I.S.S.N: 1885-6888

ECONOMIC ANALYSIS
WORKING PAPER SERIES

On the inconsistency of the Malmquist-Luenberger index

◆

Juan Aparicio, Jesús T. Pastor, and José L. Zofío

Working Paper 1/2013

UAM
UNIVERSIDAD AUTONOMA
DE MADRID

DEPARTAMENTO DE ANÁLISIS ECONÓMICO:
TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

On the inconsistency of the Malmquist-Luenberger index

Juan Aparicio[1,*], Jesus T. Pastor[1] and Jose L. Zofio[2]

(1) *Center of Operations Research (CIO)*

Universidad Miguel Hernandez, E-03202 Elche, Alicante, Spain.

(2) *Departamento de Analisis Economico*: *Teoria Economica e Historia Economica.*

Universidad Autonoma de Madrid, E-28049 Madrid, Spain.

Abstract

Apart from the well-known weaknesses of the standard Malmquist productivity index related to infeasibility and not accounting for slacks, already addressed in the literature, we identify a new and significant drawback of the Malmquist-Luenberger index decomposition that questions its validity as an empirical tool for environmental productivity measurement associated with the production of bad outputs. In particular, we show that the usual interpretation of the technical change component in terms of production frontier shifts can be inconsistent with its numerical value, thereby resulting in an erroneous interpretation of this component that passes on to the index itself. We illustrate this issue with a simple numerical example. Finally, we propose a solution for this inconsistency issue based on incorporating a new postulate for the technology related to the production of bad outputs.

Keywords: Data Envelopment Analysis; Malmquist-Luenberger productivity index; Technological change; Efficiency change; Directional distance function.

(*) Corresponding author. Email: j.aparicio@umh.es, Tel: +34-966658725, Fax: +34-966658715

1. Introduction

Chung et al. (1997) introduced the Malmquist-Luenberger index—hereafter denoted ML—as a measure of productivity change in the context of a production technology incorporating undesirable outputs production and characterized by way of the directional distance function, Chambers et al. (1996). Mirroring the decomposition of the Malmquist productivity index proposed by Färe et al. (1994), they also suggested its breakdown into two mutually exclusive components that are interpreted in terms of efficiency change and technical change, thereby allowing for the identification of the sources of productivity change. Since then, many empirical studies have adopted their theoretical framework while relying on Data Envelopment Analysis techniques to approximate the production technology and obtain empirical results, e.g. Färe et al. (2001), Murty et al. (2006) and Weber and Domazlicky (2001) in manufacturing industries, Kumar (2006) and Yoruk and Zaim (2005) for OECD countries, Barros (2008) in utilities, to name but a few.

These studies relying on the ML index draw relevant conclusions in terms of environmentally friendly productivity change as well as efficiency change and technical change. Based on their findings, the authors' conclusions are used to prescribe guidelines regarding the convenience of incentivizing the adoption of cleaner best practice technologies (i.e., efficiency gains) as well as investing in innovative and less contaminating techniques (i.e., technical progress). However, in this study, we show that under the standard technological assumptions the interpretation of the technical change component and, therefore, the index itself, can be inconsistent with the calculations that are obtained in empirical applications. In fact, the ML index is prone to several weaknesses. First, when the estimation of the shift in technology between two periods of time is based on the distance from the period t observation to the period s

technology, with $t \neq s$, infeasibility can occur. Second, linear programming techniques—i.e., Data Envelopment Analysis, DEA—are frequently used to calculate and decompose the productivity index. However, standard DEA models can leave slacks, which constitute a non-radial form of inefficiency, which is not incorporated into the analysis. Third, the ML index can incorrectly characterize technological progress, yielding inconsistent numerical values as we mentioned. In what follows, we show that while the first two issues have been addressed by the literature, the third weakness, overlooked until now, is arguably the most important since it may lead researchers to wrong analytical conclusions and result in misguided policy recommendations. The inconsistency is a consequence of the set of postulates traditionally assumed in the joint production of desirable and undesirable outputs, and we propose a redefinition of this set in order to solve the problem.

