

# THE MODIFIED D1M INTERACTIONS: NEW GOGNY FORCES ADAPTED FOR NEUTRON STAR CALCULATIONS\*

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The symmetry energy of the D1 family of Gogny interactions, which describe finite nuclei properties in good agreement with experimental data, shows too soft a behaviour above saturation density. As a consequence, the D1 family of Gogny forces often cannot describe the properties of neutron stars when extrapolated to the high-density region. To overcome this limitation, we have proposed reparametrizations of the D1M interaction with very minimal changes. The modified interactions retain the quality of the original force for dealing with finite nuclei and, at the same time, are able to describe the neutron star physics with a quality similar to the one provided by the Skyrme SLy4 force, which was specially designed for this purpose.

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## 1. Introduction

Phenomenological effective interactions are very useful tools in nuclear physics. They simulate the true but complex nucleon–nucleon interaction

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allowing to handle the nuclear many-body problem in a relatively simplified way. Usually, these interactions depend on about a dozen of parameters, which are fitted to reproduce some experimental data, as for example the nuclear masses or charge radii in a Hartree–Fock–Bogoliubov (HFB) description. The Skyrme [1] and Gogny [2] forces in the non-relativistic domain and the Relativistic Mean Field Models [3] are well-known examples of this type of effective interactions. However, they may not work necessarily well when extrapolated to a different domain from the data where the parameters of the force are fitted. In this work, we explore the performance of the D1 family of Gogny interactions in the context of neutron stars (NS).

The traditional D1 family of Gogny interactions is made of the parametrizations D1, D1S, D1N and D1M. The original D1 Gogny force was proposed in the early eighties by Gogny with the aim to describe the mean field and the pairing field simultaneously using the same interaction [2]. It was soon realized that the fission properties of D1 were not the appropriate ones, leading to the introduction of D1S [4]. Large-scale HFB calculations performed with the D1S interaction [5] using a harmonic oscillator basis show a systematic drift in the binding energy of neutron-rich nuclei. To solve this limitation of the D1S force, new parametrizations of the Gogny interaction, namely D1N [6] and D1M [7], were proposed. The key ingredient in these two parametrizations is the reproduction of the microscopic neutron matter equation of state of Friedman and Pandharipande [8]. These four parametrizations have the same functional form and only differ in the values of the parameters. They constitute the D1 family of Gogny forces, different from a new proposal, denoted D2 in the literature [9].

In this work, and for the sake of simplicity, we consider NS matter to be made of neutrons, protons and electrons in charge and  $\beta$ -equilibrium. The mass and size of an NS are obtained from the solution of the Tolman–Oppenheimer–Volkov (TOV) equations, where the equation of state (EOS) of NS matter, which is directly related to the nuclear interaction, is an essential ingredient. Gogny interactions of the D1 family, such as D1S, D1N or D1M, are unable to predict a maximum NS mass of two solar masses, as required by recent astronomical observations [10, 11]. The D1M force, with its maximum NS mass of  $\sim 1.74M_{\odot}$  provides the closest value to observational data. In the case of the D1S interaction, no solution for the TOV equations is found [12–15]. As discussed in these references, the underlying problem lies in the fact that the symmetry energy, which determines the NS equation of state (EOS) and hence the maximal mass predicted by the model, is too soft at the high-density regime. We have tried to reconcile this problem by reparametrizing one of the most recognized Gogny interactions, the D1M one, to be able to explain the properties of NS, keeping its successful predictions in the domain of finite nuclei.

The contribution is organized as follows. Firstly, the fitting procedure of the new Gogny interactions D1M\* and D1M\*\* is briefly discussed in Section 2. Next, we show some global properties concerning the core and the crust of NS evaluated with these new forces D1M\* and D1M\*\*, which are compared with the corresponding predictions of the SLy4 interaction. Finally, a summary is given in the last section.

