

Lower limits on $\mu \rightarrow e\gamma$ from new measurements on U_{e3}

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New data on the lepton mixing angle θ_{13} imply that the $e\mu$ element of the matrix $m_\nu m_\nu^\dagger$, where m_ν is the neutrino Majorana mass matrix, cannot vanish. This implies a lower limit on lepton flavor violating processes in the $e\mu$ sector in a variety of frameworks, including Higgs triplet models or the concept of minimal flavor violation in the lepton sector. We illustrate this for the branching ratio of $\mu \rightarrow e\gamma$ in the type II seesaw mechanism, in which a Higgs triplet is responsible for neutrino mass and also mediates lepton flavor violation. We also discuss processes like $\mu \rightarrow e\bar{e}e$ and $\mu \rightarrow e$ conversion in nuclei. Since these processes have sensitivity on the individual entries of m_ν , their rates can still be vanishingly small.

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I. INTRODUCTION

The observation of lepton mixing in the form of neutrino oscillations shows, without doubt, that there is physics beyond the Standard Model of elementary particles. To be precise, the presence of lepton flavor violation (LFV) has been established. While being well entrenched in the neutrino sector, the questions of how large LFV is in the charged lepton sector, and how it is connected to the quantities in the neutrino sector, arise. The power of the Glashow-Iliopoulos-Maiani mechanism [1] in the Standard Model ensures that, for instance, observation of $\mu \rightarrow e\gamma$ will be unambiguously a sign of new physics beyond the presence of “only” massive neutrinos. The question of whether this new physics is connected to neutrino mixing parameters is an extremely model-dependent one.

In this short paper, we point out an interesting new implication for scenarios in which LFV is governed by $m_\nu m_\nu^\dagger$, where m_ν is the neutrino mass matrix. In particular, the $e\mu$ entry of this matrix is of interest, as it is often responsible for $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, or muon-to-electron conversion in nuclei. The advantage of scenarios in which $m_\nu m_\nu^\dagger$ governs LFV is their predictivity; $m_\nu m_\nu^\dagger$ only depends on measurable neutrino oscillation parameters: both mass-squared differences including the sign of the atmospheric one, three mixing angles, and the Dirac CP phase. Until very recently, neutrino data allowed for the possibility that $(m_\nu m_\nu^\dagger)_{e\mu}$ vanishes, namely, when the lepton mixing matrix element $|U_{e3}|$ takes a small value around 0.015. However, recent results from T2K [2], Double Chooz [3], and finally, Daya Bay [4]¹ imply a

surprisingly large value of the lepton mixing matrix element $|U_{e3}|$ around 0.15:

$$|U_{e3}| = 0.153_{-0.015(0.055)}^{+0.014(0.039)}, \quad (1)$$

where we have given the 1σ and 3σ ranges. As we will see, this sizable value implies that $(m_\nu m_\nu^\dagger)_{e\mu}$ cannot vanish, and hence a lower limit on $(m_\nu m_\nu^\dagger)_{e\mu}$ arises. Correspondingly, lower limits on lepton flavor violating processes arise. Of course, the processes can still be unobservable because of too heavy masses of the additional particles which mediate the decays. However, the point here is that the flavor physics part of the problem cannot spoil observation anymore. Thereby, yet another possibility for LFV to hide from future experiments is ruled out.

A popular example for which the rates of LFV processes are functions of $m_\nu m_\nu^\dagger$ is the type II (or triplet) seesaw mechanism [6–11]. Here neutrino mass is generated by a Higgs triplet, which in turn can mediate LFV, and in particular leads to a branching ratio of $\mu \rightarrow e\gamma$ depending on $(m_\nu m_\nu^\dagger)_{e\mu}$. We focus here on the triplet seesaw mechanism, but point out that $m_\nu m_\nu^\dagger$ governs LFV also in classes of theories in which “minimal flavor violation” in the lepton sector is realized [12]. Minimal flavor violation assumes that Standard Model Yukawa couplings are the only sources of flavor symmetry breaking. This very economical and elegant concept was originally invented for the quark sector [13], but can be applied to the lepton sector as well [12], predictions for LFV rates depending however on the explicit operator realization. Also, for the supersymmetric triplet seesaw, with a very heavy triplet and universal boundary conditions [14], consequences of our observation arise, absolute rates depending, however, on a variety of additional parameters. Another explicit realization of $\text{Br}(\mu \rightarrow e\gamma) = f[(m_\nu m_\nu^\dagger)_{e\mu}]$ can be found in Ref. [15]; here neutrinos are Dirac particles within a particular two Higgs doublet model. There are presumably

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¹After completion of the paper, the RENO collaboration reported a new measurement [5], resulting in $|U_{e3}| = 0.163_{-0.014}^{+0.013}$ at 1σ . Our results hardly change by considering this range of values.

many more examples. For definiteness, we consider here only the triplet seesaw, where there are only two free parameters besides the ones governing neutrino oscillations, namely, the mass of the triplet and the vacuum expectation value of its neutral component.

The same result for $|U_{e3}|$ implies that $(m_\nu m_\nu^\dagger)_{e\tau}$ cannot vanish anymore, and lower limits on τe LFV processes arise. However, due to the approximate $\mu - \tau$ symmetry of lepton mixing, it holds that $(m_\nu m_\nu^\dagger)_{e\tau} \sim (m_\nu m_\nu^\dagger)_{e\mu}$. This implies that rates for τe LFV processes are of the same order as rates for μe LFV processes. Since future limits on the τe sector are expected to be less stringent than present constraints on the μe sector, those decay channels are not observable in this framework. This in turn implies that, for instance, observation of $\tau \rightarrow e\gamma$ will signal the presence of lepton flavor violation not depending on $m_\nu m_\nu^\dagger$.

