# Electronic properties and $4 f \rightarrow 5 d$ transitions in $\mathbf{C e}$-doped $\mathbf{L u}_{2} \mathbf{S i O}_{5}:$ a theoretical investigation 

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The electronic properties and the $4 f \rightarrow 5 d$ transitions of the dopant $\mathrm{Ce}^{3+}$ ions located at the two crystallographic lutetium sites of $\mathrm{Lu}_{2} \mathrm{SiO}_{5}$ (LSO) are investigated using the hybrid density functional theory (DFT) with the HSE06 functional and the wavefunction-based embedded cluster methods, respectively. The HSE06 calculations give a band gap of 6.35 eV for LSO, which agrees well with the reported experimental values between $6.4-6.8 \mathrm{eV}$. It is found that $\mathrm{Ce}^{3+}$ prefers strongly to occupy the seven-coordinated (Lu1) site over the six-coordinated (Lu2) one. The energy gaps between the occupied $\mathrm{Ce}^{3+} 4 f$ band and the valence band maximum of the host are predicted to be 2.81 and 3.07 eV for $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ substitutions, respectively, which are close to the experimental data of $2.6-2.9 \mathrm{eV}$. Based on the wavefunction-based CASSCF/CASPT2 embedded cluster calculations for the energies of the $\mathrm{Ce}^{3+} 4 f^{d}$ and $5 d^{1}$ levels, the experimentally observed $4 f \rightarrow 5 d$ transition bands are identified in association with the two substitutions. The predicted transition energy and intensity patterns for $\mathrm{Ce}_{\mathrm{Lu} 1}$ substitution are in fairly good agreement with those of the experimental absorption spectrum. The variations of the two lowest $4 f \rightarrow 5 d$ transition energies with the substitutions are finally discussed in terms of the changes of the centroid-energy difference and the crystal-field splitting with the local coordination geometries.

[^0]
## 1. Introduction

Cerium-doped optical materials have attracted a great deal of interest due to their wide applications as scintillators, light-emitting diodes, and field emission displays. ${ }^{1-3}$ Among these, cerium-doped lutetium oxyorthosilicate, $\mathrm{Lu}_{2} \mathrm{SiO}_{5}: \mathrm{Ce}(\mathrm{LSO}: \mathrm{Ce})$, has been the subject of considerable attention during the past two decades, because it has a number of appealing properties for scintillator applications in positron emission tomography and other medical imaging equipment. ${ }^{4,5}$ It has a relatively high density $\left(7.4 \mathrm{~g} \mathrm{~cm}^{-3}\right)$, an excellent light yield, a fast scintillation decay time ( $\sim 40 \mathrm{~ns}$ ), and the mechanical and chemical stabilities characteristic of oxide structures. The optical properties and luminescence mechanisms of this material have been studied extensively using various experimental methods, including emission, excitation and absorption spectra, ${ }^{6-10}$ and thermally stimulated luminescence. ${ }^{11}$ It is established that the observed optical spectra mainly come from $4 f-5 d$ electronic transitions of the dopant $\mathrm{Ce}^{3+}$, and oxygen vacancies play a major role in the scintillation afterglow.

However, the identification of the dopant sites associated with the experimentally observed $4 f \rightarrow 5 d$ transition bands in LSO:Ce has been controversial for many years. Suzuki and coworkers ${ }^{6}$ observed two distinct types of emission and the correlated excitation spectra of $\mathrm{Ce}^{3+}$ in LSO, and these results were explained as due to the existence of two sets of $\mathrm{Ce}^{3+}$ centers (referred to as Ce 1 and Ce 2 ) occupying the two crystallographically independent lutetium sites. This is known as the two-activation-center model. The emission from Ce 2 was observed to be much weaker than that from Ce 1 , and was obscured by the latter at temperatures above 80 K . Later, Naud et al. ${ }^{8}$ gave a different interpretation for the two activation centers, according to which the two types of spectra were from to the $\mathrm{Ce}^{3+}$ ions at the substitutional Lu sites and at the interstitial sites, respectively. After that, based upon the $4 f \rightarrow 5 d$ absorption measurements, Cooke and coworkers ${ }^{9}$ provided evidence in support of the previously proposed two-activation-center model by Suzuki et al., ${ }^{6}$ but also tentatively suggested that the Ce 1 and Ce 2 centers were due to the $\mathrm{Ce}^{3+}$ ions located at the six-coordinated Lu2 and the seven-coordinated Lu1 sites, respectively. A more recent investigation of LSO:Ce using electron paramagnetic resonance (EPR) spectroscopy indicated
that about $95 \%$ of the dopant $\mathrm{Ce}^{3+}$ ions occupy the Lu1 site, and the rest (about $5 \%$ ) of $\mathrm{Ce}^{3+}$ is at the Lu2 site. ${ }^{12}$ These EPR results are in agreement with the two-activation-center model, ${ }^{6}$ and attribute the Ce 1 and Ce 2 centers to the $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ sites, respectively, contrary to the assignment by Cooke et al.. ${ }^{9}$ Since the information about the location of $\mathrm{Ce}^{3+}$ ions in LSO is essential to the understanding of the relevant spectroscopic and scintillation properties, we have performed first-principles calculations to obtain this information in association with the observed $4 f \rightarrow 5 d$ transitions. Such calculations can provide information on the local geometries around $\mathrm{Ce}^{3+}$, the energy levels involved in $4 f-5 d$ transitions, and also their relationships, as shown in recent works. ${ }^{13-15}$

The energy position of localized $\mathrm{Ce}^{3+} 4 f$ states within the host band gap is an important parameter in many technological applications of Ce-doped optical materials. DFT calculations within the standard local density approximation (LDA) or the general gradient approximation (GGA) are unable to correctly predict this position due to the self-interaction errors associated with the localized $4 f$ states. ${ }^{16-18}$ One common way to address this problem is to use the so-called DFT $+U$ approach, ${ }^{19-21}$ where a Hubbard $U$ correction is applied to the localized $4 f$ states while leaving the other electronic states described by the standard DFT. DFT $+U$ can describe properly the energy position of the occupied Ce $4 f$ state relative to the top of the valence band by a suitable choice of $U$, but the band gap underestimation inherent in the exchange-correlation (XC) functionals still remains. ${ }^{15}$ Hybrid DFT, where a portion of Hartree-Fock (HF) exchange is mixed with the DFT XC functionals, has been shown to give much better band gaps than the standard DFT, and also better positions of the localized states in oxide systems. ${ }^{22,23}$ In addition, admixture of the HF exchange is a universal parameter applied to all the electronic states of the system. It is thus interesting to investigate the electronic properties of the present Ce-doped system using the hybrid DFT method, and to see the improvement in the description of the localized 4f states with respect to the standard DFT.

