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The multiregional core-periphery model: The role of the spatial topology

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Abstract

We use the multiregional core-periphery model of the new economic geography to analyze and compare the agglomeration and dispersion forces shaping the location of economic activity for a continuum of network topologies characterized by their degree of centrality, and comprised between two extremes represented by the homogenous (ring) and the heterogeneous (star) configurations. Resorting to graph theory, we systematically extend the analytical tools and graphical representations of the core—periphery model for alternative spatial configurations, and study the stability of the alternative equilibria in terms of the sustain and break points. We study new phenomena such as the absence of any stable distribution of economic activity for some range of transport costs, and the infeasibility of the dispersed equilibrium in the heterogeneous space, resulting in the introduction of the concept pseudo flat-earth as a long run-equilibrium corresponding to an uneven distribution of economic activity between regions.

KEYWORDS: New economic geography, graph theory, degree of centrality, bifurcation, equilibria and stability analysis.

JEL CODES: C62, C63, F12, R12.

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1. Introduction and motivation

Economic geography, the study of where economic activity takes place and the forces behind it, is a field of increasing interest in economics. The real world shows that economic activity is distributed unevenly across locations: nations, regions and cities, Krugman et al. (2011). One of the most important explanations for that uneven distribution is geography. At no time can the configuration of economic activity at any of the above mentioned territorial scales be dissociated from the particular geography where the activity takes place. Because economic forces are influenced by the economy's geographical characteristics, both "first nature" geographical determinants and "second nature" economic factors (market structure, pricing rules,…) shape the particular distribution of economic activity in a given space.[1] For example, if we take regions as the territorial benchmark, the distribution of economic activity and transport networks in France has given rise to a topology resembling a star network, where the central Île-de-France region presents a prominent situation, characterized by its high degree of centrality. Germany, meanwhile, presents a more even geographical distribution of economic activity, which, with the tightly woven transport grid, results in a more balanced, less centralized economy. It is clear, then, that geography, understood as a specific spatial configuration, determines the final distribution of economic activity along with economic forces.

Theoretical models explain agglomeration outcomes as a result of increasing returns to scale, and thereby depart from the perfectly competitive market assumption. Increasing returns in production and transport costs, as the opposing centrifugal force, are the main ingredients of the so-called new economic geography with respect to other approaches that study the location of economic activity in space such as location theory, Thisse (2010). Geography is introduced into the economic models by way of transport costs, normally associated to the concept of distance between locations, shaping a specific spatial configuration—to which we associate a network topology in this study.

Graph theory makes it possible to characterize the geographical configuration of economic activity with a specific spatial topology. In this context, the question naturally

[1] Cronon (1991) defines "first nature" as the local natural advantages that firms seek when settling on their location, and "second nature" as the forces arising from the presence of other firms. The first is related to geographical features and results in diverse market potential, while the second corresponds to economic interactions: i.e., Marshallian externalities.

arises on how a particular topology influences the centripetal and centrifugal forces that drive agglomeration or dispersion. In recent years several contributions have appeared that change the initial setting of the seminal core-periphery model introduced by Krugman (1991) and thoughtfully discussed in a textbook presentation by Fujita et al. (1999)—e.g., allowing for different definitions of the utility function as the model by Ottaviano et al. (2002), the existence of vertical linkages as in Puga and Venables (1195), etc.—but it is fair to say that the behavior of these models under alternative spatial configurations of the economy has not been systematically discussed. In its original version, there are two regions with the long-run distribution of economic activity either fully agglomerated in one or equally divided between the two.

Nevertheless, a few ways to generalize the model to a multiregional setting have been proposed in the literature. The core-periphery model has been extended to a greater number of regions with the assumption that they are evenly located along the rim of a circumference, in the so-called "racetrack economy", e.g., Krugman (1993), Fujita et al. 1999, Brakman et al. (2009). Whereas these authors obtain results through numerical simulation, Ago et al. (2006) analytically study a situation in which three regions are located on a line, while Castro et al. (2012) consider the case of three regions equally spaced along a circle. The former authors conclude that the central region has locational advantages and that economic activity will concentrate there as transport costs fall. With the alternative model of Ottaviano et al. (2002), however, they also show that the central region can present locational disadvantages and that price competition can make economic activity move to two or just one of the peripheral regions. Castro et al. (2012) qualify the results obtained for two regions regarding long run-equilibria, and are able to generalize some of them for a larger number of regions. In graph theory, the previous racetrack (or ring) economy and the line (star) economy represent two simple and extreme topologies of a spatial network; the former characterizing a neutral or homogeneous topology where no region has a (first nature) geographical advantage, and the latter a non-neutral heterogeneous space where the center is a privileged location.

The aim of the present study is to generalize the well-known canonical model of the new economic geography and systematically analyze the effect of different geographic configurations on the locational patterns of economic activity. To accomplish this goal we use the customary analytical and simulation tools of the new economic geography to analyze how alternative network topologies determine the long-run equilibrium of the multiregional model. In particular—and mainly with the methodology summarized as "Dixit-Stiglitz, icebergs, evolution and the computer" by Fujita and Krugman (2004), since the non-linearity of the model prevents closed analytical results for the multiregional model—we calculate the sustain and break points: i.e., the transport cost levels at which full agglomeration cannot be sustained and the symmetric dispersion is broken, and determine the existence (or absence) of alternative equilibria. We do so for a continuum of network topologies between the already mentioned extreme cases: the racetrack-ring economy (homogeneous space) and the star economy (the most uneven heterogeneous space). In fact, a racetrack-ring economy with three locations corresponds geometrically to the triangle studied by Castro et al. (2012), while the star economy corresponds to the line economy of Ago et al. (2006). Because our methodology can be extended to a larger number of regions, we can with no loss of generality study all possible network topologies (spatial configurations) for the case of four locations, which yields new results never studied in the literature.

By exploring the effect of different geographic configurations on the locational patterns of economic activity our study determines the relationship between "first" nature network characteristics and "second" nature economic forces: i.e., the underlying assumptions of the core-periphery model corresponding to CES preferences, iceberg transport costs, increasing returns and monopolistic competition. As a result we contribute to the scarce literature studying the combination—harmonization—of both first- and second-nature characteristics, and see how localization patterns change as some locations benefit from first-nature advantages, yielding endogenous asymmetries associated with short-run and long-run equilibria, as well as the dynamics associated with continuous or catastrophic changes (see the recent discussion on this matter by Picard and Zeng, 2010).

For the real case of economies with a heterogeneous network we confirm that the greater the centrality of the economy's spatial configuration, the higher the sustain points. Centripetal economic forces are reinforced by the advantage of the region with the best location, and the dissemination of economic activity therefore takes place at a higher transport cost. Alternatively, economic activity fully agglomerated in the least central region (a peripheral region) is less sustainable, because the locational disadvantage works against the agglomerating forces. Consequently, an increase in transport costs shifts economic activity in the network from regions with the lowest centrality to regions with the highest centrality. For the break points, we show that the flat-earth equilibrium is infeasible in heterogeneous space. Therefore, performing the stability analysis for break points requires the introduction of an analogous concept that we term pseudo flat-earth, for which we can determine the transport cost at which economic activity starts agglomerating. We find that this break point is higher the greater the centrality of the region with the best location. Note that these important results are observed not only for the extreme topologies represented by the ring and star networks, but also for the continuum of topologies that exists between them.

The paper is structured as follows. The multiregional core-periphery model and the characterization of network topologies by centrality index, including the extreme racetrack-ring and star space topologies, are presented in section 2. In this section we also generalize the model's dynamics relative to workers moving between existing locations. In section 3, without loss of generality, we perform the four-region analysis for the well-known racetrack economy and for its opposite spatial configuration in network topology, the star. We determine the transport cost value up to which the agglomeration of the economic activity is sustainable, the sustain point, and when the symmetry between regions gives way, the break point. We introduce and discuss new phenomena regarding the absence of long-run equilibria in the core periphery model within a homogeneous space and the infeasibility of the symmetric flat-earth equilibrium in heterogeneous space. We also show bifurcation diagrams summarizing this information for the extreme topologies. In section 4, we analyze the continuum of intermediate topologies using the network centrality index, determine the corresponding sustain and break points, and generalize the previous results for any degree of centrality. Section 5 concludes.

2. The multiregional core-periphery model and the network topology

In the multiregional core-periphery model, there are *N* regions with two sectors of production: the numéraire agricultural sector, perfectly competitive, and the manufacturing sector, with increasing returns to scale. The agricultural sector is immobile and equally distributed across regions.[2] Manufacturing workers can move

[2] Although different asymmetries can be incorporated into the model (e.g., uneven distribution of the population working in the agricultural sector, varying productivity among firms, etc.), we follow the seminal model where all locations are symmetric, and our only sources of variation in the long-run distribution of economic activity are unitary transport cost and network topology.

between regions, and λ_i is the share of manufacturing workers and manufacturing activity in region *i*. Iceberg transport costs are assumed for the manufacturing sector. Transport costs between region *i* and region *j*, τ_{ij}, depend on the unit-distance transport cost *T* and on the distance between the regions d_{ij}^{h} in the network *h*. The transport cost function defines as:

$$\tau_{ij} = T^{d_{ij}^{h}} \tag{1}$$

The income, price index, wage, and real wage equations that determine the multiregional instantaneous equilibrium are well known, Fujita et al. (1999):

$$y_i = \mu w_i \lambda_i + \left(\frac{1-\mu}{N} \right), \quad i = 1, \ldots, N \tag{2}$$

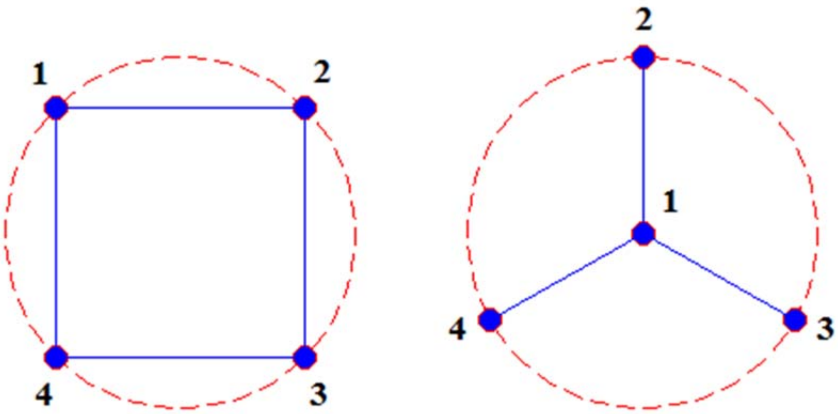
$$g_i = \left(\lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma} \right)^{1/(1-\sigma)}, \quad i = 1, \ldots, N \tag{3}$$

$$w_i = \left(y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma} \right)^{1/\sigma}, \quad i = 1, \ldots, N \tag{4}$$

$$\omega_i = w_i g_i^{-\mu}, \quad i = 1, \ldots, N \tag{5}$$

The homogeneous space is defined as a topology in which all regions have the same relative position, whereas in the heterogeneous space certain regions have better relative positions: i.e., "first nature" locational advantages. The simplest and most extensively studied case of a homogeneous topology corresponds to the afore mentioned racetrack-ring economy, where all regions are evenly situated along the rim of a circumference, Krugman (1993). The extreme heterogeneous topology is the star, where one region, the center, has the best relative position, while all the other regions, the periphery, also situated along the rim of the circumference, have the least advantageous relative positions and are connected to the center only through the spokes of the star. Figure 1 represents the four-location case for both the homogeneous ring and heterogeneous star network topologies.

Figure 1: The extreme homogeneous ring and heterogeneous star network topologies.



The network topology enters the model as the distance between regions, which determines the transport costs between them. Since we are interested in how changes to the topology affect the agglomeration and dispersion of economic activity, we normalize the absolute measures of distance and transport cost, so as to render all topologies comparable. The simplest way is with the following transport cost function replacing (1):

$$\tau_{ij} = T^{\frac{d_{ij}^h}{r}}, \tag{6}$$

where r is the radius of the circumference circumscribing all possible topologies h for a given N. To illustrate, Figure 1 shows the circumference enclosing the networks; the dotted circle denotes that regions are not connected through the circumference but through the distances within the network h, represented in these cases by straight, solid lines: i.e., the ring or star topologies.

