



Universidad Autónoma  
de Madrid

**Biblos-e Archivo**  
Repositorio Institucional UAM

**Repositorio Institucional de la Universidad Autónoma de Madrid**

<https://repositorio.uam.es>

Esta es la **versión de autor** del artículo publicado en:  
This is an **author produced version** of a paper published in:

Teaching and Teacher Education 87 (2020): 102924

**DOI:** <https://doi.org/10.1016/j.tate.2019.102924>

**Copyright:** © 2019 Elsevier. All rights reserved.

This manuscript version is made available under the CC-BY-NC-ND 4.0  
licence <http://creativecommons.org/licenses/by-nc-nd/4.0/>

El acceso a la versión del editor puede requerir la suscripción del recurso  
Access to the published version may require subscription

# **Training teachers in maintaining equity in the micro-moments of a mathematical dialogue<sup>1</sup>**

Ignacio Ramis-Conde<sup>a</sup> and Alexander Hope<sup>b</sup> (corresponding author)

The authors declare that they have no conflicts of interest with regard to this study

## **Abstract**

This article presents the analysis of a large body of video data of trainee teachers who were learning mathematical dialogue techniques. We explore some of the ways in which trainee teachers can make better use of children's own contributions to foster learning and increase pupil motivation – taking a number of analytic principles from Conversation Analysis. While the importance of dialogue in mathematics education has drawn increased attention in recent years, there is still relatively little work on how teachers may learn to make better use of dialogue in the classroom. This paper aims to address that lacunae.

**Keywords:** pre-service teacher education; constructivism; mathematics education; teaching methods; conversation analysis

- a. Universidad de Castilla-la Mancha,  
Facultad de Educación de Cuenca  
Edificio Fray Luis de León  
Campus Universitario s/n  
C.P. 16071 - Cuenca
- b. Universidad Autónoma de Madrid,  
Departamento de Filología Inglesa  
Facultad de Filosofía y Letras  
Universidad Autónoma de Madrid  
Campus de Cantoblanco  
28049 Madrid

---

<sup>1</sup> We would like to thank Ana Llinares and Natalia Evnitskaya for their suggestions on earlier versions of this manuscript



# Training teachers in maintaining equity in the micro-moments of a mathematical dialogue

## Introduction

This paper came out of a teacher training programme at the Universidad de Castilla-La Mancha. The programme was aimed at training pre-service teachers in mathematical dialogue techniques.<sup>2</sup> The training was focused on making the trainee teachers more aware of the importance of dialogue in the classroom; how they might employ different scaffolding strategies in their teaching; and on ways to incorporate students' own contributions to promote understanding and boost motivation. Mercer *et al.* (2009, p. 368) identified incorporating students' contributions as a key area of development for teacher training.

In order to analyse the data from this programme, our research methodology employs a number of key principles borrowed from conversation analysis (CA). Until recently, CA had predominantly been used by researchers interested in second language acquisition in foreign language classrooms and content and language integrated learning (CLIL) classrooms (Evnitskaya & Jakonen, 2017; Koole & Elbers, 2014, p. 59; A. Llinares, Morton, & Whittaker, 2012, pp. 52–75), as well as science classrooms (Mortimer & Scott, 2003). However, in the last few years, it has been adopted as a tool for the analysis of interactions in first-language mathematics classrooms (Bateman & Church, 2017; Cohrssen & Church, 2017; Cohrssen, Church, & Tayler, 2014). This article uses this methodology to help analyse pre-service teachers' mathematical dialogues.

During a mathematical dialogue, the teacher interacts with the student(s) by asking questions or providing information through dialogue and turn-taking. How much information s/he provides to or withholds from the student(s) delimits a personal space of choice for the student. If the dialogue is to be genuinely interactive, and students' responses incorporated, then the teacher needs to find a balance between providing enough scaffolding to enable the students to progress with the task, and solving the students' difficulties for them; or, indeed, using the scaffolding as a control method (Koole & Elbers, 2014, p. 58; Myhill & Warren, 2005; Warshauer, 2015a). From this perspective, we have defined the "Student Decision Space" as composed by the actions, discourses and thinking procedures that a student is given licence to perform during

---

<sup>2</sup> "Mathematical dialogue" is our development of Helle Alrø and Ole Skovsmose's constructivist use of "dialogue" in mathematics teaching. Their interpretation is strongly influenced by Carl Rogers and Paulo Freire (Alrø & Skovsmose, 2002, pp. 3–5).

a mathematical dialogue. This conceptual “decision space” is also related to a physical and material space – as shown by multimodal-CA researchers like Evnitskya and Jakonen (2017, p. 207). Maintaining a well-balanced student decision space means generating a dialogue based on the principles of equity. For Alrø and Skovsmose, equity is a near synonym of equality, and cite Bohm’s insistence that in a dialogue “nobody is trying to win” (2002, p. 124). We also view equity in dialogue this way; equity in mathematical dialogue requires fairness, taking into account differences of all sorts, and encouraging and respecting students’ contributions. In a well-balanced decision space, the teacher demarcates and scaffolds the space that the pupil needs to construct her knowledge – while focusing on understanding rather than mere competent production of the right answer.

In this article, we evaluate how trainee teachers manage the decision space during “critical moments” of mathematical dialogues (Myhill & Warren, 2005, p. 59). In our case, the research was based on a long-term experience of a course in practical mathematical dialogue for trainee teachers. The course focused on fostering the principle of dialogic equity by preserving the students’ decision space(s). During the two years (four full semesters) of the course, we analyzed how trainee teachers put previously prepared mathematical dialogues into practice in the classroom. We analyzed the management of the students’ decision space(s), the types of critical moments that occurred, and the potential educational consequences of the choices trainees made in their dialogues. In this way, we provide evidence of how training in mathematical dialogue can help pre-service teachers be more attentive to the dynamics of classroom interaction.

Our research questions were focused on the dynamics of the teacher-child interactions in our data:

1. How do trainee teachers perform when managing a mathematical dialogue for the first time, in terms of preserving and/or entering the students’ decision space(s)?
2. What can be learnt from trainees’ management of mathematical dialogues, as applicable both to refining and developing teacher training programmes, and to general teaching practice?

## **1. Theoretical background**

For Alrø and Skovsmose a mathematical dialogue is a mode of interaction that combines reflection and action. They draw on Paulo Freire and Carl Rogers to contrast dialogic teaching

with what Freire calls the “banking model”, in which the teacher is a repository of information that is merely transmitted to the students (Alrø & Skovsmose, 2002, p. 4). Mortimer and Scott call this model “authoritative” teaching (Mortimer & Scott, 2003, p. 39). As in any other dialogue, the richness of the interchange depends on a balance of participation between interlocutors. This pedagogical understanding is similar to social constructivism. As Ernst von Glaserfeld explains, constructivism views knowledge (including mathematics) as an intrasubjective “adaptive function”. Equally important is the notion that teaching is to promote “understanding” rather than mere “competent performance” (2005, p. xvi). As von Glaserfeld put it, “the common conviction [in constructivism] is that knowledge cannot simply be transferred ready-made [...] from teacher to student but has to be actively *built up* by each learner in his or her own mind” (2005, p. xiii; emphasis added). In this approach, building knowledge requires producing specific scenarios where students can perform research. For Mortimer and Scott this is shown by an “Interactive/dialogic” communicative approach (2003, p. 39). In order to build knowledge, students must have a space to make decisions and construct their understanding. The purpose of this research is not only to gain specific knowledge but also to develop a set of thinking structures and practices that arise within the research activity itself. Consequently, the activity is as much about learning-to-learn as gaining specific knowledge (Alrø & Skovsmose, 2002, p. 4).