In the present study, we have first performed geometry optimizations of LSO:Ce using the standard DFT-GGA calculations with the periodic supercell model, in which a cerium atom occupies either of the two lutetium (Lu1 or Lu2) sites. Hybrid DFT calculations have then been carried out on the optimized supercell geometries to investigate their electronic properties. After that, using the optimized supercell structures, we have constructed the $\mathrm{Ce}_{\mathrm{Lu} 11^{-}}$
or $\mathrm{Ce}_{\mathrm{Lu2}}$-centered embedded clusters with their lattice environments represented by ab initio model potentials (AIMP), for which the wavefunction-based CASSCF/CASPT2 calculations at the spin-orbit level have been performed to obtain the energies of $4 f^{1}$ and $5 d^{1}$ levels that are involved in the $4 f \rightarrow 5 d$ transitions. By comparing the DFT total energies of $\mathrm{Ce}_{\mathrm{Lu} 1}$ - and $\mathrm{Ce}_{\text {Lu2 }}$-doped supercells, and also the calculated and experimental $4 f \rightarrow 5 d$ transition energies, the relative preference of the $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ substitutions and their associations with the observed $4 f \rightarrow 5 d$ transition bands have been studied. The main aim of the present work is to use elaborate $a b$ initio approaches to gain insight into the identification of the dopant sites in connection with each of the experimentally observed $4 f \rightarrow 5 d$ transition bands in Ce-doped LSO, and to assess the effectiveness of the hybrid DFT in describing the electronic structure of the dopant $\mathrm{Ce}^{3+}$. The remainder of this paper is organized as follows. The methodology and computational details are described in section 2. The results for the structural properties, electronic properties, and the $4 f \rightarrow 5 d$ transition energies are presented and discussed in section 3 , with the final conclusions collected in section 4.

## 2. Methodology

The Ce-doped LSO was modeled using a $1 \times 2 \times 1$ supercell containing 128 atoms, in which one of 32 lutetium atoms was substituted by a cerium at Lu 1 or Lu 2 site ( $3.125 \%$ doping concentration). The atomic coordinates and lattice parameters were fully relaxed without constraints on symmetry or overall spin, by means of periodic $\mathrm{DFT}^{24,25}$ calculations using the pure PBE XC functional ${ }^{26,27}$ and the plane wave basis sets, as implemented the Vienna $a b$ initio simulation package (VASP)..$^{28,29}$ The $4 f^{14} 5 p^{6} 5 d^{1} 6 s^{2}$ electrons on Lu, the $3 s^{2} 3 p^{2}$ electrons on Si , the $2 s^{2} 2 p^{4}$ electrons on O , and the $5 s^{2} 5 p^{6} 4 f^{1} 5 d^{1} 6 s^{2}$ electrons on Ce , were treated as valence electrons. Their interactions with the respective cores were described by the projected augmented wave (PAW) method. ${ }^{30}$ The geometry optimizations were performed using the conjugate gradient technique, until the total energies were converged to $10^{-6} \mathrm{eV}$ and the Hellmann-Feynman forces on the atoms were less than $0.01 \mathrm{eV} \AA^{-1}$. With the PBE-optimized geometries, the electronic properties were studied using the hybrid DFT with the HSE06 functional. ${ }^{31,32}$ Due to the high computational cost of the hybrid DFT with plane wave basis sets, only single-point energy calculations were performed. In HSE06, $25 \%$ of the Fock
exchange is mixed with $75 \%$ of the PBE exchange, and the slowly decaying long-range part of the Fock exchange is replaced by the corresponding PBE counterpart, with the range separation controlled by a screening parameter $\left(=0.2 \AA^{-1}\right)$. The HSE06 functional is not free from adjustable parameters, namely, the Fock exchange contribution and the screening length, both of which can be tuned with respect to the experimental results. However, for the purposes of the present study, HSE06 was employed with the default settings. Considering the large size of the supercell, only one $k$-point $\Gamma$ was used to sample the Brillouin zone, and to achieve converged results, the cutoff energy for the plane wave basis was set to 550 eV .

Based on the geometries of the $\mathrm{Ce}_{\mathrm{Lu} 1}$ - and $\mathrm{Ce}_{\mathrm{Lu} 2}$-doped LSO supercells as optimized by the standard DFT-PBE calculations, the $4 f \rightarrow 5 d$ transition energies were computed with the wavefunction-based embedded cluster approaches. The $\left(\mathrm{Ce}_{\mathrm{Lul}^{2}} \mathrm{O}_{7} \mathrm{Si}_{4}\right)^{5+}$ and $\left(\mathrm{Ce}_{\left.\mathrm{Lu}_{2} \mathrm{O}_{6} \mathrm{Si}_{4}\right)^{7+}}\right.$ clusters were chosen for our investigation (see Fig. 1), which comprise the coordination polyhedrons around $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ ions, plus four $\mathrm{Si}^{4+}$ ions in the respective second coordination spheres. The total numbers of 693 and 678 ions within the spheres of a radius $10.0 \AA$ surrounding these two clusters were modeled using the AIMP embedding potentials, ${ }^{33}$ in order to account for the electrostatic, exchange, and Pauli interactions of the clusters with their environments. The AIMP embedding potentials for $\mathrm{Si}^{4+}$ and $\mathrm{O}^{2-}$ were taken from those produced in $\mathrm{SiO}_{2},{ }^{34}$ and for $\mathrm{Lu}^{3+}$, the potential for $\mathrm{Lu}_{2} \mathrm{O}_{3}$ was used. ${ }^{35}$ The remainders of their surroundings were simulated by 92607 and 92624 point charges located at lattice sites, respectively, generated using Lepetit's method. ${ }^{36}$ This method is an extension of the one proposed by Evjen ${ }^{37}$ and produces the same electrostatic potentials as Ewald's method. ${ }^{38}$