With regard to the shares of workers and manufacturing activity, the dynamics are as follows: (i) workers will leave region i if there is a region j with a higher real wage, eq. (5), or, equivalently, higher indirect utility; (ii) if several regions have higher real wages, workers are assumed to move to the one offering the highest value; (iii) when the highest wage is observed in several regions, workers emigrate evenly towards those regions. Therefore, from region i's perspective, workers will move according to these rules:

$$\dot{\lambda}_i < 0 \ if \ \ \omega_i < \max\left(\omega_j\right) \ \forall j \neq i \,,$$
$$\dot{\lambda}_i = 0 \ if \ \ \omega_i = \max\left(\omega_j\right), \ \forall j \ \wedge \not\exists \ \ \omega_j < \omega_i, \forall j \neq i \,, \qquad (7)$$
$$\dot{\lambda}_i > 0 \ if \ \ \omega_i = \max\left(\omega_j\right), \forall j \wedge \ \exists \ \ \omega_j < \omega_i, \forall j \neq i \,,$$

where the second line summarizes the instantaneous equilibrium: i.e., equal real wages across regions. A distribution of lambdas for which the system of equations (2) through (5) holds therefore represents an instantaneous equilibrium, while a long-run equilibrium—steady state—is one in which workers do not have an incentive to move according to (7) if there is a shock marginally increasing the share of manufactures in any region, and is denoted by $\lambda^* = (\lambda_1^*, ..., \lambda_N^*)$.

In a multiregional economy we can characterize the spatial or network topology with graph theory, which proposes several indicators that summarize the pattern of interconnections between various locations; e.g., Harary (1969). Centrality measures are particularly useful for the study of the multiregional network, as they are good indicators to characterize the space topology with.

With $\sum_{j=1}^{N} d_{ij}^h$ being the sum of the distances from location *i* to all other *j* locations within the network *h*, the centrality of location *i* corresponds to the following expression:

$$c_i^h = \frac{\min\left(\sum_{j=1}^{N} d_{ij}^h\right)}{\sum_{j=1}^{N} d_{ij}^h}, \qquad (8)$$

where $\min\left(\sum_{j=1}^{N} d_{ij}^h\right)$ corresponds to the value of the location(s) best positioned within the economy, denoted by i^*, with $c_{i^*}^h$ = 1. In a homogeneous space such as that represented by the ring topology all locations have a centrality of 1, whereas in the heterogeneous star topology the central node has a centrality of 1 and all peripheral nodes have equal centrality values lower than 1: $c_i^h < c_{i^*}^h = 1$.

The centrality of the economy—network centrality—defines as:

$$C(h)=\frac{\sum_{i=1}^{N}\left[c_{i^*}^{h}-c_{i}^{h}\right]}{\max\left[\sum_{i=1}^{N}\left[c_{i^*}^{h^*}-c_{i}^{h^*}\right]\right]}=\frac{\sum_{i=1}^{N}\left[c_{i^*}^{h}-c_{i}^{h}\right]}{\frac{(N-1)(N-2)}{(2N-3)}}, \tag{9}$$

where $\sum_{i=1}^{N}\left[c_{i^*}^{h}-c_{i}^{h}\right]$ is the sum of the centrality differences between the location with the highest centrality and all remaining locations, and $\max\left[\sum_{i=1}^{N}\left[c_{i^*}^{h^*}-c_{i}^{h^*}\right]\right]$ is the maximum sum of the differences that can exist in a network with the same number of nodes. This maximum corresponds to a heterogeneous star network with a central node and N-1 periphery nodes. The network centrality for the homogeneous ring space is $C\left(h^{HM}\right)=0$ and for the heterogeneous star space $C\left(h^{HT}\right)=1$. The two extreme topologies have the extreme network centralities.

3. Analysis of the extreme topologies: The ring and star economies

Without loss of generality, we can study a four-region economy by comparing the two opposite cases of spatial topology in terms of network centrality: the ring and the star (Figure 1). In the homogeneous space the four regions are the four vertices of a square. In the heterogeneous three-pointed star topology there is a central location, 1, and three peripheral locations connected to the center. Both spaces are circumscribed in a circumference of radius 1. The distance matrices of the four-region ring and star networks are the following:[3]

$$D^{HM}=\begin{pmatrix}0 & 1.4142 & 2.8284 & 1.4142\\ 1.4142 & 0 & 1.4142 & 2.8284\\ 2.8284 & 1.4142 & 0 & 1.4142\\ 1.4142 & 2.8284 & 1.4142 & 0\end{pmatrix},\quad D^{HT}=\begin{pmatrix}0 & 1 & 1 & 1\\ 1 & 0 & 2 & 2\\ 1 & 2 & 0 & 2\\ 1 & 2 & 2 & 0\end{pmatrix}.$$

3.1 Sustain points

The sustain point is the level of transport cost at which the agglomeration of economic activity is no longer sustainable and economic activity disperses across regions. To compute the value of the sustain point we must select the reference region,

[3] To compute the distance between two neighbor regions, we use the formula of the length of the side of a regular polygon of n sides and radius r: $d_{ij}^{HM}=2r\sin\left(\pi/n\right)$.

or regions, where the economic activity is initially agglomerated and check whether it is a feasible solution for the instantaneous equilibrium defined in eqs. (2) through (5). Next, given a particular network *h*, we use the dynamic rules set in (7) to compute the value of T for which $\dot{\lambda}_i > 0$ in each region. For example, assuming that a single location agglomerates (e.g., region 1: $\omega_1 = 1$ in (5)) and given the generalized definition of the real wages for the remaining regions ($i \neq 1$) (Appendix1):

$$\omega_i^{\sigma} = \frac{1-\mu}{N}\tau_{1i}^{\sigma-1-\mu\sigma} + \frac{1+(N-1)\mu}{N}\tau_{1i}^{1-\sigma-\mu\sigma} + \frac{1-\mu}{N}\tau_{1i}^{-\mu\sigma}\sum_{j=1}^{N-2}\tau_{1i}^{\sigma-1}\tau_{ij}^{1-\sigma}, \quad i = 2,...,N, \tag{10}$$

we compute the level of the transport cost corresponding to the sustain point $T_{1i}(S)$ for which $\omega_i > \omega_1, i \neq 1$, and determine the subsequent final instantaneous equilibrium compatible with $T > T_{1i}(S)$: i.e., a comparative statics analysis. In this section we explore the sustain point for the two extreme ring and star topologies when the region in the center starts agglomerating. In the first case all the regions in the homogeneous space are equivalent, and we need to explore only the case of one of the regions, as the long-run equilibria are symmetric: i.e., any permutation of the agglomerating location yields equal results.

3.1.1 Homogeneous-ring topology: From full agglomeration to flat-earth dispersion

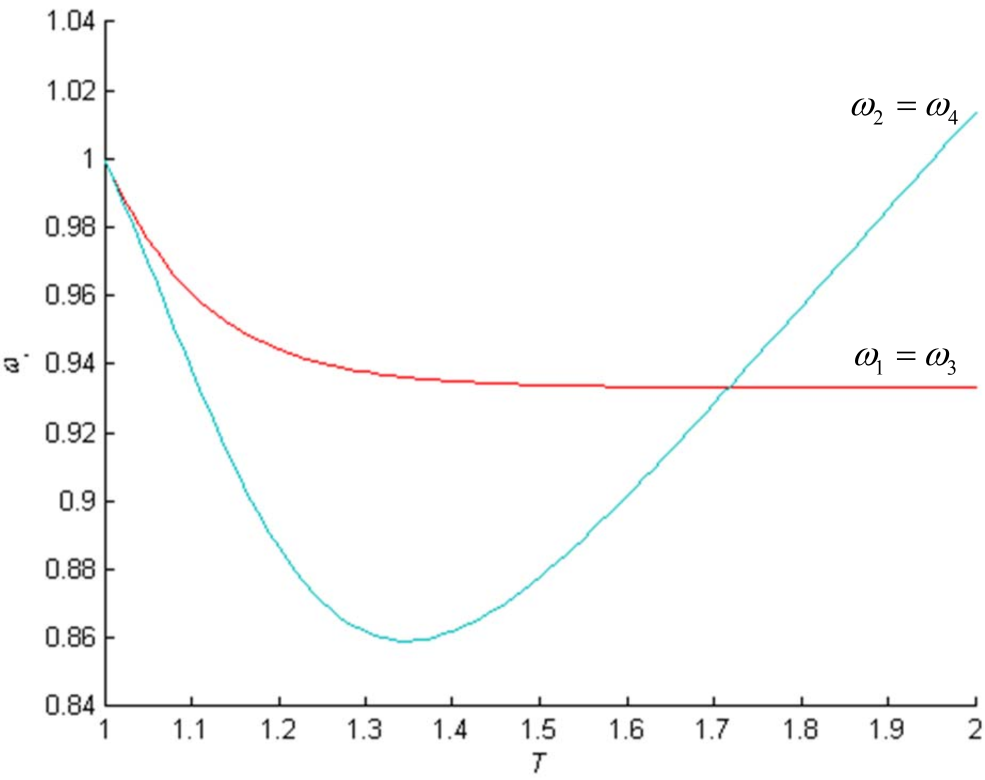
In simulations for the ring network with region 1 agglomerating ($\lambda_1^* = 1$), the sustain point for region 3 (the farthest region from 1, as d_{13}^{HM} = 2.83) is $T_{13}^{HM}(S) = 1.39$, which is lower than the value for neighbor regions 2 and 4 (separated by d_{1j}^{HM} = 1.41, j = 2, 4): *i.e.*, $T_{1j}^{HM}(S) = 1.52, j = 2,4$.[4] That is, when the transport cost T rises above 1.39 economic activity spreads to region 3, since $\omega_3 > \omega_1$, and regions 1 and 3 both produce manufactures. The sustain point, defined as $\min\left(T_{1j}^{HM}(S)\right) = 1.39, j = 2,3,4$, suggests a partial agglomeration in two regions separated by the maximum distance d_{13}^{HM} = 2.83. As a result, the configuration λ = ($\lambda_1 = 0.5, \lambda_2 = 0, \lambda_3 = 0.5, \lambda_4 = 0$) is a candidate for a

[4] To ease comparability with Fujita et al. (1999), all simulations in these sections use the parameter values $\sigma = 5$ and $\mu = 0.4$. Expressions for real wages when only one region is agglomerating and the agglomeration depends only on transport costs are presented in Appendix 1for *N* regions and in Appendix 2 for *N* = 4 regions.

stable equilibrium, since real salaries in the agglomerating regions are equal: $\omega_i = 0.9353, i = 1,3$, while those of the empty regions are $\omega_i = 0.8611, i = 2,4$. Because the minimum sustain value corresponds to the farthest regions, the balance between competition and transport costs makes it more profitable for firms and workers leaving the agglomerating region to relocate as far as possible and thereby equally serve the markets of the regions with no manufacturing activity, regions 2 and 4.

Whether the partial agglomeration (or partial dispersion) given by λ = $(\lambda_1 = 0.5, \lambda_2 = 0, \lambda_3 = 0.5, \lambda_4 = 0)$ is a long-run equilibrium depends on the corresponding stability analysis for a shock that marginally increases the share of manufactures in one or more regions, and its effect on the real salaries: i.e., $\partial\omega_i / \partial\lambda_i$, i=1, 3. This stability analysis is performed in the next section, on break points. Nevertheless, if we assume that such a shock does not take place, and since the previous distribution may represent a subsequent instantaneous equilibrium, we can further study its sustainability as transport costs keep rising. Figure 2 shows real wages for different transport-cost values when the instantaneous equilibrium corresponds to agglomeration in regions 1 and 3. The sustain point in this case is $T_{1j}^{HM}(S) = T_{3j}^{HM}(S) = 1.72, j = 2,4$. When transport cost increases beyond 1.72 manufacturing activity disperses across all regions—flat-earth. That is, a situation where all regions have the same share of manufacturing activity, $\lambda_i = 0.25\ \forall i$, emerges as a possible long-run equilibrium, as regions end up having the same real wage $\omega_i = 0.878, \forall i$. Once again, however, its steady-state assessment depends on the necessary stability analysis for long-run equilibrium.

Figure 2: Real wages for the ring topology when opposite regions agglomerate.



