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Physical Review C 106.2 (2022): 024313

DOI: https://doi.org/10.1103/PhysRevC.106.024313

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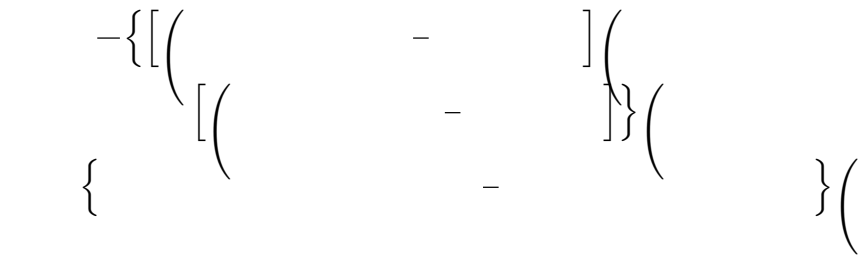
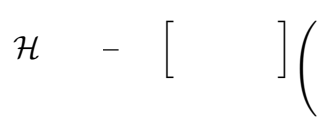
$$\hbar$$

$$\left[\quad \frac{\hbar}{} \quad \right]\Big($$





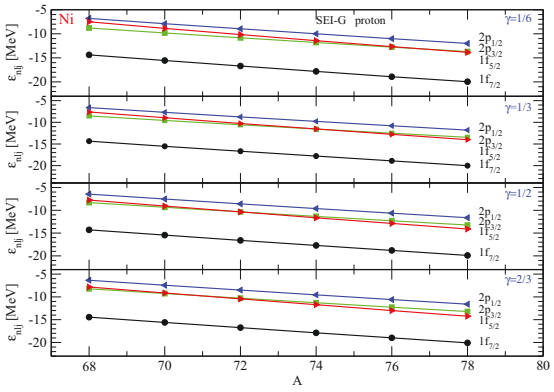


Figure 1. Proton single-particle levels around the Fermi level for Ni isotopes from A = 68 to A = 78 computed with the SEI interaction for the four EoS.

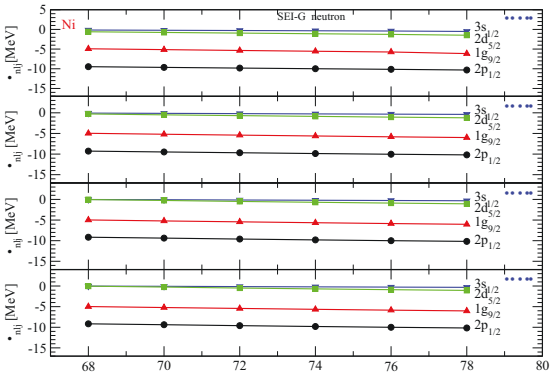


Figure 2. Neutron single-particle levels around the Fermi level for Ni isotopes from A = 68 to A = 78 computed with the SEI interaction for the four EoS.

formulation using SEI, the crossing of $1f_{5/2}$ and $2p_{3/2}$ s.p. proton levels in the isotopic chain of Ni is found to be a function of NM incompressibility, as can be appreciated from the results displayed in Fig.1 for the four considered EoSs. As it is discussed in detail in Ref.[18], some effective forces of Skyrme and Gogny types can reproduce the crossing between the $1f_{5/2}$ and $2p_{3/2}$ proton s.p. level at mass number A=74, in agreement with the experimental data, only if an additional tensor force is added to these interactions. The main effect of the tensor force in the Ni isotopes analyzed in this work is the attraction (repulsion) between the neutron $1g_{9/2}$ s.p. level, whose occupancy grows when the mass number increases, and the $1f_{5/2}$ ($2p_{3/2}$) s.p. proton level. Notice that the presence of a tensor term does not guarantee the crossing of these s.p. proton levels, as is the case of the Skyrme-Lyon force SLy5, as has been discussed recently in [18]. This fact point out the relevance of the underlying mean-field to describe the crossing of the aforementioned s.p. proton levels. For example in the case of SEI, the crossing between the $1f_{5/2}$ and the $2d_{3/2}$ s.p. proton levels is predicted at the right mass number if a EoS with a γ-value close to 1/2 (K=245 MeV) is used. SEI EoSs with smaller (larger) value of the incompressibility modulus K move the crossing point towards higher (lower) mass numbers. For instance, the EoS $\gamma = \frac{1}{6}$ (K=207 MeV), predicts the crossing at A=78 while a SEI EoS with γ-value between 1/2 (K=245 MeV) and 2/3 (K=263 MeV) at A=72.

