

Rigorous analysis of the parallel plate waveguide: From the transverse electromagnetic mode to the surface plasmon polariton

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[1] This paper presents an analysis of the parallel plate waveguide, based on a hybrid mode formulation. The nonideal metallic conductors of the waveguide are treated as a media characterized by an equivalent permittivity. The frequencies of interest in the presented analysis are at the terahertz band (from 300 GHz to 30 THz), and appropriate models are used for the correct characterization of metallic conductors at these frequencies. The behavior of the electromagnetic field of the fundamental mode is studied in a wide frequency range. At low frequencies (microwave regime) the fundamental mode is the well-known transverse electromagnetic (TEM) mode; as frequency increases, the electromagnetic field changes significantly and a surface wave or surface plasmon polariton (SPP) behavior is observed at the highest frequencies of the terahertz band. This paper shows a unified formulation that explains this transformation in the electromagnetic field behavior.

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1. Introduction

[2] The problem of surface waves was first studied by *Zenneck* [1907], demonstrating that radio waves can propagate over the surface of the ground or sea, and by *Sommerfeld* [1909] for calculating the radiation of a monopole over an infinite nonideal conducting plane. Several works came later, where surface waves have been analyzed and used in the field of microwaves [*Stratton*, 1941; *Barlow and Karbowiak*, 1953]. The surface wave is obtained at microwave frequencies by many different configurations, such as corrugated metallic surfaces or dielectric coated conductors [*Collin*, 1991].

[3] At optical frequencies the behavior of metallic conductors is far away from that of a perfect conductor and the electromagnetic field is strongly affected by the dispersive behavior of these materials. This dispersive behavior leads to electromagnetic field configurations where the field is essentially concentrated in the metal-dielectric interfaces. This field configuration is known at optical frequencies as surface plasmon polariton (SPP), [*Berini*, 2000; *Zakharian et al.*, 2007] and is the well-known surface wave at

microwave frequencies. One main feature to note is that at optical frequencies the surface wave is obtained due to the dispersive behavior of metallic conductors and at microwave frequencies it is obtained by means of the shape of the conductor and the dielectric materials. Excellent explanation about the connection of optics and microwaves are presented in the introductions of *Sihvola et al.* [2010] and *Yu et al.* [2010].

[4] The analysis of the ideal parallel plate waveguide (with perfect conductor) is carried out in several references [e.g., *Pozar*, 1997]. More elaborate studies are presented by *Mahmoud* [1991] and *Tayyar et al.* [2008] who analyze a parallel plate waveguide with different arbitrary surface impedance boundary conditions, or by *Ghamsari and Majedi* [2008] who use a nonideal metallic conductor. *Staffaroni et al.* [2011] propose an equivalent circuit of a parallel plate waveguide working at optical frequencies.

[5] Beyond the theoretical characterization, the parallel plate waveguide is often used in experiments and measurement systems. *Mendis* [2007] and *Mendis and Grischkowsky* [2001] present several systems for terahertz time domain spectroscopy, and obtain losses and dispersion of the parallel plate waveguide. It is worth noting the proposal of using the parallel plate waveguide like a lens [*Mendis and Mittleman*, 2010a], based on the concept of equivalent refractive index of a propagating mode. Other possible practical applications of the parallel plate waveguide at terahertz frequencies are presented by *Mendis and Mittleman* [2010b].

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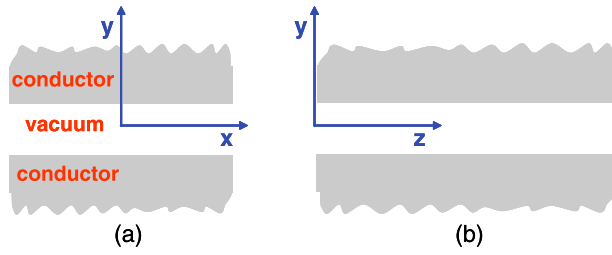


Figure 1. Parallel plate waveguide. (a) Cross-sectional view. (b) Longitudinal view.

[6] Recently, *Liu et al.* [2011] showed the experimental observation of plasmonic modes in a parallel plate waveguide. The experimental results presented by *Liu et al.* [2011] show the concentration of the electromagnetic field next to the edges caused by the corners of the finite parallel plate waveguide. The present work, also studying the same type of waveguide, is focused on the rigorous formulation that predicts the plasmonic effect of field concentration over the dielectric-conductor interface, using an accurate model description for the different media in the problem.

[7] The purpose of this paper is to present a rigorous formulation for analyzing the parallel plate waveguide in a very wide frequency range with accurate model for the conductors. This formulation predicts accurately the behavior of the electromagnetic field from microwave to optical frequencies. The change of the electromagnetic field of the fundamental mode from the classical TEM mode to the surface wave or SPP is specially emphasized. It is pointed out that the formulation shown in this paper explains the field patterns in both microwave and optical frequencies using the same underlying theory.

2. Rigorous Electromagnetic Field Formulation

[8] The parallel plate waveguide to be considered is shown in Figure 1. It consists of two metallic infinite plates separated by a distance of $2h$. The metallic plates are considered infinite in x and z directions. The propagation direction of the electromagnetic field is assumed to be z . This waveguide presents a symmetry plane at $y = 0$. Therefore, all the modes are represented by an even or an odd function in the y coordinate. This is equivalent to a perfect electric wall (PEW) or a perfect magnetic wall (PMW) at the xz plane.