In this approach, the importance of the students’ research is central but self-construction of knowledge happens as part of an accompanied process (Alrø & Skovsmose, 2002; Nathan & Knuth, 2003). From a Vygotskian perspective, the most interesting didactic opportunities are within the so-called “Zone of Proximal Development” (ZPD) (Vygotsky, 1980; Wertsch, 1984). This is the imaginary space where the student may build up knowledge herself with tailored, and interested, support from the teacher and her peers.<sup>3</sup> The manner in which this personalized support is built is defined by Wood, Bruner and Ross as “constructing Instructional Scaffolding” (Koole & Elbers, 2014; Wood, Bruner, & Ross, 1976). According to Wood *et al.*, instructional scaffolding requires reducing the degrees of freedom when carrying out a research process so that learning is facilitated. If too open, the student will not be able to construct the target knowledge; if too closed, the knowledge will be merely reproduced rather than understood. How the teacher reduces the degrees of students’ freedom to delimit their research in any given micro-environment is, therefore, paramount to understanding mathematical dialogues. To clarify, we use “micro-moment” to emphasize that teacher-pupil interaction can be analysed

---

<sup>3</sup> Mortimer and Scott do note, however, that the ZPD was originally conceived in terms of a one-to-one interaction, and that a large class scenario is rather more complicated (Mortimer & Scott, 2003, pp. 128–129)

almost second-by-second, and different approaches to the interaction observed. These strategies may be seen in verbal or paralinguistic cues and the structure of the activity itself. The tools for studying fine-grained interaction provided by conversation analysis make it ideal for evaluating such micro-moments.

Cohrssen, Church and Tayler approached the question of providing appropriately-scaffolded research space from a conversation analysis perspective. They examined teachers' use of pauses to provide pupils with opportunities to develop their skills through "academic thinking-and-doing" (Cohrssen et al., 2014, p. 99; see also 'wait-time' in Walsh, 2006). We prefer to approach student-teacher interaction in terms of how dialogue is being locally managed by the participants. Clearly, this interaction includes the spaces for reflection that the participants need to build their arguments and knowledge. Myhill and Warren (2005) identify "critical moments" where "the teacher made good use of an opportunity to develop *independent* understanding or conversely where the teacher missed or muddled such an opportunity" (2005, p. 59; emphasis added). Both Koole and Elbers and Myhill and Warren favour a more interactive style of teaching that incorporates student responses. This responsiveness is similar to the best practice proposed by Cohrssen *et al.* (2014), Mortimer and Scott's dialogic-interactive approach (2003, p. 38) and Warshauer's "probing guidance" and "affordance" (2015a, pp. 388–390).

It is important to note that our data is from pre-service teachers; as a consequence, their approaches are "work-in-progress" and part of the journey towards becoming effective teachers. Kathy Carter and Walter Doyle identified that novice teachers' personal experiences of teaching, both good and bad, heavily influence their responses to "new" teaching practices (1995, p. 193). As a result, any "muddling" of critical moments should be considered as part of the trainees' development, and their approaches to unexpected pupil responses are likely to be heavily influenced by their own experiences.

Mathematical dialogues are dynamic situations that occur in moments or even micro-moments of time, when interaction must be tailored to (re)create the scaffolding and appropriately delimit the decision space(s). The scaffolding has to be planned, but moments will always arise which require teachers to improvise, and respond to changes in the context. Teachers should be prepared to take the "risk" of ceding control of the dialogue in order to foster equity and include children's contributions (Glaserfeld, 2005, p. 4).<sup>4</sup> During these moments and micro-moments, teachers may adopt different teaching styles and switch between distinct

---

<sup>4</sup> For CA researchers and linguists "context" is a highly contested term (see McHoul, Rapley, & Antaki, 2008 for a useful overview); however, for this article we are primarily interested in the local context, i.e. how what has happened in that specific class has shaped the interaction.

genres of teaching. Each particular style involves differing degrees of freedom, and approaches to, ingression into the students' research space(s). Effectively managing different styles of mathematical dialogue is not always straightforward. It often requires teachers to be attentive to the nuances of local context, and to switch styles when necessary. Myhill and Warren (2005, p. 62) argue that teachers are often tempted to stick to a plan to achieve a "correct answer", instead of building on children's thinking. Evidently, in mixed-ability whole-class scenarios the ideal constructivist approach is not always practical, and curriculum pressures may also mitigate against more interactive teaching.

To be able to analyse how teaching interactions unfold, the great advantage of conversation analysis over other forms of analysing dialogue is that it focuses on the interpersonal aspects of the dialogue, and how the "here and now" interpretation of social actions is decided through local sequential interaction and negotiation (Evnitskaya & Jakonen, 2017, p. 202; Koole & Elbers, 2014, p. 58). Harvey Sacks calls the TEACHER + STUDENT relationship a "Collection K" pair in which the participants are not co-equal (McHoul, Rapley, & Antaki, 2008, p. 45). This power dynamic conditions the way in which context – and the students' decision space – is continually renegotiated. However, the teacher may evidently choose to cede some of that dominance to facilitate students' research and understanding. From a different perspective, Jill Adler provides an extensive discussion of how equity always involves a balancing act between "fairness and parity" (equity) and managing more disruptive pupils (2001, p. 30). This balancing act may also include aiding pupils who are struggling.

Teachers' dialogue style has been shown to be relevant to students' "productive struggle" (Hiebert & Grouws, 2007; Hiebert & Wearne, 2003). This term refers to the effortful practice that goes beyond passive learning to construct knowledge. Productive struggle can be historically framed within didactic situations of students' perplexity, confusion, or dissonance when immersed in mathematical research (Dewey, 1933, p. 12; Piaget, 1960; Polya, 1957). On the student's side, overcoming these situations requires resilience and persistence (Jackson, 2010). On the teachers' side, well-constructed scaffolding is essential. Warshauer characterized different instances of "productive struggle", and connected them with particular teacher responses to specific moments in dialogues (2015b, 2015a). She argued that it is important for teachers to balance sustaining engagement and maintaining cognitive demand; for example, providing extra information can aid task-completion but reduce cognitive demand. Warshauer calls simply providing a solution a "telling" response from the teacher (2015a, p. 388). She encourages practitioners to build on students' existing thinking through "probing guidance", or



to “afford” students more time to complete the task and provide support by making their thinking more explicit (2015a, pp. 389–390). These strategies help to maintain cognitive demand and build on existing thinking. This balance between task completion and cognitive load has clear parallels with our “decision space” for students.

## **Methods and data**

We designed a two-semester transversal course, beginning in September and ending in June, which ran across two academic years, with two different cohorts of trainee teachers. The course was based on making future teachers more aware and strategic in their approach to mathematical dialogue, and to the construction of effective scaffolding structures. For this, we focused on the following elements in which to train the pre-service teachers:

- To preserve appropriate equity in the students’ decision space(s)
- To be more aware of naturally arising and sometimes unpredictable didactic opportunities
- To become more adept at taking advantage of these opportunities

### **1.1. *Participants***

There were two types of participants in our study: the pre-service teachers and the primary school pupils. At the beginning of the study, the 25 children belonged to a 3<sup>rd</sup> year primary school class in Spain, in a fairly typical middle-class state school. The dialogues presented in this article are from the second year of the study. The children were in their 4<sup>th</sup> year of primary school (9 - 10 years old). The trainees were second-year undergraduates. The overall teaching experience of this group was negligible, as the group had not yet begun the practical modules of the degree.

### **1.2. *Course structure***

For the course to train teachers in the use of mathematical dialogue, we considered two principal aspects: conceptual and practical. The conceptual part of the course included the theoretical elements that a trainee teacher needs to manage a mathematical dialogue effectively. These are principally theoretical mathematical concepts, theoretical didactic concepts and pedagogical concepts. These theoretical lectures were interspersed with practical sessions throughout the

two terms of the course. The practical sessions of the course were composed of the three types of activities: activity design, practical activities and analysis seminars.

### 2.2.1. *Activity Design*

In the activity design classes, the trainee teachers worked in small groups (5-10) and discussed how to construct a mathematical dialogue-based activity. This session works as a virtual classroom where the teachers propose a general strategy to convey the concepts based on a series of questions. This virtual classroom is a fair representation of a teacher's work when preparing the next day's class. Didactic approaches were considered and the trainee teachers tried to envision possible interactions during the mathematical dialogues. For this, the trainees constructed a *planned* mathematical dialogue – a series of questions aimed at anticipating the pupils' responses. This *planned dialogue* formed part of the instructional scaffolding that the pre-service teachers were going to provide to their pupils during the activity.

The general framework provided to the trainee teachers was to construct a series of questions – spaces for research and scaffolding structures – so that the final construct was similar to an inquiry-based teaching class (Jaworski, 2006; Project, 2011). We focused on three principal concepts:

- (1) Pose a mathematical question
- (2) Provide research spaces where the pupils' decision space is preserved as much as possible
- (3) Consider possible outcomes during the dialogue, and propose alternative scaffolding structures if and when the pupil appears to need help.