The processes of $\mu \rightarrow 3e$ and $\mu - e$ conversion have some dependence on $(m_\nu m_\nu^\dagger)_{e\mu}$ as well. However, either the contribution of $(m_\nu m_\nu^\dagger)_{e\mu}$ is suppressed, or cancellations from other contributions can occur. Setting lower limits in the same sense as for $\mu \rightarrow e\gamma$ is not possible.

The paper is put together as follows: in Sec. II, we quantify the fact that new oscillation data for large $|U_{e3}|$ imply nonvanishing $(m_\nu m_\nu^\dagger)_{e\mu}$. Section III introduces the type II seesaw and relevant expressions for lepton flavor violating processes. A numerical study of the various constraints is performed in Sec. IV before we conclude in Sec. V.

II. NONVANISHING $|U_{e3}|$ AND NONVANISHING $(m_\nu m_\nu^\dagger)_{e\mu}$

In this section, we note the simple yet consequential fact that large $|U_{e3}|$ implies nonvanishing $(m_\nu m_\nu^\dagger)_{e\mu}$. As stated in the Introduction, a variety of scenarios and frameworks leads to LFV processes depending on the quantity $m_\nu m_\nu^\dagger$. Here, m_ν is the neutrino mass matrix which is given as

$$m_\nu = U \text{diag}(m_1, m_2, m_3) U^T, \quad (2)$$

where m_i are the three light neutrino masses and U is the Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix. Its standard parametrization is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\phi_1}, e^{i\phi_2}). \quad (3)$$

In Eq. (3), δ denotes the Dirac CP phase, while ϕ_1, ϕ_2 denote two Majorana phases. The quantities c_{ij} and s_{ij} represent $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively.

We consider here classes of theories in which LFV is governed by $m_\nu m_\nu^\dagger$. Note that this matrix is independent of the Majorana phases and the interesting off-diagonal entries are furthermore independent of the neutrino mass scale (recall that $m_\nu m_\nu^\dagger$ is the same quantity which appears in the classical Hamiltonian for neutrino oscillations). We plot in Fig. 1 the δ dependency of the off-diagonal elements of $m_\nu m_\nu^\dagger$, fixing the remaining parameters to their best-fit values. It is apparent (and well known) that $|(m_\nu m_\nu^\dagger)_{\mu\tau}|$ is larger than the other entries by 1 order of magnitude, that $|(m_\nu m_\nu^\dagger)_{e\mu}| \sim |(m_\nu m_\nu^\dagger)_{e\tau}|$, and that the variation of $|(m_\nu m_\nu^\dagger)_{\mu\tau}|$ with δ is much smaller compared to that of the other two off-diagonal entries. Such studies have been performed several times in the literature before [14,16,17] and also recently [18], and here we wish to focus on the implication of nonvanishing and sizable $|U_{e3}|$ on $|(m_\nu m_\nu^\dagger)_{e\mu}|$ and thus on $\mu \rightarrow e\gamma$.

In those cases, in which LFV depends on $m_\nu m_\nu^\dagger$, the $e\mu$ entry is of particular interest, as in the $e\mu$ sector the strongest experimental limits on LFV exist, and even stronger limits are to be expected in the near future [19,20]. The crucial flavor physics quantity is therefore $(m_\nu m_\nu^\dagger)_{e\mu}$. One might therefore wonder whether $(m_\nu m_\nu^\dagger)_{e\mu}$

can vanish in principle. This is indeed possible, and the result for $(m_\nu m_\nu^\dagger)_{e\mu} = 0$ is a rather simple formula:

$$|U_{e3}|_{(m_\nu m_\nu^\dagger)_{e\mu}=0} = \frac{1}{2} \frac{R \sin 2\theta_{12} \cot \theta_{23}}{1 \mp R \sin^2 \theta_{12}} \simeq \frac{1}{2} R \sin 2\theta_{12} \cot \theta_{23} \\ = \begin{cases} 0.0135^{+0.004(0.009)}_{-0.002(0.005)} & \text{normal} \\ 0.0141^{+0.003(0.009)}_{-0.003(0.005)} & \text{inverted} \end{cases}, \quad (4)$$

where the minus (plus) sign is for the normal (inverted) mass ordering and R is the positive ratio of the solar and the atmospheric mass-squared differences (Δm_{sol}^2 and Δm_{atm}^2 , respectively).² We have also given the implied value of $|U_{e3}|$ when the best-fit values as well as 1σ and 3σ ranges of the oscillation parameters from Ref. [22] are inserted. The value of $|U_{e3}|$ for which $(m_\nu m_\nu^\dagger)_{e\mu}$ vanishes is rather small, being of order 0.014. It has to be compared to the value of $|U_{e3}| = 0.153^{+0.039}_{-0.055}$ determined by Daya Bay, given in Eq. (1), which is significantly larger. This implies a nonzero lower limit on $(m_\nu m_\nu^\dagger)_{e\mu}$, and hence on

²Interestingly, the above condition on $|U_{e3}|$ requires in addition CP conservation, i.e., $\delta = 0$ and π , respectively. Note that with $f = m_\nu m_\nu^\dagger$ the Jarlskog invariant for leptonic CP violation in neutrino oscillations is proportional to $\text{Im}(f_{e\mu} f_{e\tau} f_{\mu\tau})$ [21]. Hence, the vanishing of an off-diagonal element of $m_\nu m_\nu^\dagger$ implies CP conservation.

