

Leptogenesis: beyond the minimal type I seesaw scenario

Thomas Hambye

Service de Physique Théorique, Université Libre de Bruxelles, B-1050
Brussels, Belgium

Departamento de Física Teórica, Universidad Autónoma de Madrid and
Instituto de Física Teórica IFT-UAM/CSIC, Cantoblanco, E-28049 Madrid,
Spain

E-mail: thambye@ulb.ac.be

New Journal of Physics **14** (2012) 125014 (39pp)

Received 3 July 2012

Published 18 December 2012

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/14/12/125014

Abstract. Recent evidence for neutrino masses has established the leptogenesis mechanism as a very natural possible explanation for the baryon asymmetry of the Universe. The explicit realization of this mechanism depends on the neutrino mass model considered. If the right-handed type-I seesaw model of neutrino masses is certainly the most straightforward, it is not the only natural one, especially in the framework of explicit GUT realizations of the seesaw. In this paper, we review in detail the various seesaw scenarios that can implement the leptogenesis mechanism successfully, beyond the paradigm of the pure standard type-I seesaw model. This includes scenarios based on the existence of scalar triplets (type-II), of fermion triplets (type-III) as well as mixed seesaw frameworks.



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1. Introduction

The origin of the baryon asymmetry of the Universe constitutes one of the most fascinating enigmas of contemporary physics. It definitely deserves an explanation! Such an explanation has not been found within the Standard Model (SM) of elementary interactions. But nothing prevents it from being found beyond the SM. Regardless of the baryogenesis enigma, there are many reasons to believe that the SM does not constitute a complete theory anyway. In particular, recent evidence for the neutrino masses (the clearest laboratory evidence one has at the moment for new physics) requires the existence of new states beyond the SM, still to be found experimentally. Especially attractive are neutrino mass models where these masses are induced by seesaw states, right-handed neutrinos ('type-I'), scalar triplets ('type-II') or fermion triplets ('type-III'). It is quite an intriguing fact that in most models of neutrino masses, and especially seesaw models, the interactions at the origin of neutrino masses do induce a baryon asymmetry in the Universe, through the leptogenesis mechanism [1]. With the exception of one scenario, the one that involves only a single scalar triplet, see below, this is basically unavoidable, unless one adopts specific assumptions and symmetries which restrict the interactions of these new states, or unless one adopts specific cosmological scenarios (e.g. a low reheating temperature at the end of the inflation era, so that the heavy states at the origin of the neutrino masses have never been produced in the thermal bath of the Universe). In particular,

in most neutrino mass models the interactions at the origin of the neutrino masses generically do satisfy the three Sakharov conditions for creating a baryon asymmetry. They break lepton number (hence baryon number together with the SM ‘sphalerons’), they break C and CP and are generally expected to be out-of-equilibrium during various epochs of the thermal history of the Universe. Moreover, even if there is unfortunately no one-to-one connection between the size of the neutrino masses and the baryon asymmetry produced, see below, and even if the model at the origin of the neutrino masses could easily lead to too a small baryon asymmetry, it is also intriguing that the size of the neutrino masses is in the ballpark of the values we need to easily induce a large enough baryon asymmetry. This constitutes too good an opportunity not to study it in detail. For the type-I seesaw model this has been reviewed at length in quite a few references, see e.g. [2] in this volume and [3–8]. This review is dedicated to leptogenesis in the framework of the seesaw models beyond the minimal type-I seesaw framework, that is to say in the type-II and type-III models, as well as in mixed seesaw scenarios, type-I + type-II, type-I + type-III, etc.

In section 2 we begin by introducing and motivating the various seesaw models. Section 3 introduces a few model-independent general considerations on the CP asymmetries. Section 4 (5) discusses in detail how leptogenesis comes into play from the decay of a type-III (type-II) seesaw state, giving in particular many explanations on how the ‘efficiency factor’ behaves in these frameworks. Many comparisons are done between these models and with the type-I seesaw model. Finally, section 6 considers mixed seesaw scenarios, in particular the ones which come naturally out of minimal Grand Unified Theory (GUT) models.

2. The various seesaw models and their motivation

The most straightforward and, in many respects, the most ‘natural’ mechanism one can conceive to generate naturally small neutrino masses is the well-known seesaw mechanism. This mechanism lays their origin in the existence of an L -violating dimension-5 Weinberg operator interaction

$$\mathcal{L}_{\text{eff}} = \frac{c_{\alpha\beta}^{d=5}}{\Lambda} (\bar{L}_\alpha^c \tilde{H}^*) (\tilde{H}^\dagger L_\beta), \quad (1)$$

with $L = (\nu_\alpha, l_\alpha^-)^T$, $H = (H^+, H^0)^T$, $\tilde{H} = i\tau_2 H^*$ and Λ an energy scale. This operator generates naturally small Majorana neutrino masses, in accordance with data, if the Λ scale is large,

$$\mathcal{M}_{\nu_{\alpha\beta}} = -\frac{v^2}{2} c_{\alpha\beta}^{d=5} / \Lambda, \quad (2)$$

with $v = 246$ GeV. The Weinberg interaction is the only gauge invariant dimension 5 interaction one can write out of SM fields. Therefore, prior to the discovery of the neutrino masses, one could have expected that its effect, that is to say the neutrino masses, would be the first laboratory manifestation one would observe of new physics at a high scale (far above the electroweak scale). This turns out to have happened (even if of course this discovery does not necessarily imply that the scale Λ is far beyond the electroweak scale, since the coefficients $c_{\alpha\beta}^{d=5}$ also come into play).

There is in principle an infinite number of new possible states which could induce such an operator with large scale Λ . Nevertheless there are only three basic ways to generate it at tree level, the most straightforward way: from the exchange of one or several right-handed

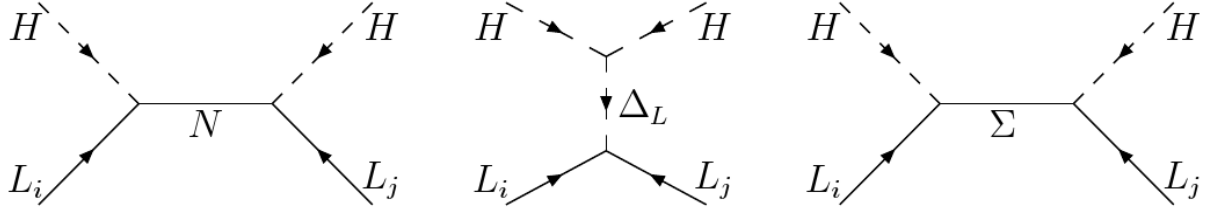


Figure 1. The three basic seesaw diagrams which can induce naturally small neutrino masses.

Table 1. Lagrangian, neutrino mass contribution and CP-asymmetry definition for each seesaw model (in matrix notation, a summation over the lepton flavor and heavy state indices is implicit). We define Σ^c as $(\Sigma^c)_{ij} = (\Sigma_{ji})^c$ with $\psi^c \equiv C\bar{\psi}^T$ and $i, j = 1, 2$ the $SU(2)_L$ indices. For the scalar triplet $\Gamma_\Delta = \Gamma_{\bar{\Delta}} = \Gamma(\Delta_L \rightarrow \bar{L}\bar{L}) + \Gamma(\Delta_L \rightarrow HH)$.

	Type of seesaw model		
	Type-I	Type-II	Type-III
Seesaw states	N	$\Delta_L = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$	$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$
Kin. term	$i\bar{N}\not{\partial}N$	$\text{Tr}[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)]$	$\text{Tr}[\bar{\Sigma}i\not{\partial}\Sigma]$
Mass term	$-\frac{1}{2}\text{Tr}[\bar{N}m_N N^c + \bar{N}^c m_N^* N]$	$-m_\Delta^2 \text{Tr}[\Delta_L^\dagger \Delta_L]$	$-\frac{1}{2}\text{Tr}[\bar{\Sigma}m_\Sigma \Sigma^c + \bar{\Sigma}^c m_\Sigma^* \Sigma]$
Interactions	$-\tilde{\phi}^\dagger \bar{N} Y_N L - \bar{L} Y_N^\dagger N \tilde{\phi}$	$-L^T Y_\Delta C i\tau_2 \Delta_L L + \mu \tilde{H}^T i\tau_2 \Delta_L \tilde{H}$	$-\tilde{\phi}^\dagger \bar{\Sigma} \sqrt{2} Y_\Sigma L - \bar{L} \sqrt{2} Y_\Sigma^\dagger \Sigma \tilde{\phi}$
ν masses	$\mathcal{M}_\nu^N = -\frac{v^2}{2} Y_N^T \frac{1}{m_N} Y_N$	$\mathcal{M}_\nu^\Delta = 2Y_\Delta v_{\Delta_L} = Y_\Delta \mu^* \frac{v^2}{m_\Delta^2}$	$\mathcal{M}_\nu^\Sigma = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{m_\Sigma} Y_\Sigma$
CP asym.	$\varepsilon_N \equiv \frac{\Gamma(N \rightarrow LH) - \Gamma(N \rightarrow \bar{L}\bar{H})}{\Gamma(N \rightarrow LH) + \Gamma(N \rightarrow \bar{L}\bar{H})}$	$\varepsilon_\Delta \equiv 2 \frac{\Gamma(\bar{\Delta}_L \rightarrow LL) - \Gamma(\Delta_L \rightarrow \bar{L}\bar{L})}{\Gamma_\Delta + \Gamma_{\bar{\Delta}}}$	$\varepsilon_\Sigma \equiv \frac{\Gamma(\Sigma \rightarrow LH) - \Gamma(\bar{\Sigma} \rightarrow \bar{L}\bar{H})}{\Gamma(\Sigma \rightarrow LH) + \Gamma(\bar{\Sigma} \rightarrow \bar{L}\bar{H})}$

neutrinos, scalar triplets or fermion triplets, see figure 1.¹ The relevant Lagrangian for each seesaw model is given in table 1, together with the neutrino mass matrix it gives. In the fermionic cases the lepton number violation comes from the coexistence of Majorana masses and Yukawa couplings, whereas for the scalar triplet option it comes from the coexistence of the coupling to two Higgs doublets ($L = 0$) and two lepton doublets ($L = 2$). In all cases the masses of the new states are not protected by any SM symmetry, hence could be naturally much larger than the electroweak scale, leading to a large Λ effective scale, and hence to naturally small neutrino masses, as observed. The seesaw states with such large mass scales are typically expected in the GUT framework, another motivation for the seesaw framework.

Among the various possibilities, the right-handed neutrino option (type-I) is the most considered, probably motivated by the intuition that the SM left-handed neutrinos, as all other fermions in the SM, should have right-handed partners. This is furthermore supported by the

¹ This can be understood easily from the fact that both L and H are doublets, and with two doublets coupling to the heavy states one can do only singlets or triplets of $SU(2)_L$: a right-handed neutrino [9–13] (i.e. LH singlet), a scalar triplet [14–17] (an LL or HH triplet), or a fermion triplet [18] (an LH triplet). As for the scalar singlet combination it does not give any neutrino masses because the HH antisymmetric singlet contribution vanishes.

usual left–right [19–22] and $SO(10)$ GUT frameworks where right-handed neutrinos come in automatically, being associated with the SM fermions within the same representation (in particular in the 16_F representation of $SO(10)$ together with a full generation of SM fermions). This is nevertheless not the only natural option, and not even necessarily the most natural one in these models. One issue is that, if one assumes right-handed neutrinos with Majorana masses, these masses constitute new scales whose origin and values must be explained. In these frameworks such scales are protected by the gauge symmetries, and therefore find their origin in the breaking of these gauge symmetries (for example of $U(1)_{B-L}$ or $SU(2)_R$ which are contained in $SO(10)$). In the $SO(10)$ context the right-handed neutrinos masses can be induced in a renormalizable way (if one does not want to induce them from higher dimensional operators, which would require the introduction of yet a new mass scale, beyond the GUT scale, or would need to rely on unknown Planck scale effects). This can be done only by introducing a 126 scalar representation². This scalar representation has the particularity of containing both a right-handed scalar triplet Δ_R , whose vacuum expectation value (VEV) gives a mass to the right-handed neutrino, and a left-handed triplet Δ_L , whose VEV induces left-handed neutrino masses through the type-II seesaw mechanism. From the point of view of $SO(10)$ a mixed type-I + type-II neutrino mass model is consequently a natural option. Similarly, from the point of view of $SU(5)$, right-handed neutrinos are not especially more natural than any other extra fermions, since they are difficult to associate with any SM fermion within the same $SU(5)$ representation. One could introduce one or several of them as singlet representation of $SU(5)$ but they could come as well from an adjoint representation of $SU(5)$, for example. The 24 representation of $SU(5)$ has the particularity to involve one fermion with the SM quantum numbers of a right-handed neutrino, but also to involve a fermionic triplet as in the type-III model. This framework offers therefore the opportunity to generate the neutrino masses from a single representation (like the type-II seesaw with only one scalar triplet). In other words $SU(5)$ does not point less towards a type-I + type-III seesaw generation of neutrino masses than it does towards a pure type-I framework.

In the next three sections we consider the possibility of inducing successful leptogenesis in the framework of the type-II and type-III seesaw models, as well as in seesaw frameworks involving several types of seesaw. For each framework we will explicitate how the two main ingredients of leptogenesis come into play, the CP asymmetry and the efficiency factor.

3. A few model-independent considerations on the CP asymmetries

Along the leptogenesis scenario of baryogenesis, a lepton asymmetry is created in the Universe when the heavy seesaw states decay at a temperature of order of their masses. The CP asymmetry is nothing but the average ΔL created each time a heavy state decays. This is simply given by the difference of the decay width to leptons and antileptons of these states, divided per their total decay width, multiplied by the number of leptons created in the decay of a single heavy state, see table 1. Note that we will start by discussing the CP asymmetry for the total lepton number, rather than the asymmetry for each flavor lepton number separately. That is to say we will not take into account the possible effect of the Yukawa interactions of the charged leptons, which differentiate these equations. For seesaw states masses below 10^{12-13} GeV this

² As is well known, as long as one assumes right-handed neutrinos in the 16_F their masses can come only from the scalar 126 representation in the product $16_F \times 16_F = 126 + 120 + 10$.

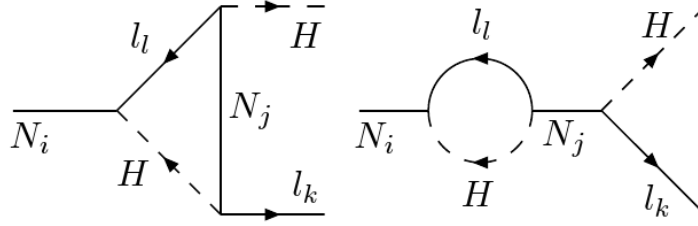


Figure 2. One-loop diagrams contributing to the CP-asymmetry in the type-I case or type-III case (replacing $N_{i,j}$ by $\Sigma_{i,j}$).

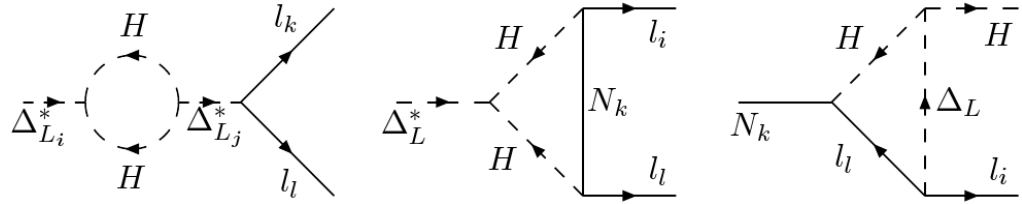


Figure 3. Diagram contributing to the asymmetry created by the decay of a Δ_L if there are several scalar triplets or an N . The third diagram also contributes to the asymmetry of a N in the presence of a heavier scalar triplet.

may induce both conceptual and quantitative differences. Various flavor effects and associated references will be discussed in sections 4.3 and 5.5. We begin by listing a number of important properties of the CP-asymmetries (some of them remain valid in the flavor case too, see below):

- (i) At lowest order, CP-violation can manifest itself only at the level of the interference between the tree level and one-loop decay width contributions. The various one-loop diagrams which could contribute in the three seesaw framework as well as in various mixed seesaw frameworks are given in figures 2 and 3. As is well known, the CP-violating imaginary part of the couplings must be associated with the absorptive part of the loop diagram. The sum of the masses of the particles involved in the loop must consequently be smaller than the mass of the decaying state.
- (ii) CP violation requires moreover (at least for the unflavored case) that the lepton number violation be explicit in the one loop diagram, i.e. the diagrams must involve intermediate and final states with different lepton numbers. For example, as shown in figure 3, in the type-II seesaw model one has to go through a HH intermediate state before going to the two lepton final state. In the type-I or type-III model one has an interference between an LH state and an $\bar{L}H^*$ state, figure 2.
- (iii) A one-loop non zero asymmetry requires flavor ‘breaking’ or more exactly at least two sources of flavor ‘breaking’, i.e. two heavy states with unequal couplings to leptons and/or scalar bosons. If there is only one heavy state or if there are several with exactly the same couplings to leptons and scalar bosons, the CP-asymmetry vanishes. In this case each coupling to leptons is automatically accompanied by its complex conjugate in the CP asymmetry. This is the reason why the pure type-II seesaw model with a single scalar triplet, which is the only one which can give two or three light neutrino masses (as required experimentally) from a single heavy state, gives a vanishing asymmetry, even though it *a priori* satisfies all Sakharov conditions, see figure 3 and section 5.3.