## 2. The modified D1M interactions. Fitting procedure

To address the problems stated earlier, we have introduced two new Gogny interactions dubbed D1M\* and D1M\*\* originally introduced in Refs. [14] and [15], respectively. The D1M\* interaction is calibrated starting from the D1M force and performing a minimal variation of its parameters in such a way that the slope of the symmetry energy is modified while retaining as much as possible the values of the nuclear binding energies and charge radii of the original D1M force. To do that, we vary the finite range strength parameters of the force, namely  $W_i$ ,  $B_i$ ,  $H_i$  and  $M_i$  ( $i = 1, 2$ ), keeping the ranges of the Gaussians, the zero-range part of the interaction and the spin-orbit force equal to their values in the original D1M interaction. Four of the eight free parameters are determined by demanding that the values of the saturation density, binding energy per particle, incompressibility modulus and effective mass in symmetric nuclear matter at saturation computed with D1M\* are the same as in the original interaction D1M. It is known that binding energies of finite nuclei constrain the symmetry energy better at an average density of about  $0.1 \text{ fm}^{-3}$  than at saturation density [16]. Therefore, we fix another parameter of the D1M\* interaction by imposing the symmetry energy at a density of  $0.1 \text{ fm}^{-3}$  to be the same as the one computed with the D1M interaction. In order to preserve pairing properties in the  $S = 0$ ,  $T = 1$  channel, we impose that the combinations  $W_i - B_i - H_i + M_i$  ( $i = 1, 2$ ) in the new force have to take the same values as in D1M. In this way, seven of the eight initially free parameters are determined as a function of the remaining parameter that we choose to be  $B_1$ . This parameter allows one to modify the slope of the symmetry energy and consequently the predicted maximum mass of NSs. We adjust  $B_1$  so that the maximum NS mass predicted by the D1M\* force is  $2M_\odot$ . With the same strategy, we have also fitted the D1M\*\* force, but imposing a constraint on the maximum NS mass of  $1.91M_\odot$ , which is close to the lower limit, within the error bars, of highly massive observed NSs [10, 11]. Finally, we perform a small variation of about  $1 \text{ MeV fm}^4$  in the strength of the zero-range term of the interaction in order to obtain a description of finite nuclei similar to the one obtained with the D1M force. The explicit values of the parameters of the D1M\* and D1M\*\* forces can be found in Refs. [14, 15].

Table I collects the symmetric nuclear matter properties predicted by different Gogny forces and by the Skyrme SLy4 interaction. The SLy4 force [17] is a Skyrme force specially designed to predict results in agreement with experimental data of finite nuclei as well as with astronomical observations. We have used this force as a benchmark throughout this work. From Table I, we see that the saturation density, binding energy per nucleon, incompressibility modulus and effective mass predicted by these forces are rather similar due to the fact that the symmetric nuclear matter at low densities is well-constrained by the masses of terrestrial nuclei. The symmetry energy is defined as  $E_{\text{sym}}(\rho) = \frac{1}{2} \partial^2 E_b(\rho, \delta) / \partial \delta^2 |_{\delta=0}$ , where  $E_b(\rho, \delta)$  is the energy per particle in asymmetric nuclear matter of density  $\rho = \rho_n + \rho_p$  and asymmetry  $\delta = (\rho_n - \rho_p) / \rho$ ,  $\rho_n$  and  $\rho_p$  being the neutron and proton densities, respectively. The symmetry energy at the saturation density computed with the different Gogny forces considered in this work and with the SLy4 interaction lies in the range of 28.55–32.00 MeV, which is basically within the window of values extracted from different laboratory data and astronomical observations (see [14] for more details). The values of the slope of the symmetry energy at saturation, which is defined as  $L = 3\rho_0 \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} |_{\rho=\rho_0}$ , where  $\rho_0$  is the saturation density, lies in the range between 22.43 and 45.96 MeV for the considered forces. This quantity has an important impact on the behaviour of the symmetry energy at high density and, consequently, on the EOS of NS matter.

TABLE I

Nuclear matter properties predicted by the D1M\*, D1M\*\*, D1M, D1N and D1S Gogny interactions and the SLy4 Skyrme force.

	$\rho_0$ [fm <sup>-3</sup> ]	$E_0$ [MeV]	$K$ [MeV]	$m^*/m$	$E_{\text{sym}}(\rho_0)$ [MeV]	$E_{\text{sym}}(0.1)$ [MeV]	$L$ [MeV]
D1M*	0.1650	-16.06	225.4	0.746	30.25	23.82	43.18
D1M**	0.1647	-16.02	225.0	0.746	29.37	23.80	33.91
D1M	0.1647	-16.02	225.0	0.746	28.55	23.80	24.83
D1N	0.1612	-15.96	225.7	0.697	29.60	23.80	33.58
D1S	0.1633	-16.01	202.9	0.747	31.13	25.93	22.43
SLy4	0.1596	-15.98	229.9	0.695	32.00	25.15	45.96

In the left panel of Fig. 1, we display the maximum mass of an NS as a function of its size for the D1M\*, D1M\*\*, D1M, D1N and SLy4 interactions. The right panel of Fig. 1 displays the nuclear symmetry energy calculated with the same forces used in the left panel. We can clearly appreciate a correlation between the predicted maximal mass and the behaviour of the