From Fig.1 we can also see that the energy gap between the $1f_{5/2}$ and $1f_{7/2}$ proton-shells remains nearly stationary for all the four EoSs of SEI. As one moves from ^{68}Ni to ^{78}Ni, the proton energy gap, $(1f_{5/2} - 1f_{7/2})$, decreases by 0.779 MeV, 0.744 MeV, 0.741MeV and 0.762 MeV for the EoSs γ=$\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$, respectively. Thus, for all

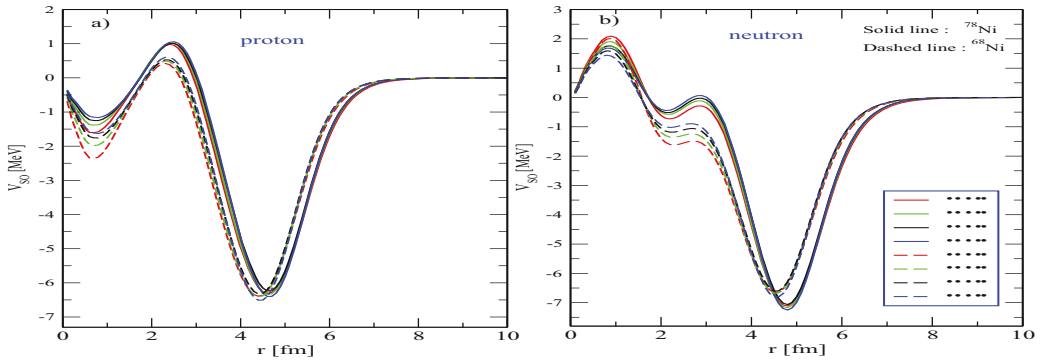


Figure 3. Proton and neutron SO-contributions in ^{68}Ni and ^{78}Ni as a function of distance r from the center of the nucleus for the four EoS.

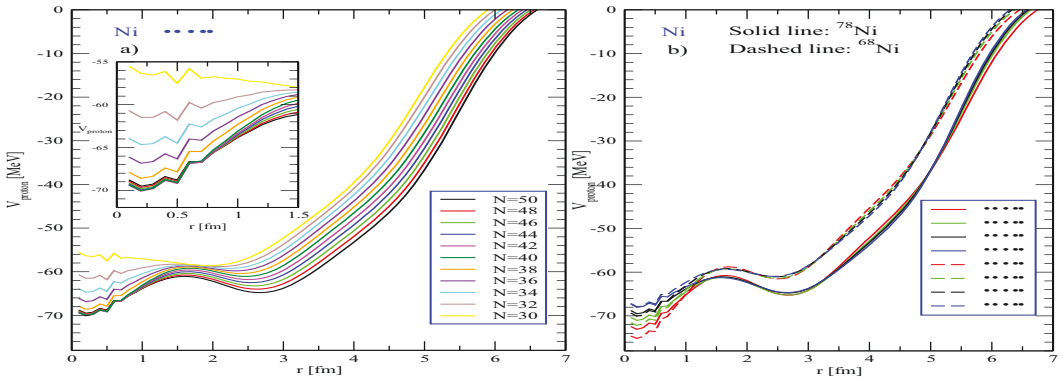
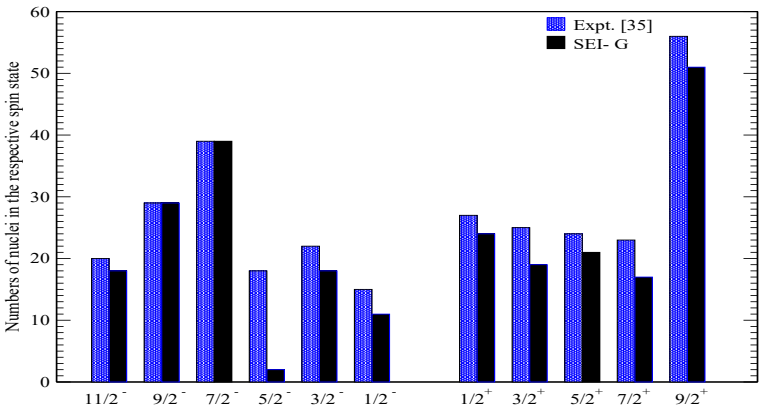