- (iv) Both heavy states must be non-degenerate in mass, otherwise the sum over all intermediate and final flavors gives a real combination of couplings leading to a vanishing asymmetry.
- (v) In the limit where the virtual seesaw state mass is much larger than that of the decaying state, i.e. if the heavy states have a hierarchical mass spectrum (as quarks and leptons in the SM), the contribution of this virtual state can be fully parameterized by its contribution to the neutrino mass matrix (neglecting the charged lepton mass effects). The CP-asymmetry is proportional to it. This stems from the simple argument that the CP asymmetry comes from the absorptive part of the loop diagram, where particles inside the loop are on-shell. Therefore, to calculate the absorptive part, the heavy intermediate state can be integrated out, even though there is a loop diagram. As a result the CP asymmetry is necessarily proportional to the neutrino mass matrix induced by the heavy states, since in the effective Lagrangian picture all the L violation is encoded in the single Weinberg operator, no matter what is the heavy state [23]. This can be clearly seen from the one-loop diagrams, which all have as subdiagram one of the seesaw diagrams of figure 1. Moreover, since both tree level and one-loop diagrams, put together, form a closed loop, the L violation from the heavy state must be accompanied by L violation from the decaying heavy state, which in order to close the loop can come only through the combination of Yukawa couplings producing the neutrino masses. This implies that the asymmetry must be proportional to the neutrino mass matrix induced by the decaying state too³. Therefore, for the three possible types of decaying particles one gets a CP asymmetry quadratic in neutrino mass matrix contributions and inversely proportional to $v^4 \Gamma_D$, where Γ_D is the total decay width of the decaying state. This means that an extra M_D^3 factor must come, with M_D the mass of the decaying state. For instance, for the three seesaw cases, we get the simple form:

$$\varepsilon_N = -\frac{3}{32\pi^2} \frac{m_N^3}{\Gamma_N v^4} \text{Im}[(\mathcal{M}_\nu^N)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger], \quad (3)$$

$$\varepsilon_\Delta = -\frac{1}{16\pi^2} \frac{m_\Delta^3}{\Gamma_\Delta v^4} \text{Im}[(\mathcal{M}_\nu^\Delta)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger] \quad [30, 81], \quad (4)$$

$$\varepsilon_\Sigma = -\frac{1}{32\pi^2} \frac{M_\Sigma^3}{\Gamma_\Sigma v^4} \text{Im}[(\mathcal{M}_\nu^\Sigma)_{\beta\alpha} (\mathcal{M}_\nu^H)_{\alpha\beta}^\dagger] \quad [7] \quad (5)$$

with \mathcal{M}^N , \mathcal{M}^Σ , \mathcal{M}^Δ and \mathcal{M}^H the neutrino mass matrix induced by the decaying N , the decaying Σ , the decaying Δ_L and whatever heavier state in the loop diagram, respectively. For the type-I and -III cases, the cubic $M_{N,\Sigma}^3$ has an important consequence. Given the fact that $\Gamma_{N,\Sigma}$ goes parametrically as $M_{N,\Sigma}^2 \mathcal{M}_\nu^{N,\Sigma}/v^2$, the CP-asymmetry is linear in the mass of the lighter state (and linear in neutrino masses), which, as noticed long ago [7, 24–28], is at the origin of the fact that leptogenesis in the hierarchical case requires heavy states, with upper bounds precisely determined in [28] and [7] for the type-I and -III case respectively, see below. For the type-II case this is more complicated because the total decay width receives contributions from two different decays (to LL and HH) but there also exists a lower bound on m_Δ [29, 30], see below.

³ Note nevertheless that obviously the decaying state, being on-shell, cannot be integrated-out, which means that the numerical prefactor in front of both neutrino mass matrices in the CP asymmetry has no reason to be the same in the three cases. Both the kinetic and mass terms of the decaying state contribute and $SU(2)_L$ index contractions can be different.

Below we will also give, in specific models, the CP asymmetries one gets when the virtual states are not much heavier than the decaying particle.

4. Leptogenesis from the decay of a fermion triplet

We begin by considering the decay of a fermion triplet [7, 31, 32, 36–38]. This case is similar to the one of the decay of a right-handed neutrino except for one important difference: triplets can be thermalized by gauge interactions. For clarity, it is natural, as we will do here, to consider the fermion triplet possibility before the scalar triplet one because the latter option involves not only gauge interactions but also several decays, whose interplay introduces extra subtleties.

4.1. Hierarchical case

In this subsection, we consider the case of a fermion triplet much lighter than the other seesaw states. In this case the discussion is greatly simplified for three main reasons. Firstly, as discussed in point (v) of the previous section, the CP-asymmetry takes in this case a universal form, no matter what the heavy state, equation (5). Secondly, the effects of the lepton number violating processes involving the heavier states can be neglected. These processes are either Boltzmann suppressed if these heavy states are on-shell in these processes, or suppressed by the power of the masses of these heavy states if they are off-shell. Thirdly the lepton asymmetry produced by the decay of these heavier states can be generally ignored because it is washed-out by the interactions of the lightest one before this state becomes out of thermal equilibrium (although this is not always the case, especially in the flavor case, see below).

A fermion triplet, as a right-handed neutrino, decays only through Yukawa couplings. Its neutral component is a Majorana right-handed neutrino and has consequently the same decay width as a function of these Yukawa couplings

$$\Gamma_{\Sigma^0} = \Gamma(\Sigma^0 \rightarrow LH) + \Gamma(\Sigma^0 \rightarrow \bar{L}\bar{H}) = \frac{1}{8\pi} m_{\Sigma} |Y_{\Sigma i}|^2. \quad (6)$$

As for the charged states, the (right-handed) Σ^{\pm} and the (left-handed) conjugated states of the Σ^{\pm} form a Dirac spinor ψ^{\pm} with, from $SU(2)_L$ invariance, the same decay width as in equation (6). The CP-asymmetry, given in equation (5), turns out to be three times smaller than for a right-handed neutrino, equation (3). By $SU(2)_L$ invariance, it is equal for each of the three triplet components, Σ^0 , ψ^+ , ψ^- .

The lepton asymmetry produced by the decay of a heavy triplet is given by the average ΔL produced per decay, ε_{Σ} , times the number density of triplets which have decayed, n_{Σ}/s , times the ‘efficiency factor’, η ,

$$\frac{n_L}{s} = \varepsilon_{\Sigma} \eta \frac{n_{\Sigma}}{s} \Big|_{T \gg m_{\Sigma}}, \quad (7)$$

with $n_{\Sigma} = n_{\Sigma^0} + n_{\psi^+} + n_{\psi^-}$ the total number of triplets (both particles and antiparticles). The η factor results from the integration of the Boltzmann equations, see below. It takes its maximum value, unity, if all decays occur out-of-equilibrium and if there is no wash-out effect from $\Delta L \neq 0$ processes. It is smaller otherwise. In this way the baryon asymmetry produced is

$$\frac{n_B}{s} = 3 \frac{135\zeta(3)}{4\pi^4 g_*} C_{L \rightarrow B} \varepsilon_{\Sigma} \eta = -0.0041 \varepsilon_{\Sigma} \eta, \quad (8)$$

where $s = g_*(2\pi^2/45)T^3$ is the entropy density and $C_{L \rightarrow B} = -28/79$ [39] is the L to B asymmetry conversion factor one gets in the SM from the sphaleron processes. The factor of 3 in the last equation accounts for the sum of the asymmetries of the three triplet states. For the situation where, beside the seesaw states, there are only SM states in the thermal bath of the Universe at $T \sim M_\Sigma$ when the triplets decay, the number of relativistic degrees of freedom g_* is equal to 106.75.

As mentioned above, in the type-I seesaw case the CP asymmetry, equation (3), involves the same number of Yukawa couplings and seesaw state mass as the combination $m_N \mathcal{M}_\nu / v^2$. As a result the CP-asymmetry of equation (3) turns out to be bounded from above as [28]

$$\varepsilon_N \leq \frac{3}{8\pi} \frac{M_N}{v^2} (m_{\nu_3} - m_{\nu_1}) = \frac{3}{8\pi} \frac{M_N}{v^2} \frac{\Delta m_{\text{atm}}^2}{m_{\nu_3} + m_{\nu_1}}, \quad (9)$$

which gives the bound

$$m_N \gtrsim 6 \times 10^8 \text{ GeV}, \quad (10)$$

with $\Delta m_{\text{atm}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$, the atmospheric neutrino squared mass splitting (see e.g. [40]).

Exactly the same expression holds for the triplet, except for the $1/3$ factor in the CP-asymmetry

$$\varepsilon_\Sigma \leq \frac{1}{8\pi} \frac{M_\Sigma}{v^2} (m_{\nu_3} - m_{\nu_1}) = \frac{1}{8\pi} \frac{M_\Sigma}{v^2} \frac{\Delta m_{\text{atm}}^2}{m_{\nu_3} + m_{\nu_1}}. \quad (11)$$

This factor is nevertheless compensated by the factor 3 in equation (8). To calculate the lower bound it gives on m_Σ , one cannot simply assume a situation where the efficiency is unity, as one can for a right-handed neutrino decay. Gauge scatterings give a suppression of the efficiency which, to get this lower bound, must be calculated from the Boltzmann equations. Since different components of the fermion triplet can be involved in the same gauge scattering, it is convenient to consider the Boltzmann equation for the sum of all three triplet components, $Y_\Sigma \equiv n_\Sigma/s$. One gets [7]

$$sH z \frac{dY_\Sigma}{dz} = - \left(\frac{Y_\Sigma}{Y_\Sigma^{\text{eq}}} - 1 \right) \gamma_D - 2 \left(\frac{Y_\Sigma^2}{Y_\Sigma^{2\text{eq}}} - 1 \right) \gamma_A, \quad (12)$$

$$sH z \frac{dY_{B-L}}{dz} = -\gamma_D \varepsilon_\Sigma \left(\frac{Y_\Sigma}{Y_\Sigma^{\text{eq}}} - 1 \right) - \frac{Y_{B-L}}{Y_l^{\text{eq}}} \left(\frac{\gamma_D}{2} + 2\gamma_\Sigma^{\text{sub}} \right). \quad (13)$$

In these equations $z \equiv m_\Sigma/T$ is the evolution variable, $H = 1.66\sqrt{g_*}T^2/m_{\text{Pl}}$ is the Hubble constant and the suffix eq denotes the equilibrium value. Y_l^{eq} stands for the equilibrium number of a two degrees of freedom fermion ($Y_l^{\text{eq}} = Y_\nu$). The γ_i are the reaction densities for the various processes (i.e. the number of reactions occurring per unit volume per unit time). $\gamma_D = n_\Sigma^{\text{eq}} \Gamma_\Sigma K_1(z)/K_2(z)$ is the decay/inverse decay reaction rate. $\gamma_\Sigma^{\text{sub}}$ comes from the $\Delta L = 2$ scattering processes, $LL \leftrightarrow H^*H^*$ and $LH \leftrightarrow \bar{L}H^*$, see [7]. γ_A is the gauge scattering reaction density from the $\Sigma\bar{\Sigma}' \leftrightarrow GG'$, $f\bar{f}$, $H\bar{H}$ processes, with $G^{(\prime)}$ all possible gauge bosons. Summing over the 12 SM fermions we get the following reduced cross section [7, 32, 41]:

$$\hat{\sigma}_A = \frac{6g^4}{72\pi} \left[\frac{45}{2}\beta - \frac{27}{2}\beta^3 - (9(\beta^2 - 2) + 18(\beta^2 - 1)^2) \ln \frac{1+\beta}{1-\beta} \right], \quad (14)$$

with $\beta = \sqrt{1 - 4/x}$, $x = s/m_\Sigma^2$,

$$\gamma(a b \leftrightarrow 1 2) = \frac{T}{64 \pi^4} \int_{s_{\min}}^{\infty} ds \sqrt{s} \hat{\sigma}(s) K_1\left(\frac{\sqrt{s}}{T}\right), \quad (15)$$

and $\hat{\sigma} \equiv 2 s^{-1} \lambda^2 [s, m_a^2, m_b^2] \sigma(s)$ with $\lambda[a, b, c] = \sqrt{(a - b - c)^2 - 4bc}$. Since the gauge scatterings do not violate L , they appear only in the Y_Σ Boltzmann equation, equation (12).

In the absence of the γ_A term the efficiency is exactly the same as for the right-handed neutrinos, except that all interaction rate terms are multiplied by 3, in equation (13). This holds also for the inverse decay term since a lepton has three times more probability to encounter a Higgs particle to produce a heavy triplet (i.e. it can inverse decay in three ways instead of one). For a given value of

$$\tilde{m} = \frac{1}{2} \sum_j |Y_{\Sigma_{1j}}|^2 \frac{v^2}{m_\Sigma} = 8\pi \Gamma_\Sigma \frac{v^2}{m_\Sigma}, \quad (16)$$

the efficiency suppression due to inverse decay is therefore the same as the one obtained for a right-handed neutrino with an \tilde{m} value three times larger.

Let us now discuss in detail what happens when one adds the γ_A gauge term. The gauge processes do not bring any new unknown parameter. They are fully known as a function of m_Σ . On the contrary they render leptogenesis more predictive since they can thermalize the decaying triplet, giving a baryon asymmetry independent of the initial condition on the triplet population. At large $T \gg m_\Sigma$, gauge interactions are not in thermal equilibrium because dimensionally, from the Planck scale dependence of the Hubble constant, $\gamma_A/n_\Sigma^{\text{eq}} H$ necessarily goes like m_{Pl}/T . Similarly at $T \ll m_\Sigma$ one has [32]

$$\hat{\sigma}_A = c_s \beta + c_p \beta^3 + \mathcal{O}(\beta^5), \quad (17)$$

$$\gamma_A = \frac{m_\Sigma T^3}{32\pi^3} e^{-2m_\Sigma/T} \left[c_s + \frac{3T}{2m_\Sigma} \left(c_p + \frac{c_s}{2} \right) + \mathcal{O}(T/m_\Sigma)^2 \right], \quad (18)$$

where $c_s = 111g^4/8\pi$ and $c_p = 51g^4/8\pi$ refer to the s and p wave components. The gauge reaction rate is doubly Boltzmann suppressed, since it involves two external heavy particles, which gives a thermalization rate $\gamma_A/n_\Sigma^{\text{eq}} H \propto e^{-m_\Sigma/T} m_{\text{Pl}}/\sqrt{m_\Sigma T}$ which has a single Boltzmann suppression. As a result $\gamma_A/n_\Sigma^{\text{eq}} H$ reaches a maximum value at $T \sim m_\Sigma$, which turns out to be

$$\frac{\gamma_A}{n_\Sigma^{\text{eq}} H} \sim 2 \times \frac{10^{14} \text{ GeV}}{m_\Sigma}. \quad (19)$$

Therefore as soon as m_Σ is below $\sim 10^{14} \text{ GeV}$ the triplet thermalizes and the asymmetry produced is independent of the initial condition on the triplet population, and the smaller m_Σ is the more the triplet is thermalized. Actually one could believe that, as a consequence, leptogenesis quickly becomes hopeless below this value. As was shown in [7, 29, 30] this is nevertheless not the case at all, because, for $T < m_\Sigma$, the gauge interaction rate, which is doubly Boltzmann suppressed, drops more quickly than the decay/inverse decay γ_D rate which is simply Boltzmann suppressed. Therefore gauge processes decouple faster than the γ_D term, still leaving the possibility to create a substantial asymmetry from the decay once the gauge scatterings become slower than the decay/inverse decay processes (or slightly before, as we will see below). All this can be seen in figure 4, which shows the reaction rates for various values of m_Σ and \tilde{m} . Note that the $\gamma/n_\Sigma^{\text{eq}} H$ rates are the relevant ones for equation (12) whereas the

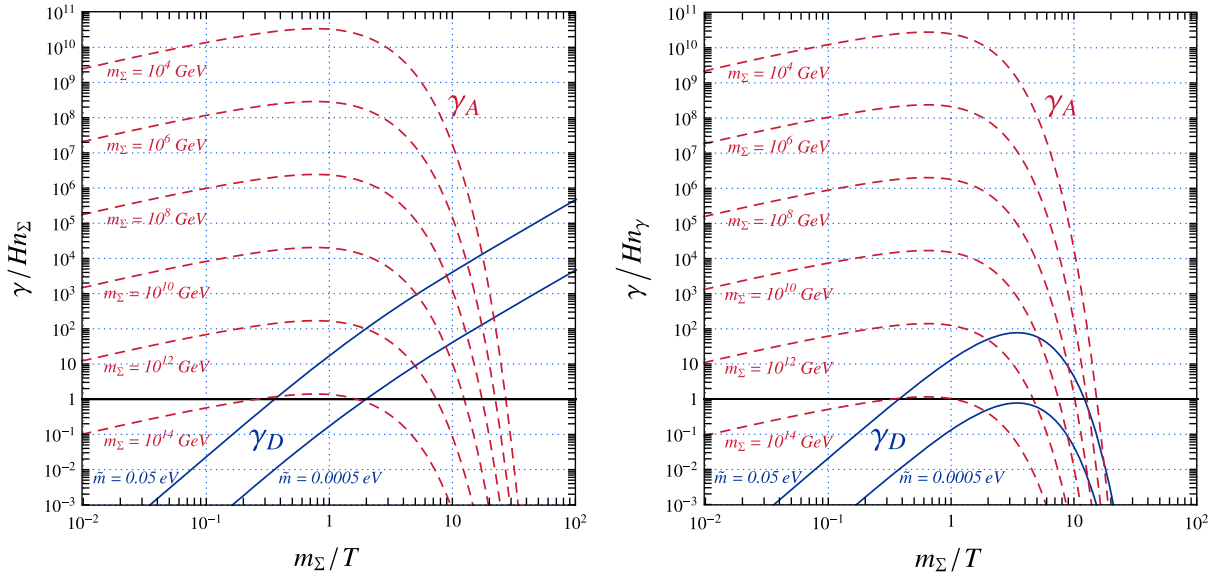


Figure 4. $\gamma/Hn_{\Sigma}^{\text{eq}}$ and γ/Hn_{γ} thermalization rates, as relevant for equations (12) and (13) respectively. $\gamma_D/Hn_{\Sigma,\gamma}^{\text{eq}}$ depends only on \tilde{m} , whereas $\gamma_A/Hn_{\Sigma,\gamma}^{\text{eq}}$ depends only on m_{Σ} .