In the particular dataset presented in this article, the didactic activity was designed by the students to introduce tetrahedra to a 4th grade class, part of the official curriculum in this area. The principal objectives of the activity were: 1. to build a tetrahedron with spaghetti and clay by looking at a 3D tetrahedron model made with Frameworks; 2. to differentiate the basic elements of the tetrahedron; and 3. to provide an introduction to the two different 2D mappings of a tetrahedron. The *planned dialogue* was as follows:

1. Clay and spaghetti are provided to the students.
2. Students are allowed to investigate and play with the materials.
3. A model tetrahedron built with Frameworks is presented to the students.
4. **Question:** Can you build something like this?

5. Scaffolding is provided via hints and suggestions, if they cannot overcome the difficulties
  6. **Question:** The students are asked about basic characteristics of tetrahedra: *what have you built? How many faces does it have? How many edges does it have?*, etc.
  7. The model tetrahedron is dismantled in front of the pupil, leaving each piece separate.
  8. **Question:** The students are asked to put together a 2D shape with Frameworks that will form a tetrahedron when closed; and, before closing the shape, they are asked to say whether they think their 2D mappings will close to form a tetrahedron.
- (3) Scaffolding is provided via hints and suggestions, should the students appear to hit an impasse

### **2.2.2. Practical activities**

At the beginning of the activity, a set of trainee teachers (5 to 10 people) entered the classroom and were warmly presented to the children. After this, each trainee was assigned a small group of pupils. During varying setups of the practical activities, we observed that small numbers (2-4 students) provided a well-balanced social environment. The environment was designed to minimize feeling artificial to the children. To this end, the activities were embedded within the school mathematics timetable. Working in small groups allowed the trainee teachers to reduce the cognitive load involved in managing a large class considerably (primary classes in Spain are about 25 pupils). "Social scaffolding" demands are significantly reduced in small groups (Nathan & Knuth, 2003, p. 178); this social scaffolding includes keeping the class quiet (or focused), attending to multiple students' needs, and providing varying teaching rhythms. In this way, the trainee teachers could focus on how particular teaching dynamics develop. Across the two-year period, the practical activities took place with a frequency of about 1 activity every 2-3 weeks. The activities were filmed and children's hand-written/represented material was recovered during the sessions for later analysis.

### **2.2.3. Analysis seminars**

After each practical session, there was a reflective debate between the trainee teachers and their lecturer. The purpose of this was to allow them to receive feedback from their peers and lecturer, and evaluate their own experiences. Practical situations allowed the students to apply their teaching proposals and learn from them in real-time. However in real-time it is evidently near impossible for the students to properly analyse their practice. The different didactic choices, the

structure of the dialogue used to interact with the children, and the management of their decision space(s) are better evaluated in a different framework. The audio-visual material collected played a central role at this stage; the trainees were able to see and analyze their practical sessions, with the help of their lecturer and peers.

### **2.3 Data Collection**

Across the two academic years, we conducted frequent practical activities in the school. In total, over 100 trainees performed at least one practical activity – following the setup outlined above. The researcher was always present in the class during these activities, positioned at a suitable angle and distance to minimize influence on the dynamics of trainee-pupil dialogues. Observational notes were collected for 10 activities, and video recordings were made of another 13. Each practical session was scheduled within a standard 45-minute mathematics class. In addition, trainees' conversations and in-depth interviews were recorded during the subsequent *Analysis Seminar* sessions – both in small groups and individually. In total, more than two hours of group conversations and in-depth interviews were collected.

Focusing on the data presented in this article, we tried to make the filming process a part of the pupils' daily routine as much as possible. The results presented here come from the second year of a series of videos of the same class. After a few recording sessions, the pupils accepted the camera as just another part of the activities. It was necessary to rotate the trainees to allow them all to visit the school. Therefore, the filming process may be considered as having been a first-time experiment for each trainee. The results presented in this article come from the second year of a series of videos recorded in the same class. After a few sessions, the children accepted the camera as just another part of the activities. Data were recovered from notes, observations and more than twelve hours of real classroom scenario recordings.

### **1.3 Data analysis**

We used standard qualitative analysis methods to collect and evaluate the data (Schutt, 2018). We collated the notes and video recordings, and then identified those clips where styles of dialogue most obviously contrasted, and coded these using the same conversation analysis notation as Cohrsen, Church and Tayler (Cohrsen et al., 2014; Jefferson, 1984). We categorized the teaching styles after observing various different *modus operandi* that were characteristic of

the trainee-pupil interactions. The total sample was obtained across 12 activities focused on different topics in geometry. Table 1 shows the different data collection methodologies applied to each different activity.

Methodology	Data
Practical activity	- In situ notes Video recordings Post - activity notes
Analysis Seminar	- Trainees group conversations video recordings Trainees in depth interviews video recordings In - situ notes
Data analysis	- 1) Notes analysis 2) Video analysis 3) Video conversation sorting 4) CA analysis

Table 1. Data acquisition and analysis

#### 1.4.1 Coding the teaching styles

The principles of conversation analysis work in terms of “unmotivated looking”, which focuses on the phenomena that appear to be relevant to the participants in the dialogue (Evnitskaya & Jakonen, 2017, p. 202). As a consequence, when analyzing the data, we considered different dimensions of the teacher-pupil relationship without pre-formed theories about the dialogues. Initially, we examined how efficiently the trainees were conveying mathematical concepts. However, this dimension was not always representative because the quality of the trainees’ dialogue did not necessarily correlate with students’ acquisition of concepts. For instance, more advanced pupils were likely to still provide good answers in response to trainees who managed the dialogues less effectively. Conversely, some trainees did not “deliver” the concept, despite effectively managing the dialogue. Hence, we decided to focus on the strategies for managing the dialogue as separate from the apparent result. From this perspective, we developed the concept of a “decision space”. This provided a tool to analyse different patterns of utterances: some

where the scaffolding allows the pupil to discover the concept by herself, and utterances in which this process is stymied in some way.

## **2. Results**

We could identify that trainee teachers used different mathematical dialogue styles in the critical moments we are presenting, characterized by preserving/ingressing into the students' decision space(s). These styles were not specific to particular trainees. Teachers may change from one style to another in different moments, depending on their specific aims; or, may adopt a particular style because of their prior beliefs rather than as a conscious choice (S. Llinares, 2002). As noted above, pre-service teachers may be particularly influenced by their prior experiences (Carter & Doyle, 1995). However, as Warshauer (2015a) has shown, simply providing the answer can reduce the cognitive load and therefore bypass productive struggle. In our sample, we could differentiate the following dialogue styles related to the trainee teachers' management of the situation: non-ingressive aware, ingressive aware, and ingressive non-aware.

### *Non-Ingressive Aware style:*

This style is characterized by a teacher who focuses on allowing the student to perform personal research from which she self-extracts knowledge. By student research, we mean that students are encouraged to "enter an area to be researched" and there must be at least some openness to this activity (Alrø & Skovsmose, 2002, p. 52). If all results and the exact process of obtaining them are determined in advance, this is not student research. This is comparable to Warshauer's "affordance" (2015a, p. 388). For example, the teacher may model an activity, then monitor the students until the students complete the activity and then give feedback. The teacher shows their awareness during the dialogue by managing the personal spaces of the relationship by using a small number of short utterances. This allows the student(s) ample research space. The social component of the scaffolding in this style is often determined by how the teacher supplies a physical presence and encouragement so that the children feel that they are performing a meaningful task. This style is closer to what is viewed as good practice in constructivism and tends to incorporate more pauses, as suggested by Cohrssen *et al.* (2014, p. 101). Within Conversation Analysis pauses often act as Transition Relevance Place (TRP), a general name for conversational devices that (potentially) cede the floor to another party, for example questions

and question intonation are some of the most obvious examples (Have, 2007, p. 128; Sacks & Jefferson, 2006, p. 33; Schegloff, 2007, p. 4). A non-ingressive aware teacher is likely to use few utterances with often less circumscribed<sup>5</sup> TRPs. That is to say, the teacher's responses do not frame a "preferred answer" (Koole & Elbers, 2014; Schegloff, 2007, pp. 58–96). This provides appropriate scaffolding which guides whilst offering the students opportunities to contribute and return to independent research – through TRPs. To clarify, a TRP is not the same as student's decision space(s), rather a specific linguistic feature that shows how a dialogue is being managed locally. TRPs invite other participants to take a turn in the dialogue. The use of TRPs with minimal additional information helps students reflect on their own thinking, while maintaining cognitive load (Warshauer, 2015a).