Figure 4. (a)Proton mean-field for Ni(N=30-50)isotopes predicted by the SEI for $\gamma = \frac{1}{2}$ under the QLDFT formulation.(b)Proton mean-field for ^{68}Ni and ^{78}Ni isotopes predicted by the SEI for $\gamma = \frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$ under the QLDFT formulation.

the EoSs of SEI the magicity of Z=28 is preserved. The energy of the $1g_{9/2}$ and $2d_{5/2}$ s.p. neutron levels predicted by the different EoSs of SEI remain almost stationary as one moves from ^{68}Ni to ^{78}Ni, as can be seen in Fig.2. This implies that the magicity of the neutron number N=50 is also preserved. To get more insight about the evolution of shell structure predicted by SEI in Ni isotopes, we shall analyze the central and spin-orbit contributions to the neutron and proton mean-fields. In panels (a) and (b) of Fig.3 we display the SO contributions to p- and n- mean-fields as a function of the distance from the center for the nuclei ^{68}Ni and ^{78}Ni calculated with the four SEI EoSs used in this work. For all these four EoSs, the results are similar in quality with closely lying values. This behaviour is different from the one obtained with the D1M Gogny interaction at QLDFT level. In that case, the SO-contribution to the p-mean-


Expt. [35]
SEI- G
Numbers of nuclei in the respective spin state
0
10
20
30
40
50
60
11/2 -
9/2 -
7/2 -
5/2 -
3/2 -
1/2 -
1/2 +
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7/2 +
9/2 +

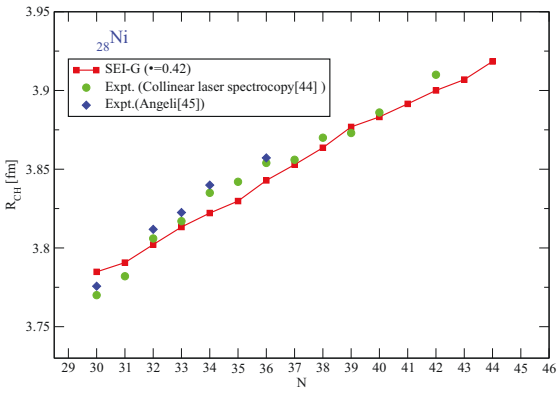

Figure 6. Nuclear charge radii R_{CH} using SEI-G ($\gamma = 0.42$) compared with the experimental data [44] and [45]).

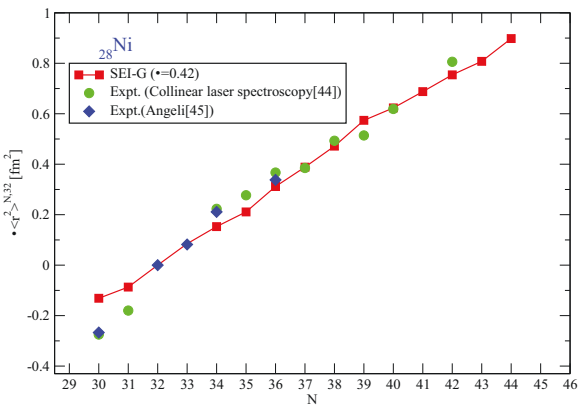

Figure 7. Isotopic shift using SEI-G ($\gamma = 0.42$) compared with the experimental data [44] and [45]).