$\gamma/n_{\gamma}^{\text{eq}}H$ are the relevant ones for equation (13). If they are smaller than unity the corresponding interactions do not thermalize in the corresponding Boltzmann equation and can be neglected. Figure 5 gives the corresponding asymmetry we get, as a function of z , once we integrate the Boltzmann equations. To better understand these results let us proceed step by step in the following way:

- To begin with, let us neglect all terms in the Boltzmann equations except the first term of both Boltzmann equations, i.e. no gauge scattering term, no inverse decay washout term and no $\Delta L = 2$ scattering terms. In this case, no matter what the value of γ_D is, by putting the first Boltzmann equation into the second one, one obviously gets $Y_L = \int \varepsilon_{\Sigma} (dY_{\Sigma}/dz) dz = \varepsilon_{\Sigma} Y_{\Sigma}^{\text{eq}}|_{T \gg m_{\Sigma}}$, that is to say an efficiency equal to unity.
- Adding now the gauge term, clearly as long as $\gamma_A/n_{\Sigma}^{\text{eq}}H > 1$, Y_{Σ} tracks Y_{Σ}^{eq} . Moreover as long as $4\gamma_A > \gamma_D$, the gauge term dominates the thermalization of the triplet (i.e. the triplets scatter before they decay), see equation (12). In this case putting equation (12) in equation (13) one simply gets

$$\frac{dY_L}{dz} = \varepsilon_{\Sigma} \frac{dY_{\Sigma}^{\text{eq}}}{dz} \frac{\gamma_D}{4\gamma_A}. \quad (20)$$

This equation remains correct until a value of $z = z_A$ where either $\gamma_A/n_{\Sigma}^{\text{eq}}H$ gets below one (in the case the gauge scattering is no longer in thermal equilibrium) or $4\gamma_A$ gets below γ_D . As a result the lepton asymmetry contains two pieces: a lepton asymmetry produced when $z \lesssim z_A$, which is suppressed by the $\gamma_D/4\gamma_A$ factor, and an asymmetry produced afterwards, when $z \gtrsim z_A$. The latter is not suppressed by any wash-out effect, i.e. all triplets left at $z = z_A$ produce leptons without any suppression, but it is obviously suppressed by the fact that the number of triplets left at $z = z_A$ is Boltzmann suppressed. Summing these two

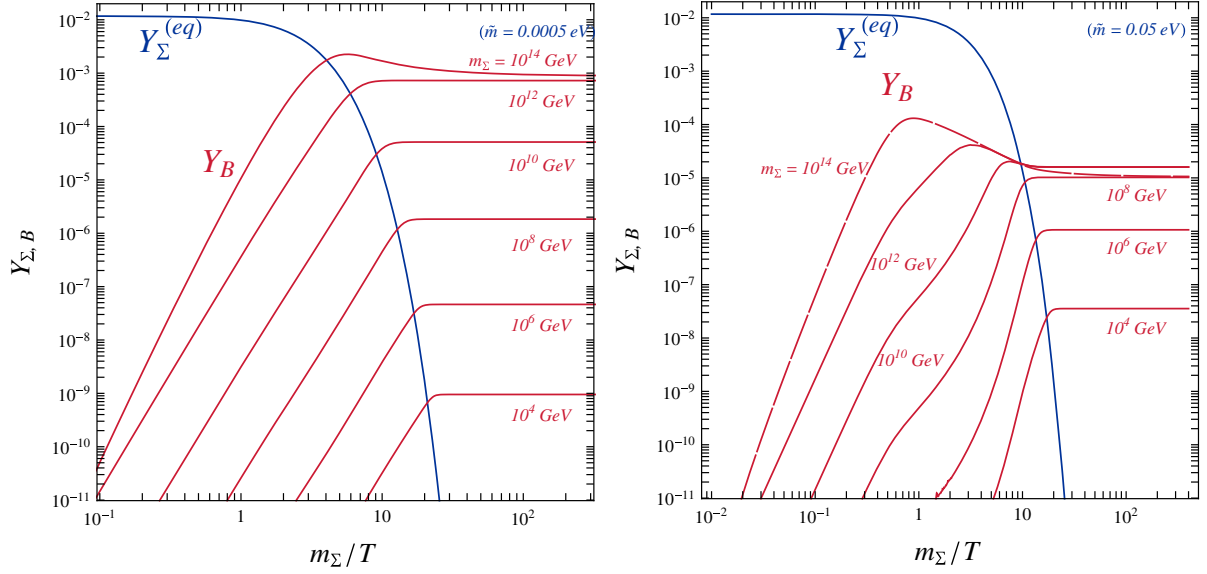


Figure 5. Y_{Σ}^{eq} and Y_B as a function of $z = m_{\Sigma}/T$, for various values of m_{Σ} and $\tilde{m} = 0.0005$ eV (left panel) and $\tilde{m} = 0.05$ eV (right panel). The CP-asymmetry has been taken equal to unity. For the first panel one lies always in the gauge regime, except for $m_{\Sigma} = 10^{14}$ GeV where the suppression effect comes from the $\Delta L = 2$ scattering processes. For $\tilde{m} = 0.05$ eV instead, one lies in the gauge regime below $\sim 10^7$ GeV, see figure 6. Above this mass value the Yukawa regime leads to an asymmetry more and more independent of m_{Σ} , except above $\sim 10^{14}$ GeV (due to the $\Delta L = 2$ scattering processes here too).

contributions one gets

$$Y_L \simeq \varepsilon_{\Sigma} \int_{z_{\text{in}}}^{z_A} \frac{dY_{\Sigma}^{\text{eq}}}{dz} \frac{\gamma_D}{4\gamma_A} + \varepsilon_{\Sigma} Y_{\Sigma}^{\text{eq}}(z_A). \quad (21)$$

It is important to stress that unless \tilde{m} is quite small, $\gamma_D/(n_{\Sigma}^{\text{eq}} H)$ (which is not a Boltzmann suppressed quantity!) is larger than one when γ_A gets Boltzmann suppressed, at $z \sim$ a few. Thus, the value of z_A is determined by the condition $4\gamma_A = \gamma_D$ rather than by the condition $\gamma_A/(n_{\Sigma}^{\text{eq}} H) = 1$. This means that the gauge scatterings become irrelevant already before they get out-of-equilibrium, i.e. for $z > z_A$ the triplets decay before they scatter even if their scattering rate is still fast. That is precisely the reason why the suppression effect of the efficiency from the gauge scatterings is not as dramatic as we could have expected at first sight. Note also that in the integrand of equation (21) the various Boltzmann suppressions of the various terms cancel each other, leaving an integrand which scales as $1/T^3$. As a result we get

$$Y_L \simeq \varepsilon_{\Sigma} Y_{\Sigma}^{\text{eq}}(z_A) \int_{z_{\text{in}}}^{z_A} \frac{z^3}{z_A^3} dz + \varepsilon_{\Sigma} Y_{\Sigma}^{\text{eq}}(z_A) \simeq \varepsilon_{\Sigma} Y_{\Sigma}^{\text{eq}}(z_A) (z_A/4 + 1). \quad (22)$$

Since z_A is a few, that is to say lies between ~ 1 for $m_{\Sigma} \simeq 10^{14}$ GeV and ~ 25 for $m_{\Sigma} \simeq 1$ TeV, this means that a good part of the asymmetry is produced just before $z = z_A$ and the rest just after. This can be seen in figure 5(a) which, with $\varepsilon_{\Sigma} = 1$, gives Y_L of order

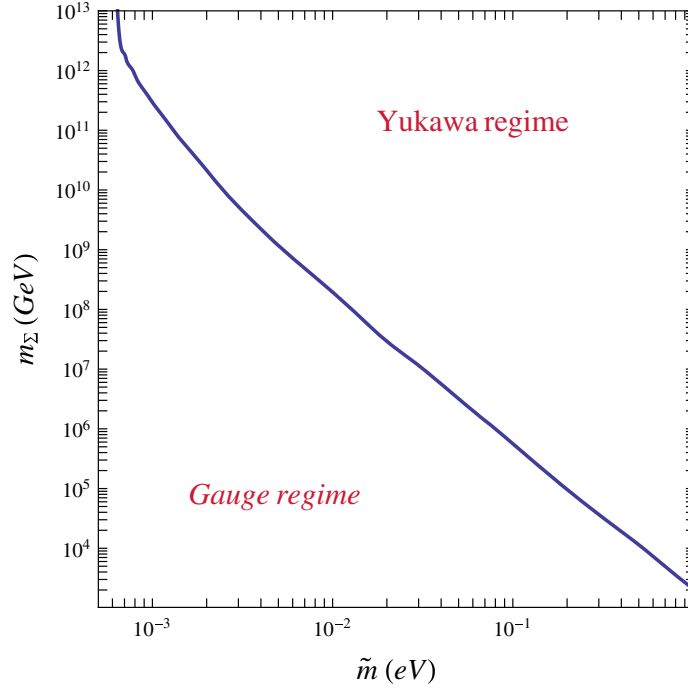


Figure 6. Gauge versus Yukawa regime. In the gauge regime the inverse decay washout term can be neglected and the asymmetry is given by equation (22). In the Yukawa regime instead, the gauge annihilation terms can be neglected and one gets an efficiency which has the same behavior as for a decaying right-handed neutrino. Only close to the transition line, which gives the maximum asymmetry for fixed m_Σ , can both terms be relevant.

$Y_\Sigma^{\text{eq}}(z = z_A)$ (and similarly in figure 5(b) for low enough m_Σ). In all cases note that the smaller \tilde{m} is, the smaller γ_D/γ_A is, the larger z_A is, the more $Y_\Sigma^{\text{eq}}(z = z_A)$ is Boltzmann suppressed, and hence the more Y_L is Boltzmann suppressed.

- (c) Now let us add the $\Delta L = 2$ scattering term, $\gamma_\Sigma^{\text{sub}}$. As in the type-I framework these terms are proportional to the seesaw mass scale, m_Σ , and, since they involve the neutrino mass Feynman diagram, to the neutrino mass scale. For neutrino masses in agreement with the data (i.e. below the $\sim \text{eV}$ scale) their effect is small for m_Σ below a scale which depends on \tilde{m} . For a hierarchical or inverse hierarchical neutrino mass spectrum, with $\tilde{m} \lesssim 10^{-1} \text{ eV}$ there is no effect below $\sim 10^{14} \text{ GeV}$ whereas, for $\tilde{m} \sim 1 \text{ eV}$, there is a sizeable effect down to 10^{12} GeV . For a quasi-degenerate spectrum of light neutrino the mass where they come to have a sizeable effect scales as the inverse of the neutrino mass scale.
- (d) Let us now add the $\propto Y_{B-L}\gamma_D$ inverse decay washout term in equation (13). What is its effect? Well, the relevant quantity for the inverse decay washout is the $\gamma_D/(n_\gamma H)$ thermalization rate (which unlike $\gamma_D/(n_\Sigma^{\text{eq}} H)$ is Boltzmann suppressed at $z > 1$!). Therefore, if at $z = z_A$, the inequality $\gamma_D/(n_\gamma H) < 1$ holds, clearly equation (22) remains valid. This is true clearly if $\tilde{m} \lesssim 3 \times 10^{-3} \text{ eV}$ since in this case $\gamma_D/(n_\gamma H)$ never reaches unity. More generally this is also true for larger values of \tilde{m} , since at $z = z_A > 1$, $\gamma_D/(n_\gamma H)$

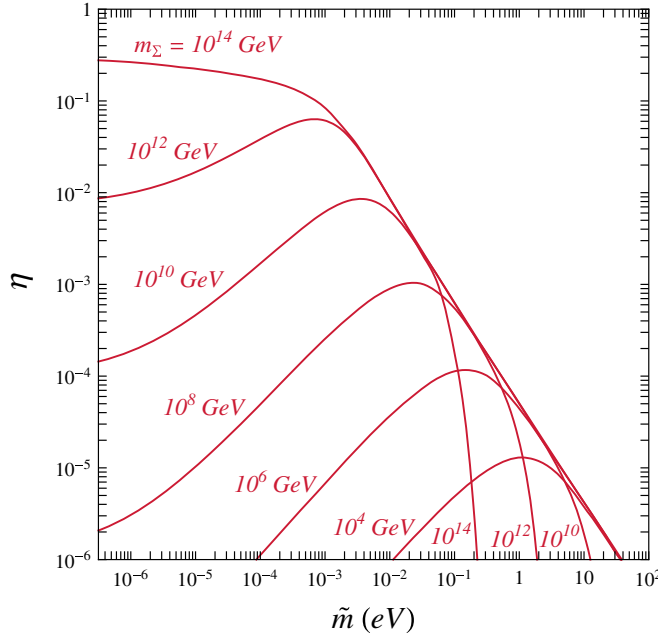


Figure 7. Efficiency as a function of \tilde{m} for various values of m_Σ .

is already Boltzmann suppressed. The value of $\tilde{m} \equiv \tilde{m}_A$ which gives

$$\left. \frac{\gamma_D}{n_\gamma H} \right|_{z=z_A} = 1 \quad (23)$$

is given in figure 6. It delimits the lower \tilde{m} ‘gauge regime’ region where inverse decays can be neglected, so that equation (22) holds, from the larger \tilde{m} ‘Yukawa regime’ region where inverse decays are important. These two regions present totally different behavior of the efficiency as a function of \tilde{m} , see figure 7 (a similar plot can be found in [31, 36]). In the gauge regime, as explained above, the efficiency increases with \tilde{m} . In the Yukawa regime instead the decay/inverse decay processes dominate and the efficiency decreases with \tilde{m} (similar to what happens in the type-I scenario). Consequently a maximum efficiency is obtained at the junction between both regimes, for $\tilde{m} \simeq \tilde{m}_A$. Note that at this maximum the gauge scattering processes have still nevertheless a suppression effect, but it is already moderate, of order $\simeq 3$ –5. Consequently, as figure 7 shows, at this maximum point the efficiency still depends on m_Σ . The gauge scattering effect, and hence the m_Σ dependence, becomes totally negligible for a value of \tilde{m} equal to about ten times \tilde{m}_A , see figure 7. At this point even if the gauge scatterings still have an effect at early times, they do not affect the final asymmetry produced. This is similar to the known feature of the type-I leptogenesis scenario, that in the strong washout regime (i.e. for $\tilde{m} \gg 10^{-3}$ eV), the final lepton asymmetry produced does not depend on the reheating temperature as soon as this temperature is somewhat larger than the temperature where the inverse decays decouple. For $\tilde{m} > \tilde{m}_A$ one is therefore in a regime totally dominated by the Yukawa interaction, as with right-handed neutrinos⁴.

