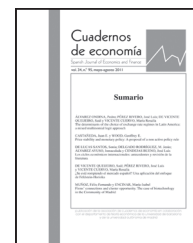




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ARTÍCULO

Predictability and self-similarity in demand maturity of tourist destinations: The case of Tenerife

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JEL CODES

D8;
E3;
R5;
Z1;
Z3

KEYWORDS:

Demand modeling;
Mature tourist
destinations;
ARFIMA

Abstract: This work aims to explore the behavior of a mature island tourist destination, Tenerife, according to the Autoregressive Fractional Differencing Integrated Moving Average (ARFIMA) model. This behavior will be compared to several other destinations, in an increasing geographical scale. Relevant lessons will arise for policy making at the short and long run. *Predictability* seems to be available but only for a limited horizon. *Self similarity* across geographic level seems to arise from the results. These findings complement previous conceptualizations like classic TALC (tourism area life cycle) based on long term predictability.

CÓDIGOS JEL

D8;
E3;
R5;
Z1;
Z3

PALABRAS CLAVE:

Modelización de la
demanda;
Destinos turísticos
maduros;
ARFIMA

Resumen: El objetivo de este trabajo es explorar el comportamiento de un destino turístico maduro, Tenerife, de acuerdo con el modelo Auto Regresivo y de Medias Móviles Fraccionalmente Integrado (ARFIMA por sus siglas en inglés). Este comportamiento se comparará con el de otros destinos en una escala geográfica creciente. Se podrán extraer diferentes conclusiones para la política turística a corto y largo plazo. La predictibilidad parece posible en el corto plazo. La auto-semejanza entre diferentes escalas geográficas también parece estar presente. Este artículo complementa las conceptualizaciones previas de los modelos TALC (Ciclo de Vida de las Áreas Turísticas) basadas en predicciones a largo plazo.

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<https://doi.org/10.32826/cude.v42i118.16>

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1. Introduction

Tourism is a major source of income in many regions but is also a source of concern because of its uncertain long-term sustainability. A key aspect of that sustainability is the way in which tourism activities fit in the economic portfolio of a given territory, and how this role is perceived by the local population. As in other aspects of policy making and management, the short and the long run are often presented as antagonistic values.

Today, after the global financial crisis, voices have arisen around the world asking for more emphasis on the long-term, while *short-termism* is pointed out as the most significant dangerous behavior that promotes corruption and crisis. In the context of tourism management, the short-term is highly related to the sinusoidal shape of demand, and the use of revenue management tools to improve income smoothness. Seasonality is often a reason for local communities to be sceptical of tourism activities as a relevant source of wealth and job quality. Baggio and Sainaghi (2016) developed several empirical works highlighting the importance of non-linear models and methods for the analysis of tourism demand series. These authors also confirmed that the mainstream tools in this arena are the well-known integrated autoregressive moving-average models (ARIMAs) proposed by Box and Jenkins (1970), or the SARIMA variation, which considers seasonal patterns (Cho, 2001). The focus is on modelling and forecasting demand, as long as, in the context of tourism, this ability is a key aspect in business development to reduce uncertainty (Li and Jiang, 2017).

There are both theoretical and practical questions about if these series are, to a certain extent, suitable for modelling and forecasting using the seasonal Autoregressive Fractional Differencing Integrated Moving Average (ARFIMA) model, which emphasizes the long-term. In this paper, we explore tourist demand data for the Canary Islands archipelago (an ultra-peripheral region of the European Union [EU]) in general, and Tenerife island (a major destination) in particular, as evolving destinations. Tenerife is a unique tourist area in the global context. It has fewer than 1 million inhabitants, but it receives more than 5 million tourists every year. Additionally, due to its political situation in the context of the EU, it combines a high level of self-government with a heavy integration with the countries where tourists originate (with the UK and Germany as the most popular places of origin). Despite the need to compete with other tourist destinations, in the Canaries, and particularly in Tenerife, a specific control on supply was implemented, a *moratorium* on new hotel establishments since 2001. The argument for this policy was mainly a diagnosis about expected demand. The idea was that, given a *predicted* decline in demand, it was necessary to prevent the accommodation sector from an excess of capacity. This policy and its effects are under intense debate even today, when it is gradually deactivated. For that reason, this article is relevant because we were able to collect sufficient evidence (a longer time series) of the development of this destination. This analysis will also be applied to other destinations, in an increasing geographical scale for the Canary Islands archipelago and Spain. Relevant lessons will arise for policy making at the short- and long-term. *Predictability* seems to be available,

but only for a limited horizon. *Self-similarity* across geographic level seems to arise from the results. These findings complement previous conceptualizations like classic TALC (tourism area life cycle) based on long-term predictability and a focus on long-term behaviour.