#### *Ingressive Aware Style:*

This style is characterized by the teacher performing a controlled ingression into the students' research space to provide further scaffolding. This style is often observable during a situation where the teacher feels that the student(s) need additional help or to be corrected. The correction process is generally composed of questions, hints, and suggestions rather than statements or direct instruction (and as such could be considered similar to Mortimer and Scott's "dialogic-interactive" (2003, p. 38) and Warshauer's "probing guidance" (2015a, p. 389)). The teacher aims to build scaffolding that tries not to explicitly correct but rather provide relevant clues to help the student solve the difficulty herself. As the teacher tries to re-direct the students' thinking, the utterances are more frequent than in the *Non-Ingressive Aware Style*, and the act of preserving the students' decision space tends to occur within the dialogue (providing frequent TRPs to encourage participation, reflection and action) and not only in the pauses. However, this style may slip into end-focused scaffolding that restricts students' own research, despite the teacher's best intentions (see below).

#### *Ingressive Non-Aware style:*

This style is characterized by a teacher entering in the students' decision space, resulting in a loss of dialogic equity and reduced cognitive load. The ingression process generally happens as a consequence of two possible processes. The first occurs after the teacher switches genres to a direct instruction approach; and the second, after the teacher explicitly corrects a student's

---

<sup>5</sup> i.e. where the preferred answer is not specifically framed by the first half of the adjacency pair (Schegloff, 2007, pp. 58–81).

claim. This correction changes the genre to a more closed question-answer-correction structure. Myhill and Warren critique the potential restrictions placed on children's research by answer-focused scaffolding (2005); Warshauer shows how "directed guidance" tends not to connect to pupils' existing thinking and "telling" bypasses struggle completely (2015a); Koole and Elbers demonstrate how this switch may take place linguistically from a CA perspective, with the concept of "preference" clearly framing responses to lead the students to a "correct" answer (2014, p. 60; cf. Schegloff, 2007, pp. 58–96). In general, this style is characterized by the teacher focusing on extracting "correct" answers, or providing them, rather than maintaining students' decision spaces.

### 2.1. *Examples from the practical activities*

In the dataset presented in this article, the didactic activity was designed by the trainees in order to introduce tetrahedra to a 4<sup>th</sup> year class. The principal objectives of the activity were: to convey the concepts of tetrahedral construction, to differentiate the elements of the tetrahedron, and to provide an understanding of the different 2D mappings (nets). As noted above, the trainees had devised a set of questions for the children. They had tried to guess possible responses and propose different scaffolding alternatives based on how dialogues might unfold. This was defined as the *Planned Dialogue*.

### 2.2. *Situation 1: Tetrahedron building*

The children were set the task building a tetrahedron from Framework pieces. The concepts of rotations and symmetries arose naturally, which was not predicted by the trainee teachers. Therefore, it represented an opportunity to improvise and build up *in-situ* scaffolding. Hence, the trainees needed to judge the degree of freedom appropriate to each particular moment. Examples A and B show two different ways of managing the dialogue. We categorised the first teacher as using a Non-Ingressive Aware style, the second an Ingressive Non-Aware:

#### *Example A: Non-ingressive Aware teacher*

This dialogue can be seen at the following link:

<https://vimeo.com/287247882>. Password:teaching styles

1. Teacher: A ver si puedes monta:::r algo como est::o  
Let's see if you can build something like this



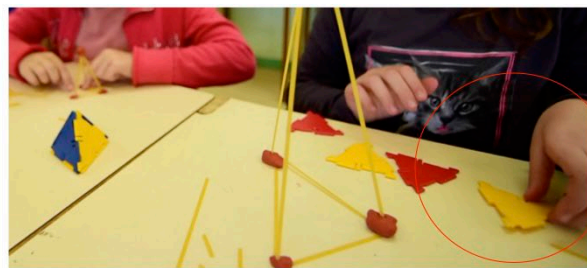
2. (10) (Moment 2)
3. Student 2: algo igual que esto? ↑  
Something like this? ↑
4. Teacher : ha↑, con esto de aquí.  
ha ↑ with this here.  
((teacher gestures towards built Frameworks model))
5. (1.3) (Moments 3 - 6)
6. Student: se puede? ↑  
Is it possible? ↑
7. (0.1)
8. Teacher: Prueba, prueba. Puedes mirar este también para verlo.  
Try it, try it. You might want to look at this one  
((referring to the mounted tetrahedron))

In this situation the student explores the material by performing movements and symmetries. Concomitantly, the teacher takes on a receptive position, characterized by providing neither relevant information nor instruction but just answering the questions that the child poses. The *Moments* refer to the temporal development shown in Figure 1. In *Moment 1* the teacher proposes reproducing the model, in this case he accompanies his scaffolding with a gesture. In the following moments, the teacher preserves the student's decision space and as a consequence new didactic opportunities arise (rotations and symmetries). These opportunities had not been anticipated by the trainees. In *Moments 2* to *5*, the child investigates the material by performing rotations and symmetries (observe the hand positions which show the rotations in Moments 4 and 6). The scaffolding used by the teacher is mostly of a social nature. This is expressed in the language ("You can try, you can try... You might want to look at this one"). He encourages the student to continue her research, but without framing what the student *should* discover. During the later analysis seminar, when questioned about the most difficult parts of the activity, he did not mention anything related to the management of the children's decision space. For him, the most difficult part was introducing new specific concepts: "the most difficult part was to how introduce new words, such as vertex, tetrahedron". While we have called this style "non-ingressive aware", it may be that this trainee teacher did not feel the need to enter the decision

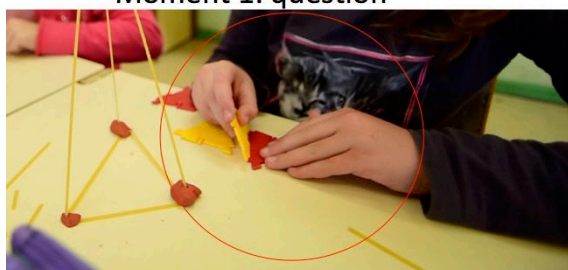
space while monitoring. Although the student initially appears to reach an impasse, she is afforded more opportunities to discover the solution herself.



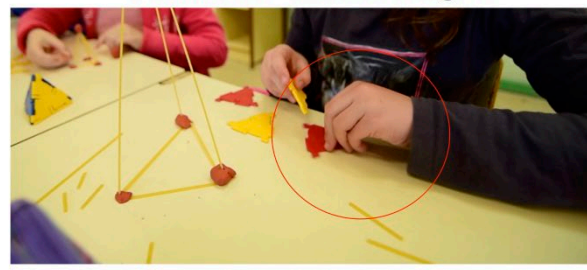
Moment 1: question



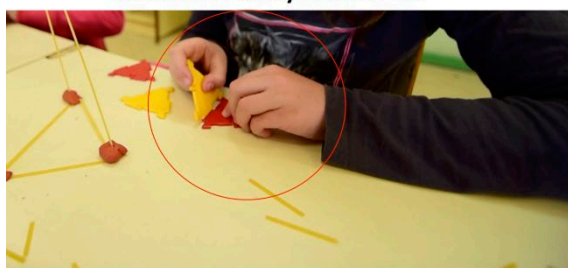
Moment 2: research begins



Moment 3: symmetries



Moment 4: rotation (left hand)



Moment 5: symmetries



Moment 6: rotation (right hand)

Figure 1. Temporal development of the mathematical dialogue sequence shown in Example A

### Example B: Ingressive Non-Aware teacher

<https://vimeo.com/287247882#t=47s>. Password: teaching styles

1. ((Student 1 presents some difficulties when trying to connect two pieces))
2. (0.5) (Moments 1 and 2)
3. Teacher (Moment 3) : Busca el hueco de una (0.1) con:n con lo que sobresale de otra

Look for the hole of one and what sticks out of the other (Moment 3)  
((pointing to the piece))

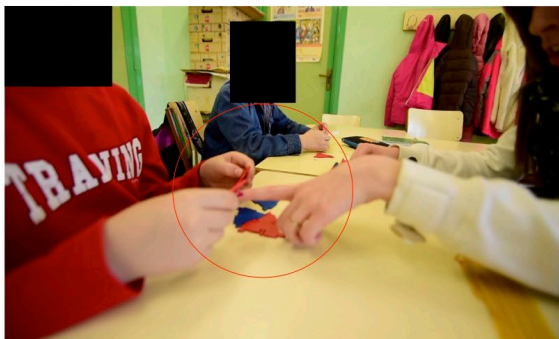
4. (0.2) (Moment 4)



Moment 1: research



Moment 2: ingression begins



Moment 3: ingression



Moment 4: recovery

**Figure 2.** Temporal development of Example B. The teacher invades the initial student decision space by a physical gesture in *Moment 3*.