⁴ This separation between Yukawa and gauge regimes has already been quantified analytically in [36], taking the different prescription for defining \tilde{m}_A that $\gamma_D/(n_\gamma H) = 1$ when $\gamma_A/(n_\Sigma^{\text{eq}} H) = 1$. This results in values of \tilde{m}_A that,

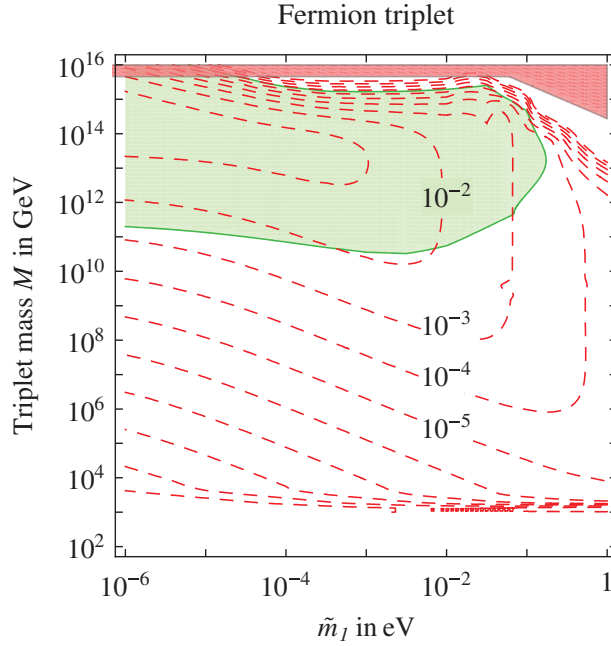


Figure 8. Efficiency for a decaying fermion triplet, from [32]. Note that this figure differs from a similar one obtained in [7] by the fact that Sommerfeld enhancement effects for non-relativistic triplets in gauge scatterings have been taken into account. These effects modify the final baryon asymmetry by less than 20%.

All the properties above can also be seen in figure 8 which shows a contour plot of the efficiency one obtains, as a function of \tilde{m} and m_Σ . Look in particular at figure 6 to understand the change of efficiency behavior in figures 6 and 8. These figures also show that the efficiency quickly drops for m_Σ below a few TeV or above $\sim 10^{12-14}$ GeV (depending on the value of \tilde{m}). In the first case this is due to the sphalerons which get out-of-equilibrium at the electroweak scale. The value of the decoupling temperature one gets for $m_h = 125$ GeV is $T_c = (139.5 \pm 2.5)$ GeV, see figure 3 of [42]. It is a good approximation to simply use a sharp step function for this decoupling. In the second case this is due to the fact that above $\sim 10^{12-14}$ GeV the Yukawa induced $\Delta L = 2$ scattering processes (which are proportional to m_Σ) cease to have a negligible wash-out effect.

Figure 8 also shows the region of parameter space that, from the CP asymmetry, one gets in the hierarchical case, equation (5), gives a baryon asymmetry large enough to account for the observed one. It gives the lower bound [7, 31, 32]

$$m_\Sigma \gtrsim 3 \times 10^{10} \text{ GeV}, \quad (24)$$

for low values of m_Σ , can be orders of magnitudes larger. For example for $m_\Sigma = 10^6$ GeV our prescription gives $\tilde{m}_A \simeq 0.1$ eV, to be compared with the value $\tilde{m}_A = 25$ eV one gets adopting the prescription of [36]. Figure 5(b) shows in this case that the lepton asymmetry is produced when $\gamma_A \simeq \gamma_D$, which occurs at $z \simeq 13$, when $\gamma_A \simeq \gamma_D$, see figure 4, rather than when $\gamma_A/(n_\gamma^{\text{eq}} H) = 1$ which occurs at $z = 20$ (where all quantities are much more Boltzmann suppressed). Nevertheless, this does not affect the numerical values of Y_L obtained in [36] with which we agree. The prescription of [36] holds for a situation where the effects of gauge scatterings are already very suppressed, see figure 7.

which, due to the gauge scatterings, is about two orders of magnitude larger than for the decay of a right-handed neutrino, equation (10). Taking into account the fact that the CP asymmetry is linear in m_Σ one can easily understand this bound from the discussion above. This bound is obtained for $\tilde{m} \simeq \tilde{m}_A$, which for this value of the triplet mass has the value 2.2×10^{-3} with $z_A = 7.2$. Thus, in this case the gauge scatterings suppress the asymmetry produced by a factor $\sim Y_\Sigma^{\text{eq}}(z = z_A)/Y_\Sigma^{\text{eq}}(z \ll 1) = 1.3 \times 10^{-2}$, that is to say by two orders of magnitude. That must be compensated by a two orders of magnitude m_Σ increase in the CP asymmetry, equation (11). As for an upper bound on m_Σ , it is of order 10^{16} GeV and is not far from the one we would get from requiring perturbative Yukawa coupling (if the decaying triplet gives a neutrino mass of the order of the atmospheric value 0.05 eV).

Finally note that in the supersymmetric context, where the fermion triplet comes together with its scalar superpartner, all the discussion above remains unchanged, up to the factor of order unity for the CP-asymmetries and the efficiency factor. Equation (24) holds in this case up to a factor of order unity (which to our knowledge has not been determined exactly in the literature). This bound implies a leptogenesis scale above the upper bound that in the supersymmetric context, the gravitino implies on the reheating temperature, $T_{\text{reh}} \lesssim 10^{6-10}$ GeV [43–45]. This basically means that triplets with a mass large enough to insure successful leptogenesis would never have been created in the thermal bath of the Universe. The bound of equation (24) can nevertheless be relaxed by orders of magnitude if there is more than one triplet fermion and if they have a similar mass, see next subsection. One could also think about possible additional effects from supersymmetry breaking soft terms involving the triplets. For a recent extensive review on type-I seesaw soft leptogenesis [86–93] see [94]. To our knowledge, for the type-III seesaw framework, this possibility has also not been explored in detail in the literature.

4.2. Resonant case: multiple fermion triplets with quasi-degenerate spectrum

The bounds of equations (11) and (24) are strictly valid only in the limit where the mass of the heaviest seesaw states goes to infinity (with neutrino masses fixed). For a less hierarchical case, for example if the seesaw state masses differ by a factor of 10, these bounds can be relaxed for particular structures of the Yukawa and seesaw state mass matrix, see [7]. In this reference this has been discussed for the right-handed neutrino case. This discussion also applies to the fermion triplet case. For a quasi-degenerate spectrum instead all the discussion changes because in this case the self-energy diagram involving several triplets, figure 2, becomes resonant, through the propagator of the virtual triplet (for references on resonant leptogenesis in the type-I model, see e.g. [46–58]). With several fermion triplets the CP-asymmetry of, say, Σ_1 , takes the form [7]

$$\varepsilon_{\Sigma_1} = - \sum_j \frac{3}{2} \frac{m_{\Sigma_1}}{m_{\Sigma_j}} \frac{\Gamma_{\Sigma_j}}{m_{\Sigma_j}} I_j \frac{V_j - 2S_j}{3}, \quad (25)$$

with

$$S_j = \frac{m_{\Sigma_j}^2 \Delta m_{1j}^2}{(\Delta m_{1j}^2)^2 + m_{\Sigma_1}^2 \Gamma_{\Sigma_j}^2}, \quad V_j = 2 \frac{m_{\Sigma_j}^2}{m_{\Sigma_1}^2} \left[\left(1 + \frac{m_{\Sigma_j}^2}{m_{\Sigma_1}^2} \right) \log \left(1 + \frac{m_{\Sigma_1}^2}{m_{\Sigma_j}^2} \right) - 1 \right] \quad (26)$$

and

$$I_j = \frac{\text{Im}[(Y_\Sigma Y_\Sigma^\dagger)_{1j}^2]}{|Y_\Sigma Y_\Sigma^\dagger|_{11} |Y_\Sigma Y_\Sigma^\dagger|_{jj}}, \quad \Delta m_{ij}^2 = m_{\Sigma_j}^2 - m_{\Sigma_i}^2. \quad (27)$$

In the hierarchical limit both the self energy and vertex diagram contributions take the value unity, $S_j = V_j = 1$. In the extreme opposite case where the mass splitting would be equal to the decay width of the intermediate triplet, $m_{\Sigma_j} - m_{\Sigma_1} \simeq \Gamma_{\Sigma_j}$, the resonance is maximum, $S_j \simeq m_{\Sigma_j} / \Gamma_{\Sigma_j}$, leading to a CP asymmetry which can be as large as 1/2 [7]. Together with the CP-asymmetry of the second quasi-degenerate triplet, one obtains a maximum CP-asymmetry equal to unity, which can lead to successful leptogenesis provided the following lower bound is satisfied [32]:

$$m_{\Sigma} > 1.6 \text{ TeV}. \quad (28)$$

This bound results from the interplay of the gauge scatterings, which for $m_{\Sigma} \sim 1 \text{ TeV}$ allows a production of the asymmetry only for $z > 15\text{--}20$, and of the sphaleron decoupling scale which forbids any sizable production below it.

Finally note that for the type-III framework, as for the type-I framework, there exist upper bounds on neutrino masses from the constraint of successful leptogenesis, but none of them appear to be very relevant, when compared to the other bounds which exist already on the neutrino mass scale, for example the direct experimental bound from the Mainz experiment, $m_{\nu}^{\text{max}} < 2.2 \text{ eV}$ [33]. For instance, it is true that, as in the type-I case [6, 7, 34, 35], an ‘infinite’ hierarchy between the triplet seesaw states does lead to an upper bound [7]

$$m_{\nu} < 0.12 \text{ eV}. \quad (29)$$

This bound results from two effects which suppress the asymmetry when the neutrino mass scale increases. Firstly, the upper bound on the CP-asymmetry decreases with this mass scale, equation (11). Secondly, in the type-III model, as in the type-I model, the \tilde{m} parameter is bounded from below by the value of the lightest neutrino mass

$$\tilde{m} \geq m_{\nu}^{\text{min}}. \quad (30)$$

This inequality results from the fact that \tilde{m} , equation (16), and the neutrino mass matrix seesaw formula, table 1, differ only by a flavor index summation and absolute values on the Yukawa couplings. However given the structure of the seesaw formula, $\mathcal{M}_{\alpha\beta} \propto \sum_{N_i} Y_{N_{i\alpha}} Y_{N_{i\beta}} / m_{N_i}$, it does not appear at all easy to get a quasi-degenerate spectrum of light neutrinos, which applies for this bound, from a hierarchical spectrum of heavy triplets. That would require a precise cancellation between the hierarchy of heavy state masses and the hierarchy of Yukawa couplings. To get a quasi-degenerate spectrum of light neutrinos it is much easier to assume that the heavy states would be themselves quasi-degenerate. In this case, as discussed above, a resonance effect occurs, and there is no more upper bound on neutrino masses for successful leptogenesis [7] (and this even without talking about flavor effects which further relax these bounds, even for hierarchical states [59]).

4.3. Effect of flavor

Above we have made the assumption that the three Boltzmann equations for each $L_{e,\mu,\tau}$ flavor lepton number can be reduced to a single Boltzmann equation, for the sum of them, i.e. for the total lepton number. As is well known this is fully justified for a decaying right-handed neutrino with a mass above 10^{12-13} GeV . However, below this scale this is not necessarily justified because the τ SM Yukawa coupling interaction is in thermal equilibrium. As a result one gets two Boltzmann equations, for $Y_{L_{\tau}}$ and $Y_{L_{\mu}} + Y_{L_e}$, which summed do not reduce to the single total lepton number one. Similarly for T below $\sim 10^9 \text{ GeV}$ the μ Yukawa coupling is

also in thermal equilibrium and one gets three independent $Y_{L_{e,\mu,\tau}}$ Boltzmann equations. For a discussion of the flavor effects in the type-I seesaw model, see e.g. [5, 59–62]. For earlier references on the subject, see [24, 63]. Flavor effects bring in general a moderate correction to the lepton flavor ‘aligned’ case, but not always. Two clear examples are in particular:

- (a) *Flavor hierarchy.* A substantially less suppressed efficiency can be obtained if the lightest triplet has in the same lepton flavor channel a large CP asymmetry and a small decay width (i.e. typically with $\varepsilon_{\Sigma}^{l_j} \gg \varepsilon_{\Sigma}^{l_i}$ and $\Gamma(\Sigma \rightarrow L_i H) \gg \Gamma(\Sigma \rightarrow L_j H)$). Note that the neutrino mass matrix CP-violating phases, which have no effect on leptogenesis in the unflavored case, can now contribute [64–66].
- (b) *N_2 leptogenesis.* If the second lightest seesaw state produces an asymmetry, this asymmetry eventually cannot be erased by the lightest one. This does not allow the reduction of the leptogenesis scale but allows a ‘spectator’ triplet well below the bounds above. This situation can already work in the unflavored case, obviously if all couplings of the lightest states are suppressed. In the flavor case this can work also [67–69] if some of the couplings of the lightest seesaw state are large, as long as the coupling to the flavor in which the asymmetry has been created is suppressed.

In all cases the leptogenesis scale bound obtained in the hierarchical case, equation (10), remains essentially unchanged.

For fermion triplets the situation is different, once again due to the gauge scatterings. This depends on the regime we consider:

- (a) *Yukawa regime.* In this regime, as explained above, the gauge scatterings have no effect in the unflavored case. However they can have an effect in the flavor case. To understand this let us consider for example a hierarchical seesaw spectrum with a triplet mass equal to the bound value of equation (24). As already mentioned above, for this value of the mass, the Yukawa regime holds for $\tilde{m} \geq \tilde{m}_A = 2.2 \times 10^{-3}$ eV and the maximum value of the asymmetry (giving the bound of equation (24)) is obtained for $\tilde{m} \simeq \tilde{m}_A$. The gauge scatterings suppress the efficiency by a factor of $\sim Y_{\Sigma}^{\text{eq}}(z = z_A) / Y_{\Sigma}^{\text{eq}}(z \ll 1) \sim 10^{-2}$ with $z_A = 7.2$ the value of z where $4\gamma_A$ becomes smaller than γ_D . Now, as also explained above, for $\tilde{m} \gtrsim \tilde{m}_A$ ($\tilde{m} \gg \tilde{m}_A$) the gauge scatterings have a moderate (no) effect on the efficiency, not because they are out-of-thermal equilibrium but because they are slower than the decays/inverse decays, $\gamma_D > 4\gamma_A$. Now suppose that playing with flavor we render the inverse decay term harmless. Thus, in this case one has no more suppression from the inverse decay but still one from the (flavor blind) gauge scattering processes [36], since they are in thermal equilibrium. As a result one understands that, for $\tilde{m} \gtrsim \tilde{m}_A$, one can get a larger asymmetry than in the unflavored case, but still a gauge scattering suppressed result (unless one takes such a large value of \tilde{m} that $z_A \lesssim 1$). For numerical examples of this kind see figure 5 of [36]. A consequence of this behavior we want to point out here is that the absolute lower bound on the mass of the decaying triplet for a hierarchical spectrum is essentially the one we get in the type-I case, equation (10),

$$m_{\Sigma} > \sim 10^9 \text{ GeV}, \quad (31)$$

but this requires large values of \tilde{m} of order at least ~ 10 eV. For $\tilde{m} = 0.05$ eV one can reduce the bound of equation (24) only by a factor ~ 3 . The same effect could be used in principle for a quasi-degenerate triplet spectrum.

- (b) *Gauge regime.* As explained above, in this regime all the suppression of the asymmetry comes from the (flavor blind) gauge scatterings and the inverse decay term can be neglected. Therefore there are no flavor effects, as discussed in [36]. In particular the lower bound on the leptogenesis scale of equation (28) still holds, because this bound is obtained deeply in the gauge regime. The latter statement can be understood from the fact that, for example, for a one TeV triplet mass, sphalerons decouple at $z \sim 7.5$, and at this temperature γ_D is much smaller than γ_A (unless \tilde{m} is really large, above $\sim \text{keV}$).

4.4. Consequences for the Large Hadron Collider

Given the smallness of neutrino masses, a seesaw state with a mass accessible at colliders has typically many suppressed Yukawa couplings, far too small to produce them in large enough numbers. However, a fermion triplet, unlike a right-handed neutrino, can be produced at colliders via gauge interactions, via Drell–Yan pair production. This has been analyzed in [70–78]. For a luminosity of 10 fb^{-1} and 14 TeV energy one should be able to produce at the Large Hadron Collider (LHC) about 30 (10^4) pairs of triplets with mass equal to 1 TeV (250 GeV). There are three main types of processes one can look at, see for instance [73]: (a) $pp \rightarrow \bar{\nu} W^+ W^\pm l^\mp \rightarrow 4\text{jets} + \text{missing energy} + \text{a charged lepton}$, the signal with largest rate but largest background too, (b) lepton number violating signature, $pp \rightarrow \Sigma^\pm \Sigma^0 \rightarrow l_1^\pm l_2^\pm Z W^\pm$, with smaller rate but much smaller background and (c) lepton flavor violating processes like $pp \rightarrow l_1 \bar{l}_2 4\text{jets}$, with a rate similar to the lepton number violating signals but with larger background. All possibilities are currently under study at the LHC and the most promising channels are probably the lepton violating ones. An integrated luminosity of 10 fb^{-1} (100 fb^{-1}) with 14 TeV of energy should allow to see this signal for m_Σ up to $\sim 0.8 \text{ TeV}$ (1.2 TeV). This is below the absolute leptogenesis scale lower bound of equation (28). To establish the fermion triplet leptogenesis framework at LHC consequently appears hopeless. At most one could still observe a lighter fermion triplet playing no role for leptogenesis (along the N_2 leptogenesis framework mentioned above). Note that in this case one can take advantage of the fact that at such low scales, neutrino masses typically require small Yukawa couplings, which means slow triplet decay. As a result low seesaw scale models lead to displaced vertices [73]. One gets $\tau_\Sigma \simeq 0.3 \text{ mm} (m_\Sigma / 100 \text{ GeV})^2 (0.05 \text{ eV} / \tilde{m})$. The ATLAS and CMS detectors could in principle detect such kinds of displaced vertices as long as they are above $\sim 0.1 \text{ mm}$. In this way it could also be that a light fermion triplet is observed with Yukawa interactions that would be too large to be compatible with the N_2 leptogenesis scenario (i.e. leading to too much washout of any preexisting L asymmetry). Such an observation could exclude any baryogenesis scenario occurring at a higher scale.