2. Background

Tourism is one of the most robust and resilient industries in the global economic portfolio. A comparison between its 2007-2009 global deceleration and the subsequent recovery within the same period for other economic sectors is highly illustrative. As stated by Butler (2009), it is not difficult to predict how global tourism is going to behave, as it would merely consist of an increment on previous observations on expenditure, international arrivals, or another relevant variable. The difficult task is to model, explain and forecast the behavior of concrete destinations. In the context of such a task, a key milestone is indeed the TALC (tourism area life cycle) proposed by Butler (1980), which identifies several stages in the development of tourist resorts from exploration to stagnation stages and subsequent further alternatives from rejuvenation to decline. But to what extent is it possible to offer a long-term forecast of tourist demand? The study of long memory processes is particularly relevant in this arena.

The Canary Islands archipelago is situated 100 km west of the African coast in the Atlantic Ocean. As part of Spain, it is also considered an ultra-peripheral region of the EU, but with a high level of autonomy to develop their own tourism policy. It comprises seven major islands with an especially mild climate that allows tourism activities in every season. As a tourist destination, it offers sandy beaches, volcanoes, natural parks and other protected areas, event scenarios, sport facilities and cultural and social attractions (Santana-Jimenez & Hernández, 2011). With around 2.1 million inhabitants in 2017, it registered about 94 million nights spent at tourist accommodation establishments, according to Eurostat (2017). This is largely due to their lower seasonality (Peña-Alonso, Ariza, Hernández-Calvento, & Pérez-Chacón, 2018) as their climate allows tourism activities all year. Tenerife and Gran Canaria are the two islands that lead these figures, prominently with sun and sand forms of tourism. Tourism in the islands is focused on the warm weather, friendly people and their image as a safe destination for travellers (Parra-López & Oreja-Rodríguez, 2014). Nowadays, the impact of their massive tourism implies that public authorities are developing policies to incorporate the necessary amount of sustainability in the islands' tourism (Tiago, Faria, Cogumbreiro, Couto, & Tiago, 2016). Several studies have focused on the Canary Islands archipelago in general and Tenerife island in particular as evolving destinations, using both reflections on subsequent policies and statistical methods, for different time intervals (Bianchi, 2004; Díaz-Pérez, Bethencourt-Cejas, & Álvarez-González, 2005; Garín-Muñoz, 2004; Gil-Alana, Cunado, & Perez de Gracia, 2008; González, 1999; Hernández-Martín, Álvarez-Albelo, & Padrón-Fumero, 2015; Ledesma-Rodríguez, Navarro-Ibáñez, & Pérez-Rodríguez, 2001, 2005; Martín-Santana, Beerli-Palacio, & Nazzareno, 2017; Oreja-Rodríguez, Parra-López, & Yanes-Estévez, 2008; Parra-López & Oreja-Rodríguez, 2014; Santana-Gallego, Ledes-

ma-Rodríguez, & Pérez-Rodríguez, 2011; Santana-Jimenez & Hernández, 2011). The global financial crisis, the rise of political and social instability in key competing destinations like those of the Middle East and North Africa, along with the availability of comprehensive publicly available statistical datasets make this study relevant.

Tenerife is the most populated island in the Canary Islands, with around 900,000 inhabitants in 2017. Tenerife, like the Canary Islands generally, is a well-known tourist brand in Europe due to the importance of British and German international arrivals, which are the most relevant markets “highly dependent on air transport” (Hernández Luis, 2004, p. 235). Unlike other tourist destinations around the Mediterranean or the Caribbean, Tenerife has no seasonality (Garau-Vadall, Díaz-Armas, & Gutierrez-Taño, 2013, p. 580). Additionally, Tenerife is the Canary Island that has received the most international visitors according to the Canary Islands Statistical Institute with more than 5,000,000 in 2017 (Instituto de Estadística de Canarias, 2018). The island tourism public authorities in the Tenerife Island Council (*Cabildo Insular de Tenerife*) focus their international marketing strategy in these markets to support stable growth despite the influence of the Spanish economic crisis of 2008 (Turismo de Tenerife, 2012, 2017). One special characteristic of the tourists in Tenerife is the high percentage of second and third visits (Ledesma et al., 2005, p. 2058), and having very loyal tourists is very important for Tenerife’s image (Martín-Santana et al., 2017). Tenerife has the peculiarity of being one of the first small islands in the world with two sites designated as World Heritage Sites: Mount Teide inside the National Park, the highest peak in Spain, and the city of San Cristóbal de La Laguna, which contribute to Tenerife being a “striking amalgam of landscapes” (Oreja-Rodríguez et al., 2008, p.57). In this sense, academic researchers agree that the peculiarity of Tenerife began with the arrival of mass tourism at the end of the 1960s and the beginning of the 1970s (González, 1999), leading the island to become highly dependent on tourism (Santana-Gallego et al., 2011, p. 107), with some negative side effects as a poor cultural tourism attractive offer (Parra-López & Oreja-Rodríguez, 2014). Currently, the sustainability of tourism income is a concern for the public authorities due to the impacts of mass tourism in the local population. Sustainability has been in the political agenda since 2001, when the government of the Canary Islands passed a law to restrict the construction of new hotels unless they were 5-star hotels or 4-star hotels with golf courses (Hernández-Martín et al., 2015) because tourism was in a stage of “infrastructural obsolescence and excessive degradation in some natural areas” (Santana-Jimenez & Hernández, 2011, p. 417). A related problem was created when tourism started to grow without a clear public strategy for sustainability because it was “secondary to the main need to design new and maintain existing tourism products” (Oreja-Rodríguez et al., 2008, p.56) and the worst tourism impacts were on the environment (Garau-Vadall et al., 2013, p. 584). This analysis and the subsequent policies were also based on the belief that there would be an imminent decline in demand.