As anticipated, it is worth noting that the first two weaknesses apply also to the standard Malmquist index, and in this respect, several solutions have been proposed in the literature. Regarding the infeasibility problem, Pastor and Lovell (2005) introduced the concept of a global Malmquist productivity index as a way of using a base period technology to estimate and decompose productivity change. Following this line of research, Oh (2010) adapted the same idea to the ML index, incorporating the negative effect of environmentally harmful by-products. As for the problem related to slacks, as we are aware, two different solutions exist. On the one hand, we can find approaches based on non-radial measures (Grifell-Tatje et al., 1998 and Chen, 2003) that allow the incorporation of slacks into the efficiency measures comprising the Malmquist index. On the other hand, as in the standard definition of the Malmquist index, other authors prefer the use of radial measures and avoid the existence of slacks by resorting to assurance regions (see Dharmapala, 2010). However, the third drawback, the inconsistency issue, is a problem exclusively related to the ML index and has not been identified or solved in the existing literature.

Regarding the inconsistency issue, and focusing on the technical change component, we show that while the ML index may signal a decline in the environmental productivity, precisely the opposite may actually be occurring (environmental productivity growth based on technical progress). This erroneous result represents a serious drawback and casts important doubts on the correctness and robustness of the results obtained in the empirical literature, as well as on the conclusions that have been drawn upon them, including policy recommendations. In this sense, we propose a solution to the inconsistency issue that avoids the problems with the interpretability of the ML index. The key to our approach lies in assuming a new postulate for the technology when good and bad outputs are produced.

The paper is structured as follows: In the next section, we briefly characterize the production technology and present the definition of the ML index as the geometric mean of two adjacent period indices, including its decomposition, and the normal interpretation in terms of efficiency change and technical change. Section 3 discusses the inconsistency issue. In section 4, we propose a solution in order to overcome this problem through the incorporation of a new postulate for the technology. Section 5 concludes.

2. The productivity measurement

Let us assume a set of $k = \{1,...,K\}$ observations transforming a set of inputs $x \in \mathbb{R}^{N}_{+}$ into a set of outputs, of which $y \in \mathbb{R}^{M}_{+}$ are good (desirable) and $b \in \mathbb{R}^{I}_{+}$ are bad (undesirable). The production technology can be represented by way of the following output correspondence P: $\mathbb{R}^{N}_{+} \to P(x) \subseteq \mathbb{R}^{M+I}_{+}$, $P(x) = \left\{(y,b) : x \text{ can produce } (y,b)\right\}$.

Given $x \in R_{+}^{N}$, we assume that (A1): $0_{M+I} \in P(x)$; (A2): $P(x)$ is compact; (A3) if $x' \geq x$, then $P(x) \subseteq P(x')$; (A4) $(y,b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta b) \in P(x)$; (A5) if $(y,b) \in P(x)$ and $b = 0$, then $y = 0$; and (A6) $(y,b) \in P(x)$ and $y' \leq y$ imply $(y',b) \in P(x)$ (see Färe et al., 2007).

The ML index used to measure productivity change is based on the directional distance function, which seeks the largest feasible increase in desirable outputs compatible with a reduction in undesirable outputs (see Chung et al., 1997):

$$\vec{D}_o(x, y, b; g) = \sup\left\{\beta : (y,b) + \beta g \in P(x)\right\}, \tag{1}$$

where *g* is the directional vector setting the particular orientation in which outputs are scaled. A standard choice of orientation corresponds to the observed values of the desirable and undesirable outputs: *g* = (*y*, −*b*), with the latter expressed in negative values, thereby allowing for their reduction.[1]

We now turn to the definition of the ML index and its decomposition. Following Färe et al. (2001), the index based on period *s* technology is:[2]

$$ML^{s} = \frac{1 + \vec{D}_{o}^{s}\left(x^{t}, y^{t}, b^{t}; y^{t}, -b^{t}\right)}{1 + \vec{D}_{o}^{s}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}, \quad s = t, t+1, \tag{2}$$

which can be decomposed into efficiency change and technical change:

---

[1] See Figure 1 in Chung et al. (1997) for a graphical illustration of the directional distance function in a context with good and bad outputs.