5. Leptogenesis from the decay of a scalar triplet

A scalar triplet, as a fermion triplet, undergoes gauge interactions which can thermalize it and reduce the efficiency. In addition it has two properties which distinguish it from the fermion seesaw frameworks:

- A scalar triplet has two totally different types of decay, to a pair of leptons or to a pair of Higgs doublets. At tree level one obtains

$$\Gamma(\Delta_L \rightarrow \bar{L} \bar{L}) = \frac{m_\Delta}{8\pi} \text{Tr}[Y_\Delta Y_\Delta^\dagger] = B_L \Gamma_\Delta, \quad (32)$$

$$\Gamma(\Delta_L \rightarrow HH) = \frac{1}{8\pi} \frac{|\mu_\Delta|^2}{m_\Delta} = B_H \Gamma_\Delta, \quad (33)$$

with Γ_Δ the total decay width of the scalar triplet, and $B_{L,H}$ the branching ratios ($B_L + B_H = 1$). At one loop, from the definition of the CP asymmetry in table 1, and taking into account CPT symmetry one gets

$$\Gamma(\bar{\Delta}_L \rightarrow LL) = \Gamma_\Delta(B_L + \varepsilon_\Delta/2), \quad (34)$$

$$\Gamma(\Delta_L \rightarrow \bar{L}\bar{L}) = \Gamma_\Delta(B_L - \varepsilon_\Delta/2), \quad (35)$$

$$\Gamma(\bar{\Delta}_L \rightarrow \bar{H}\bar{H}) = \Gamma_\Delta(B_H - \varepsilon_\Delta/2), \quad (36)$$

$$\Gamma(\Delta_L \rightarrow HH) = \Gamma_\Delta(B_H + \varepsilon_\Delta/2). \quad (37)$$

The fact that the scalar triplet has two different decays and that lepton number is violated only if both types of decay coexist has important consequences for the efficiency. As we will see below this allows no suppression of the efficiency, even at low scale where the gauge scatterings are very fast⁵.

- A scalar triplet is not a self-conjugate state. Therefore a triplet–antitriplet asymmetry can be generated, leading to an extra Boltzmann equation, see below. Actually for a fermion triplet one could believe at first sight that an asymmetry could be generated between the charged triplets of opposite signs. However, this is not the case, at least in the $SU(2)_L$ symmetric limit, since in this limit one should get the same result as for the neutral triplet component. The latter cannot have any $\Sigma^0 - \bar{\Sigma}^0$ asymmetry since it is a Majorana fermion. More explicitly, in this $SU(2)_L$ limit the equalities $\Gamma(\Sigma^+ \rightarrow \nu\phi^+) = \Gamma(\Sigma^- \rightarrow l\phi^0)$ and $\Gamma(\bar{\Sigma}^+ \rightarrow \bar{\nu}\phi^-) = \Gamma(\bar{\Sigma}^- \rightarrow \bar{l}\phi^{0*})$ hold. Therefore if one calls these two decay widths ‘ a ’ and ‘ b ’ respectively, the lepton number is violated because $a \neq b$ but the number of positively charged triplets remains equal to the number of negatively charged triplets (simply because both components change by the same $\propto (a+b)$ amount). And there is no $\Sigma^+ - \Sigma^{-c}$ asymmetry either, since these two components mix through their Dirac mass. Only for the scalar triplet can we have an ‘unprotected’ particle–antiparticle asymmetry for the seesaw states. The extra Boltzmann equation it implies has important consequences, see below.

Let us start with a few general formulas which apply for a scalar triplet. The lepton asymmetry produced by the decay of a heavy triplet is given by the general formula

$$\frac{n_L}{s} = \varepsilon_\Delta \eta \frac{n_\Delta + n_{\bar{\Delta}}}{s} \Big|_{T \gg m_\Delta}, \quad (38)$$

so that the baryon asymmetry produced is

$$\frac{n_B}{s} = 3 \frac{135\zeta(3)}{4\pi^4 g_\star} C_{L \rightarrow B} \varepsilon_\Delta \eta = -0.0041 \varepsilon_\Delta \eta. \quad (39)$$

The factor of 3 in the last equation accounts for the fact that each triplet component produces an asymmetry ε_Δ . Again the η efficiency factor is defined in such a way that it takes the value unity if all decays occur out-of-equilibrium and if there is no wash-out effect from $\Delta L \neq 0$ processes.

⁵ Actually in the type-I and -III framework one can also have a similar structure with similar properties [79] if one assumes an approximately conserved L symmetry, in the case one has two types of Yukawa couplings, L -breaking and L -violating. For the scalar triplet this is generic.

The total decay width of a scalar triplet can be written as

$$\Gamma_{\Delta} = \frac{1}{8\pi} \frac{m_{\Delta}^2 \tilde{m}_{\Delta}}{v^2} \frac{B_L + B_H}{\sqrt{B_L B_H}}, \quad (40)$$

where

$$\tilde{m}_{\Delta}^2 \equiv \text{Tr}[\mathcal{M}_{\nu}^{\Delta\dagger} \mathcal{M}_{\nu}^{\Delta}] = \sum_i (m_{\nu_i}^{\Delta})^2 = \text{Tr}[Y_{\Delta}^{\dagger} Y_{\Delta}] |\mu_{\Delta}|^2 \frac{v^4}{m_{\Delta}^4} \quad (i = 1, 2, 3), \quad (41)$$

with $m_{\nu_i}^{\Delta}$ the eigenvalues of the neutrino mass matrix induced by the scalar triplet. As a result the following bound applies

$$\Gamma_{\Delta} \geq \frac{1}{4\pi} \frac{m_{\Delta}^2 \tilde{m}_{\Delta}}{v^2}. \quad (42)$$

This bound is saturated for $B_L = B_H = 1/2$. Equation (40) divided by the Hubble constant can be written as

$$\frac{\Gamma_{\Delta}}{H} \Big|_{T=m_{\Delta}} = \frac{\sqrt{\sum (m_{\nu_i}^{\Delta})^2}}{0.001 \text{ eV}} \frac{1}{2} \frac{B_L + B_H}{\sqrt{B_L B_H}}. \quad (43)$$

It means that if the triplet contributes for ν masses above the $\sim 10^{-3}$ eV level, the triplet decays anyway in thermal equilibrium. It shows also that for fixed contribution to the neutrino masses, the further we are from the $B_L = B_H$ case, the more the triplet is in thermal equilibrium. In this case the triplet decay/inverse decay processes can be in thermal equilibrium even if this triplet contributes very little to the neutrino masses. The reason why one gets the sum of the neutrino mass squared, rather than the lightest neutrino mass as for the type-I and -III cases, equation (30), stems from the fact that $\mathcal{M}_{\nu}^{\Delta}$ could be any generic matrix, while \mathcal{M}_{ν}^N has rank 1.

For a hierarchical spectrum of heavy states, the CP asymmetry given in equation (4) is bounded from above. Rewriting it as

$$\varepsilon_{\Delta} = -\frac{1}{2\pi} \frac{m_{\Delta}}{v^2} \sqrt{B_L B_H} \frac{\text{Im Tr}[\mathcal{M}_{\nu}^{\Delta} \mathcal{M}_{\nu}^{H\dagger}]}{\tilde{m}_{\Delta}}, \quad (44)$$

one gets the bound [30]

$$|\varepsilon_{\Delta}| \leq \frac{1}{2\pi} \frac{m_{\Delta}}{v^2} \sqrt{B_L B_H \sum_i m_{\nu_i}^2}, \quad (45)$$

which holds for the decay of a heavy triplet, no matter what is the nature of the heavy seesaw states. The bound of equation (45) is a model independent bound. If extra information on how the neutrino mass matrix decomposes as the sum of $\mathcal{M}_{\nu}^{\Delta}$ and \mathcal{M}_{ν}^H is known (as e.g. in particular neutrino mass models) the bound of equation (45) can be strengthened since one gets in this case

$$|\varepsilon_{\Delta}| \leq \frac{1}{2\pi} \frac{m_{\Delta}}{v^2} \sqrt{B_L B_H \text{Min} \left[\sum_i m_{\nu_i}^2, \tilde{m}_H^2 \right]}, \quad (46)$$

with $\tilde{m}_H^2 \equiv \text{Tr}[\mathcal{M}_{\nu}^{H\dagger} \mathcal{M}_{\nu}^H]$. Here too the reason why one gets the sum of the neutrino mass squared, rather than $m_{\nu_3} - m_{\nu_1}$ as in equation (9), is associated with the fact that $\mathcal{M}_{\nu}^{\Delta}$ could be any generic matrix, while \mathcal{M}_{ν}^N has rank 1. This bound implies a lower bound on m_{Δ} which we will determine from the Boltzmann equation below. Strictly speaking, equations (45) and (46)

are valid only in the hierarchical limit. In the general case there exists also a ‘perturbativity’ bound on the CP-asymmetry, which one obtains by requiring that the one loop contribution to the various decay widths does not exceed the tree level one,

$$\varepsilon_{\Delta} \leq 2 \text{Min}(B_L, B_H). \quad (47)$$

5.1. Efficiency of a decaying scalar triplet

As explained above, if L , C and CP are violated there is no symmetry principle which could prevent that a triplet–antitriplet asymmetry, $\Delta_T \equiv Y_{\Delta} - Y_{\bar{\Delta}}$, would be created when the triplets decay. As a result there are three independent coupled Boltzmann equations instead of two. The set of equations one gets is

$$sHz \frac{d\Sigma_T}{dz} = - \left(\frac{\Sigma_T}{\Sigma_T^{\text{eq}}} - 1 \right) \gamma_D - 2 \left(\frac{\Sigma_T^2}{\Sigma_T^{2\text{eq}}} - 1 \right) \gamma_A, \quad (48)$$

$$sHz \frac{d\Delta_L}{dz} = \gamma_D \varepsilon_L \left(\frac{\Sigma_T}{\Sigma_T^{\text{eq}}} - 1 \right) - 2\gamma_D B_L \left(\frac{\Delta_L}{Y_L^{\text{eq}}} + \frac{\Delta_T}{\Sigma_T^{\text{eq}}} \right) + X, \quad (49)$$

$$sHz \frac{d\Delta_H}{dz} = \gamma_D \varepsilon_L \left(\frac{\Sigma_T}{\Sigma_T^{\text{eq}}} - 1 \right) - 2\gamma_D B_H \left(\frac{\Delta_H}{Y_H^{\text{eq}}} - \frac{\Delta_T}{\Sigma_T^{\text{eq}}} \right) + X, \quad (50)$$

$$sHz \frac{d\Delta_T}{dz} = -\gamma_D \left(\frac{\Delta_T}{\Sigma_T^{\text{eq}}} + B_L \frac{\Delta_L}{Y_L^{\text{eq}}} - B_H \frac{\Delta_H}{Y_H^{\text{eq}}} \right), \quad (51)$$

with $z \equiv m_{\Delta}/T$, $\Sigma_T = (n_{\Delta} + n_{\bar{\Delta}})/s$, $\Delta_L = (n_L - n_{\bar{L}})/s$, $\Delta_H = (n_H - n_{\bar{H}})/s$ and

$$X = -2 \left(\frac{\Delta_L}{Y_L^{\text{eq}}} + \frac{\Delta_H}{Y_H^{\text{eq}}} \right) (\gamma_{Ts}^{\text{sub}} + \gamma_{Ti}). \quad (52)$$

Out of these four equations, only three are independent, due to the decay sum rule $2\Delta_T + \Delta_H - \Delta_L = 0$, which follows from the fact that a Δ_L triplet can decay to HH or $\bar{L}\bar{L}$ but not to their conjugated states, equations (34)–(37). The decay/inverse decay reaction rate is $\gamma_D = (n_{\Delta}^{\text{eq}} + n_{\bar{\Delta}}^{\text{eq}}) \Gamma_{\Delta} K_1(z)/K_2(z)$. γ_A comes from all gauge scattering processes of the triplets, $\delta\delta' \leftrightarrow GG'$, $f\bar{f}$, $H\bar{H}$ with $G^{(\prime)}$ all possible gauge bosons. One has [30, 41]

$$\hat{\sigma}_A = \frac{6}{72\pi} \left[(15C_1 - 3C_2)\beta + (5C_2 - 11C_1)\beta^3 + 3(\beta^2 - 1)[2C_1 + C_2(\beta^2 - 1)] \ln \frac{1+\beta}{1-\beta} \right], \quad (53)$$

where $x = s/m_{\Delta}^2$, $\beta = \sqrt{1 - 4/x}$, $C_1 = 3g^4/2 + 3g_Y^4 + 12g^2g_Y^2$ and $C_2 = 6g^4 + 3g_Y^4 + 12g^2g_Y^2$. The other reaction rates parametrize the effects of $\Delta L = 2$ scatterings, $LL \leftrightarrow \bar{H}\bar{H}$ and $L\bar{H} \leftrightarrow \bar{L}H$. Their analytical expressions can be found in [30].

To understand the structure of the Boltzmann equations it is useful to proceed step by step, in a way similar to the discussion of the fermion triplet above:

- (a) To start, let us consider only the decay/inverse decay term without washout from inverse decays and without scattering processes, i.e. consider only the first term of the first three Boltzmann equations, equations (48)–(50). In this case, no matter what γ_D is, the efficiency is unity. If $\gamma_D/(n_{\Delta}^{\text{eq}}H) \gg 1$, i.e. $\Gamma_{\Delta} \gg H$, the decay/inverse decay processes are in deep thermal equilibrium, i.e. $\Sigma_T/\Sigma_T^{\text{eq}} - 1 \ll 1$, but this gives the same result as the opposite case ($\Gamma_{\Delta} \ll H$) because all equations involve the same $\gamma_D(\Sigma_T/\Sigma_T^{\text{eq}} - 1)$ product, which remains constant.

- (b) Now let us add the γ_A gauge term in equation (48). As for a fermion triplet $\gamma_A/n_\Delta^{\text{eq}}H$ scales as m_{Pl}/T for $T \gg m_\Delta$. For $T \ll m_\Delta$ it is a good approximation to consider just the s-wave contribution which gives equations (17) and (18) with m_Σ replaced by m_Δ and

$$c_s = \frac{9g^4 + 12g^2g_Y^2 + 3g_Y^4}{2\pi}. \quad (54)$$

This gives a doubly Boltzmann suppressed reaction rate, $\gamma_A \propto e^{-2m_\Delta/T}$, since it involves two external heavy particles, so that the thermalization rate involves only one Boltzmann suppression power, $\gamma_A/n_\Delta^{\text{eq}}H \propto e^{-m_\Delta/T} m_{\text{Pl}}/\sqrt{m_\Delta T}$. The maximum value of $\gamma_A/n_\Delta^{\text{eq}}H$ is reached for $T \sim m_\Delta$, where it takes the value

$$\frac{\gamma_A}{n_\Delta^{\text{eq}}H} \sim 7 \times \frac{10^{13} \text{ GeV}}{m_\Delta}. \quad (55)$$

Therefore, similar to the fermion triplet case, as soon as m_Δ is below $\sim 10^{14}$ GeV, $\gamma_A/(n_\Delta^{\text{eq}}H) > 1$ at $T \sim m_\Delta$, so that $\Sigma_\Delta \simeq \Sigma_\Delta^{\text{eq}}$ at this temperature, and the asymmetry produced is independent of the initial scalar triplet population we start with. And the smaller m_Δ is the more the triplet is thermalized at a temperature around its mass. The discussion is the same as for the fermion triplet above. Above $m_\Delta \sim 10^{14}$ GeV, this gives an efficiency equal to unity. Below this value, if for any T where $\gamma_A/(n_\Delta H) > 1$, the $4\gamma_A \leq \gamma_D$ inequality holds, the gauge scattering term can be neglected in equation (48) and we are back to case (a) above with efficiency unity, even if the gauge scattering terms are in thermal equilibrium for some time. The triplets decay before they get the opportunity to annihilate through gauge interactions. If on the contrary $4\gamma_A \geq \gamma_D$ down to a temperature T_A below $\sim m_\Delta$, the γ_D term can be neglected in equation (48), and $(\Sigma_T/\Sigma_T^{\text{eq}} - 1)$ (and hence the production of an asymmetry) is suppressed down to this temperature. This gives, similar to equation (22),

$$Y_L \simeq \varepsilon_\Delta Y_\Delta^{\text{eq}}(z_A)(z_A/4 + 1), \quad (56)$$

with $z_A \equiv m_\Delta/T_A$, the value of z where $4\gamma_A$ gets below γ_D (remember that γ_A involves one Boltzmann suppression power more than γ_D).