3.1.2 Heterogeneous-star topology: From full agglomeration to "pseudo" flat-earth

We now examine the star topology when the location with the highest centrality—the center of the star: $\max c_i^{HT} = c_{i*}^{HT} = c_1^{HT} = 1$—begins agglomerating: $\lambda_1^* = 1$. As shown Section 4, this extreme heterogeneous network topology has the highest sustain point of all possible spatial configurations–network centralities, with $T_{c_{i*}j}^{HT}(S) = 2.58,\ j = 2,3,4$. Above this value of transport cost, agglomeration is no longer sustainable and manufacturing activity disperses to the three peripheral regions. Once again, the question is whether the dispersion of economic activity can result in an equal distribution of the manufacturing industry: i.e., whether $\lambda_i = 0.25\ \forall i$ corresponds to a long-run equilibrium.

Once again, we must resort to stability analysis, but it turns out that we can immediately prove that this spatial configuration does not represent a stable equilibrium, because it simply cannot exist. That is, the flat-earth long-run equilibrium is infeasible in any heterogeneous space with the system of equations (2) through (5) characterizing it, because it requires transport costs to be equal for all regions (i.e., a homogeneous space topology is a necessary condition). Indeed, symmetric equilibrium is possible only if all regions have the same real wage: $\omega_i = w_i g_i^{-\mu} \;\; \forall i$. If all regions have the same share of manufacturing, $\lambda_i = 1/N$, the nominal wage in all agglomerating regions is $w_i = 1$, as the following condition holds (cf. Robert-Nicoud (2005, 11) for N=2, as well as Ago et al. (2006, 822) and Castro et al. (2012, 406) for $N = 3$):

$$\sum_{i=1}^{n} \lambda_i w_i = 1 \tag{11}$$

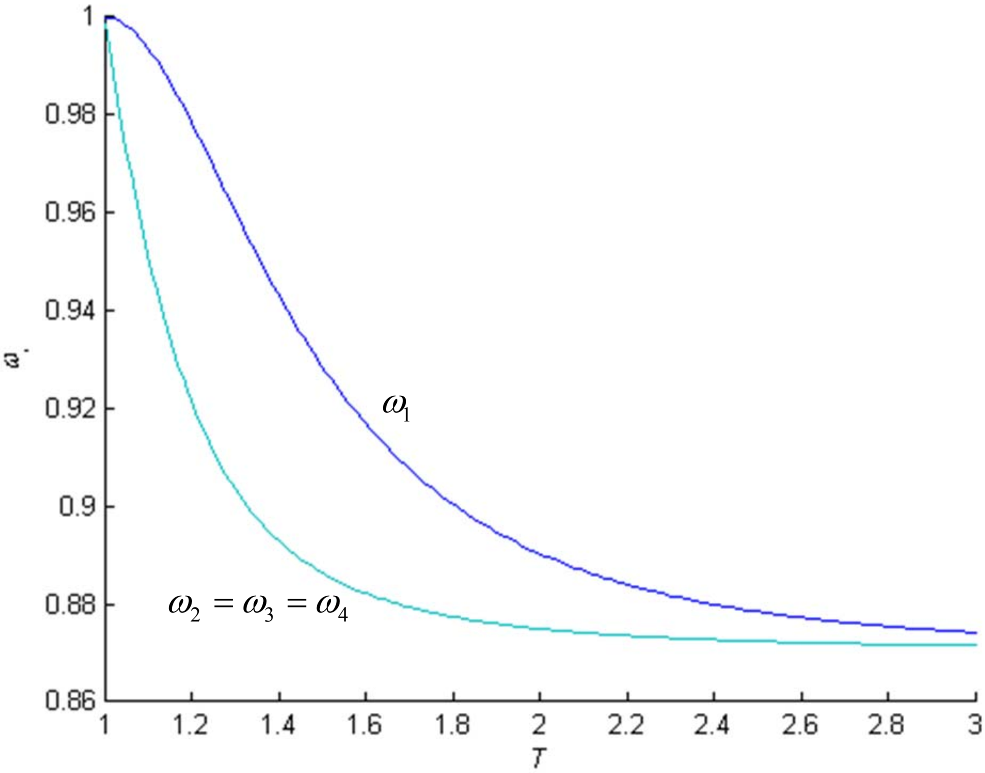
Therefore, real wages are equal in all regions only if price indices are equal in all regions. Since the price index of a region i depends on the transport cost between all agglomerating regions and region i, the price index will be equal across regions if and only if all the regions have the same relative position in the network economy.

Proposition 1. Inexistence of the flat-earth equilibrium in a heterogeneous space: *Symmetric equilibrium, flat-earth, is feasible only if all locations have the same relative position in the network. Therefore, symmetric equilibrium is feasible only in a homogeneous space.*

Proof: Equality of real wages across regions: $\omega_i = \omega_j, \forall\, i, j$, agglomerating an even share of manufacturing activity $\lambda_i = 1/N \, \forall i$, requires that price indices be equal: $g_i = g_j, \forall\, i, j$. Substituting this even share of manufacturing and $w_i = 1 \;\forall\, i$—from (11)—in (3), real salaries are (not) equal if all bilateral transport costs—centralities—are the same; this is (not) verified in the homogeneous (heterogeneous) space. ■

Proposition 1 can be easily illustrated. Real wages when the four regions of the star hypothetically have the same share of manufacturing activity: $\lambda_i = 0.25 \, \forall i$, are represented in Figure 3. For all levels of transport cost, the real wage of the central region 1 is higher than the real wages of the remaining regions except in the unreal case when transport is costless: $T = 1$. This illustrates that economic activity moves from the periphery to the center and that the flat-earth equilibrium is not feasible in the heterogeneous space.

Figure 3: Real wages for the star topology when all regions have an equal share of economic activity.



Therefore, with region 1 agglomerating, once transport costs overcome the (single) sustain point $T_{c_i*j}^{HT}(S) = 2.58,\ j = 2,3,4$, manufacturing activity will disperse across regions and reach a configuration that we define and characterize in the following section and name pseudo flat-earth. As we show, for a pseudo flat-earth the central region's share of manufacturing is above 0.25, while peripheral regions' shares are below 0.25. Figure 3 illustrates that the hypothetical flat-earth situation is not a stable equilibrium for all transport costs, including the sustain point $T_{c_i*j}^{HT}(S) = 2.58,\ j = 2,3,4$, as the real salary is higher in the central region than in any other: $\omega_1 = 0.8774$ > $\omega_i = 0.8772, i = 2,3,4$.

3.1.3 Comparing sustain points in ring and star network topologies

The differences in the sustain points between the homogeneous and the heterogeneous space lead to the following result—as long as the "no-black-hole" condition holds: [5]

Result 1. The sustain point in a heterogeneous space is higher (lower) than in the homogeneous space for central (peripheral) regions. There is a transport-cost level in the homogeneous ring topology and the heterogeneous star topology at which dispersion forces outweigh agglomeration forces are outweighed by the dispersion forces. Regarding this level of the transport cost, the sustain point for the central region (peripheral region) is higher (lower) in a heterogeneous space than in a homogeneous space, because agglomeration forces are higher (lower) in regions that have a locational advantage (disadvantage), i.e., that exhibit a better (worse) relative position: [6]

$$T_{c_i * j}^{HT}(S) > T_{ij}^{HM}(S) > T_{j\,c_i *}^{HT}(S) \tag{12}$$

The values of the sustain point for the different situations already examined are presented in Table 1. Beginning with the homogeneous space we have the initial equilibrium, E^{HM} = 1, in which only one region is agglomerating. When transport cost reaches $T_{13}^{HM}(S) = 1.39$ half of the economic activity moves to the farthest region, thereby reaching a second—unstable—equilibrium, E^{HM} = 2. If transport cost continues to increase beyond $T_{1j}^{HM}(S) = T_{3j}^{HM}(S) = 1.72,\ j = 2,4$ economic activity disperses across all regions, attaining a final long-run equilibrium, E^{HM} = 3. In a heterogeneous star topology, starting at an equilibrium in which the center is agglomerating economic activity, E^{HT} = 1, when transport cost rises above $T_{1j}^{HT}(S) = 2.58,\ j{=}2{,}3{,}4$, economic activity disperses across all regions, attaining a pseudo flat-earth long-run situation, E^{HT} = 2.

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[5] The "no-black-hole" condition in the multiregional model can be obtained from eq. (10). It can be shown that all summands except the second tend to infinity as transport costs increase—regardless of network configuration—as long $(\sigma\text{-}1)/\sigma < \mu$: i.e., as long as the original two-region condition holds. For the particular N =4 case shown in Appendix 2, the first summand coincides with that of the two-region case, while the third and fourth terms are positive for the values of σ and μ previously assumed: $\sigma > 1$ and $\mu \in [0, 1]$.

[6] We have also studied the sustain point for one of the peripheral regions with lowest centrality: $\lambda_2 = 1$ with c_i^{HT} = 0.6, i = 2, 3, 4 (top region in Figure 1). In this case, the central region defines the lowest value for the sustain point: min $T_{2\,c_i *}^{HT}(S) = 1.44$. Complete results for the full range of alternative simulations are available upon request.

Table 1: Sustain-point values for different network topologies: From agglomeration to dispersion

Region	Homogeneous ring topology			Heterogeneous star topology	
	One region agglomerating $E^{HM} = 1$ (1)	Opposite regions agglomerating $E^{HM} = 2$ (2)	Dispersion $E^{HM} = 3$ (3)	Central Region agglomerating $E^{HT} = 1$ (4)	Dispersion $E^{HT} = 2$ (5)
1	Agglomeration: $\lambda_1 = 1$	Partial agglomeration: $\lambda_1 = 0.5$	Dispersion: $\lambda_1 = 0.25$	Agglomeration: $\lambda_1 = 1$	Dispersion: $\lambda_1 > 0.25$
2	1.52	1.72	Dispersion: $\lambda_2 = 0.25$	2.58	Dispersion: $\lambda_2 < 0.25$
3	1.39	Partial agglomeration: $\lambda_2 = 0.5$	Dispersion: $\lambda_3 = 0.25$	2.58	Dispersion: $\lambda_3 < 0.25$
4	1.52	1.72	Dispersion: $\lambda_4 = 0.25$	2.58	Dispersion: $\lambda_4 < 0.25$

3.2 Break points

Studying the break point involves determining when a symmetric equilibrium is broken. To obtain the break point analytically we generalize the procedure set out in Fujita et al. (1999), which requires defining an initial distribution for the stability analysis. We start with a symmetric equilibrium—either flat-earth in the homogeneous ring topology or pseudo flat-earth in the heterogeneous star topology—in which all regions have the same share of manufacturing activity ($\lambda_i = 1/N$) and evaluate the derivative of the real wage with respect to the change in a region's share of manufacturing activity *i*: $\partial\omega_i/\partial\lambda_i$. A break point is the transport cost at which the derivative of the real wage equals zero and the symmetric equilibrium is unstable, because the right derivative is positive and the left derivative negative. If the equilibrium is unstable, a small shock increasing a region's share of manufacturing activity triggers agglomeration in that region.[7]

[7] This is normally illustrated in the literature with the so-called "wiggle" diagram, which presents the value of the derivative $\partial\omega_i/\partial\lambda_i$ for the full range on lambda values: $\lambda \in [0, 1]$. In this diagram, instantaneous equilibria are characterized by equality of real wages. The instability (stability) of these

The system of nonlinear equation derivatives of (2) through (5) that allows us to determine the value of $\partial\omega_i/\partial\lambda_i$ is the following:[8]

$$dy_i = \mu dw_i\lambda + \mu w_i d\lambda_i \,, \tag{13}$$

$$\begin{aligned}(1-\sigma)\frac{dg_i}{g_i^{-\sigma}} &= w_i^{1-\sigma}d\lambda_i + (1-\sigma)\lambda_i w_i^{-\sigma}dw_i + \\ &+\sum_{j=1}^{N-1}\left(\left(w_j\tau_{ji}\right)^{1-\sigma}d\lambda_j + (1-\sigma)\lambda_j\tau_{ji}^{\;1-\sigma}w_j^{-\sigma}dw_j\right),\end{aligned} \tag{14}$$

$$\begin{aligned}\sigma\frac{dw_i}{w_i}w_i^{\sigma} &= g_i^{\sigma-1}dy_i + (\sigma-1)y_i\sigma_i^{\sigma-2}dg_i + \\ &+\sum_{j=1}^{N-1}\left(g_j^{\sigma-1}\tau_{ij}^{1-\sigma}dy_j + (\sigma-1)y_i\tau_{ij}^{1-\sigma}g_j^{\sigma-2}dg_j\right),\end{aligned} \tag{15}$$

$$g_i^{\mu}d\omega_i = dw_i - \mu w_i\frac{dg_i}{g_i}\,. \tag{16}$$

3.2.1 Homogeneous ring topology: From flat-earth to agglomeration

At the symmetric equilibrium we calculate the break point corresponding to a first simulation (S1) characterized by: (i) an equal distribution of manufacturing activity corresponding to the following—transposed—vector: $\lambda^{S1} = (\lambda_i^*)' = (0.25, 0.25, 0.25, 0.25)$, and (ii) the evaluation of the system on non-linear equations $\partial\omega_i/\partial\lambda_i \;\forall i$ under a shock of the following magnitude: $d\lambda^{S1}$ = $d\lambda'_{4\cdot 1}$ = $(d\lambda_1 = 0.001,\; d\lambda_2 = -d\lambda_1/3,\; d\lambda_3 = -d\lambda_1/3,\; d\lambda_4 = -d\lambda_1/3)$. This corresponds to the standard setting in the literature for break-point evaluation: a flat-earth configuration and a shock in one of the regions. The value of the break point is $T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}} = 1.45$, meaning that when transport cost falls below 1.45 the symmetric equilibrium breaks as $\partial\omega_1/\partial\lambda_1$ > 0, and the agglomeration of economic activity starts. Given the differentials, this positive value is observed in the region whose share of manufacturing increases: $d\lambda_i$> 0, which in this case is region 1. However, a long-run equilibrium characterized

interior equilibria depends on whether the right and left derivatives are positive (negative) and negative (positive), respectively.