[2] The definition of the Malmquist–Luenberger index is such that when the direction *g* is (*y*, *b*) rather than (*y*, -*b*), it coincides with the standard Malmquist index. However, since the direction (*y*, *b*) is not suitable for dealing with the production of bad outputs (see Chung et al., 1997), we have that in this context practitioners use the direction (*y*, -*b*) and, consequently, the values of the ML index will differ from those of the standard Malmquist index.

$$ML^{t}=\underbrace{\frac{1+\vec{D}_{o}^{t}\left(x^{t},y^{t},b^{t};y^{t},-b^{t}\right)}{1+\vec{D}_{o}^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}}_{MLEFFCH^{t}}\cdot\underbrace{\frac{1+\vec{D}_{o}^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}{1+\vec{D}_{o}^{t}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}}_{MLTECH^{t}},\tag{3}$$

$$ML^{t+1}=\underbrace{\frac{1+\vec{D}_{o}^{t}\left(x^{t},y^{t},b^{t};y^{t},-b^{t}\right)}{1+\vec{D}_{o}^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}}_{MLEFFCH^{t+1}}\cdot\underbrace{\frac{1+\vec{D}_{o}^{t+1}\left(x^{t},y^{t},b^{t};y^{t},-b^{t}\right)}{1+\vec{D}_{o}^{t}\left(x^{t},y^{t},b^{t};y^{t},-b^{t}\right)}}_{MLTECH^{t+1}}.\tag{4}$$

To avoid the use of an arbitrary reference technology, the geometric mean of the two based period indices is considered, thereby defining $ML_{t}^{t+1}=\left(ML^{t}\cdot ML^{t+1}\right)^{1/2}$. ML_{t}^{t+1} credits producers for simultaneously increasing good outputs and reducing the production of bad outputs. Also, from (3) and (4), ML_{t}^{t+1} can be decomposed into the same two components, accounting for efficiency change and technical change. Noting that $MLEFFCH^{t}=MLEFFCH^{t+1}$, one obtains the following breakdown:

$$ML_{t}^{t+1}=\underbrace{\frac{1+\vec{D}_{o}^{t}\left(x^{t},y^{t},b^{t};y^{t},-b^{t}\right)}{1+\vec{D}_{o}^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}}_{MLEFFCH_{t}^{t+1}}\cdot\underbrace{\left[MLTECH^{t}\cdot MLTECH^{t+1}\right]^{1/2}}_{MLTECH_{t}^{t+1}}.\tag{5}$$

Any improvement in productivity, efficiency and technical change corresponds to a value greater than one. On the contrary, values less than one indicate regress.

3. The inconsistency of the Malmquist-Luenberger index

We now focus on the theoretical weaknesses of the ML index and its decomposition. In particular, we are referring to the inability of the technical change component to correctly characterize technological progress as the shift in the production possibility frontier, yielding inconsistent numerical values, *e.g.* technical regress is compatible with values of $MLTECH_t^{t+1}$ greater than one. First, we recall the interpretation of the technical change component given in the literature. Following, for example, Färe et al. (2001) or Kumar (2006), $MLTECH_t^{t+1}$ measures the shift in the production possibility frontier and, therefore, the technical change in the joint production of desirable and undesirable outputs. Citing the former authors, “Shifts of the production possibilities frontier in the direction of ‘more goods and fewer bads’ results in the value of the $MLTECH_t^{t+1}$ index exceeding unity. If an $MLTECH_t^{t+1}$ index equals unity, this indicates that there was no shift in the production possibilities frontier. Finally, an $MLTECH_t^{t+1}$ index value of less than unity indicates a shift of the production possibilities frontier in the direction of ‘fewer goods and more bads’”, Färe et al. (2001; 391).[3] In this way, $MLTECH_t^{t+1} > 1$ is associated to environmental ‘technical progress’, while $MLTECH_t^{t+1} < 1$ would signal environmental ‘technical regress’. We illustrate how these interpretations of the technical change component are at odds with the values obtained from a simple example.