In this case, the initial conditions were almost identical to the previous example but with quite a different development. The approach of this teacher is qualitatively different. As soon as she sees the student having trouble, she *tells* the student how to overcome the problem. In this case, the teacher uses a gesture to encroach on the student's decision space. The teaching approach may be defined as "heuristic" (Gigerenzer & Gaissmaier, 2011, p. 454) as she adopts a preference to "move ahead" instead of allowing the child to explore unexpected research spaces. Warshauer makes clear that this approach also reduces cognitive load (2015a, p. 388). As a result, the trainee loses the opportunity to allow the student to investigate rotations and symmetries, and then build on these to develop a more complete understanding of the shape. During the later analysis seminar, she did not express awareness of having encroached on the student's decision space. Figure 2 presents a time sequence for the dialogue. In *Moment 1* the trainee observes the student manipulating the material. From *Moments 2* to *4* the teacher invades the student's

decision space to tell him how to connect the pieces; the ingression is both verbal (“Look for the hole of one an:nd and the connector of the other”) and physical (hand pointing the model).

The diagram shown in Figure 3 depicts the different *in-situ* scaffoldings used by Teachers A (lines in blue) and B (lines in green). Teacher A, by preserving a Non-Ingressive approach, managed to allow the planned concepts to develop further: the rotations and the symmetries investigated when trying to connect the framework pieces had not been considered. In contrast, the Teacher B’s ingression misses this opportunity.

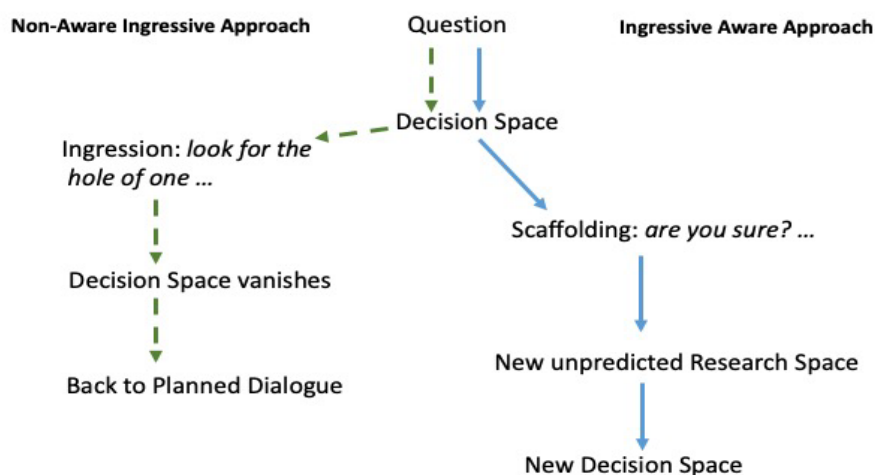


Figure 3: Diagram of different approaches to the interaction

## Situation 2: 2D mapping – analysis

This was the second part of the activity, after the children had built up model tetrahedra with spaghetti and clay. The trainee teachers then posed analytical questions about the model. The initial question may be considered rhetorical – “what have you built?” – as it refers to information the teacher had provided when introducing the activity (the shape’s name). However, from a child’s perspective the word *tetrahedron* may not be the only answer. In this situation, it would be normal to obtain descriptive answers rather than the technical name. These descriptive answers can elucidate children’s reasoning, which provides an opportunity to develop those ideas further.

This question is also interesting from a dialogue-construction perspective. In the following conversations, how the student interpreted the shape did not fit with the expectations from the planned activity. In both examples, the children stated that the tetrahedron was a triangle. We have observed this to be a very common answer from children analysing 3D-shapes for the first time. The most obvious way to construct *in-situ* scaffolding here would be to ask why she thinks it is a triangle, and thus re-direct the dialogue to open a new space for research – based on the students’ own thinking. The differences in approach can be seen in the provision of scaffolding as a “repair” to the planned dialogue. In conversation analysis, these are “other-initiated” repairs, as the teacher uses interrogatives to try to induce self-correction of the terminology or concept (Schegloff, 2007, p. 102). In these examples, the first teacher (Teacher C) gets some benefit from this unexpected situation whilst entering into the student’s decision space; the second teacher (Teacher D) performs a different type of ingression that limits the opportunity to open a new space for research.

*Teacher C: Ingressive-Aware Teacher*

The following dialogue can be seen at: <https://vimeo.com/287247882#t=1m09s>. Password: teaching styles

1. Student (Moment 1): Estamos construyendo eso.  
We are building that.
2. Teacher (Moment 2): ((holding the model to show the student))  
Esto qué era?↑  
And what was this?
3. Student: Pue:es (0.5) un tri::angulo  
It wa:as (0.5) a tri::angle
4. Student 2: Tri:angulo  
Tri:angle
5. Teacher: Estás seguro?↑  
Are you sure?↑
6. Student: S:[i→]  
Ye[:es →]  
Student 2: [pues, claro↓]

[well, of course↓]

7. (Moment 3) ((Dismantling the tetrahedron to show the student one of the faces))

8. Teacher: Y esto qué era?

And what was this?

9. Student (Moment 4): una piez- (pieza), un tri:angulo.

A piec- (piece), a tri:angle

10. Student 2: tri:angulo. ↓ ((referring to the triangle))

tri:angle ↓

11. Teacher (Moment 6): Y es lo mismo esto que esto?↑ =

And is it the same as that? ↑

((referring to the tetrahedron and (Moment 5) remounting the triangular face))

12. Student: (.) = No, porque eso es -¡NO ESO ES UN CUBO!

No, because that is – NO THAT'S A CUBE! ((pointing to the tetrahedron))

13. Teacher: (0.1) El cubo de qué es?. Cuadrado ↓ . Y el triángulo...?↑

What's a cube? Square ↓. And the triangle?