[8] Equation (13) is obtained directly by totally differentiating the income equation (2). The differentiation process yielding (14) through (16) is presented in Appendices 3 through 5, respectively.

by $\lambda_1^* = 1$ is not reached, because, as shown in the previous section, the sustain point for this configuration is $\min T_{ij}^{HM}(S) = T_{13}^{HM}(S) = 1.39$—a value that situates below the previous break point $T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}} = 1.45$. In ring topology, therefore, neither full agglomeration nor symmetric dispersion represents long-run equilibria for values of $T \in \left(\min T_{ij}^{HM}(S); T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}}\right) = (1.39;1.45)$. This contrasts with the usual configurations of stable equilibria in two- or three-region economies, where at least one or both of the equilibria exist (cf. Ago et al (2006) and Castro et al. (2012)). Given the relevance of this situation we stress the following result.

Result 2. In the multiregional homogeneous ring network topology, core-periphery and symmetric flat-earth equilibria do not exist if the break point is greater than the minimum sustain point. In the ring topology, there exist transport costs in the range $T \in \left(\min\left(T_{ij}^{HM}(S)\right); T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}}\right)$ for which full agglomeration and symmetric dispersion of manufacturing activity are not stable equilibria.

This result holds when the following inequality is verified:

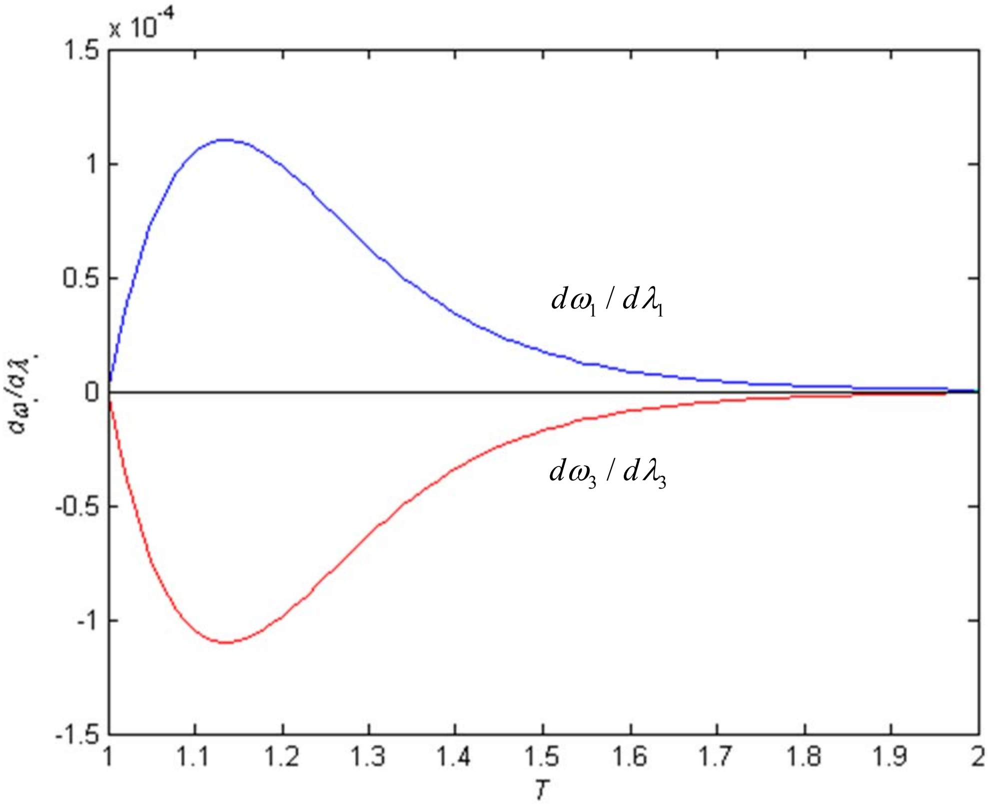
$$T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}} > \min\left(T_{ij}^{HM}(S)\right) \tag{16}$$

As shown below, the existence of an intermediate distribution of manufacturing activity λ^* representing a stable long-run equilibrium for the previous range of transport cost: $T \in \left(\min\left(T_{ij}^{HM}(S)\right); T^{HM}(B)\big|_{\lambda^{S1},d\lambda^{S1}}\right)$, can be ascertained through a stability analysis evaluating the equality of real wages and by the sign of the derivative $\partial\omega_i / \partial\lambda_i$.

The previous evaluation of the stability of the symmetric equilibrium produced by the shock $d\lambda^{S1}$ in the first region is not the only possible one. Complementing the existing literature on three-region models, in the multiregional model there can be shocks affecting any number of regions as long as $\sum_{i=1}^{N} d\lambda = 1$. Let us now consider a simulation with the initial distribution of manufacturing activity: $\lambda^{S2} = \lambda^{S1} = (\lambda_i^*)' = (0.25, 0.25, 0.25, 0.25)$, but in which a shock hits two opposite regions, so that: $d\lambda^{S2}$ = $d\lambda_{4\cdot 1}^{'}$ = $\left(d\lambda_1 = 0.0005, d\lambda_2 = -0.0005, d\lambda_3 = 0.0005, d\lambda_4 = -0.0005\right)$. This results in a break point value of $T^{HM}\left(B\right)\big|_{\lambda^{S2},d\lambda^{S2}} = 1.59,$ which is higher than the previous one. Thus the second result above still holds. This second break point is larger than the first—$T^{HM}\left(B\right)\big|_{\lambda^{S2},d\lambda^{S2}}$ > $T^{HM}\left(B\right)\big|_{\lambda^{S1},d\lambda^{S1}}$—because as the agglomerating shock spreads to more and more regions (from region 1 to include region 3) the other regions remaining at the symmetric equilibrium decrease in number accordingly, and the centrifugal forces associated with them weaken. As a result, the symmetric equilibrium breaks at a higher transport cost. For this second simulation, the range of transport costs for which neither full agglomeration nor symmetric dispersion exists is T ∈ $\left(\min\left(T_{ij}^{HM}\left(S\right)\right);T^{HM}\left(B\right)\big|_{\lambda^{S2},d\lambda^{S2}}\right)$ = $\left(1.39;1.59\right)$. In this case, regions 1 and 3 both start agglomerating and a partial symmetric dispersion—or partial asymmetric agglomeration—occurs in the two regions: $\lambda^{'}$ = $(\lambda_1 = 0.5, \lambda_2 = 0, \lambda_3 = 0.5, \lambda_4 = 0)$. As a result, we reach the instantaneous equilibrium as characterized in the previous section, with the first region's departure from full agglomeration, and we can now perform the stability analysis to check if this represents a long-run equilibrium.

Specifically, we check for each region whether the derivative of its real wage with respect to its manufacturing share is positive, signaling unstable equilibrium, or negative, signaling stability. We have calculated these derivatives for the whole range of transport costs assuming the distribution of economic activity $\lambda^{S3} = (\lambda_i)^{'} = (0.5, 0, 0.5, 0)$ and a subsequent shock to the two farthest regions exhibiting manufacturing activity: i.e., $d\lambda^{S3}$ = $d\lambda_{4\cdot 1}^{'}$ = $\left(d\lambda_1 = 0.001, d\lambda_3 = -0.001, d\lambda_2 = 0, d\lambda_4 = 0\right)$. In this case, we find that for region 1 the derivative is always positive for *any* transport-cost level and no break point exists, since the instantaneous equilibrium characterized by λ^{S3} is never stable and brakes in favor of the region experiencing the positive shock. Figure 4 shows the real wage derivatives for the previous shock and partial equilibrium. This means that in this partial symmetric equilibrium if a shock were to hit one on the regions that agglomerates, further agglomeration would start in that region regardless of transport cost; and this distribution of economic activity is therefore never a long-run equilibrium. However, whether region 1 ends up in stable full agglomeration ($\lambda_1^* = 1$) depends on the particular transport cost at which the shock takes place. For transport costs below the sustain point $T < \min\left(T_{ij}^{HM}(S)\right)$, economic activity agglomerates in that region, while for transport costs above this threshold $T > \min\left(T_{ij}^{HM}(S)\right)$, the change in other regions' real salaries eventually reverses the agglomeration process, with the real salary in the third region overcoming that in the first region. As we discuss in the following section when commenting on the bifurcation diagram, for $T > \min\left(T_{ij}^{HM}(S)\right)$ any positive shock in one of the regions results in the redistribution of manufacturing activity between different instantaneous equilibria.

Figure 4: Real-wage derivative for a positive shock in region 1 under partial equilibrium.



3.2.2 Heterogeneous star topology: From "pseudo" flat-earth to agglomeration

In any heterogeneous network topology like the star the flat-earth equilibrium, with all regions having the same share of manufacturing activity, is as stated in proposition 1, infeasible. Therefore, to analyze the break point we must first characterize the stable long-run equilibrium that best captures the idea of symmetric dispersion: i.e., a spatial configuration where no region lacks manufacturing production: $\lambda^{*'} = (\lambda_1^*,...,\lambda_N^*),\ \lambda_i^* > 0$. In general, then, what we call pseudo flat-earth is a situation in which all locations have some level of manufacturing but some (central) regions have a greater share. Given this criterion we can introduce a further qualification that allows us to determine the bounds for the set of lambdas λ^* for which long-run equilibria exist. The lowest bound can be defined according to the principle of least difference, by which the sum of the differences in manufacturing shares is the lowest: $\min \sum_i^N \left(\max(\lambda_i) - \lambda_i\right)$—denoted by $\lambda^{*L'} = (\lambda_1^{*L},...,\lambda_N^{*L}),\ \lambda_i^{*L} > 0$ and named minimum pseudo flat-earth. Opposite to this, the upper bound corresponds to that distribution for which the sum of differences is the highest: $\lambda^{*H'} = (\lambda_1^{*H},...,\lambda_N^{*H}),\ \lambda_i^{*H} > 0$, termed maximum pseudo flat-earth: max $\sum_i^N \left(\max(\lambda_i) - \lambda_i\right)$. The introduction of pseudo flat-earth (including its maximum and minimum qualifications) is a novel outcome of the present multiregional core-periphery model, which, unlike the two- and three-region models, allows us to characterize a steady state where all regions produce manufacturing but have different shares. In pseudo flat-earth, each region's particular share of manufacturing depends on its relative position in the network.