For the two embedded clusters, we performed state-average CASSCF (SA-CASSCF) 39-42 plus CASPT2 ${ }^{43-45}$ calculations with the scalar relativistic many-electron Hamiltonian, which take into account the bonding, static and dynamic correlation effects. Then, with the wavefunctions from SA-CASSCF and energies from CASPT2, the AMFI approximation of the DKH spin-orbit coupling operator ${ }^{46-48}$ was added to the Hamiltonian and a restricted active space state-interaction spin-orbit (RASSI-SO) method ${ }^{49}$ was employed to account for the spin-orbit effect. These wavefunction-based calculations were performed using the program MOLCAS ${ }^{50}$

In the SA-CASSCF calculations, a $[4 f, 5 d, 6 s]$ complete active space was adopted. The CASSCF wavefunctions come from interactions of all configurations in which the single unpaired electron occupies one of the thirteen molecular orbitals of main characters $\mathrm{Ce}^{3+} 4 f$, $5 d$, and $6 s$. The molecular orbitals are optimized by minimizing the average energy of the
thirteen states. Since no symmetry (or $C_{1}$ symmetry) was found for $\mathrm{Ce}_{\mathrm{Lu} 1}$ or $\mathrm{Ce}_{\mathrm{Lu} 2}$ site in the DFT-PBE geometry optimization, these states may be labeled by the irreducible representation $(A)$ of the $C_{1}$ point group, each prefixed by a number $i(i=1-13)$ in order of increasing energy. The states are thus denoted by $1-13^{2} A$, where the superscript " 2 " indicates the doublet state with spin $S=1 / 2$. With the CASSCF wavefunctions and the occupied and virtual orbitals, CASPT2 calculations were carried out, where dynamic correlation effects of the $\mathrm{Ce}^{3+} 5 s, 5 p, 4 f$ and $5 d$ electrons and the $\mathrm{O} 2 s, 2 p$ electrons are considered. Further inclusion of spin-orbit coupling mixes all of these states, leading to thirteen Kramer's doublets that belong to the $\Gamma_{2}$ irreducible representation of the $C_{1}{ }^{*}$ double group. In these calculations, a relativistic effective core potential ( $[\mathrm{Kr}]$ core) with a $(14 s 10 p 10 d 8 f 3 g) /[6 s 5 p 6 d 4 f 1 g]$ Gaussian valence basis set from ref. 51 was used for Ce , and a $[\mathrm{He}]$ core effective core potential with a $(5 s 6 p 1 d) /[2 s 4 p 1 d]$ valence basis set from ref. 52 was used for O . For Si , we used a [ Ne ] core effective potential and a $(7 s 6 p 1 d) /(2 s 3 p 1 d)$ valence basis set from ref. 52. These basis sets were further augmented by the respective auxiliary spin-orbit basis sets for a proper description of the inner core region in the spin-orbit calculations.

## 3. Results and discussion

### 3.1 Structural properties

The structure of the undoped LSO (monoclinic $C 2 / c$ symmetry) was first optimized using the standard DFT-PBE method, and the results are listed in Tables 1 and 2 for the lattice and internal parameters, respectively. The optimized lattice parameters, which were obtained with the $\mathrm{Lu} 4 f^{14}$ electrons treated in the core (labeled by Lu- $4 f$ core in Table 1), agree very well with the experimental data, ${ }^{53}$ while the results with the $\mathrm{Lu} 4 f^{14}$ electrons treated as valence electrons (labeled by Lu- $4 f$ val) are clearly overestimated. Similar observations have been reported in the DFT-PBE calculations of the geometries for the $\mathrm{Lu}_{2} \mathrm{O}_{3}$ and $\mathrm{LuAlO}_{3}$ crystals, ${ }^{15,54}$ traced to the insufficient cancellation of the self-interaction errors associated with the localized $\mathrm{Lu} 4 f$ electrons in the PBE XC functional. For the two crystallographically different types of lutetium atoms (Lu1 and Lu2), the silicon atom, and the five distinct types of oxygens (O1-O5), which are all located at sites of $C_{1}$ symmetry, the calculated internal
parameters in the $\mathrm{Lu} 4 f$-core and $\mathrm{Lu} 4 f$-val schemes are both in good agreements with the experimental data, ${ }^{53}$ see Table 2. In light of these results, the $\mathrm{Lu} 4 f$-core scheme has been employed in the following geometry optimizations for the Ce-doped LSO.

Table 3 summarizes the calculated lattice parameters for the $\mathrm{Ce}_{\mathrm{Lu} 1}-$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$-doped LSO supercells, along with those of the undoped LSO for comparison. From the table, we see that the substitution of a Ce into the Lu 1 or Lu 2 site produces a very small increase (by $0.76 \%$ or $0.68 \%$, respectively) of the supercell volume, and slightly distorts the monoclinic phase of the undoped LSO into a triclinic one with the deviations of the angles no larger than $\pm 0.08^{\circ}$. Table 4 gives the selected distances for the local geometries of the $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ sites. Compared with the undoped system, the substitution of Ce at Lu 1 site increases the distances to the six nearest oxygens by between 0.125 to $0.177 \AA$, while leaving the distance to the remaining (most distant) O2b atom almost unchanged. The substitution at Lu2 site increases the distances to the six coordinated oxygens by between 0.126 to $0.169 \AA$. The lengthening of these bonding distances is qualitatively consistent with the larger ionic radius of $\mathrm{Ce}^{3+}$ than that of $\mathrm{Lu}^{3+}$ by $\sim 0.16 \AA$ in the same coordination. ${ }^{55}$ The changes of the distances to the four Si atoms in the second coordination shell upon the substitutions are in the ranges of -0.036 to $0.056 \AA$ for $\mathrm{Ce}_{\mathrm{Lu} 1}$ and -0.017 to $0.090 \AA$ for $\mathrm{Ce}_{\mathrm{Lu} 2}$. The structures of the $\mathrm{SiO}_{4}$ moieties are somewhat flexible, with slight tetrahedral deformations and the changes of Si-O distances no larger than 0.020 Å.