Sustainability is being integrated into the public policies of tourism in Tenerife in order to address the locals’ negative opinion about the impact of tourism (Corral, Hernández, Navarro Ibáñez, & Rivero Ceballos, 2016). To achieve

this goal, it is necessary to develop a tourism public planning group (Farmaki, Altinay, Botterill, & Hilke, 2015, p. 181; Hatipoglu, Alvarez, & Ertuna, 2016, p. 306; Pjerotić, Radenović, & Tripković-Marković, 2016; Yuksel, Bramwell, & Yuksel, 1999, p. 351) with objectives and activities for the short-term because electoral cycles and economic evolution does not allow planning for the long-term (Agiomirgianakis, Serenis, & Tsounis, 2017, p. 36; Jamal & Getz, 1995, p. 199). In the planning process it is necessary to take into account the opinion of the private sector, social groups and citizens (Currie, Seaton, & Wesley, 2009, p. 56; Healy, van Riper, & Boyd, 2016, p. 582), as well as public sector stakeholders without forgetting the collaboration and cooperation between all of them (Beritelli, 2011; Jamal & Getz, 1995, p. 198; Jepsen & Eskerod, 2009, p. 335; Merinero-Rodríguez & Pulido-Fernández, 2016, p. 127; Pjerotić et al., 2016, p. 120; Remoaldo et al., 2017, p. 509; Williams, Penrose, & Hawkes, 1998, p. 884). Unlike “lack of coordination and cohesion within the highly fragmented tourism industry is a well-known problem to destination planners and managers” (Jamal & Getz, 1995, p. 186). Collaboration and cooperation in the public sector is more complex than it seems, because we must consider the positions of state governments, autonomous governments and local governments. According to Clarkson (1995), we differentiate between primary stakeholders, that are essential for the planning process, and secondary stakeholders, that can be seen as important due to their participation in the plan but without a direct presence in the planning organization (Domínguez-Gómez & González-Gómez, 2017, p. 137; Todd, Leask, & Ensor, 2017, p. 497). Working together for the same objectives is easier when all the stakeholders know the benefits of the planning process and every actor “identif[ies] and articulate[s] collective interests, establish[es] links as well as coordinate[s] negotiations” (Volgger & Pechlaner, 2014, p. 66). Collaboration and cooperation are the best ways to avoid future conflicts between stakeholders (Yuksel et al., 1999, p. 351). In this case, a tourism public plan should be led by the local government due to its proximity to the citizens (Bornhorst, Ritchie, & Sheehan, 2010, p. 572; Cheng, Hu, Fox, & Zhang, 2012, p. 33; Selcuk Can, Alaeddinoglu, & Turker, 2014, p. 191), as it works in Tenerife, without losing sight of the fact that the government of Spain and the government of the Canary Islands have much to say in the maintenance of infrastructure and in the integration of Tenerife into the Spanish public tourism strategy, among other initiatives (Gobierno de España, 2012). The main example in the recent years is the creation in 2010 of the Urbanist Consortium for the Rehabilitation of Puerto de la Cruz, the first tourist city of Tenerife, with the support of the Government of Spain but led by the island local government. In this case it is essential to have the support of the different stakeholders because the local governments do not have enough economic resources to adequately plan for tourism (Panyik, Costa, & Rätz, 2011, p. 1360; Williams et al., 1998, p. 884). There are many competences that local governments must develop, apart from tourism, and “local authorities are often the best placed organizations for (...) setting a strategy and balancing the interest of tourism enterprises, tourist and residents” (Selcuk Can et al., 2014, p. 193). Tourism public planning has been extensively analysed in different contexts, such as Cancún (Ambrosie, 2015), China (Cheng et al., 2012), Cyprus (Farmaki et al.,

2015), Ireland (Healy et al., 2016), Montenegro (Pjerotić et al., 2016), Portugal (Remoaldo et al., 2017), Singapore (Agiomirgianakis et al., 2017) and Turkey (Hatipoglu et al., 2016; Yuksel et al., 1999), due to its importance in ensuring the success of tourism in a given destination. Tourism needs a consistent tourism public policy that takes into account the growth possibilities of the destination, but it can be difficult to evaluate the decisions made in a specific place and a specific time, because public administrations do not always prepare a plan for this purpose. Besides “if not properly planned and managed, tourism can realise its negative impacts on host destinations” (Pjerotić et al., 2016, p. 120).