As in the existing empirical applications, we use the standard Data Envelopment Analysis approximation of the production technology consistent with A1-A6. The environmental output sets for any time period s, $s = t, t+1$, can be modeled in the following way—see Chung et al. (1997):

$$P^s(x) = \left\{ (y,b) : \sum_{k=1}^{K} z_k y_k^s \geq y, \sum_{k=1}^{K} z_k b_k^s = b, \sum_{k=1}^{K} z_k x_k^s \leq x, z_k \geq 0, k = 1,\dots,K \right\}. \qquad (6)$$

---

[3] Kumar (2006; 284-285) states that “If technical change enables more production of good and less production of bad output, then $MLTECH_t^{t+1} > 1$, whereas if $MLTECH_t^{t+1} < 1$, there has been a shift in the frontier in the direction of fewer good outputs and more bad outputs”.

Regarding the numerical example, we consider two observations: *A* and *B* in the *t* and *t*+1 time periods, which use an equal amount of a single input (*x*) to produce one good output (*y*) and one bad output (*b*)—Table 1. The environmental output production sets are illustrated in Figure 1. The bold solid line corresponds to the frontier of the production possibility set for period *t*, whereas the thick solid line corresponds to the frontier of the production possibility set for period *t*+1. Focusing the analysis on *B*, we see that this observation is efficient in periods *t* and *t*+1, and therefore $\vec{D}_o^t\left(x_B^t, y_B^t, b_B^t; y_B^t, -b_B^t\right)$ = $\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)$ = 0, resulting in $MLEFFCH_t^{t+1}$ = 1, and any improvement or decrease in productivity must be a consequence of technological shifts.

In this situation associated to observations leading the change in the production frontier, we show that the technical change component does not measure the actual shift in the production possibility set properly. Environmentally friendly technical progress is depicted in Figure 1 since the shift is in the direction of ‘more goods and fewer bads’. Nevertheless, this progress is associated with a value of *MLTECH* < 1 that indicates unreal technological regress. In order to illustrate this, we calculate the technical change component for the ML index based on period *t* as the reference technology. In this way, we obtain:

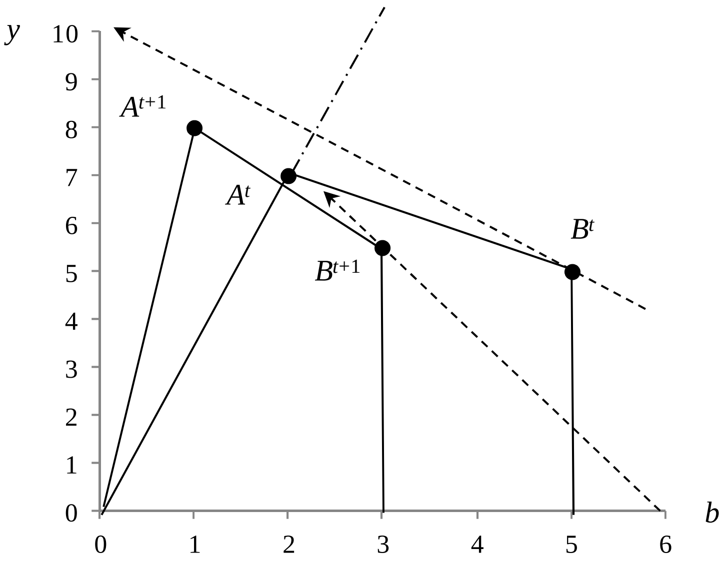
$$MLTECH^t = \frac{1+\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)}{1+\vec{D}_o^{t}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)} < 1, \tag{7}$$

since $\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) = 0$ and $\vec{D}_o^{t}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) > 0$.

Table 1. Data.