### 3.2 Electronic properties

The electronic properties of the undoped LSO were first investigated in the Lu $4 f$-core and Lu $4 f$-val schemes, to see the effects of the hybrid DFT on the calculated band gap and of the Lu $4 f^{4}$ electrons on the other electronic states. Fig. 2 shows the calculated total and orbital projected densities of states (DOS) using the pure PBE and hybrid HSE06 functionals, based on the structure optimized in the Lu 4 -core scheme. Comparing Figs. 2(a) and 2(b), one can immediately notice that the band gap energy ( $E_{\text {gap }}$ ) with HSE06 $(6.35 \mathrm{eV}$ ) is much larger than that with PBE ( 4.73 eV ), and is close to the experimental values of $6.4-6.8 \mathrm{eV}$ as estimated from excitation and absorption spectral measurements. ${ }^{9,11,56-58}$ On the other hand, the
comparison of Figs. 2(b) and 2(c) indicates that the inclusion of the full $\mathrm{Lu} 4 f^{4}$ shell as valence electrons does not substantially change the electronic properties of the other states, apart from a slight narrowing of the band gap by $\sim 0.1 \mathrm{eV}$.

In the orbital projected DOS obtained for the undoped LSO with HSE06 (Fig. 2(b)), the O 2 p states form a valence band about 6.62 eV wide just below the Fermi level $\left(E_{\mathrm{F}}\right)$, while the $\mathrm{Lu} 5 d$ states mainly constitute the conduction band lying $7.35-14.01 \mathrm{eV}$ above $E_{\mathrm{F}}$. The O 2 s bands lie in an energy range of $16.40-19.85 \mathrm{eV}$ below $E_{\mathrm{F}}$. There is a small peak at 6.35 eV above $E_{\mathrm{F}}$, which constitutes the conduction band edge. This peak is mainly composed of Lu $6 s$ and $\mathrm{O} 2 s$ states; it has similar characteristics as Yb -trapped excitons in $\mathrm{SrCl}_{2}: \mathrm{Yb}^{59}$ and it might well correspond to a Lu-trapped exciton. It is noted that the orbital characters of valence and conduction bands with HSE06 are basically the same as those obtained with the pure PBE (comparing Figs. 2(a) and 2(b)), although their energy positions relative to $E_{\mathrm{F}}$ are different.

For $\mathrm{Ce}_{\mathrm{Lu} 1{ }^{-}}$and $\mathrm{Ce}_{\mathrm{Lu2} 2}$-doped LSO , the total energy calculations for the two optimized structures indicate that the former substitutional doping is more favorable than the latter by 415 meV with PBE and 452 meV with HSE06. The strong tendency of Ce to occupy the Lu1 site compared to the Lu2 site is in support of the conclusions drawn from analysis of the experimental EPR spectra. ${ }^{12}$

In Fig. 3, we show the total and orbital-projected DOS for $\mathrm{Ce}_{\mathrm{Lu} 1}-$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$-doped LSO obtained with PBE and HSE06 in the Lu $4 f$-core scheme. One observes that the incorporation of Ce into LSO leads to formation of electronic states in the band gap, and their properties depends significantly on the choice of the XC functional. With the pure PBE functional, the $\mathrm{Ce}_{\mathrm{Lu1}}{ }^{-}$or $\mathrm{Ce}_{\mathrm{Lu} 2}$-doped LSO are predicted to be nearly metallic with $E_{\mathrm{F}}$ positioned within the Ce $4 f$ bands, see Figs. 3(a) and 3(b). With the hybrid HSE06 functional, an energy gap opens between the occupied and unoccupied Ce $4 f$ states, and the two doped systems are correctly predicted to be insulating, see Figs. 3(c) and 3(d). The unoccupied $4 f$ states are positioned above the bottom of the conduction band, and are broadened with respect to those with PBE due to the hybridization with the other empty states. In the DOS with both the pure and hybrid functionals, the sharp peak at the valence band edge corresponds to a lone Ce $4 f$ electron, indicating a $3+$ oxidation state of the Ce cation.

In Figs. 3(a)-(d), we also show the values of the computed gap $\left(\Delta E_{4 f}\right)$ between the occupied Ce $4 f$ band and the top of the valence band. For $\mathrm{Ce}_{\mathrm{Lu}^{-}}$(and $\mathrm{Ce}_{\mathrm{Lu} 2^{-}}$) doped LSO, the value of $\Delta E_{4 f}$ is decreased from 4.05 eV (and 4.24 eV ) with PBE to 2.81 eV (and 3.07 eV ) with HSE06. Experimentally, the value of $\Delta E_{4 f}$ was estimated to be about $2.6-2.9 \mathrm{eV}^{11}$ based on the wavelength-resolved thermally stimulated luminescence measurements. Thus, the hybrid HSE06 gives a value for $\Delta E_{4 f}$ in much closer agreement with the experimental data than the standard PBE functional, especially for the more preferential $\mathrm{Ce}_{\mathrm{Lu} 1}$ doping. In addition, the calculated HSE06 value of $\Delta E_{4 f}$ for $\mathrm{Ce}_{\mathrm{Lu} 1}$ is larger than that for $\mathrm{Ce}_{\mathrm{Lu} 2}$ (by 0.19 $\mathrm{eV})$. This relative position is qualitatively consistent with the experimental results from the temperature and spectrally-resolved photoconductivity study. ${ }^{60}$