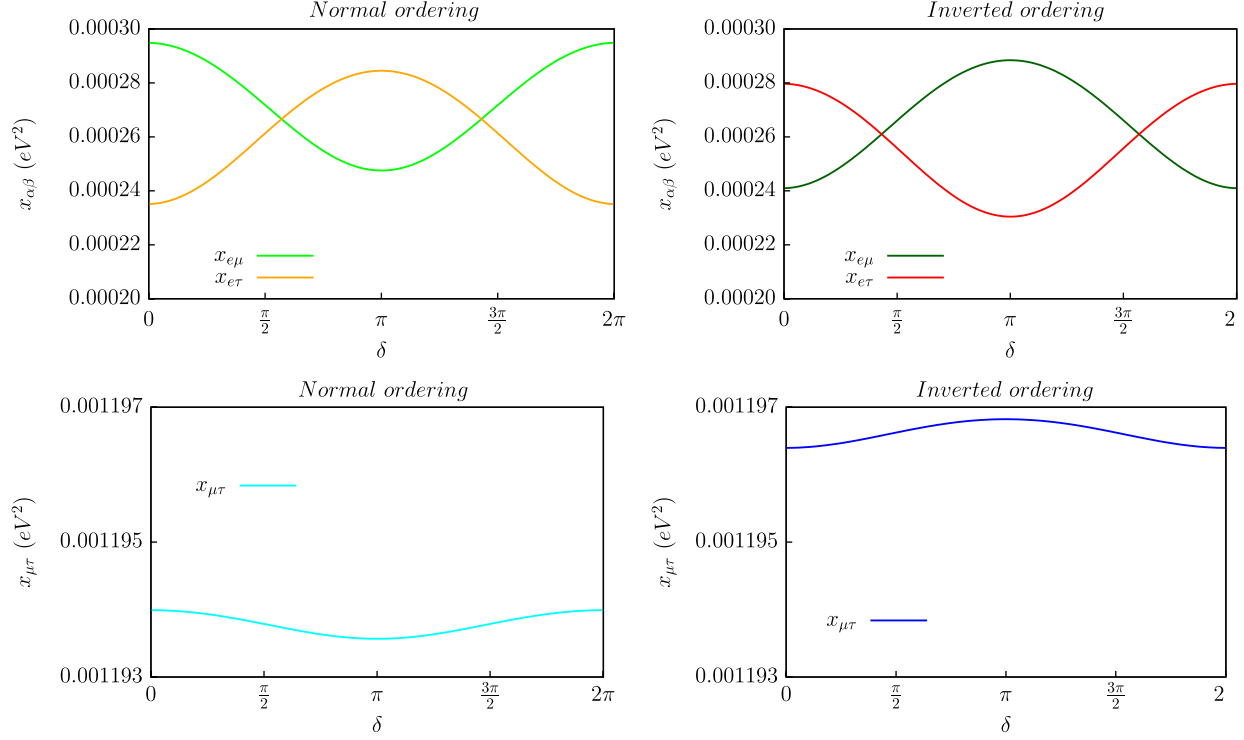


FIG. 1 (color online). Plots showing the effect of the Dirac CP -phase δ on various $x_{\alpha\beta}$ where $x \equiv |(m_\nu m_\nu^\dagger)|$. The remaining oscillation parameters are fixed at their best-fit values (see Table I).

branching ratios for lepton flavor violating processes in a variety of scenarios. This is the main point of this paper, and we will quantify this for the example of Higgs triplets in the type II seesaw mechanism. Values of the oscillation parameters in the 1σ and 3σ range are given in Table I. Using those values, the explicit range at 1σ and 3σ of $(m_\nu m_\nu^\dagger)_{e\mu}$ reads

$$|(m_\nu m_\nu^\dagger)_{e\mu}| [\text{eV}^2] = \begin{cases} 1.9 \times 10^{-4} - 4.5 \times 10^{-4} & (1\sigma) \\ 1.0 \times 10^{-4} - 3.5 \times 10^{-4} & (3\sigma), \end{cases} \quad (5)$$

with differences between the normal and inverted ordering not showing up before the second decimal place. Using the recent RENO result [5] would give minimal (maximal) values smaller (larger) by about $0.2 \times 10^{-4} \text{ eV}^2$.

In the same spirit, the large value of $|U_{e3}|$ has implications for LFV in the $e\tau$ sector. The condition for $(m_\nu m_\nu^\dagger)_{e\tau} = 0$ gives the following result:

$$|U_{e3}|_{(m_\nu m_\nu^\dagger)_{e\tau}=0} = \frac{1}{2} \frac{R \sin 2\theta_{12} \tan \theta_{23}}{1 \mp R \sin^2 \theta_{12}} \simeq \frac{1}{2} R \sin 2\theta_{12} \tan \theta_{23} \\ = \begin{cases} 0.0146^{+0.004(0.010)}_{-0.003(0.006)} & \text{normal} \\ 0.0153^{+0.003(0.009)}_{-0.003(0.006)} & \text{inverted} \end{cases}. \quad (6)$$

Similar to $(m_\nu m_\nu^\dagger)_{e\mu}$, one can evaluate the right-hand side of Eq. (6), giving similar numbers.