[9] In order to take into account the behavior of the metallic conductors accurately, these are considered as another medium. The frequencies of interest in the presented analysis are in the terahertz band and thus the Drude model is used for the correct characterization of the metallic conductors [*Cai and Shalaev*, 2010; *Leal-Sevillano et al.*, 2010]. This model is based on the approximation of considering the metals like a free electron gas, and in that way, metallic conductors are modeled as dispersive media. The relative effective permittivity given by the Drude model is:

$$\epsilon_r = 1 + \frac{\sigma}{j\omega\epsilon_0(1 + j\omega\tau)} \quad (1)$$

The model assumes that there are no magnetic effects and thus the magnetic permeability is equal to the permeability of free space μ_0 .

[10] In the following analysis silver is assumed as reference metal with the parameters $\sigma = 2.463 \cdot 10^7$ (S/m) and $\tau = 1.5965 \cdot 10^{-14}$ (s) given by *Zakharian et al.* [2007]. Values of the Drude model parameters for other metals are given by *Cai and Shalaev* [2010].

[11] In that way, the waveguide to characterize is a nonhomogeneous waveguide with two different regions, due to the symmetry plane xz . In the first region ($i = 1, 0 < y < h$) vacuum is assumed and in the second region ($i = 2, h < y < \infty$) the equivalent dielectric permittivity of the Drude model (equation (1)) is used. Since the waveguide is nonhomogeneous a hybrid mode formulation is required; that is, each mode has, in principle, both longitudinal electric and magnetic components.

[12] The longitudinal components are obtained by solving the Helmholtz equations:

$$\left. \begin{aligned} \Delta_t \psi_{Ei} - \gamma_{ci}^2 \psi_{Ei} &= 0 \\ \Delta_t \psi_{Hi} - \gamma_{ci}^2 \psi_{Hi} &= 0 \end{aligned} \right\} \rightarrow \left. \begin{aligned} E_{zi} &= \psi_{Ei} e^{-\gamma z} \\ H_{zi} &= \psi_{Hi} e^{-\gamma z} \end{aligned} \right\} \quad i = 1, 2, \quad (2)$$

where $\gamma = \alpha + j\beta$ is the propagation constant and $\gamma_{ci}^2 = -\omega^2 \mu_i \epsilon_i - \gamma^2$, being ω the angular frequency and ϵ_i and μ_i the electric permittivity and magnetic permeability in medium i , respectively.

[13] The parallel plate waveguide is infinite in the x coordinate and only electromagnetic fields with y variation are considered ($\Delta_t = \frac{\partial^2}{\partial y^2}$). Then, the Helmholtz equations (equation (2)) reduce to simple second-order differential equations, which can be solved analytically:

$$\left. \begin{aligned} \psi_{E1} &= A_1 e^{-\gamma_{c1} y} + B_1 e^{\gamma_{c1} y} \\ \psi_{H1} &= C_1 e^{-\gamma_{c1} y} + D_1 e^{\gamma_{c1} y} \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \psi_{E2} &= A_2 e^{-\gamma_{c2} y} + B_2 e^{\gamma_{c2} y} \\ \psi_{H2} &= C_2 e^{-\gamma_{c2} y} + D_2 e^{\gamma_{c2} y} \end{aligned} \right\}$$

where $A_1, B_1, C_1, D_1, A_2, B_2, C_2$ and D_2 are, in principle, unknown complex constants, which will be determined by the boundary conditions. In order to ensure the proper behavior of the electromagnetic field when $y \rightarrow \infty$ and taking into account that γ_{c2} is a complex value with positive real part, the constants B_2 and D_2 must be zero.

[14] The transverse components are calculated from the longitudinal ones as:

$$\begin{bmatrix} \vec{E}_{ti} \\ \vec{H}_{ti} \end{bmatrix} = \frac{1}{\gamma_{ci}^2} \begin{bmatrix} \gamma \nabla_t & -j\omega \mu_i \hat{z} \times \nabla_t \\ j\omega \epsilon_i \hat{z} \times \nabla_t & \gamma \nabla_t \end{bmatrix} \begin{bmatrix} E_{zi} \\ H_{zi} \end{bmatrix}, \quad i = 1, 2. \quad (4)$$

The subindex t in the operators refers to the transverse coordinates.

[15] The continuity of the tangential components of the electromagnetic field at the interface $y = h$ is finally imposed in order to obtain the dispersion or characteristic equation of the waveguide. For the modes with PEW symmetry plane,

Table 1. Dispersion or Characteristic Equations

	PEW ^a Symmetry Plane	PMW ^b Symmetry Plane
TM modes	$\frac{\epsilon_1}{\gamma_{c1}} \cosh(\gamma_{c1}h) + \frac{\epsilon_2}{\gamma_{c2}} \sinh(\gamma_{c1}h) = 0$	$\frac{\epsilon_1}{\gamma_{c1}} \sinh(\gamma_{c1}h) + \frac{\epsilon_2}{\gamma_{c2}} \cosh(\gamma_{c1}h) = 0$
TE modes	$\frac{\mu_2}{\gamma_{c2}} \cosh(\gamma_{c1}h) + \frac{\mu_1}{\gamma_{c1}} \sinh(\gamma_{c1}h) = 0$	$\frac{\mu_2}{\gamma_{c2}} \sinh(\gamma_{c1}h) + \frac{\mu_1}{\gamma_{c1}} \cosh(\gamma_{c1}h) = 0$

^aPerfect electric wall.^bPerfect magnetic wall.