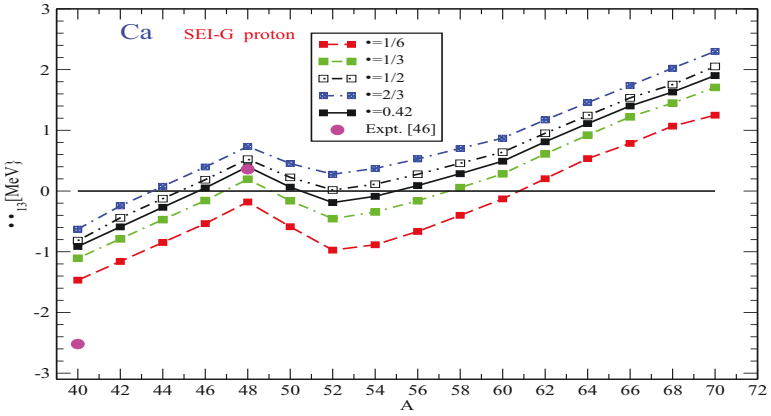

Figure 8. Energy differences of $2s_{1/2}$ and $1d_{3/2}$ proton levels ($\Delta\epsilon_{13}$) of the Ca isotopes predicted by the SEI for $\gamma = \frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$ and 0.42. The experimental data is also shown which are taken from the Ref.[46].

most of the effective interactions needs an additional tensor term to reproduce the level crossing effect. However, there are some interactions, for example, the SKI5 and SGII Skyrme forces, the D1M Gogny interaction, and the DDME1 and NL3 relativistic mean field sets, which predict the inversion of the $2s_{1/2}$ and $1d_{3/2}$ proton s.p. levels in ^{48}Ca without considering an extra tensor contribution. In Refs.[49, 50] it is found that the evolution of the orbitals and hence the energy difference Δ_{13}=$2s_{1/2}$-$1d_{3/2}$ as neutron number increases in the Ca-isotopic chain follows a similar trend for different interactions, irrespective of the inversion is predicted at ^{48}Ca or not. The energy difference Δ_{13} in Ca isotopes is shown as a function of N in Fig.8 for the four sets of SEI,

the radial directions [6].

The tensor effects on the gap $1h_{11/2}$-$1g_{7/2}$ of the unoccupied proton states along the

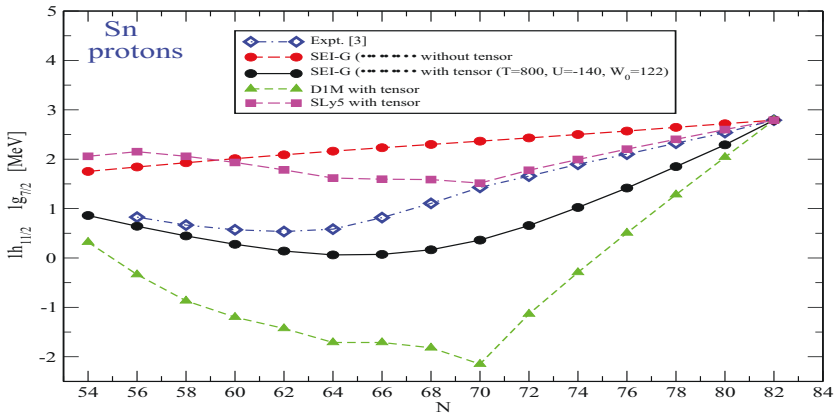

Figure 9. Energy differences between $1h_{11/2}$ and $1g_{7/2}$ proton s.p.levels in Sn-isotopes for SEI with and without Tensor. The experimental data are taken from Ref.[3]. Theoretical results have been shifted so that splittings coincide with ^{132}Sn. The corresponding results for D1M and SLy5 interaction sets with tensor are also given for comparison.

