Einstein equations, they split into transverse and longitudinal components to the ETW brane, giving two independent equations, subsequently combined into the last two equations.

The analysis of these equations is more amenable in terms of a new quantity, the tunneling potential introduced in [198, 199] (see also [200–205])

$$V_t(\phi) \equiv V(\phi) - \frac{1}{2}\phi'^2 \, . \tag{6.7}$$

[3] Even though our analysis holds for general potential, we often refer to the scalar as modulus, and its field space as moduli space.

Using it to eliminate the scalar from the eoms we get

$$(d-1)\sqrt{2\,(V-V_t)}\,\sigma' - \partial_\phi V_t = 0\,, \tag{6.8}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V_t = 0\,, \tag{6.9}$$

$$(d-2)\sigma'' - 2\,(V-V_t) = 0\,. \tag{6.10}$$

Finally, combining the first two equations to eliminate σ we get

$$\frac{1}{4}(d-2)\left(\partial_\phi V_t\right)^2 + (d-1)\,(V-V_t)\,V_t = 0\,. \tag{6.11}$$

This is a d-dimensional generalization of a condition found in [203] in the context of domain walls.

Now, given a potential $V(\phi)$, one can use this equation to solve for the tunneling potential $V_t(\phi)$, and then use (6.7) and (6.9) to solve for $\phi(y)$ and $\sigma(y)$ respectively. In addition, one should check that (6.10) is also satisfied.

Before moving on, let us comment on the implications that these equations have for the signs of the relevant quantities. From equation (6.9) we learn that $V_t \leq 0$. In addition, from (6.7) we get that $V - V_t \geq 0$. Notice that these two facts are consistent with equation (6.11). Finally, combining the last inequality with (6.10) we learn that $\sigma'' \geq 0$. When solving our system of equations we will systematically pick signs so that these inequalities are satisfied.

A nice way of parametrizing the freedom of choosing the potential is by writing

$$V(\phi) = a(\phi)V_t(\phi)\,, \tag{6.12}$$

where we have to impose that $a(\phi) \leq 1$ for the reason explained above. Plugging this into (6.11) one can easily get to the solution

$$\log\left(\frac{V_t}{V_t^0}\right) = \pm 2\sqrt{\frac{d-1}{d-2}}\int_{\phi_0}^{\phi}\sqrt{1-a(\tilde{\phi})}\,d\tilde{\phi}\,, \tag{6.13}$$

where we are taking $V_t^0 \equiv V_t(\phi_0)$ as boundary condition.

6.1.2 Local description of End of The World branes

As explained in the introduction, we are interested in solutions for which the scalar attains infinity in field space i.e. $\phi \to \pm\infty$ at a point at finite distance in spacetime, defining an ETW brane. Without loss of generality we take this boundary to be $y = 0$, and the infinity in field space as $\phi \to \infty$.

We note that, if $\phi \to \infty$ at a finite spacetime distance, then we also have $|\phi'| \to \infty$ as $y \to 0$. Using $\phi'^2 = 2(V - V_t)$ from (6.7), this tell us that the difference of the potentials goes to infinity at $y \to 0$. One possibility would seemingly be that $|V_t| \ll |V|$ (i.e., $a(\phi) \to -\infty$). However, although one can cook up potentials realizing this possibility, we have not encountered it in any of the string theory examples in later sections. We therefore ignore this possibility in what follows, leaving open the question about the consistency of such behaviors from the point of view of UV fundamental theories.

From (6.13), it is then clear that the asymptotic behavior as $y \to 0$, $\phi \to \infty$ is controlled by the asymptotic constant value of a, so we restrict to constant a. Recalling the constraint $a \leq 1$, note that it includes $a = 0$, which corresponds to solutions with potential negligible with respect to the kinetic energy for the scalar (at least asymptotically).

Taking constant a, (6.13) gives

$$V_t(\phi) \simeq -c\, e^{\delta\,\phi}\,, \tag{6.14}$$

where $c > 0$ is related to the boundary condition used before. As explained in Appendix D, we also allow c to hide some ϕ-dependence, corresponding to subleading corrections. The leading behaviour is an exponential controlled by the critical exponent δ, given by

$$\delta = 2\sqrt{\left(\frac{d-1}{d-2}\right)(1-a)}\,. \tag{6.15}$$

Here we choose the plus sign for δ. As we will see later this will imply that ETW brane explores $\phi \to \infty$ as explained above.

The critical exponent δ controls the structure of the local solution, in particular the asymptotic profile of fields as $y \to 0$, and the scaling relations among different physical local quantities.

Recall that the freedom of choosing a potential is parametrized by a. It is then interesting to ask how the potential itself looks like when approaching the end of the world. Plugging (6.14) into (6.12) we find

$$V(\phi) \simeq -a\, c\, e^{\delta\,\phi}\,. \tag{6.16}$$

Note that we get an exponential dependence, except for $a = 1$, in which case the potentials V may take different forms e.g. power-like, growing strictly slower than exponentials.

Also notice that, since $c > 0$, the sign of the potential is completely determined by that of a. Moreover, using the relation between a and the critical exponent δ in (6.15), we can put bounds on the latter depending on the sign of the potential. Namely, for $V > 0$ we must have $a < 0$, which implies $\delta > 2\sqrt{\frac{d-1}{d-2}}$, while if $V < 0$ then $0 < a < 1$, yielding $\delta < 2\sqrt{\frac{d-1}{d-2}}$. We thus neither have negative potentials whose exponential behaviour is arbitrarily strong, nor positive potentials whose exponential behaviour is arbitrarily mild. The explanation is that such exponentials would lead to $\phi'^2 \gg V$ as we approach the ETW brane, and therefore they correspond to the $a = 0$ case of our analysis.

It is interesting that we have derived fairly generically an exponential shape of the potential near infinity in moduli space, from the requirement that the theory contains ETW branes, namely configurations reaching infinity in moduli space at finite spacetime distance. In section 6.5.1 we will study its interplay with a variety of swampland constraints on scalar potentials. We note however that theories with milder growth of the potential (most prominently, theories with vanishing potential and exact moduli spaces) are still included in the analysis, and correspond to $a(\phi) \to 0$. The corresponding statement that $V \to 0$ in this case actually means that the theory can have any potential as long as it grows slower than ϕ'^2.

From (6.7) we can obtain the asymptotic profile of ϕ as $y \to 0$

$$\phi(y) \simeq -\frac{2}{\delta}\log\left(\frac{\delta^2}{4}\sqrt{\left(2c\,\frac{d-2}{d-1}\right)}\,y\right)\,. \tag{6.17}$$

Here we are ignoring an additive integration constant, irrelevant in the $\phi \to \infty$ limit. We have also fixed another integration constant by demanding that the function blows up for $y \to 0$. The leading term as $y \to 0$ is

$$\phi(y) \simeq -\frac{2}{\delta} \log y \,. \tag{6.18}$$

Hence the scalar goes off to infinity as we approach the end of the world. This motivates the appearance of a lowered cutoff as we approach the wall, above which a more complete microscopic description simply resolves the singularity; this resonates with the swampland distance conjecture, as we discuss in section 6.5.1.

Plugging (6.17) into (6.9) we can also solve for $\sigma(y)$. The final result is

$$\sigma(y) \simeq \pm \frac{4}{(d-2)\delta^2} \log y \,. \tag{6.19}$$

Here we ignore an integration constant which can be reabsorbed by a change of coordinates. Note that, to comply with (6.10), we only need to pick the minus sign.

Furthermore, the d-dimensional scalar curvature is given by

$$R = (d-1)\left(2\sigma'' - d\sigma'^2\right) \sim \frac{1}{y^2} \,. \tag{6.20}$$

We thus recover that the curvature blows up as we approach the end of the world, leading to a naked singularity in the effective field theory description.

Notice that we have ignored a prefactor that, interestingly, vanishes for the special case $\delta^2 = \frac{2d}{d-2}$. For that value one should consider the next-to-leading order term in the $y \to 0$ expansion. In what follows we ignore this case and keep the generic one.

Since the scalar ϕ is normalized canonically, the field space distance D_ϕ as $y \to 0$ is (6.18). Also, the distance in spacetime to the singularity is given by y. Hence from (6.18) and (6.20) we obtain the universal relations

$$\Delta \sim e^{-\frac{\delta}{2} D_\phi} \quad , \quad |R| \sim e^{\delta\, D_\phi} \,. \tag{6.21}$$

The solutions provides a simple universal description of dynamical cobordism in terms of the effective field theory. The microscopic description of the cobordism defect is available only in the UV complete theory, and is thus model-dependent (but known in many cases, see our explicit examples in later sections). From our present perspective, the only microscopic information we need is the very existence of such defects, guaranteed by the swampland cobordism conjecture [63]. It is thus remarkable that, the simple requirement that scalars go to infinity at finite spacetime distance leads to a complete local description of the EFT behaviour near a dynamical cobordism. Moreover, it constrains the structure of the theory, in particular it naturally yields an exponential behavior of the scalar potential near infinity in field space.

The above local description can be used to prove a general relation, introduced in [158], between the dynamical tadpole (defined as the derivative of the potential $\mathcal{T} = \partial_\phi V(\phi)$) at a given point and the spacetime distance Δ to the ETW brane, which in our examples is given by

$$\Delta \sim (\mathcal{T})^{-\frac{1}{2}} \,. \tag{6.22}$$

Indeed, using (6.16) and (6.18), we obtain $\mathcal{T}$ evaluated at a point y^*:

$$\mathcal{T}|_{y=y^*} = \partial_\phi V|_{y=y^*} = -a\, c\, \delta\, e^{\delta\phi}|_{y=y^*} = -a\, c\, \delta\, (y^*)^{-2}\,, \tag{6.23}$$

Δ is constructed as the distance from a point y^* to the singularity at $y = 0$, we therefore have $\Delta = y^*$. We hence have a general relation[4]

$$\Delta = \left(\frac{-\mathcal{T}}{a\, c\, \delta}\right)^{-\frac{1}{2}} \sim (\mathcal{T})^{-\frac{1}{2}}\ . \tag{6.24}$$

This relation places a bound on the spacetime extent of a solution whose running is induced by a dynamical tadpole, as emphasized in [158], due the dynamical appearance of an end of spacetime. We would nevertheless mention that there exist solutions with spacetime boundaries even in situations with no dynamical tadpole. The simplest example is Horava-Witten theory, which corresponds to M-theory on an interval with two boundaries. Even in our present context of scalars running off to infinity at finite spacetime distance, it is possible to find ETW branes in cases with vanishing potential $V = 0$ (or asymptotically negligible potentials, $a = 0$).

6.2 Some 10d Examples

In this section we revisit the examples of 10d theories with Dynamical Cobordism solutions in chapter 5, and use the above local description to easily derive their structure. The results nicely match the asymptotic behavior of the complete solutions in the literature.

6.2.1 The 10d massive type IIA theory

We consider the 10d massive type IIA theory. The effective action in the Einstein frame for the relevant fields is

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left\{R - (\partial\phi)^2 - \frac{1}{2}e^{\frac{5}{2}\sqrt{2}\phi}F_0^2 - \frac{1}{2}e^{\frac{\sqrt{2}}{2}\phi}|F_4|^2\right\}\,, \tag{6.25}$$

where F_0 denotes the Romans mass parameter. The $\sqrt{2}$ factors in the exponents ensure that the normalization of the scalar agrees with our conventions.

This theory has a potential

$$V = \frac{1}{2}e^{\frac{5}{\sqrt{2}}\phi}F_0^2, \tag{6.26}$$

hence it does not admit 10d maximally symmetric solutions. On the other hand there are 9d Poincaré invariant (and in fact 1/2 supersymmetric) running solutions of the equations of motion in which the dilaton (and other fields) depend on a space coordinate, e.g. x^9. The metric and dilaton profile read

$$\begin{aligned} ds_{10}^2 &= Z\left(x^9\right)^{1/12}\eta_{\mu\nu}dx^\mu dx^\nu\,,\\ e^{\sqrt{2}\phi} &= Z\left(x^9\right)^{-5/6}\,, \end{aligned} \tag{6.27}$$

[4] For the particular case of the warped throat in 6.3.4 this corrects the statement in [158].

where the coordinate function is $Z\left(x^9\right) = -F_0 x^9$. This solution hits a singularity at $x^9 = 0$, which was proposed to correspond to an end of the world brane in [158]. In the microscopic theory, it corresponds to an O8-plane (possibly with D8-branes), as in one of the boundaries of the interval of type I' theory [170].

In the following we show how the local structure of the Dynamical Cobordism can be obtained from the analysis in the previous section.

The only input of the local analysis is the potential (6.26). Matching it with the local analysis expression (6.16), we obtain the following values for δ and, using (6.15) for a:

$$\delta = \frac{5}{\sqrt{2}} \quad , \quad a = -\frac{16}{9}. \tag{6.28}$$

Plugging this into (6.18) we obtain the dilaton profile

$$\phi \simeq -\frac{2\sqrt{2}}{5} \log y. \tag{6.29}$$

We can now obtain the profile for σ (6.19)

$$\sigma \simeq -\frac{1}{25} \log y \ , \tag{6.30}$$

which determines the metric via (6.3). As usual, the local description predicts the scalings

$$\Delta \sim e^{-\frac{5}{2\sqrt{2}} D_\phi} \quad , \quad |R| \sim e^{\frac{5}{\sqrt{2}} D_\phi} . \tag{6.31}$$

These results from the local analysis are in agreement with the scaling relations obtained in the paper [158] from the complete solution. In fact, this can be done very easily from (6.27), by a change of coordinates

$$y = \int_0^{x^9} \left(-F_0 \tilde{x}^9\right)^{1/24} d\tilde{x}^9, \tag{6.32}$$

in terms of which the solution acquires the form of

$$dw - ansatz \tag{6.33}$$

$$ds_{10}^2 = \left[\frac{25}{24} (-F_0)\, y\right]^{2/25} ds_9^2 + dy^2 \,,$$
$$e^{\sqrt{2}\phi} = \left[\frac{25}{24} (-F_0)\, y\right]^{-4/5} .$$

This indeed corresponds to profiles for σ (via (6.3)) and ϕ in agreement with (6.30) and (6.29) respectively.

6.2.2 The 10d non-supersymmetric $USp(32)$ string

Let us consider a second example in the same spirit, but in the absence of supersymmetry. We consider the 10d non-supersymmetric $USp(32)$ theory, built in [187] as a type IIB orientifold with a positively charged O9-plane and 32 anti-D9-branes. The 10d Einstein frame action for the relevant fields is

$$S_E = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left\{ R - (\partial\phi)^2 \right\} - T_9^E \int d^{10}x \sqrt{-G}\, 64\, e^{\frac{3}{\sqrt{2}}\phi} . \tag{6.34}$$

We have introduced factors of $\sqrt{2}$ relative to the conventions in [187], to normalize the scalar as in previous sections.

This theory has a dilaton tadpole, due to the uncanceled NSNS tadpoles, and hence does not admit maximally symmetric 10d solution. On the other hand, there are 9d Poincaré invariant running solutions of its equations of motion [159], given by

$$\begin{aligned} ds_E^2 &= |\sqrt{\alpha_E} r|^{\frac{1}{9}}\, e^{-\frac{\alpha_E r^2}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + |\sqrt{\alpha_E} r|^{-1} e^{-\frac{3\phi_0}{\sqrt{2}}} e^{-\frac{9\alpha_E r^2}{8}}\, dr^2\,, \\ \phi &= \frac{3}{4\sqrt{2}}\alpha_E r^2 + \frac{\sqrt{2}}{3}\log|\sqrt{\alpha_E} r| + \phi_0\,, \end{aligned} \tag{6.35}$$

where $\alpha_E = 64\kappa^2 T_9^E$, and ϕ_0 is a reference value for the dilaton. The coordinate r was denoted by y in [159] but here, we preserve y for the coordinate of the local analysis near end of the world branes.

The solution hits two singularities, at $r \to 0$ and at $r \to +\infty$, which are at finite spacetime distance, yet the scalar attains infinity in fields space ($\phi \to -\infty$ at $r \to 0$, and $\phi \to \infty$ at $r \to \infty$, respectively). As discussed in section 5.5, it thus describes a Dynamical Cobordism with two end of the world branes.

Let us now exploit the local analysis to display the scalings near these walls, with the scalar potential in (6.34) as sole input.

6.2.2.1 $r \to 0$

From equation (6.35), we see that $r \to 0$ corresponds to the limit $\phi \to -\infty$. The potential in (6.34) vanishes in that limit. As a consequence, we have an ETW brane in which the potential becomes negligible, i.e., the critical exponents in for the local model are

$$\delta = \frac{3}{\sqrt{2}} \quad , \quad a = 0. \tag{6.36}$$

The local analysis then leads to the dilaton and radion profiles

$$\phi \simeq \frac{2\sqrt{2}}{3}\log y \quad , \quad \sigma \simeq -\frac{1}{9}\log y\,. \tag{6.37}$$

Note that we have chosen the sign of $\phi \to -\infty$ as $y \to 0$.

These results allow to obtain the universal scalings for the curvature and spacetime distance with the field space distance (6.21), namely

$$\Delta \sim e^{-\frac{3}{2\sqrt{2}}D_\phi} \quad , \quad |R| \sim e^{\frac{3}{\sqrt{2}}D_\phi}\,. \tag{6.38}$$

It is easy to check that the above profiles and scaling reproduce the behaviour of the complete solution (6.35). This can be shown by the following coordinate change to bring it into the ansatz (6.3):

$$y = \int \sqrt{|\sqrt{\alpha_E} r|^{-1} e^{-\frac{3\phi_0}{\sqrt{2}}} e^{-\frac{9\alpha_E r^2}{8}}}\, dr \sim \left[\Gamma\left(\frac{1}{4}, \frac{9\alpha_E}{16} r^2\right) - \Gamma\left(\frac{1}{4}, 0\right)\right] \sim \sqrt{r}\,. \tag{6.39}$$

In the last step we have taken the leading behaviour as $r \to 0$. By also taking the leading behaviour in (6.35), plugging in y, and reading off σ as it appears in (6.3) we finally recover the profiles predicted by the local analysis in (6.37).

6.2.2.2 $r \to \infty$

This should be described by a local model where $\phi \to +\infty$ at $y \to 0$, i.e. the origin of a new local coordinate (which corresponds to $r \to \infty$). In this case the potential in (6.34) is blowing up, hence via (6.16) and (6.15), we get $\delta = 3/\sqrt{2}$, $a = 0$, just as in (6.36). The result $a = 0$ may seem puzzling, since from (6.16) this would seem to imply $V \to 0$. However, one should recall that in the local description $a = 0$ simply means that $V \ll \phi'^2$. Indeed, it may happen that c blows up as $\phi \to \infty$ in such a way that it compensates having $a \to 0$ in this same limit. We will explicitly check this later on.

The dilaton and radion profiles read

$$\phi \simeq -\frac{2\sqrt{2}}{3}\log y \quad , \quad \sigma \simeq -\frac{1}{9}\log y \,. \tag{6.40}$$

The dilaton sign differs from (6.37) in order to have $\phi \to +\infty$ as $y \to 0$. We also recover the scalings for Δ and R with D_ϕ, which are again given by (6.38).

Let us now show that the above local model indeed reproduces the $r \to \infty$ regime of (6.35). The required change of variables is now

$$y = \int_r^\infty |\sqrt{\alpha_E}\tilde{r}|^{-1/2} e^{-\frac{3}{4}\phi_0} e^{-\frac{9\alpha_E r^2}{16}} d\tilde{r} \sim \Gamma\left(\frac{1}{4}, \frac{9\alpha_E}{16}r^2\right) \sim r^{-\frac{3}{2}} e^{-\frac{9\alpha_E}{16}r^2} \,. \tag{6.41}$$

The integration limits are chosen so that the finite distance singularity at $r \to \infty$ is located at the origin for the new coordinate. In the last step we have taken the leading behaviour of the Gamma function as $r \to \infty$.

Taking the logarithm of this expression and keeping the leading behaviour we get

$$\log y \simeq -\frac{9\alpha_E}{16} r^2 \,. \tag{6.42}$$

Finally, by also taking the leading behaviour in (6.35), reading off σ as it appears in (6.3) and plugging in our previous expression for y, we recover the profiles anticipated by the local analysis in (6.40).

Let us now come back to the issue of having $a = 0$ while not having vanishing potential. First, let us check that indeed $\phi'^2/V \to \infty$ as we approach the ETW brane. We can compute it, with no approximations, as

$$\frac{\phi'^2}{V} \sim \left(\frac{3\alpha_E}{2\sqrt{2}} r + \frac{\sqrt{2}}{3}\frac{1}{r}\right)^2 , \tag{6.43}$$

where we are ignoring irrelevant numerical prefactors. Importantly, for this computation one has to remember that ϕ' is the derivative with respect to y, not with respect to r. As advanced, we find that this blows up to infinity in both $r \to 0$ and $r \to \infty$ limits.

Moreover, using this result one can compute the tunneling potential as $\phi \to \infty$ as

$$V_t \simeq \frac{\phi'^2}{2} \sim r^2 V \sim \phi\, e^{\frac{3}{\sqrt{2}}\phi} \sim e^{\frac{3}{\sqrt{2}}\phi + \log\phi} , \tag{6.44}$$

where we have plugged in the value of V from (6.34) and $r^2 \sim \phi$ from the $r \to \infty$ limit of $\phi(r)$ in (6.35). As advertised, we find a case in which the coefficient c in (6.14) blows up as we approach the wall of nothing. This is consistent with our local analysis because, as we see in the last equality, c does not blow up faster than the exponential, i.e., it gives subleading corrections to $\log V_t$ (see Appendix D for more details).

6.3 Branes as cobordism defects

The local analysis of Section 6.1 provides a general framework to describe effective ETW branes, encapsulating Dynamical Cobordisms of the underlying theory. An interesting observation is that, in compactified theories with fluxes, the cobordism requires the introduction of charged objects. Namely, those required to break the corresponding cobordism charge to avoid a global symmetry, which should be absent in Quantum Gravity. A typical example is the introduction of NS5- and D-branes in bubbles of nothing in compactifications with NSNS and RR fluxes (see [66] for a recent discussion on bubbles of nothing).

Therefore it is interesting to explore the description of such objects in the local picture of section 6.1. As a simple illustrative setup, in this section we describe the geometry around a stack of Dp-branes in the language of the local analysis of section 6.1. In local terms, it corresponds to regarding the Dp-brane supergravity solution as a compactification of the 10d theory on $\mathbf{S}^{8-p}$ with flux, yielding a $d = (p+2)$-dimensional running solution along one of the coordinates (morally the radial coordinate), which has finite extent and end on an effective ETW brane. The microscopic description of the latter is actually given by the Dp-brane in the UV.

The above idea generalizes the description in 5.3 of the EFT strings solutions in [55] as cobordism defects of $\mathbf{S}^1$ compactifications of the underlying 4d $\mathcal{N} = 1$ theory with axion flux along the $\mathbf{S}^1$.

We note that the compactification of the 10d theory on the $\mathbf{S}^{8-p}$ around a Dp-brane actually corresponds to a truncation onto the $SO(9-p)$-invariant sector. Sphere truncations have long been studied in the literature, in particular in the holographic context, see [206] for a discussion for Dp-brane solutions. However, in our context we should regard the sphere truncation as a fair local description of Dynamical Cobordisms in actual compactifications, including those with scale separation, allowing for a more physical notion of lower-dimensional effective theory. Our local analysis should be regarded as part of the latter. This is depicted in figure 6.1, and is illustrated quantitatively in a similar example for Witten's bubble of nothing in appendix 3.2.

Finally, although we phrase our discussion in terms of Dp-branes, notice that other string theory branes admit similar analysis; in fact, the NS5-brane is essentially the same as the D5-brane, since we are working in the Einstein frame, in which S-duality acts manifestly.