## 3.3. $4 f \rightarrow 5 d$ transition energies

Using the relaxed atomic structures of $\mathrm{Ce}_{\mathrm{Lu} 1}-$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$-doped LSO supercells from DFT-PBE calculations in the Lu 4 -core scheme, we constructed the Ce-centered embedded clusters, $\left(\mathrm{Ce}_{\mathrm{Lu}^{2}} \mathrm{O}_{7} \mathrm{Si}_{4}\right)^{5+}$ and $\left(\mathrm{Ce}_{\mathrm{Lu}_{2}} \mathrm{O}_{6} \mathrm{Si}_{4}\right)^{7+}$ (denoted hereafter by $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ for brevity), with the surroundings represented by AIMPs and point charges to account for the electrostatic, exchange, and Pauli interactions of the clusters with the embedding environments. The wavefunction-based CASSCF/CASPT2 calculations were then carried out to obtain the energy levels for $\mathrm{Ce}^{3+} 4 f^{1}, 5 d^{1}$, and $6 s^{1}$ configurations. In Table 5 we list the level energies calculated at both the spin-orbit-free and spin-orbit levels. One can see that, besides its expected importance for the $4 f^{d}$ levels, the inclusion of the spin-orbit coupling increases the $5 d^{1}$ and $6 s^{1}$ level energies uniformly by around $\sim 1000 \mathrm{~cm}^{-1}$ for both $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ clusters. A similar observation was found before for $\mathrm{Ce}^{3+}$ doped in $\mathrm{Y}_{3} \mathrm{Al}_{5} \mathrm{O}_{12} .{ }^{61}$ When comparing the calculated $4 f \rightarrow 5 d$ transition energies with those estimated from the experimental absorption or excitation band maxima, the calculated results for the three lowest $5 d^{1}$ levels of $\mathrm{Ce}_{\mathrm{Lu} 1}$ and the two lowest $5 d^{1}$ levels of $\mathrm{Ce}_{\mathrm{Lu} 2}$ agree respectively with the experimental results for Ce 1 and Ce 2 activation centers. The respective average deviations are 430 and $550 \mathrm{~cm}^{-1}$ at the spin-orbit-free level, and 610 and $1010 \mathrm{~cm}^{-1}$ at the spin-orbit level. For the $4 f^{1}, 6 s^{1}$ and the other $5 d^{1}$ levels, the firm experimental data are lacking and thus no comparison can be made
with experiments. Table 6 lists the relative oscillator strengths computed using the CASSCF wavefunctions and CASPT2 energies for the zero-phonon transition lines from the $4 f^{1}$ ground level $\left(1 \Gamma_{2}\right)$ to the five $5 d^{1}$ levels $\left(8-12 \Gamma_{2}\right)$, and for $\mathrm{Ce}_{\mathrm{Lu} 1}$ a schematic representation is given in Fig. 4. Its intensity pattern shows a good agreement with the experimental absorption spectrum of the Ce 1 center in ref. 4.

The above comparison of calculated and experimental $4 f \rightarrow 5 d$ transition energies provide direct evidence that the experimentally designated Ce 1 and Ce 2 centers are due to the $\mathrm{Ce}^{3+}$ ions residing in the two crystallographically inequivalent Lu sites, in support of the two-activation-model proposed by Suzuki et al.. ${ }^{5}$ Furthermore, it unambiguously identifies the Ce 1 and Ce 2 centers to the seven-coordinated $\mathrm{Ce}_{\mathrm{Lu} 1}$ and the six-coordinated $\mathrm{Ce}_{\mathrm{Lu} 2}$ sites, respectively. This indicates that the dopant $\mathrm{Ce}^{3+}$ ions preferentially occupy the larger substitutional sites, which is consistent with results of the DFT supercell total-energy calculations and also the EPR spectral measurements. ${ }^{12}$ Along this line of thinking, the dopant $\mathrm{Ce}^{3+}$ is not likely to occupy the much smaller interstitial sites instead of the larger substitutional sites, before the latter are almost filled.

Table 5 also shows that, for the two lowest transitions, $4 f^{1} 1 \Gamma_{2} \rightarrow 5 d^{1} 8,9 \Gamma_{2}$, the calculated transition energies for $\mathrm{Ce}_{\mathrm{Lu} 1}$ are greater than those for $\mathrm{Ce}_{\mathrm{Lu} 2}$ by 269 and $3648 \mathrm{~cm}^{-1}$, respectively. These values are in quite good agreement with the corresponding experimental data of 931 and $3184 \mathrm{~cm}^{-1}$, as estimated from the excitation or absorption band maxima. The lowering of the two transition energies from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$ may intuitively be explained by the increase of the crystal-field splitting of $5 d^{1}$ levels due to a reduction of the size of the coordination polyhedron, with the average Ce-O distance decreasing from $2.463 \AA$ in $\mathrm{Ce}_{\mathrm{Lu}}$ to $2.371 \AA$ in $\mathrm{Ce}_{\text {Lu2 }}$. Considering that the change of the centroid energy might also contribute the reduction of the transition energies, in the following we will discuss the lowering of these two transition energies in a little more detail.

According to the analysis in ref. 14 , each of the transition energies from the lowest $4 f^{1}$ level $\left(1 \Gamma_{2}\right)$ to the two lowest $5 d^{1}$ levels $\left(i \Gamma_{2}, i=8,9\right)$ may be decomposed into two components, i.e., the centroid-energy difference (ced) and the crystal-field stabilization (cfs) energies,

$$
\Delta E\left(4 f^{1} 1 \Gamma_{2} \rightarrow 5 d^{1} i \Gamma_{2}\right)
$$

$$
\begin{aligned}
& =\Delta E_{\mathrm{ced}}\left(4 f^{\mathrm{d}} \rightarrow 5 d^{1}\right)+\Delta E_{\mathrm{cfs}}\left(4 f^{1} 1 \Gamma_{2} \rightarrow 5 d^{1} i \Gamma_{2}\right) \\
& =\Delta E_{\mathrm{ced}}\left(4 f^{\mathrm{d}} \rightarrow 5 d^{1}\right)+\Delta E_{\mathrm{cfs}}\left(4 f^{\mathrm{d}} 1 \Gamma_{2}\right)-\Delta E_{\mathrm{cfs}}\left(5 d^{1} i \Gamma_{2}\right)
\end{aligned}
$$

where $\Delta E_{\text {ced }}\left(4 f^{1} \rightarrow 5 d^{1}\right)$ is the centroid-energy difference between $5 d^{1}$ and $4 f^{d}$ configurations, and $\Delta E_{\text {cfs }}\left(4 f^{1} 1 \Gamma_{2}\right)$ and $\Delta E_{\text {cfs }}\left(5 d^{1} i \Gamma_{2}\right)$ are the crystal-field stabilization energies of the $4 f^{1} 1 \Gamma_{2}$ and $5 d^{1} i \Gamma_{2}$ levels, relative to their respective $4 f^{d}$ and $5 d^{1}$ centroid energies. A schematic representation of these quantities is shown in Fig. 5, and their values for the $4 f^{d} 1 \Gamma_{2} \rightarrow 5 d^{1} i \Gamma_{2}$ $(i=9,10)$ transitions of $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ are given in Table 7. From this table, we see that the centroid-energy difference, $\Delta E_{\text {ced }}\left(4 f^{f} \rightarrow 5 d^{1}\right)$, shifts only by $7 \mathrm{~cm}^{-1}$ from $\mathrm{Ce}_{\text {Lu1 }}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$, which may be rationalized qualitatively using Judd-Morrison model. ${ }^{62,63}$ According to this model, from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}, \Delta E_{\text {ced }}\left(4 f^{1} \rightarrow 5 d^{1}\right)$ is decreased with the shortening of the average Ce-O distance, but at the same time it is increased with the decrease of the coordination number (from seven to six), and the two effects nearly cancel. Therefore, for the reduction of the $4 f^{1}$ $1 \Gamma_{2} \rightarrow 5 d^{1} 8,9 \Gamma_{2}$ transition energies from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$, the contribution from the change of the centroid-energy difference is negligible, and hence the contribution from the crystal-field stabilization dominates.