DMU	x	y	b
A^t	1	7	2
B^t	1	5	5
A^{t+1}	1	8	1
B^{t+1}	1	5.5	3

Figure 1. Output sets in *t* and *t*+1 (good and bad outputs).



This value suggests that *B* has experienced technological regress, i.e., a shift in the direction of ‘fewer goods and more bads’. However, the change is exactly in the opposite direction, i.e., ‘more goods and fewer bads’, exemplifying the claimed inconsistency. Consequently, the ML index can yield wrong results.

Using the same example, we also illustrate the other two weaknesses inherited from the standard Malmquist productivity index: infeasibility and existence of slacks.

As for the infeasibility problem, we point out that the technical change component cannot be calculated. Mathematically,

$$MLTECH^{t+1} = \frac{1+\vec{D}_o^{t+1}\left(x_B^t, y_B^t, b_B^t; y_B^t, -b_B^t\right)}{1+\vec{D}_o^{t}\left(x_B^t, y_B^t, b_B^t; y_B^t, -b_B^t\right)} = \frac{1+\text{'infeasible'}}{1+0}, \qquad (8)$$

since no $\beta \in R$ exists for B such that $\left(x_B^t, y_B^t\right) + \beta\left(y_B^t, -b_B^t\right) \in P^{t+1}\left(x\right)$. This is graphically shown in Figure 1 by the dashed lines, representing the projected directions for *B* in both periods. In particular, we do not identify a reference benchmark in the *t*+1 output set for B^t. As a direct consequence of the infeasibility problem, we have that it is not possible to determine the value of the adjacent ML index as the geometric mean of the two based period indices.

On the other hand, and regarding the problem associated with the slacks, we note that the directional distance function frequently neglects this type of inefficiency (see Ray, 2004, p. 95), underestimating the actual distance to the relevant Pareto-Koopmans efficient subset of the frontier of the technology. This is graphically illustrated in Figure 1 by the dashed line corresponding to the projected direction for unit *B* in period *t*+1 with respect to the *t* output set. It is worth mentioning that for period *t*, the Pareto-Koopmans efficient subset corresponds to the semi-ray generated from unit A^t.

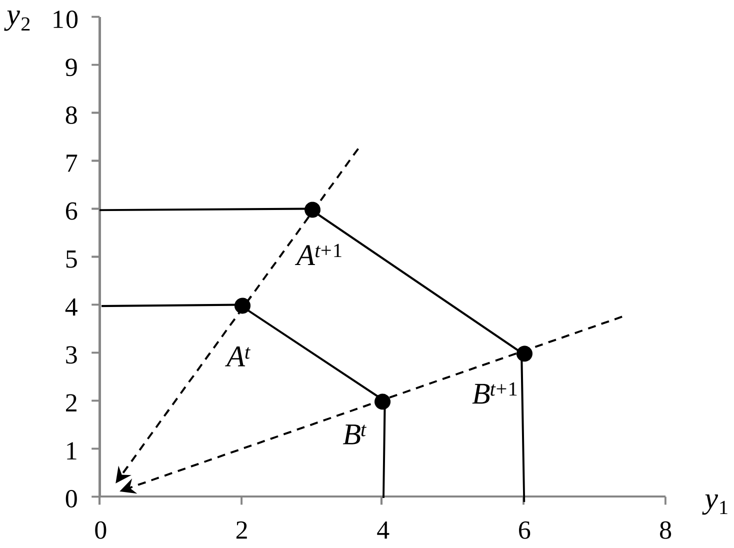
4. Overcoming the inconsistency problem of the ML index

In this section, we introduce a way to overcome the problem related to the inconsistency issue of the ML index. Our approach is based on assuming a new postulate on the environmental technology additional to those usually accepted in the related literature.