6.3.1 Compactification to a running solution

Let us begin with a precise description of the general procedure of compactifying a codimension $(n+1)$ brane-like solution in $d+n$ dimensions down to a running solution (codimension 1) in d dimensions. In next sections, we will apply this reasoning to the Dp-branes as cobordism defects of $\mathbf{S}^{8-p}$ compactifications.

Take the general metric of a codimension n object in $d+n$-dimensions:

$$ds^2 = e^{-2\mu(r)} ds_{d-1}^2 + e^{2\nu(r)} (dr^2 + r^2 d\Omega_n^2) \,. \tag{6.45}$$

The directions in ds_{d-1}^2 span the worldvolume of the object, while we have split the transverse directions into radial and angular ones.

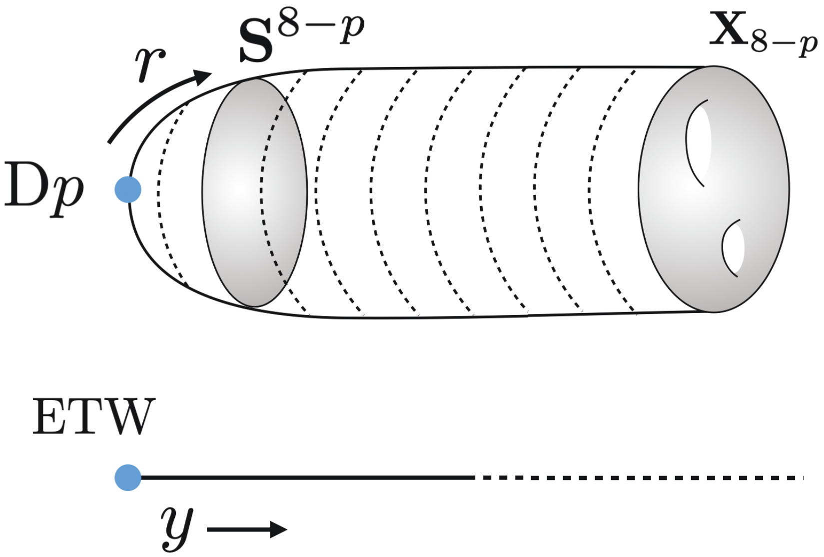

Figure 6.1: Dp-branes as cobordism defect in theories with $(8-p)$ compact dimensions from the higher and lower dimensional perspective. Our local $(p+2)$-dimensional description of the $\mathbf{S}^{8-p}$-truncation corresponds to the local structure of a Dynamical Cobordism of a more general compactification on $\mathbf{X}_{8-p}$.

We want to perform an $\mathbf{S}^n$ truncation to look at this solution from the d-dimensional perspective. We thus take the compactification ansatz

$$ds^2 = e^{-2\alpha\omega(r)} ds_d^2 + e^{2\beta\omega(r)} r_0^2 ds_n^2 \,, \tag{6.46}$$

where r_0 is a reference scale. By requiring that the d-dimensional action is in the Einstein frame and has canonically normalized kinetic term for the radion ω we get the following constraints for α and β:

$$\gamma \equiv \frac{\alpha}{\beta} = \frac{n}{d-2} \qquad \beta^2 = \frac{d-2}{n(d+n-2)} \,. \tag{6.47}$$

The first one implements the Einstein frame requirement, while in the second one we already apply both conditions. Note that for $d=2$ we recover the familiar statement that there is no Einstein gravity in 2 dimensions. We will deal with reductions to 2d in section 6.4, and consider $d>2$ in what follows.

By matching the compactification ansatz (6.46) with the metric in (6.45) we obtain the profile for the radion

$$e^{2\beta\omega(r)} = e^{2\nu(r)} \left(\frac{r}{r_0}\right)^2 \,, \tag{6.48}$$

as well as the lower-dimensional metric

$$ds_d^2 = e^{2\alpha\omega(r)} \left(e^{-2\mu(r)} ds_{d-1}^2 + e^{2\nu(r)} dr^2 \right) \,. \tag{6.49}$$

In order to put solutions in the general form (6.3) used for the local description in section 6.1, we introduce a new coordinate

$$y = \int e^{\alpha\omega(r)} e^{\nu(r)} dr \,, \tag{6.50}$$

in terms of which we can borrow the results (6.16)-(6.21) from the local description.

From the viewpoint of the d-dimensional theory, there is a non-trivial potential arising from the curvature of the $\mathbf{S}^n$, and possibly other sources (such as fluxes, etc). Generically the total potential does not have a minimum, hence the running solutions can be regarded as induced by a dynamical tadpole. Applying the results in [158], the d-dimensional solution must describe a Dynamical Cobordism ending on an ETW brane, to which we can apply the local analysis in section 6.1.

Note that however there are cases with a non-trivial minimum. A prominent example is the $\mathbf{S}^5$ compactification with a large number N of RR 5-form field string flux units (see [115] for a discussion in similar terms). The minimum corresponds to a setup with no tadpole, and admits a maximally symmetric solution, namely the celebrated $\mathrm{AdS}_5 \times \mathbf{S}^5$. Because of this, we will not consider the D3-branes in our discussion, and focus on genuinely running solutions.

6.3.2 D-branes as Dynamical Cobordisms

In this section we regard the 10d Dp-brane solutions as $\mathbf{S}^{8-p}$ compactifications and re-express them in terms of the local description of ETW branes of the $(p+2)$-dimensional theory of section 6.1. Note that, in contrast with section 6.2, we do not intend to *derive* the local solutions from a $(p+2)$-dimensional scalar potential; rather we take the familiar 10d solutions and express their near brane asymptotics as local $(p+2)$-dimensional ETW brane solutions.

Consider the Dp-brane solution in the 10d Einstein frame, with $0 \leq p \leq 8$. The 10d metric and dilaton profile take the form

$$ds^2_{10} = Z(r)^{\frac{p-7}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{p+1}{8}} (dr^2 + r^2 d\Omega^2_{8-p}) \,, \tag{6.51}$$

$$\Phi = \frac{(3-p)}{4\sqrt{2}} \log(Z(r)) \,, \tag{6.52}$$

where the warp factor is given by the harmonic functions

$$Z(r) = 1 + \left(\frac{\rho}{r}\right)^{7-p} \quad \text{for } 0 \leq p \leq 6 \,, \tag{6.53}$$

$$Z(r) = 1 - \frac{N}{2\pi} \log\left(\frac{r}{\rho}\right) \quad \text{for } p = 7 \,, \tag{6.54}$$

$$Z(r) = 1 - \frac{|r|}{\rho} \quad \text{for } p = 8 \,. \tag{6.55}$$

Here $\rho > 0$ is a length scale. For the cases $p \neq 7$ it depends on the number of Dp-branes, N, while for $p = 7$ this dependence does not enter in ρ but has been made explicit in the solution.

As we have explained, these formulas should be regarded as the local description near the D-branes in possibly more general compactifications, namely the above $Z(r)$ should be though of as local expansions around the D-brane location of the warp factor in more general compactification spaces, c.f. Figure 6.1.

We immediately see that for $p \neq 3$, the dilaton reaches infinite values near the point $r = 0$, the core of the Dp-brane. As explained above, we do not consider the case $p = 3$, since it relates to AdS minimum of the theory. The solution does not run towards an ETW brane but towards a minimum in the potential. Similarly, for $p = 8$, the dilaton reaches finite values at $r = 0$. This fits with the identification of $D8$-branes as interpolating walls instead of walls of nothing in section 5.1. In the following we restrict to $p \neq 3$ and $1 \leq p \leq 7$, the lower bound to avoid reduction to 2d (postponed until section 6.4), and the upper bound to have non-trivial sphere compactification.

The Dp-brane is a solution of the following generic type II theory with a dilaton and RR field:

$$S_{10} \sim \frac{1}{2} \int d^{10}x \sqrt{-g_{10}} \left\{ R_{10} - (\partial\Phi)^2 - \frac{1}{2n!} e^{a\Phi} |F_n|^2 \right\} . \tag{6.56}$$

where $n = 8 - p$. This 10d theory does not have a scalar potential. However, once compactified on $\mathbf{S}^{8-p}$ with N units of F_{8-p} flux, the curvature of the sphere as well as the flux itself will generate dynamical tadpoles for the ensuing radion and $(p+2)$-dimensional dilaton. Indeed, let us perform this compactification explicitly and show that we find ourselves in an end-of-the-world scenario.

Taking a compactification ansatz of the form (6.46) we obtain the $d = (p+2)$-dimensional Einstein frame metric:

$$ds_d^2 = \left(\frac{r^2}{r_0^2} Z(r)^{\frac{p+1}{8}} \right)^{\frac{8-p}{p}} \left\{ Z(r)^{\frac{p-7}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{p+1}{8}} dr^2 \right\} , \tag{6.57}$$

where the Greek indices correspond to directions along the world volume of the p-brane. The $(p+2)$-dimensional dilaton inherits the same profile as the original one and one obtains the radion's profile through matching:

$$e^{2\beta\omega(r)} = \frac{r^2}{r_0^2} Z(r)^{\frac{p+1}{8}} . \tag{6.58}$$

The radion is canonically normalized if $\beta^2 = \frac{p}{8(8-p)}$.

The solution has a spacetime singularity at $r = 0$, at which both the dilaton and radion blow up. We can now compute the relevant scaling quantities, namely the spacetime distance Δ_d to the singularity, the curvature scalar $|R_d|$ near the singularity, and the distance D_ϕ traversed in field space. For the former two we obtain:

$$\Delta_d \sim \begin{cases} r^{\frac{(p-3)^2}{2p}} & \text{for } p \in [1,6] \text{ and } p \neq 3 , \\ r^{8/7} & \text{for } p = 7 . \end{cases} \tag{6.59}$$

$$|R_d| \sim \begin{cases} r^{-\frac{(p-3)^2}{p}} & \text{for } p \in [1,6] \text{ and } p \neq 3 , \\ r^{-16/7} & \text{for } p = 7 . \end{cases} \tag{6.60}$$

For the field space distance near the singularity, we obtain the following by plugging in the profiles of the radion (6.58) and dilaton (6.51):

$$D_\phi(r) = \int (d\omega^2 + d\Phi^2)^{1/2} dr \simeq \begin{cases} -\frac{|3-p|}{2}\sqrt{\frac{9-p}{p}} \log r & \text{for } p \in [1,6] \text{ and } p \neq 3\,, \\ -\frac{4}{\sqrt{14}} \log r & \text{for } p = 7\,. \end{cases} \tag{6.61}$$

The solution thus describes Dynamical Cobordisms with the following scaling relations:

$$\Delta_d \sim e^{\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}} D_\phi}\,, \quad |R_d| \sim e^{\frac{2|p-3|}{p}\sqrt{\frac{p}{9-p}} D_\phi} \text{ for } p \in [1,6] \text{ and } p \neq 3\,, \tag{6.62}$$

and

$$\Delta_9 \sim e^{-\frac{2\sqrt{14}}{7} D_\phi}\,, \quad |R_9| \sim e^{\frac{4\sqrt{14}}{7} D_\phi} \text{ for } p = 7\,. \tag{6.63}$$

This shows that Dp-brane are cobordism defects, which reduced on the surrounding $\mathbf{S}^{8-p}$ can be described as ETW branes. In the following we describe their structure in terms of the local description of section 6.1.2. This will allow us a much simpler computation of the above scaling relations.

The objective is to put the d-dimensional metric in domain-wall form (6.3). In the notation of section 6.3.1, one obtains:

$$\sigma(r) = -\alpha\omega(r) + \mu(r) = -\frac{8-p}{p} \log\left(Z(r)^{\frac{p+1}{16}} \left(\frac{r}{r_0}\right)\right) - \frac{p-7}{16} \log\left(Z(r)\right). \tag{6.64}$$

The new coordinate y is obtained as

$$y = \int^r e^{\alpha\omega(r)} e^{\nu(r)} dr = \int^r Z(r)^{\frac{p+1}{2p}} \left(\frac{r}{r_0}\right)^{\frac{8-p}{p}} dr\,. \tag{6.65}$$

For a general Dp-brane with $p \neq 3, 7$, in the limit $r \to 0$ we have

$$y = \int^r \left(\frac{\rho}{r}\right)^{(7-p)\frac{p+1}{2p}} \left(\frac{r}{r_0}\right)^{\frac{8-p}{p}} dr \sim r^{\frac{(p-3)^2}{2p}}\,. \tag{6.66}$$

Using equation (6.64), this yields

$$\sigma(r) \simeq \sigma\left(y^{\frac{2p}{(p-3)^2}}\right) \simeq -\frac{(9-p)}{(p-3)^2} \log y\,. \tag{6.67}$$

We may compare this to the profile for σ put forward by the local description described in section 6.1.2:

$$\sigma(y) \simeq -\frac{4}{p\,\delta^2} \log y\,. \tag{6.68}$$

We can thus extract the value of δ and, for completeness, that of a:

$$\delta^2 = \frac{4(p-3)^2}{p(9-p)}\,, \qquad 1 - a = -\frac{(p-3)^2}{(p-9)(p+1)}\,. \tag{6.69}$$

Thus, we have, from equation (6.18):

$$D_\phi(y) \simeq -\frac{2}{\delta} \log y \simeq -\frac{|p-3|\sqrt{9-p}}{2\sqrt{p}} \log r\,. \tag{6.70}$$

We have thus recovered exactly the profile (6.61), without having to use the explicit scalar profile. From (6.21), we also recover the scaling relations (6.62), namely:

$$\Delta_d = y \sim e^{-\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}}D_\phi}\,, \qquad |R_d| \sim e^{2\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}}D_\phi}\,. \tag{6.71}$$

Hence, in this case we have used the local description to recover the field-space distance and scaling relations near the singularity without knowing the full details of the d-dimensional theory. In fact, we can use the local description to derive the asymptotic behaviour of interesting d-dimensional quantities. For instance, the scalar potential scales near the singularity as (6.16):

$$V(D_\phi) = -c\left(1 - \frac{(p-3)^2}{(9-p)(p+1)}\right)\left(e^{\frac{2(p-3)}{\sqrt{p(9-p)}}D_\phi}\right). \tag{6.72}$$

This is a very interesting bottom-up approach. In the actual d-dimensional action, the potential would depend on the radion and dilaton with contributions from the curvature of the sphere and the flux traversing it. However, the local description encapsulates only the dependence on the effective scalar dominating the field distance D_ϕ near the ETW brane, erasing any other irrelevant UV information. From the previous equation we find that the potential is negative as we approach the ETW brane (recall $c > 0$). With the extra input that the curvature and the flux contributions to the potential are negative and positive respectively, the local description is then telling us that it is the curvature term the one that dominates in this limit.

For the D7-brane, the coordinate y is given by

$$y = \int e^{\alpha\omega(r)}e^{\nu(r)}dr = \int \left(\left(-\frac{N}{2\pi}\log\left(\frac{r}{\rho}\right)\right)^{\frac{4}{7}}\left(\frac{r}{r_0}\right)^{\frac{1}{7}}\right)dr \sim r^{\frac{8}{7}}\,, \tag{6.73}$$

where we have neglected the logarithmic contribution compared to the polynomial one. Similarly, we have:

$$\sigma(r) \simeq \sigma(y^{\frac{7}{8}}) \simeq -\alpha\omega(y^{\frac{7}{8}}) \simeq -\frac{1}{8}\log y\,. \tag{6.74}$$

Hence, comparing this to equation (6.19), we find:

$$\delta^2 = \frac{32}{7}\,, \qquad a = 0\,. \tag{6.75}$$

This means that the asymptotic potential vanishes, in the sense of $\phi'^2 \gg V$. Plugging this value of δ^2 into equation (6.18) and (6.21), we recover the same field space distance and scaling relations as in the computations of the previous section:

$$D_\phi(y) \simeq -\sqrt{\frac{7}{8}}\log y \simeq -\frac{4}{\sqrt{14}}\log r\,, \tag{6.76}$$

$$\Delta_9 = y \sim e^{-\sqrt{\frac{8}{7}}D_\phi}\quad,\quad |R_9| \sim e^{2\sqrt{\frac{8}{7}}D_\phi}\,. \tag{6.77}$$

6.3.3 Revisiting the EFT strings

In [39, 55] it was proposed that in 4d $\mathcal{N}=1$ theories the limits in which saxionic scalars go to infinity in moduli space can be studied as radial flows in 4d supersymmetric EFT string solutions magnetically charged under the corresponding axionic partners. In section 5.3 the result was recovered by considering running solution of the compactification of the theory to 3d with axion fluxes along the $\mathbf{S}^1$: the solutions implement a Dynamical Cobordism ending spacetime along the running direction, and the EFT string arises as the cobordism defect required to get rid of the axion flux. In this section we check the universal scaling relations from the local description. As expected, the analysis is fairly similar to the 10d D7-brane example in the previous section; indeed, upon compactification of the 10d theory on a CY3, the wrapped D7-branes turns into the simplest avatar of the EFT strings in [39, 55].

In the 4d EFT string solution [39, 55], the profile for the scalars is given by

$$s(r) = s_0 - \frac{q}{2\pi}\log\frac{r}{r_0}\,, \tag{6.78}$$

$$a(\theta) = a_0 + \frac{\theta}{2\pi}q\,. \tag{6.79}$$

In our 3d interpretation, equation (6.79) describes the axionic flux over the $\mathbf{S}^1$, and equation (6.78) solves the dynamical tadpole for the saxion.

The 4d metric takes the form

$$ds_4^2 = -dt^2 + dx^2 + e^{2D}dzd\bar{z}\,, \tag{6.80}$$

with $z = re^{i\theta}$. The warp factor is given by

$$2D = -K + K_0 = \frac{2}{n^2}\log\frac{s}{s_0}\,, \tag{6.81}$$

where the Kähler potential is $K = -\frac{2}{n^2}\log s$.

Matching the 4d metric (6.80) to the setup in section 6.3.1 with $n=1$, we obtain the 3d coordinate y:

$$y = \int^r e^{\alpha\omega(r)}e^{\nu(r)}dr = \int^r \left(1 - \frac{q}{2\pi s_0}\log\frac{r}{r_0}\right)^{\frac{2}{n^2}}\frac{r}{r_0}dr \sim r^2\,, \tag{6.82}$$

where we have once more neglected the logarithm compared to the polynomial contribution. Then, we can put the 3d metric in the domain-wall form (6.3), in the $r\to 0$ limit, with:

$$\sigma(y^{\frac{1}{2}}) = -\gamma\beta\omega(y^{\frac{1}{2}}) \simeq -\log\left(\left(1 - \frac{q}{2\pi s_0}\log\frac{y^{\frac{1}{2}}}{r_0}\right)^{\frac{1}{n^2}}\frac{y^{\frac{1}{2}}}{r_0}\right) \simeq -\frac{1}{2}\log y\,. \tag{6.83}$$

Comparing this to (6.19), we obtain

$$\delta^2 = 8 \quad , \quad a = 0\,. \tag{6.84}$$

We can use these parameters to recover the profiles and scaling of the local solution. For instance, we obtain that $\phi'^2 \gg V$, as in the D7-brane case. We also obtain the field-space profile and scaling relations from (6.18) and (6.21):

$$D_\phi(y) \simeq -\sqrt{\frac{1}{2}}\log y\,, \tag{6.85}$$

$$\Delta = y \sim e^{-\sqrt{2}\,D_\phi}\,, \qquad |R| = e^{2\sqrt{2}\,D_\phi}\,. \tag{6.86}$$

We thus find that the full solution can be described in terms of the local description, with the EFT string described in terms of an ETW brane.

6.3.4 The Klebanov-Strassler throat

In the previous examples we have shown that D-branes can play the role of ETW branes in running solutions of compactifications with fluxes. We would like to mention, however, an alternative mechanisms in which Dynamical Cobordisms can get rid of fluxes in the compactification, namely when the running involves axion monodromy[5]. This is most clearly illustrated in the celebrated Klebanov-Strassler (KS) solution [112], related to the compactification of type IIB theory on the 5d Sasaki-Einstein space $T^{1,1}$ with N units of RR 5-form flux and M units of RR 3-form flux on an $\mathbf{S}^3 \subset T^{1,1}$.

As shown in [158], the KS solution can be regarded as a Dynamical Cobordism, in which the tip of the throat ends spacetime at finite spacetime distance in the radial direction, smoothing out (or UV completing) the singularity of the related Klebanov-Tseytlin (KT) solution [111]. In this section we show that the structure of the KT solution is indeed that of an ETW brane from the viewpoint of the 5d effective theory.

Consider the KT solution [111], whose 10d Einstein frame metric reads:

$$ds_{10}^2 = h^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(r)\left(dr^2 + r^2 ds_{T^{1,1}}^2\right), \tag{6.87}$$

with

$$h(r) = b_0 + \frac{M^2 \log\left(r/r_*\right)}{4r^4}\,. \tag{6.88}$$

The singularity is at r_s such that $h(r_s) = 0$, signalling the location of the ETW brane. One can show that $\partial_r h \neq 0$ at $r = r_s$, hence we may expand this harmonic function near this point as

$$h(r) \sim r - r_s \equiv \tilde{r}\,. \tag{6.89}$$

We now take the compactification ansatz

$$ds_{10}^2 = L^2\left(e^{-5q}ds_5^2 + e^{3q}ds_{T^{1,1}}^2\right) \tag{6.90}$$

with L an overall scale. Matching with (6.87) we get the profile for the breathing mode

$$q(r) = \frac{1}{6}\log\left(\left(\frac{r}{L}\right)^4 h(r)\right) \simeq \frac{1}{6}\log\tilde{r}\,, \tag{6.91}$$

where in the last equality we have taken the near ETW limit. We also get the 5d Einstein frame metric:

$$L^2 ds_5^2 = \left(\left(\frac{r}{L}\right)^2 h^{\frac{1}{2}}\right)^{\frac{5}{3}}\left(h^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}dr^2\right). \tag{6.92}$$

From it we can derive the relation between $\tilde{r}$ and the radial coordinate y in the local analysis, which is

$$\tilde{r} \sim y^{\frac{3}{5}}\,. \tag{6.93}$$

[5] For axion monodromy in inflation, see [97–99, 101, 103, 104, 119, 207]

Reading off the warp factor

$$e^{-2\sigma} = \left(\left(\frac{r}{L}\right)^2 h^{\frac{1}{2}}\right)^{\frac{5}{3}} h^{-\frac{1}{2}} \sim \tilde{r}^{\frac{1}{3}} \sim y^{\frac{1}{5}}\,, \tag{6.94}$$

we finally find

$$\sigma(y) \simeq -\frac{1}{10}\log y\,. \tag{6.95}$$

Hence, the 5d KT solution near the singularity fits with the form of an ETW brane in our local description with

$$\delta = \frac{2\sqrt{30}}{3} \quad , \quad a = -\frac{3}{2}\,. \tag{6.96}$$

We can also check that the solution for the scalars also fits in the local model description. The NSNS axion is given by

$$T(r) = \tilde{T} + M\log r \simeq T_s + \frac{M}{r_s}\tilde{r}\,, \tag{6.97}$$

again in the near ETW brane limit. Here $T_s = T(r_s)$, which we can keep arbitrary. The field space metric from the 5d action in [111] is given by

$$dD_\phi^2 = 30(\partial q)^2 + \frac{1}{2}g_s^{-1}e^{-6q}(\partial T)^2\,. \tag{6.98}$$

Using the profiles for q and T in the $\tilde{r}\to 0$ limit, we have

$$(\partial q)^2 \simeq \frac{1}{36\tilde{r}^2} \qquad , \qquad e^{-6q}(\partial T)^2 \simeq \left(\frac{M}{r_s}\right)^2\frac{1}{\tilde{r}}\,. \tag{6.99}$$

For $\tilde{r}\to 0$, the breathing modes dominates the field space distance in field-space. Following the discusion in chapter 4, it is then an asymptotically geodesic trajectory. This is in contrast with the $r\to\infty$ limit, for which the field-space trajectory was shown to be highly non-geodesic in chapter 3. Hence we have

$$dD_\phi^2 \simeq 30(\partial q)^2 \simeq \frac{5}{6}\tilde{r}^{-2}\,. \tag{6.100}$$

Upon integration and using (6.93) we obtain

$$D(y) \simeq -\frac{\sqrt{30}}{10}\log y\,. \tag{6.101}$$

This again takes the form found in our local analysis, for the above coefficients (6.96).