Definition 1. In multiregional heterogeneous network topology, pseudo flat-earth is a stable long-run equilibrium characterized by: i) $\lambda_i^* > 0\ \forall i$**, ii)** $\omega_i = \omega_j\ \forall\, i, j$**, and iii)** $\partial\omega_i / \partial\lambda_i \leq 0\ \forall i$**.**

In the particular case of the heterogeneous star network topology, derivative of the real wage should be zero for the central region and negative for peripheral regions. Pseudo flat-earth is therefore given by the set of lambdas $\lambda_{1\cdot N}^{*'} = (\lambda_1^*,...,\lambda_N^*),\ \lambda_i^* > 0$, the upper and lower bounds being the values that solve the following optimization programs for all transport-cost levels, corresponding to the maximum and minimum pseudo-flat-earth distributions of manufacturing production, respectively. Considering the system of equations (2) through (5) and its associated system of derivatives (13) through (16) , we determine the upper bound associated with the maximum lambda of the region of highest centrality (maximum pseudo flat-earth distribution) by solving the following program:

$$\max\ \lambda_{c_{i^*}}^{H} \tag{17}$$

$$s.t.\begin{cases}\lambda_i > 0, \forall i, \\ \omega_i = \omega_j \ \ \forall i, j, \\ \dfrac{\partial \omega_1}{\partial \lambda_1} = 0, \\ \dfrac{\partial \omega_j}{\partial \lambda_j} < 0, \quad \forall j \neq 1, \end{cases}$$

where the first set of restrictions characterizes the new pseudo-flat-earth definition (no emptiness), the second set ensures that an instantaneous equilibrium exists, and the third and fourth sets determine its stability. Precisely, the upper bound corresponds to third restriction, which determines the largest value of lambda λ_1^{*H} for which the pseudo flat-earth still holds, thereby signaling the associated transport cost corresponding to the break-point value.

The minimum value of lambda for which the dispersed equilibrium holds—i.e., characterizing the minimum pseudo flat-earth distribution—is:

$$\min\ \lambda_{c_{i^*}}^{L} \tag{18}$$

$$s.t.\begin{cases}\lambda_i > 0, \forall i, \\ \omega_i = \omega_j \ \ \forall i, j, \\ \dfrac{\partial \omega_i}{\partial \lambda_i} < 0 \quad \forall i. \end{cases}$$

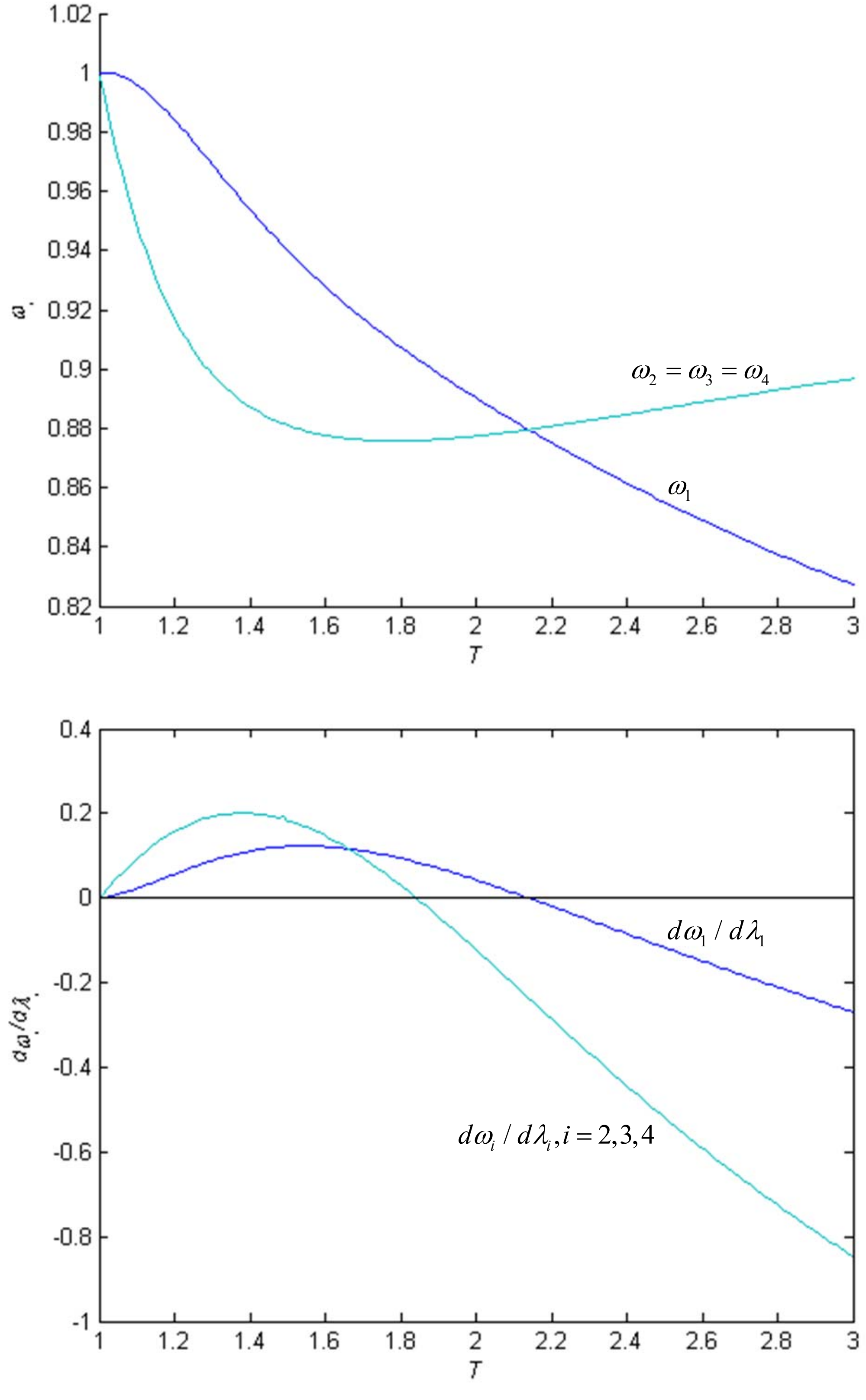
We let δ denote the distance between the maximum and minimum shares of manufacturing that the central region can have for pseudo flat-earth equilibria to be stable.

$$\delta = \max \lambda_{c_{i^*}}^{*H} - \min \lambda_{c_{i^*}}^{*L} \tag{19}$$

As for the stability analysis, since the central region tends to attract and agglomerate economic activity as a result of its privileged "first nature" situation—see proposition 1 in Ago et al. (2006)—we consider once again the first shock: $d\lambda^{S1}$ = $d\lambda'_{4\cdot 1}$ = $(d\lambda_1 = 0.001,\ d\lambda_2 = -d\lambda_1/3, d\lambda_3 = -d\lambda_1/3, d\lambda_4 = -d\lambda_1/3)$, when evaluating $\partial\omega_i/\partial\lambda_i$. In this analysis, maximum pseudo flat-earth corresponds to the transport cost and its associated distribution of manufacturing shares for which $\partial\omega_1/\partial\lambda_1 = 0$ constitutes a break point $T^{HM}(B)\big|_{\lambda^{*H}, d\lambda^{S1}}$. Conversely, minimum pseudo flat-earth is asymptotic to the traditional flat-earth definition, with manufacturing production approaching equal distribution as transport cost tends to infinity.

For our particular four-region star network topology, the combination of shares that solves the maximization problem given by (17) is $\lambda_{c_{i*}}^{*H} = \lambda_1^{*H} = 0.3376,\ \lambda_j^{*H} = 0.2208, j = 2,3,4$, yielding a break point value of $T^{HT}(B)\big|_{\lambda^{*H}, d\lambda^{S1}}$ = 2.14, at which real wages across regions are equal $\omega_i = \omega_j\ \forall\ i, j$—as illustrated in Figure 5a—and $\partial\omega_1/\partial\lambda_1 = 0$, with the right derivative being positive and the left derivative negative. In contrast to a "wiggle" diagram representing $\partial\omega_i/\partial\lambda_i$ for different λ and a given *T*, Figure 5b illustrates these values for different *T* and a given lambda, and therefore the equilibrium is unstable when $\partial\omega_i/\partial\lambda_i$ is negative for a marginal increment in *T*. This means that for a transport cost value lower than $T^{HT}(B)\big|_{\lambda^{*H}, d\lambda^{S1}}$ = 2.14, the maximum pseudo flat-earth is no longer stable and manufacturing production starts agglomerating in the central region 1. Instead, the combination of shares of manufacturing that solves the minimization problem given by (18) is $\lambda_{c_{i*}}^{*L} = \lambda_1^{*L} \approx 0.25$, slightly over 0.25 for the central region, and $\lambda_2^{*L} = \lambda_3^{*L} = \lambda_4^{*L} \approx 0.25$, slightly under 0.25 for the peripheral regions. The distance between the maximum and the minimum is δ=0.0875. Consequently, pseudo flat–earth exists for $\lambda_{c_{i*}}^{*} = \lambda_1^{*} \in \left(\lambda_1^{*L}; \lambda_1^{*H}\right]$ = $(0.25; 0.3376], \lambda_j^{*} \in [0.2208; 0.25)$, $j = 2,3,4$, and for this range of transport costs $T \in [2.139; +\infty)$. For each level of transport cost we find a unique combination of shares of manufacturing that produces stable long-run pseudo-flat-earth equilibrium; Figures 5a and 5b illustrate the real wage and real-wage derivatives for maximum pseudo flat-earth.

Figure 5a-b: Real wages and real-wage derivatives for the maximum pseudo flat-earth



3.2.3 Comparing the break point in homogeneous and heterogeneous spaces

The differences in break points between homogeneous and heterogeneous spaces lead to the following result—as long as the "no-black-hole" condition holds:

Result 3. The break point is greater in heterogeneous than in homogeneous space. There is a transport-cost level in the homogeneous ring topology and the heterogeneous star topology below which the long-run dispersed equilibrium (either flat-earth or pseudo flat-earth, respectively) becomes unstable. This level of transport cost is higher in the star than in the ring topology, because regions with locational advantage—i.e., better relative position—start agglomerating economic activity for higher values of transport costs.

This result can be summarized in the following inequality:

$$T^{HT}\left(B\right)\Big|_{\lambda^{*H},d\lambda^{S1}} > T^{HM}\left(B\right)\Big|_{\lambda^{S1},d\lambda^{S1}} \tag{19}$$

The value of the break points for the two network topologies are summarized in Table 2. Beginning with the ring topology, the full dispersion equilibrium characterized by $\lambda_i^* = 0.25, \forall i$, E^{HM} = 1, is stable for transport costs from $T^{HT}\left(B\right)\Big|_{\lambda^{*H},d\lambda^{S1}}$ = 1.45 to infinity if the positive shock affects only central region 1: $d\lambda^{S1}$ = $d\lambda^{'}$ = $(d\lambda_1 = 0.001,\ d\lambda_2 = -d\lambda_1/3,\ d\lambda_3 = -d\lambda_1/3,\ d\lambda_4 = -d\lambda_1/3)$, and from $T^{HT}\left(B\right)\Big|_{\lambda^{*H},d\lambda^{S2}}$ = 1.59 to infinity if the exogenous shock applies to the two farthest regions $d\lambda^{S2}$ = $d\lambda_{4\cdot 1}^{'}$ = $\left(d\lambda_1 = 0.0005, d\lambda_2 = -0.0005, d\lambda_3 = 0.0005, d\lambda_4 = -0.0005\right)$. In Table 2 we report the latter simulation. For the obtained partial dispersion (or partial agglomeration) instantaneous equilibrium: $\lambda_1 = \lambda_3 = 0.5$, E^{HM} = 2, the partial derivative of real salaries with respect to manufacturing share for region 1 is always positive (Figure 4), so it is unstable. And together with the sustain-point results, for transport-cost range $T \in$ $\left(\min\left(T_{ij}^{HM}\left(S\right)\right);T^{HM}\left(B\right)\Big|_{\lambda^{S2},d\lambda^{S2}}\right)$ = $\left(1.39;1.59\right)$ full-agglomeration and partial-symmetric-dispersion equilibria are unfeasible (result 2), and only short-run instantaneous equilibria exist. For transport-cost levels below sustain point $T_{13}^{HM}\left(S\right)$ = 1.39, full-agglomeration long-run equilibrium in one region is stable.