- (c) Now let us add the X term, i.e. the $\Delta L = 2$ scattering terms. As for the fermion triplet case these terms are important only for masses above $\sim 10^{12-14}$ GeV.
- (d) Finally let us add the wash-out term from the inverse decay, i.e. all remaining terms in equations (49)–(51), which are all proportional to γ_D .

If $B_L \simeq B_H$ the discussion of the γ_A – γ_D interplay is just the same as for the fermion triplet. This can be seen easily from the fact that if $B_L = B_H$, equation (51) vanishes and one gets a set of two independent Boltzmann equations similar to the fermion triplet case. Just as for a fermion triplet one can determine a critical value of \tilde{m}_Δ below (above) which one lies in the ‘gauge’ (‘Yukawa’) regime. This value is given in figure 9 as a function of m_Δ . For $B_L = B_H = 1/2$ it gives about the same efficiency as for a fermion triplet, that is to say about the same efficiency as for the type-I case in the Yukawa regime and an efficiency given by equation (56) in the gauge regime.

If instead $B_L \gg B_H$ or $B_H \gg B_L$ the situation is different. In this case it turns out that one gets larger efficiencies. One can even get an efficiency of order unity, even if the triplet mass is orders of magnitudes below 10^{14} GeV. To understand this feature let us consider a value of \tilde{m}_Δ large enough to ensure that the triplet decays before it annihilates for any

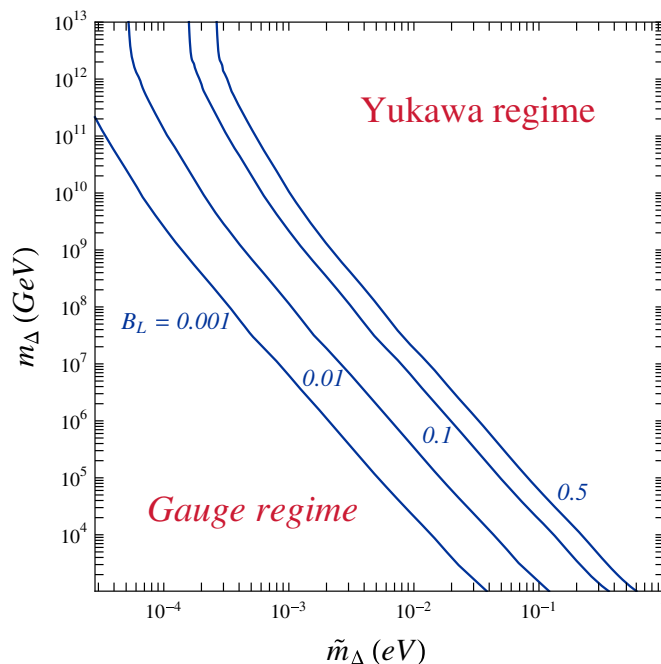


Figure 9. Gauge versus Yukawa regimes for various values of B_L , as a function of \tilde{m}_Δ and m_Δ . As a function of B_H the separation between both regimes is the same, simply replacing B_L by B_H .

value of $z \gtrsim 1$. In this case, even if the decay/inverse decay γ_D term is deeply in thermal equilibrium, one does not get necessarily a suppression of the efficiency. The point is that in the type-II seesaw model, lepton number violation is, as said above, due to the coexistence of scalar triplet couplings to a pair of leptons and to a pair of Higgs doublets. Therefore, even if the total decay rate is much larger than the Hubble rate, if $B_H \ll 1$ or $B_L \ll 1$ (so that either the decay to Higgs doublets or the decay rate to leptons is out of thermal equilibrium), the lepton number is not washed-out by the inverse decay. In this case the scalar triplet has effectively lepton number 2 or 0 respectively, and the lepton number is broken only by the subleading decay process which is out-of thermal equilibrium. In other words, even if the gauge scatterings and the total decay rate are in thermal equilibrium, there is no suppression from the γ_A gauge scattering term if $\gamma_D > 4\gamma_A$ and there is no suppression either from γ_D if one of the decay rates is much smaller than the other one (so that it is out of thermal equilibrium). The third Sakharov condition, i.e. that ‘the creation of an asymmetry requires that the decaying particles are out-of-equilibrium’, is therefore not exact, if taken literally. The decaying particle can be deeply in thermal equilibrium with the Universe thermal bath without any washout being induced. What is precisely required is that the inverse decays that are in thermal equilibrium do not break the lepton number. Note that this is possible only because one has three Boltzmann equations for the asymmetries. If there were only two, for Δ_L and Δ_H , the sum rule $\Delta_H - \Delta_L = 0$ (which holds in type-I and type-III leptogenesis) would necessarily lead to a suppression of both asymmetries.

To see better how effectively a large asymmetry can develop in such a case, in figure 10 we give the rates and asymmetry evolution we get for an example of a situation where the

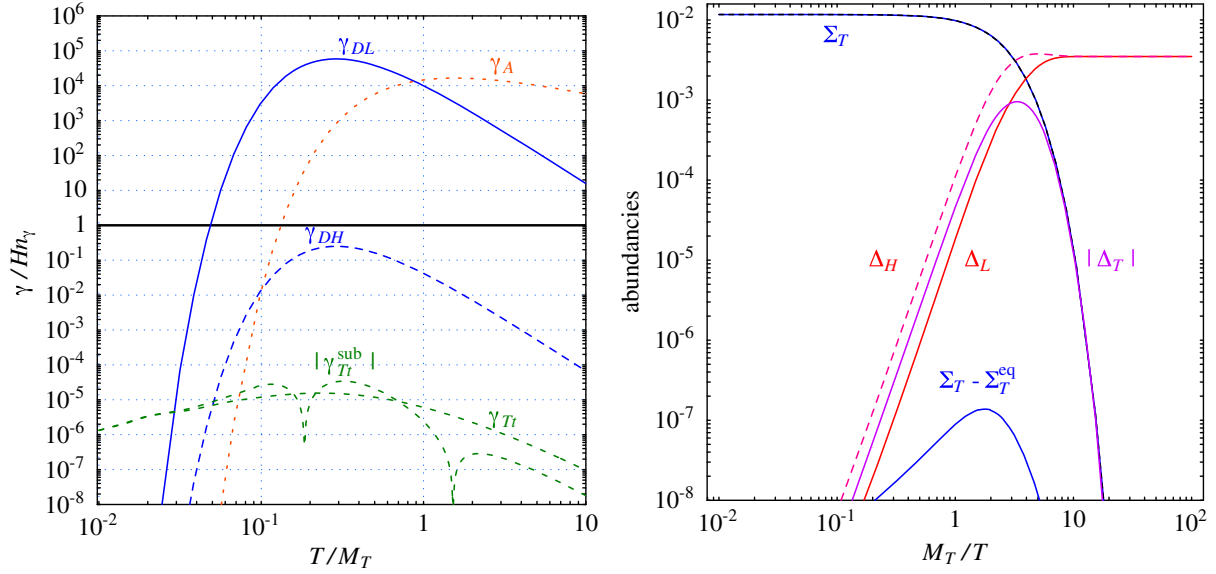


Figure 10. From [30], thermalization rates γ_i/Hn_γ (left panel) and corresponding asymmetry evolutions (right panel) for an example case where $m_\Delta = 10^{10}$ GeV, $\tilde{m}_\Delta = 0.05$ eV, $\lambda_L = 0.1$ giving $B_H = 3 \times 10^{-6}$ and efficiency $\eta \simeq 0.38$.

gauge scatterings as well as the total decay rate are much larger than the Hubble rate, with $B_H \ll 1$ (so that the decay to Higgs doublets is never in thermal equilibrium). This example shows that the first asymmetry to become large is the Δ_H one, because, if $B_H \ll 1$, the washout term proportional to $\gamma_D B_H$ in equation (50) is ineffective. The Higgs asymmetry produced is not suppressed and increases rapidly. One gets approximately $\Delta_H(T) \simeq \varepsilon_\Delta(\Sigma_T^{\text{eq}}(T \gg m_\Delta) - \Sigma_T^{\text{eq}}(T))$. Due to the sum rule $2\Delta_T + \Delta_H - \Delta_L = 0$ an equally large $2\Delta_T - \Delta_L$ develops. Note that since in this case the scalar triplet has effectively $L = -2$, the $2\Delta_T - \Delta_L$ asymmetry is nothing else than the effective lepton asymmetry. This combination is not washed-out since there is no fast decay breaking this effective lepton number. On the other hand, the Δ_L asymmetry is washed-out largely by the inverse decay term proportional to $\gamma_D B_L$ in equation (49), since this term is large. This means that in a first step $2\Delta_T - \Delta_L \simeq 2\Delta_T$. The lepton number asymmetry is therefore stored in the triplet asymmetry. Finally, later on, as all triplets will ultimately decay (to leptons mostly since $B_L \gg B_H$), all the effective lepton asymmetry stored in the triplet asymmetry will be transferred to Δ_L , i.e. to a lepton asymmetry in terms of leptons. Or in other words, since the Δ_H asymmetry produced is large anyway since it is not affected by any washout term, and since ultimately, after all triplets have decayed, the sum rule $\Delta_H - \Delta_L = 0$ applies, all the $2\Delta_T - \Delta_L$ effective lepton asymmetry is necessarily transferred to the Δ_L asymmetry. This pattern can be clearly seen in figure 10.

From all these considerations one can understand the behavior of the efficiency one obtains for a scalar triplet, figures 11 and 12. The efficiency depends on three parameters, m_Δ , Γ_Δ and B_L . Equivalently, as has been done in these figures [30, 32], it may be expressed as a function of m_Δ , \tilde{m}_Δ and λ_L with $\lambda_L^2 \equiv 2\text{Tr}[Y_\Delta Y_\Delta^\dagger]$, so that $\Gamma_\Delta = (m_\Delta/16\pi)(\lambda_L^2 + 4\tilde{m}_\Delta^2 m_\Delta^2/(\lambda_L^2 v^4))$ and $B_L = \lambda_L^2 m_\Delta/(16\pi\Gamma_\Delta)$. Figure 11 shows how the efficiency varies as a function of m_Δ and λ_L .

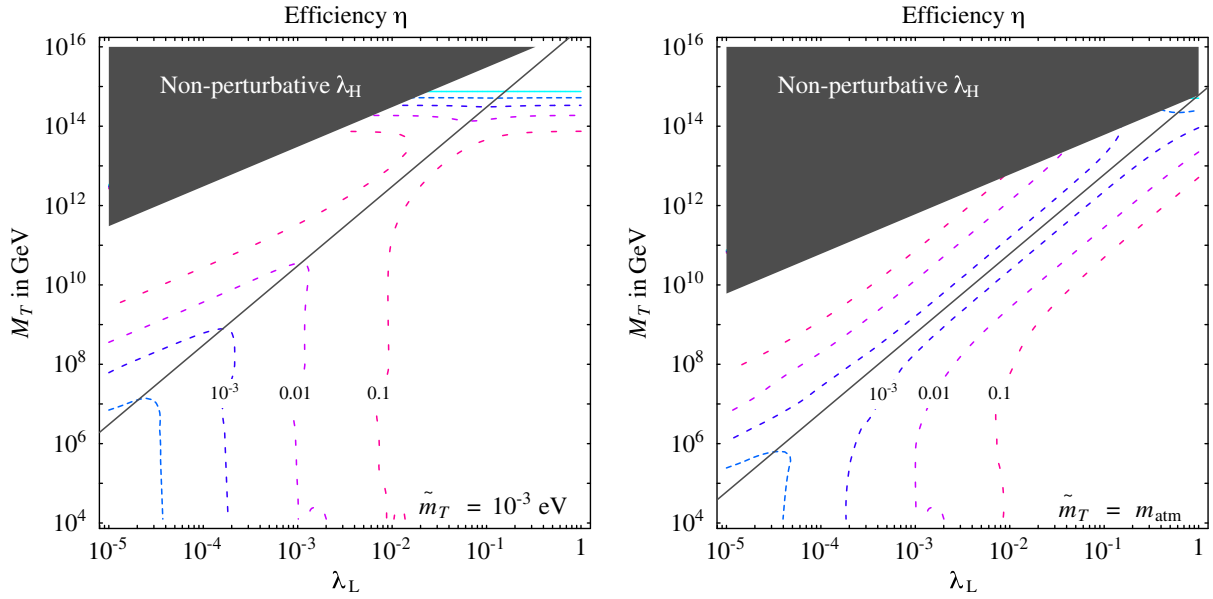


Figure 11. From [30], scalar triplet efficiency obtained as a function of λ_L and m_Δ for $\tilde{m}_\Delta = 10^{-3}$ eV (left panel) and $\tilde{m}_\Delta = 0.05$ eV (right panel). The diagonal line corresponds to the $B_L = B_H = 1/2$ case. The shaded regions are regions where $\lambda_H \equiv \sqrt{2}\mu/m_\Delta$ is larger than one.

for $\tilde{m}_\Delta = 0.05$ eV and $\tilde{m}_\Delta = 10^{-3}$ eV. The first case holds in particular for the situation where the decaying triplet dominates neutrino masses with a hierarchical spectrum of light neutrinos. The second situation corresponds to a subleading triplet contribution to neutrino masses (with leading contribution coming from the heavier seesaw states). The efficiency is also given in figure 12 as a function of \tilde{m}_Δ for $B_L = B_H = 1/2$. Let us first consider this case.

$B_L = B_H = 1/2$ case. As anticipated, the efficiency displayed in figures 11 and 12 has essentially the same properties as for the fermion triplet, see section 3. Both efficiencies do not differ by more than a factor of a few, compare for example figures 12 and 8. For $\tilde{m}_\Delta = 10^{-3}$ eV one always lies in the ‘gauge regime’, unless $m_\Delta \gtrsim 10^{11}$ GeV, whereas, for $\tilde{m}_\Delta = 0.05$ eV, one always lies in the ‘Yukawa regime’, unless m_Δ is below $\sim 10^6$ GeV. Remember the transition line between the gauge and Yukawa regimes for $B_L = B_H = 1/2$ is given in figure 9. Therefore for $\tilde{m}_\Delta = 0.05$ eV and unless $m_\Delta < 10^6$ GeV, the efficiency one gets is similar to the one gets in the type-1 case. In first approximation the efficiency scales as $\sim 1/\tilde{m}_\Delta$, almost independently of m_Δ . For $\tilde{m}_\Delta = 10^{-3}$ eV, on the other hand, one gets only a very mild suppression from inverse decay at high mass but one does get a suppression from gauge scatterings. This gives an efficiency which scales as $\sim m_\Delta$ for m_Δ below 10^{11} GeV. $\Delta L = 2$ scatterings induce a suppression only for $m_\Delta > 10^{14}$ GeV.

$B_L \neq B_H$ case. As expected above, figures 11 and 12 display an efficiency which increases off the $B_L = B_H$ diagonal. Note that equation (40) can be rewritten as

$$\frac{\Gamma_\Delta}{H(T = m_\Delta)} = \left(\frac{\tilde{m}_\Delta}{0.0012 \text{ eV}} \right) \frac{1}{2} \frac{B_L + B_H}{\sqrt{B_L B_H}}, \quad (57)$$

which means that the total inverse decay rate is in thermal equilibrium at $T \sim m_\Delta$ provided $\tilde{m}_\Delta = \sqrt{(m_{\nu_i}^\Delta)^2} \gtrsim 0.002 \text{ eV} \frac{\sqrt{B_L B_H}}{B_L + B_H}$. Therefore, for fixed value of \tilde{m}_Δ and m_Δ , as one increases

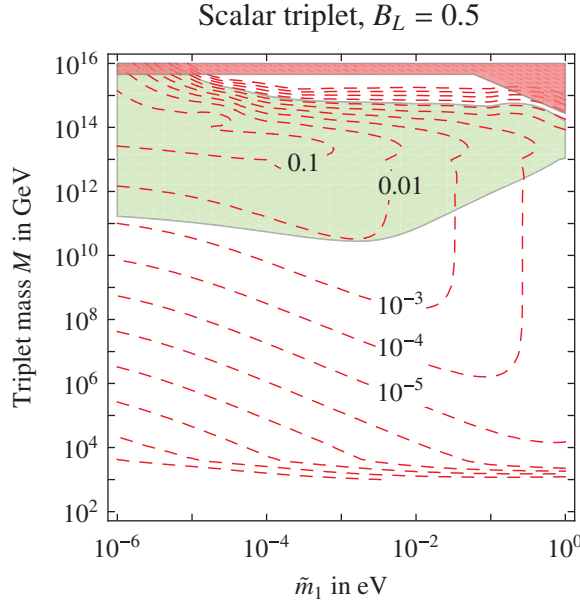


Figure 12. Scalar triplet efficiency obtained in the case $B_L = B_H = 1/2$, as a function of \tilde{m}_Δ and m_Δ , from [32]. Similar results can also be found in [30, 80].