Finally, we also check that the 5d scalar potential from [111] scales as predicted by the local model. The complete potential is

$$V(\phi) = -5e^{-8q} + \frac{1}{8}g_sM^2e^{-14q} + \frac{1}{8}(N+MT)^2e^{-20q}\,. \tag{6.102}$$

Plugging in $T = T_s$ and $D \simeq -\sqrt{30}\,q$ as dictated by (6.100), we get

$$V(D_\phi) = -5e^{\frac{4\sqrt{30}}{15}D_\phi} + \frac{1}{8}g_sM^2e^{\frac{7\sqrt{30}}{15}D_\phi} + \frac{1}{8}(N+MT_s)^2e^{\frac{2\sqrt{30}}{3}D_\phi}\,. \tag{6.103}$$

For $N + MT_s \neq 0$, we find that the last term dominates as $D_\phi \to \infty$. As predicted by our local analysis, it has an exponential behaviour with D_ϕ with the coefficient δ given in (6.96). Moreover, as predicted by finding $a < 0$, the coefficient in front of this exponential is positive.

We hope these examples suffice to convince the reader that the local description provides a simple and efficient framework to discuss the structure of Dynamical Cobordisms near the ETW brane.

6.4 Small Black Holes as Dynamical Cobordisms

The analysis of the previous section for single-charge D-brane solutions can be similarly carried out for systems of multiple charges, namely combining D-branes of different dimensionalities. Such systems have been extensively employed in the construction and microscopic understanding of black holes, both with finite horizon, starting with [14], or with vanishing classical horizon area (small black holes) (see [196, 197] for some reviews). In this section we describe brane configurations, closely related to the celebrated D1/D5 system, leading to small black holes, and describe them as cobordism defects of suitable sphere compactifications of the underlying theory. The resulting dimensionally truncated theory corresponds to a 2d theory of gravity and an effective scalar (2d dilaton gravity), for which we find scaling relations analogous to the higher dimensional cases. This description relates the Dynamical Cobordisms to the realization of the Swampland Distance Conjecture in small black holes[6] in [57].

6.4.1 The D2/D6 system on $\mathbf{T}^4$

We consider a configuration of D6- and D2-branes in the following (1/4 susy preserving) configuration

$$D6: \; 0 \; 1 \; 2 \; \times \; \times \; \times \; 6 \; 7 \; 8 \; 9 \tag{6.104}$$
$$D2: \; 0 \; 1 \; 2 \; \times \; \times \; \times \; \times \; \times \; \times \; \times \tag{6.105}$$

where the numbers correspond to directions spanned by the brane worldvolumes and $\times$'s mark transverse directions. We consider all branes to coincide in the mutually transverse directions 345. We moreover smear the D2-branes in the direction 6789. Eventually these directions will be taken to be compact, so the smeared description is valid for small compactification size.

In the 10d Einstein frame the metric and dilaton profile are given by harmonic superposition (see [211] for background)

$$ds^2 = Z_6(r)^{-\frac{1}{8}} Z_2(r)^{-\frac{5}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z_6(r)^{\frac{7}{8}} Z_2(r)^{\frac{3}{8}} (dr^2 + r^2 d\Omega_2^2) + Z_6(r)^{-\frac{1}{8}} Z_2(r)^{\frac{3}{8}} dx^m dx^m \quad ,$$
$$\Phi(r) = \frac{1}{2\sqrt{2}} \log \left(Z_6(r)^{-\frac{3}{2}} Z_2(r)^{\frac{1}{2}} \right) , \tag{6.106}$$

where r is the radial coordinate in 345, $d\Omega_2^2$ is the volume of a unit $\mathbf{S}^2$ in this $\mathbf{R}^3$, and $m = 6, 7, 8, 9$. The harmonic functions are

$$Z_6(r) = 1 + \frac{\rho_6}{r} \quad , \quad Z_2(r) = 1 + \frac{\rho_2}{r} \, . \tag{6.107}$$

[6] For other approaches to Swampland constraints using (large and small) black holes, see e.g. [208–210].

As announced, we now consider compactifying the directions 6789 on a $\mathbf{T}^4$ (similar results hold for K3 compactification, as usual), with the compactification ansatz

$$ds^2 = e^{-\frac{t}{\sqrt{2}}} ds_6^2 + e^{\frac{t}{\sqrt{2}}} ds_{T^4}^2 \,. \tag{6.108}$$

Matching this ansatz to (6.106), we obtain the canonically normalized radion

$$t(r) = \sqrt{2} \log \left(Z_6(r)^{-\frac{1}{8}} Z_2(r)^{\frac{3}{8}} \right) \,. \tag{6.109}$$

The 6d Einstein frame metric reduces to:

$$\begin{aligned} ds_6^2 &= e^{\frac{t}{\sqrt{2}}} \left(Z_6(r)^{-\frac{1}{8}} Z_2(r)^{-\frac{5}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z_6(r)^{\frac{7}{8}} Z_2(r)^{\frac{3}{8}} (dr^2 + r^2 d\Omega_2^2) \right) \\ &= Z(r)^{-\frac{1}{4}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{3}{4}} (dr^2 + r^2 d\Omega_2^2) \end{aligned} \tag{6.110}$$

where $Z(r) = Z_6(r) Z_2(r)$.

One can see that the dilaton and radion are both blowing up upon reaching the point $r = 0$, which is at finite spacetime distance, hence the configuration can be dubbed a 6d small black 2-brane.

As in section 6.3, we can describe the configuration as a Dynamical Cobordism of the 6d theory compactified on an $\mathbf{S}^2$ with suitable 2-form fluxes (for the RR 2-form field strength and the $\mathbf{T}^4$ reduction of the RR 6-form field strength). To implement this, we take the general ansatz:

$$ds_6^2 = e^{-2\alpha\sigma} ds_4^2 + r_0^2 e^{2\beta\sigma} d\Omega_2^2 \,. \tag{6.111}$$

In the resulting 4d theory, there are non-trivial potential terms for the new radion σ arising from the curvature of $\mathbf{S}^2$ and the 2-form fluxes. Imposing the Einstein frame in 4d comes down to setting $\gamma = \frac{\alpha}{\beta} = 1$. One can then choose β such that the radion $\sigma(r)$ has a canonically normalized kinetic term and one obtains $\beta = \frac{1}{2}$. From matching this compactification ansatz to equation (6.110), we obtain the canonically normalized radion σ,

$$\sigma(r) = \log \left(\frac{r^2}{r_0^2} Z(r)^{\frac{3}{4}} \right) , \tag{6.112}$$

and the following 4d Einstein frame metric

$$ds_4^2 = e^{\sigma} \left(Z(r)^{-\frac{1}{4}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{3}{4}} dr^2 \right) = \left(\frac{r}{r_0} \right)^2 \left(Z(r)^{\frac{1}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{3}{2}} dr^2 \right) .$$

This solution is a 4d Dynamical Cobordism, with the D2/D6-brane system playing the role of cobordism defect. The solution has the structure of an ETW brane; there are 3 running scalars going off to infinite distance at the singularity at $r = 0$, which is straightforward to show lies at finite spacetime distance. Indeed, near $r = 0$, we have

$$\Delta = \int_0^r \left(\frac{r}{r_0} \right) \left(Z_6(r) Z_2(r) \right)^{\frac{3}{4}} dr \sim \sqrt{r} \,. \tag{6.113}$$

Furthermore, near the singularity, the distance in field space goes like:

$$dD_\phi^2 = d\Phi^2 + d\sigma^2 + dt^2 \simeq \frac{1}{2} \frac{dr^2}{r^2} \to D_\phi \simeq -\frac{1}{\sqrt{2}} \log(r) \tag{6.114}$$

Near the singularity, the Ricci scalar in 4d behaves as:

$$|R| \sim r^{-1} \tag{6.115}$$

These lead to the familiar scaling relations near $r = 0$:

$$|R|^{-\frac{1}{2}} \sim \Delta \sim e^{-\frac{1}{\sqrt{2}} D_\phi} \,. \tag{6.116}$$

Since the above full solution has the structure of a Dynamical Cobordism, it should be possible to express it in the framework of our local description, with the D2/D6-brane system playing the role of the ETW brane. Let us define the new coordinate:

$$y = \int^r \left(\frac{r}{r_0}\right) \left(Z_6(r) Z_2(r)\right)^{\frac{3}{4}} dr \sim \sqrt{r} \,, \tag{6.117}$$

where we have considered the leading behaviour near $r = 0$.

Using equation (6.48), we have:

$$\sigma(y^2) = -\frac{1}{2} \log \left(\frac{y^4}{r_0^2} Z(y^2)^{\frac{1}{2}} \right) \simeq -\log y \,. \tag{6.118}$$

Matching this to the profile in (6.18), we see that $\delta^2 = 2$ and $a = \frac{2}{3}$. Then we automatically fall back on the previous field-space distance and scaling relations using equations (6.18) and (6.21):

$$D_\phi(y) \simeq -\sqrt{2} \log y \,, \tag{6.119}$$

$$\Delta = y \sim e^{-\frac{1}{\sqrt{2}} D_\phi} \sim |R|^{-\frac{1}{2}} \,. \tag{6.120}$$

This gives yet another nice check of the usefulness of the local analysis.

Beyond the Einstein frame

One last remark that will be relevant in the next sections is that scaling relations similar to those of (6.116) can be found, independent of the frame chosen during the compactification. Indeed, if one insists on keeping γ (and thus, also β) general and tracking it throughout the computations, one obtains the new coordinate near $r = 0$:

$$\Delta = y \sim r^{\frac{1}{4}(\gamma+1)} \quad \text{and} \quad |R| \sim r^{-\frac{1}{2}(\gamma+1)} \,. \tag{6.121}$$

Note that, if $\gamma < -1$, then these scalings behave opposite to those we have seen for ETW branes. This illustrates that the scalings mentioned rely on using the Einstein frame metric to describe the ETW brane.

In setups where one needs (or finds convenient) to use general frames, the condition for an ETW brane is that the picture of a scalar going off to infinity at finite spacetime distance can be attained by a suitable change of frame. In this respect, we note that there is an extra subtlety in dealing with the field space distance in general frames. Indeed, not being in the Einstein frame implies that the radion is multiplying the Einstein-Hilbert term in the action:

$$S_4 \supset \frac{1}{2} \int d^4x \sqrt{-g_4} e^{2\beta\sigma(1-2\gamma)} \{ e^{2\beta\gamma\sigma} (R_4 - (\partial t)^2 - (\partial \Phi)^2 - \beta^2 (6\gamma^2 - 8\gamma + 6)(\partial\sigma)^2) \} \,. \tag{6.122}$$

It thus makes sense to define the field space distance measured in units set by this coefficient of the Ricci scalar in the action. This field space distance near the singularity in this general frame reads:

$$\begin{aligned} dD_\phi^2 &= d\Phi^2 + \beta^2(6\gamma^2 - 8\gamma + 6)d\sigma^2 + dt^2 \\ D_\phi &\simeq -\frac{\sqrt{6\gamma^2 - 8\gamma + 10}}{4}\log r\,. \end{aligned} \tag{6.123}$$

Hence, we can derive the following universal scaling relations in a general frame:

$$\Delta \sim e^{-\frac{\gamma+1}{\sqrt{6\gamma^2-8\gamma+10}}D_\phi} \sim |R|^{-\frac{1}{2}}\,. \tag{6.124}$$

Note that these reduce to those of (6.116) when setting $\gamma = 1$, as required by the Einstein frame. As a side note, one cannot recover this result in the local description detailed in section 6.1.2 as it was constructed in the Einstein frame. We leave such a more general formulation of the local construction for future work.

6.4.2 Small Black Holes from the D2/D6 system on $\mathbf{T}^4 \times \mathbf{T}^2$

Let us now consider turning our D6/D2-brane systems into a (small) black hole, by a further compactification on $\mathbf{T}^2$.

We take the ansatz

$$ds_6^2 = e^{-q}ds_4^2 + e^{q}ds_{T^2}^2\,. \tag{6.125}$$

By matching this ansatz to the 6d metric obtained previously (6.110), we get the 4d Einstein frame metric:

$$\begin{aligned} e^{q(r)} &= Z(r)^{-\frac{2}{8}}\,, \\ ds_4^2 &= (g_4)_{ij}dx^i dx^j = e^{q(r)}\left(-Z(r)^{-\frac{2}{8}}dt^2 + Z(r)^{\frac{6}{8}}(dr^2 + r^2 d\Omega_2^2)\right) \\ &= -Z(r)^{-\frac{1}{2}}dt^2 + Z(r)^{\frac{1}{2}}(dr^2 + r^2 d\Omega_2^2)\,. \end{aligned} \tag{6.126}$$

This solution describes a small black hole (in fact, equivalent to the celebrated D1/D5-brane one, by T-duality in one of the $\mathbf{T}^2$ directions), of the kind considered in [57].

To motivate the relation with the more general discussion in the next section, let us make the following heuristic argument. Although our solution has three scalar fields, the radial evolution can be reduced to one effective scalar as follows. Near $r = 0$, all three scalars have the same profile, so we may combine them in one effective scalar D_ϕ whose effective action near $r = 0$ is of the form

$$S_4 \sim \frac{1}{2}\int d^4x\sqrt{g_4}\{R_4 - (\partial D_\phi)^2 - \frac{1}{4}e^{\frac{1}{\sqrt{2}}D_\phi}|F_2|^2\} \tag{6.127}$$

where we have restricted to the $U(1)$ linear combination under which the D2/D6 system is charged.

With this proviso, we can frame this particular example with the more general class of small Black Holes considered in [57], to be discussed next.

6.4.3 General small Black Holes

In the context of the swampland program, [57] proposed the use of 4d small black hole solutions to provide further evidence for a number of a number of Swampland conjectures. A particularly important property is that the 4d solutions contain scalars going off to infinite field space distance at the black hole core. In the spirit of previous sections, in this section we show that these 4d solutions can be turned into 2d Dynamical Cobordisms upon reducing on the $\mathbf{S}^2$, with the small black hole playing the role of the ETW brane. In fact we will check that the 2d running solution satisfies the familiar scaling relations (for a general frame, since there is no Einstein frame in 2d).

Let us briefly review the key features of such solutions. We consider 4d Einstein-Maxwell coupled to a scalar controlling the gauge coupling. We take the action

$$S_{4d} \sim \frac{1}{2}\int d^4x\,\sqrt{-g_4}\,\left(R_4 - (\partial\phi)^2 - e^{2a\phi}|F_2|^2\right)\,. \tag{6.128}$$

We focus on exponential dependence, since it provided the most explicit class considered in [57]. It also fits with the special role of exponential functions in local descriptions of ETW branes.

Without loss of generality, we take $a > 0$ so that $\phi \to \infty$ corresponds to weak coupling for the $U(1)$ gauge field. Note that this a should not be confused with the parameter in (6.12), and we trust the reader to distinguish them by the context.

In this theory, electrically charged extremal black holes take the form

$$ds_4^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2R(r)^2 d\Omega_2^2\,, \tag{6.129}$$

where

$$R(r) = \left(1 - \frac{r_h}{r}\right)^{\frac{a^2}{1+a^2}}\,,\quad f(r) = \left(1 - \frac{r_h}{r}\right)^{\frac{2}{1+a^2}}\,. \tag{6.130}$$

In addition, the profile for the scalar is given by

$$\phi(r) = \phi_0 - \frac{\sqrt{2}\,a}{1+a^2}\log\left(1 - \frac{r_h}{r}\right)\,. \tag{6.131}$$

The scalar goes off to infinity at the horizon $r = r_h$, which is however not smooth, since the $\mathbf{S}^2$ shrinks to zero size, leading to a small black hole.

In the string theory context, small black holes can be easily built by using D-branes. In fact, we now recast the above solution in a form closer to the solution (6.126), which described our system of D2- and D6-branes on $\mathbf{T}^4 \times \mathbf{T}^2$. This was already anticipated when we obtained (6.127), which has the structure of (6.128) (for $a = \frac{1}{2\sqrt{2}}$).

Carrying out the coordinate change $r \to r + r_h$, the metric (6.129) becomes

$$ds_4^2 = -\left(1 + \frac{r_h}{r}\right)^{-\frac{2}{1+a^2}} dt^2 + \left(1 + \frac{r_h}{r}\right)^{\frac{2}{1+a^2}}\left(dr^2 + r^2 d\Omega_2^2\right)\,. \tag{6.132}$$

Similarly, the scalar reads

$$\phi(r) = \phi_0 + \frac{\sqrt{2}\,a}{1+a^2}\log\left(1 + \frac{r_h}{r}\right)\,. \tag{6.133}$$

This has the structure of (6.126) with $Z(r) = (1 + r_h/r)^{\frac{4}{1+a^2}}$. Note that the core of the small black hole now lies at $r = 0$.

We now perform the reduction on $\mathbf{S}^2$ to express these solutions as 2d running solutions describing a local Dynamical Cobordism, with the small black hole playing the role of the ETW brane. We will also recover the corresponding (general frame) scaling relations.

Since there is no Einstein frame in 2d, we perform the $\mathbf{S}^2$ reduction with the following general ansatz:

$$ds_4^2 = e^{-2\alpha\omega} ds_2^2 + e^{2\beta\omega} r_0^2 d\Omega_2 \,. \tag{6.134}$$

The 2d action obtained from the compactification contains the terms

$$S_{2d} \supset \frac{1}{2} \int d^2x \sqrt{-g_2}\, e^{2\beta\omega} \left(R_2 - (\partial\phi)^2 - 6\beta^2 (\partial\omega)^2 \right) \,. \tag{6.135}$$

These expressions already show the impossibility to define an Einstein frame: it would require $\beta = 0$, and this would kill the radion's kinetic term. We therefore keep β general, so we deal with a dilaton-gravity theory. By matching the ansatz (6.134) with the 4d metric (6.132) we get the profile for the radion

$$\omega(r) = \frac{1}{\beta} \log \left(\frac{r}{r_0} \left(1 + \frac{r_h}{r} \right)^{\frac{1}{1+a^2}} \right) \,, \tag{6.136}$$

and the 2d metric

$$ds_2^2 = \left(\frac{r}{r_0} \right)^{2\gamma} \left(- \left(1 + \frac{r_h}{r} \right)^{-\frac{2(1-\gamma)}{1+a^2}} dt^2 + \left(1 + \frac{r_h}{r} \right)^{\frac{2(1+\gamma)}{1+a^2}} dr^2 \right) \,, \tag{6.137}$$

where $\gamma = \frac{\alpha}{\beta}$.

Computing the 2d Ricci scalar and taking the leading order in $r \to 0$ we get

$$|R| \sim r^{-2\frac{(\gamma+1)a^2}{1+a^2}} \,, \tag{6.138}$$

where we are ignoring a constant prefactor[7].

Similarly, the spacetime distance from a given r to the singularity, at leading order in $r \to 0$, scales as

$$\Delta \sim r^{\frac{(\gamma+1)a^2}{(1+a^2)}} \,. \tag{6.139}$$

We note that, as expected, the scaling is the familiar ETW one if $\gamma > -1$. As explained above, the fact that 2d gravity is topological means that the criterion for an ETW brane in a solution should be that the usual relations hold in *some* suitable frame.

Let us now recover the usual scalings with the field distance. Recalling the latter is measured in units set by the coefficient of the Ricci scalar in the action, we can it read off from (6.135) as:

$$dD_\phi^2 = d\phi^2 + 6\beta^2 d\omega^2 \,. \tag{6.140}$$

[7]This prefactor vanishes for either $a^2 = 1$ or $a^2 = -2\gamma$. We will skip these cases without further discussion.

Plugging the profiles (6.131) and (6.136) at leading order as $r \to 0$ and integrating the line element we recover

$$D_\phi(r) \simeq -\frac{a\sqrt{2+6a^2}}{1+a^2} \log r \,. \tag{6.141}$$

Finally, together with the previous results for the distance to the end of the world and the curvature, we obtain the scalings

$$\Delta \sim e^{-\frac{\delta}{2} D_\phi} \,, \quad |R| \sim e^{\delta\, D_\phi} \,, \tag{6.142}$$

with

$$\delta = \frac{2(\gamma+1)a}{\sqrt{2+6a^2}} \,. \tag{6.143}$$

Hence, we recover the general frame scaling relations introduced in section 6.4.1. This shows that small black hole solutions can be regarded as just another instance of Dynamical Cobordism, and that they admit local scaling relations identifying the small black hole core with ETW branes in 2d.

6.5 Swampland constraints and Surprises from the UV

In this section we discuss interesting interplays of the scalar running off to infinity in field space in Local Dynamical Cobordisms and the Swampland constraints.

6.5.1 Swampland Distance Conjecture and other constraints

Many studies of Swampland constraints are related to infinity in scalar moduli/field space. Since Dynamical Cobordisms explore infinite field space distances, in this section we discuss the interplay with different Swampland constraints, especially the Distance Conjecture [35] (see [36, 45, 47, 106, 135, 139, 141, 212–215] and the reviews above for other approaches).

Let us focus on the simplest expression of the Distance Conjecture, which states that, when the scalars are taken to infinite field space distance D_ϕ (in an adiabatic approach, namely, by changing the spacetime independent vevs), there is a tower of states becoming exponentially light, and thus the cutoff of the effective theory is lowered as

$$\Lambda \sim e^{-\alpha D_\phi} \,, \tag{6.144}$$

with some positive order 1 coefficient α.

This scaling can be combined in an interesting way with our scalings near ETW branes. For instance, using (6.21), we have

$$\Lambda \sim \Delta^{\frac{-2\alpha}{\delta}} \,. \tag{6.145}$$

This matches with our intuition that the full description of the ETW brane requires UV completing the effective theory. It is important to note that the appearance of an infinite tower in the adiabatic version of the Distance Conjecture does not necessarily imply the appearance of a tower in the present Dynamical Cobordism context. On the other hand, the lowered cutoff certainly signals that there could be situations where the naive ETW brane picture as described in effective theory may be corrected. We will see explicit examples in section 6.5.2.

Using also (6.21), we get that the cutoff scale relates to the spacetime curvature as

$$|R| \sim \Lambda^{\frac{\delta}{\alpha}}, \tag{6.146}$$

(where we have taken the generic case $\delta \neq (2d/(d-2))^{1/2}$ for concreteness). This relation is reminiscent of (although admittedly different in spirit from) that in [215] for AdS vacua.

From this perspective, the correlation between the appearance of the naked singularity and the running of the scalar going off to infinity suggests that the lowered cutoff of the swampland distance conjecture is responsible for regulating the singularity, which would be resolved in a more complete microscopic UV description. This remark is in the spirit of [55] (see also [39]) and [57], where the singular behaviour of certain defects (EFT strings or small black holes, respectively) is related to scalars going off to infinite distance.

From our perspective, the relation follows from the Dynamical Cobordism Distance Conjecture put forward in chapter 5. In our present terms: Every infinite field distance limit of an effective theory consistent with quantum gravity can be realized as a solution running into a cobordism ETW brane (possibly in a suitable compactification of the theory).

In particular, in Sections 6.3 and 6.4 we provided a description of general defects as ETW branes of Dynamical Cobordisms. This general framework encompasses the defects in [55, 57] as particular examples.

An interesting spin-off of our local analysis is that it constrains the asymptotic form of the potential. Namely, whenever it is not vanishing (actually, negligible as compared with the scalar kinetic energy) it has an exponential form with a critical exponent δ, c.f. (6.16). It is thus interesting to compare this asymptotic form of the potential with Swampland constraints expected to hold near infinity in scalar field space.