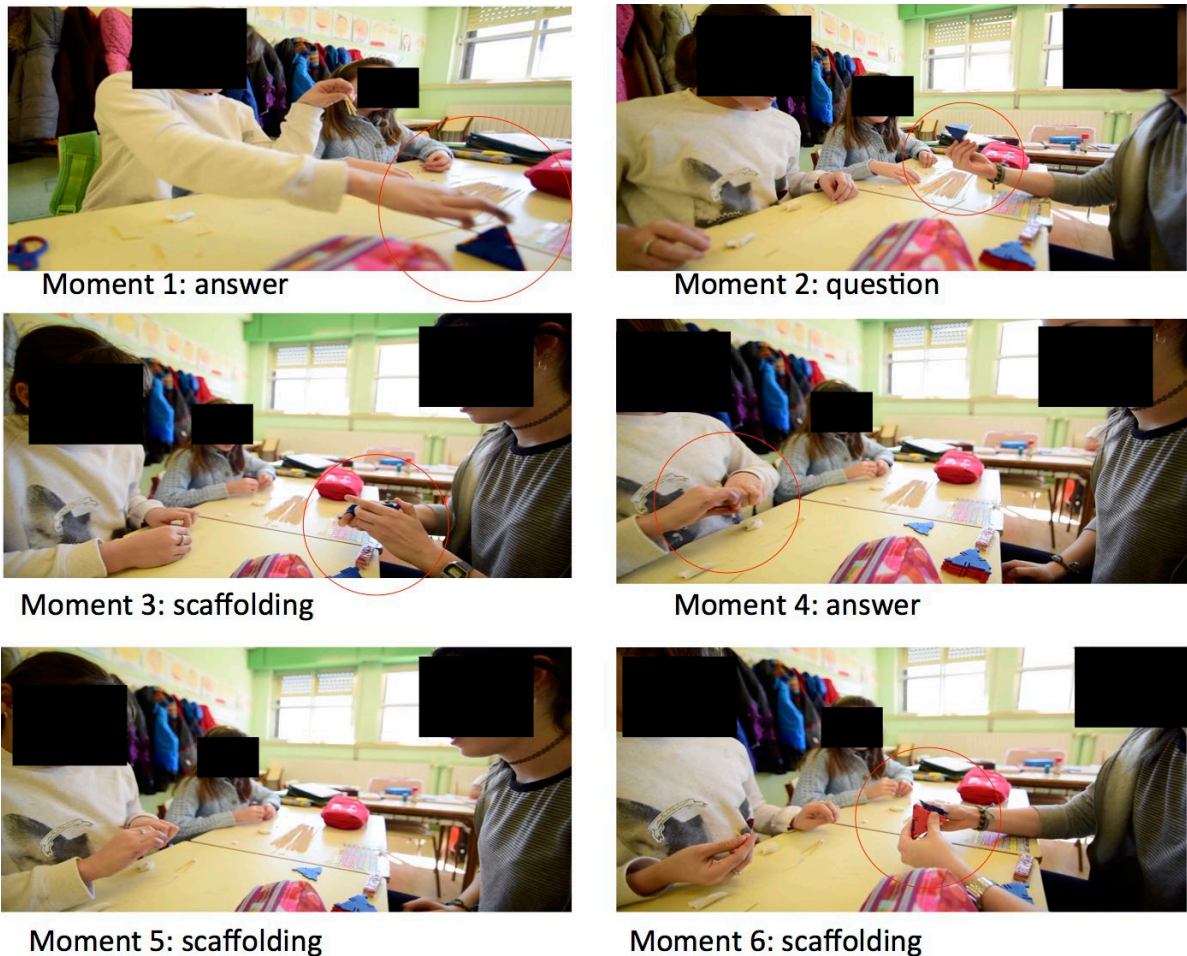


Figure 4. Temporal sequence showing the development of Teacher C's dialogue

Figure 4 shows the sequence of the previous dialogue. In *Moment 1* the student points towards the model and asserts that it is a triangle. At this point the teacher could have simply corrected the student, but instead questioned her assertion. To do this, in *Moment 2* she provided analytical scaffolding by holding the figure (and softly twisting it, line 6, “And what’s this?”). This gesture attracted attention to the figure, and helped the student realize that it was different to the triangle. The teacher’s response indicates a non-preferred answer, but does not frame a preferred answer. As the student persisted with her claim, the teacher provided further analytical scaffolding by dismantling the model (*Moments 3 and 4*), and showing one of the triangular faces of the model separately (*Moment 5*). The teacher then reconstructed the tetrahedron and asked if the two were the same (triangle and tetrahedron) (*Moment 6*). The dialogue continued with the student making a similar mistake (confusing tetrahedron and cube) and the teacher again implicitly asking the students to reconsider their answers.

The adjacent pairs of the dialogue show that the trainee opened a reflective space regarding the child's claims – maintaining equity. She did not make an explicit repair via an overt correction. Observe that within the whole dialogue the trainee provides an assertive statement only once (“What’s a cube? Square ↓.”); the remaining sentences are composed of a series of open and semi-open questions (TRPs inviting further research or contributions). In this way, the teacher preserved more of the student’s decision space.

However, this shows that preserving the student’s decision space does not always occur as an introduction of pauses into the dialogue, as proposed by Cohrssen et al. (2014). In contrast to *Teacher A* where the teacher’s pauses predominate, in this dialogue, the teacher’s utterances do – a marker of an Ingressive-Aware Style. As a consequence, the dialogue is more dynamic (note the latching in lines 11 and 12). The teacher uses the students’ own observations to indicate that “triangle” is non-preferred, but does not frame a preferred “right” answer to be parroted back, or give clues that would make the task too easy. She preserves a delimited decision space by using non-restrictive first-part pairs and TRPs, which indicate non-preferred answers and invite student contributions – rather than via pauses.

### 3.3.1. Example D: Ingressive Non-aware teacher

The following dialogue can be seen at <https://vimeo.com/287247882#t=1m35s>. Password: teaching styles

1. Student (Moment 1): Pues estoy hacie:endo (.) algo parecido a este cuadrado.

Well I’m mak:ing (.) something similar to this square.

2. (1.0)

3. Teacher (Moment 2): ¿Pero cómo se llama esa figura? ↑

But what’s the name of that shape? ↑.

4. Student 2: Triangulo

Triangle

5. Student (Moment 3): °Tria\*ngulo:::° ↓

°Tria\*ngle:::° ↓

6. (2.0)

7. Teacher (Moment 4): La figura completa se llama tetrahedro



The complete shape is called a tetrahedron

8. Student: Tetrahedro ↓  
Tetrahedron ↓

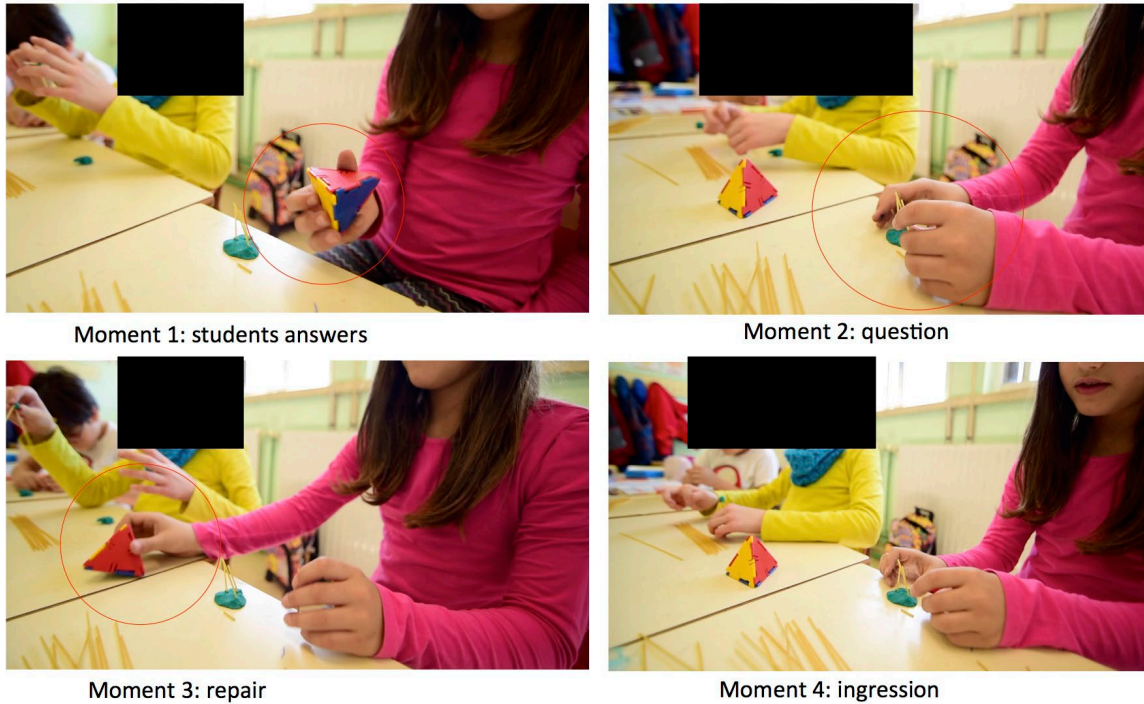


Figure 5. Temporal sequence of Teacher D's dialogue.