As stated previously, the public sector must develop adequate tourism public planning to ensure the continuous growth of a destination. In the first stage of the planning process, it is necessary to know more about the future, so forecasting changes in the number of tourists of a destination is a good option. Many studies have focused on illuminating the future on a short-term basis (Cole & Razak, 2009, p. 339) and facilitating the role of the public sector in the elaboration of a tourism policy (Chang, Sriboonchitta, & Wiboonpongse, 2009, p. 1743; Chu, 2009, p. 740; Chu, 2011, p. 1419; Peng, Song, & Crouch, 2014; Song & Hyndman, 2011, p. 817; Wan, Song, & Ko, 2016, p. 27). To make accurate predictions, authors have used different mathematical models that allow us to know what is going to happen over the next several years. Forecasting implies that the “future is a unique, linear, evolutionary process based upon past experiences” (Fernández-Güell & Collado, 2014, p. 83). In this sense, we highlight some papers that elaborate models in real time forecasting (Guizzardi & Stacchini, 2015) or papers that use web search analytics to make a successful forecast (Gunter & Önder, 2016; Li, Pan, Law, & Huang, 2017; Yang, Pan, Evans, & Lv, 2015). Despite the relevant mathematical models elaborated, the public sector is aware of the difficulty of forecasting because the economic, social and political changes that can occur in a destination can modify a forecast very quickly (Cole & Razak, 2009, p. 339; Yang et al., 2015, p. 387). The main mathematical models are based on time series algorithms like ARIMA, SARIMA or ARFIMA (Athanasopoulos, Hyndman, Song, & Wu, 2011; Brierley, 2011; Chang et al., 2009; Chu, 2009; Chu, 2011; Divino & McAleer, 2010; Vergori, 2012; Wong, Song, Witt, & Wu, 2007), multivariate regressions (Athanasopoulos & Hyndman, 2008; Hirashima, Jones, Bonham, & Fuleky, 2017) and the ARMAX model (Akal, 2004). It is important to remember that “no definitive criteria can be used to determine which forecasting method should be employed when a particular demand forecasting task is performed” (Wong et al., 2007, p. 1068).

The aim of this article is to explore a publicly available dataset using the seasonal ARFIMA(p, d, q; P, D, Q) estimation model, as proposed by Giraitis and Leipus (1995) and Reisen and Rodriguez (2006), which is also connected to ideas derived from chaos theory. Tourist demand has experienced a strong increase in recent years (2010-2018), partially explained by the flow of tourists unable to visit destinations in the Middle East and North Africa, because of political instability in these regions. Questions have arisen regarding the exactitude, usefulness and relevance of the maturity diagnosis, when confronting recent statistical data, and the precise impact of such phenomena.

Our discussion gravitates around the concept of predictability. If market demand follows the random walk hypothesis, then making forecasts is an unnecessary exercise, as long as the current demand levels reflect all available information and are open to change in the light of unexpected news. If a certain degree of predictability is possible, further questions arise: to what extent is it possible to make reliable forecasts? Could relevant breakpoints be efficiently detected? Are there long-term impacts of certain events in the studied phenomena? This leads to *nonlinearity* and the emergence of *chaos theory*, according to which a certain level of modelling and forecasting is possible. Self-similarity is another key concept in this scenario, as long as some links can be found across geographic frameworks.

Since the developments of Lorenz (1963), chaos theory has been present in diverse fields, connected to notions of limited short-term only predictability (a popular concept in meteorology), and self-similarity. Without being exhaustive, we consider it important to mention some details about these concepts:

- Lorenz presents a model (of atmospheric behavior) in which, from relatively simple equations, given certain initial conditions and specific values for the parameters, an apparently random behavior can be understood. This provides a new interesting view for researchers in many fields: perhaps complex behavior can have some hidden deterministic structure behind it, and this structure may be simpler than expected.
- These new types of systems tend to be very sensitive to initial conditions; therefore, they have a long memory.
- Other authors (e.g., Mandelbrot, 1983) developed a geometry based on the concept of fractal dimension. In fact, under this approach, dimensions do not need to take integer values (0 in a point, 1 in a line, 2 in a plane figure, 3 in a body, like in Euclidean geometry), but can present fractional values that can be accurately measured as ‘fractal’ dimensions. The profile presented by many economic and financial time series resembles this kind of graphical object.
- Another interesting feature of chaotic systems is self-similarity (e.g., smaller parts resemble the shape and properties of a bigger piece of it). This feature could help to link short-term behavior to long-term trends.