Let us consider the de Sitter conjecture in the version of [157] (see [213, 216] for the refined one), namely $|\nabla V|/V > \mathcal{O}(1)$. From (6.16) we have

$$\frac{V'}{V} = \delta \,. \tag{6.147}$$

Since in general the critical exponent $\delta \sim \mathcal{O}(1)$, the potential satisfies the de Sitter conjecture. This fits nicely with the idea that the latter is expected to hold near infinity in moduli/field space.

Moreover, let us compare with the Transplanckian Censorship Conjecture [37]

$$|\nabla V| \geq \frac{2}{\sqrt{(d-1)(d-2)}} V \,. \tag{6.148}$$

When $V < 0$, the constraint is trivial; on the other hand, when $V > 0$, in our setup we must have $a < 0$, and the expression (6.15) for δ guarantees that the above inequality is satisfied. A caveat for the above statements is that both the de Sitter and the Transplanckian Censorship conjectures involve the gradient ∇V, whereas our local description provides the potential only along one direction, the effective scalar dominating the running near the ETW brane. Hence, the comments above would hold under the assumption that the effective scalar in the local description follows a gradient flow. It would be interesting to assess this point in explicit models, and we leave this as an open question for future work.

6.5.2 Large N surprises from the UV

In the previous section we have discussed that the Distance Conjecture implies a lowered cutoff as one approaches the ETW brane. Indeed, as mentioned at several points, the microscopic description of the ETW branes lies in the underlying UV completion. In most of our examples, the corresponding cobordism defect is known, so that the end of the world picture can be confirmed in the full theory. However, it is conceivable that in some specific cases there exist UV effects hidden at the core of the ETW brane potentially modifying this picture. In this section we present two examples, where such corrections exist and lead to large backreactions, ultimately turning the candidate ETW brane into a domain wall interpolating to a new region beyond the apparent singularity. A further interesting observation is that both examples are related to large N physics and holography.

Large number of M2-branes

Consider as our first example a stack of N D2-branes in flat 10d spacetime (or at a smooth point in any other compactification). Locally around the D2-brane location the $\mathbf{S}^6$ truncation yields a 4d theory with an ETW brane, at which a scalar (a combination of the radion and the dilaton) goes to infinity in field space. One may follow the theory in this limit and, as noted in [206], realize that the strong coupling is solved by lifting to M-theory, and turning the D2-branes into M2-branes. For small N, the UV completion of the effective ETW brane is thus merely a stack of M2-branes removing the flux and allowing spacetime to end, as befits a Dynamical Cobordism.

On the other hand, for N large we have a different behavior: the large number of M2-branes backreact on the geometry and generate an infinite $\mathrm{AdS}_4 \times \mathbf{S}^7$ throat. The effective theory ETW brane has a UV description with so many degrees of freedom that it actually generates a gravity dual beyond the the wall.

From the perspective of the running scalars, the $\mathrm{AdS}_4 \times \mathbf{S}^7$ represents a minimum of the ($\mathbf{S}^7$ radion) potential. Hence the full D2/M2 solution describes the running of the theory from the slope of the potential down to a stable minimum, at which the theory relaxes to a maximally symmetric solution, instead of hitting an end of the world. The location of the minimum in field space is hidden near infinity in the original D2-brane effective description. Hence, the large N allows for the appearance of a minimum at strong coupling, which is nevertheless tractable[8].

Moreover, the full D2/M2 solution describes a dynamical cobordism from the M-theory perspective. Far away from the stack of branes we can use the description in terms of D2-branes. As described above the 4d theory would be obtained by compactifying Type IIA on an $\mathbf{S}^6$. This would be further lifted to M-theory on $\mathbf{S}^6 \times \mathbf{S}^1$. On the other hand, we have just argued that close to the stack of branes the 4d theory is given by M-theory on $\mathbf{S}^7$. We then see that this solution describes a dynamical cobordism between to different compactifications. Notice that this is not a cobordism to nothing, described by ETW brane solutions.

Warped KS throat with large number of D3-branes

Our second example is based on the warped throat considered in section 6.3.4. Recall we have type IIB theory compactified on $T^{1,1}$ with N units of RR 5-form flux and M units of RR 3-form flux on the $\mathbf{S}^3$, and we focus on the choice of parameters $N = KM + P$. At

[8] This is reminiscent of the argument [155] that the scale separation (and hence the tractability) of the AdS minima in [217, 218] is controlled by a large number of flux units.

the level of the 4d effective theory, we recover a KT solution with a singularity at a finite spacetime distance, at which a scalar (a combination of the $T^{1,1}$ radion and the dilaton, but dominated by the former) goes off to infinite field space distance.

The UV smoothing of this singularity is slightly trickier than the $N = KM$ case of section 6.3.4. It involves the smoothing of the singular conifold geometry into a deformed conifold, with a finite size $\mathbf{S}^3$, but there remain P D3-branes at the tip of the throat. This can be shown using the holographic dual field theory, as follows. There is a Seiberg duality cascade from the initial $SU(N) \times SU(N+M)$ theory in which N effectively decreases in multiples of M; hence, in the last step of the cascade we have an $SU(P) \times SU(M+P)$ gauge theory, whose strong coupling dynamics leads to an remnant $\mathcal{N} = 4$ $SU(P)$ theory, as befits the above mentioned P probe D3-branes.

Hence, for small P the ETW brane of the 5d theory is microscopically described by the smooth Klebanov-Strassler throat dressed with P explicit D3-branes, required to absorb the remnant 5-form flux and allow spacetime to end.

On the other hand, for P large we have a different behavior: the large number of D3-branes backreact on the geometry and generate an infinite $\mathrm{AdS}_5 \times \mathbf{S}^5$ throat. The effective theory ETW brane has a UV description with so many degrees of freedom that it actually generates a gravity dual beyond the the wall. The interpretation of this strong correction in terms of the running scalars is similar to the one mentioned above, as the apperance of an AdS minimum hidden near the infinite field space distance limit of the effective description.

We have seen two examples in which a naive ETW brane in the effective description has a UV description encoding large backreactions on the geometry recreating a geometry beyond the wall. Alternatively, the corrections generate minima in the scalar potential in the region near field space infinity of the effective description. It would be interesting to explore in more detail these and other possible classes of examples exhibiting this phenomenon. We hope to report on this in the future.

6.6 Summary

In this chapter we have studied Dynamical Cobordism solutions in which theories of gravity coupled to scalars develop an end of spacetime. The latter is encoded in the effective theory as the appearance of a singularity at finite spacetime distance, at which some scalars run off to infinite field space distance. We have provided a local description of the configurations in the near ETW brane regime, and shown that the solutions are largely simplified, and fall in universality classes characterized by a critical exponent δ, which controls the profiles of the different fields and the scaling relations among the field space distance D_ϕ, spacetime distance Δ and scalar curvature R.

We have studied several explicit models of ETW branes and characterized them in the local description, computing their critical exponent. The different examples and their key parameters are displayed in Table 6.1. This list is intended to illustrate typical values of these parameters. It would be interesting to explore more examples and to explore possible connections among ETW branes described by the same parameters.

We have moreover shown that small black holes can also be regarded as Dynamical Cobordisms, and satisfy similar scaling laws. It would be interesting to explore from the

Example	d	δ	a
Massive IIA	10	$\frac{5}{\sqrt{2}}$	$-\frac{16}{5}$
Non-susy $USp(32)$ *string*	10	$\frac{3}{\sqrt{2}}$	0
D7 branes	9	$\frac{4\sqrt{14}}{7}$	0
D6 branes	8	$\sqrt{2}$	$\frac{4}{7}$
D5 branes	7	$\frac{2}{\sqrt{5}}$	$\frac{5}{6}$
D4 branes	6	$\frac{1}{\sqrt{5}}$	$\frac{24}{25}$
Klebanov-Strassler	5	$\frac{2\sqrt{30}}{3}$	$-\frac{3}{2}$
Bubble of Nothing	4	$\sqrt{6}$	0
D2 branes	4	$\frac{\sqrt{14}}{7}$	$\frac{20}{21}$
D2/D6 on $T^4 \times S^2$	4	$\sqrt{2}$	$\frac{2}{3}$
D1 branes	3	$\sqrt{2}$	$\frac{3}{4}$
EFT string	3	$2\sqrt{2}$	0

Table 6.1: Table of examples in this chapter, with the corresponding parameters for the local description near the ETW brane.

cobordism perspective the recent applications of small black holes to the derivation of swampland constraints.

Part IV

The Swampland Distance Conjecture in AdS/CFT

7
Tackling the SDC in AdS with CFTs

As discussed in 2.2, the SDC has been mainly studied in the context of four- [45, 46, 50, 51, 71, 81, 212, 219] or six-dimensional [52] Minkowski theories with eight or more supercharges obtained by dimensional reduction of type II string theories, or their lifts to strong coupling. Using the beautifully-intricate web of dualities of string theory, it was proposed that the tower of massless states corresponds to either a decompactification limit or a tensionless weakly-coupled fundamental string in disguise [50, 51, 81], although it may be required to take quantum corrections into account to make them manifest [48, 49, 51, 56]. Note that, in the latter case, the tower of states generically contains arbitrarily large higher-spin fields. See [220] for implications in (quasi-)dS spaces.

A variation of this framework is the inclusion of a potential [51, 54, 58, 95, 106, 139–141, 213, 221–224], which may lift the flat spacetime geometry to an AdS space. In the limit of large AdS radius in Planck units, $LM_{Pl} \to \infty$, a similar behaviour is expected, with an infinite tower of states also becoming massless, behaving as $m/M_{Pl} \sim (LM_{Pl})^{-\lambda}, \lambda > 0$ [215]. In supersymmetric cases a strong version of the conjecture suggests $\lambda = \frac{1}{2}$, usually interpreted as a consequence of the no-scale-separation condition between the internal manifold and the AdS radius. In string-theoretic realisations of these AdS geometries, the tower is often identified with a sector of Kaluza–Klein modes. Part of the internal manifold and the AdS space are stabilised by the same fluxes and, as a consequence, the AdS radius and a breathing mode of the compact space are linked together. The limit of large radius will then also lead to a decompactification. For recent works, see [155, 225–228].

This proposal is somewhat different from what one would naively call the Swampland Distance Conjecture for moduli spaces of AdS vacua. Even though it is exploring the possible AdS vacua of the theory, it is not about the continuously-connected part parametrised by massless scalars, which we refer to in this work as the moduli space. It is rather about the different branches of vacua parametrised by massive scalars. In string theory constructions, the presence of fluxes will give masses to the scalars controlling these limits and can therefore no longer be considered as moduli in the usual sense. Typically one consider different branches of vacua in this setup by changing the flux quanta.

This raises the question of whether the SDC extends to moduli spaces of AdS vacua in the sense described above, and what kind of towers of states can be expected to appear. In those setups, the AdS scale in Planck units, LM_{Pl}, remains fixed throughout all the moduli space. This is the kind of trajectories we want to tackle in this work.

In this context, an open question is whether it is possible to consider decompactification limits. Such trajectories would imply the possibility of tuning the size of an internal dimension without changing the AdS radius at all. Current models featuring a separation of scales always link the AdS radius and the internal dimensions in some way, while the limits we are interested in would require them to be independent. Although inconclusive, current understanding of AdS vacua seems to disfavour such trajectories and leads to the intriguing possibility that equi-dimensional and non-equi-dimensional limits in AdS are distinguished, being called SDC and ADC directions, respectively.

In the case of equi-dimensional limits it is reasonable to expect the appearance of tensionless strings. However, an immediate challenge one faces is that those points are out of the regime of parametric control of the usual supergravity description of these vacua. Indeed, the tension of these strings will eventually fall below the AdS scale, leading to the notoriously difficult problem of quantising strings in highly-curved backgrounds. To retain control over the theory one conversely assumes a weakly-curved background, corresponding to a semi-classical approximation. A possible way to go around the issue is to make use of the AdS/CFT correspondence [15]. Our aim here is to analyse a possible extension of the SDC by studying the evolution of physical quantities through their CFT duals.

Using holography as a tool to study the Swampland programme has already bore fruits. Proofs of no-go theorems applied to global symmetries as well as the Weak Gravity Conjectures in AdS were established by relating black hole quantities to conformal data [22,23,80]. More recently, positivity bounds were related to moduli stabilisation constraints in AdS_4/CFT_3, as well as possible connections to the SDC [229]. Closer to our setup, the classical moduli space has further been shown to be a coset in the case of AdS_5 gauged supergravity with sixteen (real) supercharges [230].

Moduli and Marginal Deformations

In AdS/CFT, each field of mass m in the bulk is associated on the boundary to a conformal operator of dimension Δ. The dictionary between scalars and ℓ-symmetric traceless tensors is given in AdS_5/CFT_4 by:

$$m^2L^2 = \Delta(\Delta-4)\,, \qquad \text{(scalars)}\,; \tag{7.1}$$
$$m^2L^2 = (\Delta+\ell-2)(\Delta-\ell-2)\,, \qquad (\ell\text{-symmetric traceless tensors}). \tag{7.2}$$

The variation of any mass as a function of the moduli in the bulk can thus be controlled by tuning parameters of the CFT. We note that, as we demand the AdS radius, L, to be fixed in Planck units, we can use these expressions to evaluate the mass of a given state in Planck units up to some numerical coefficients which are irrelevant to our analysis.

The moduli space, parametrised by the vacuum expectation value (vev) of the moduli, z^i, is then identified with the conformal manifold, the space of exactly marginal deformations, λ^i, of the CFT, see e.g. [231,232]. This conformal manifold is endowed with the so-called Zamolodchikov metric, χ, that is the dual of the bulk moduli space metric, G:

$$\big(\mathcal{M}_{\text{mod}}, G_{ij}(z)\big) \longleftrightarrow \big(\mathcal{M}_{\text{CFT}}, \chi_{ij}(\lambda)\big)\,. \tag{7.3}$$

More specifically, the Zamolodchikov metric is mapped to the metric in moduli space measured in AdS units up to a constant prefactor, such that in Planck units we have:

$$(LM_{Pl})^3\, G_{ij}(z) \sim \chi_{ij}(\lambda)\,. \tag{7.4}$$

The specific form of the metric can be computed as a series in large N, dual to the weakly-coupled quantum-gravity expansion in the bulk [233]. As the partition function of the bulk on the boundary is identified with the generating functional of correlation functions of the CFT, corrections on either side are guaranteed to match on the other. This means that we can use (7.4) to compute, at least in principle, the metric in moduli space from the Zamolodchikov metric in any regime of the theory.

As it is well known, unitarity furthermore imposes constraints on the CFT data. For instance the dimension is bounded from below, and an operator with spin ℓ must satisfy:

$$\begin{aligned} &\Delta \geq 1\,, \qquad && \ell = 0\,; \\ &\Delta \geq \ell + 2\,, \qquad && \ell > 0\,. \end{aligned} \tag{7.5}$$

For $\ell = 0$ the bound is saturated by a free scalar field and maps to tachyonic fields in the bulk, while the flavour currents and the energy-momentum tensor sit at the bound for $\ell = 1\,, 2$, respectively. For $\ell > 2$, the bound is associated to so-called higher-spin conserved currents. By the Maldacena–Zhiboedov theorem [234] and its extensions [235–239], having a single higher-spin conserved current implies the existence of an infinite number of them. Moreover this can only occur in the presence of (generalised) free fields. Such higher-spin currents will be a central part of this work.

With this framework, the study of the Swampland Distance Conjecture for AdS spacetime can then be rephrased as an analysis of the possible infinite-distance points of conformal manifolds. We will focus on four-dimensional $\mathcal{N} = 2$ SCFTs, corresponding to supergravity theories with sixteen (real) supercharges on AdS_5, although we will comment on implications in other dimensions. In those theories, ($\mathcal{N} = 2$)-preserving exactly marginal deformations can only correspond to variations of (complexified) gauge couplings [240,241], and it is then easy to give a physical interpretation to the towers of states in terms of gauge data.

In particular, we will be able to track how the dimension of certain operators behaves near a class of infinite-distance points corresponding to weakly-coupled gauge subsectors of the SCFT. Among them, we find the above-mentioned infinite tower of higher-spin currents that saturate the unitarity bound (7.2) in the presence of free fields. In the bulk, this leads to an infinite tower of higher-spin fields becoming massless and satisfying the condition

$$\frac{M_\ell}{M_{Pl}} \sim e^{-\alpha D_\phi}\,. \tag{7.6}$$

We will further estimate the decay rate α, that will be shown to be *at least* of order one.

This chapter is structured as follows: in section 7.1 we describe how to use AdS/CFT to study the SDC in the bulk by working in the moduli space of $\mathrm{AdS}_5 \times \mathrm{S}^5$ vacua of type IIB string theory. The high degree of supersymmetry is enough to compute the metrics exactly and therefore constitutes the simplest example to study. In section 7.2 we review some important properties of the conformal manifolds associated to four-dimensional $\mathcal{N} = 2$ SCFTs and study the SDC on general grounds. In section 7.3 we make contact again with the bulk by examining a specific family of $\mathcal{N} = 2$ SCFTs with known bulk duals. We give a summary and discuss applications to other dimensions in section 7.4. Additionally, we briefly review unitary representations of the $\mathcal{N} = 2$ superconformal algebra in appendix E.

7.1 A Warm-up: Type IIB $\mathrm{AdS}_5 \times \mathrm{S}^5$ Vacua

As a first example and to set our nomenclature and conventions, we consider the family of AdS_5 vacua obtained by compactifying Type IIB on S^5 with N units of F_5-flux. It is the most celebrated example of the holographic principle, being dual to $\mathcal{N} = 4$ super-Yang–Mills theory with gauge group $SU(N)$ in four dimensions. Due to the high amount of supersymmetry, non-renormalisation theorems makes it possible to compute relevant quantities exactly.

This case will be useful to exemplify the inclusion of the moduli space into the scalars manifold, including the stabilised scalar fields. It will make a clear distinction between the limits we want to explore and the "ADC directions", where the AdS scale, L, is allowed to vary [215]. It will also serve to illustrate the issues of parametric control that occur when approaching infinite-distance points in AdS moduli space and how the dual CFT picture allows one to circumvent them.

This family of solutions is parametrised by N and the complex axio-dilaton,

$$\tau = C_0 + i\frac{1}{g_s}\,, \tag{7.7}$$

made out of the string coupling, g_s, and the Type IIB axion, C_0. The AdS radius, L, is forced to coincide with the radius of the five-sphere and is set to:

$$L^4 = 4\pi g_s N \alpha'^2 = \frac{1}{4\pi^3} g_s N M_s^{-4}\,. \tag{7.8}$$

In terms of the five-dimensional Planck mass, it is rewritten as

$$L\, M_{Pl} \sim N^{2/3}\,, \qquad M_{Pl} \sim g_s^{-1/4} N^{5/12} M_s\,, \tag{7.9}$$

so that keeping the AdS scale fixed in Planck units corresponds to fixing N. Thus, the moduli space of AdS vacua is parametrised solely by the axio-dilaton.

From the perspective of compactification, this is understood as a stabilisation of the scalar associated to the breathing mode of the sphere through fluxes, such that it is no longer a modulus. Note that with the nomenclature established in the introduction, this stabilised scalar is associated to the ADC direction and not part of the moduli space of massless scalars.

Computing the moduli space metric usually requires one to perform the dimensional reduction of type IIB supergravity on S^5, including kinetic terms for the axio-dilaton and the breathing mode, and substitute (7.8).[1] However, we can here take full advantage of type IIB S-duality which is preserved in this background, and constrains the metric to be:

$$dD_\phi^2 \sim \frac{d\tau d\bar{\tau}}{\mathrm{Im}(\tau)^2}\,, \tag{7.10}$$

up to numerical factors that will be irrelevant for our purposes. It is then obvious that there are only two infinite-distance points: $\mathrm{Im}\tau \to 0, \infty$, which are physically equivalent. We focus the rest of the discussion on the latter.

One could naively expect the SDC to work exactly as it does in flat space: using the metric (7.10) any geodesic approaching $\mathrm{Im}\tau \to \infty$ is forced to move only along the $\mathrm{Im}\tau$-direction, and as a consequence the distance behaves logarithmically with g_s. The associated tower of states is then identified with string excitations controlled by the string

[1] According to the generalised SDC [215], this should also include the contribution to the distance due to the change of the AdS scale, which is irrelevant here as it is kept fixed.

scale, M_s, which falls polynomially to zero in Planck units. Putting the two together, we find the expected exponential behaviour.

However, for this argument to work a key point is to remain under parametric control along the trajectory. In particular the quantisation of the string with the usual methods requires one to be in the *weakly-curved* regime. This imposes the string scale to be above the AdS scale:

$$LM_s \sim (g_s N)^{1/4} \gg 1\,. \tag{7.11}$$

As the AdS scale is fixed along the trajectory, we are no longer under parametric control as $\mathrm{Im}\tau \to \infty$. This extra condition does not arise when considering the moduli spaces of Minkowski vacua such as those considered in the usual compactification to flat backgrounds.

We are therefore leaving the phase of the moduli space where the supergravity description is valid, making a qualitative assessement of the SDC impossible, as the two infinite-distance points, $\tau = 0, i\infty$, are both inaccessible in that regime. However, we are conversely entering a phase where a weakly-coupled description in terms of the conformal theory is appropriate. There, the bulk axio-dilaton is identified with the complexified gauge coupling,

$$\tau = \tau_{\mathrm{YM}} = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{\mathrm{YM}}^2}\,, \tag{7.12}$$

and parametrises the only possible ($\mathcal{N} = 4$)-preserving marginal deformation. Due to the amount of supersymmetry, the Zamolodchikov metric, χ, is found to be quantum exact and can be computed through usual diagrammatic methods, or by localisation techniques reviewed next section:

$$\chi_{\tau\bar{\tau}} \sim \frac{N^2 - 1}{\mathrm{Im}(\tau)^2}\,. \tag{7.13}$$

The numerator is set by the dimension of the gauge group $\mathcal{G} = SU(N)$, and we once again ignored irrelevant order one prefactors. As expected from the bulk, there are also two physically-equivalent infinite-distance points related by S-duality, and the bulk limit $\mathrm{Im}\tau \to \infty$ corresponds to a free theory, $g_{\mathrm{YM}} = 0$, on the CFT side.

The operators of the CFT are gauge-invariant composite operators made out of fields in the $\mathcal{N} = 4$ vector multiplet.[2] Their conformal dimensions are given by the sum of the free value and their anomalous dimension, γ:

$$\Delta = \Delta_{\mathrm{free}} + \gamma(\tau)\,. \tag{7.14}$$

In the free limit, the conformal dimension is obtained by naive dimensional analysis. For instance, the lowest-lying operators is given by $\mathrm{Tr}\phi^2$ and is of dimension $\Delta = 2$ in the limit $\mathrm{Im}\tau \to \infty$. Using the dictionary (7.1), it therefore corresponds to a field at the Breitenlohner-Freedman (BF) bound in the bulk.

From the SDC one expects a tower of states becoming massless exponentially with the distance at the infinite-distance point. To see what happens on the CFT side, we can use perturbation theory to write the leading contribution in the $g_{YM} \to 0$ limit as

$$\Delta = \Delta_{\mathrm{free}} + \eta\, g_{\mathrm{YM}}^{\beta} + \mathcal{O}(g_{\mathrm{YM}}^{\beta+1}) = \Delta_{\mathrm{free}} + \eta\, e^{-\alpha_\chi\, D_\chi(\tau)}\,, \tag{7.15}$$

[2]For simplicity, we do not take into consideration the R-symmetry structure of these fields, and generically denote any of the scalars transforming in the **6** of $SU(4)_R$, or later any other scalar, by ϕ. For our purpose, it will be irrelevant.

where $\alpha_\chi\,, \beta\,, \eta$ are coefficients depending on the type of operator considered. In the second equality we have used the expression for the distance with respect to the Zamolodchikov metric in terms of the Yang–Mills coupling. We can easily see that—with the exception of operators whose dimensions are protected by a selection rule—the conformal dimension falls exponentially fast to its free value. An important class of such operators are spin-ℓ operators of the form:

$$J_{\mu_1\ldots\mu_\ell} = \bar{\phi}\, \overleftrightarrow{\partial}_{(\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_\ell)}\phi - (\text{traces})\,. \tag{7.16}$$

These operators have an anomalous dimension at a generic point of the conformal manifold but—using the equations of motion—become conserved in the free limit and saturates the unitarity bound (7.5). The presence of these higher-spin conserved currents in a CFT in fact implies that the theory is free by the Maldacena–Zhiboedov theorem [234].