Sn-isotopic chain, shown in Fig.9, strongly depend on the position and occupancy of the neutron s.p. levels, which in the case of SEI are displayed in Figs. 10 and 11, respectively. From Fig. 10, we see that the impact of the tensor force is more important on states of large orbital angular momentum, such as $1g_{7/2}$ or $1h_{1172}$, whose s.p. energies are clearly shifted with respect to the values computed without the tensor interaction. Above N=50, the neutron levels $1g_{7/2}$ and the $2d_{5/2}$ predicted by SEI lie very close to each other. The same happens with the $3s_{1/2}$ and $2d_{5/2}$ levels, while $1h_{11/2}$ remains isolate at higher s.p. energy. As can be seen from Fig. 11, from A=100 to A=114, neutrons in the Sn isotopic chain mainly populate the $1g_{7/2}$ and the $2d_{5/2}$ levels almost with the same occupation probability, which reaches at 80% at A=114. Above this mass number the occupancy of the $3s_{1/2}$ and $2d_{3/2}$ levels increase remarkably until about 60% in competition with the filling of the $1h_{11/2}$, which has a small occupation up to A=120, but from this mass number onward increases till saturate at A=132. The gap between the unoccupied $1h_{11/2}$ and $1g_{7/2}$ proton levels along the Sn-isotopic chain computed with and without the tensor interaction are shown in Fig.9 where we have shifted the theoretical results to coincide with the experimental value at ^{132}Sn. This gives a better insight into the evolution of the orbitals under the influence of the tensor force along the Sn-isotopic chain [56]. We have also shown the corresponding results of the D1M and SLy5 interaction sets with tensor. For SEI the shift in the cases of with and without tensor part are δ=2.99 MeV and 1.85 MeV, respectively. In

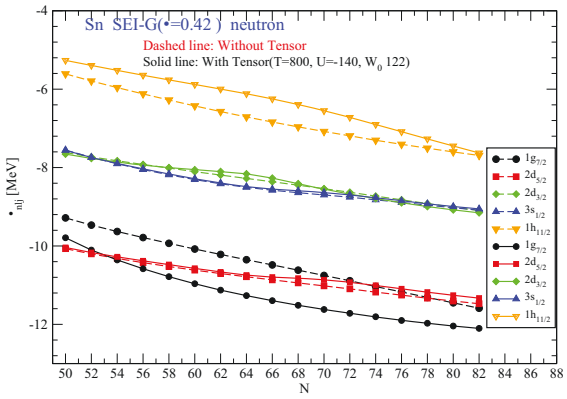

Figure 10. Neutron levels of Sn isotopes in the N=50 to N=82 major shell. Solid (dashed) lines correspond to the s.p. energies computed with (without) tensor force.

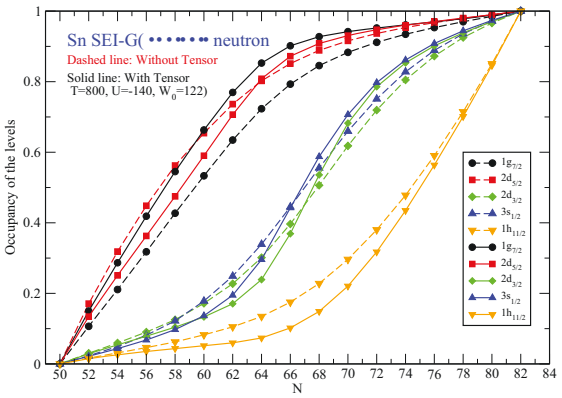

Figure 11. Occupation probability of the neutron levels of the Sn isotopes in the N=50 to N=82 major shell. Solid (dashed) lines correspond to the occupations computed with (without) tensor force.