LFV processes in the $\tau\mu$ sector also have lower limits, since the relevant flavor quantity $(m_\nu m_\nu^\dagger)_{\mu\tau}$ cannot vanish. This was true even before the recent results on U_{e3} . At leading order, one finds

$$|(m_\nu m_\nu^\dagger)_{\mu\tau}| \simeq \frac{1}{2} \Delta m_{\text{atm}}^2 \sin 2\theta_{23} (1 - R \cos^2 \theta_{12}), \quad (7)$$

which is always nonzero. The order of magnitude of $(m_\nu m_\nu^\dagger)_{\mu\tau}$ is always larger than the one of $(m_\nu m_\nu^\dagger)_{e\mu}$:

$$\frac{|(m_\nu m_\nu^\dagger)_{e\mu}|^2}{|(m_\nu m_\nu^\dagger)_{\mu\tau}|^2} \simeq \frac{|U_{e3}|^2}{\cos^2 \theta_{23}} + 2|U_{e3}| \cos \delta \frac{\sin 2\theta_{12}}{\sin 2\theta_{23}} R. \quad (8)$$

TABLE I. Best-fit, 1σ and 3σ ranges of the oscillation parameters. Values of all the parameters (except $\sin^2 \theta_{13}$) are taken from Ref. [22]. For θ_{13} , the results of Daya Bay [4] have been used. Results applying to the inverted mass ordering are in square brackets.

Parameters	Best-fit	1σ range	3σ range
$\Delta m_{\text{sol}}^2 [\text{eV}^2] \times 10^5$	7.59	7.41–7.79	7.09–8.19
$\Delta m_{\text{atm}}^2 [\text{eV}^2] \times 10^3$	2.50	2.34–2.59	2.14–2.76
	$[-2.40]$	$[-(2.31-2.48)]$	$[-(2.13-2.76)]$
$\sin^2 \theta_{12}$	0.312	0.297–0.329	0.27–0.36
$\sin^2 \theta_{23}$	0.52	0.45–0.58	0.39–0.64
	$[0.52]$	$[0.46-0.58]$	$[0.39-0.64]$
$\sin^2 \theta_{13}$	0.0236	0.0190–0.0279	0.0097–0.0369
δ	-0.61π	$-1.26\pi-0.14\pi$	$0-2\pi$
	$[-0.41\pi]$	$[-1.11\pi-0.24\pi]$	$[0-2\pi]$

We will continue with a study focusing on the decay $\mu \rightarrow e\gamma$ in the type II seesaw, leaving a more detailed study of other decays and other scenarios for a future study. In general, however, the necessary existence of LFV in the $e\mu$ (and $e\tau$ sector) adds to the known existence of LFV in the $\tau\mu$ sector, and guarantees the presence of all three channels.

III. NONVANISHING BRANCHING RATIOS: EXAMPLE OF THE HIGGS TRIPLET

As mentioned before, we focus here on the type II or triplet seesaw mechanism. In this framework, neutrino masses are generated by interactions of lepton doublets L_α , with $\alpha = e, \mu, \tau$, with a weak triplet, hypercharge 2 scalar:

$$\mathcal{L} = h_{\alpha\beta} \bar{L}_\alpha^c i\tau_2 \Delta L_\beta + \text{H.c.}, \quad \text{where} \quad \Delta = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}. \quad (9)$$

Upon acquiring a vacuum expectation value (VEV) $\langle H^0 \rangle = v_\Delta/\sqrt{2}$, the neutrino mass matrix for light Majorana neutrinos is

$$(m_\nu)_{\alpha\beta} = \sqrt{2} v_\Delta h_{\alpha\beta}, \quad (10)$$

where $h_{\alpha\beta}$ are the neutrino Yukawa couplings. The interesting and potentially substantial part of this mechanism is that the members of the triplet induce LFV with couplings given in terms of Eqs. (9) and (10), i.e., in terms of, in principle, measurable parameters [16]. These parameters, together with the masses of the triplet members which are in principle accessible at colliders [17], allow for a scenario that is fully determinable and makes definite predictions for LFV.

Let us recapitulate the well-known formulas for the branching ratios [16]. For $\mu \rightarrow e\gamma$ one has

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{27\alpha}{256\pi G_F^2 M_{H^{++}}^4} \frac{|(m_\nu m_\nu^\dagger)_{e\mu}|^2}{v_\Delta^4} \text{Br}(\mu \rightarrow e\bar{\nu}\nu), \quad (11)$$

with $M_{H^{++}}$ as the triplet mass and $\text{Br}(\mu \rightarrow e\bar{\nu}\nu) \simeq 100\%$. The branching ratio for $\tau \rightarrow e\gamma$ is given by

$$\text{Br}(\tau \rightarrow e\gamma) = \frac{27\alpha}{256\pi G_F^2 M_{H^{++}}^4} \frac{|(m_\nu m_\nu^\dagger)_{e\tau}|^2}{v_\Delta^4} \text{Br}(\tau \rightarrow e\bar{\nu}\nu), \quad (12)$$

where $\text{Br}(\tau \rightarrow e\bar{\nu}\nu) = 17.82 \pm 0.04\%$ [23]. The analogous formula for $\text{Br}(\tau \rightarrow \mu\gamma)$ depends on $(m_\nu m_\nu^\dagger)_{\mu\tau}$. At this stage, combining Eqs. (11) and (12), we can rewrite Eq. (12) as

$$\text{Br}(\tau \rightarrow e\gamma) = 0.1782 \times \frac{|(m_\nu m_\nu^\dagger)_{e\tau}|^2}{|(m_\nu m_\nu^\dagger)_{e\mu}|^2} \text{Br}(\mu \rightarrow e\gamma). \quad (13)$$

TABLE II. List of constraints on different lepton flavor violating decays that have been used in our numerical analysis.