The crystal-field stabilization energies of the $4 f^{1} 1 \Gamma_{2}$ and $5 d^{1} i \Gamma_{2}(i=9,10)$ levels all increase from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$ with the values of 236,512 , and $3891 \mathrm{~cm}^{-1}$, respectively. This is consistent with the expectation that the size reduction of the coordination polyhedron from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\text {Lu2 }}$ should lead to an increased crystal-field stabilization of $4 f^{1}$ and $5 d^{1}$ levels. This size effect is especially pronounced for the second $5 d^{1} 10 \Gamma_{2}$ level, which leads to a reduction of the transition energy to this level by $3648 \mathrm{~cm}^{-1}$ from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$, as compared to the value of only $269 \mathrm{~cm}^{-1}$ for the lowest $4 f^{1} 1 \Gamma_{2} \rightarrow 5 d^{1} 8 \Gamma_{2}$ transition.

## 4. Conclusions

We have investigated the electronic properties and $4 f \rightarrow 5 d$ transitions of Ce-doped LSO using the hybrid DFT calculations with the periodic supercell model and the wavefunction-based CASSCF/CASPT2 calculations with the embedded cluster model, respectively. It was found that the hybrid HSE06 functional provides a Kohn-Sham band-gap value and the energy
position of the occupied $\mathrm{Ce}^{3+} 4 f$ states in much better agreement with experiments than the standard GGA-PBE functional. At the same time, the total energy calculations reveal that the Ce substitution into the seven-coordinated Lu1 site is strongly preferred over the six-coordinated Lu2 site, consistent with the EPR spectral observations. Based on the previously determined supercell structures, the embedded $\left(\mathrm{Ce}_{\left.\mathrm{Lul} \mathrm{O}_{7} \mathrm{Si}_{4}\right)^{5+} \text { and }\left(\mathrm{Ce}_{\left.\mathrm{Lu}_{2} \mathrm{O}_{6} \mathrm{Si}_{4}\right)^{7+}}{ }^{7+} \text {. }\right.}\right.$ clusters were constructed, with their embedding environments represented by AIMPs. The CASSCF/CASPT2 calculations with the spin-orbit coupling effects were then performed to study the $4 f \rightarrow 5 d$ transitions. From comparisons of the calculated and experimental transition energies, the two distinct types of $4 f \rightarrow 5 d$ excitation bands as observed experimentally have been identified as due to the $\mathrm{Ce}^{3+}$ ions located in the two substitutional Lu sites, with the experimentally designated Ce 1 and Ce 2 activation centers assigned to the $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ sites, respectively. Finally, the changes of the two lowest $4 f \rightarrow 5 d$ transition energies from CeLu1 to $\mathrm{Ce}_{\mathrm{Lu} 2}$ have been analyzed in terms of the centroid-energy difference and the crystal-field splitting.

The present study demonstrates that the DFT calculations with the screened hybrid HSE06 functional may represent an effective way in describing the position of the $\mathrm{Ce}^{3+} 4 f$ states within the host band gap, which are usually difficult to obtain directly using the conventional X-ray or UV photoelectron spectroscopy due to the weakness of the $4 f$ signal of the dopant ion. It also shows the ability of high-level $a b$ initio calculations in elucidating the experimentally observed $\mathrm{Ce}^{3+} 4 f \rightarrow 5 d$ transitions, which have broad applications in the field of optical materials. A combination of the hybrid DFT supercell approach and the wavefunction-based embedded cluster method could aid in the prediction of the optical properties of Ce-doped optical materials from the minimal information of the host composition.

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Table 1 Calculated and experimental lattice parameters for the LSO crystal.

| Method | Reference | $a(\AA)$ | $b(\AA)$ | $c(\AA)$ | $\beta(\mathrm{deg})$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| DFT-PBE <br> (Lu 4f-core) | This work | 14.282 | 6.669 | 10.284 | 122.019 |
| DFT-PBE <br> (Lu 4f-val) | This work | 14.384 | 6.725 | 10.380 | 122.079 |
| Experiment | ref. 53 | 14.277 | 6.640 | 10.247 | 122.224 |

Table 2 Calculated internal parameters for the LSO crystal. The experimental (Expt.) data from ref. 53 are also listed for comparison.

|  | Method | $x$ | $y$ | $z$ |  | Method | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lu1 | Lu 4f-core | 0.5382 | 0.7567 | 0.4711 | O2 | Lu 4f-core | 0.3782 | 0.7885 | 0.1698 |
|  | Lu $4 f$-val | 0.5383 | 0.7564 | 0.4705 |  | Lu $4 f$-val | 0.3780 | 0.7867 | 0.1705 |
|  | Expt. | 0.5373 | 0.7559 | 0.4671 |  | Expt. | 0.3802 | 0.7883 | 0.1762 |
| Lu2 | Lu 4f-core | 0.1411 | 0.3806 | -0.1612 | O3 | Lu 4f-core | 0.2010 | 0.6508 | 0.1783 |
|  | Lu $4 f$-val | 0.1417 | 0.3816 | -0.1614 |  | Lu $4 f$-val | 0.2020 | 0.6508 | 0.1786 |
|  | Expt. | 0.1409 | 0.3774 | -0.1636 |  | Expt. | 0.2023 | 0.6490 | 0.1768 |
| Si | Lu 4f-core | 0.3175 | 0.5921 | 0.1936 | O4 | Lu 4f-core | 0.2972 | 0.4234 | 0.0654 |
|  | Lu $4 f$-val | 0.3177 | 0.5919 | 0.1942 |  | Lu $4 f$-val | 0.2973 | 0.4245 | 0.0668 |
|  | Expt. | 0.3179 | 0.5917 | 0.1931 |  | Expt. | 0.2984 | 0.4289 | 0.0630 |
| O1 | Lu 4f-core | 0.4120 | 0.5130 | 0.3655 | O5 | Lu 4f-core | 0.0166 | 0.4042 | -0.1040 |
|  | Lu $4 f$-val | 0.4112 | 0.5129 | 0.3647 |  | Lu $4 f$-val | 0.0167 | 0.4054 | -0.1039 |
|  | Expt. | 0.4112 | 0.5062 | 0.3620 |  | Expt. | 0.0177 | 0.4034 | -0.1025 |