$C_1 = D_1$, $A_1 = -B_1$ and the following system of equations is obtained:

$$\begin{bmatrix} -e^{-\gamma_{c2}h} & 2 \sinh(\gamma_{c1}h) & 0 & 0 \\ \frac{\epsilon_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\epsilon_1}{\gamma_{c1}} \cosh(\gamma_{c1}h) & 0 & 0 \\ 0 & 0 & \frac{\mu_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\mu_1}{\gamma_{c1}} \sinh(\gamma_{c1}h) \\ 0 & 0 & -e^{-\gamma_{c2}h} & 2 \cosh(\gamma_{c1}h) \end{bmatrix} \cdot \begin{bmatrix} A_2 \\ B_1 \\ C_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (5)$$

This system can be divided into two uncoupled systems:

$$\underbrace{\begin{bmatrix} -e^{-\gamma_{c2}h} & 2 \sinh(\gamma_{c1}h) \\ \frac{\epsilon_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\epsilon_1}{\gamma_{c1}} \cosh(\gamma_{c1}h) \end{bmatrix}}_{A_{TM}} \begin{bmatrix} A_2 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and } C_2 = D_1 = 0$$

$$\underbrace{\begin{bmatrix} \frac{\mu_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\mu_1}{\gamma_{c1}} \sinh(\gamma_{c1}h) \\ -e^{-\gamma_{c2}h} & 2 \cosh(\gamma_{c1}h) \end{bmatrix}}_{A_{TE}} \begin{bmatrix} C_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and } A_2 = B_1 = 0 \quad (6)$$

The above systems of equations correspond to solutions of type TM and TE, respectively. The determinant of each system must be zero in order to obtain a nontrivial solution. This leads to the dispersion equations, $|A_{TM}(\gamma)| = 0$ and $|A_{TE}(\gamma)| = 0$. A similar derivation can be done for the modes with PMW symmetry plane ($A_1 = B_1$, $C_1 = -D_1$). The systems obtained for this symmetry are:

$$\underbrace{\begin{bmatrix} -e^{-\gamma_{c2}h} & 2 \cosh(\gamma_{c1}h) \\ \frac{\epsilon_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\epsilon_1}{\gamma_{c1}} \sinh(\gamma_{c1}h) \end{bmatrix}}_{A_{TM}} \begin{bmatrix} A_2 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and } C_2 = D_1 = 0$$

$$\underbrace{\begin{bmatrix} \frac{\mu_2}{\gamma_{c2}} e^{-\gamma_{c2}h} & 2 \frac{\mu_1}{\gamma_{c1}} \cosh(\gamma_{c1}h) \\ -e^{-\gamma_{c2}h} & 2 \sinh(\gamma_{c1}h) \end{bmatrix}}_{A_{TE}} \begin{bmatrix} C_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and } A_2 = B_1 = 0 \quad (7)$$

The dispersion equations are summarized in Table 1. It is worth noting that all the modes obtained are TM or TE, although, the formulation has started from the general case, assuming hybrid modes.

3. Results

(6) [16] The propagation constant of each mode is obtained from the dispersion equations by means of root finding in the

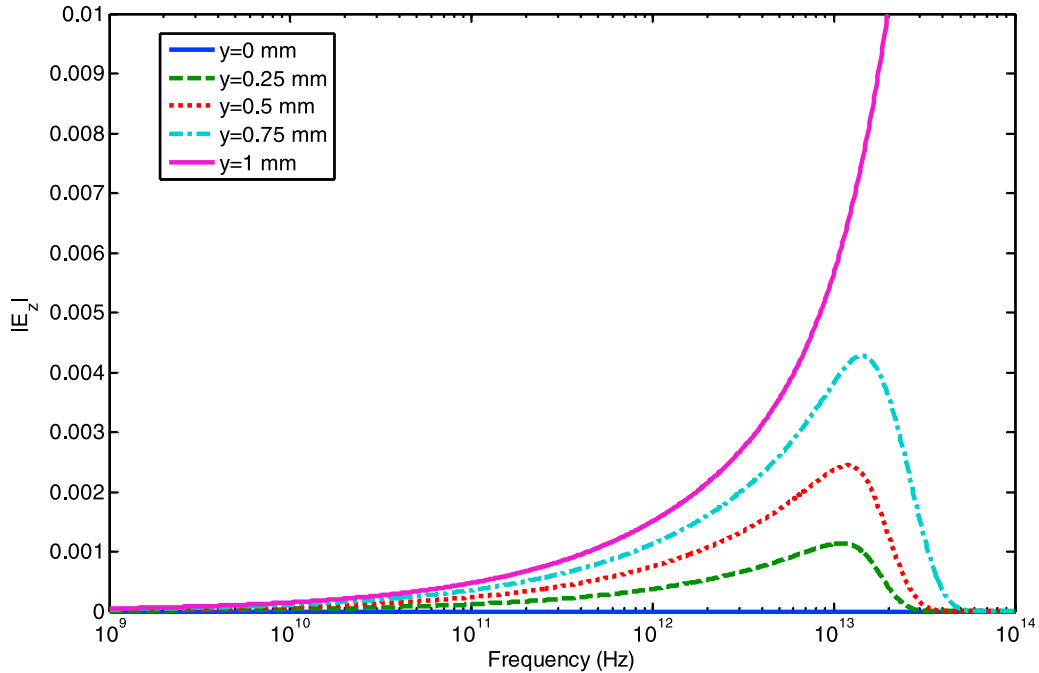


Figure 2. Magnitude of the longitudinal electric field for the fundamental mode of a parallel plate waveguide with 2 mm of height. The field is normalized by the maximum value of $|E_y|$ (transverse component).