The key to the numerical example illustrating the inconsistency issue in the last section lies in assuming that A^{t+1} and B^{t+1} are clearly better production situations than the same units in the previous period t. Mathematically speaking, unit A uses the same quantity of inputs to produce more good outputs and less bad outputs in period $t+1$ than unit A in period t, and the same for unit B; let us denote these situations as $A^{t+1}>A^t$ and $B^{t+1}>B^t$, respectively. Clearly, this implies that environmentally friendly technical progress is taking place in the direction of more goods and fewer bads. However, we showed that the technical change component corresponding to the ML index took such a value that it did not measure the actual shift in the technology over time properly.

In order to propose a solution for the inconsistency problem, we first analyze what happens in the same situation: $A^{t+1}>A^t$ and $B^{t+1}>B^t$, but working with the standard Malmquist output-oriented definition and technology related to the production of exclusively good outputs (see Figure 2).

Figure 2. Output sets in t and t+1 (good outputs only).



In this case, we conclude that $P^t(x) \subseteq P^{t+1}(x)$. In other words, the production possibility set corresponding to period *t* is nested within the production possibility set of period *t*+1. Then, using the well-known expression of the decomposition of the standard Malmquist index (Färe et al., 1994), it is not hard to prove that the technical change component takes a value greater than one, showing the actual shift in the production possibility set in the direction of 'more goods'. In other words, thanks to the nested technologies, the standard Malmquist index does not suffer from the inconsistency issue that we showed for the ML index. Relying on this background, we follow the same argument in the case of the production of good and bad outputs to ensure that $P^t(x) \subseteq P^{t+1}(x)$. This is achieved through the addition of a new postulate to those already assumed (see Section 2). To do so, let us introduce some new notation.

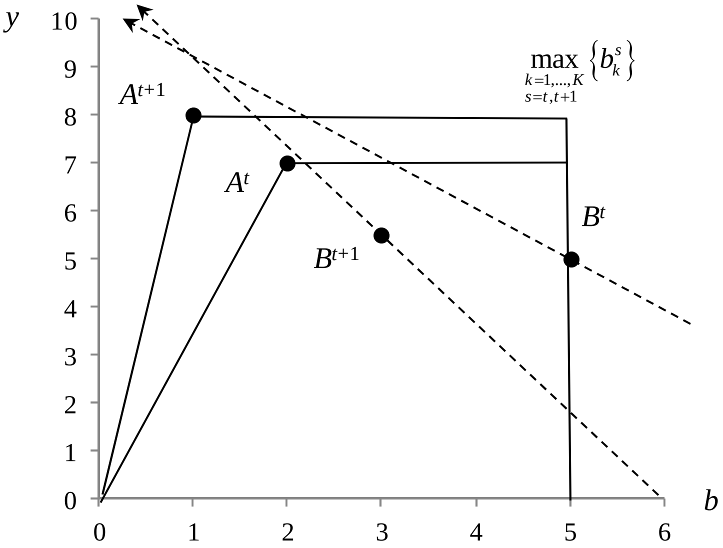
Given $x \in \mathbb{R}_+^N$, let $\bar{b}(x): \mathbb{R}_+^N \to \mathbb{R}_{++}^I$ be a correspondence representing the upper bound for the generation of each considered bad output from the input vector x. In other words, if given x, the vector (y,b) is feasible, then $b \leq \bar{b}(x)$. In our example, see Figure 1, $\bar{b}^t(x) = 3$ and $\bar{b}^{t+1}(x) = 5$ for period *t* and *t*+1, respectively.

Under the assumption of good and bad outputs, the production technology is usually constructed from axioms A1-A6 (see Section 2). We are now ready to introduce a new axiom in order to solve the inconsistency problem:

(A7) If $(y,b) \in P(x)$ and $b \leq b' \leq \bar{b}(x)$, then $(y,b') \in P(x)$.

In words, A7 establishes that if x can produce outputs (y,b), then it is feasible to produce more contaminants up to a certain limit, $\bar{b}(x)$. Graphically, the effects of including the new postulate are depicted in Figure 3. In contrast to the original Figure 1, in Figure 3 the environmental technologies are nested. In this respect, we would like to highlight two points. First, thanks to the production possibility set of period *t* being a subset of the production possibility set of period t+1, we will be able to prove that the technical change component corresponding to the ML index properly measures the shift in the technology over time. Second, in order to achieve that $P^t(x) \subseteq P^{t+1}(x)$, we have additionally assumed that $\bar{b}^t(x) = \bar{b}^{t+1}(x) = \max_{\substack{k=1,\dots,K \\ s=t,t+1}} \{b_k^s\}$.