B_L or B_H , the total decay rate increases, so that γ_D/γ_A increases (suppressing the gauge effect) and at the same time L gets less and less broken as $B_H\gamma_D$ or $B_L\gamma_D$ decreases (suppressing the inverse decay effect), see equation (40). For $\tilde{m}_\Delta = 0.05$ eV this is the second effect which is mostly responsible for the increase of the efficiency off the diagonal line in figure 12. For $\tilde{m}_\Delta = 10^{-3}$ eV this is mostly the first one. In fact for $B_L \neq B_H$ one gets out of the gauge regime (i.e. gauge effects cease to be relevant, see section 3) as soon as

$$\tilde{m}_\Delta < \tilde{m}_\Delta^A \frac{\sqrt{B_L B_H}}{B_L + B_H} \times 2, \quad (58)$$

with \tilde{m}_Δ^A the transition value one gets for $B_L = B_H = 1/2$, see figure 9. In this figure one can find, for various values of B_L , the values of \tilde{m}_Δ separating both regimes. Similarly there is no sizable inverse decay effect as soon as

$$\text{Min}[B_L, B_H] \frac{\Gamma_\Delta}{H(T = m_\Delta)} \lesssim 1. \quad (59)$$

For B_L smaller (larger) than B_H , this inequality can be rewritten as $\lambda_L^2/m_\Delta < 17 \times 16\pi/M_{\text{Pl}}$ ($\mu_\Delta^2/m_\Delta^3 < 17 \times 8\pi/M_{\text{Pl}}$). In the Yukawa regime, when $K_{\text{Min}} \equiv \text{Min}[B_L, B_H]\Gamma_\Delta/H|_{T=m_\Delta}$ is larger than unity, the efficiency due to the inverse decay processes scales in first approximation as $1/K_{\text{Min}}$, that is to say as $(\lambda_L^2/m_\Delta)^{-1}$ and $(\mu_\Delta^2/m_\Delta^3)^{-1} = (\lambda_L^2/m_\Delta)(v^4/\tilde{m}_\Delta^2)/2$ for $B_L \ll 1$ and $B_H \ll 1$ respectively.

5.2. Baryon asymmetry from a decaying scalar triplet

Using the efficiency results above, figure 13 gives the region of m_Δ and λ_L which can lead to successful baryogenesis, dark green region in figure 13. To this end one has considered in equation (39) the bound on the CP-asymmetry which holds for a hierarchical seesaw state

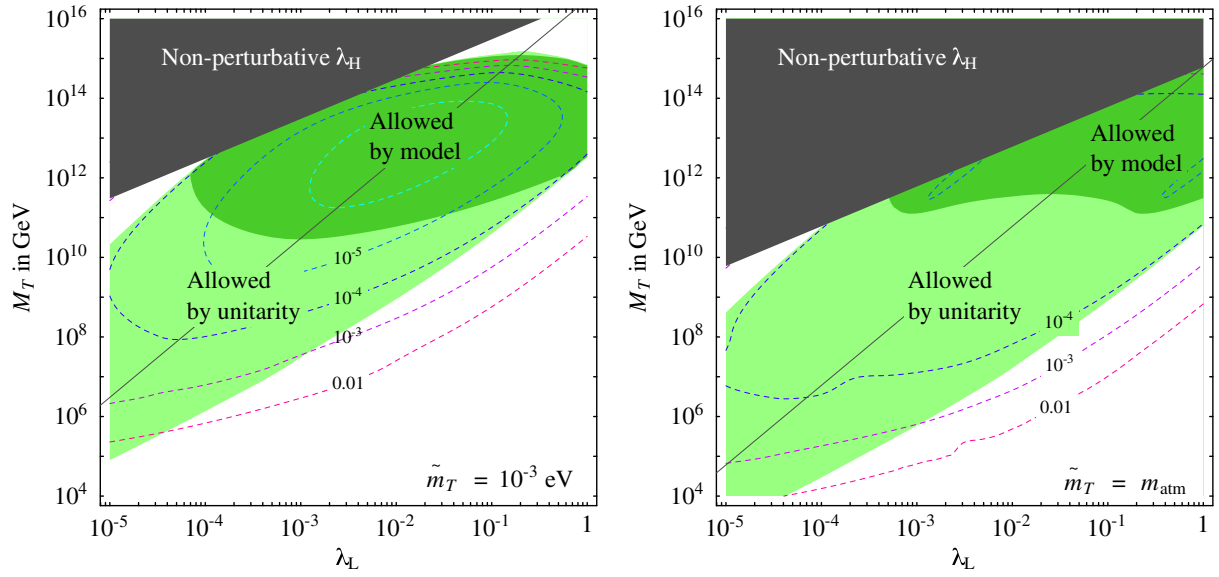


Figure 13. From [30], values of λ_L and m_Δ which allow successful leptogenesis for a hierarchical spectrum (dark green, equation (46)) and from equation (47) (light green). The dashed lines are isocurves of $\varepsilon_\Delta/\sqrt{4B_L B_H}$. The left (right) panel is obtained for $\tilde{m}_\Delta = 10^{-3}$ eV ($= 0.05$ eV). The shaded regions are regions where $\lambda_H \equiv \sqrt{2}\mu/m_\Delta$ is larger than one.

spectrum, equation (46). As one could expect, this CP asymmetry decreases off the $B_L = B_H$ case (i.e. $\varepsilon_\Delta \propto \sqrt{B_L B_H}$), since in this case the amount of L violation decreases. Consequently the behavior of the CP-asymmetry is opposite to that of the efficiency. Thus, one could wonder whether both effects compensate each other exactly in the baryon asymmetry produced, equation (39). In fact they do not cancel exactly. As explained above, in the regime where the decay term dominates the washout (as is the case for $\tilde{m}_\Delta = 0.05$ eV and $m_\Delta \gtrsim 10^6$ GeV), the efficiency goes approximately as $1/K_{\min} \propto 1/\text{Min}[B_L, B_H]$. Therefore unless we are so far away from the $B_L = B_H = 1$ case that the efficiency is unity (giving $n_B/s \propto \sqrt{B_L B_H}$), n_B/s goes like $\sqrt{B_L B_H}/\text{Min}[B_L, B_H]$. As a result, for fixed value of m_Δ , the maximum baryon asymmetry is obtained off the $B_L = B_H = 1/2$ case. For $\tilde{m}_\Delta = 0.05$ eV, this can be seen in figure 13. For the $\tilde{m}_\Delta = 10^{-3}$ eV case on the other hand there is no washout from the decays/inverse decays and the efficiency is independent of $B_{L,H}$. It depends only on m_Δ through the gauge scattering effect. Thus, the baryon asymmetry is proportional to $\sqrt{B_L B_H}$ in the same way as the CP-asymmetry and one gets a maximum baryon asymmetry for the $B_L = B_H = 1/2$ case, see figure 13.

Since, for a hierarchical spectrum of seesaw states, the upper bound on the CP asymmetry, equation (4), is proportional to m_Δ , successful leptogenesis requires a heavy scalar triplet. Moreover the thermalization suppression effect from the gauge scattering increases as $1/m_\Delta$. Assuming a hierarchical spectrum of light neutrinos, this leads to the bounds [30]

$$\begin{aligned} m_\Delta &> 2.8 \times 10^{10} \text{ GeV} \quad (\tilde{m}_\Delta = 0.001 \text{ eV}), \\ m_\Delta &> 1.3 \times 10^{11} \text{ GeV} \quad (\tilde{m}_\Delta = 0.05 \text{ eV}). \end{aligned} \quad (60)$$

As for the fermion triplet case these bounds are stronger than for the type-I seesaw case due to the gauge scattering effects. These bounds apply using equation (45). For small contribution of

the heavy seesaw states to the neutrino mass matrix a stronger bound applies. In this case the bounds of equations (60) have to be multiplied by a factor $\sqrt{\sum m_{\nu_i}^2}/\tilde{m}_H$, see equation (46). For example, for $\tilde{m}_H = (\Delta m_{\text{sun}}^2)^{1/2} = 0.007 \text{ eV}$ this gives: $m_\Delta > 8 \times 10^{11} \text{ GeV}$ (with $\tilde{m}_\Delta = 0.05 \text{ eV}$). These precise constraints have the same order of magnitude than the ones which were estimated in [8, 29, 81].

For larger values of $\sum_i m_{\nu_i}^2$ the constraints of equations (60) get relaxed by a factor $(\Delta m_{\text{atm}}^2 / \sum_i m_{\nu_i}^2)^{1/2}$, i.e. by one order of magnitude for quasi-degenerate neutrinos with $m_\nu \approx 0.5 \text{ eV} \approx 10 m_{\text{atm}}$. This allows going down to $m_\Delta \sim 10^9 \text{ GeV}$. In fact, by increasing the neutrino mass scale, keeping m_Δ fixed, the asymmetry increases but the efficiency remains unchanged [81].

5.3. Leptogenesis possibilities with neutrino masses induced by a single scalar triplet

The type-II seesaw model with a single scalar is the only seesaw model which can generate any possible neutrino mass matrix on the basis of a single state beyond the SM. As a result, in full generality, this model involves ‘only’ 11 parameters, the triplet mass, its μ_Δ coupling and 6 real parameters plus 3 phases in the Y_Δ Yukawa coupling matrix, see table 1. Out of these parameters, three of them are normalization parameters (m_Δ , μ_Δ and $y_\Delta = (\text{Tr}[Y_\Delta Y_\Delta^\dagger])^{1/2}$) and eight are ‘flavor’ parameters. This is similar to what one has with two type-I or -III states, with the important difference that there are fewer decoupling parameters in the type-II option. In particular this model has the attractive property that knowledge of the full low energy neutrino mass matrix M_ν would allow determination of the full flavor high energy structure in Y_Δ , both matrices being just proportional to each other. In this sense it has a ‘minimal flavor structure’, see e.g. [82], that is to say that the knowledge of the flavor structure of the lowest dimension operator (i.e. the dimension-4 Y_Δ interaction), allows knowledge of the flavor structure of all higher dimensional effects (i.e. of the L -violating dim-5 neutrino mass matrix, of the seesaw induced L -conserving dim-6 operators generating rare lepton flavor violating processes such as $\mu \rightarrow e\gamma$, etc). This is not the case with two right-handed neutrinos, except in very particular cases [82]. As for the three normalization parameters, the neutrino mass matrix allows us to know their combination $y_\Delta \mu_\Delta / m_\Delta^2$. Only two parameters decouple. This model would therefore constitute a perfect minimal leptogenesis model, with all CP-violating phases which could be determined at low energy, from the neutrino mass matrix. However, unfortunately, it is so minimal that it does not work at all for leptogenesis, as already mentioned above and explained in [7, 29, 83, 84]. Since the triplet is not a self-conjugated particle, there is no vertex diagram and leptogenesis could come only from a self-energy diagram involving two leptons in the final state and two Higgs doublets in the self-energy, first diagram of figure 3. This diagram, with just one triplet, is real and does not bring any CP-violation. Flavor effects do not help on this issue, and at the higher loop level the asymmetries would be too suppressed anyway (and this model involves only one source of flavor breaking anyway too). Therefore the SM extended by just one scalar triplet leads in a ‘minimal’ way to neutrino masses but does not lead to successful leptogenesis.

There are nevertheless possibilities to have successful leptogenesis with neutrino masses induced only by a single triplet. One possibility arises within the supersymmetric version of this single triplet model. This model involves an additional triplet of opposite hypercharge (to insure that the theory is anomaly free) but still only one scalar triplet contributing to the neutrino mass matrix. In this case successful leptogenesis is possible from a self-energy diagram involving

both triplets and extra soft SUSY breaking sources of L violation [84, 85]. The CP asymmetry in this case is determined by both the flavor structure of the Y_Δ coupling and of the L violating soft terms. To have a non-vanishing CP asymmetry in this framework, it is sufficient that the various superpotential terms and soft terms differ only by an overall non-vanishing phase and that both triplets mix and have a non-vanishing mass splitting due to soft terms (bilinear in the triplets).

Another interesting possibility, which does not rely on soft SUSY breaking terms, arises in a $SO(10)$ model based on a particular set of representations [95]. It consists in assuming, in addition to the usual 10_H and $16_{F_{1,2,3}}$ $SO(10)$ representations, $10_{F_{1,2,3}}$ fermions representations, a 54_S scalar representation and a 10_S scalar representation. The ordinary SM lepton doublets are in the 10_F multiplets. As a result there is no type-I contribution to neutrino masses (i.e. the usual $Y_N 16_F 16_F 10_H$ do not involve the left-handed neutrinos) but a pure type-II one from the exchange of the scalar triplet in the 54_S between a pair of Higgs doublets in the 10_H and a pair of lepton doublets (i.e. a $f_{ij} 54_S 10_{F_i} 10_{F_j}$ coupling). In the supersymmetric version of this model leptogenesis is induced exclusively by an asymmetry involving four powers of the f_{ij} couplings (through a vertex type diagram where the triplet decays to a pair of leptons (sleptons) with a virtual particle from the 54_S). The second source of flavor breaking, required to have a CP asymmetry, is provided by the mass hierarchy between the various beyond the SM particles in the 10_{F_i} representations. If we take for example the two heaviest ones to be heavier than m_Δ there is no GIM cancellation and we get an asymmetry proportional to $\text{Im}[f_{11}(f^* f f^*)_{11}]$ (where ‘1’ refers to the lightest 10_{F_i} particle) times a function of the masses of the various particles. That is basically the maximum connection one could get between the neutrino masses and leptogenesis: a CP asymmetry totally determined by the neutrino mass matrix (here proportional to four powers of the neutrino mass matrix with, in particular, CP-violation determined by it) times an unknown normalization constant.

5.4. Quasi-degenerate case: the double type-II seesaw model

If there is more than one heavy scalar triplet contributing to the neutrino masses, leptogenesis can be easily induced by the decay of the triplets to two leptons with a one-loop self-energy diagram involving two different triplets [29, 83], first diagram of figure 3. This is actually the first model which has been proposed to induce leptogenesis from the decay of a scalar triplet. The asymmetry in this case is given by

$$\varepsilon_{\Delta_i} = -\frac{1}{2\pi} m_{\Delta_i} \frac{\text{Im}[(\mu_i^* \mu_j (Y_{\Delta_i})_{kl} (Y_{\Delta_j}^*)_{kl})]}{|(Y_{\Delta_i})_{kl}|^2 m_{\Delta_i}^2 + |\mu_i|^2} \frac{M_{\Delta_j}^2 \Delta m_{ij}^2}{(\Delta m_{ij}^2)^2 + m_{\Delta_i}^2 \Gamma_{\Delta_j}^2}, \quad (61)$$

where there is now a triplet scalar index on the couplings of table 1, with $m_{ij}^2 = m_{\Delta_j}^2 - m_{\Delta_i}^2$. For hierarchical triplets, all the results above hold. In particular the CP asymmetry of equation (61) reduces to the one of equation (4) for $m_{\Delta_2} \gg m_{\Delta_1}$. Similarly the bounds of equation (60) hold. For quasi-degenerate triplets, however, a resonance effect occurs in equation (61), similar to equation (25). This effect can lead to a large enhancement of the CP-asymmetry. As a result successful leptogenesis can be obtained for much smaller values of the triplet masses. Applying the bound of equation (47), one gets the absolute bound [32]

$$m_\Delta > 1.6 \text{ TeV}. \quad (62)$$

Similar to what happens in the fermion triplet quasi-degenerate case above, this scale results from the interplay of the electroweak scale, below which there is no more $L \rightarrow B$ conversion,

and of the gauge scatterings (which at those scales give a suppression of the efficiency of more than five orders of magnitudes, see figure 12). The value of equation (62) is too high to be reachable at LHC. Similar to the fermion triplet case too, the LHC experiments should be able to discover a scalar triplet with a mass up to $\sim \text{TeV}$ [96–101] (through Drell–Yan pair production). As of today the preliminary LHC lower bound on a doubly charged scalar boson is $\sim 400 \text{ GeV}$.

Note also that with several triplets, and whatever is the triplet mass spectrum, there is no relevant upper bound on neutrino masses for successful leptogenesis because one can always increase the neutrino mass contribution of the virtual triplet in the self-energy diagram, leading to a larger CP asymmetry, without affecting the efficiency. For specific leptogenesis models involving several scalar triplets see [102, 103].

The supersymmetric version of this double type-II scenario has been considered in [29]. It involves two pairs of opposite hypercharge triplets (in order that the model is anomaly free). It differs from the non-supersymmetric version by factors of order unity for the CP-asymmetry and efficiency. In particular, for a hierarchical spectrum of triplet pairs, the bound of equation (60) remains valid and lies above the gravitino upper bound on the reheating temperature. Successful leptogenesis would therefore require in this framework a quasi-degenerate spectrum of triplet pairs, or extra L -violating soft SUSY breaking terms as proposed in [84, 85].

5.5. Effect of flavor

Finally note that effects of flavor on the production of a lepton asymmetry from the decay of a scalar triplet are in general expected to be somewhat smaller than for the right-handed neutrino, if one assumes that this triplet dominates the neutrino masses, simply because in this case a single seesaw state gives several neutrino masses. But, still, one could have interplays of flavor asymmetries similar to the ones one can have with right-handed neutrinos. This has been studied for a specific model in [104]. Note that in the same way as for a fermion triplet above, it is interesting to point out that, by invoking flavor effects (reducing the washout from the inverse decays), one could go below the upper bounds of equation (60). But flavor effects do not allow going below the bound of equation (62) because this bound is obtained deeply in the gauge regime where flavor effects are negligible (in the same way as for the fermion triplet above, equation (28)).