In this empirical paper, an exploration of the relationship between several tourist destinations, in increasing geographic scale, are attempted, in order to contribute to this ongoing discussion. The present article complements this view using ARFIMA processes (Granger & Joyeux, 1980) instead of Hurst exponents (Hurst, 1951) to approach the fractal nature of the processes involved.

3. Method

Several methods are available to analyse relevant economic time series data. In recent years, due to the inspiration of chaos theory, fractal geometry and several other instrumental advancements, along with a significantly growing availability of data, this type of empirical study has flourished. Mandelbrot (1983) dimensions relate to time series analysis by means of long run dependence systems or long memory processes. Given this, chaos theory concepts have a certain connection with a robust methodology for the analysis of time series: The Box-Jenkins methodology (1971) or ARIMA estimation process. Abraham-Frois (1998) offers a didactic view of this connection, which is not free of controversy (Graves et al., 2017), and that is the view we follow here.

Let y_t be a relevant time series for our purposes (the stock market price for a given firm was already mentioned, other proxies will be introduced later). When performing ARMA, the first step is to evaluate if the series is stationary, that is, if the mean and autocovariances of the series do not depend on time. Therefore, the time series first needs to be differenced until it is stationary. The number of times the series needs to be differenced to achieve stationarity is reflected in the d parameter (Box & Jenkins, 1976). When d is allowed to be a non-integer, then the result is a fractionally integrated, autoregressive and moving average estimation model (ARFIMA; see Appendix for some extra details on notation). Originally proposed by Granger and Joyeux (1980), the ARFIMA model follows the expression:

$$\phi_p(B)(1-B)^d y_t = \theta_q(B)\varepsilon_t \quad (1)$$

or

$$(1 - \sum_{i=1}^p \rho_i B^i) y_t (1-B)^d = (1 + \sum_{j=1}^q \theta_j B^j) \varepsilon_t \quad (2)$$

where $(1-B)^d$ allows for the fractional differencing of y_t in pursuit of stationarity, being ρ_i and θ_j , respectively, the p and q correspond to AR(p) and MA(q) estimations, B is the 'lag' operator and ε_t is the usual random residual.

$$B y_t = y_{t-1}; B^p y_t = y_{t-p} \quad (3)$$

$$B \varepsilon_t = \varepsilon_{t-1}; B^q \varepsilon_t = \varepsilon_{t-q} \quad (4)$$

Capital D , P and Q are also added to account for seasonal effects, like in the classic ARIMA approach, being B^S such that $B^S y_t = y_{t-S}$, with $S=12$ in this paper with monthly data.

$$\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D y_t = \theta_q(B)\Theta_Q(B^S)\varepsilon_t \quad (5)$$

As noted by Peters (1994), a non-integer value for d is connected to the concept of the fractal dimension D developed by Mandelbrot (not to be confused with the D for seasonal effects), as follows:

$$D = 3/2 - d \quad (6)$$

The *fractal* dimension is related to a set of objects for which dimension is not an integer, evolving from the classic Euclidean idea of dimension compared to the one studied

in fractal geometry. Demand data series are among candidates in social sciences to be analysed using this novel approach. The d parameter is also related to the popular *Hurst exponent* (Hurst, 1951), which is a measure of long memory in time series (Nile river behavior). This relationship allows researchers to establish certain boundaries and to some extent decipher the behavior of the corresponding time series, depending on the estimated d for the ARFIMA, as follows:

- If $d = 0$, the process does not present long memory, only short-term.
- If $0 < d < 0.5$, the process is persistent and presents long memory.

Estimating d in demand time series is relevant because, if significantly different from, it is related to long memory and to a certain degree of predictability. Additionally, the correct modelling of a time series allows for a more efficient detection of structural breaks. This is particularly interesting in the tourism industry, which is subject to political instability and to the entry into force of several regulatory frameworks. Several other studies attempt to find nonlinear and/or chaos behavior in market dynamics, as summarized by Barnett and Serletis (2000). We compare classic SARIMA processes with the corresponding ARFIMA ones. The comparison is made between the class of seasonal ARIMA models, SARIMA($p,d,q; P,D,Q$), with an integer nonnegative value of d , and the alternative given by the seasonal ARFIMA specification with a non-integer value of d . This study considers the analysis of monthly time series of tourist demand to three main destinations, Tenerife, Canary Islands and Spain, where tourist demand is measured by the total number of international monthly tourist arrivals obtained from public online databases of Eurostat, along with the Spanish National Institute of Statistics and the Institute of Statistics of the Canary Islands. Sample period is January 1996 to December 2017, which means 264 for estimation plus 3 'out of sample' observations till March 2018, for forecasting evaluation. Model selection is based on the usual test for individual significance of parameters (usual 1% and 5% levels), normality of resulting residuals under Jarque-Bera test, global significance of the model using F statistics along with minimizing information criteria among models. Out of sample forecasts will be also analysed. 1% of significance is used in all statistical tests.