Process	Constraints	
	Present	Future
$\text{Br}(\tau \rightarrow e\bar{e}e)$	2.7×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow e\bar{e}\mu)$	1.8×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow e\bar{\mu}\mu)$	1.7×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow \mu\bar{\mu}\mu)$	2.1×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow \mu\bar{e}\mu)$	1.8×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow \mu\bar{e}e)$	1.5×10^{-08} [23]	1.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow \mu\gamma)$	4.4×10^{-08} [23]	2.0×10^{-09} [20]
$\text{Br}(\tau \rightarrow e\gamma)$	3.3×10^{-08} [23]	3.0×10^{-09} [20]
$\text{Br}(\mu \rightarrow e\gamma)$	2.4×10^{-12} [24]	1.0×10^{-13} [20]
$\text{Br}(\mu \rightarrow e\bar{e}e)$	1.0×10^{-12} [23]	1.0×10^{-13} [20]
$\text{R}(\mu \rightarrow e)$	7.0×10^{-13} ($^{197}_{97}\text{Au}$) [23]	2.0×10^{-17} ($^{27}_{13}\text{Al}$) [20,25]

In general, as stated earlier, $\text{Br}(\mu \rightarrow e\gamma)$ and $\text{Br}(\tau \rightarrow e\gamma)$ are of the same order of magnitude since $(m_\nu m_\nu^\dagger)_{e\mu} \sim (m_\nu m_\nu^\dagger)_{e\tau}$ due to the approximate $\mu - \tau$ symmetry of lepton mixing. The current limit on $\text{Br}(\tau \rightarrow e\gamma)$ is 3.3×10^{-8} [23], with a potential improvement to 3.0×10^{-9} in the SuperB facility [20] still being way below the current $\mu \rightarrow e\gamma$ limit. Recall that this was recently improved to [24]

$$\text{Br}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}, \quad (14)$$

and future limits to values down to 10^{-13} are foreseen [19]. Exact $\mu - \tau$ symmetry would result in $\frac{|(m_\nu m_\nu^\dagger)_{e\tau}|^2}{|(m_\nu m_\nu^\dagger)_{e\mu}|^2} = 1$ and thus $\frac{\text{Br}(\tau \rightarrow e\gamma)}{\text{Br}(\mu \rightarrow e\gamma)} \simeq 0.2$. In this case, a limit on $\text{Br}(\mu \rightarrow e\gamma) < 10^{-12}$ would correspond to $\text{Br}(\tau \rightarrow e\gamma) < 10^{-13}$, beyond the reach of upcoming experiments (see Table II). A careful study including the variation of the oscillation parameters shows that $\frac{\text{Br}(\tau \rightarrow e\gamma)}{\text{Br}(\mu \rightarrow e\gamma)} \simeq 0.15\text{--}0.21$ (both for 1σ and 3σ), and hence this conclusion remains valid. Thus, any evidence of $\tau \rightarrow e\gamma$ in near future experiments will rule out triplet seesaw models or any model in which $m_\nu m_\nu^\dagger$ governs LFV.

We should remark that $\mu \rightarrow 3e$ is also a very interesting process being mediated at tree level. The branching ratio for $\mu \rightarrow 3e$ is given by

$$\text{Br}(\mu \rightarrow 3e) = \frac{1}{16G_F^2 M_{H^{++}}^4} \frac{|(m_\nu)_{\mu e}|^2 |(m_\nu)_{ee}|^2}{v_\Delta^4} \text{Br}(\mu \rightarrow e\bar{\nu}\nu). \quad (15)$$

Unlike $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ or $\tau \rightarrow \mu\gamma$, the process $\mu \rightarrow 3e$ can yield an experimentally inaccessible branching ratio even with a recent θ_{13} value and low triplet masses, namely, when the ee or $e\mu$ elements of the Majorana neutrino mass matrix vanish. In this case, one-loop diagrams can provide the dominant contribution, depending on $(m_\nu m_\nu^\dagger)_{e\mu}$, the same flavor quantity that governs $\mu \rightarrow e\gamma$. Assuming that the decay is generated by e^+e^- pair creation from a virtual photon, the following ratio of branching ratio is found:

$$\frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e\gamma)} = \frac{\alpha_{em}}{3\pi} \left[\log \frac{m_\mu^2}{m_e^2} - \frac{11}{4} \right] \approx 1.5 \times 10^{-3}. \quad (16)$$

Thus, $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-12}$ implies $\text{Br}(\mu \rightarrow 3e) \sim 10^{-15}$. This illustrates the importance of experiments focusing on the dramatic improvement of limits on $\mu \rightarrow 3e$. We note that two proposals are under discussion, which aim to go down to 10^{-16} : one at PSI and one at the MuSIC facility in Osaka. Our finding applies to those possible experiments. However, there may be cancellations of this loop-suppressed $m_\nu m_\nu^\dagger$ contribution with a small tree-level contribution, so that lower limits on $\text{Br}(\mu \rightarrow 3e)$ are not as straightforward as the ones for $\mu \rightarrow e\gamma$. Since, in addition, the projects on $\mu \rightarrow 3e$ are not as advanced as the other LFV search experiments, we will not discuss this issue further, leaving it for further study.

A related remark concerns the supersymmetric type II seesaw case [14], in which a super-heavy supersymmetric triplet exists at the scale of $B - L$ breaking, and universal boundary conditions for the slepton masses, trilinear couplings, and gaugino masses are present. Renormalization group evolution from the grand unified scale to the triplet mass generates LFV that depends solely on $m_\nu m_\nu^\dagger$. Therefore, $\mu \rightarrow 3e$ depends now also on $(m_\nu m_\nu^\dagger)_{e\mu}$, i.e., on the same quantity as the branching ratio of $\mu \rightarrow e\gamma$. The relative factor between the two branching ratios is the same as in Eq. (16). In this case, the decay $\mu \rightarrow 3e$ is guaranteed to happen. To quantify the order of magnitude of the LFV rates one would need to specify the various supersymmetry parameters, which we will not do here.