In the bulk they are identified with higher-spin fields that become massless exponentially fast:

$$M_\ell^2 L^2 = (\Delta + \ell - 2)(\Delta - (\ell+2)) \sim e^{-\alpha\, D_\phi}\,. \tag{7.17}$$

We therefore indeed have a tower of massless modes in the bulk when going to the infinite-distance point, behaving according to the Swampland Distance Conjecture.

However, the unitarity bound (7.5) implies that fields dual to scalar operators of the CFT—e.g. single-trace operators, $\mathcal{O} \sim \text{Tr}\phi^n$—remain massive in the bulk and are regularly spaced for sufficiently large n:

$$M^2_{\text{scal.}} L^2 \sim n^2 + \mathcal{O}(e^{-D_\phi})\,. \tag{7.18}$$

This is a striking difference with respect to the usual results of the SDC for Minkowski backgrounds: in this case the tower is formed by higher-spin modes, which are in principle interacting,[3] but there are only a small number of massless scalar fields. In flat space, the tower always contains an infinite number of massless scalars. These residual masses in our setup are likely related to the presence of curvature and fluxes.

The origin of the tower is however clear: as $g_s \to 0$, the higher-spin fields are those expected from a tensionless fundamental string. The density of the tower is moreover linear, $M_\ell^2 \sim \ell$, which agrees with the flat space expectation, while that of a Kaluza–Klein tower is $M_k^2 \sim k^2$ for sufficiently large k [53]. As we elaborate in the following section, this is a very generic behaviour when a tower of higher-spin conserved currents appears in the CFT, and lends credence to the expectation that infinite-distance points at fixed AdS radius should not be decompactification limits.

One can also ask about the order of magnitude of α in equation (7.17). From (7.13) we see that α_χ in equation (7.16) is, up to order one factors, given by $\alpha_\chi \sim \dim(SU(N))^{-1/2}$. However, we recall that the relation between the moduli space metric and that of the conformal manifold (7.4) introduces a dependence on (LM_{Pl}), which in turn depends on N (7.9). This factor enters in the relation between α_χ and α, which is nothing but taking into account that they are measured in AdS and Planck units. All in all, we find that the exponential rate is order one in Planck units:

$$\alpha \sim (LM_{Pl})^{3/2}\,\alpha_\chi \sim \mathcal{O}(1)\,. \tag{7.19}$$

[3] For a review of the higher-spin/CFT duality, see e.g. [242]

Before closing this section, let us come back to the issue of a well-defined supergravity description. Parametric control is lost when $LM_s < 1$, the scale at which the tower of higher-spin modes falls below the one set by the AdS radius. To obtain an effective description in that regime, it would be required to integrate out these fields in a consistent way, and the new cut-off of the theory would be below the AdS scale. This does not seem to be a meaningful description of the physics in AdS. This suggests that an infrared description of quantum gravity in terms of AdS_5 supergravity is not appropriate to describe an infinite-distance limit, and such a point has to be located at the boundary of the quantum moduli space. One is instead forced to go to the CFT dual to probe such a limit, where the theory is free.

While $\mathcal{N}=4$ super-Yang–Mills and type IIB $\text{AdS}_5 \times S^5$ vacua are very constrained by symmetries and their respective metrics can be understood throughout the entire moduli space, they illustrate a behaviour that is quite universal: when a subsector of the theory becomes free, an infinite number of higher-spin conserved currents always appear at that point in the conformal manifold. In addition, single-trace operators, as the ones we used in equation (7.18), are omnipresent in conformal gauge theories. It is also general that there can only be a small number of scalar fields sitting at the BF bound, as the dual operators must take the form $\text{Tr}(\phi^2)$.

7.2 $\mathcal{N}=2$ Conformal Manifolds in Four Dimensions

Strengthened by the observations made in the previous section, we would now like to extend these arguments to theories with less supersymmetry. We will focus on theories with sixteen real supercharges in the bulk, in particular those obtained by compactifying on an orbifold of S^5. In the SCFT dual, half of the five-dimensional supercharges are mapped to superconformal generators, and one obtains four-dimensional $\mathcal{N}=2$ SCFTs. Before studying the infinite-distance points in both description in more details, let us review some well-established facts about $\mathcal{N}=2$ theories.

As mentioned above, to be able to define a notion of distance on the conformal manifold, $\mathcal{M}_{CFT}$, the relevant object is the so-called Zamolodchikov metric, χ. Denoting the set of all exactly marginal operators by $\mathcal{O}_i$ and their associated coupling constants by τ_i, it is defined as the coefficient of the two-point correlators of marginal operators:

$$\left\langle \mathcal{O}_i(x) \mathcal{O}^\dagger_{\bar{j}}(y) \right\rangle = \frac{\chi_{i\bar{j}}(\tau)}{|x-y|^8}\,. \tag{7.20}$$

Supersymmetry as well as the number of spacetime dimensions constrain the structure of the superconformal multiplets, the possible marginal deformations, and the properties of the metric. For $\mathcal{N}=2$, a relevant class of multiplets are the chiral (resp. anti-chiral) multiplets, denoted $\mathcal{E}_r$ (resp. $\bar{\mathcal{E}}_{-r}$), with r their $U(1)_R$ charge. They have the property of being annihilated by four of the supercharges:[4]

$$[\bar{Q}_{\dot{\alpha}a}, \mathcal{E}_r] = 0\,, \qquad \Delta = r\,. \tag{7.21}$$

Our convention for the quantum numbers and the relevant notions pertaining to superconformal multiplets and their primaries are reviewed briefly in appendix E.

[4] We use the nomenclature of $\mathcal{N}=2$ superconformal multiplets of [243] and, by abuse of notation, also denote their superconformal primaries by $\mathcal{E}_r$.

These multiplets form a ring under the operator product expansion, and can be used to probe a host of properties of a given SCFT. For us, their importance comes from the fact that the chiral ring contains the only possible marginal operator preserving eight supercharges.

To be able to define the $\mathcal{N} = 2$ Zamolodchikov metric (7.20), an operator, $\mathcal{O}$, must satisfy the following properties: be exactly marginal, $\Delta_{\mathcal{O}} = 4$; be a singlet under the R-symmetry group, $SU(2)\times U(1)$, namely $(R, r) = (0, 0)$; and annihilated by all supercharges, $Q\,,\bar{Q}$ (up to total derivatives). Note that such an operator need not be a *super*conformal primary, but simply a conformal primary. It turns out that in the case of four-dimensional $\mathcal{N} = 2$ theories the only such operator is the bottom component of $\mathcal{E}_2$ (or its conjugate) [240, 241]. This operator is indeed by definition annihilated by all anti-chiral supercharges, $\bar{Q}$, and is reached from the superconformal primary by successive applications of the four remaining supercharges, Q. As each application of a supercharge increases the conformal dimension by $1/2$, it is also exactly marginal, $\Delta = 4$. One can further verify that all these marginal operators are then proportional to θ- or gauge kinetic terms,

$$\mathcal{O} = Q^4\mathcal{E}_2 \sim \mathrm{Tr}(F \wedge *F + iF \wedge F)\,. \tag{7.22}$$

When a Lagrangian description is available, the deformation term is obtained by integrating the multiplet over superspace and corresponds to an F-term:

$$\delta\mathcal{L} = \tau^i \int d^4\theta(\mathcal{E}_2)_i + c.c. \tag{7.23}$$

As such, the only possible marginal deformations preserving $\mathcal{N} = 2$ correspond to a modification of Yang–Mills couplings.

7.2.1 The Zamolodchikov Metric

From the discussion above, the conformal manifold metric is therefore related to the two-point functions of the superconformal primaries of the chiral multiplets, $\langle\mathcal{E}_{2,i}\,\bar{\mathcal{E}}_{-2,j}\rangle$. The structure of the conformal manifold of four-dimensional $\mathcal{N} = 2$ SCFTs is extremely constrained. It was indeed shown that superconformal symmetry imposes the conformal manifold to be Hodge–Kähler, and that its Kähler potential, K, is related to the partition function on the four-sphere [244–246]:

$$\chi_{i\bar{j}} = 192\,\partial_i\bar{\partial}_{\bar{j}}K\,, \qquad K = 12\log(Z_{S^4})\,. \tag{7.24}$$

This is a very powerful statement, as the four-sphere partition function of such theories can then be computed via localisation techniques [247]. Indeed, if the SCFT has a Lagrangian description anywhere in the conformal manifold, the partition function can be written as a an integral over the Cartan subalgebra, $\mathfrak{h}$, of the gauge group:

$$Z_{S^4}(\tau_i, \bar{\tau}_i) = \int_{\mathfrak{h}} da\,\Delta(a)\,\left|Z_\Omega(a, \tau_i)\right|^2\,, \tag{7.25}$$

where $\Delta(a)$ is the Vandermonde determinant, and the integrand factorises as

$$Z_\Omega = Z_{\Omega,\mathrm{cl}}(a, \tau_i)\cdot Z_{\Omega,\mathrm{loop}}(a)\cdot Z_{\Omega,\mathrm{inst}}(a, \tau_i)\,. \tag{7.26}$$

The classical contribution is universal,

$$Z_{\Omega,\mathrm{cl}}(a)\;^{2} = \exp\left(2\pi \mathrm{Im}(\tau)\mathrm{Tr}a^2\right), \tag{7.27}$$

while the one-loop and instanton contributions depend on the spectrum of the theory under consideration. For the special subset of $\mathcal{N}=4$ super-Yang–Mills theories, there are no one-loop or instanton contributions, and the computation is reduced to performing a Gaussian integral:

$$Z_{S^4}^{\mathcal{N}=4}(\tau,\bar{\tau}) \sim (\mathrm{Im}\tau)^{-\dim(\mathcal{G})/2}\,. \tag{7.28}$$

Taking derivatives, one arrives again to the result advertised in equation (7.13).

For a generic $\mathcal{N}=2$ theory one can obtain the metric as a formal power series by performing an expansion with respect to marginal couplings. This technique has been used to find the perturbative expansion of the Zamolodchikov metric to high order in SQCD [245, 248] and the large-N limit of necklace theories [249].

7.2.2 The SDC and Weakly-gauged Points

It is easy to see that at any point of the manifold for which a subset $\{\tau_a\}$ of the marginal couplings go to the free limit, $\mathrm{Im}\tau_a \to \infty$, the four-sphere partition function is dominated by the classical term (7.27). After performing a change of variable, the contribution from one-loop and instanton terms is negligible, and one recovers the same Gaussian integral obtained for $\mathcal{N}=4$ (7.28) for each sector:

$$Z_{S^4}(\tau_i) \sim \prod_a Z_{S^4}^{\mathcal{N}=4}(\tau_a)\,, \qquad \text{as } \mathrm{Im}\tau_a \to \infty\,. \tag{7.29}$$

We can now see that the behaviour we observed for $\mathcal{N}=4$ is very generic in this limit. We can again construct infinite towers of composite operators out of all possible fields of the theory. In the $\mathcal{N}=2$ case, we now have as many directions as there are gauge couplings—or equivalently chiral multiplets of R-charge two—setting the dimension of the conformal manifold. The relevant operators will be those made out of combinations of ℓ appropriately-symmetrised derivatives and n scalars coming from the vector multiplets.[5] Their conformal dimensions is then

$$\Delta_{\mathcal{O}_{n,\ell}} = n + \ell + \gamma(\tau_1,\ldots,\tau_{\dim\mathcal{M}})\,. \tag{7.30}$$

A particularity of operators constructed out of scalars coming from vector multiplets is that any interaction term involving them will either come from gauged kinetic terms or from F-terms, and therefore always involve powers of the coupling. In the limit where $\mathrm{Im}\tau_a \to \infty$, the anomalous dimensions will be proportional to the gauge couplings, and using the form of the Zamolodchikov metric (7.29) one finds:

$$\Delta_{\mathcal{O}_{n,\ell}} \sim n + \ell + \eta\, e^{-\alpha_\chi D_\chi(\tau_a)}\,. \tag{7.31}$$

Note that, while for $\mathcal{N}=4$ super-Yang–Mills the case-dependent coefficient η was a pure number, it now can depend on the other couplings that are not taken to the free limit.

[5] Note that these operators must be *bona fide* conformal operators, i.e. eigen-operators of the dilatation generator, and there will in general be mixing between operators with the same quantum numbers. This point is irrelevant to our analysis, as we are only interested in the qualitative behaviour near the infinite-distance point.

We therefore obtain the same qualitative behaviour observed for $\mathcal{N}=4$: in the bulk there is an infinite tower of scalar fields of mass $(m_n L)^2 \sim n^2$, but more importantly, and as required by Maldacena–Zhiboedov theorem [234], there is also an infinite number of higher-spin currents of the form (7.16) that become conserved. The latter class of operators are again mapped to the bulk as an infinite tower of higher-spin modes becoming exponentially light with the distance as in (7.17), as required by the SDC. Similarly, the density of the tower being linear with the spin, we can again expect that these limits do not correspond to a partial decompactification in the string theory description.

Since the tower of states satisfies the SDC, moving away from the "interior" of the moduli space towards an infinite-distance point, the mass of the higher-spin states will eventually fall below the constant AdS radius. Similarly to what happens in the case of the moduli space of $\mathrm{AdS}_5\times S^5$ discussed in the previous section, the supergravity regime will again break down, and the appropriate description will be that of a weakly-coupled CFT. As will be discussed shortly, superconformal representation theory severely restricts the possible infinite-distance points of the conformal manifold. This means that moduli spaces of consistent AdS_5 supergravity theories with sixteen supercharges does not contain any infinite-distance points—at least of the type considered here—where an effective description does not completely breaks down when taking into account quantum corrections.

This breakdown is in spirit similar to what happens in flat space when trying to reach the small-volume point of Calabi–Yau moduli spaces. As one tries to approach it, one leaves geometric phase of the moduli space, and the usual classical moduli are not appropriate quantum variables, leading to a quantum obstruction. Such examples have been studied in the context of the SDC in [49, 53].

Conversely, an obvious difference with $\mathcal{N}=4$ is that many of the operators will not go to their free value, even when they contain fields charged under the gauge group that decouples. The anomalous dimension of such composite operators will generically not be proportional to the associated couplings and there can be mixing with fields of another sector, if for instance they are in the bifundamental representation of groups whose coupling does not go in the free limit.

Having found a tower of states compatible with the SDC, we can now inquire about the order of magnitude of the exponential rate, α. One can consider several sectors decoupling at different paces, and this will be reflected in the value of α. Let us introduce a parameter, t, describing the fastest gauge couplings satisfying $\mathrm{Im}\tau_a \to \infty$. Those going to the same limit, but slower, can be described similarly by introducing an exponent, p_a:

$$\mathrm{Im}(\tau_a) = t^{p_a}\,, \quad 0 < p_a \leq 1\,. \tag{7.32}$$

Of course, $p_a = 1$ only for the parameter—or family of parameters—going to the free limit the fastest. We note that all these trajectories are geodesics, as can be seen by using flat coordinates $\Phi_a \sim \log(\mathrm{Im}\tau_a)$ with respect to the Zamolodchikov metric derived from (7.29) and checking that they are straight lines.

Using (7.29) and (7.24) one can estimate the distance in the Zamolodchikov metric in terms of this parameter, and then translate it to a distance in the moduli space using (7.4). One proceeds in the same fashion as for $\mathcal{N}=4$ to obtain the usual logarithmic behaviour, and a decay rate,

$$\alpha \sim \left(\frac{(LM_{Pl})^3}{\sum_a p_a^2 \dim(\mathcal{G}_a)}\right)^{\frac{1}{2}}\,. \tag{7.33}$$

When the theory admits a point in the moduli space where a supergravity description in terms of Einstein gravity is available, we can estimate the value of LM_{Pl} by computing the trace-anomaly coefficients of the CFT, a, c. Going through the usual holographic computation, one obtains that at leading order in N, $(LM_{Pl})^3 \sim a$. The coefficients further agree up to linear corrections in N, $24(a - c) = n_v - n_h = \mathcal{O}(N)$, where $n_h\,, n_v$ correspond to the number of $\mathcal{N} = 2$ hyper- and vector multiplets, respectively. For large-enough values of N, the standard formulas therefore yield:

$$a = \frac{5n_v + n_h}{24} \sim \frac{n_v}{4} = \frac{1}{4}\dim(\mathcal{G})\,. \tag{7.34}$$

Further gravitational corrections in the bulk will modify the value of the trace-anomaly coefficients, but those will always be subleading in N and will not modify the overall scaling. For our purpose, we can therefore use them to estimate the scaling of $(LM_{Pl})^3$ in terms of the dimension of the total gauge group:

$$\alpha \sim \left(\frac{\dim(\mathcal{G})}{\sum_a p_a^2 \dim(\mathcal{G}_a)}\right)^{\frac{1}{2}}. \tag{7.35}$$

We see that the denominator is bounded between $\dim(\mathcal{G}_{dec.})$, the dimension of the gauge subgroup decoupling the fastest which by assumption has $p_a = 1$, and the dimension of the total gauge group. For any free limit and large-enough groups, we find the bounds:

$$\mathcal{O}(1) \lesssim \alpha \lesssim \left(\frac{\dim(\mathcal{G})}{\dim(\mathcal{G}_{\mathrm{dec.}})}\right)^{\frac{1}{2}}. \tag{7.36}$$

This means that the exponential rate is always of order one in Planck units, or larger. It is thus very easy to engineer limits with large α. For example a theory with gauge group $SU(N)^K$ in the limit where a single $SU(N)$ becomes free leads to:

$$\alpha \sim \sqrt{K}\,. \tag{7.37}$$

Note that while we focussed on $\mathcal{N} = 2$ four-dimensional theories where the relations between the trace-anomaly coefficients and the gauge group data are simple, estimating LM_{Pl} in terms of group theoretical data of the gauge theory can be adapted *mutatis mutandis* to studies in other dimensions, trading a, c for the appropriate quantities. We therefore expect similar bounds in more general cases whenever a sector of the CFT decouples.

7.2.3 Beyond Free Points

If the Swampland Distance Conjecture is true, we expect infinite-distance points to be associated with an infinite towers of massless states in the bulk. As we have seen, those associated to a weak-gauge-coupling limit on the boundary CFT will have an infinite number of higher-spin conserved currents, as required by the Maldacena–Zhiboedov theorem. One may then ask what type of behaviour one can expect beyond those where a sector becomes free, if any.

For instance, one can consider a limit in which a tower of scalars become massless in the bulk and whether it is at infinite distance. Via the dictionary (7.1) such a tower can only appear if there are points with additional marginal operators in the boundary, which by $\mathcal{N} = 2$ superconformal representation theory only exists when there is a gauge symmetry enhancements of the CFT. We can always move slightly away from the conformal manifold onto the Coulomb branch by giving a vacuum expectation value to scalar fields inside the vector multiplet. As the dimension of the Coulomb branch is an invariant of the theory, the total rank of the gauge group is fixed and an infinite tower of scalars is not possible. Beyond $\mathcal{N} = 2$, we are not aware of any CFT exhibiting loci in the conformal manifold where an infinite number of new marginal deformations appear. These towers would be ideal candidates for Kaluza–Klein towers in the bulk and their apparent absence again provides support to the expectation that the ADC and SDC directions in moduli space are separate limits.

Another possibility is an enhancement of the flavour group of the CFT. This requires a would-be flavour current to be part of a long multiplet that becomes short. As shown in [250, 251], an analysis of the recombination rules of long multiplets at threshold reveal the only such possibility to be a superconformal multiplet of type $\widehat{\mathcal{C}}_{0,(\frac{1}{2},\frac{1}{2})}$. This multiplet contains a higher-spin conserved current, implying that there is again a sector of the SCFT that will decouple. It in turn means that the associated gauge enhancements in the bulk are at infinite distance.

There are also strongly-coupled points in the conformal manifold that are at infinite distance. These points are often free points in disguise, as there exists a duality transformation to a frame where there is a weakly-coupled sector. Such examples are plentiful in class S constructions, and we will consider specific cases in the next section.

While we are not able to show that there are no infinite-distance point that do not correspond to a decoupling limit of a $\mathcal{N} = 2$ SCFT, we are not aware of such a case. Using localisation techniques, it is in principle feasible to compute the Zamolodchikov metric in a non-perturbative regime by taking into account all loop and instanton corrections in (7.26).

Finally, there cannot be compact smooth conformal manifolds with $\mathcal{N} = 2$ supersymmetry [246], thereby excluding cases that do not admit any free limit at all. There is furthermore a conjecture stating that any n-dimensional $\mathcal{N} = 2$ conformal manifold can be obtained by gauging n simple factors of the flavour symmetry associated to SCFTs with no marginal deformations [251]. In that sense, all the infinite-distance points studied in this work correspond to reversing (partially or completely) the process by returning to a flavour symmetry.

We close this section by noting that the results we have obtained carry to cases with lower dimensions and supersymmetry. Whenever a sector of the CFT becomes free there will always be a tower of massless higher-spin fields in the bulk. However, this does not mean that sending any marginal coupling to zero will involve a tower of the form (7.16). Indeed let us imagine an $\mathcal{N} = 1$ SCFT depending on two marginal parameters. As marginal operators need not be gauge deformations in that case, sending one of the parameters to zero does not imply that the anomalous dimensions of would-be conserved currents also vanish. It might still depend non-trivially on the other parameter, depending on the structure of the CFT, and the decoupled point could be at finite distance. While there is a possibility that it may be at infinite distance and an SDC tower still exists, it requires a further analysis of $\mathcal{N} = 1$ superconformal representations, which we leave for future works.

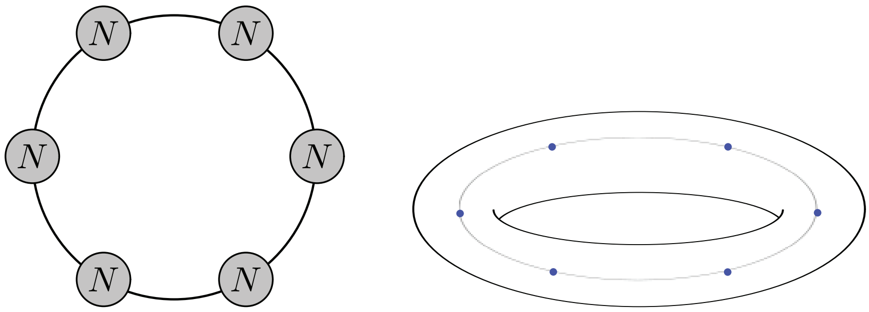

Figure 7.1: Left: quiver representation for the $\hat{A}_{K-1}$ necklace theory, with each node corresponding to a $\mathcal{N} = 2$ vector multiplet and each line a hypermultiplet in the bifundamental of the adjacent groups. Right: the torus with K minimal punctures, T^2_K, of the associated class S construction.