the same figure we also show the experimental data extracted from (α, t) reaction on tin isotopes displayed in the upper panel of Fig.3 of Ref. [3]. Due to the tensor force, the filling of the $1g_{7/2}$ neutron state enhances the spin-orbit splittings of the h and g s.p. proton states, thereby decreasing the gap between the $1h_{11/2}$ and the $1g_{7/2}$ proton levels. The effect of $2d_{5/2}$ neutron level on these two proton levels is just the opposite. As the occupancy of the $1g_{7/2}$ and $2d_{5/2}$ neutron levels is quite similar, there is a partial cancellation between the tensor effects of these two neutron levels and the $1h_{11/2}$ and $1g_{7/2}$ proton levels. Due to the large overlap between the s.p wavefunction of the $1g_{7/2}$ neutron state and the $1h_{11/2}$ or $1g_{7/2}$ proton states as compared with the overlap in the case of the $2d_{5/2}$ neutron state, the SEI calculation including tensor force predicts that the $1h_{11/2}$-$1g_{7/2}$ proton gap decreases when A increases from 100 to 114, which is in agreement with the experimental data [3]. When the mass number of the isotope increases above A=114, the occupancy of the $2d_{3/2}$ and $3s_{1/2}$ neutron levels becomes progressively important reaching about a 60% at A=120. The tensor interaction between the $2d_{3/2}$ neutron state and the $1h_{11/2}$ and $1g_{7/2}$ proton states should reduce the proton gap, however, owing to the $1h_{11/2}$ neutron level, which attracts the $1g_{7/2}$ and repels the $1h_{11/2}$ proton levels, the combined effect produces an increasing of the $1h_{11/2}$-$1g_{7/2}$ proton gap. In spite of the small occupancy of the $1h_{11/2}$ neutron level in the range between A=114 and A=120 (see Fig. 11), the tensor interaction of this state with the $1h_{11/2}$ and $1g_{7/2}$ proton states is strong enough, due to its principal quantum number and large orbital angular momentum, to reverse the reduction of the gap due to the $2d_{3/2}$ neutron state. Finally above A=120, the occupancy of the $1h_{11/2}$ grows increasing the gap between the $1h_{11/2}$ and $1g_{7/2}$ proton states in agreement with the experimental

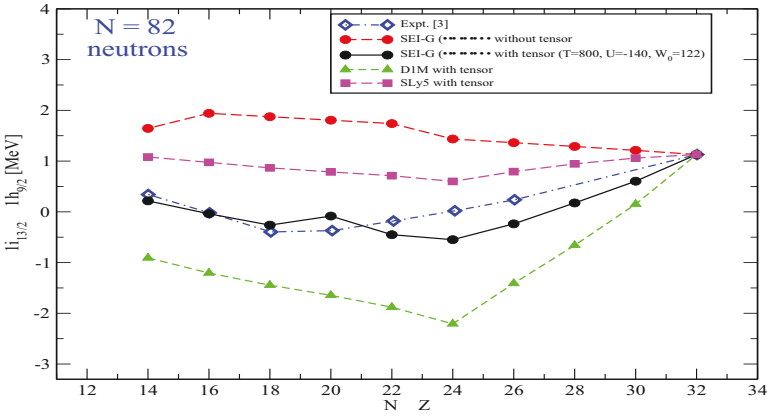

Figure 12. Energy differences between $1i_{13/2}$ and $1h_{9/2}$ neutron s.p. levels in $N = 82$ computed for SEI with and without tensor interaction. The experimental data are taken from Ref.[3]. Theoretical results have been shifted so that splittings coincide with ^{132}Sn. The corresponding results for D1M and SLy5 interaction sets with tensor are also given for comparison.

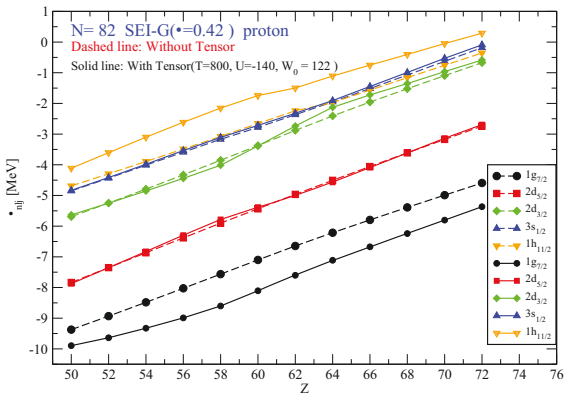

Figure 13. Proton levels of the N=82 isotones in the Z=50-Z=72 major shell. Solid (dashed) lines correspond to the s.p. energies computed with (without) tensor force.