For the star topology, pseudo-flat-earth dispersed equilibrium, E^{HT} = 1, is stable only for transport costs from $T^{HT}(B)\Big|_{\lambda^{*H},d\lambda^{S1}}$ = 2.14 to infinity. If transport costs fall below 2.14, only full agglomeration in the central region, E^{HT} = 2, is possible, because the sustain point for the star network topology is $T^{HT}_{c_i*j}(S) = 2.58,\ j = 2,3,4$, and therefore $T^{HT}(B)\Big|_{\lambda^{*H},d\lambda^{S1}} < T^{HT}_{c_i*j}(S),\ j = 2,3,4$ (i.e., result 2 above does not hold).

Table 2: Break-point values for different network topologies: From dispersion to agglomeration

Region	Homogeneous ring topology			Heterogeneous star topology	
	Flat-earth E^{HM} = 1	**Partial dispersion E^{HM} = 2**	**Full agglomeration E^{HM} = 3**	**Pseudo flat-earth E^{HT} = 1**	**Full agglomeration E^{HT} = 2**
1	$\lambda_1^* = 0.25$	$\lambda_1 = 0.5$	$\lambda_1^* = 1$	λ_1^* =(0.25;0.3376)	$\lambda_1^* = 1$
2	$\lambda_2^* = 0.25$	$\lambda_2 = 0$	$\lambda_2^* = 0$	λ_2^* = [0.2208;0.25)	$\lambda_2^* = 0$
3	$\lambda_3^* = 0.25$	$\lambda_3 = 0.5$	$\lambda_3^* = 0$	λ_3^* =[0.2208;0.25)	$\lambda_3^* = 0$
4	$\lambda_4^* = 0.25$	$\lambda_4 = 0$	$\lambda_4^* = 0$	λ_4^* =[0.2208;0.25)	$\lambda_4^* = 0$
***T*(*B*)**	1.59	$\nexists$	$\nexists$	2.14	$\nexists$
Stability range	1.59 — +∞	Unstable (Result 2)	1 —1.39	2.14 — +∞	1 — 2.58

3.3 Bifurcation diagrams

The bifurcation diagrams summarizing the information on the sustain and break points for both the homogeneous ring and heterogeneous star network topologies are shown in Figures 6a-b, respectively. The horizontal axis shows the different transport-cost values, and the vertical axis the share of manufacturing for region 1. Solid lines represent stable long-run equilibria and dotted lines only short-run stable equilibria.

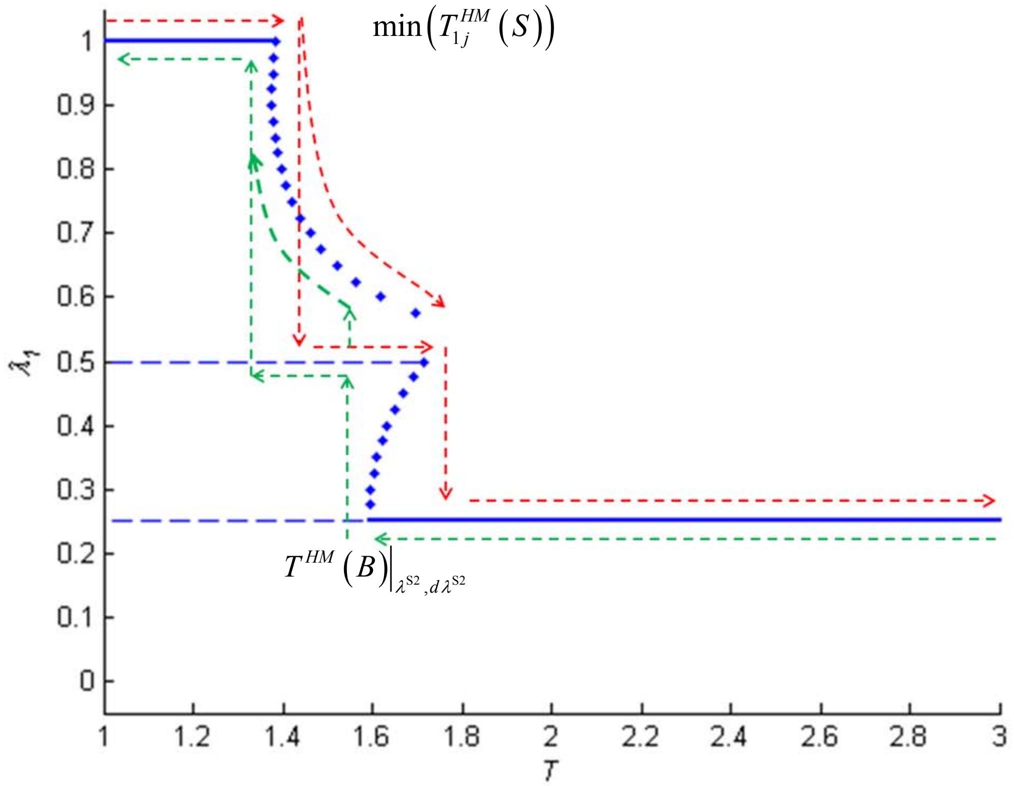
3.3.1 Bifurcation diagram of the homogeneous ring topology

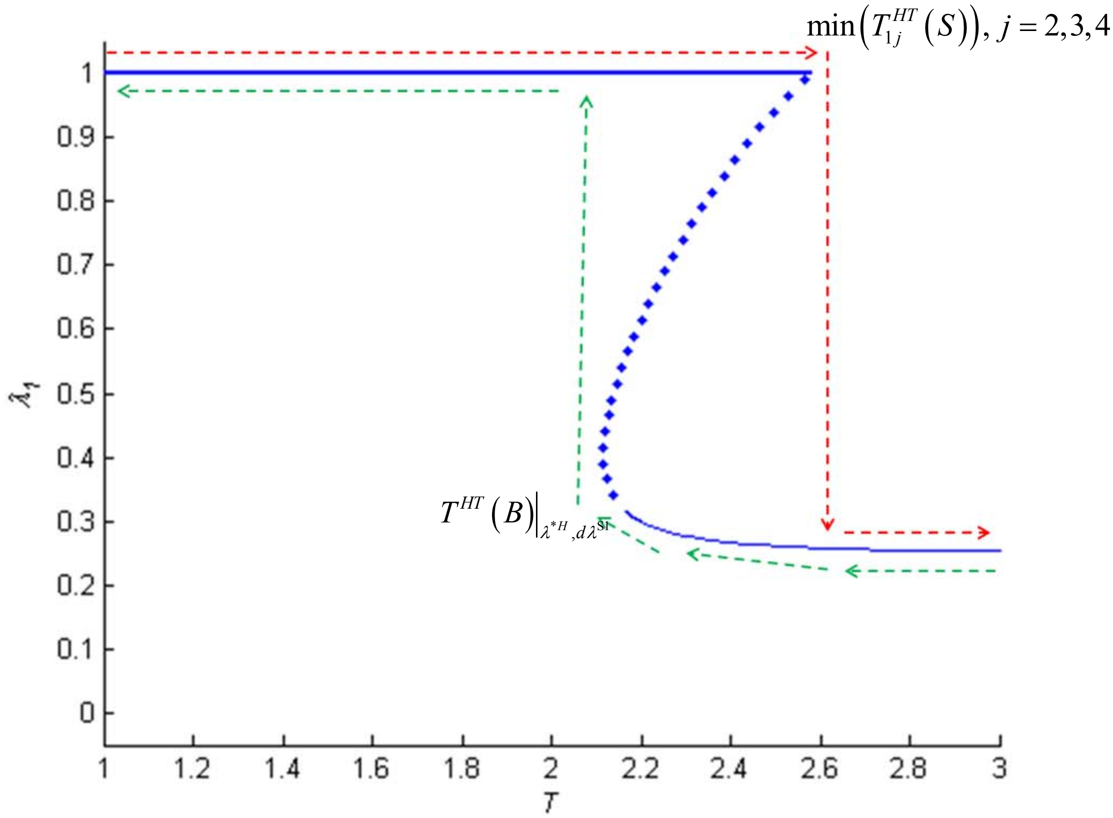
In the bifurcation diagram of the homogeneous ring topology (Figure 6a), full agglomeration in region 1 is stable until transport cost reaches $T^{HM}_{13}(S) = 1.39$. Beyond this threshold economic activity disperses to the two opposite regions 1 and 3, sharing half of the manufacturing activity: $\lambda_1 = \lambda_3 = 0.5$. This is nevertheless not a stable equilibrium (Figure 4), and thus any shock to the manufacturing share in one of the two regions triggers a redistribution between instantaneous short-run equilibria. This situation holds for any simulation where $T \in \left(\min\left(T_{ij}^{HM}(S)\right); T^{HM}(B)\big|_{\lambda^{S2}, d\lambda^{S2}}\right) = (1.39; 1.59)$ if the shock for the break point is given by $d\lambda^{S2} = d\lambda'_{4\cdot 1} = (d\lambda_1 = 0.0005, d\lambda_2 = -0.0005, d\lambda_3 = 0.0005, d\lambda_4 = -0.0005)$, and for different distributions of economic activity between regions 1 and 3. However, as transport cost increases over break point $T^{HM}(B)\big|_{\lambda^{S2}, d\lambda^{S2}} = 1.59$, manufacturing activity spreads over all regions, each attaining the same share: $\lambda_i^* = 0.25$.

Indeed, with a departure from the flat-earth equilibrium and a shock on the two opposite regions: $d\lambda_1 = d\lambda_3 = 0.0005$ and $d\lambda_2 = d\lambda_4 = -0.0005$, the symmetric equibrium is stable for transport-cost values higher than $T^{HM}(B)\big|_{\lambda^{S2}, d\lambda^{S2}} = 1.59$ but unstable for lower values. That said, the intermediate instantaneous equilibrium $\lambda_1 = \lambda_3 = 0.5$ is not stable, and, once again, we find no long-run equilibria for transport costs in the range $T \in \left(\min\left(T_{ij}^{HM}(S)\right); T^{HM}(B)\big|_{\lambda^{S2}, d\lambda^{S2}}\right) = (1.39; 1.59)$.[9] As transport costs fall below 1.39, an exogenous shock to the share of manufacturing in two opposite regions leads to an agglomeration process in one of the regions: e.g., region 1. Note that there are levels of the transport cost at which different equilibria are possible. For example, there are several *unstable* equilibria between the highest break point, 1.59, and the second sustain point, 1.72 , as well as between that break point and the first sustain point.

[9] In fact, the partial dispersion or agglomeration, instantaneous equilibrium, represented by $\lambda_1 = \lambda_3 = 0.5$ exists only in the transport cost range from 1 to 1.72.

Figure 6a-b: Bifurcation diagrams of four-region ring and star topologies





3.3.2 Bifurcation diagram of the heterogeneous star topology

In the bifurcation diagram of the heterogeneous star space (Figure 6b) agglomeration in the central region, region 1, is stable for transport costs lower than $T_{1j}^{HT}\left(S\right)=2.58, j=2,3,4$. If transport cost rises over 2.58, economic activity is dispersed between all regions, resulting in a pseudo-flat-earth long-run equilibrium with a manufacturing share over 0.25 in the central region and slightly under 0.25 in the peripheral regions. On the other hand, the pseudo-flat earth-long-run equilibrium is stable with transport costs over $T^{HT}\left(B\right)\Big|_{\lambda^{*H},d\lambda^{S1}}$ = 2.14 and the following ranges of manufacturing shares: $\lambda_{c_{i^*}}^{*}=\lambda_1^{*}\in\left(\lambda_1^{*L};\lambda_1^{*H}\right]$ = $\left(0.25;0.3376\right]$, $\lambda_j^{*}\in\left[0.2208;0.25\right), j=2,3,4$. Under 2.14, the only long-run stable equilibrium is the agglomeration in the central region. We note in passing that in this case, contrary to the two- and three-region models, there is continuous change in manufacturing shares as we reduce transport costs within the range from infinity to 2.14. This change corresponds to successive equilibria (in favor of regions with the highest centrality). We therefore observe smooth changes in equilibria, as opposed to catastrophic agglomeration or dispersion. For transport-cost values from $T^{HT}\left(B\right)\Big|_{\lambda^{H},d\lambda^{S1}}$ = 2.14 to $T_{1j}^{HT}\left(S\right)=2.58, j=2,3,4$ there are three possible equilibria. Two represent stable long-run equilibria—full agglomeration in the central region and the pseudo flat-earth—and the other is unstable: i.e., short-run equilibrium.