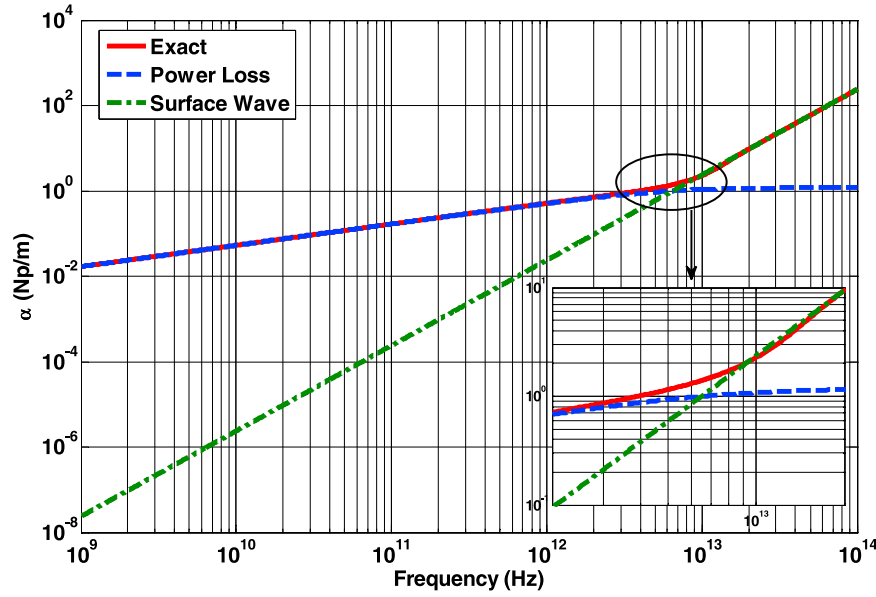


Figure 3. Real part of the propagation constant (α) for the fundamental mode of a parallel plate waveguide with 2 mm of height. Comparison of the exact, power loss, and surface wave solutions.

complex plane. It is highlighted that the propagation constants of a nonhomogeneous waveguide can present an arbitrary behavior with frequency and the root finding must be done for each frequency. The root finding is carried out by means of the Muller method [Press *et al.*, 1992] and the results of the ideal parallel plate waveguide (with perfect conductor) are used as initial guesses. Once the propagation constant of a mode is obtained, the field pattern can be derived by solving the appropriate system of equations (6) or (7).

[17] The fundamental mode is a TM mode (with PEW symmetry plane). The longitudinal component of the electric field of this fundamental mode tends to zero when the metallic conductor tends to be a perfect conductor. At low frequencies the metallic conductors behave as good conductors, thus the fundamental mode is accurately represented by the classical solution, the TEM mode. This behavior of the longitudinal component of the fundamental mode can be seen in Figure 2. Hence, the fundamental mode is a quasi-TEM mode, this is, at low frequencies tends to be a TEM mode.

[18] When frequency increases the behavior of metallic conductors change significantly. In the asymptotic limit (high frequency and electrically large height), the dispersion equation for the TM modes can be simplified. If the dispersion equation is expressed with exponential functions and the negative exponentials are neglected (large $\gamma_{c1}h$) the following dispersion equation is obtained:

$$\frac{\epsilon_1}{\gamma_{c1}} + \frac{\epsilon_2}{\gamma_{c2}} = 0. \quad (8)$$

The only solution of this characteristic equation is:

$$\gamma_{SW} = j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\frac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}}}. \quad (9)$$

This is the propagation constant of a surface wave or SPP propagating over a metal-dielectric interface [Sihvola *et al.*, 2010; Collin, 1991].

[19] In Figure 3 the real part of the exact propagation constant (attenuation constant) is compared with the classical approach obtained by the power loss method [Pozar, 1997; Collin, 1991], and with the surface wave, equation (8). As can be seen, the exact solution is quite similar to the power loss solution at frequencies until some terahertz, where a significant change in the behavior of the real part of the propagation constant is observed. At highest frequencies the exact solution fits perfectly with the SPP solution.

[20] For the imaginary part of the propagation constant, all the solutions (ideal, exact and surface wave) are quite similar in the analyzed frequency range, as can be seen in Figure 4. Thus, the imaginary part can be approximated by that of a plane wave propagating in vacuum.

[21] It is highlighted how the real part of the propagation constant affects the magnitude of the longitudinal component of the fundamental mode (see Figure 2), and also to the rest of the components of the electromagnetic field. This is further discussed now.

[22] Although the real part of the propagation constant of the fundamental mode always increases with frequency (see Figure 3), in such a wide frequency range it is more convenient, to represent the attenuation over a distance of one wavelength, this is:

$$Att_\lambda = 8.686 \alpha \lambda = 8.686 2\pi \frac{\alpha}{\beta}, \text{ [dB/wavelength]}, \quad (10)$$

where the factor 8.686 is for considering the attenuation in decibels and λ is the wavelength of the mode ($\frac{2\pi}{\beta}$).