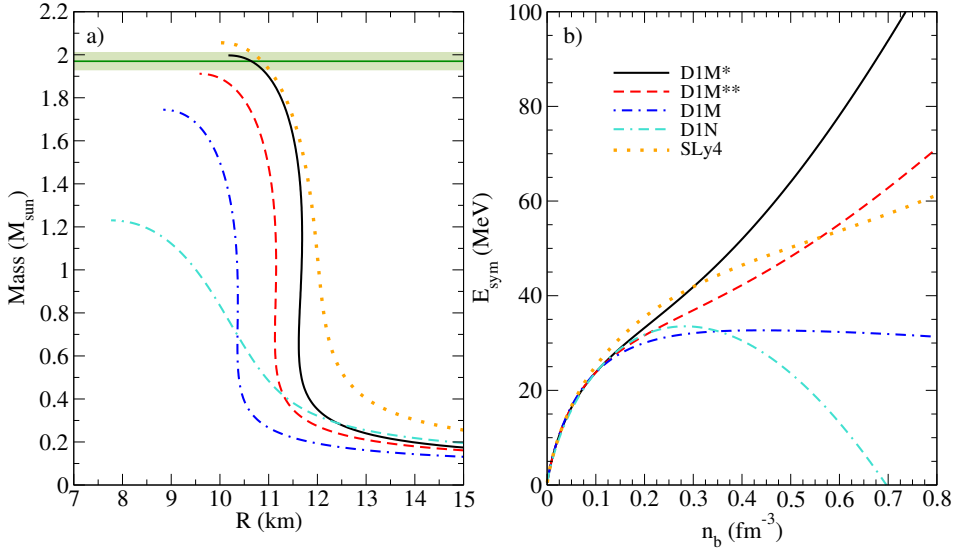


Fig. 1. (a) Mass-radius relation for a few Gogny forces and the SLy4 Skyrme interaction. Constraints of Refs. [10, 11] are also plotted. (b) Symmetry energy as a function of the baryon density for the same interactions as in panel (a).

symmetry energy at high density. Looking at Table I, we observe that models with parameter  $L$  about 45 MeV are able to predict an NS maximum mass of  $2M_{\odot}$ . We notice, however, that the maximum mass predicted by a given model not only depends on the value of the parameter  $L$  but also on the behaviour of the EOS of NS matter at high density. For example, the D1N and D1M\*\* forces have almost the same value of  $L$  (see Table I), but the maximum mass predicted by D1N is much smaller than the one predicted by D1M\*\*. This fact shows the importance of the high-density region of the EOS in the solution of the TOV equations.

### 3. Properties of the D1M\* and D1M\*\* interactions

#### 3.1. Bulk properties of neutron stars

In the previous section, we have included in the fitting procedure of D1M\* (D1M\*\*) the constraint of a maximum mass of  $2M_{\odot}$  ( $1.91M_{\odot}$ ). The solution of the TOV equations also provides the radius of the NS. In D1M\*, the radii corresponding to the NS of maximum mass of  $2M_{\odot}$  and to the NS of canonical mass of  $1.4M_{\odot}$  are 10.2 km and 11.6 km, respectively. These values are similar to the predictions of the SLy4 model and are in harmony with recent extractions of the NS radius from low-mass X-ray binaries and X-ray bursters (see [14] for more details). To solve the TOV equations, we

need the full EOS from the center to the surface of the star. For the inner crust, we follow the prescription used in Ref. [14] and consider that the EOS in this region can be described by a polytropic form of the type  $P = a + b\varepsilon^{4/3}$ , where  $P$  is the pressure and  $\varepsilon$  is the mass-energy density.

Another global property of interest in an NS is its moment of inertia. The reason is that information about its properties may be extracted from observations of binary pulsars (see [14] for more details). In the slow-rotation regime, the NS moment of inertia can be computed by solving the TOV equations together with the differential equation for the moment of inertia in general relativity [18]. The left panel of Fig. 2 displays the moment of inertia of NSs calculated using the EOS associated to the different Gogny forces considered in this work along with the SLy<sub>4</sub> interaction. We see that the moment of inertia predicted by the D1M\* and SLy<sub>4</sub> interactions is very similar and larger than the D1M result. The value obtained using D1M\*\* lies in between, although closer to the prediction of D1M\* than to the one of D1M. Assuming a mass of  $1.34M_{\odot}$  for the pulsar A of the binary system PSR J0737-3039, whose moment of inertia is expected to be available within a few years, SLy<sub>4</sub> and D1M\* predict a moment of inertia of  $1.29 \times 10^{45} \text{ g cm}^2$ , while D1M\*\* and D1M predict  $1.19 \times 10^{45} \text{ g cm}^2$  and  $1.04 \times 10^{45} \text{ g cm}^2$ , respectively.