4. Intermediate topologies: Centrality and critical points

In this section we explore the sustain and break points for a continuum of topologies between the already studied extremes: the homogeneous ring configuration, exhibiting a centrality measure $C\left(h^{HM}\right)$ = 0, and the heterogeneous star configuration, with $C\left(h^{HT}\right)$ = 1. First, we determine the number of intermediate topologies, or steps, that we want to study between these two cases. If we recall the distance matrices in section 3, the differences between these extreme topologies are given by a linear transition matrix:

$$D_{Dif}=\frac{\left(D^{HM}-D^{HT}\right)}{S}, \tag{20}$$

where D^{HM} is the distance matrix of the ring topology e, D^{HT} the distance matrix of the star topology and *S* stands for the total number of steps.

For our four-region case, the difference matrix is:

$$D_{Dif} = \begin{pmatrix} 0 & 0.4142/S & 1.8284/S & 0.4142/S \\ 0.4142/S & 0 & -0.5858/S & 0.8284/S \\ 1.8284/S & -0.5858/S & 0 & -0.5858/S \\ 0.4142/S & 0.8284/S & -0.5858/S & 0 \end{pmatrix}. \tag{21}$$

In our simulation we determine the sustain and break points for a hundred network topologies: $S = 100$, each corresponding to the following matrices: $D^{HT(h)} = D^{HT} + hD_{Dif}$, $h = 0,\ldots,100$, where $D^{HT(h)}$ varies as the matrix of the star topology gets successively one step closer to that of the ring topology: i.e., for *h*=100, $D^{HT(h)} = D^{HM}$.

Given the linear transition schedule represented by the difference matrix (21), we determine the extension of the economy represented by the circle circumscribing each topology, so as to adjust transportation cost by (6). This ensures that transportation costs are normalized and we can disentangle the effect on changes in the unit transport cost and each network's centrality.

4.1 Sustain points for the continuum of network topologies

Figure 7a shows the sustain point for intermediate space topologies from $C\left(h^{HM}\right)$ = 0 to $C\left(h^{HT}\right)$ = 1. Generalizing the first result, we see that the underlying function that defines the sustain point increases as the network centrality increases. Moreover, it is convex, implying that as the uneven spatial configuration associated with first-nature characteristics reduces, the reduction in the sustain point gets smaller. Assuming that the "no-black-hole" condition holds, we can summarize this finding as follows:

Result 4: The higher (lower) the centrality of the network, the higher (lower) the sustain point. There exists a transport-cost level at which the forces agglomerating economic activity are outweighed by the forces dispersing manufacturing activity. This transport-cost level—the sustain point—rises (falls) as the centrality of the network, $C(h)$, rises (falls).

This result can be summarized in the following inequality:

$$\min\left(T_{1j}^{C(h)}(S)\right) > \min\left(T_{1j}^{C(h')}(S)\right), C(h) > C(h') \tag{22}$$

4.2 Break point values for the continuum of network topologies

To compute the break point for each intermediate topology and its associated maximum pseudo-flat-earth distribution: $\lambda_{c_{i^*}}^{*H}$, we once again evaluate the system of equations (2) through (5) along with its associated system of derivatives (13) through (16), for the following vectors of differentials, which correspond to the previous analyses of ring and star topologies.

$$d\lambda^{HM} = \begin{pmatrix} d\lambda_1 \\ d\lambda_2 \\ d\lambda_3 \\ d\lambda_4 \end{pmatrix} = \begin{pmatrix} 0.0005 \\ -0.0005 \\ 0.0005 \\ -0.0005 \end{pmatrix}; \quad d\lambda^{HT} = \begin{pmatrix} d\lambda_1 \\ d\lambda_2 \\ d\lambda_3 \\ d\lambda_4 \end{pmatrix} = \begin{pmatrix} 0.001 \\ -0.001/3 \\ -0.001/3 \\ -0.001/3 \end{pmatrix}. \tag{23}$$

The difference vector of the shock from one topology to the next is given by:

$$d\lambda_{Dif} = \frac{\left(d\lambda^{HM} - d\lambda^{HT}\right)}{S}. \tag{24}$$

As for the distance matrices, the vector of differentials for each simulation is $d\lambda^{HT(h)} = d\lambda^{HT} + hd\lambda_{Dif}$, h = 0,…,100, where $d\lambda^{HT(h)}$ varies as the star topology's associated matrix gets one step closer to that of the ring topology, and $d\lambda^{HT(h)} = d\lambda^{HM}$ for h=100.

The break point values for intermediate topologies $T(B)\big|_{\lambda_{c_{i^*}}^{*H}, d\lambda^{HT(h)}}$ are shown in Figure 7a and the shares of manufactures for those break points in Figure 7b. As with the sustain points, the function underlying the break point shows increasing network centrality and is convex. This implies that decreasing network centrality makes the full dispersed equilibrium stable over a larger range of transport costs. Once again, if the “no-black-hole” condition holds, this generalizes the third result, relating the break points for the two extreme topologies.

Result 5. The higher (lower) the centrality of the network, the higher (lower) the break point. There exists a transport-cost level at which long-run dispersed equilibrium becomes unstable. This level rises (falls) as the centrality of the network rises (falls).

Again, this result can be summarized in the following inequality:

$$T^{C(h)}(B)\Big|_{\lambda^{*H}_{c_{i^*}},d\lambda^{HT(h)}} > T^{C(h')}\left(B\right)\Big|_{\lambda^{*H}_{c_{i^*}},d\lambda^{HT(h)}},\; C(h) > C(h') \tag{25}$$

Figures 7a allows us to disentangle the effects of changes in network topology, $C(h)$, and the unit-distance transport cost T. For a given value of transport cost between the minimum (ring) and maximum sustain (star) points: $T \in \left(\min\left(T^{HM}_{ij}\left(S\right)\right);T^{HT}_{c_{i^*}j}\left(S\right)\right)$, and with a departure from a fully agglomerated equilibrium (below the sustain point line), reducing the centrality of the network will eventually result in a dispersed spatial configuration as the sustain point is reached eventually. Alternatively, for a given value of transport cost between the minimum (ring) and maximum (star) break points: $T \in \left(T^{HM}\left(B\right)\Big|_{\lambda^{S2},d\lambda^{S2}};\max\left(T^{HT}(B)\Big|_{\lambda^{*H}_{c_{i^*}},d\lambda^{HT(h)}}\right)\right)$, and with a departure from a dispersed pseudo-flat-earth equilibrium (above the break point line), increasing the centrality of the network will break the equilibrium eventually and shift the economy toward a more agglomerated outcome.

Also, Figure 7a illustrates the previous result 2, regarding the inexistence of either fully agglomerated or dispersed equilibria. For zero-degree centrality we noted that in the range $T \in \left(\min\left(T^{HM}_{ij}\left(S\right)\right),T^{HM}\left(B\right)\Big|_{\lambda^{S2},d\lambda^{S2}}\right) = \left(1.39, 1.59\right)$ none of these equilibria exists. Now we confirm that this situation holds for a range of centrality from $C\left(h^{HM}\right)$ = 0 to $C\left(h^{HT}\right)$ = 0.6975 (for this latter value $\min\left(T^{HM}_{ij}\left(S\right)\right) = T^{HM}\left(B\right)\Big|_{\lambda^{S2},d\lambda^{S2}}$), and that beyond this level of centrality both long-run as well as other, intermediate, unstable equilibria exist, as presented in the bifurcation diagram for the star topology (Figure 6b). The expected outcome with regards to the final long-run situation that is eventually reached as network centrality varies is also illustrated in Figure 7a, where A represents an economy exhibiting a degree of centrality and unitary transport cost given by

$(C(h),T)$. In this situation neither fully agglomerated nor fully dispersed equilibria are steady states, and reducing network centrality (e.g., by infrastructure policy) favors the dispersed outcome, whereas if network centrality were increased the agglomerated outcome would emerge.

Finally, Figure 7b allows us to picture the gap between the maximum and minimum pseudo flat-earth for a given network centrality: $\delta = \max \lambda_{c_{i^*}}^{*H} - \min \lambda_{c_{i^*}}^{*L}$. The largest and smallest gaps are observed for the extreme star and ring topologies, respectively.[10]

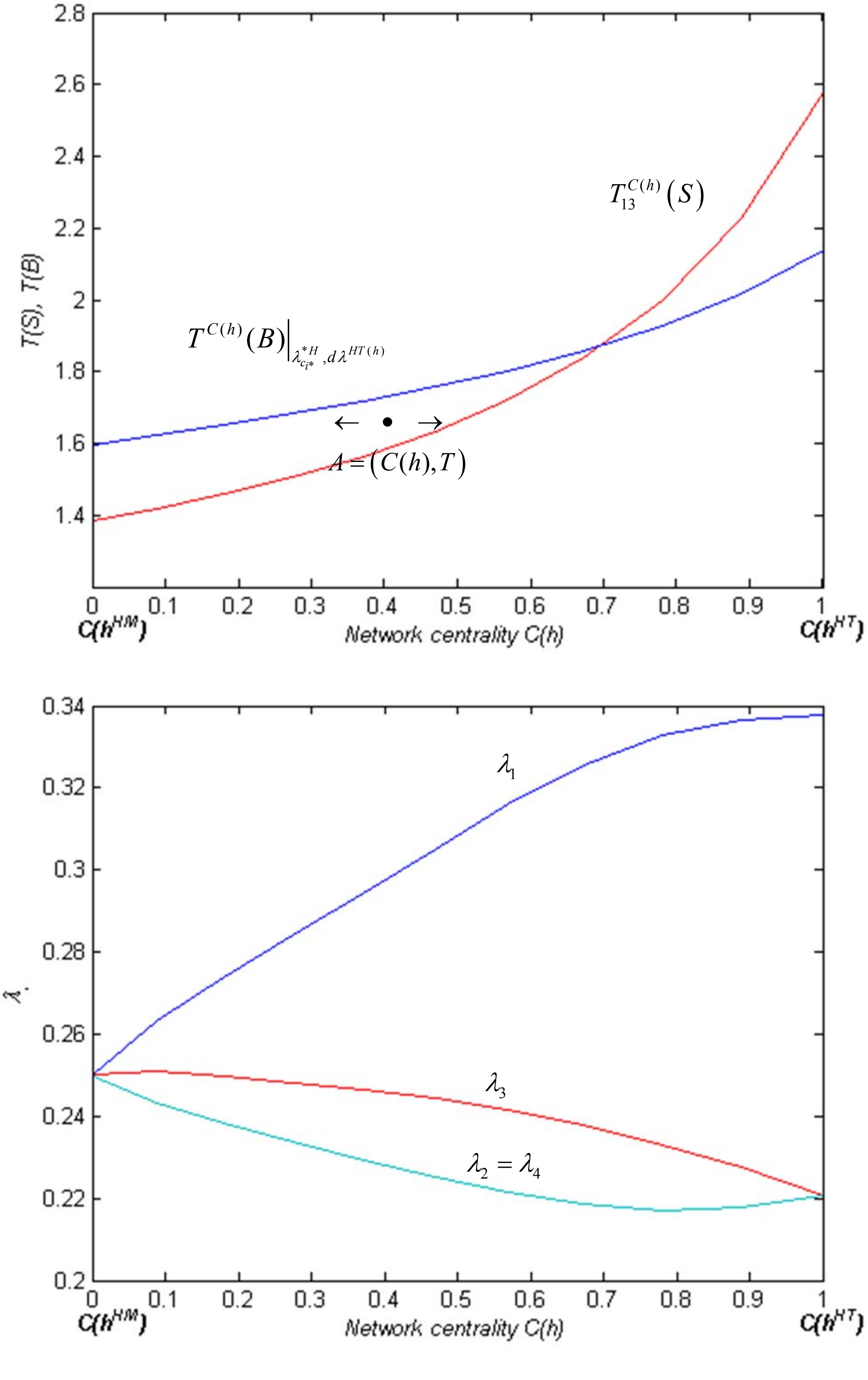
5. Conclusions

The relative position of a location—nation, region or city—in space plays a critical role in the agglomeration and dispersion of economic activity. Whereas transport cost is one of the elements that shapes the current distribution of economic activity, geographical topology must also be taken into account, since the effects of a change in transport costs on the distribution of economic activity (e.g., the triggering of alternative processes of agglomeration or dispersion) differ depending on the economy's spatial configuration. Thus the relative position of a region in space determines the final result of these processes.