Relationships between these datasets are explored, with theoretical implications for the fields of Economics and Tourism Management. Estimation of d was performed using EViews9 (based on Sowell, 1992; Doornik & Ooms, 2003). In particular, estimation was performed using the standardized Maximum Likelihood (ML). For ARFIMA estimation, the fractional difference parameter is initialized using the Geweke and Porter-Hundlak (1983) log periodogram regression (Automatic), a fixed value of 0.1. The information matrix estimate is computed using the outer product of the gradients (OPG). Optimization algorithm included in Eviews is also based on the Berndt-Hall-Hausman (BHHH) algorithm. At the time of this writing, this software does not offer the technical capabilities to perform fractional D (for the seasonal component), so D remains integer in both SARIMA and seasonal ARFIMA specifications, with $D=3$, 6 or 12 in several models (monthly data). This is a limitation of

this empirical work that will be subsequently overcome in future research. This research contributes to the literature because it shows the potential of a new tool for the analysis of relevant time series in order to monitor the behavior of tourists.

4. Results

Preliminary results are provided below. Figure 1a. shows the three analysed time series and provides a view of their interconnection. Figure 1b. offers a zoom in order to highlight the interrelated seasonality these data present. According to this, high season for Spain corresponds to low demand in both Canaries and Tenerife. Table 1 offers a first view of the estimation of d for a set of suitable ARFIMA models, corresponding to the three geographic scales used. It is relevant to note that for the three destinations, estimations situate the differencing parameter d in the proximity of 0.45, corresponding to long memory. *Predictability* seems to be available, at least for a limited horizon 'beyond short term'. Seasonality is present at all levels and seems to be captured by both ARIMA models (summarized in Table 2) and its ARFIMA counterparts. ARFIMA offers better fit (62% Tenerife, 96% the Canaries, 99% for Spain, compared to their counterparts ARIMA, 47%, 60% and 98%, respectively), along with the possibility to consider the estimations at the 1%. Regarding the Akaike info criterion and the Schwarz criterion, these are very similar, slightly lower for ARIMA. Figure 2 illustrates the goodness of fit of the seasonal ARFIMA model to data in Tenerife.

Figure 1.a. Tourism demand in Tenerife, Canary Islands and Spain.

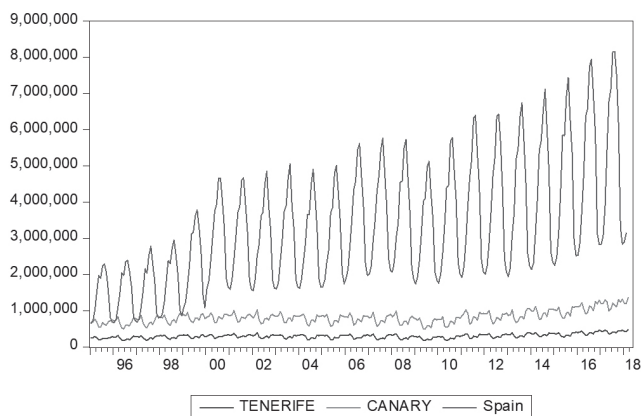


Figure 1.b. Seasonality in tourism demand: Tenerife, Canary Islands and Spain.

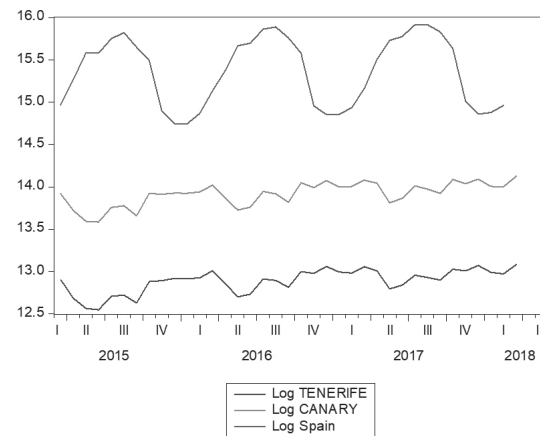
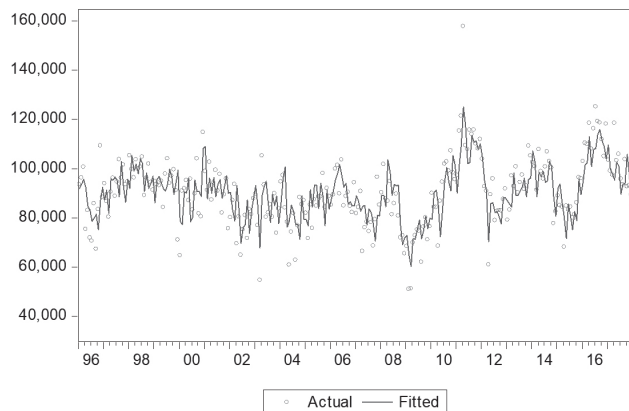


Table 1. d estimation at different geographical levels. Significance at 1% level.