Finally, the $\mu - e$ conversion rate in nuclei is given by [26]

$$R(\mu N \rightarrow e N^*) = \frac{\alpha^5 m_\mu^5 Z_{\text{eff}}^4 Z |F(q)|^2}{16\pi^4 M_{H^{\pm\pm}}^4 v_\Delta^4 \Gamma_{\text{capt}}} \left| \sum_{k=e,\mu,\tau} \frac{(m_\nu^\dagger)_{ek} (m_\nu)_{k\mu} F(r, s_k)}{3} - \frac{3(m_\nu^\dagger m_\nu)_{e\mu}}{8} \right|^2, \quad (17)$$

where

$$F(r, s_k) = \ln s_k + \frac{4s_k}{r} + \left(1 - \frac{2s_k}{r}\right) \sqrt{\left(1 + \frac{4s_k}{r}\right) \ln \frac{\sqrt{1 + \frac{4s_k}{r}} + 1}{\sqrt{1 + \frac{4s_k}{r}} - 1}}, \quad (18)$$

with $r = -\frac{q^2}{M_{H^{\pm\pm}}^2}$, $s_k = \frac{m_k^2}{M_{H^{\pm\pm}}^2}$, $k = e, \mu, \tau$. For $\mu N \rightarrow e N^*$ in different nuclei corresponding values of Z_{eff} , Γ_{capt} , $F(q^2 \simeq -m_\mu^2)$ can be obtained from Ref. [27]. The best current limit on the μ to e conversion ratio $R(\mu \rightarrow e)$ is 7×10^{-13} for $^{197}_{79}\text{Au}$ [23]. Future experiments (Mu2e, COMET, using $^{27}_{13}\text{Al}$) [25] are expected to reach a sensitivity of 2×10^{-17} in the near future. In the far future using $^{48}_{22}\text{Ti}$, the ratio is expected to be probed down to values of 10^{-18} [28]. As is obvious from Eq. (17), there are two contributions to the process, and it turns out that setting a lower limit on the rate of $\mu - e$ conversion is not possible, even with large $|U_{e3}|$. While the second contribution in $R(\mu N \rightarrow e N^*)$ is the same expression as in $\mu \rightarrow e\gamma$ and has a lower limit, it can be cancelled by the more complicated first term, which depends, in a complicated way, on the individual neutrino masses and Majorana phases. In fact, the rate of $\mu - e$ conversion under certain assumptions, can vanish for certain parameter values, as was recently shown in Ref. [18]. We will therefore not study this process anymore and will rather focus on the minimal $\text{Br}(\mu \rightarrow e\gamma)$ as implied by recent data on U_{e3} .

IV. RESULTS OF NUMERICAL ANALYSIS

Our observation is here that the large observed value of $|U_{e3}|$ implies that the branching ratio of the decay $\mu \rightarrow e\gamma$ cannot vanish, and hence a lower limit on its branching ratio arises. We quantify this finding now as a function of

the triplet VEV v_Δ and the triplet mass $M_{H^{\pm\pm}}$. When evaluating the minimal (and maximal) value of $\mu \rightarrow e\gamma$, we vary the neutrino oscillation parameters within the ranges given in Table I; their 1σ and 3σ ranges are from Ref. [22], and for θ_{13} we have considered the 1σ and 3σ ranges from Daya Bay [4]. The three CP phases were also varied in their allowed ranges. We took the current constraints on a large number of LFV processes into account, which are listed in Table II. Moreover, we also considered the case of when all processes obey limits obtainable in future experiments; most of the future limits have been taken from Ref. [20].

We have studied the variation of the lowest possible branching ratio for $\mu \rightarrow e\gamma$ with the triplet mass $M_{H^{\pm\pm}}$ for four different triplet VEVs, $v_\Delta = 0.5, 1.0, 5.0$, and 10.0 eV. In the course of investigation we have also considered the impact of the absolute neutrino mass scale (m_1 for normal hierarchy and m_3 for inverted hierarchy) for three different values, namely, $0.003, 0.05$, and 0.2 eV. These values are chosen in a fashion in which they not only covered the pure normal and inverted hierarchical ($m_{1(3)} = 0.003$ eV) scenarios, but also the quasidegenerate and intermediate cases. While the branching ratio of $\mu \rightarrow e\gamma$ does not depend on those masses, as well as on the Majorana phases, there is an indirect influence from the limits on the other LFV processes.

It is well understood from Eqs. (11), (12), (15), and (17) that the branching ratios will decrease for larger $M_{H^{\pm\pm}}$

and v_Δ . Consequently, if we ask that the stronger future constraints are obeyed, larger $M_{H^{\pm\pm}}$ and v_Δ are more favorable. Further, with light v_Δ , larger triplet masses are favorable. Of course, for sufficiently large values of triplet mass and VEV, some of these branching ratios will be inaccessible to the ongoing and even to the future experiments. In addition, there may arise situations when some of the processes remain unobserved while others have been seen. Such more complicated situations will be discussed elsewhere.