Our results show that alternative network topologies result in different behaviors for agglomerating and dispersing forces and thus for alternative spatial configurations of economic activity. Indeed, results 1 and 3 show that for the two polar cases—homogeneous ring topology and heterogeneous star topology—both the sustain and break points are higher in the latter. The existence of a "first nature" advantage in favor of the central region makes agglomeration in that region more sustainable (and therefore less sustainable in peripheral regions). For the exact same reason, if we were to depart from symmetric equilibrium, regions with higher centralities would start drawing economic activity at a higher transport-cost level than if the network were neutral, with no region presenting a locational advantage. We generalize the results for extreme topologies to any pair of network configurations, showing in results 4 and 5 that the sustain and break points are higher in networks presenting higher centralities.

[10] Given the transition matrix (21), regions 2 and 4 present the same centrality index (8) for all network topologies, and therefore have the same shares of manufacturing activity.

Figure 7a-b: Sustain points, break points and manufacturing shares for intermediate network topologies.



The systematic study of sustain and break points results in several interesting results never studied in the literature. Firstly, for homogeneous networks with a zero degree of centrality we find a range of transport costs for which neither full agglomeration nor full dispersion produces stable long-run equilibrium; only instantaneous, short-run equilibria exist (as opposed to the existing two- and three-region cases literature where this result is not reported). This result is observed for transport-cost values between the minimum sustain points and the break point, with the particularity that the former is lower than the latter (result 2). Secondly, for heterogeneous networks exhibiting a positive degree of centrality, we stress that the dispersed flat-earth equilibrium, which is the initial configuration of manufacturing activity when studying break points, is infeasible (proposition 1). Therefore, to perform the stability analysis associated with the break points, we introduce the concept of pseudo flat-earth. We define pseudo flat-earth as a steady-state equilibrium in which all regions produce manufacturing. As there are various values of manufacturing shares that satisfy this stability criterion, we further qualify this concept in terms of inequality between shares. We thereby introduce maximum pseudo flat-earth as an economy where the share difference between the central region and the peripheral regions is at its largest, and the minimum pseudo flat-earth as an economy where the difference is at its smallest. Thirdly, we find that both the sustain and break points are convex on the degree of centrality. As the centrality of the network increases, therefore, the transport-cost thresholds for which full agglomeration and symmetric dispersion are no longer stable increase to a higher rate.

These results have important implications for policies aiming to increase territorial cohesion between regions by way of infrastructure investment (e.g., in terms of accessibility, which in our network framework corresponds to a reduction of network centrality). With a departure from a heterogeneous space, full cohesion between regions can be achieved only if all regions have the same relative position in terms of transport costs. Because in real geographical patterns some locations are better situated than others (i.e., have first-nature advantages), full cohesion is not possible unless transport costs are made equal across all regions (e.g., with infrastructure investments). Infrastructure policies should take this into account. And because in the real world it is impossible with infrastructure policies to transform a heterogeneous space into a homogeneous space like the "racetrack economy," policymakers should bear in mind that there might be situations where the first-nature advantages of some locations are so large that any feasible reduction in the centrality of network topology may not be enough to trigger a dispersion of economic activity. In other words, at existing levels of unit-transport costs, using infrastructure policy to reshape the economy's spatial configuration in terms of network centrality may not be enough to substantially change the distribution of economic activity. In the same vein, given network centrality, a reduction in unitary transport cost driven by lower market prices (e.g., as expected from a liberalization of labor and capital markets) or by technological improvements (e.g., vehicle fuel efficiency) may not be enough to overcome the privileged position of some locations.[11]

For our model, we have normalized the distance between regions by the radius of the circumference circumscribing the alternative topologies. Our results are therefore based on relative transport-cost differences, regardless of their absolute values. This allows us to disentangle the effects of changes in transport cost and in the network topology's degree of centrality. Nevertheless, it is clear that both elements end up configuring total transport costs. In fact, distance as cost in economics, and even in geography, is not represented solely by the obvious geographical distance between two locations. There are other measures of distance besides it: for instance, distance as travel time, generalized transport cost. All of these can be expressed in unit-distance terms (e.g., per kilometer, minute, dollars), and thus our distinction between these two elements can be maintained in empirical applications. Still other clear alternatives for the introduction of transport costs would be weighted networks, where distance matrices capture more sophisticated definitions of the cost function. This opens the possibility of using weighted links—e.g., distances weighted by generalized transport costs—within network theory (e.g., Opsahl et al. (2010)). In any case, it would be possible to simulate the effect on particular economies of transport policies aimed at reducing network centrality, thereby predicting whether such investments would in fact increase territorial cohesion. For example, as previously suggested, a country's network topology could be such that no investment whatsoever would change the existing geographical distribution

[11] Note that we do not favor a particular locational pattern, since the superiority of dispersion or agglomeration as a social outcome depends on transport costs and the alternative social functions defined (Charlot et al., 2006). Nevertheless, it is widely accepted that transport-infrastructure policies aim to increase territorial cohesion in terms of per-capita income. When promoting infrastructure improvements, therefore, public officials take for granted that a reduction in network centrality favors less-developed (peripheral) regions: i.e., their expected long-run outcome is territorial cohesion through reduction of income differentials.

of economic activity, thanks to a network so central that no sustain point could ever be reached.

Finally, for the multiregional model in this study we have considered only the canonical core-periphery model of Krugman (1991), but we could extend the analysis and introduce network theory in other simple models of the new economic geography, like the linear model by Ottaviano et al. (2002) or more elaborated models as the one with vertical linkages by Puga and Venables (1995).

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Appendix 1: Real wages in a multiregional economy when one region is agglomerating

When only one region—say, region 1—is agglomerating we set $\lambda_1 = 1$ and $\lambda_i = 0 \ \forall i \neq 1$ in equation (2), thereby obtaining:

$$y_1 = \frac{1 + (N-1)\mu}{N}; \, y_i = \frac{1-\mu}{N}, \, i = 2, \ldots, N.$$

Since by equation (11) the real wage of region 1 is equal to 1, we can substitute and get price indices (3):

$$g_1 = 1; \, g_i = \tau_{1i}, \, i = 2, \ldots, N.$$

Inserting the price indices and income we obtain nominal wages (4):

$$w_1 = 1; \, w_i = \left(\frac{1-\mu}{N} \tau_{1i}{}^{\sigma-1} + \frac{1+(N-1)\mu}{N} \tau_{1i}^{1-\sigma} + \sum_{j=1}^{N-2} \frac{1-\mu}{N} \tau_{1i}^{\sigma-1} \tau_{ij}{}^{1-\sigma} \right)^{1/\sigma}, \, i = 2, \ldots, N,$$

as well as real wages (5):

$$\omega_1^\sigma = 1; \, \omega_i^\sigma = \frac{1-\mu}{N} \tau_{1i}{}^{\sigma-1-\mu\sigma} + \frac{1+(N-1)\mu}{N} \tau_{1i}^{1-\sigma-\mu\sigma} + \frac{1-\mu}{N} \tau_{1i}^{-\mu\sigma} \sum_{j=1}^{N-2} \tau_{1i}^{\sigma-1} \tau_{ij}{}^{1-\sigma}, \, i = 2, \ldots, N$$

Appendix 2: Real wages in a multiregional economy with N = 4.

Following the same procedure as in Appendix 1 and setting $N = 4$, we obtain the following expressions of real wages:

$$\omega_1^\sigma = 1$$

$$\omega_2^\sigma = \frac{1-\mu}{4} \tau_{12}{}^{\sigma-1-\mu\sigma} + \frac{1+3\mu}{4} \tau_{12}^{1-\sigma-\mu\sigma} + \frac{1-\mu}{4} \tau_{12}^{-\mu\sigma} \left(\tau_{12}^{\sigma-1} \tau_{23}{}^{1-\sigma} + \tau_{12}^{\sigma-1} \tau_{23}{}^{1-\sigma} \right)$$

$$\omega_3^\sigma = \frac{1-\mu}{4} \tau_{13}{}^{\sigma-1-\mu\sigma} + \frac{1+3\mu}{4} \tau_{13}^{1-\sigma-\mu\sigma} + \frac{1-\mu}{4} \tau_{13}^{-\mu\sigma} \left(\tau_{13}^{\sigma-1} \tau_{32}{}^{1-\sigma} + \tau_{13}^{\sigma-1} \tau_{34}{}^{1-\sigma} \right)$$

$$\omega_4^\sigma = \frac{1-\mu}{4} \tau_{14}{}^{\sigma-1-\mu\sigma} + \frac{1+3\mu}{4} \tau_{14}^{1-\sigma-\mu\sigma} + \frac{1-\mu}{4} \tau_{14}^{-\mu\sigma} \left(\tau_{14}^{\sigma-1} \tau_{42}{}^{1-\sigma} + \tau_{14}^{\sigma-1} \tau_{43}{}^{1-\sigma} \right)$$

Appendix 3: Price-index derivative

Raising the price- ndex equation (3) to $1-\sigma$ yields:

$$g_i^{1-\sigma} = \lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma} .$$

Taking logs:

$$\left(1-\sigma\right) \log g_i = \ln \left(\lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma} \right).$$

Taking the derivative:

$$\left(1-\sigma\right) \frac{dg_i}{g_i} = \frac{\mathrm{d}\left(\lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma} \right)}{\lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma}}.$$

The denominator of the right-hand side is $g_i^{1-\sigma}$, which can be brought to the left side:

$$\left(1-\sigma\right) \frac{dg_i}{g_i} g_i^{1-\sigma} = \mathrm{d}\left(\lambda_i w_i^{1-\sigma} + \sum_{j=1}^{N-1} \lambda_j \left(w_j \tau_{ji} \right)^{1-\sigma} \right).$$

Totally differentiating the right-hand side, we obtain:

$$\left(1-\sigma\right) \frac{dg_i}{g_i} g_i^{1-\sigma} = w_i^{1-\sigma} d\lambda_i + \left(1-\sigma\right) \lambda_i w_i^{-\sigma} dw_i + \sum_{j=1}^{N-1} \left(\left(w_j \tau_{ji} \right)^{1-\sigma} d\lambda_j + \left(1-\sigma\right) \lambda_j \tau_{ji}^{1-\sigma} w_j^{-\sigma} dw_j \right),$$

and we arrive at equation (14):

$$\left(1-\sigma\right) \frac{dg_i}{g_i^{-\sigma}} = w_i^{1-\sigma} d\lambda_i + \left(1-\sigma\right) \lambda_i w_i^{-\sigma} dw_i + \sum_{j=1}^{N-1} \left(\left(w_j \tau_{ji} \right)^{1-\sigma} d\lambda_j + \left(1-\sigma\right) \lambda_j \tau_{ji}^{1-\sigma} w_j^{-\sigma} dw_j \right).$$

Appendix 4: Wage derivative

Raising wage equation (4) to σ yields:

$$w_i^{\sigma} = y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma}.$$

Taking logs:

$$\sigma \log w_i = \log\left(y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma} \right).$$

Taking derivatives:

$$\sigma \frac{dw_i}{w_i} = \frac{d\left(y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma} \right)}{y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma}}.$$

The denominator of the right-hand side is w_i^{σ}, so it can be brought to the left side:

$$\sigma \frac{dw_i}{w_i} w_i^{\sigma} = d\left(y_i g_i^{\sigma-1} + \sum_{j=1}^{N-1} y_j g_j^{\sigma-1} \tau_{ij}^{1-\sigma} \right).$$

Totally differentiating the right-hand side, we get equation (15):

$$\sigma \frac{dw_i}{w_i} w_i^{\sigma} = g_i^{\sigma-1} dy_i + (\sigma - 1) y_i g_i^{\sigma-2} dg_i + \sum_{j=1}^{N-1} \left(g_j^{\sigma-1} \tau_{ij}^{1-\sigma} dy_j + (\sigma - 1) y_j \tau_{ij}^{1-\sigma} g_j^{\sigma-2} dg_j \right)$$

Appendix 5: Real-wage derivative

Totally differentiating equation (5) yields:

$$d\omega_i = dw_i g_i^{-\mu} - \mu w_i dg_i g_i^{-\mu-1}.$$

Multiplying both sides by g_i^{μ}:

$$g_i^{\mu} d\omega_i = dw_i - \mu w_i dg_i g_i^{-1},$$

results in equation (16):

$$g_i^{\mu} d\omega_i = dw_i - \mu w_i \frac{dg_i}{g_i}.$$