[23] The attenuation over one wavelength is shown in Figure 5. As can be seen, the frequency where this attenuation is at a minimum is the inflection point where the behavior of the fundamental mode changes from the TEM to the surface wave (see Figure 3). This point can only be determined by means of the rigorous formulation here presented.

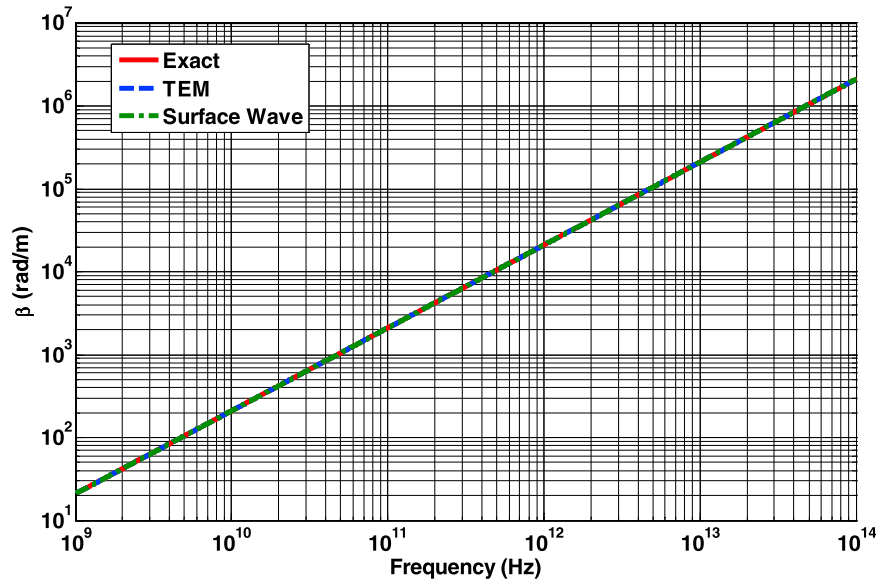


Figure 4. Imaginary part of the propagation constant (β) for the fundamental mode of a parallel plate waveguide with 2 mm of height. Comparison of the exact, power loss, and surface wave solutions.

[24] The power flux density of the fundamental mode is shown at different frequencies in Figure 6. At low frequency the power flux density in the longitudinal direction (z) is uniform in the vacuum region, thus corresponding to a TEM mode, as can be seen in Figure 6a. As frequency increases this behavior changes significantly to a surface wave, hence most of the power flux is concentrated over the metal-dielectric interface. In Figure 6b the power flux in the transverse direction is represented. This power flux represents the loss of the fundamental mode. In addition, it can be seen that the continuity of the normal component of the Poynting vector is fulfilled at the metal-dielectric interface.

[25] In Figure 7, the electric field (\hat{y} component) of the fundamental mode at different frequencies is shown. The change of the electric field with frequency clearly explains the aforementioned behavior of the fundamental mode. At low frequencies the fundamental mode is the classical TEM and at the highest frequency the SPP behavior of the electromagnetic field is observed.

[26] It is interesting to point out that the imaginary part of the propagation constant is very accurately represented by that of a plane wave propagating in vacuum. Thus, the real part of the propagation constant is representing the significant change in the field pattern of the fundamental mode. At low frequencies, the attenuation

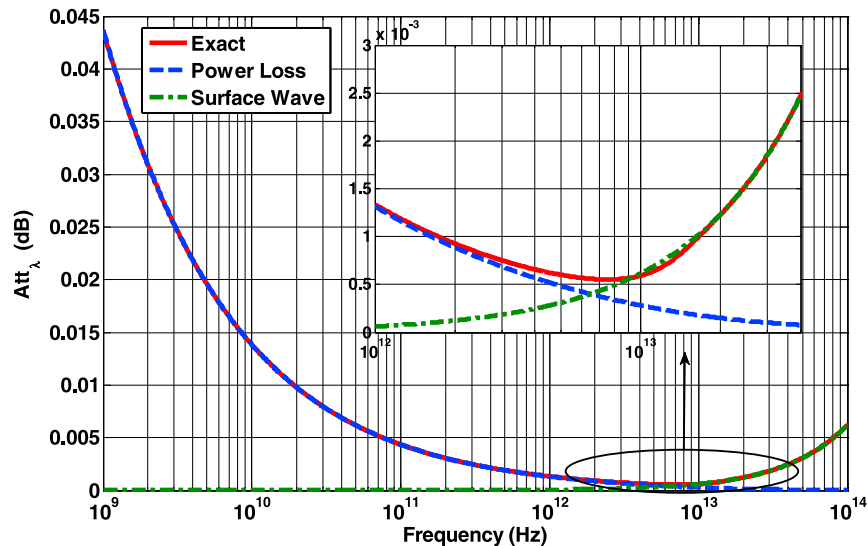


Figure 5. Attenuation in one wavelength for the fundamental mode of a parallel plate waveguide with 2 mm of height. Comparison of the exact, power loss, and surface wave solutions.