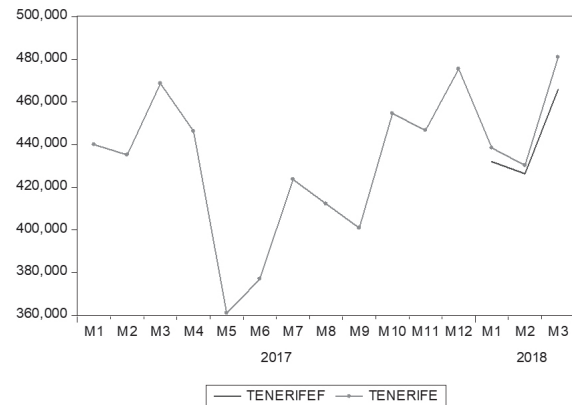
SARFIMA(p, d, q; P, D, Q)	Tenerife	Canary Islands	Spain
p	0	0	0
d	0.483884	0.494849	0.392802
q	0	0	0
P	6	6	12
D	12	6	3
Q	12	12	6
R2	0.619160	0.959103	0.990185
F Statistic =	105.2690	0.959103	6759.425
	Prob=0.000	Prob=0.000	Prob=0.000
Akaike info criterion	22.32780	24.14445	27.34976
Schwarz criterion	22.32780	24.21312	27.41587

Table 2. Comparative estimation using classic SARIMA. Significance at 1% level.

SARIMA(p, d, q; P, D, Q)	Tenerife	Canary Islands	Spain
p	2	0	0
d	1	0	0
q	0	3	3
P	12	12	12
D	12	12	9
Q	12	6	0
R2	0.479630	0.600990	0.988424
F Statistic =	47.37580	64.51574	4457.211
	Prob=0.000	Prob=0.000	Prob=0.000
Akaike info criterion	22.28275	24.29263	27.48565
Schwarz criterion	22.36424	24.38745	27.56626

Figure 2. Seasonal ARFIMA modelling of Tenerife.

Regarding forecast evaluation, an 'out of the sample' forecast has been performed, extending the sample till March 2018 (3 observations more), and including actual data for these months from the same databases. Forecasting outperforming of seasonal ARFIMA is summarized in table 3 and confirmed specifically for Tenerife, where all measurements are smaller for ARFIMA than for the corresponding ARIMA. The other forecasts were more precise (for Canaries and Spain) using ARIMA. This result is not surprising, as long as being ARIMA more capable to capture short term behaviour. But for Tenerife, seasonal ARFIMA provided the best value for the Theil coefficient (better when closer to 0). Additionally, figure 3 offers a view of the best forecast provided by seasonal ARFIMA for Tenerife against the 3 additional actual observations for 2018, as it presents the lower value of the Theil Inequality Coefficient.

Figure 3. Forecasting from seasonal ARFIMA for Tenerife, against actual observations for 2018.

In general, ARFIMA models were acceptable in comparison with their corresponding ARIMA estimations. After this exploratory stage, much more empirical work is required in order to extend the analysis to other island destinations, and to try to clarify the relationship between scales of analysis. The evolving capacity of mainstream statistical software and tests will allow us to consider further complexity in these models and to apply them to actual data of tourist destinations in order to better support policymaking.

5. Discussion and final remarks

Tenerife is unique due to the importance of tourism, which is thanks to the warm weather that allows tourist activities throughout the year. Tenerife is in the Canary Islands, a well-known brand in Europe, and is the island with the most year-round international arrivals. Tenerife has no seasonality, unlike many destinations in the Mediterranean and the Caribbean. Nowadays sustainability is a public concern for the public sector because of a sand and sun tourism model.

Table 3. Forecasting evaluation for seasonal ARIMA and ARFIMA estimations.