Varying over the oscillation parameters, one expects very similar behavior for the normal and inverted ordering (there are only tiny differences because the indirect constraints from other LFV processes depend on the mass ordering). Therefore, we only plot the normal ordering

case in Fig. 2. As can be seen, with lighter $v_\Delta = 0.5$ and 1.0 eV, the region with lighter triplet mass is excluded by the other LFV constraints. With the present constraints, there exists no allowed region for $v_\Delta = 0.5$ eV and $m_{1(3)} = 0.2$ eV. Obviously with a heavier triplet mass ($M_{H^{\pm\pm}} > 1$ TeV), such a conclusion no longer remains valid. Nevertheless, the scenario with a very heavy triplet has less appealing collider phenomenology. We have noted that throughout all the parameter space $\mu - e$ conversion possesses the most stringent bounds. With the future constraints, exclusion of the entire region with any values of the triplet mass and for $v_\Delta = 0.5$ and 1.0 eV, is solely due to the very stringent future $\mu - e$ conversion constraint [25]. As can be seen from Fig. 2, pushing the branching ratio of $\mu \rightarrow e\gamma$ down to 10^{-13} makes it

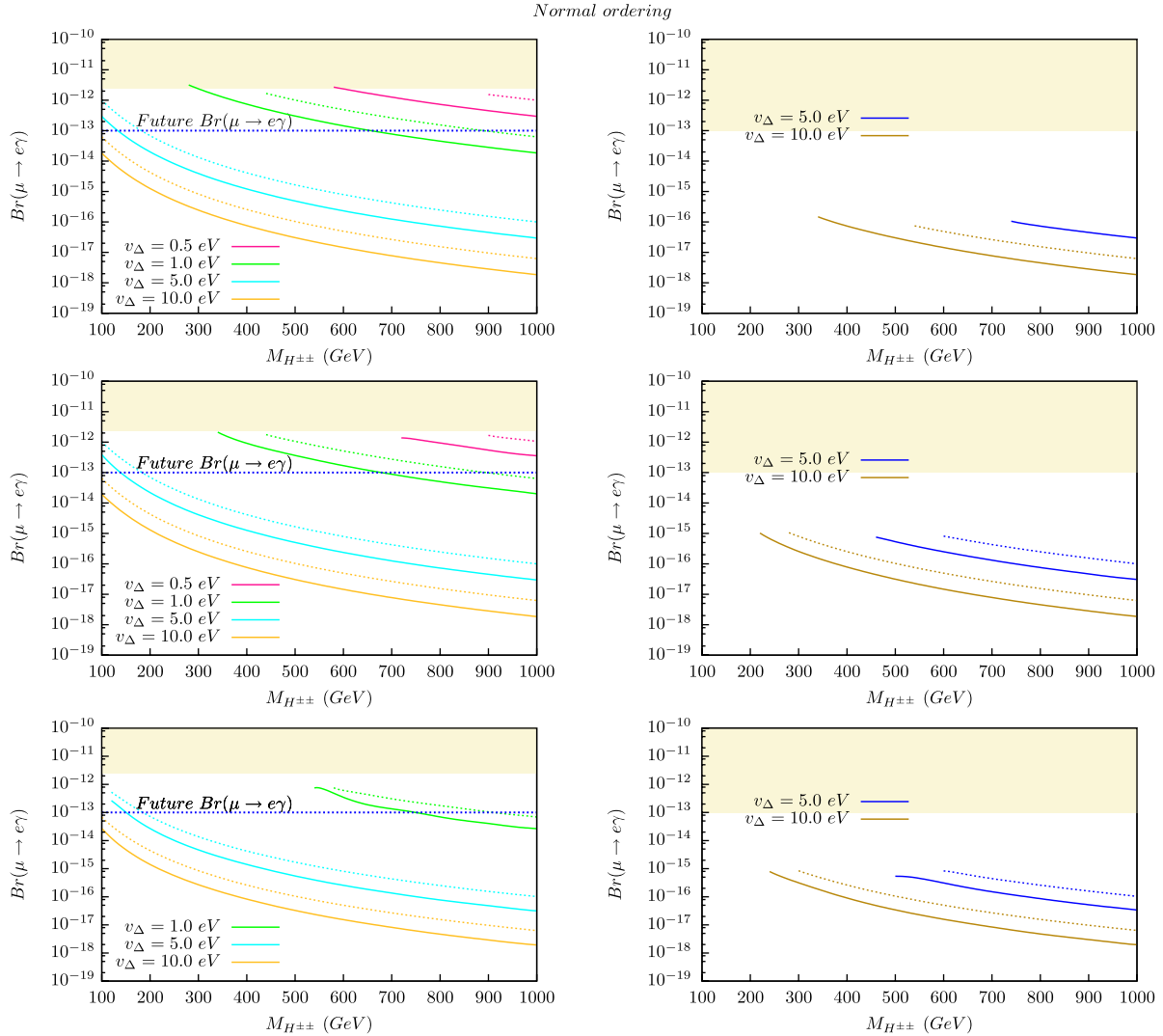


FIG. 2 (color online). Plots showing the variation of the lowest possible $\text{Br}(\mu \rightarrow e\gamma)$ vs $M_{H^{\pm\pm}}$ with different values of v_Δ for the normal neutrino mass ordering. The left plots are considering the present constraints on different LFV processes and the right plots are with the future constraints. Plots in the upper row are with the lightest neutrino mass $m_1 = 0.003$ eV, the middle row is for $m_1 = 0.05$ eV, and the lower row is for $m_1 = 0.2$ eV. The solid (dotted) line corresponds to the 3σ (1σ) range of the oscillation parameters. The colored (dark) band corresponds to the exclusion region as suggested by present and future experimental bounds. All constraints are listed in Table II. The corresponding plots for the inverted ordering look basically identical.