Figure 3. New output sets in t and t+1 under (A7).



Additionally, it is worth mentioning that the implications of assuming the new postulate (the generation of nested technologies) are closely related to the notion of sequential frontiers introduced by Tulkens and Vanden Eeckaut (1995), that was extended to the Malmquist productivity index context by Shestalova (2003). The latter author justified the use of nested technologies for measuring productivity change in production sectors where technological regress is unlikely to occur. Applying this methodology to calculate Malmquist productivity indices for a set of OECD industrial activities in the traditional context (good outputs), she showed that DEA results based on sequential frontiers provides a more reliable measure than standard DEA. As a result, we believe that the sequential approach fits well our production framework with good and bad outputs, because technical progress, and not regress, is empirically observed as a result of environmentally friendly innovations–i.e., the possibility of producing more desirable output for any given amount of undesirable output increases with time, and also because bad outputs are essentially a byproduct of manufacturing activities (in fact, most environmental applications of the ML index study manufacturing sectors, as reflected in the bibliography). Applying all these notions to the context of production of goods and bads, we assume that in any period $t+1$ the technology of the previous period, *t*, is still feasible. Consequently, all preceding technologies are feasible as well. Particularly, this is the underlying idea for axiom A7 in our approach. Moreover, we would like to stress the idea that DEA with sequential frontiers goes well back in time and constitutes a consolidated methodology, with recurring contributions to the literature as early as the already cited reference by Tulkens and Vanden Eeckaut (1995).

We turn next to show that if $P^t\left(x\right)\subseteq P^{t+1}\left(x\right)$, as in Figure 3, we have that $MLTECH_t^{t+1}\geq 1$. By definition, $MLTECH_t^{t+1}$ is the geometric mean of $MLTECH^t$ and $MLTECH^{t+1}$. Analyzing what happens with respect to, for example, $MLTECH^t=\dfrac{1+\vec{D}_o^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}{1+\vec{D}_o^{t}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)}$, it is verified that $\vec{D}_o^{t+1}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)\geq\vec{D}_o^{t}\left(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}\right)$ as a direct consequence of $P^t\left(x\right)\subseteq P^{t+1}\left(x\right)$ and, therefore, $MLTECH^t\geq 1$. The same can be proved for the other subcomponent of the technical change, $MLTECH^{t+1}$. In this way, we finally have that $MLTECH_t^{t+1}\geq 1$, as initially required.

In our numerical example, $MLTECH^{t}$ is strictly greater than one for unit *B* since $\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) > \vec{D}_o^{t}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)$ in Figure 3, in contrast to the original value we found. The same happens with respect to $MLTECH^{t+1}$ and $MLTECH_t^{t+1}$.

Finally, it is worth mentioning that even if we correct the inconsistency issue related to the ML index by means of the assumption of a new postulate, the two other weaknesses inherited from the standard expression of the Malmquist productivity index could still occur. In other words, at this point it would be possible to resort to the solutions previously introduced in the literature in order to overcome the problems associated with infeasibility and the existence of slacks (see, for example, Grifell-Tatje et al., 1998; Chen, 2003; Pastor and Lovell, 2005; and Dharmapala, 2010).