6. Mixed seesaw models

6.1. The type-I + type-II model: decay of the scalar triplet

In the left–right models [139–143] spontaneous breaking of parity finds its origin in the VEV, ‘ v_R ’, of a ‘ Δ_R ’ hypercharge two scalar boson, which is a triplet of the $SU(2)_R$ gauge symmetry. This VEV induces the right-handed neutrino Majorana masses. The parity partner of Δ_R is nothing but the type-II seesaw Δ_L triplet. Similarly in $SO(10)$ models which give right-handed neutrino masses in a renormalizable way, one also finds a type-II seesaw field on top of the right-handed neutrinos (in the 126_S multiplet which contains both the Δ_L and Δ_R fields). In all these cases the relevant Lagrangian for what concerns the neutrino masses is just the sum of the type-I and -II Lagrangians of table 1. To discuss this possibility it is necessary to consider two cases, depending on which particle is the lightest one, the scalar triplet Δ_L or the lightest right-handed neutrino N_1 [81].

If the scalar triplet is the lightest seesaw state, its decay naturally dominates the production of the asymmetry from a diagram of vertex type with a virtual right-handed neutrino, second diagram of figure 3, first displayed in [81, 105]. Calculating explicitly its contribution we get [30, 81]

$$\varepsilon_{\Delta} = -\frac{1}{16\pi^2} \frac{m_{N_k}^2 m_{\Delta}}{\Gamma_{\Delta}}. \quad (63)$$

For $m_{N_{1,2,3}} \gg m_{\Delta}$ this asymmetry reduces to the one of equation (4) with $\mathcal{M}_v^H = \sum_i \mathcal{M}_v^{N_i}$, and the bounds of equation (60) hold. In this case there are obviously no relevant bounds on the neutrino masses [81] because the asymmetry is quadratic in couplings which do not induce any washout, i.e. in the type-I Yukawa couplings. This feature also allows reduction of the lower bound on m_{Δ} of equation (60), because if one increases these couplings, keeping fixed the scalar triplet neutrino mass contribution, one increases the CP-asymmetry without changing the efficiency. In this case, from equation (46), the bounds of equation (60) are relaxed by a factor $0.05 \text{ eV} / (\text{Min}[\sum m_{v_i}^2, \sum m_{v_i}^{\Delta^2}])^{1/2}$. For neutrino masses below the eV scale this does not allow to go below $\sim 10^{10} \text{ GeV}$. One could eventually go a few times lower by invoking additional flavor effects in the same way as for the fermion triplet above. Since the diagram of figure 3 does not display any resonant feature, to take $m_{\Delta} \sim m_{N_{1,2,3}}$ does not give any enhancement and the bounds above remain valid also for this case, up to factors of order unity.

6.2. The type-I + type-II model: decay of a right-handed neutrino

If the lightest right-handed neutrino mass is lighter than the scalar triplet, one must consider the opposite case where the asymmetry production is dominated by the decay of the lightest right-handed neutrino. In this case leptogenesis can be produced from the pure type-I usual diagrams⁶. However, there is an additional contribution to leptogenesis which comes from the third diagram of figure 3, involving a virtual scalar triplet [23, 81, 105, 110, 111]. This diagram gives an extra contribution to the CP asymmetry of the lightest right-handed neutrino [23, 81]

$$\varepsilon_{N_1} = -\frac{3}{8\pi^2} \frac{m_{N_1} m_{\Delta}^2}{\Gamma_{N_1}} \frac{1}{v^4} \text{Im}[(M_v^{N_1})_{\beta\alpha} (M_v^{\Delta})_{\alpha\beta}^{\dagger}] \left(1 - \frac{m_{\Delta}^2}{m_{N_1}^2} \log[1 + m_{N_1}^2/m_{\Delta}^2]\right), \quad (64)$$

which, in the hierarchical limit $m_{N_1} \ll m_{\Delta}$, reduces to equation (3). In fact in this hierarchical limit, and with $m_{N_1} \ll m_{N_{2,3}}$, the respective contributions of $N_{2,3}$ and Δ_L are proportional to their contribution to the neutrino masses. If the neutrino mass matrix is dominated by the scalar triplet, the scalar triplet contribution of equation (64) will naturally dominate the production of the baryon asymmetry. If instead $N_{2,3}$ gives a larger contribution to the neutrino mass matrix, the asymmetry of equation (3) with \mathcal{M}_v^H replaced by $\mathcal{M}_v^{N_{2,3}}$ will naturally dominate. Of course, since we are dealing with matrices, this depends on the interplay of the various complex flavor entries in \mathcal{M}_v^I and in \mathcal{M}_v^{Δ} or $\mathcal{M}_v^{N_{2,3}}$. Consequently there is no one to one correspondence between the size of neutrino masses induced by each seesaw state and their contribution to the CP asymmetry, but this is the natural pattern which emerges [81]. For instance, the scalar triplet term of equation (64) could perfectly do the job alone. This contribution, here too, does not decrease with neutrino masses, since it is quadratic in the scalar triplet couplings which do not cause any washout. As a result, here too, there is no relevant upper bound on neutrino masses,

⁶ The contribution of the usual pure type-I CP-asymmetry in the context of type-I + II seesaw models has been studied in detail in [106–109].

even though the seesaw states have a hierarchical spectrum. Moreover given the fact that the CP-asymmetry is linear in M_{N_1} this allows decreasing the bound on the lightest right-handed neutrino mass linearly in the neutrino masses [23, 81]. The precise bound one gets in this case on the CP-asymmetry is [23]

$$\varepsilon_{N_1} < \frac{3}{8\pi} \frac{M_{N_1}}{v^2} m_v^{\max}, \quad (65)$$

with m_v^{\max} the highest neutrino mass eigenvalue. This gives

$$M_{N_1} > 6.0 \times 10^8 \text{ GeV} \times \left(\frac{0.05 \text{ eV}}{m_v^{\max}} \right). \quad (66)$$

For neutrino masses below the eV scale this allows going down to \sim a few 10^7 GeV. Here too since the associated one-loop diagram is non-resonant one cannot go below this value. For various studies of the type-I + II leptogenesis frameworks, including within the context of specific flavor models, see [38, 80, 112–131]. A study of flavor effects in the type-I + type-II model producing the asymmetry from the right-handed neutrino decays (i.e. with $m_{N_1} \ll m_\Delta$) can be found in [138].

6.3. Implications for left–right symmetric and $SO(10)$ models

The most important prediction of the left–right models, for what concerns neutrino masses and leptogenesis, is that right- and left-handed neutrino mass matrices are proportional to each other, i.e. $(Y_\Delta)_{ij} = \frac{1}{2} \delta_{ij} m_{N_i} / v_R$, with v_R the $SU(2)_R$ breaking scale. As a result from equations (63) and (64) one gets

$$\varepsilon_\Delta = -\frac{1}{4\pi} \frac{m_{N_i}}{m_{N_k}} \frac{\text{Im}[(Y_N)_{ki}^2 \mu_\Delta]}{\left(\frac{1}{4} \frac{m_{N_j}^2}{v_R^2} + \frac{|\mu_\Delta|^2}{m_\Delta^2} \right)}, \quad (67)$$

$$\varepsilon_{N_1} = \frac{3}{16\pi} \frac{m_{N_1} m_{N_i}}{m_\Delta^2 v_R} \frac{\text{Im}[(Y_N)_{1i}^2 \mu_\Delta]}{|Y_{N1j}|^2}, \quad (68)$$

which depends only on the normalization parameters μ_Δ , m_Δ and v_R , and on the right-handed neutrino parameters, $(Y_N)_{ki}$ and m_{N_i} . Thus, apart from normalization factors this framework involves in the CP-asymmetries the same number of parameters as the pure type-I framework. This must be taken into account for quantitative analysis of the asymmetry, even if it has basically no effect on the above lower bounds on the mass of the decaying state.

In the left–right models the mass hierarchy of seesaw states is not predicted. One could have nevertheless the prejudice that the Yukawa couplings are hierarchical and that m_Δ gets contributions of order of the $SU(2)_R$ breaking scale v_R (provided the associated scalar coupling would be of order unity, for example the $\Delta_R^\dagger \Delta_R \Delta_L^\dagger \Delta_L$ coupling). This prejudice would point towards the $m_{N_1} < m_\Delta$ scenario. Note that in this case there is no real reason to believe that the diagram involving the scalar triplet, equation (64), would contribute less than the pure type-I diagrams. On the contrary one could argue that to have a dominant type-II contribution to the neutrino masses is a particularly attractive way to obtain large neutrino mixing angles despite the fact that the quark mixings are small (especially in the $SO(10)$ framework where quarks and lepton Yukawa couplings are related, see e.g. [144]).

Similarly in the left–right model the mass of the second Higgs doublet in the scalar bidoublet is not predicted. It is generally expected to be large as it receives contributions

proportional to v_R from the scalar potential, but nothing prevents it from being below m_{N_1} . In this case both scalar doublets appear in all diagrams, so that in the CP asymmetries the Y_N couplings carry now a scalar index, Y_N^i , $i = 1, 2$, and μ_Δ becomes a 2×2 matrix.

Note also that in the left–right model the right-handed gauge bosons could also have an effect if their mass is similar to m_{N_1} . Since $m_{W_R} = g_R v_R / 2 \sim v_R$, and since the $SU(2)_R$ coupling g_R is *a priori* of order one (or equal to the $SU(2)_L$ coupling in the left–right symmetric case), this is the case only if $m_{N_{1,2,3}} \sim v_R$. In this case the main effect of the W_R and Z_R is a washout effect [145, 146]. This effect is especially dangerous for the efficiency because W_R gauge interactions induce scattering processes involving only one external right-handed neutrino: $Ne_R \rightarrow \bar{u}_R d_R$, $N\bar{u}_R \rightarrow \bar{e}_R d_R$, $Nd_R \rightarrow e_R u_R$. Thus, unlike the gauge scatterings induced by the left-handed gauge boson above, which involve two external seesaw states, these scatterings have a reaction rate γ_A^R which is suppressed by only one Boltzmann suppression power instead of two. That means that they have a thermalization rate $\gamma_A^R / n_N^{\text{eq}} H$ which is not Boltzmann suppressed at all [146]! As a result if $m_{W_R} \sim m_{N_1}$ these scatterings maintain the N_R in thermal equilibrium until $T \ll m_{N_1}$ where n_N^{eq} is hugely suppressed. In this case the bulk of the asymmetry is produced (in a very suppressed way) before this decoupling. Only for $m_{N_1} \gtrsim 10^{14}$ GeV are these scatterings never in thermal equilibrium. Consequently successful leptogenesis in this scenario basically requires $m_{N_1} \ll m_{W_R}$ or $m_{N_1} \gtrsim 10^{12}$ GeV or $m_{\Delta_L} \lesssim m_{N_1}$. This actually has consequences for what would imply the discovery of a right-handed neutrino at the LHC. The right-handed neutrino production channel usually considered at LHC is through an intermediate W_R produced from quarks. Such a process leads to a measurable signal only if the scale $m_{W_R} \sim v_R$ is not much above the mass of the right-handed neutrino, and does not exceed the ~ 4 eV scale, see e.g. [147, 148]. The discovery of a right-handed neutrino and/or of a W_R at LHC would therefore exclude the right-handed neutrino decay leptogenesis scenario [146]. Successful leptogenesis requires m_{W_R} above the ~ 10 TeV scale.

For various leptogenesis studies in the frameworks of left–right models, see [122–127, 149]. Similarly for $SO(10)$, see [128–130, 150]. In $SO(10)$ models where the Y_N couplings have the same flavor structure as the one of up quark mass matrix, the type-I seesaw formula leads to a strong hierarchy between the m_{N_i} (such that $m_{N_{1,2,3}} \propto m_{u,c,t}^2$). Given the GUT scale value, this gives a value of m_{N_1} well below the bound of equation (10). This scenario has been analyzed in detail in the context of pure type-I $SO(10)$ inspired models [130, 132–134]. This problem can be cured in various ways, in particular if one has also a type-II contribution dominating the neutrino masses [150], or in the N_2 -leptogenesis framework mentioned above, invoking flavor effects [135–137]. For a related model based on E_6 grand unification, see [151, 152].

6.4. The type-I+type-III model and implications for the adjoint $SU(5)$ models

As mentioned in section 2, a type-I + type-III neutrino mass generation setup is a quite straightforward possibility in the framework of grand-unification [31, 71, 72, 153–155]. In the adjoint representation of $SU(5)$ there is a particle which has the quantum numbers of a seesaw fermion triplet and another particle which has the ones of a right-handed neutrino. This leads to a particularly minimal model where all seesaw states are in a single representation. For what concerns neutrino masses it is similar in many respects to the two right-handed neutrinos seesaw models. It can easily fulfil the neutrino mass data constraints. Moreover it may lead to gauge unification if the fermion triplet is light, below the ~ 2 – 3 TeV scale. As mentioned

above, a triplet fermion with mass below $\sim \text{TeV}$ could be discovered at LHC. At such scale, for what concerns leptogenesis, one may think about the resonant self-energy possibility [72], especially for the decay of the right-handed neutrino whose efficiency is not suppressed by any gauge scatterings. That would require that both fermions have masses very close to each other. Beyond the fact that in this model there is no reason for such a pattern, this anyway does not work because such a self-energy diagram requires that a singlet fermion turns into a triplet fermion at the one loop level, which requires $SU(2)_L$ breaking. This turns out to lead to a very suppressed asymmetry at temperatures above the electroweak scale [154].

7. Conclusion

The seesaw framework allows explanation within a common framework of two experimental facts that, at first sight, one would have expected to be disconnected: the neutrino masses and the baryon asymmetry of the Universe. It is important to determine the main seesaw scenarios through which this could be achieved and to determine their main general properties. It turns out that there exist quite a few attractive scenarios of this kind! Beside the usual right-handed neutrino decay scenario, the fermion triplet and scalar triplet decay scenarios could work perfectly as well. Each of these two scenarios can be realized in various ways, depending on the nature of the other seesaw states contributing to the neutrino masses. In these scenarios, as in the type-I framework, the neutrino masses have values within a range which turns out to be perfect for the generation of a large asymmetry. An important difference, nevertheless, is that, if their mass is below $\sim 10^{14}$ GeV, these triplet states do have gauge interactions which thermalize them with the Universe thermal bath, at a temperature of the order of their masses. We devoted a large part of this review to explaining in detail how the involved interplay of these interactions and the Yukawa interactions takes place in the calculation of the efficiency. Constraints that this interplay imply on the seesaw parameters are determined and compared with the type-I usual constraints.

An important question which arises is whether these attractive possible explanations of the baryon asymmetry of the Universe could be tested in the near future. There exist clear possibilities of falsifying the leptogenesis mechanism. For instance, it is well known that an inverse hierarchical or quasi-degenerate spectrum of light neutrinos implies a lower bound on the size of the neutrinoless double beta decay signal [156]. If in the future one of these two mass spectra turns out to be established experimentally, the non-observation of such a process with such a minimum magnitude would basically rule out the seesaw mechanism as the main origin of the neutrino masses. In a different vein, the observation of a TeV scale right-handed W_R boson would basically rule out the type-I leptogenesis, due to the huge washout of the L -asymmetry this state would imply. Similarly, a fermion or a scalar triplet discovered at LHC, could easily wash-out any preexisting asymmetry. More generally, the observation of new particles at LHC, other than seesaw particles, with lepton number violating interactions, could rule out leptogenesis scenarios. As for establishing a leptogenesis scenario experimentally, this appears to be a tremendously difficult task. Triplet states, unlike right-handed neutrinos, do have the advantage that, from SM gauge interactions, they can be Drell–Yan pair produced at colliders. Unfortunately the efficiency suppression that gauge interactions imply at such low scales, leads, for successful leptogenesis, to a lower bound on the triplet state mass which is slightly above the reach of the LHC. In any case it must be kept in mind that even if at LHC one could observe a seesaw state with interactions which could produce the baryon asymmetry of

the Universe, this would still not preclude the possibility that the reheating temperature has been low enough to forbid this possibility, or that the baryon asymmetry which we observe today is the product of a dynamics which took place afterwards, for example along the supersymmetric Affleck–Dine mechanism (closely related to the dynamics of inflation). For all these reasons, to establish the leptogenesis mechanism is even more difficult than to establish the seesaw mechanism as the origin of neutrino masses. In this sense one could say that leptogenesis remains more a fascinating paradigm—that could well have been realized in Nature!—than a framework that one could test and study on and on with always greater accuracy.

Acknowledgments

I warmly thank D Aristizabal Sierra, S Blanchet, R Franceschini, J-M Frère, M Frigerio, F-X Josse-Michaux, E Ma, M Papucci, M Raidal, G Senjanovic, U Sarkar, A Strumia, M Tytgat and G Vertongen for many discussions and collaborations on leptogenesis. This work is supported by the FNRS-FRS, the IISN and the Belgian Science Policy (IAP VI-11). Many thanks also to the Departamento de Física Teórica (UAM-Madrid) and the IFT-Madrid for hospitality, and to the Comunidad de Madrid (Proyecto HEPHACOS S2009/ESP-1473).

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