7.3 Orbifolds and $\mathcal{N} = 2$ Necklace Quivers

In Section 7.2 we have reviewed the machinery of four-dimensional $\mathcal{N} = 2$ SCFTs to learn about the possible behaviour of SDC towers in AdS_5 vacua with sixteen supercharges. To understand the mechanisms responsible for the associated infinite-distance points in the bulk, as well as exploring points that are a priori not free, we now turn to an explicit construction in string theory, namely the family of AdS_5 vacua obtained by type IIB compactification on an orbifold of the form S^5/Γ. In order to conserve sixteen supercharges we are forced to consider the orbifold action to be an ADE-type discrete subgroup, $\Gamma \subset SU(2)$. For simplicity we focus on the A-type series, that is, on backgrounds of the form $AdS_5 \times S^5/\mathbb{Z}_K$.[6] Other cases can be generalised straightforwardly.

The dual four-dimensional $\mathcal{N} = 2$ SCFT are the well-known necklace quiver theories with gauge group $\mathcal{G} = SU(N)^K$ [252, 253], which are represented by the affine Dynkin diagram $\hat{A}_{K-1}$, as depicted in figure 7.1. In addition to vector multiplets associated to each gauge factor, there are also hypermultiplets transforming in bifundamental representations of each pair of adjacent gauge factors. At large N, their Zamolodchikov metric was studied in [249].

Necklace theories can be obtained by projecting out modes of $\mathcal{N} = 4$ super-Yang–Mills theory with gauge group $SU(KN)$ and it is natural that the complexified gauge coupling of each gauge sector, τ_i, is related to that of the parent theory, τ_0 [252]:

$$\tau_i = \frac{\tau_0}{K}\,, \qquad \tau = \sum_{i=1}^{K} \tau_i\,. \tag{7.38}$$

Recalling that $\tau_0 = \tau$ is the holographic dual of the axio-dilaton, we see that it is controlling the overall complex gauge coupling in this setup. As each gauge coupling, τ_i, is again associated to a marginal deformation, we expect $K - 1$ additional complex moduli in the bulk. To separate these from the axio-dilaton, we choose the following basis of marginal couplings:

$$\tau_i \in \{\tau_0, \tau_a\}\,, \qquad a = 1, \ldots, K-1. \tag{7.39}$$

Since the axio-dilaton is the only moduli of the unorbifolded theory, the other moduli should come from the twisted sector. Indeed, $\mathbb{Z}_K \subset SU(2)$ does not act freely on the

[6] Seen as a constant-radius hypersurface on $\mathbb{C}^3$, the precise action of $\mathbb{Z}_k$ on the S^5 is given by $(z_1, z_2, z_3) \to \left(e^{2\pi i/K} z_1, e^{-2\pi i/K} z_2, z_3\right)$.

five-sphere, but leaves a circle $S^1 \subset S^5$ of singular fixed points after orbifolding. This leads to a twist sector related to the $K-1$ blow-up 2-cycles resolving these singularities. One finds five scalars for each 2-cycle: two axions and three of geometric origin. It turns out that the latter are stabilised by the potential, and consequently are not part of the moduli space. Thus, as proposed in [252], the $K-1$ complex moduli we are looking for are the axions coming from the periods of the RR and NSNS 2-forms on the 2-cycles, b_a and c_a respectively. We will assume them to be normalised to have period one. All in all, one finds the following dictionary between the moduli and the marginal couplings:

$$\tau_a = c_a + \tau_0 \, b_a \,, \tag{7.40}$$

or in terms of Yang–Mills gauge couplings and theta-angles,

$$\frac{\theta_a}{2\pi} = c_a + b_a C_0 \,, \qquad \frac{4\pi}{g_a^2} = \frac{b_a}{g_s} \,. \tag{7.41}$$

Additional details on the duality between axions in the twisted sector and marginal couplings can be found in [254–256].

Notice that tuning the vevs of these twist-sector moduli corresponds to a deformation of the theory away from the one obtained by orbifolding $\mathrm{AdS}_5 \times \mathrm{S}^5$. Indeed, (7.38) is only recovered when the vev of these axions correspond to the orbifold point [257],

$$c_a = 0, \qquad b_a = \frac{1}{K}. \tag{7.42}$$

There are various ways to reach an infinite-distance point by tuning the different moduli, which as we will see will lead us away from the orbifold point to regions where the supergravity description breaks down. The situation is similar to that of four-dimensional $\mathcal{N}=2$ theories in flat space, as we can tune one or more moduli at the same time, leading to different behaviours. We now analyse various limits and try to identify the associated infinite towers of states, both in the bulk and the CFT.

7.3.1 Overall Free Limit

From the definition of the moduli τ_i it is straightforward to see that the simplest limits are the ones for which all the gauge sectors are becoming free at the same rate. In the bulk, this is controlled by the limit

$$\mathrm{Im}(\tau) \to \infty \,, \tag{7.43}$$

while b_a, c_a and C_0 are fixed. From the expression of the Zamolodchikov metric (7.29), we see that this point is indeed at infinite distance, and we are in a situation similar to that of section 7.1, where the fundamental Type IIB string becomes tensionless and weakly coupled. In this particular case, we can also make use of type IIB $SL(2,\mathbb{Z})$ duality to argue that the same behaviour occurs when reaching the overall strong coupling limit, $\mathrm{Im}\tau \to 0$ with all the other moduli constant.

There is again an infinite tower of higher-spin fields, dual to generalised currents, $J_{\mu_1 \ldots \mu_\ell}$, becoming exponentially massless with the distance in Planck units:

$$\frac{M_\ell}{M_{Pl}} \sim e^{-\alpha D_\phi(\tau)} \,. \tag{7.44}$$

These massless higher-spin currents have an obvious interpretation in the bulk: they originate from higher-spin excitations of tensionless fundamental strings. Moreover, being an overall free limit in the CFT with all gauge sectors decoupling at the same rate, we know from the discussion in section 7.2.1 that α is of order one.

Relaxing the condition $b_a = \text{const}$, it is possible to also explore limits in which different $SU(N)$ sectors decouple at different rates. For this more general case we know from (7.36) that α will be bounded between an order one number and $\sqrt{K}$, and can as a result be parametrically large.

As in $\mathcal{N} = 4$ SYM, but contrary to Minkowski backgrounds, only a small number of scalar fields become massless, while using (7.31) and the AdS/CFT dictionary, all other scalars have masses that are regularly spaced near the infinite-distance point:

$$\frac{M_n}{M_{Pl}} \sim n^2 \,. \tag{7.45}$$

This case is however quite special, as everything is controlled by the value of the axio-dilaton and all fundamental fields of the CFT are free in that limit. We can reach a much richer network of infinite-distance point in moduli space by demanding that only a strict subset of the moduli change over the path, which in the CFT corresponds to taking only some of the gauge couplings to be free.

7.3.2 Strong-coupling Points and Dualities

Demanding the axio-dilaton to remain constant, tuning some—or all—of the $K - 1$ remaining moduli to zero, we can reach a variety of new points in the conformal manifold:

$$\tau = \text{const}\,, \qquad b_a, c_a \to 0\,. \tag{7.46}$$

One could naively expect this path to lead to a finite-distance point, as we are moving in axionic directions. However, moving away from the orbifold point we cannot trust the usual supergravity description, but we can use the dual CFT to nonetheless learn about what happens in its neighbourhood.

Using the dictionary (7.41), sending the moduli to zero corresponds to keeping the overall complex coupling fixed while taking the remaining $K - 1$ gauge sectors to strong coupling:

$$\tau_0 = \text{const}\,, \qquad g_a \to \infty\,, \qquad \theta_a \to 0\,. \tag{7.47}$$

Note that this limit also demands the θ-angles to fall to zero. This setup was studied in details in [258] using Gaiotto's class S construction. In that framework, the necklace quivers are realised as a compactification of the six-dimensional $\mathcal{N} = (2, 0)$ SCFT of type A_{N-1} on a torus with K minimal regular punctures, T^2_K, depicted in figure 7.1.

From this point of view, a trajectory in the conformal manifold in which the axio-dilaton is kept fixed corresponds to changing the relative position of the punctures, and the limit above brings them all together, possibly at different rates. A key point of the class S framework is that as one brings two punctures together, the torus develops a throat and there exist an S-duality frame in which a sector of the theory becomes a weakly-coupled gauge theory. Furthermore, this process is local, and does not depend on what happens in the rest of the surface [259].

In the case at hand, one obtains a sector consisting of a strongly-interacting SCFT, where the $\mathcal{N} = (2, 0)$ theory is compactified on a torus with a single puncture, connected to a weakly-coupled gauge theory with gauge group [258]:

$$\mathcal{G}_{\text{dec.}} \subset \widehat{\mathcal{G}} = SU(2) \times SU(3) \times \cdots \times SU(K)\,. \tag{7.48}$$

We are therefore once again in the situation described around equation (7.31): the conformal dimensions of operators built out of fields in the weakly-coupled vector multiplets have an anomalous dimension that is exponentially suppressed with the distance around the point (7.46) in this particular duality frame, which is at infinite distance. The Maldacena–Zhiboedov theorem again requires the presence of an infinite tower of higher-spin conserved currents.

In the bulk, we also have an infinite tower of higher-spin operators that become massless exponentially fast in Planck units, $M_\ell/M_{Pl} \sim e^{-\alpha\, D_\phi}$. The decay rate can then be estimated using the bounds (7.36) in terms of the group theory data of the dual.

Let us comment on the stringy origin of these states. As b_a and c_a become small, so does the tension of D3-branes wrapped on the blow-up cycles:

$$T_{\text{D3}} \sim \iint_{\Sigma_a} |C_2 + \tau B_2| \xrightarrow{b_a, c_a \to 0} 0\,. \tag{7.49}$$

One might be tempted to conclude that the massless higher-spin fields in the bulk come from such tensionless strings, particularly in the context of the Emergent String Conjecture [50]. However, little is known about the spectrum of these strings in AdS—in particular, in flat space they are non-critical and do not give rise to an infinite number of massless states—and we are therefore unable to make such a claim. The origin of the tower of states remains elusive in these limits.

Intriguingly, it was proposed in [258] that there might be a dual description where there is no tower of higher-spin modes in the bulk. There, part of the SCFT is associated with a four-dimensional strongly-interacting gauge theory living on the boundary of AdS_5, which is then coupled to the rest of the bulk, i.e. to the rest of the type IIB spectrum. At the point where the boundary SCFT becomes free, this gauge theory becomes weakly-coupled. Therefore the higher-spin conserved currents in the boundary are not mapped to massless higher-spin modes in the bulk, but to the higher-spin conserved currents of this four-dimensional gauge theory. This possibility however involves choosing non-standard boundary conditions, and the usual AdS/CFT dictionary does not apply. We leave an exploration of such a description and its relation to the SDC for future works.

Classically, the limits we have been considering would be at finite distance and we expect the infinite distance to be driven by quantum corrections.[7] We note that, similarly to what happens in the case of $\text{AdS}_5 \times S^5$, as we move away from the orbifold point by tuning the axions, we leave the phase of the moduli space where the supergravity regime is valid, and enter a phase where the CFT description is more appropriate.

The behaviours described above generalise to a wide zoo of class S examples. Given a $\mathcal{N} = (2, 0)$ six-dimensional SCFT of ADE type, one can reach a four-dimensional $\mathcal{N} = 2$ SCFT by compactifying on a punctured compact Riemann surface [260], see [261] for a

[7] This can be seen by considering the moduli space as a truncation of the theory obtained by placing the orbifold in flat space. For the latter, the moduli space is classically exact and our limits are known to be at finite distance [258].

review. The surfaces can then be constructed as 3-punctured spheres glued by tubes, called "tinkertoys" [259, 262]. For SCFTs of type A_N, the allowed collisions of punctures leading to a weakly-coupled gauge sector have been studied in [263]. In some cases, one can construct a weakly-curved holographic gravity dual from M-theory on a background of the form $\mathrm{AdS}_5 \times X_6$. As for $\mathrm{AdS}_5 \times S^5$ and its orbifolds, the infinite-distance points will be on the boundary of the moduli space where there supergravity regime has broken down because the tower of state has reached a scale smaller than the AdS radius.

7.4 Summary

By studying the behaviour of states near a family of infinite-distance points in the moduli space of AdS vacua, we have taken a first step towards a possible extension of the Swampland Distance Conjecture to curved backgrounds. To that end, we have used the power of the dual four-dimensional $\mathcal{N} = 2$ superconformal symmetry, which allows one to reduce large classes of infinite-distance points to a case where a subsector of the SCFT becomes free. While we are unable to claim that all infinite-distance points correspond to free limits, we are not aware of possible counterexamples. In the bulk, there is then always an infinite number of higher-spin modes becoming exponentially massless playing the rôle of the SDC tower for AdS moduli spaces.

In flat space, the tower of light states indicates that the effective field theory description is no longer valid, and the SDC is parametrising how this breakdown occurs. By contrast, we find that in the bulk this always happens before reaching the infinite-distance point. For instance, this limit for $\mathrm{AdS}_5 \times \mathrm{S}^5$ vacua is located in the highly-curved regime. On general grounds, the SDC predicts that a tower of states will eventually fall below the AdS scale and an effective description would need a lower cut-off. Should this not be an appropriate description, it would mean that the landscape of AdS vacua in quantum gravity cannot admit an effective field theory description when getting close to infinite-distance points in moduli space, thereby strongly constraining the possible theories which can be coupled to quantum gravity. Note that how close to the infinite-distance point one can go with an effective theory depends on the AdS radius. In particular, it would be interesting to relate the lack of effective description to the species scale, as is done in flat space [25]. In this context, the species scale controls the effective gravity coupling, and thus the size of a typical quantum fluctuation around the background metric. If this logic applies to AdS, when the species scale becomes smaller than the AdS scale, they are large compared to the background, making a geometric description inconsistent.

For theories described by Einstein gravity at a point of the moduli space we have also been able to find bounds for the exponential decay constant. It must always be at least of order one in Planck units and is bounded from above by the ratio between the dimensions of the total gauge group and the decoupled sector. It is therefore possible to obtain a parametrically-large decay constant by engineering a small sector decoupling from a large gauge group.

We have applied this analysis to orbifolds of S^5, with the associated CFT being described by necklace quivers. When all gauge nodes are decoupled, one finds a tensionless fundamental string in the bulk. Using the class-S description of that SCFT we were further able to relate the behaviour of the SDC for individual strong-coupling points to that of free limits via S-duality and found no other infinite-distance points. However, the stringy interpretation of the tower of states is less obvious in these cases: at these points, D3-branes wrapped on blow-up two-cycles become tensionless, but their flat-space avatars are non-critical strings and at finite distance in moduli space. Further, the effective description has broken down well before reaching that point. A relation with the Emergent String Conjecture [50] is therefore not conclusive, and calls for further analysis.

Many of the arguments we discussed here generalize to more arbitrary cases, with and without supersymmetry, that admit a CFT dual. The Maldacena–Zhiboedov theorem does not require supersymmetry and there will always be an infinite tower of higher-spin states in a limit leading to subsector of the CFT becoming free. If the marginal deformation is identified with a gauge coupling along the conformal manifold, it will be at infinite distance by looking at the Zamolodchikov metric and there will be exponentially light states accompanying it. However, the structure of conformal manifolds greatly depends on the spacetime dimension and number of supercharges. For instance, there are no supersymmetry-preserving marginal deformations in six dimensions, which in the bulk translates to all moduli being stabilised [264]. In lower dimensions however, there can be marginal deformations that go beyond changing gauge coupling constants. One would therefore expect to have a far richer network of infinite-distance points in these cases. It might be very interesting to look for similar structures as the ones used in the context of Hodge theory, see e.g. [45, 46, 141, 148].

Furthermore, there also exists conformal manifolds which are compact, see e.g. [265]. While their holographic duals are not well understood, it would be interesting to see if the requirements needed to have a compact manifold can be related to swampland constraints in the bulk.

Unlike the Weak Gravity Conjecture, where black hole physics plays an important rôle, the current understanding of the SDC comes principally from string theory. Using the AdS/CFT correspondence therefore opens new avenues to explore this part of the Swampland programme. For instance, unitarity constraints and other features of superconformal symmetry might shed new light on the origin of the various conjectures and how they are related.

Part V

Conclusions

8

Conclusions and Outlook

The goal of this thesis has been to push the limits of the Swampland Distance Conjecture by studying it from two new perspectives. Parts II and III have been dedicated to two types of spacetime varying configurations exploring infinite field distance. The motivation is to extend the SDC to this context, where it becomes phenomenologically relevant. In part IV the conjecture was put into test in AdS/CFT. Importantly, this opens up a new window for getting evidence and understanding it. In what follows, we summarize the main results of each part, putting emphasis in the future directions that are left open.

We started in chapter 3 with a first example of running solution. This one is particularly interesting since it goes against the naive extension of the SDC to this context. Indeed, there is no tower of states becoming exponentially light as infinite field distance is explored. This however did not translate to a violation of the conjecture in its usual formulation. The SDC is satisfied for geodesics in the field space of the effective field theory, but the running solution explores a highly non-geodesic trajectory that delays the falloff of the tower. For this, it is was crucial that the dilaton could not be integrated out from the effective field theory describing the dynamics of the running solution. Motivated by this interesting mechanism, we proposed in chapter 4 that consistency of the SDC along the RG flow of the theory imposes constraints on the potentials that are attainable in quantum gravity. It should not happen that a potential has a valley exploring a non-geodesic trajectory as the one in chapter 3. Otherwise one could obtain an EFT containing only a scalar and no tower of exponentially light states at infinite field distance. In order to characterize these forbidden trajectories we introduced a geometric formulation of the SDC. Interestingly, we also showed that this is equivalent to a convex hull condition à la Convex Hull WGC and in the spirit of the Scalar WGC. These results appearing in part II leave several interesting avenues for future research:

- The solution in chapter 3 has negative vacuum energy and space-like dependent profiles for the fields. For phenomenological reasons, it would be interesting to find similar ones but with positive vacuum energy and with time-like dependence.
- It would also be exciting to strengthen the connection between the Convex Hull SDC introduced in chapter 4, the Convex Hull WGC [82] and the Scalar WGC [71]. This could be a way of clarifying the nature of the extremal region in the Convex Hull SDC, and could eventually provide a bottom-up rationale for the SDC.
- Finally, further exploring the constraints put by the SDC on potentials can lead to connections to other Swampland conjectures, such as the de Sitter conjectures [157, 213, 216] or the TCC [37].

In part III we moved to another type of running solutions dubbed dynamical cobordisms. They were first introduced in [158] as dynamical realizations of the cobordisms of the Cobordism Conjecture [63]. In chapter 5 we argued for a relation between the type of cobordism and the field space distance explored in setups with dynamical tadpoles. It is natural that interpolating domain walls, in which spacetime continues across the wall, are related to scalars staying at finite distance. On the other hand, walls of nothing capping off spacetime beyond them correspond to scalars exploring infinite field distance. The latter kind of solutions were further studied in chapter 6. The universal scaling relations

$$\Delta \sim e^{-\frac{1}{2}\delta D_\phi}, \quad |R| \sim e^{\delta D_\phi} \tag{8.1}$$

were found in several examples in string theory. Moreover, a bottom-up effective field theory approach revealed a relation between these scalings and having exponential potentials at infinite field distance limits. Potentials with this property are ubiquitous in string theory as well. There are several interesting open question for the future:

- We have mainly focused on space-dependent running solutions. It is clearly interesting to consider time-dependent solutions, extending existing results in the literature [159–165, 188], and exploit them in applications, in particular with an eye on possible implications for inflationary models or quintessence.
- The appearance of an universal behaviour as infinite field distance is explored in dynamical cobordisms to nothing calls for a relation to the universal behaviour of the SDC. For instance, the lowered cutoff of the SDC could be related to the resolution of the singularity in the effective field theory description of the dynamical cobordism. On the other hand, the relation between these scalings and the presence of exponential potentials reveals a tantalizing link to the de Sitter conjectures [157, 213, 216] or the TCC [37]. It would be interesting to further explore these connections.
- When combining the exponential behaviour of the SDC with the universal scalings for the scalar curvature, we obtained an ADC-like scaling relation in (6.145). Its appearance possibly signals an underlying improved understanding of infinite distance limits in dynamical (rather than adiabatic) configurations. As shown in chapter 3, the $r \to \infty$ limit in the Klebanov-Strassler solution [112] avoids the appearance of a tower of states becoming massless exponentially with the distance. However, as dictated by the lack of separation of scales in this model, an ADC-like scaling is yet respected as the scalar curvature goes to zero in this limit. This could point to a more universal way of writing the SDC in dynamical configurations.

Changing topics, in part IV we started the study of the SDC in the context of AdS/CFT. From the CFT perspective, all the infinite distance limits that we considered were such that a subsector of the theory decoupled. In these cases, the tower of states is given by the higher-spin conserved currents that are always present in these limits. We showed that this is very generic in the context of 4d $\mathcal{N} = 2$ theories. For those with Einstein gravity duals, we were able to show that the exponential decay rate is always larger than an order one constant. Unlike in the flat space case, it is however easy to build examples in which it is parametrically large. We studied in more detail some particular examples whose gravity duals are known in string theory. They realized the possibility of having parametrically large exponential decay rate α for the SDC. Furthermore, there were limits in which the infinite field space distance is generated by quantum corrections, in the sense that they are at finite distance in the classical supergravity description. The stringy object giving rise to the tower of states in these same limits remained unclear, but we proposed some interesting candidates. The results of this part leave very promising future directions:

- It would be very interesting to exploit the CFT machinery to test (or even prove) the SDC in AdS/CFT. For example, there are conformal perturbation theory techniques describing how the CFT data changes along the conformal manifold. They are very general, as no specific number of dimensions or amount of supersymmetry is assumed, and seem to be a perfect fit for testing the SDC from the CFT.

- In a similar fashion, one can try to import techniques that has been useful to study the SDC in flat space to the conformal manifold. A lot of research has been focused in exploiting the consequences of monodromies around the infinite distance loci (see e.g. [45,46]). It would be interesting to find and describe these kind of monodromies around weak coupling loci in conformal manifolds. This could lead to an useful description of weakly-coupled CFTs.

- Finally, it would be exciting to clarify the string theory origin of the tower of states in the orbifold theories discussed in section 7.3. The only candidate that was found is a non-critical string when propagating in 6d flat space. It would be extremely interesting to check whether it can give rise to an infinite number of states when being put in $AdS_5 \times S^1$.

Let us close this thesis remarking how fruitful it is to push the limits of the Swampland conjectures. By testing and studying the SDC outside of its strict regime of validity, we have been able to find unexpected implications, obtain promising connections to other conjectures, and even open a whole new window to test and understand the conjecture. Looking for quantum gravity imprints at low energies is at the very core of the Swampland program. This can not only grant us with a deeper understanding about quantum gravity, but also with some plausible experimental test for these ideas. For this reason, extending the limits of validity of the Swampland conjectures is crucial both from the theoretical and the phenomenological side. We hope that the results presented in this thesis serve as motivation to keep pushing these limits in future research.

Conclusiones y Perspectivas

El objetivo de esta tesis ha sido empujar los límites de la Conjetura de la Distancia estudiándola desde dos nuevas perspectivas. En las partes II y III hemos estudiado dos tipos de configuraciones dinámicas que exploran distancia de campos infinita en espacio-tiempo. La motivación es extender la SDC a este contexto, donde se vuelve relevante para modelos fenomenológicos. En la parte IV hemos testeado la conjetura en AdS/CFT. Esto abre una nueva serie de escenarios en los que buscar evidencia y entender la conjetura. A continuación resumimos los resultados principales de cada parte, poniendo énfasis en las preguntas que han quedado abiertas para el futuro.