5. Conclusions

We demonstrate that the ML index is a dubious definition of environmental productivity change because the customary interpretation of its technical change component in terms of production frontier shifts can be inconsistent with its numerical values. This is particularly relevant in environmental productivity studies where the magnitude of the technical change is driven by efficient observations, thereby representing the benchmark for the remaining inefficient firms. In fact, we have shown that productivity change for efficient firms is equal to technical change. Since both the interpretative and numerical results regarding technical change are questionable, overall conclusions with respect to the entire industry are also in jeopardy, and the prescription of policy guidelines based on them could be risky. Nevertheless, in order to make the ML index consistent when measuring environmental productivity change, we introduce a new postulate that enhances the usual set of axioms. We show that under this new assumption the inconsistency problem is solved. Finally, since the ML index inherits other weaknesses corresponding to the standard Malmquist index, specifically infeasibility and existence of slacks, when undertaking empirical studies the solutions already proposed in the literature to address these issues should complement our theoretical proposal. All this renders the ML index a reliable definition upon which to measure environmental productivity change.

Acknowledgements. This research has been supported by the *Ministerio de Ciencia e Innovacion, Gobierno de España*, under grants ECO2008/00510, ECO2011-25349 and MTM2009/10479, as well as by the *Conselleria de Educacion, Generalitat Valenciana*, under grant ACOMP/2012/144.

References

Barros, C.P., 2008. Efficiency analysis of hydroelectric generating plants: A case study for Portugal. Energ. Econ. 30(1), 59–75.

Chambers, R.G., Chung, Y., Färe, R., 1996. Benefit and Distance Functions. J. Econ. Theory 70(2), 407–19.

Chen, Y., 2003. A non-radial Malmquist productivity index with an illustrative application to Chinese major industries. Int. J. Prod. Econ. 83, 27–35.

Chung, Y., Färe, R., Grosskopf, S., 1997. Productivity and undesirable outputs: a directional distance function approach. J. Environ. Manage. 51, 229–240.

Dharmapala, P.S., 2010. The issue of input/output slacks in validating data envelopment analysis-based Malmquist productivity index: an example from banking. Int. J. Mathematics in Operational Research. 2(3), 302–319.

Färe, R., Grosskopf, S., Lindgren, B., Roos, P., 1994. Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach, in: Charnes, A., Cooper, W.W., Lewin, A.Y., Seiford, L.M. (Eds.), Data Envelopment Analysis: Theory, Methodology and Applications. Kluwer Academic Publishers.

Färe, R., Grosskopf, S., Pasurka, C.A., 2001. Accounting for air pollution emissions in measures of state manufacturing productivity growth. J. Regional Sci. 41(3), 381–409.

Färe, R., Grosskopf, S., Pasurka, C.A., 2007. Environmental production functions and environmental directional distance functions. Energy 32, 1055–1066.

Grifell-Tatje, E., Lovell, C.A.K., Pastor, J.T., 1998. A Quasi-Malmquist Productivity Index. J. Prod. Anal. 10, 7–20.

Kumar, S., 2006. Environmentally sensitive productivity growth: a global analysis using Malmquist-Luenberger index. Ecol. Econ. 56, 280–293

Murty, M.N., Kumar, S., Paul, M., 2006. Environmental regulation, productive efficiency and cost of pollution abatement: a case study of the sugar industry in India. J. Environ. Manage. 79(1), 1–9.

Oh, D.H., 2010. A global Malmquist-Luenberger productivity index. J. Prod. Anal. 34, 183–197.

Pastor, J.T., Lovell, C.A.K., 2005. A Global Malmquist Productivity Index. Econ. Lett. 88, 266–271.

Ray, S.C., 2004. Data envelopment analysis. Theory and techniques for economics and operations research. Cambridge University Press.

Shestalova, V., 2003. Sequential Malmquist Indices of Productivity Growth: An Application to OECD Industrial Activities. J. Prod. Anal. 19, 211–226.

Tulkens, H., Vanden Eeckaut, P., 1995. Non-Parametric Efficiency, Progress and Regress Measure for Panel Data: Methodological Aspects. Eur. J. Oper. Res. 80, 474–499.

Weber, W.L., Domazlicky, B., 2001. Productivity growth and pollution in state manufacturing. Rev. Econ. Stat. 83(1), 195–199.

Yoruk, B.K., Zaim, O., 2005. Productivity growth in OECD countries: A comparison with Malmquist indices. J. Comp. Econ. 33(2), 401–420.