In this repair situation, the trainee provided the specific term (tetrahedron) without realizing they were missing an opportunity to build on the children's thinking. Figure 6 shows the temporal sequence of the dialogue. In *Moment 1* the student takes the model to show what she was building ("I am building something similar to this square"). In *Moment 2* the teacher asks "but what's the name of the figure"? However, the question also reframes and limits the students' decision space by focusing on the name as an end-point and preferred answer. The student provides an answer (triangle). Although this answer is not "correct", it contains a tacit understanding of the tetrahedron as a figure closely related to the triangle. The student provides a second "wrong", clearly non-preferred, answer (Line 4, 2 second pause shows non-preference), perhaps triggered by Student 2's answer (Line 3). In response, the teacher decides to provide the technical name of the figure directly instead of opening a new space for reflection. The teacher swings her dialogue style into direct instruction by introducing a correction-repair (line

5: “the complete figure is called a tetrahedron”). The adjacent pair composed by a teacher’s assertion and a student’s utterance repeating this assertion can be considered as a sign of an unbalanced dialogue, and a hierarchical student-teacher relationship (a true “Collection K”). The utterance in Line 5 (“the complete figure is called...”) moves the dialogue into a situation where the preferred response is simply repetition. As a consequence, the student’s decision space and the potential to investigate other aspects of tetrahedra is limited. This is a version of the initiation-response-follow-up-feedback structure (IRF) structure found in many whole class scenarios, with a closed question eliciting a minimal answer (Heritage & Atkinson, 1984, p. 53; Mercer et al., 2009, p. 356; Sinclair & Coulthard, 1975). From a multimodal perspective, it is clear that there is also a disjunction between the teacher’s focus on the correct name and the research the students are continuing to carry out. Furthermore, while the quietness and downward intonation of the student may be her personality, the comparison with the enthusiasm shown by the student in the previous extract is notable.

The diagram shown in 6 shows the differences between the mathematical dialogues constructed by Teacher C (blue) and Teacher D (green lines). While Teacher C builds an *in-situ* scaffolding structure that intends to redirect the student’s thinking so that she realises her own mistakes, the approach of Teacher D is a straightforward correction. Both approaches enter the decision space. However, the first ingress opens new didactic opportunities. In the second, *telling* reduces cognitive load and restricts the decision space. Nevertheless, it is important to note that these were pre-service teachers and these interactions were part of their learning journey. Below, we include some of trainees’ self-evaluation of these kinds of interaction.

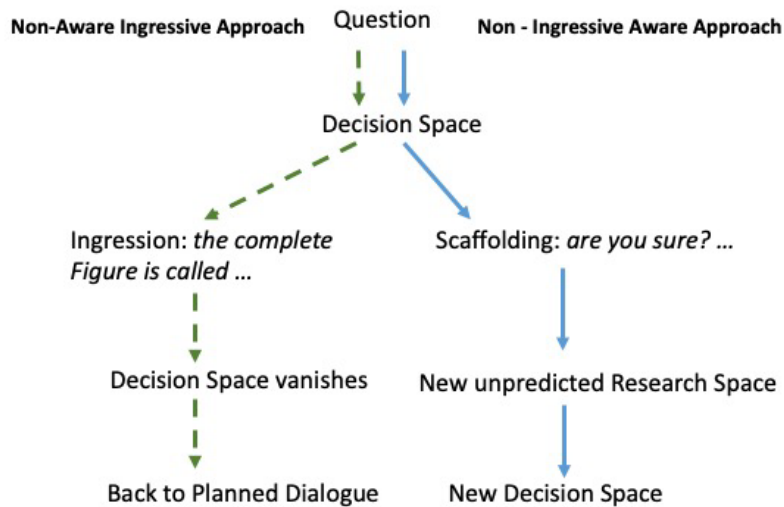


Figure 6. Diagram showing the different approaches of Teachers C (lines in green) and D (lines in blue).

### 3. Discussion

In the dialogues presented above, we can see the effects of different forms of ingression into students' decision spaces. Generally, maintaining a well-balanced mathematical dialogue means avoiding *unnecessary* ingressions so that the students' self-construction of mathematical concepts is preserved as much as possible. This is not immediate and requires awareness of the specific context so that the teacher directs her attention towards what is meaningful in each micro-moment of the dialogue. When a teacher is mindful of preserving of the student's decision space, this allows for the development new unexpected didactic situations that build on children's thinking. It can also help to maintain the cognitive load of the activity. We can see this clearly in the differences between the different teaching styles just outlined.

Some trainees explained they were aware of preserving the students' decision space, but fell into a situation where they did not do so. Some intentionally entered the space to re-direct the mathematical dialogue. In both cases, the explanation that they most frequently provided regarding this behaviour was a need to transmit the concept. A trainee teacher who was recorded entering the decision space, explained during the subsequent interview as follows:

I was feeling a failure somehow as I felt the concept was not being properly delivered because the children thought they were only playing... So occasionally... I told them what they had to do too much, invading their space. But in the moment you don't think about

it. Maybe after, when reflecting, you think you shouldn't have done it but in the moment you don't realize.

(trans. from analysis seminar recording )

Managing the dialogue was difficult for trainees as they seemed to have an impulse to “deliver the concept”. As with Myhill and Warren’s analysis of scaffolding (2005), the trainee teachers seemed to begin the activity with a focus on the end point (“deliver the concept” – i.e. the “banking model”) rather than on the process. This limited their ability to capitalize on opportunities provided by the dialogue as it unfolded as an interaction. As noted above, this is likely to be heavily influenced by trainees’ own educational experiences (Carter & Doyle, 1995), and there probably is no “magic bullet” to convince trainees to feel comfortable trying out alternative teaching strategies to those which worked for them in the past.

Creating a respectful and balanced mathematical dialogue does require longer pauses (cf. Cohrssen et al., 2014; Walsh, 2006) where students can perform research and construct their own theories. The reason for this is justified by the difficulties of an abstract thinking-procedure such as mathematics. Partly, this is a question of cognitive load, which can be seen as another way of approaching the constructivist distinction between understanding and competent performance. Productively struggling can aid understanding and improve motivation. This can be seen in the content and confident expressions shown by the primary students at the ends of the “aware” dialogues, and the lower confidence exhibited by the ingressive non-aware teacher’s students.

However, preserving the students’ decision space(s) does not always mean interacting less with the students, or only using pauses to provide spaces for research. Corrections and instruction are needed too, and the swing between a Non-Ingressive Style and an Ingressive Style can re-orientate the students to a more productive didactic scenario. Students need confidence that the task is achievable, and so leaving students who have hit an impasse to work it out for themselves can be counter-productive. Warshauer shows that switching to a style that uses “probing guidance” involving ingression into the decision space can still be productive, even though the cognitive load may potentially be lower. Furthermore, Mazenod *et al.* (2018) have highlighted the very real long-term effects that the dependency encouraged by providing too much scaffolding may induce. Mazenod’s study (2018) – of the dependency produced by teachers’ expectations of students in lower streams – forcefully illustrates Myhill and Warren’s earlier concerns about constrictive uses of scaffolding (2005).

For constructivists, an ongoing dialogue should involve an in-the-moment reflection-action process that evaluates how the dialogue is being built. This process should aim to foster understanding rather than competent performance. The teacher focuses on the students' claims and provides suggestions, hints or questions to open new research spaces and afford students' space to develop their understanding. We can see this in the density of TRPs provided by the Ingressive-Aware trainee teachers, and the way in which open questions do not over-restrict the scope of preferred answers in the "aware" dialogues. These TRPs encourage participation but still give students room for their own research. In this way, entering into the students' decision space is not an action in which the teacher imposes her way of thinking on the student. Instead, it represents a moment for common reflection and knowledge building.

The choice to swing between ingressive and non-ingressive styles in a mathematical dialogue is not always straightforward. Although a dialogue should be a respectful and balanced relationship between interlocutors, a mathematical dialogue aims to help students to build specific mathematical knowledge. This intention means that teachers using non-ingressive styles may switch to more ingressive approaches to re-direct students. How this intervention occurs affects the self-scaffolding implemented by students, and indeed their long-term ability to learn-to-learn – a metacognitive characteristic of higher-achieving, and life-long learners (De La Harpe & Radloff, 2000, p. 170). As Myhill and Warren put it, it is easy for the teacher "to slide from scaffolding as a learning support mechanism to scaffolding as a device to enable pupils to complete a task successfully, without necessarily grasping the learning at the heart of the task" (2005, p. 58). We can see this occur with the teacher in the ingressive non-aware micro-moment; in this critical moment she had the opportunity to develop the children's understanding, but this was missed because of a focus on getting to the end-point. Unfortunately, the productive struggle to understand was partly bypassed too. When teachers enter the decision space without building on the contributions of the pupils, they can easily lose dialogic equity. Warshauer and Myhill and Warren both show that experienced teachers can also struggle with striking this balance, not only pre-service teachers. Our trainees were not subject to the same curriculum pressures as in-service teachers, but they nonetheless felt a need to "deliver a concept".