Empezamos en el capítulo 3 con un primer ejemplo de solución dinámica. Esta es especialmente interesante, ya que va en contra de la extensión más directa de la conjetura a este contexto. No hay una torre de estados volviéndose ligera exponencialmente al explorar distancia de campos infinita. Sin embargo, esto no se tradujo en una violación de la conjetura en su formulación original. La SDC se cumple para las geodésicas en el espacio de campos de la teoría efectiva, pero la solución dinámica explora una trayectoria altamente no geodésica que retrasa la caída de la torre. Para que esto funcionara, fue crucial que el dilatón no se pudiera integrar fuera de la teoría efectiva de campos que usamos para describir la dinámica de la solución. Con este mecanismo como motivación, en el capítulo 4 se propuso que el que la SDC sea consistente a lo largo del flujo de renormalización de la teoría impone ciertos requisitos sobre los potenciales que pueden aparecer en teorías de gravedad cuántica. No debe ocurrir que el potencial tenga un valle que explore una trayectoria no geodésica como la que apareció en el capítulo 3. De lo contrario, uno podría hallar una teoría efectiva de campos que solo contenga un escalar y sin ninguna torre de estados exponencialmente ligera a distancia de campos infinita. Para describir estas trayectorias prohibidas introdujimos una formulación geométrica de la SDC. Un aspecto interesante de esta es que es equivalente a la condición sobre la envolvente convexa de la Convex Hull WGC pero usando cantidades que aparecen en la Scalar WGC. Estos resultados de la parte II dejan varias direcciones abiertas para investigaciones futuras:

- La solución del capítulo 3 tiene energía de vacío negativa y los campos tienen dependencia de tipo espacial. Por razones fenomenológicas, sería interesante encontrar soluciones similares con energía de vacío positiva y dependencia de tipo temporal.
- También es sugerente tratar de fortalecer la conexión entre la Convex Hull SDC introducida en el capítulo 4, la Convex Hull WGC [82] y la Scalar WGC [71]. Esto podría llevar a entender mejor la naturaleza de la región extremal de la Convex Hull SDC, y eventualmente podría llevarnos a una razón fundamental por la que la SDC debería cumplirse.
- Por último, explorar más a fondo los requisitos puestos por la SDC sobre los potenciales puede llevar a conexiones con otras conjeturas tales como la del de Sitter [157, 213, 216] o la TCC [37].

En la parte III pasamos a otro tipo de solución dinámica conocida como cobordismos dinámicos. Estos fueron introducidos por primera vez en [158] como descripciones dinámicas de los cobordismos de la Conjetura del Cobordismo [63]. En el capítulo 5 presentamos una relación entre el tipo de cobordismo y la distancia de campos explorada en teorías con renacuajos dinámicos. Los muros de dominio, en los que el espacio-tiempo continua tras el muro, están relacionados con escalares que permanecen a distancia finita. Por otro lado, los muros de la nada que acaban con el espacio-tiempo corresponden a escalares explorando distancia de campos infinita. Soluciones de este último tipo fueron estudiadas más a fondo en el capítulo 6. Comprobamos las relaciones de escala universales

$$\Delta \sim e^{-\frac{1}{2}\delta D_\phi}, \quad |R| \sim e^{\delta D_\phi} \tag{8.2}$$

en muchos modelos en teoría de cuerdas. Más aún, un análisis en teorías efectivas de campos genéricas nos reveló que estas relaciones están conectadas con tener potenciales exponenciales en los límites a distancia de campos infinita. Este tipo de potenciales son omnipresentes en teoría de cuerdas también. Hay varias preguntas sin resolver para el futuro:

- Hasta ahora nos hemos centrado en soluciones con dependencia de tipo espacial. Sería interesante considerar soluciones con dependencia de tipo temporal, extendiendo los resultados ya existentes en la literatura [159–165, 188]. El objetivo es buscar posibles aplicaciones, en particular prestando atención a posibles implicaciones para modelos inflacionarios o de quintaesencia.

- Haber encontrado un comportamiento universal al explorar distancia de campos infinita en cobordismos dinámicos a la nada sugiere una relación con el comportamiento universal que predice la SDC. Por ejemplo, la validez reducida asociada a la SDC podría estar relacionada con la resolución de la singularidad que aparece en la descripción del cobordismo dinámico en teoría efectiva de campos. Por otro lado, la relación entre las relaciones de escala y la presencia de potenciales exponenciales revela una posible conexión con las conjeturas del de Sitter [157, 213, 216] o la TCC [37]. Sería interesante explorar estas conexiones más en profundidad.

- Al combinar el comportamiento exponencial en la SDC con la relación de escala para el escalar de curvatura, obtuvimos una relación de escala como la de la ADC en (6.145). Esto posiblemente apunta hacia una mejor manera de comprender los límites a distancia infinita en configuraciones dinámicas (en vez de adiabáticas). Tal y como mostramos en el capítulo 3, el límite $r \to \infty$ en la solución Klebanov-Strassler [112] evita la aparición de la torre de estados volviéndose ligera de forma exponencial con la distancia. Sin embargo, tal y como dicta la falta de separación de escalas en este modelo, la relación de escalas del tipo ADC sigue satisfaciéndose en este límite en el que el escalar de curvatura va a cero. Esto podría señalar a una forma más universal de escribir la SDC en configuraciones dinámicas.

Cambiando de tema, en la parte IV comenzamos a estudiar la SDC en AdS/CFT. Desde la perspectiva de la CFT, todos los límites a distancia infinita que consideramos eran tal que un sector de la teoría se desacoplaba. En estos casos, la torre de estados viene dada por las corrientes conservadas de espín alto que siempre están presentes en estos límites. Mostramos que esto es muy genérico en teorías en 4d y con $\mathcal{N} = 2$. Para aquellas con duales gravitacionales de tipo Einstein, fuimos capaces de poner una cota inferior de orden uno sobre la velocidad de caída exponencial. A diferencia de en el caso de espacio plano, es bastante fácil construir ejemplos en los que esta cantidad puede ser arbitrariamente grande. Estudiamos más a fondo algunos ejemplos cuyo dual gravitacional es conocido en teoría de cuerdas. En ellos se podía encontrar velocidades de caída exponencial α paramétricamente grandes para la SDC. Además, había límites en los que la distancia infinita era generada por correcciones cuánticas. Esto quiere decir que estos mismos límites están a distancia finita en la descripción clásica en supergravedad. No quedó claro cual es el objeto cuerdoso del que nace la torre de estados en estos mismos límites, pero propusimos un candidato muy interesante. Los resultados de esta parte dejan líneas de investigación muy prometedoras para el futuro:

- Sería interesante explotar las herramientas desarrolladas para estudiar CFTs para poner a prueba la SDC en AdS/CFT. Por ejemplo, la teoría de perturbaciones conformes describe como la CFT cambia al moverse por la variedad conforme. Estas técnicas son muy generales, ya que no especifican el número de dimensiones ni asumen ninguna cantidad de supersimetría, y parecen perfectas para testear la SDC desde la CFT.

- De la misma manera, se puede intentar importar técnicas que han sido muy útiles para estudiar la SDC en espacio plano a la variedad conforme. Se ha desarrollado mucha investigación alrededor de las consecuencias que tienen las monodromías alrededor de los puntos a distancia infinita (véase por ejemplo [45,46]). Sería fascinante encontrar y describir este tipo de monodromías alrededor de puntos de acople débil en variedades conformes. Esto podría resultar en una descripción muy útil para CFTs débilmente acopladas.

- Por último, sería fascinante aclarar el origen en teoría de cuerdas de la torre de estados en las teorías tipo órbifold que comentamos en la sección 7.3. El único candidato que encontramos es una cuerda no crítica cuando se propaga en espacio plano. Sería extremadamente interesante comprobar si esta puede dar lugar a un número infinito de estados cuando es puesta en $AdS_5 \times S^1$.

Cerramos esta tesis haciendo hincapié en lo fructífero que es empujar los límites de las conjeturas de la Ciénaga. Poniendo a prueba y estudiando la SDC fuera de su régimen de validez estricto, hemos sido capaces de encontrar implicaciones inesperadas, de obtener prometedoras conexiones con otras conjeturas e incluso de establecer un nuevo contexto en el que aprender sobre ella. Buscar consecuencias que tiene gravedad cuántica en la física de bajas energías es algo central en el programa Ciénaga. Esto no solo nos puede ayudar a entender mejor como funciona la gravedad a nivel cuántico, sino que también nos puede revelar algún experimento que sea plausible de llevar a cabo y que ponga a prueba estas ideas. Por este motivo, extender los límites de validez de las conjeturas de la Ciénaga es crucial tanto desde el punto de vista teórico como fenomenológico. Esperamos que los resultados presentados en esta tesis sirvan de motivación para seguir empujando estos límites en futuras investigaciones.

Part VI

Appendices

A

Periodic crossing and the dual Hanany-Witten picture

In this section we discuss a T-dual realization of the KS duality cascade, in terms of the NS5- and D4-brane configurations [255] realizing 4d gauge theories à la Hanany-Witten [266]. The picture is similar to that mentioned in [97], albeit with additional relevant refinements. The configuration is flat 10d space with one dimension, labelled 6, compactified on an $\mathbf{S}^1$. There is one NS5-brane along the directions 012345 (and at the origin in 89), and one NS5-brane (denoted NS5') along the directions 012389 (and at the origin in 45), with D4-branes along 0123 and suspended among them in 6 (and at the origin in 4589), in a compact version of [267]. The positions of all branes in the directions 7 are taken equal. The numbers of D4-branes at each side of the interval are N and $N+M$ respectively. The scalar ϕ corresponds to the distance (in units of 2π the radius of $\mathbf{S}^1$) between the NS and the NS'-branes, so it has periodicity $\phi \sim \phi + 1$. In a naive description, as the scalar winds around its period, the crossings of the NS and NS'-branes produce Seiberg dualities that complete a full cycle in the duality cascade. This naive picture would seem to suggest that each crossing leads to additional light degrees of freedom, which could spoil the axion monodromy, or at least its description in terms of an effective action not including these new modes. However, the actual picture is somewhat more intricate and is free of these problems. The answer lies in the phenomenon of brane bending in [255], which implies that the M additional D4-branes on one of the intervals forces the NS- and NS'-branes to bend. This bending has a logarithmic dependence, and is a long distance result of the description of the whole system as a single M5-brane in a holomorphic curve in the M-theory lift of the configuration [255, 268]. In $\mathcal{N}=2$ 4d theories, this corresponds in a precise manner to the field theory running of gauge couplings on the Coulomb branch. In the present $\mathcal{N}=1$ setup, the RG direction (to become the radius in the gravitational dual side) can be thought of as the radial distance away from the point $x^4 = x^5 = x^8 = x^9 = 0$ at which all branes are located. Then, there is a logarithmic bending of the positions of the NS- and NS'-branes in the directions 6, which matches the above naive description. However, the other positions of the NS- and NS'-branes in the other directions do not coincide, hence no actual crossing of branes occurs. The discussion of Seiberg dualities carries over but in this more precise sense. The phenomenon is similar to the discussion in [269].

B

Holographic Examples of Dynamical Cobordisms

In [158] it was shown that Dynamical Cobordism underlies the structure of the gravity dual of the $SU(N) \times SU(N+M)$ conifold theory, namely fractional brane deformation of $\mathrm{AdS}_5 \times T^{1,1}$. This in fact explains the appearance of a singularity at finite radial distance [111] and its smoothing out into a configuration capping of the warped throat [112], as a cobordism wall of nothing. In this appendix we provide some examples of other warped throat configurations which illustrate the appearance of other cobordism walls of nothing, and cobordism domain walls interpolating between theories corresponding to compactification on horizons of different topology. The discussion is strongly inspired by the constructions in [124] (see also [129]).

2.1 Domain wall to a new vacuum

As a first example we consider a configuration in which a running of the conifold theory hits a wall (given by the tip of a KS throat) interpolating to an $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum. The latter is the maximally symmetric solution of a theory at the bottom of its potential, i.e. with no dynamical tadpole. We carry out the discussion in terms of the dual field theory, which translates easily into the just explained gravity picture. The dilaton is constant in the whole configuration, so we skip factors of g_s.

Consider the conifold theory with $SU(N) \times SU(N+M)$ at some scale, i.e. at some position r there are N units of 5-form flux and M units of 3-form flux. The Klebanov-Tseytlin solution [111] gives a running for the effective flux

$$N(r) = N + M^2 \log(r)\,, \tag{B.1}$$

and we get a singu at a value r_0 defined by

$$N + M^2 \log r_0 = 0 \quad \Rightarrow \quad r_0 = e^{-N/M^2}\,. \tag{B.2}$$

Naively, the singularity would seem to be smoothed out into a purely geometric background with a finite size $\mathbf{S}^3$. Indeed, this is the full story if N is multiple of M, namely $N = KM$: in the field theory, the $SU(KM) \times SU(KM+M)$ theory suffers a cascade of K Seiberg dualities in which K decreases by one unit in each step. Morally, the cascade ends when the effective $K = 0$ and then we just have a pure $SU(M)$ SYM, which confines and develops a mass gap. This is the end of the RG flow, with no more running, hence the spacetime in capped off in the IR region of the dual throat.

However, as also noticed in [112], the story is slightly different if $N = KM + P$. After the K steps in the duality cascade, one is left over with an $SU(P)$ gauge theory with three complex scalar degrees of freedom parametrizing a deformed mesonic moduli space corresponding to (the symmetrization of P copies of) the deformed conifold. This gauge theory flows to $\mathcal{N}=4$ $SU(P)$ SYM in the infrared, which is a conformal theory. In the parameter range $1 \ll P \ll M \ll N$, the whole configuration admits a weakly coupled supergravity dual given by a KS throat at which infrared region we have a finite size $\mathbf{S}^3$, at which P D3-branes (which we take coincident) would be located; however, since P is large, they backreact and carve out a further $\mathrm{AdS}_5 \times \mathbf{S}^5$ with P units of RR 5-form flux, which continues the radial direction beyond the KS throat endpoint region. Hence, this region acts as an interpolating domain wall between two different (but cobordant) theories, namely the conifold throat (with a dynamical tadpole from the fractional brane charge), and the $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum (with no tadpole, and preserving maximal symmetry). The picture is summarized in Figure B.1

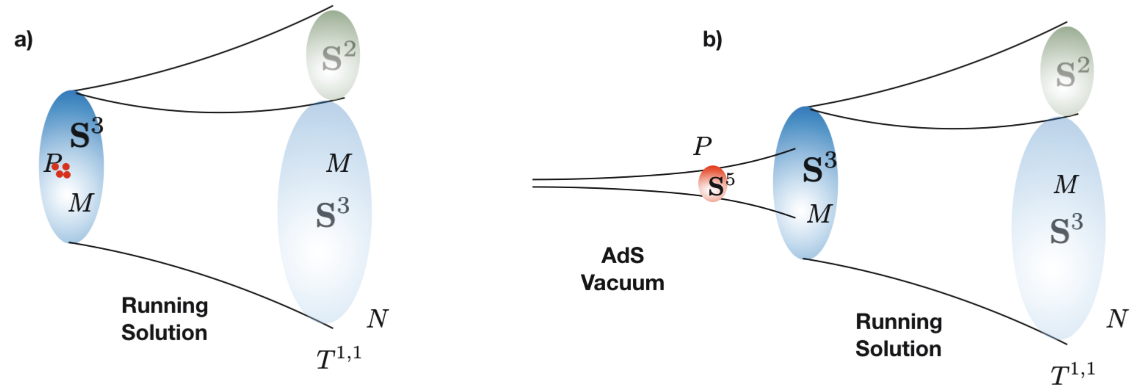

Figure B.1: Domain wall interpolating between the conifold theory with fractional branes, and an AdS vacuum. Figure a) shows a heuristic intermediate step of a KS solution with a number P of left-over probe D3-branes. If P is large, the appropriate description requires including the backreaction of the D3-branes, which lead to a further AdS throat, to the left of the picture in Figure b). Hence the running of the dynamical tadpole in the right hand side ends in a domain wall separating it from an AdS vacuum.

2.2 Domain wall to a new running solution

Running can lead to an interpolating domain wall, across which we find not a vacuum, but a different running solution (subsequently hitting a wall of nothing, other interpolating domain walls, or just some AdS vacuum). We now illustrate this idea with an example of a running solution A hitting a domain wall interpolating to a second running solution B, which subsequently hits a wall of nothing. The example is based on the multi-flux throat construction in [124] (whose dimer picture is given in [270]). It is easy to devise other generalizations displaying the different behaviours mentioned above.

Consider the system of D3-branes at the singularity given by the complex cone over dP_3. The gauge theory is described by the quiver and dimer diagrams[1] in Figure B.2.

[1]For references, see [129, 271–273].

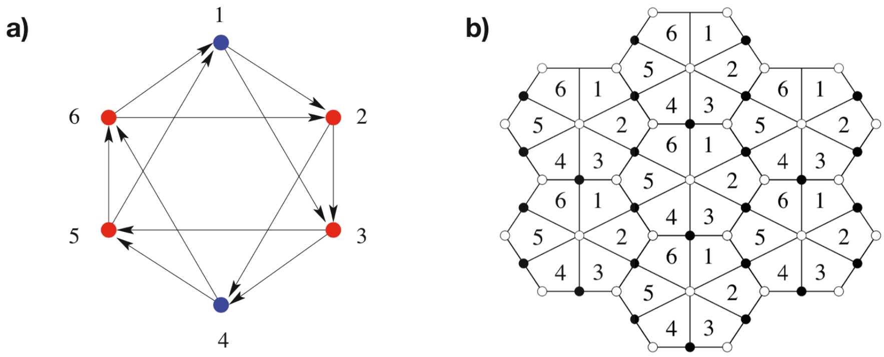

Figure B.2: The quiver and dimer diagrams describing the gauge theory on D3-branes at the tip of the complex cone over dP_3.

We can add fractional branes, i.e. rank assignments compatible with cancellation of non-abelian anomalies. There are several choices, corresponding to different fluxes on the 3-cycles in the dual gravitational theory. Some of them correspond to 3-cycles which can be grown out of the singular origin to provide a complex deformation of the CY. These are described as the splitting of the web diagram into sub-webs in equilibrium, see [270]. In particular we focus on the complex deformation of complex cone over dP_3 to a conifold, see the web diagram in Figure B.3.

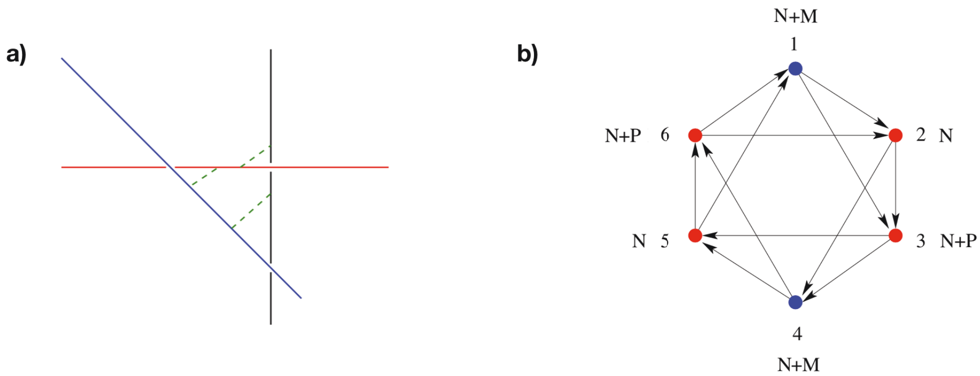

Figure B.3: a) Web diagram of the complex cone over dP_3 splitting into three sub-webs. b) Rank assignment (fractional branes) that trigger those complex deformations.

There are two kinds of fractional branes, associated to M and P. In the gravity dual, these correspond to RR 3-form fluxes on 3-cycles (obtained by an $\mathbf{S}^1$ fibration over a 2-cycle on dP_3), and there are NSNS 3-form fluxes in the dual 3-cycles. These are non-compact, namely they span a 2-cycle (dual to the earlier 2-cycle in dP_3) and the radial direction. For more details about the quantitative formulas of this kind of solution, see Section 5 of [123].

If we focus in the regime[2] $P \ll M$, then the larger flux M implies a larger corresponding component of the H_3 flux, which means a faster running of the corresponding 5d NSNS axion. The axion associated to the flux P also runs, but more slowly. In the field theory, the duality cascade is controlled by M, so that N is reduced in multiples of M (at leading order in P/M). When N is exhausted we are left with a rank assignment as given in Figure B.4a. The result of the strong dynamics triggered by M can be worked out in field theory as in [124] or using dimers as in [270]. All the info about this last description is in Figure B.4b.

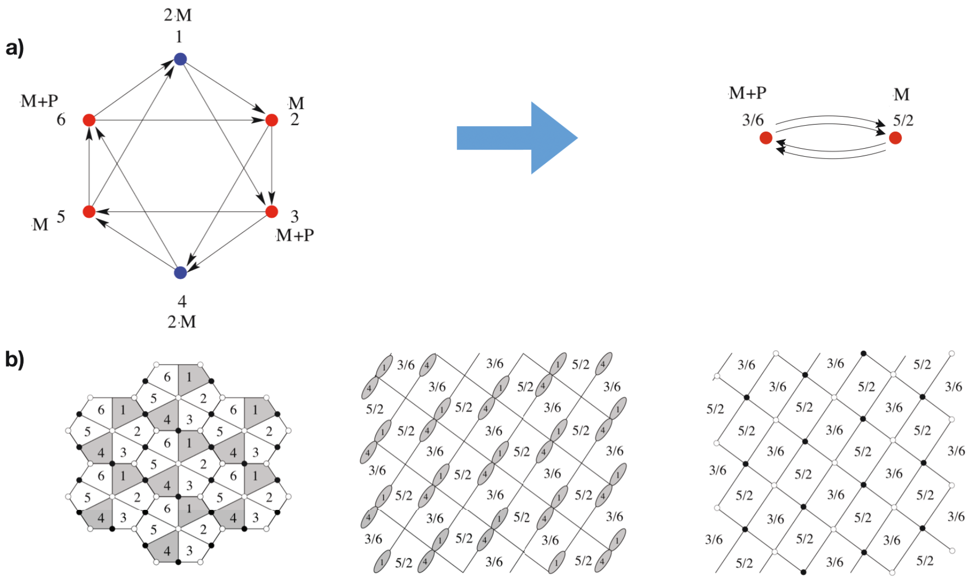

Figure B.4: a) Quiver of the dP_3 theory in the last step of the first cascade, which turns into the conifold upon strong dynamics of the nodes 1 and 4. b) Same story in the dimer picture.

The result is a conifold theory with M regular branes and P fractional branes. This is the standard KS story (with just different labels for the branes): M decreases in sets of P until it is exhausted, then the running stops due to strong dynamics. In the gravity dual, we have a KS throat sticking out and spacetime ends on the usual $\mathbf{S}^3$ (alternatively, if M is not a multiple of P, there is a number P of leftover D3-branes, which, if large, can trigger a further AdS throat as in Section 2.1. A sketch of the gravity dual picture is shown in Figure B.5.

Note that this kind of domain wall interpolates into two topologically different compactifications. As we cross it, the compactification space changes, and the spectrum of light fields changes (at the massless level, one of the axions ceases to exist). In this sense, it is a cobordism domain wall connecting two different quantum gravity theories [63].

[2] Note that in [124] the regime is the opposite, but both kinds of fractional branes are similar, so the result is the same up to relabeling.