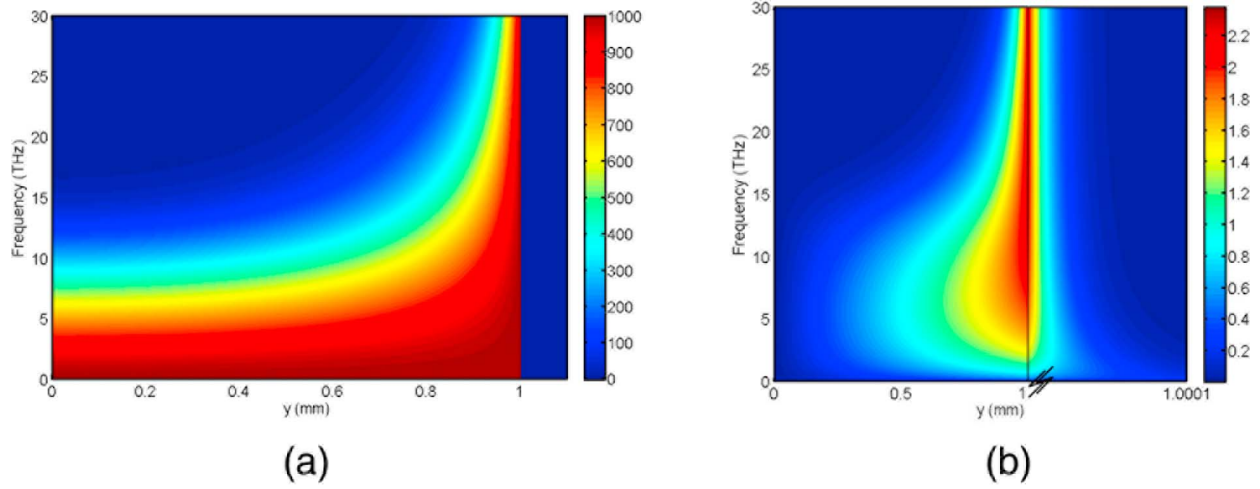


Figure 6. Power density of the fundamental mode of a parallel plate waveguide with 2 mm of height. (a) Longitudinal component of Poynting vector (S_z) in both the vacuum and the conductor regions. (b) Transverse component of Poynting vector (S_y) in both the vacuum and the conductor regions.

constant is that of a TEM mode. At highest frequencies, the attenuation constant is that of a SPP or surface wave. Between these two behaviors, where the attenuation over one wavelength reaches its minimum value, the evolution

from the TEM mode to the SPP is observed (10 and 15 THz in Figure 7).

[27] It is interesting to establish a formal definition of the frequency where the mode passes from a TEM-like behavior to a surface wave or SPP, f_{TEM-SW} . Liu *et al.* [2011]

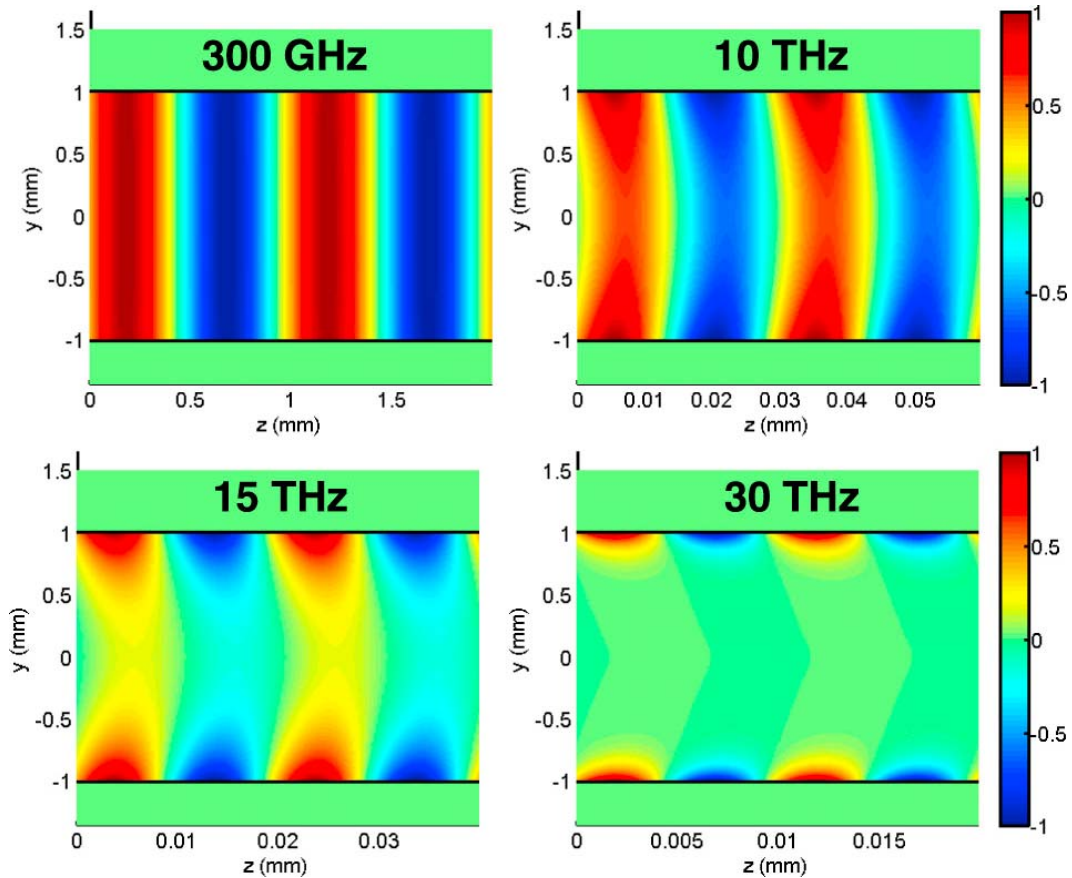


Figure 7. Electric field (\hat{y} component) for the fundamental mode of a parallel plate waveguide with 2 mm of height.