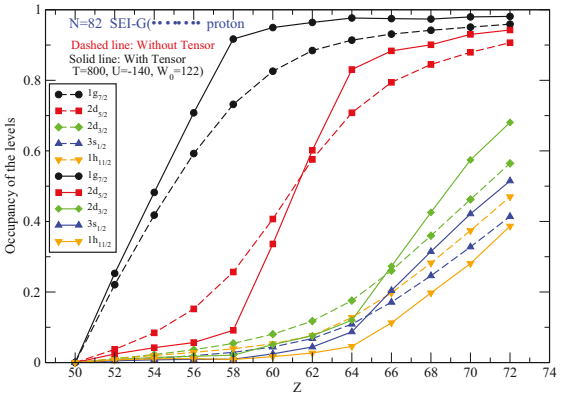

Figure 14. Occupation probability of the proton levels of the N=82 isotones in the Z=50-Z=72 major shell. Solid (dashed) lines correspond to the occupations computed with (without) tensor force.

owing to the larger overlap of the wave function of the $1g_{9/2}$ proton level with the one of the $1g_{7/2}$ neutron level as compared with the overlap with the wavefunction of the $2d_{3/2}$ level.

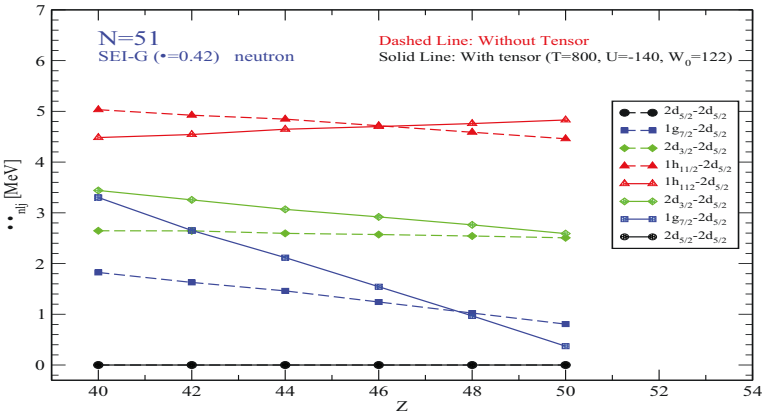

Figure 15. Neutron s.p levels in N=51 isotones relative to $2d_{5/2}$ with and without Tensor.

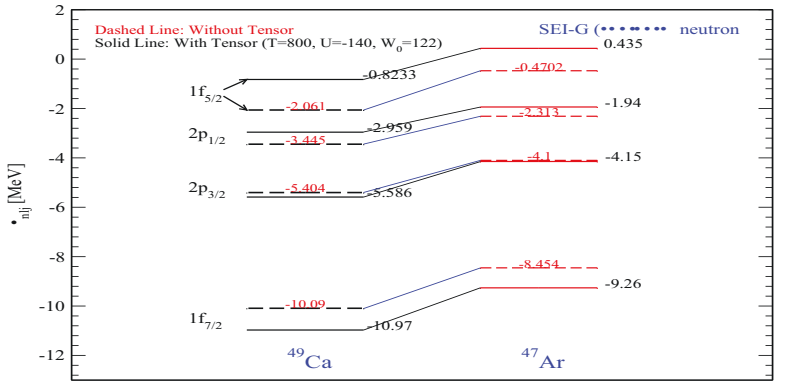

Figure 16. Neutron single-particle energies (SPE) of the fp orbitals for the $^{47}Ar_{29}$ and $^{49}Ca_{29}$ nuclei with and without Tensor

The N=28 gap For the nucleus ^{47}Ar, the SEI model with γ=0.42 predicts the binding energy and spin-parity of the ground-state as well as the ordering of the first excited levels, assumed of s.p. nature, in agreement with the experimental values [4]. Although this model also predicts a reduction of 332 keV of the N=28 gap in passing from ^{49}Ca to ^{47}Ar [4], it fails in the estimation of the reduction of the splitting of the f and p s.p. levels, which are predicted by SEI to be 45 KeV and 172 KeV, respectively, in comparison with the experimental values of 875 KeV and 890 KeV [4]. This failure can be cured, at least partially, by including a tensor component in the effective interaction. The neutron s.p. energies in the nuclei ^{49}Ca and ^{47}Ar are shown in Figure 16. We can