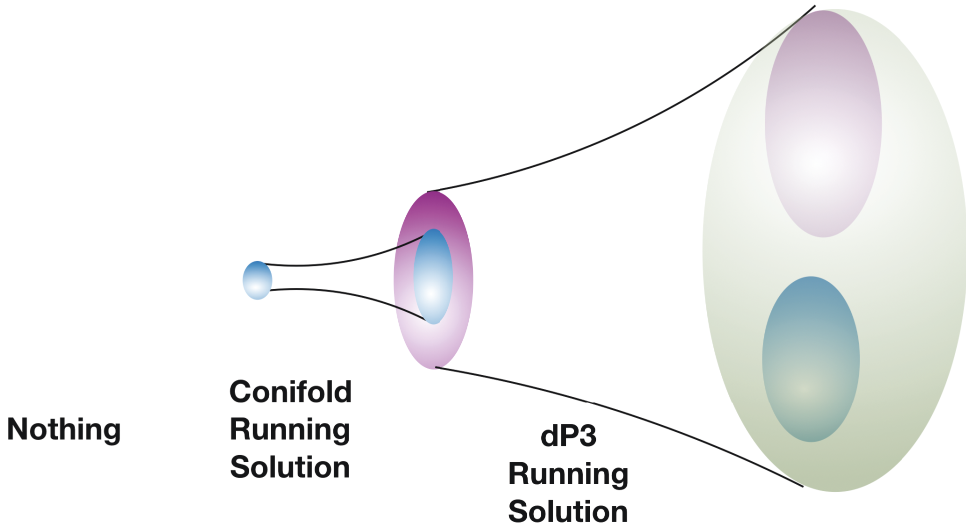

Figure B.5: Domain wall interpolating between the theory on dP_3 with $(M + P)$ fractional branes, and a conifold theory with M regular branes and P fractional ones. The running of one of the dynamical tadpoles in the dP_3 theory stops at the wall but the other continues running until it reaches the $\mathbf{S}^3$ at the bottom of the KS throat.

2.3 Cobordism domain walls to disconnected solutions

The construction of singularities admitting complicated patterns of complex deformations (or resolutions) can be carried out systematically for toric singularities, using the techniques in [129]. This can be used to build sequences of domain walls realizing a plethora of possibilities. For our last class of examples, we consider cobordism domain walls to disconnected theories.

This has already been realized in the geometry used in [105] to build a bifid throat, i.e. two throats at the bottom of a throat, see Figure B.6. These had been proposed in [98] as possible hosts of axion monodromy inflation models (see [97, 101, 102, 119, 207] for additional references).

Actually, a far simpler way of getting a running solution with a domain wall to a disconnected set of e.g. vacua is to consider the KS setup in Section 2.1, with the P leftover D3-branes split into two stacks P_1 and P_2 of D3-branes at separated locations on the $\mathbf{S}^3$ (with $P_1, P_2 \gg 1$). This corresponds to turning on a vev v for a Higgsing $SU(P) \to SU(P_1) \times SU(P_2)$ (with $P_1 + P_2 = P$) with a scale for v much smaller than the scale of confinement Λ of the original $SU(KM + P) \times SU(KM + M + P)$ theory. In the gravity dual, we have a running solution in the holographic direction, towards low energies; upon reaching Λ, we have the $\mathbf{S}^3$ domain wall, out of which we have one $\mathrm{AdS}_5 \times S^5$-like vacuum (with flux P), until we hit the scale v, and the single throat splits into two $\mathrm{AdS}_5 \times \mathbf{S}^5$ throats (with fluxes P_1, P_2). If $v \simeq \Lambda$, the splitting of throats happens in the same regime as the domain wall ending the run of the initial solution. This is depicted in Figure B.7.

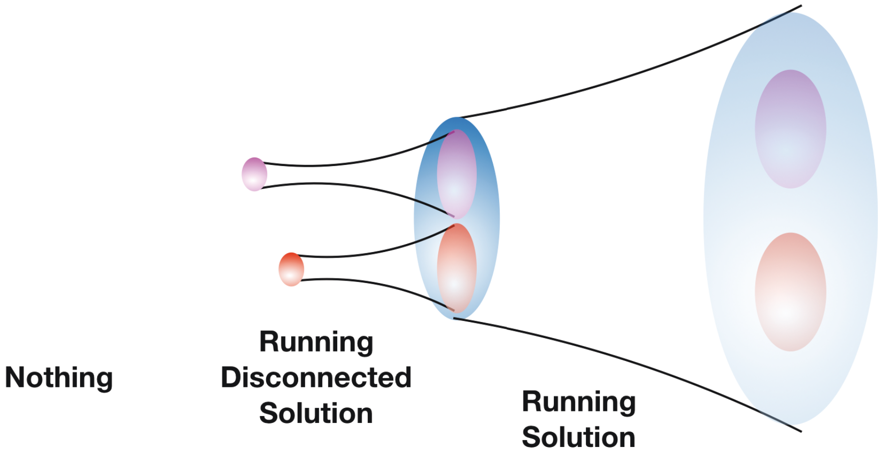

Figure B.6: Picture of a bifid throat. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two quantum gravity theories.

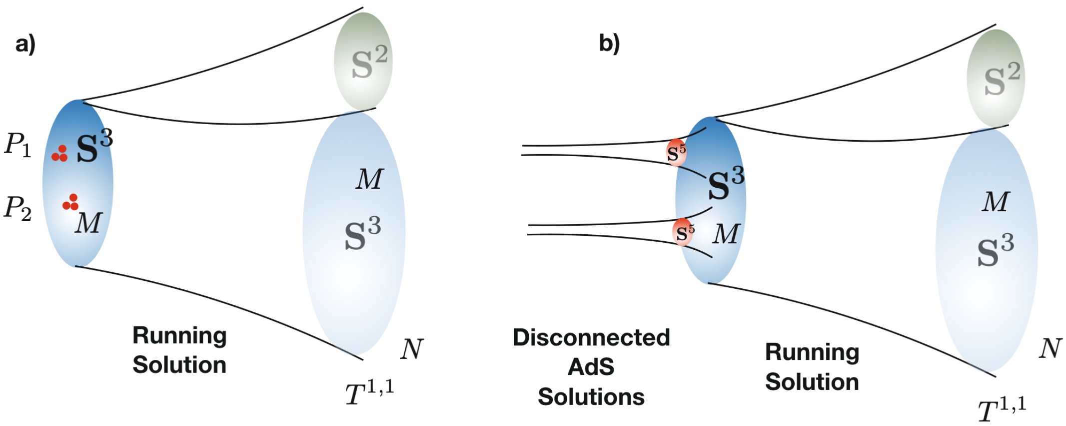

Figure B.7: Picture of a bifid throat with two AdS tongues. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two AdS theories.

C
Local Dynamical Cobordisms with curved $(d-1)$-dimensional slices

3.1 General analysis for curved slices

We can generalize the discussion in Section 6.1 to the case in which the ETW brane has constant internal curvature R_d. Namely we take the foliation ansatz (6.3) with ds^2_{d-1} describing a constant curvature $(d-1)$-dimensional metric. The equations of motion read

$$(d-1)\sqrt{2\,(V-V_t)}\,\sigma' - \partial_\phi V_t = 0\,, \tag{C.1}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^{\,2} + V_t - \frac{1}{2}e^{2\sigma}R_d = 0\,, \tag{C.2}$$

$$(d-2)\sigma'' - 2\,(V-V_t) - \frac{1}{d-1}e^{2\sigma}R_d = 0\,, \tag{C.3}$$

where we have again introduced the tunneling potential defined in (6.7).

For $R_d \neq 0$, it is still possible to eliminate σ by combining the first two equations (and their derivatives):

$$\left(d\,\partial_\phi V_t - (d-1)\partial_\phi V\right)\partial_\phi V_t = 2(d-1)(V_t - V)\left[\left(\partial^2_\phi V_t + \frac{2}{d-2}((d-1)V - (d-2)V_t)\right]\,. \tag{C.4}$$

Importantly, in this derivation we need to assume $R_d \neq 0$, so that we do not expect to necessarily recover the results in section 6.1.

Restricting to the case $V = aV_t$, with a a constant, we find that the solution to this equation is

$$V_t(\phi) = -c\left(\cosh\left(\frac{(a(d-1)+2-d)\phi}{\sqrt{(1-a)(d-2)(d-1)}}\right)\right)^{2-\frac{2}{a(d-1)+2-d}}\left(\right.\,, \tag{C.5}$$

where we have ignored an integration constant that is irrelevant for the $\phi \to \infty$ limit.

Notice that, for $a > 1$, the coefficient in front of ϕ becomes imaginary and then what we have is a cosine, rather than a hyperbolic cosine. As we are not interested in this behaviour we from now on require $a < 1$. From computing ϕ'^2 from this solution and requiring that it must be positive, we then learn that we must have $c > 0$.

In addition, as we are interested in ETW branes, we want to require that ϕ'^2 blows up as $\phi \to \infty$. This is equivalent to having $|V_t| \to \infty$ in this same limit, which in turn implies that the power in (C.5) must be positive. This gives us that the only ETW brane solutions are for $a < \frac{d-2}{d-1}$. For this range of a, we can approximate the hyperbolic cosine by an exponential (as we are interested in the limit $\phi \to \infty$) and we have

$$V_t(\phi) \simeq -c\left(\exp\left(\frac{(a(d-1)+2-d)\phi}{\sqrt{(1-a)(d-2)(d-1)}}\right)\right)^{2-\frac{2}{a(d-1)+2-d}} = -c\, e^{\delta\,\phi}\,. \tag{C.6}$$

The coefficient δ is

$$\delta = 2\sqrt{\frac{d-1}{d-2}(1-a)}\,. \tag{C.7}$$

So for $a < \frac{d-2}{d-1}$ the case of a ETW brane with internal curvature coincides with the case studied in the paper. Interestingly, this case turns out to be more restrictive than the $R_d = 0$ one, for which any $a < 1$ described an ETW brane.

This solution was also assuming that $a \neq \frac{d-2}{d-1}$. Plugging that particular value in (C.4), we find that the equation of motion simplifies to

$$(\partial_\phi V_t)^2 = V_t \cdot \partial_\phi^2 V_t\,. \tag{C.8}$$

This equation has the solution

$$V_t = -c\, e^{\delta\,\phi}\,, \tag{C.9}$$

with c and δ arbitrary constants. In order to describe an ETW brane we require $\delta > 0$. Interestingly, for this special value of a with $R_d \neq 0$, we find that we recover the exponential behaviour, but with the freedom of choosing the critical exponent δ.

In both cases we find the same exponential behaviour for V_t. Therefore, just as in section 6.1.2, we find that the potential takes the form

$$V(\phi) \simeq -a\, c\, e^{\delta\,\phi}\,. \tag{C.10}$$

However, here we uncover that, for a given potential of this form, the setup with $R_d \neq 0$ allows for two possible values of a, namely the $a < \frac{d-2}{d-1}$ given in (C.7)), or the value $a = \frac{d-2}{d-1}$, with δ and a independent. For this reason, from now on we keep a and δ as different variables when solving the rest of the equations, and at the end we comment on the two possibilities.

Using (6.7) we can obtain the profile for ϕ

$$\phi(y) \simeq -\frac{2}{\delta}\log\left(\frac{\delta}{2}\sqrt{2(1-a)c}\, y\right)\,. \tag{C.11}$$

Notice that this is the equivalent to (6.17), but with a and δ kept independent. The leading behaviour is then given by

$$\phi(r) \simeq -\frac{2}{\delta}\log y\,, \tag{C.12}$$

and thus the field only depends on the critical exponent.

We can now use (C.1) to get the profile for the warp factor σ:

$$\sigma \simeq -\frac{1}{(d-1)(1-a)}\log y\,, \tag{C.13}$$

where we have set an integration constant to zero without loss of generality. We recover the equivalent to (6.19), albeit with a and δ kept independent. We see that the warp factor doesn't depend on δ, but specifically on the prefactor a of the potential.

Finally, we have to check that the solution is compatible with (C.3). From it we obtain the condition

$$\frac{4}{\delta^2} - \frac{d-2}{(d-1)(1-a)} + \frac{R_d}{d-1} y^{2-\frac{2}{(d-1)(1-a)}} = 0\,. \tag{C.14}$$

Let us now apply it for the two possible values for a:
• For $a < \frac{d-2}{d-1}$, the power of y in the last term is positive, so that it is subleading in the $y \to 0$ limit. Moreover, recall that in this case δ relates to a via (C.7), which is the precise the value for which the first two terms cancel each other. In conclusion, for $a < \frac{d-2}{d-1}$ having $R_d \neq 0$ becomes irrelevant as we approach the ETW and we basically recover the same results as in the $R_d = 0$ case.
• For $a = \frac{d-2}{d-1}$, the exponent of y vanishes , and hence the R_d term is relevant. In this case, consistency of the equations requires

$$\delta = 2\left(d-2-\frac{R_d}{d-1}\right)^{-\frac{1}{2}}\,. \tag{C.15}$$

Therefore, for this case δ is also fixed, but in terms of R_d. Notice that this quantity must satisfy $R_d < (d-2)(d-1)$. Provided this condition, we find that δ can take any positive value.

This case corresponds to a metric $ds^2 = dy^2 + y^2 ds^2_{d-1}$, hence it describes a conical singularity. The singularity is absent in the case $R_d = (d-1)(d-2)$, namely the curvature of ds^2_{d-1} is that of $\mathbf{S}^{d-1}$, and the geometry is locally smooth, and we have $\delta = 0$ and no exponential growth of the potential. Also, in order to have an ETW brane, the $(d-1)$-dimensional curvature must be lower than that of $\mathbf{S}^{d-1}$.

In conclusion, given a potential with an exponential behaviour as $\phi \to \infty$, in the $R_d \neq 0$ case there exist two different kind of solutions. In the first one the value of R_d is irrelevant and we recover the same behaviour as in the $R_d = 0$ case (but with a more constrained critical exponent, $\delta > \frac{2}{\sqrt{d-2}}$). In the second, the curvature R_d is relevant and it must be fixed by the critical exponent by (C.15).

3.2 Witten's Bubble of Nothing

To illustrate the above general formulation for curved $(d-1)$-dimensional slices, we consider the example of the celebrated Witten's bubble of nothing [67] (see [66, 274–276] for other recent realization of bubbles of nothing). We show it admits a description in an effective 4d theory of gravity coupled to an scalar with zero potential, as a 4d Dynamical Cobordism, and characterize its local description and critical exponent δ.

Related discussion of a 4d effective description of the configuration have appeared in [277] (recently revisited in the context of bubbles in de Sitter space in [167, 168]).

Since we have restricted our discussion to dependence on spatial coordinates, we actually consider the euclidean 5d Schwarzschild black hole solution, before the Wick rotation to the expanding bubble solution. The 5d metric reads

$$ds^2 = \left(1 - \frac{R^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 + \left(1 - \frac{R^2}{r^2}\right) d\phi^2 \,. \tag{C.16}$$

Here ϕ parametrizes an $\mathbf{S}^1$ fibered over the radial coordinate r, times and $\mathbf{S}^3$; the radial coordinate is constrained to the range $r \geq R$, and the $\mathbf{S}^1$ shrinks to zero size at the euclidean horizon $r = R$ (in a smooth way for the periodicity $\phi \sim \phi + 2\pi R$).

We would like to perform a reduction to 4d along the $\mathbf{S}^1$. This is a sphere reduction analogous to those in Section 6.3.1. Hence, we match this metric with (6.46), for $n = 1$, $d = 4$, and, using (6.47), $\alpha = -\sqrt{1/6}$ and $\beta = -\sqrt{2/3}$. We obtain that the radion ω in (6.46) is:

$$\omega = -\sqrt{\frac{3}{8}} \log \left(1 - \frac{R^2}{r^2}\right) . \tag{C.17}$$

The 4d metric is given in (6.49) and reads

$$ds_4^2 = \left(1 - \frac{R^2}{r^2}\right)^{-\frac{1}{2}} dr^2 + \left(1 - \frac{R^2}{r^2}\right)^{\frac{1}{2}} r^2 d\Omega_3^2 \,. \tag{C.18}$$

We would now like to zoom into the location of the ETW brane, the euclidean horizon $r = R$. So we introduce the coordinate $\tilde{r} = 1 - \frac{R^2}{r^2}$. Near $r \to R$ the metric scales as

$$ds_4^2 \sim \tilde{r}^{-\frac{1}{2}} d\tilde{r}^2 + \tilde{r}^{\frac{1}{2}} d\Omega_3^2 \,. \tag{C.19}$$

Now, we make the change (6.50):

$$y = \int \frac{d\tilde{r}}{\tilde{r}^{1/4}} \simeq \tilde{r}^{3/4} \,. \tag{C.20}$$

Replacing $\tilde{r} \simeq y^{\frac{4}{3}}$ in (C.19) we get the 4d metric as a foliation of $\mathbf{S}^3$ slices:

$$ds_4^2 \sim dy^2 + y^{\frac{2}{3}} d\Omega_3^2 \,. \tag{C.21}$$

This corresponds to a metric of the kind (6.3) for curved 3d slices, namely of the kind studied in appendix 3.1. Using (C.13) we can see that $a = 0$, and from (C.7) $\delta = \sqrt{6}$. Interestingly, this corresponds to the case in which the curvature of the slices is irrelevant, and the solution is similar to the $R_d = 0$ case.

We could have also obtained the same result from the profile for the radion,

$$\omega = -\sqrt{\frac{3}{8}} \log \tilde{r} \simeq -\sqrt{\frac{2}{3}} \log y \,. \tag{C.22}$$

By using (C.12), $\omega \simeq -\frac{2}{\delta} \log y$, we read that $\delta = \sqrt{6}$, hence $a = 0$.

Hence Witten's bubble of nothing is described by a 4d Dynamical Cobordism running solution with the scalar reaching off to infinite distance in fields space at a rate controlled by the critical exponent $\delta = \sqrt{6}$. This provides a simple local description in terms of an

ETW brane. From this perspective, the 5d solution provides the UV completion of the ETW brane, which in this case is purely a geometrical closing-off of the geometry.

We would like to emphasize that this example provides an explicit realization of the picture discussed in Section 6.3, in particular Figure 6.1 (albeit, with no brane dressing at the tip). Namely, the complete solution involves a genuine compactification on a finite size $\mathbf{S}^1$, yet it is described by a local EWT brane model identical to that obtained as an $\mathbf{S}^1$ reduction on a flat $\mathbf{R}^2$ (which, given the vanishing potential, straightforwardly leads to $a = 0$, hence $\delta = \sqrt{6}$). This supports the picture in Section 6.3 that the sphere reductions in the flat space transverse to the D-branes suffices to provide the local description even in the (physically more interesting case) in which the transverse space is globally given by a more involved geometry, implementing the actual compactification to the lower-dimensional theory.

D

Subleading corrections to the local description

In section 6.1.2 we took constant a as a proxy for the leading behaviour of $a(\phi)$ as $\phi \to \infty$. Here we consider the role of possible subleading corrections. We notice that these corrections do not necessarily go to zero as $\phi \to \infty$ in (6.13). For example, let us take

$$\sqrt{1 - a(\phi)} = \sqrt{1-a} + \frac{b}{\phi}\,. \tag{D.1}$$

It is clear that $a(\phi)$ asymptotes to a as $\phi \to \infty$, but after doing the integral in (6.13) the correction to the leading behaviour given by the second term behaves as $\log \phi$. Indeed, ignoring constant prefactors we get

$$V_t \sim \phi^{2\sqrt{\frac{d-1}{d-2}}\,b} e^{\delta\,\phi}\,, \tag{D.2}$$

with δ defined in (6.15). Comparing with (6.14) we see that we can describe this example with our leading order analysis if we allow for $c \sim \phi^{2\sqrt{\frac{d-1}{d-2}}\,b}$. Notice that the example in section 6.2.2 precisely realise this behaviour (see equation (6.44)).

As a general lesson, we can include these kind of corrections that do not vanish in the $\phi \to \infty$ limit by promoting c from just a constant to a ϕ-dependent quantity that may hide subleading corrections. In this way, it may happen that $c \to \infty$ as $\phi \to \infty$ as long as it blows-up slower than an exponential (otherwise it would not represent a subleading behaviour).

This remark is specially interesting in the $a(\phi) \to 0$ case. From (6.16) we would conclude that $V \to 0$ if c is a finite constant. However, if allowing $c \to \infty$ because of possible subleading terms, it can happen that a times c remains finite in the $\phi \to \infty$ limit. In this way, we describe a solution in which $\phi'^2 \gg V$ (i.e., $a(\phi) \to 0$) without requiring that V vanishes asymptotically.

E

Superconformal Representations of $SU(2,2|2)$

We summarise in this appendix the basic notions of superconformal representations useful in this work. The $\mathcal{N}=2$ superconformal group is $SU(2,2|2)$, its bosonic subgroup being constituted of both the conformal and R-symmetry groups, $SO(2,4)\times SU(2)_R\times U(1)_R$. The algebra contains the usual conformal generators, namely those of translations, rotations, and so-called special conformal transformations, $M_{\mu\nu}, P_\mu, K_\mu$, as well as the dilatation operator, $\mathcal{D}$. With the R-symmetry generators, it is supplemented by the super- and superconformal charges:

$$Q_{\alpha A}\,,\bar{Q}_{\dot{\alpha}A}\,,\quad S^{\alpha A}\,,\bar{S}^{\dot{\alpha}A}\qquad \alpha\,,\dot{\alpha}=1,2\,,\quad A=1,\ldots,4\,. \tag{E.1}$$

One can then label any operator of the theory by the following quantum numbers:

$$[\Delta;j,\bar{\jmath};R;r]\,. \tag{E.2}$$

In addition to the usual conformal dimension, Δ, which is the charge of the operator under dilatations, and the Lorentz Dynkin indices, $(j,\bar{\jmath})$, of $\mathfrak{so}(1,3)=\mathfrak{su}(2)\oplus\mathfrak{su}(2)$, we also define the R-charges, (R,r), of $SU(2)_R\times U(1)_R$.

The spectrum of the theory organises itself into superconformal multiplets of $SU(2,2|2)$ whose highest weights are called superconformal primaries. A superconformal primary, $\mathcal{O}$, is then by definition annihilated by special conformal transformation generators—as for all usual conformal primaries—and superconformal charges:

$$[K_\mu,\mathcal{O}]=0\,,\qquad \left[S^{\alpha A},\mathcal{O}\right\}=0=\left[\bar{S}^{\dot{\alpha}A},\mathcal{O}\right\}\Big(,\qquad \alpha,\dot{\alpha}=1,2\,,\quad A=1,\ldots,4 \tag{E.3}$$

where the commutator or anti-commutator is used depending on whether $\mathcal{O}$ is fermionic or bosonic. Given a superconformal primary, the rest of the multiplet (called descendants), is then generated by successive applications of the translation operator, P_μ, and the regular supercharges, $Q_{\alpha A}\,,\bar{Q}_{\dot{\alpha}A}$. Note that due to their fermionic nature, the number of states generated by the supercharges is finite.

Moreover using the conformal algebra it possible to show that applying the shift of conformal dimension of descendants from its primary is (half-)quantised:

$$\Delta_{[Q,\mathcal{O}]}=\Delta_{\mathcal{O}}+\frac{1}{2}\,,\qquad \Delta_{[P,\mathcal{O}]}=\Delta_{\mathcal{O}}+1\,. \tag{E.4}$$

Schematically for a bosonic superconformal primary, the descendants and their dimensions are found to be:

$$\begin{array}{llllllll}\mathcal{O}\,, & [Q_{\alpha A},\mathcal{O}]\,, & [\bar{Q}_{\dot{\alpha}A},\mathcal{O}]\,, & \ldots & [P_\mu,\mathcal{O}]\,, & [P_\nu,[P_\mu,\mathcal{O}]]\,, & \ldots \\ \Delta\,, & \Delta+\frac{1}{2}\,, & \Delta+\frac{1}{2}\,, & \ldots & \Delta+1\,, & \Delta+2\,, & \ldots\end{array} \tag{E.5}$$

Conversely, for superconformal charges and special conformal transformations, the sign of the shift is reversed.

A complete analysis of the representations then separates superconformal multiplets into two classes: long and short multiplets. The latter corresponds to cases where the superconformal primary is annihilated by a combination of the supercharges, in which case its dimension is set by the rest of the quantum numbers. In this work, we are mainly interested in a class of multiplet whose superconformal primary is annihilated by half of the supercharges, such as the chiral multiplets, $\mathcal{E}_r$. Long multiplets are unconstrained and their conformal dimensions are only bounded from below by unitarity.

Additional details and a complete classification of unitary superconformal multiplets can be found in e.g. [243, 278].

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