Table 2. Frequency Where the Mode Passes From a TEM-Like Behavior to a Surface Wave or Surface Plasmon Polariton for Different Heights

h (mm)	f_{TEM-SW} (THz)
5	2.5
1	6.5
0.1	22
0.03	41

proposed a definition of f_{TEM-SW} . Nevertheless, this definition is based on an arbitrary criterion rather than on the physical behavior of the electromagnetic field. A new definition is proposed here.

[28] As has been shown in the previous results, the change of the electromagnetic field from TEM-like to SPP is produced at frequencies close to the aforementioned inflection point. Thus, this is a change point in both the field pattern and in the propagation constant of the mode. Since there is not a significant change in the imaginary part of the propagation constant (the TEM mode and the surface wave, asymptotic solutions, have very similar phase constants), it is more convenient to define f_{TEM-SW} looking at the attenuation constants. Therefore, the proposed formal definition of f_{TEM-SW} is this inflection point, where both asymptotic limits intersect. Based on this definition it is possible to obtain a transcendental equation: f_{TEM-SW} is the frequency where the attenuation constant obtained by the power loss method and the one obtained for a surface wave are equal:

$$\alpha_{PL}(f_{TEM-SW}, h, \epsilon_{metal}) - \text{Re}\{\gamma_{SW}(f_{TEM-SW}, \epsilon_{metal})\} = 0, \quad (11)$$

where α_{PL} is the attenuation constant calculated by the power loss method and γ_{SW} is given by equation (9).

[29] Equation (11) provides the f_{TEM-SW} for a given parallel plate waveguide. As can be seen this frequency point

depends on the height of the waveguide and on the behavior of the metallic conductor. In Table 2 the calculated f_{TEM-SW} for different heights is provided.

4. Final Remarks

[30] With the exception of certain surface waves obtained in closed hollow metallic corrugated waveguides, the surface wave or SPP behavior of the electromagnetic field is usually obtained in open waveguides. In this case a surface wave has been obtained for the parallel plate waveguide. This fact needs some explanations.

[31] When the frequency increases, the separation between the metallic plates increases electrically. This affects to the electromagnetic field configuration and the aforementioned change is obtained, as is shown in Figure 7. The surface wave behavior is reached at highest frequencies, or, in other words, the electromagnetic field splits into two uncoupled surface waves. In fact, each new electromagnetic field corresponds to a surface wave propagating over a metal-dielectric interface, as was analytically demonstrated in section 3. This is, when the parallel metallic plates do not affect each other, the electromagnetic field configuration is equivalent to that of a surface wave on a metal-dielectric interface (an open waveguide). All these are illustrated in Figure 8.

[32] The reasoning is discussed now from optics to microwaves. Two independent SPPs propagate over each of the metal-dielectric interfaces. When frequency decreases, the extension of the electromagnetic field enhances into the air region. Finally, the two different fields are coupled leading to a new field configuration. This new field configuration tends to a TEM mode at microwave frequencies.

5. Conclusion

[33] The parallel plate waveguide has been analyzed by means of a rigorous hybrid mode formulation. The nonideal behavior of metallic conductors has been completely taken

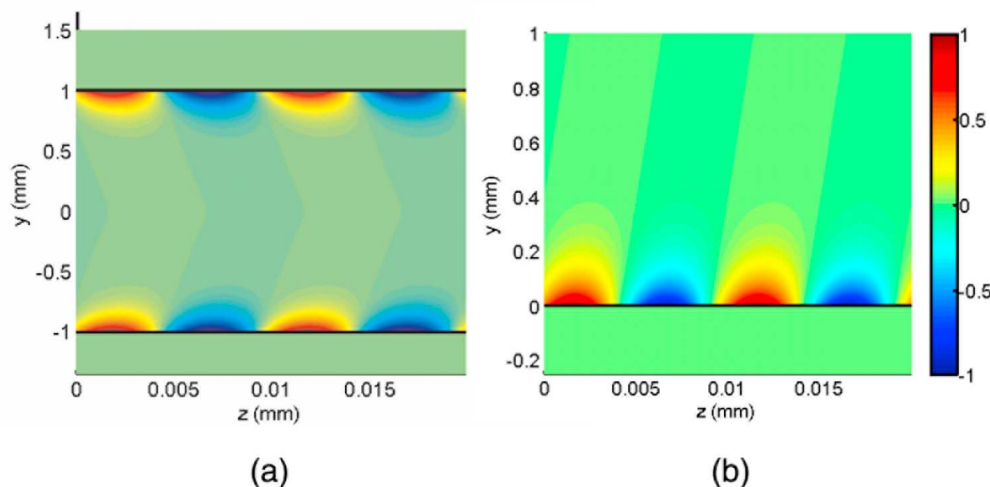


Figure 8. (a) Electric field (\hat{y} component) for the fundamental mode of a parallel plate waveguide with 2 mm of height at 30 THz. (b) Electric field (\hat{y} component) of a surface wave propagating over a metal-air interface at 30 THz.

into account. The analysis has been carried out at terahertz frequencies and the Drude model has been used for the correct modeling of metallic conductors at these frequencies. It is worth noting that the parallel plate waveguide is a canonical problem and the complete formulation has been derived analytically.