Q)	Tenerife	Canary Islands	Spain
SARFIMA(p, d, q; P, D, Q)	Root Mean Squared Error	Root Mean Squared Error	Root Mean Squared Error
	9703.639	90103.19	304614.4
	Mean Absolute Error	Mean Absolute Error	Mean Absolute Error
	8461.141	86435.32	279861.4
	Mean Abs. Percent Error	Mean Abs. Percent Error	Mean Abs. Percent Error
	1.834574	6.769337	9.439822
	Theil Inequality Coefficient	Theil Inequality Coefficient	Theil Inequality Coefficient
	0.010876	0.036975	0.048126
Q)	Tenerife	Canary Islands	Spain
SARIMA(p, d, q; P, D, Q)	Root Mean Squared Error	Root Mean Squared Error	Root Mean Squared Error
	13941.50	32983.79	296158.2
	Mean Absolute Error	Mean Absolute Error	Mean Absolute Error
	12739.93	31928.38	221096.7
	Mean Abs. Percent Error	Mean Abs. Percent Error	Mean Abs. Percent Error
	2.890617	2.553700	7.599518
	Theil Inequality Coefficient	Theil Inequality Coefficient	Theil Inequality Coefficient
	0.015324	0.013004	0.047400

As an island that is highly dependent on tourism, the current situation allows us to extrapolate ideas to other insular destinations

In this article, an initial exploration on the value of d for seasonal ARFIMA processes is attempted, using tourist demand data of a mature destination, the Tenerife island, along with several other geographic frameworks in increasing scale. These estimations of d are initially consistent with long memory, which suggests that these demand series deserve further exploration for modelling and forecasting. Some similarities are also found among destinations, which could lead to a deeper study on self-similarity across increasing geographic scale, along with the connection between the study of seasonality and cycle in mature destinations. Even admitting the fact that predictability is not an exclusive phenomenon characterizing long memory processes, and that both under short memory or nonstationarity, many different models can capture some existing degrees of dependencies that may help to predict, it is also truth that, under policymaking and corporate strategy, long memory and more ambitious forecasting horizons can be useful.

In harmony to previous works, in particular Chu's (2008) studies of Singapore, our findings also account for an outperforming of more sophisticated models like ARFIMA with respect to the classic ARIMA. We then contribute to the growing empirical findings in this domain, applied to tourist areas. Our study also contributes to the literature because it was developed in a post-crisis scenario and applied to a more specialized holiday coastal destination. For the specific case of the Canary Islands, we also add to the previous work of Gil-Alana and colleagues (2008), who focused on the seasonal component of the series before the global crisis. As explained before, the public sector must develop adequate tourism public planning to ensure the continuous growth of a destination. In particular, two main insights arise from both the reflection on the evolution of Tenerife and from our empirical results:

- *Predictability* is possible for tourist demand in Tenerife, due to the different suitable models that were possible to construct using the available data. This modelization suggests the presence of some long memory in this demand behaviour. This article, and further empirical works, could reinforce the idea that rigorous technical analysis is required before certain public policies are implemented.
- Regarding *self-similarity*, our results also find some evidence across geographic scenarios. This result suggests that a more efficient collaboration between different public bodies (at local, regional and national levels) is also required (Volgger & Pechlaner, 2014; Yuksel et al., 1999).

These findings, to some extent, can be considered as an additional insight to conceptualizations like those of the TALC (tourism area life cycle; Butler, 1980). Butler himself recognized the potential relevance of chaos theory, among other alternative views, to complete the toolkit of tourism demand modelling (Butler, 2009).

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Appendix

AR(p)

$$y_t = \rho_1 y_{t-1} + \dots + \rho_p y_{t-p} + \varepsilon_t$$

MA(q)

$$y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

ARMA(p,q)

$$y_t = \rho_1 y_{t-1} + \dots + \rho_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

$$y_t - \rho_1 y_{t-1} - \dots - \rho_p y_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

$$By_t = y_{t-1}; B^p y_t = y_{t-p}$$

$$B\varepsilon_t = \varepsilon_{t-1}; B^q \varepsilon_t = \varepsilon_{t-q}$$

$$\left[y_t - \rho_1 y_{t-1} - \dots - \rho_p y_{t-p} \right] = \left[\varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} \right]$$

$$\left[y_t - \rho_1 By_t - \dots - \rho_p B^p y_t \right] = \left[\varepsilon_t + \theta_1 B\varepsilon_t + \dots + \theta_q B^q \varepsilon_t \right]$$

$$\left[1 - \rho_1 B - \dots - \rho_p B^p \right] y_t = \left[1 + \theta_1 B + \dots + \theta_q B^q \right] \varepsilon_t$$

$$\left(1 - \sum_{i=1}^p \rho_i B^i \right) y_t = \left(1 + \sum_{j=1}^q \theta_j B^j \right) \varepsilon_t$$

$$\phi_p(B) y_t = \theta_q(B) \varepsilon_t$$

ARFIMA(p,d,q)

If y_t is not stationary, it is required to differentiate it d times. d is allowed to be a fractional number.

$$\left(1 - \sum_{i=1}^p \rho_i B^i \right) y_t (1-B)^d = \left(1 + \sum_{j=1}^q \theta_j B^j \right) \varepsilon_t$$

$$\phi_p(B) (1-B)^d y_t = \theta_q(B) \varepsilon_t$$