Table 3 Calculated lattice parameters and volumes for the $\mathrm{Ce}_{\mathrm{Lu1}}$ - and $\mathrm{Ce}_{\mathrm{Lu2}}$-doped $\mathrm{LSO}(1 \times 2$ $\times 1)$ supercells. The percent changes in parentheses were taken with respect to the volume of the undoped LSO.

|  | LSO | LSO:Ce $\mathrm{Ce}_{\mathrm{Lu}}$ | LSO:Ce $\mathrm{Lu}^{2}$ |
| :--- | ---: | ---: | ---: |
| $a(\AA)$ | 14.282 | 14.285 | 14.292 |
| $2 b(\AA)$ | 13.337 | 13.364 | 13.375 |
| $c(\AA)$ | 10.284 | 10.331 | 10.311 |
| $\alpha(\mathrm{deg})$ | 90.000 | 90.047 | 89.945 |
| $\beta(\mathrm{deg})$ | 122.019 | 121.940 | 121.961 |
| $\gamma(\operatorname{deg})$ | 90.000 | 89.852 | 89.878 |
| Volume $\left(\AA^{3}\right)$ | 1660.991 | 1673.609 | 1672.296 |
|  |  | $(+0.76 \%)$ | $(+0.68 \%)$ |

Table 4 Calculated distances (in $\AA$ ) from the dopant site to the atoms in the first and second coordination shells before and after the $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ substitutions in LSO. The values in parentheses are the differences taken with respect to the data before substitution. See Fig. 1 for the definition of atomic labels.

|  | $\begin{gathered} \text { LSO } \\ \text { M = Lu1 } \end{gathered}$ | $\begin{aligned} & \text { LSO:Ce }{ }_{\text {Lu1 }} \\ & \mathrm{M}=\mathrm{Ce}_{\mathrm{Lu} 1} \end{aligned}$ |  | $\begin{gathered} \text { LSO } \\ \mathrm{M}=\mathrm{Lu} 2 \end{gathered}$ | $\begin{aligned} & \text { LSO:Ce } \mathrm{Ce}_{\mathrm{L} 2} \\ & \mathrm{M}=\mathrm{Ce}_{\mathrm{Lu} 2} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M-O1a | 2.235 | 2.378 (+0.143) | M-O2 | 2.226 | 2.376 (+0.150) |
| O1b | 2.302 | 2.426 (+0.125) | O3 | 2.234 | 2.376 (+0.142) |
| O2a | 2.325 | 2.489 (+0.164) | O4a | 2.215 | 2.384 (+0.169) |
| O2b | 2.713 | 2.710 (-0.003) | O4b | 2.230 | 2.389 (+0.159) |
| O3 | 2.254 | 2.412 (+0.159) | O5a | 2.154 | 2.281 (+0.126) |
| O5a | 2.141 | 2.296 (+0.154) | O5b | 2.271 | 2.424 (+0.152) |
| O5b | 2.353 | 2.530 (+0.177) |  |  |  |
| M-Si1 | 3.122 | 3.169 (+0.047) | M-Si1 | 3.262 | 3.245 (-0.017) |
| Si2 | 3.468 | 3.432 (-0.036) | Si2 | 3.547 | 3.585 (+0.038) |
| Si3 | 3.532 | 3.582 (+0.049) | Si3 | 3.606 | 3.695 (+0.090) |
| Si4 | 3.743 | $3.799(+0.056)$ | Si4 | 3.451 | 3.523 (+0.072) |

Table 5 Calculated energy levels of the $4 f^{1}, 5 \mathrm{~d}^{1}$, and $6 \mathrm{~s}^{1}$ configurations for the $\left(\mathrm{Ce}_{\left.\mathrm{LuI}_{1} \mathrm{O}_{7} \mathrm{Si}_{4}\right)^{5+}}\right.$ and $\left(\mathrm{Ce}_{\mathrm{Lu}_{2}} \mathrm{O}_{6} \mathrm{Si}_{4}\right)^{7+}$ clusters embedded in LSO. The experimental data were taken from the average of the excitation and absorption peak energies in refs. 4,6,7,9 and 10 for Ce 1 center and refs. 4 and 6 for Ce 2 center. All values are in units of $\mathrm{cm}^{-1}$.

| $4 f^{1}$ levels | Calculation without spin-orbit coupling |  |  | Calculation with spin-orbit coupling |  |  | Experiments |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{Ce}_{\text {Lu1 }}$ | $\mathrm{Ce}_{\text {Lu2 }}$ |  | $\mathrm{Ce}_{\text {Lu1 }}$ | $\mathrm{Ce}_{\mathrm{Lu} 2}$ | Cel | Ce 2 |
|  | $1^{2} \mathrm{~A}$ | 0 | 0 | $1 \Gamma_{2}$ | 0 | 0 |  |  |
|  | $2^{2} \mathrm{~A}$ | 282 | 149 | $2 \Gamma_{2}$ | 373 | 430 |  |  |
|  | $3^{2} \mathrm{~A}$ | 369 | 379 | $3 \Gamma_{2}$ | 676 | 1116 |  |  |
|  | $4^{2} \mathrm{~A}$ | 466 | 541 | $4 \Gamma_{2}$ | 2227 | 2173 |  |  |
|  | $5^{2} \mathrm{~A}$ | 534 | 1050 | $5 \Gamma_{2}$ | 2447 | 2648 |  |  |
|  | $6^{2} \mathrm{~A}$ | 869 | 1738 | $6 \Gamma_{2}$ | 2742 | 3252 |  |  |
|  | $7^{2} \mathrm{~A}$ | 1849 | 2349 | $7 \Gamma_{2}$ | 3480 | 3978 |  |  |
| $5 d^{1}$ levels | $8^{2} \mathrm{~A}$ | 27908 | 27748 | $8 \Gamma_{2}$ | 28872 | 28603 | 28070 | 27139 |
|  | $9^{2} \mathrm{~A}$ | 33905 | 30208 | $9 \Gamma_{2}$ | 34905 | 31257 | 33884 | 30700 |
|  | $10^{2} \mathrm{~A}$ | 36945 | 32376 | $10 \Gamma_{2}$ | 38027 | 33518 | 38017 |  |
|  | $11^{2} \mathrm{~A}$ | 46604 | 50846 | $11 \Gamma_{2}$ | 47812 | 51966 |  |  |
|  | $12^{2} \mathrm{~A}$ | 51433 | 56967 | $12 \Gamma_{2}$ | 52592 | 58082 |  |  |
| $6 s^{1}$ level | $13^{2} \mathrm{~A}$ | 72192 | 72188 | $13 \Gamma_{2}$ | 73277 | 73248 |  |  |