[34] The results obtained for the fundamental mode show that at low frequencies is perfectly characterized by the TEM mode of the ideal waveguide (with perfect conductors). When the frequency increases the electromagnetic field distribution changes significantly. At the highest frequencies of the terahertz band a surface wave or SPP behavior of the electromagnetic field is obtained. The surface wave behavior in the electromagnetic field pattern agrees with the asymptotic solution of the dispersion equation: this is the propagation constant of a surface wave propagating over a metal-dielectric interface. The evolution of the fundamental mode from the TEM mode to the SPP has been obtained. Furthermore, based of the obtained results with a rigorous formulation a general transcendental equation has been defined for determining the frequency where the mode changes from a TEM-like behavior to a SPP.

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References

- Barlow, H. M., and A. E. Karbowiak (1953), An investigation of the characteristics of cylindrical surface waves, *Proc. IEE*, 100, 321–328.
- Berini, P. (2000), Plasmon-polariton wave guided by thin lossy metal films of finite width: Bound modes of symmetric structures, *Phys. Rev. B*, 61(15), 10,484–10,503, doi:10.1103/PhysRevB.61.10484.
- Cai, W., and V. Shalaev (2010), *Optical Metamaterials: Fundamentals and Applications*, Springer, New York.
- Collin, R. E. (1991), *Field Theory of Guided Waves*, 2nd ed., IEEE Press, New York.
- Ghamsari, B. G., and A. H. Majedi (2008), Terahertz transmission lines based on surface waves in plasmonic waveguides, *J. Appl. Phys.*, 104, 083108, doi:10.1063/1.3000444.
- Leal-Sevillano, C. A., J. A. Ruiz-Cruz, J. R. Montejó-Garai, and J. M. Rebollar (2010), Simple models for the analysis of waveguiding systems at the terahertz band using classical microwave approaches, paper presented at 4th European Conference on Antennas and Propagation, Eur. Assoc. on Antennas and Propag., Barcelona, Spain, 12–16 Apr.
- Liu, J., R. Mendis, and D. M. Mittleman (2011), The transition from a TEM-like mode to a plasmonic mode in a parallel-plate waveguides, *Appl. Phys. Lett.*, 98, 231113, doi:10.1063/1.3598404.
- Mahmoud, S. F. (1991), *Electromagnetic Waveguides: Theory and Applications*, IEEE Electromagn. Waves Ser., vol. 32, Peter Peregrinus, London, doi:10.1049/PBEW032E.
- Mendis, R. (2007), THz transmission characteristics of dielectric-filled parallel plate waveguides, *J. Appl. Phys.*, 101, 083115, doi:10.1063/1.2719669.
- Mendis, R., and D. Grischkowsky (2001), Undistorted guided-wave propagation of subpicosecond terahertz pulses, *Opt. Lett.*, 26, 846–848.
- Mendis, R., and D. M. Mittleman (2010a), A 2-D artificial dielectric with $0 < n < 1$ for the terahertz region, *IEEE Trans. Microwave Theory Tech.*, 58(7), 1993–1998, doi:10.1109/TMTT.2010.2050386.
- Mendis, R., and D. M. Mittleman (2010b), Multifaceted terahertz applications of parallel plate waveguide: TE₁ mode, *Electron. Lett.*, 46(26), 40–44, doi:10.1049/el.2010.3318.
- Pozar, D. M. (1997), *Microwave Engineering*, John Wiley, New York.
- Press, W., S. Teukolsky, W. Vetterling, and B. Flannery (1992), *Numerical Recipes in Fortran: The Art of Scientific Computing*, Cambridge Univ. Press, New York.
- Sihvola, A., J. Qi, and I. Lindell (2010), Bridging the gap between plasmonics and Zenneck waves, *IEEE Trans. Antennas Propag.*, 52(1), 124–136, doi:10.1109/MAP.2010.5466406.
- Sommerfeld, A. (1909), Über die Ausbreitung der Wellen in der drahtlosen Telegraphie, *Ann. Phys.*, 333(4), 665–736, doi:10.1002/andp.19093330402.
- Staffaroni, M., J. Conway, S. Vedantam, J. Tang, and E. Yablonovitch (2011), *Circuit Analysis in Metal-Optics*, SPIE, San Francisco, Calif.
- Stratton, J. A. (1941), *Electromagnetic Theory*, McGraw-Hill, New York.
- Tayyar, İ. H., A. Büyükkaksoy, and A. Işıkyer (2008), A Wiener-Hopf analysis of the parallel plate waveguide with finite length impedance loading, *Radio Sci.*, 43, RS5005, doi:10.1029/2007RS003768.
- Yu, N., Q. J. Wang, M. A. Kats, J. A. Fan, F. Capasso, S. P. Khanna, L. Li, A. G. Davies, and E. H. Linfield (2010), Terahertz plasmonics, *Electron. Lett.*, 46(26), 52–57, doi:10.1049/el.2010.2131.
- Zakharian, A., J. Moloney, and M. Mansuripur (2007), Surface plasmon polaritons on metallic surfaces, *Opt. Express*, 15, 183–197.
- Zenneck, J. (1907), Über die Fortpflanzung ebener elektromagnetischer Wellen längs einer ebenen Leiterfläche und ihre Beziehung zur drahtlosen Telegraphie, *Ann. Phys.*, 328(10), 846–866, doi:10.1002/andp.19073281003.
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