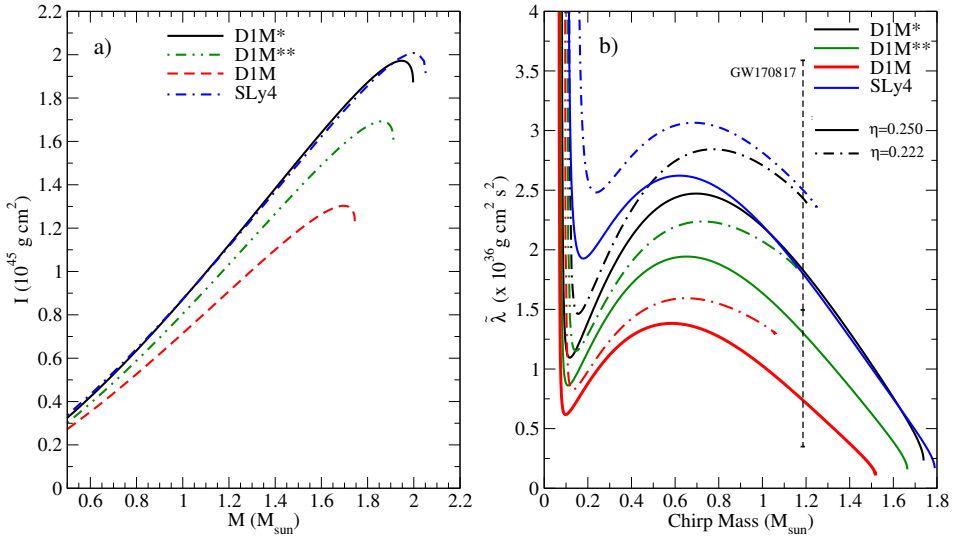


Fig. 2. (a) Total moment of inertia as a function of the total mass of the NS for the D1M\*, D1M\*\*, D1M and SLy<sub>4</sub> forces. (b) Weighted tidal deformability as a function of the chirp mass for the same interactions as in panel (a).

Relevant information about the EOS of NS from astronomical observations may be extracted from the analysis of the early part of the gravitational wave signal of binary NS inspirals. The influence of the internal structure of the star on the wavefront is provided by the tidal distortion of the star in the extremely strong gravitational field of the companion during the inspiral [19]. As it has been pointed out before [20], the relatively clean effects at the early times of the binary dynamics could, potentially, be measured. The tidal deformability is characterized by a single parameter,  $\lambda$ , which measures the quadrupole deformation of the star as a response to the perturbing tidal field of the companion. In the right panel of Fig. 2, we display the weighted tidal deformability  $\tilde{\lambda}$  [19], computed with the different models considered in this work as a function of the so-called chirp mass, defined as  $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ ,  $m_1$  and  $m_2$  being the masses of the binary. The vertical dashed line at  $\mathcal{M} = 1.186 M_\odot$  corresponds to the constraint of the weighted tidal deformability provided by the recent GW170817 event [21]. From Fig. 2, we see that for a symmetric mass binary system of  $\eta = 0.250$ , where  $\eta = m_1 m_2 / (m_1 + m_2)^2$ , the values of  $\tilde{\lambda}$  obtained with SLy4, D1M\*, D1M\*\* and D1M lie inside the constraint. On the other hand, of the considered forces, only the SLy4 and D1M\* interactions fit inside the constraint if a very asymmetric binary system with  $\eta = 0.222$  is considered.

### 3.2. Neutron star crust properties

As discussed in more detail in Refs. [14, 15], HFB calculations of the binding energy of 620 even–even nuclei (which are those with experimentally known masses [22]) performed in a harmonic oscillator basis using the D1M\* force give an r.m.s. deviation of 1.34 MeV. This value is very close to the r.m.s. deviation of 1.36 MeV obtained with the original D1M interaction for the same set of nuclei and the same calculation protocol. The r.m.s. deviation of charge radii for 320 nuclei where experimental data is known [23] and computed with the D1M, and D1M\* interactions are very similar in the two cases: 0.028 fm and 0.031 fm, respectively. However, due to the larger value of the parameter  $L$  in the D1M\* force, the neutron radii calculated with this force are larger than when computed with the original D1M interaction.