3σ , $m_1 = 0.003$ eV, $v_\Delta = 0.5$ eV, Normal ordering, Lowest of $Br(\mu \rightarrow e\gamma)$

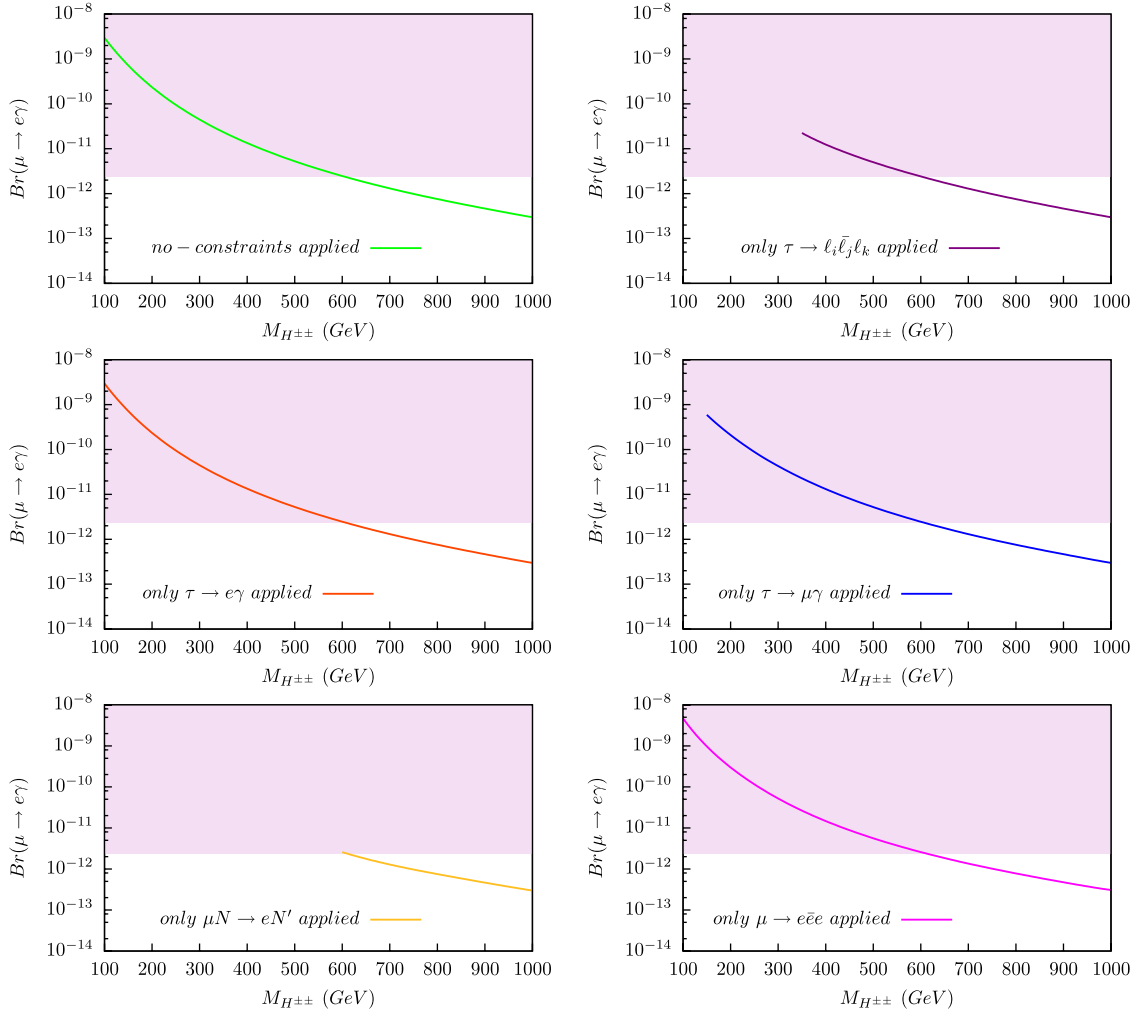


FIG. 3 (color online). Plots showing the effects of the constraints on other LFV processes on the minimal branching ratio of $\mu \rightarrow e\gamma$. The colored (dark) band corresponds to the exclusion region as suggested by the present experimental bound.

possible to definitely probe regions of the parameter space of v_Δ and $M_{H^{\pm\pm}}$. Examples are if $v_\Delta \lesssim 5$ eV and $M_{H^{\pm\pm}} \lesssim 200$ GeV, or when $v_\Delta \lesssim 1$ eV and $M_{H^{\pm\pm}} \lesssim 700$ GeV. We stress again that before the recent results on large U_{e3} were obtained, this was not possible. The effects of the constraints of the other LFV modes on the minimal value of $Br(\mu \rightarrow e\gamma)$ can be seen in Fig. 3. Two implications result when one switches on the other LFV limits: (i) the scale of $M_{H^{\pm\pm}}$ is set to larger values, and (ii) the lower limit on the branching ratio is increased by a moderate amount.

V. SUMMARY

Lepton flavor violation (LFV) may be connected directly or indirectly to neutrino oscillation parameters. In this paper, we worked in scenarios with presumably the most direct connection, in which the quantity $m_\nu m_\nu^\dagger$ is responsible for LFV in the charged lepton sector. Minimal flavor violation in the lepton sector, as well as

other frameworks and scenarios, has such a feature. We noted that recent results on the lepton mixing parameter U_{e3} imply that the $(m_\nu m_\nu^\dagger)_{e\mu}$ cannot vanish. Consequently, lower limits on lepton flavor violation arise, and we have quantified this with the example of $\mu \rightarrow e\gamma$ in the type II seesaw mechanism, in which a Higgs triplet is responsible for neutrino mass. We stress that many more examples in which our finding applies can be discussed.

We also shortly discussed processes as $\mu \rightarrow 3e$ and $\mu - e$ conversion, where $(m_\nu m_\nu^\dagger)_{e\mu}$ is also of relevance. However, either the contribution of $(m_\nu m_\nu^\dagger)_{e\mu}$ is suppressed, or cancellations from other contributions can occur. Setting lower limits in the same sense as for $\mu \rightarrow e\gamma$ is not possible.

While searches for lepton flavor violation do not need further motivation, we feel that our observation closes yet another loophole that would allow LFV to hide, and adds additional interest to study LFV in the $e\mu$ sector.

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