Table 6 Computed relative oscillator strengths of $4 f \rightarrow 5 d$ transitions of $\mathrm{Ce}_{\mathrm{Lu} 1}$ and $\mathrm{Ce}_{\mathrm{Lu} 2}$ in LSO at the spin-orbit level.

|  | $\mathrm{Ce}_{\text {Lu1 }}$ |  |  | $\mathrm{Ce}_{\text {Lu2 }}$ |
| ---: | :---: | :---: | :---: | :---: |
|  | Transition <br> energy | Relative oscillator <br> strength | Transition <br> energy | Relative oscillator <br> strength |
| $1 \Gamma_{2} \rightarrow 5 d^{1} 8 \Gamma_{2}$ | 28872 | 1.00 | 28603 | 0.93 |
| $9 \Gamma_{2}$ | 34905 | 0.45 | 31257 | 0.58 |
| $10 \Gamma_{2}$ | 38027 | 0.40 | 33518 | 0.25 |
| $11 \Gamma_{2}$ | 47812 | 0.14 | 51966 | 0.31 |
| $12 \Gamma_{2}$ | 52592 | 0.36 | 58082 | 0.18 |

Table 7 Analysis of the shifts of the two lowest $4 f \rightarrow 5 d$ transition energies from $\mathrm{Ce}_{\mathrm{Lu} 1}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$ in LSO in terms of the changes of the centroid-energy difference (ced) and the crystal-field stabilization (cfs) energy. All numbers are in units of $\mathrm{cm}^{-1}$.

|  | $\mathrm{Ce}_{\text {Lul }}$ | $\mathrm{Ce}_{\text {Lu } 2}$ | Shift from Ce ${ }_{\text {Lu1 }}$ to $\mathrm{Ce}_{\mathrm{Lu} 2}$ |
| :---: | :---: | :---: | :---: |
| $\Delta E_{\text {ce }}\left(4 f^{\text {d }}\right.$ ) | 1706 | 1942 | 236 |
| $\Delta E_{\mathrm{ce}}\left(5 d^{1}\right)$ | 40442 | 40685 | 243 |
| $\Delta E_{\text {ced }}\left(4 f^{\prime} \rightarrow 5 d^{1}\right)$ | 38736 | 38743 | 7 |
| $\Delta E_{\mathrm{cfs}}\left(4 f^{\mathrm{d}} 1 \Gamma_{2}\right)$ | 1706 | 1942 | 236 |
| $\Delta E_{\text {cfs }}\left(5 d^{1} 8 \Gamma_{2}\right)$ | 11570 | 12082 | 512 |
| $\Delta E_{\text {cfs }}\left(5 d^{1} 9 \Gamma_{2}\right)$ | 5537 | 9428 | 3891 |
| $\Delta E_{\text {cfs }}\left(4 f^{\mathrm{d}} 1 \Gamma_{2} \rightarrow 5 d^{1} 8 \Gamma_{2}\right)$ | -9864 | -10140 | -276 |
| $\left.\rightarrow 5 d^{1} 9 \Gamma_{2}\right)$ | -3831 | -7486 | -3655 |
| $\Delta E\left(4 f^{\prime} 1 \Gamma_{2} \rightarrow 5 d^{1} 8 \Gamma_{2}\right)$ | 28872 | 26110 | -269 |
| $\left.\rightarrow 5 d^{1} 9 \Gamma_{2}\right)$ | 34905 | 30503 | -3648 |

## Figure captions

Fig. 1 Schematic representations of the local coordination structures of $\mathrm{Ce}^{3+}$ at the two lutetium (Lu1 and Lu2) sites in LSO. The numerical labels on oxygen atoms specify the crystallographic types, with "a" and "b" used to characterize the atoms of the same type. The labels on silicon are only for differentiation of the atoms.

Fig. 2 Total and orbital-projected DOS for the LSO crystal calculated using DFT with the PBE and HSE06 functionals. The labels "Lu $4 f$-core" and "Lu $4 f$-val" denote the calculations with the $\mathrm{Lu} 4 f^{14}$ electrons treated as core and valence electrons respectively, and the DOS for the $\mathrm{Lu} 4 f$ states in (c) are shaded in light gray. The derived band-gap energies ( $E_{\text {gap }}$ ) are indicated in the legends. The Fermi energy is set as zero energy.

Fig. 3 Total and orbital-projected DOS for the $\mathrm{Ce}_{\text {Lu1 }}$ - and $\mathrm{Ce}_{\text {Lu2 }}$-doped LSO crystals obtained using DFT with the PBE and HSE06 functionals in the Lu 4f-core scheme. The energies of the gaps ( $\Delta E_{4 f}$ ) between the occupied $\mathrm{Ce}^{3+} 4 f$ bands and the tops of O 2 p valence bands are shown in the legends. The DOS for the $\mathrm{Ce}^{3+} 4 f$ states have been enlarged by a factor of two, and are shaded in light gray. The Fermi energies are indicated by the dashed lines.

Fig. 4 Schematic diagram for the calculated energies and relative oscillator strengths of the $4 f \rightarrow 5 d$ transitions in $\mathrm{Ce}_{\text {Lu1 }}$-doped LSO . The inset shows the experimental absorption spectrum adapted from ref. 4 for comparison.

Fig. 5 Schematic representation for the energy levels of the $4 f^{1}$ and $5 d^{1}$ configurations of $\mathrm{Ce}^{3+}$ in LSO. $\Delta E_{\text {ced }}$ denotes the centroid-energy difference between the two configurations, and $\Delta E_{\mathrm{cfs}}$ the crystal-field stabilization energy of the levels.

FIG. 1


FIG. 2


FIG. 3


## FIG. 4



FIG. 5



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