Myhill and Warren note that practitioners are often influenced by their beliefs about what teaching and learning means. Elements such as teachers' unwillingness to deviate from the teaching agenda (Hilton, 2002; Myhill & Warren, 2005) may have been adopted by the trainees from personal experiences as students (Carter & Doyle, 1995). They may have entered the students' decision space as a consequence of understanding mathematical dialogue in the way

they themselves experienced – instead of choosing to intervene for pedagogical reasons. A trainee explained her feelings about keeping to the plan in one of the analysis sessions:

the most difficult part was to *reproduce the activity* as we had previously set it up [referring to the *planned dialogue*] ... I was nervous but finally I decided to observe and just see what they were doing [emphasis added]

Trainees were often uncomfortable deviating from their planned structures and allowing the children to develop their own research. As Alrø and Skovsmose point out, citing Rogers, “risk is frightening” (2002, p. 4). Consequently, trainees need to be encouraged to take risks constructively. When they start teaching larger groups, they will then have experience of letting a situation develop, responding to their students, and making the most of students’ contributions. The dialogues presented in this article show some of the ways in which trainee teachers may be made more aware of how they are interacting with students, and how this may influence those students’ understanding and motivation. This should, in principle, enable trainees to employ different styles of dialogue and intervention strategically. Training pre-service teachers in conversation analysis might be too much to add to an already heavily-loaded curriculum. However, perhaps additional training in conversation and discourse analysis may assist lecturers in helping trainee teachers to be more attentive to the interactional dynamics of each micro-moment. More widespread use of video analysis and peer-feedback as used during this programme could also be beneficial.

During the two years of the study, we concluded that training in constructivist approaches to dialogue provides an interesting and innovative approach to teacher education. The type of ingressive/non-ingressive styles described in our research were common to many of the trainees, with particular differences depending on their personal teaching profiles. Teaching the trainees how to differentiate styles in real classroom activities, and encouraging reflection about their uses provided a new dimension to their practical education. In addition, linking linguistics to mathematics highlighted the importance of how trainees relate to and interact with their pupils – to create effective scaffolding together. This shifted away from the conventional view of teaching, based on methodologies and concepts. Instead it encouraged a paradigm based on understanding how interaction between teachers and students delimit the learning environment. In this scenario, the trainees were learning how to create and maintain equity when constructing a mathematical dialogue. Finally, the study encouraged reflective practice and peer-learning on the part of the trainee teachers, not just their students.

## References

- Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht ; Boston: Kluwer Academic Publishers.
- Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht ; Boston: Kluwer Academic.
- Bateman, A., & Church, A. (2017). Children's Knowledge-in-Interaction: An Introduction. In A. Bateman & A. Church (Eds.), *Children's Knowledge-in-Interaction* (pp. 1–11). <https://doi.org/10.1007/978-981-10-1703-2>
- Carter, K., & Doyle, W. (1995). Preconceptions in Learning to Teach. *The Educational Forum*, 59(2), 186–195. <https://doi.org/10.1080/00131729509336385>
- Cohrssen, C., & Church, A. (2017). Mathematics Knowledge in Early Childhood: Intentional Teaching in the Third Turn. In A. Bateman & A. Church (Eds.), *Children's Knowledge-in-Interaction*. <https://doi.org/10.1007/978-981-10-1703-2>
- Cohrssen, C., Church, A., & Tayler, C. (2014). Purposeful pauses: Teacher talk during early childhood mathematics activities. *International Journal of Early Years Education*, 22(2), 169–183.
- De La Harpe, B., & Radloff, A. (2000). Informed Teachers and Learners: The importance of assessing the characteristics needed for lifelong learning. *Studies in Continuing Education*, 22(2), 169–182. <https://doi.org/10.1080/713695729>
- Evnikskaya, N., & Jakonen, T. (2017). Multimodal conversation analysis and CLIL classroom practices. In A. Llinas & T. Morton (Eds.), *Language Learning & Language Teaching* (Vol. 47, pp. 201–220). <https://doi.org/10.1075/llt.47.12evn>
- Gigerenzer, G., & Gaissmaier, W. (2011). Heuristic decision making. *Annual Review of Psychology*, 62, 451–482.

- Glaserfeld, E. von. (2005). *Radical constructivism in mathematics education*. Retrieved from <http://accesbib.uqam.ca/cgi-bin/bduqam/transit.pl?&noMan=25127639>
- Have, P. ten. (2007). *Doing conversation analysis* (2nd ed). Los Angeles: Sage.
- Heritage, J., & Atkinson, J. M. (1984). Structures of social action. *Studies in Conversation Analysis*.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics* (Vol. 1, pp. 371–404). Charlotte, NC: Information Age Pub.
- Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. *Teaching Mathematics through Problem Solving: Grades, 6(12)*, 3–14.
- Hilton, M. (2002). *Intricate complexities*. Taylor & Francis.
- Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9(2), 187–211.
- Jefferson, G. (1984). Transcript notation. *Structures of Social Action: Studies in Conversation Analysis*, 346–369.
- Koole, T., & Elbers, E. (2014). Responsiveness in teacher explanations: A conversation analytical perspective on scaffolding. *Linguistics and Education*, 26, 57–69.
- Llinares, A., Morton, T., & Whittaker, R. (2012). *The roles of language in CLIL*. Cambridge: Cambridge Univ. Press.
- Llinares, S. (2002). Participation and reification in learning to teach: The role of knowledge and beliefs. In *Beliefs: a hidden variable in mathematics education?* (pp. 195–209). Springer.
- Mazenod, A., Francis, B., Archer, L., Hodgen, J., Taylor, B., Tereshchenko, A., & Pepper, D. (2018). Nurturing learning or encouraging dependency? Teacher constructions of students in



- lower attainment groups in English secondary schools. *Cambridge Journal of Education*, 1–16. <https://doi.org/10.1080/0305764X.2018.1441372>
- McHoul, A., Rapley, M., & Antaki, C. (2008). You gotta light? *Journal of Pragmatics*, 40(1), 42–54. <https://doi.org/10.1016/j.pragma.2007.03.006>
- Mercer, N., Dawes, L., & Staarman, J. K. (2009). Dialogic teaching in the primary science classroom. *Language and Education*, 23(4), 353–369. <https://doi.org/10.1080/09500780902954273>
- Mortimer, E. F., & Scott, P. (2003). *Meaning making in secondary science classrooms*. Maidenhead: Open University Press.
- Myhill, D., & Warren, P. (2005). Scaffolds or straitjackets? Critical moments in classroom discourse. *Educational Review*, 57(1), 55–69.
- Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction*, 21(2), 175–207.
- Project, L. M. for T. (2011). Measuring the mathematical quality of instruction. *Journal of Mathematics Teacher Education*, 14, 25–47.
- Sacks, H., & Jefferson, G. (2006). *Lectures on conversation: Volumes I & II* (1. publ. in one paperback volume 1995, [Nachdr.]). Oxford: Blackwell.
- Schegloff, E. A. (2007). *Sequence organization in interaction: A primer in conversation analysis*. Cambridge ; New York: Cambridge University Press.
- Schutt, R. K. (2018). *Investigating the social world: The process and practice of research*. London: SAGE Publications.
- Sinclair, J. M., & Coulthard, M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils*. Oxford Univ Pr.
- Vygotsky, L. S. (1980). *Mind in society: The development of higher psychological processes*. Harvard university press.

- Walsh, S. (2006). Talking the talk of the TESOL classroom. *ELT Journal*, 60(2), 133–141.  
<https://doi.org/10.1093/elt/cci100>
- Warshauer, H. K. (2015a). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, 18(4), 375–400. <https://doi.org/10.1007/s10857-014-9286-3>
- Warshauer, H. K. (2015b). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20(7), 390. <https://doi.org/10.5951/mathteacmidscho.20.7.0390>
- Wertsch, J. V. (1984). The zone of proximal development: Some conceptual issues. *New Directions for Child and Adolescent Development*, 1984(23), 7–18.
- Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17(2), 89–100.