As an application of the D1M\* interaction to calculate the properties of finite nuclei in the NS scenario, we compute the energy per baryon in the inner crust of NSs as a function of the average density using this force and we compare the results obtained with the predictions provided by other calculations available in the literature [24]. To this end, we have performed Variational Extended Thomas–Fermi calculations [25] using trial densities of the type proposed in [26] within the Wigner–Seitz (WS) approximation. Details about the way of including the  $\hbar^2$  corrections to the energy in the case of finite-range interactions can be found in Ref. [27]. Although shell

effects are not included in this simple approach, the semi-classical calculations in the inner crust of NS are realistic enough to estimate the EOS in this region of the star [24]. The left panel of Fig. 3 displays the energy per baryon in the inner crust as a function of the average density in a WS cell. As expected from our fitting procedure, the energy per baryon computed with the D1M and D1M\* forces are very similar, and in excellent agreement with the values obtained using the SLy4 interaction [28]. Due to the larger value of the slope of the symmetry energy in D1M\* than in D1M, the crust–core transition density is estimated to be smaller with the D1M\* force than with D1M [13]. We have also calculated the energy per baryon in the inner crust using the D1S force. In this case, we observe significant differences, in particular at high densities, with the results computed with the D1M and D1M\* interactions. The reason may lie in the limitations of the D1S force in describing neutron-rich nuclei. In the left panel of Fig. 3, we also show the energy per particle in the inner crust provided by other calculations of different nature. As representative examples, we show the results obtained using the BCPM energy density functional based on microscopic calculations, the Compressible Liquid-Drop Model of Baym–Bethe–Pethick, the quantal calculation of Negele and Vautherin, and the calculation performed using the Skyrme force BSk21 of the Brussels group (see [24] and references therein for more details). Although the energy per baryon in the inner crust computed with different methods can show non-negligible differences, its impact on the EOS of the inner crust, which is basically provided by the neutron and electron gases, is smaller.

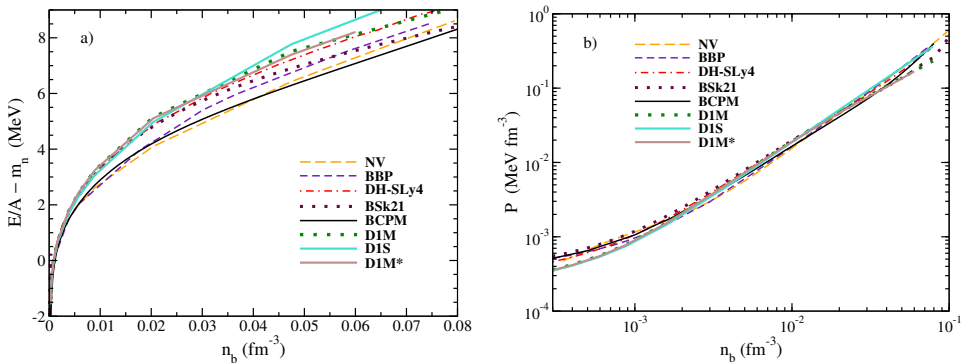


Fig. 3. (a) Energy per particle as a function of the baryon density for different theoretical models. (b) Pressure as a function of baryon density, both in logarithmic scale, for the same set of models as in panel (a).



#### 4. Conclusions

We have discussed some new parametrizations of Gogny forces based on the D1M interaction, which describes successfully finite nuclei properties. By a suitable modification of the isovector sector of this force, these new forces, namely D1M\* and D1M\*\*, are able to describe finite nuclei at the same level of accuracy to that of D1M interaction and, at the same time, predict neutron star properties, as for instance the maximum mass, in harmony with recent astronomical observations. On top of the mass–radius relation, we have also analyzed the moment of inertia and the tidal deformability in neutron stars finding good agreement with the predictions of the Skyrme SLy4 interaction. We have also estimated, using the Extended Thomas–Fermi approximation within the Wigner–Seitz picture, the energy per baryon and the EOS in the inner crust using the D1M and D1M\* forces. We find, as expected, that the energy per baryon computed with these two forces are very similar. We have also estimated the EOS in this region of the star, which is determined by the neutron and electron gases. We find that the EOS obtained with the D1M and D1M\* forces are again very similar between them and also lie very close to other EOS in the inner crust computed using other models of different nature.

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