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Moduli fixing and SUSY-breaking in Type II String Theory with fluxes

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Chapter 1

Introducción.

1.1 El paradigma actual de la física teórica.

El gran progreso tecnológico del último siglo ha permitido acceder a rangos de energías y longitudes inusuales, poniendo de manifiesto la extrema complejidad de la Naturaleza. El concepto de simetría ha adquirido especial relevancia, dando sustento a la antigua idea de unificación. Producto de ello ha sido el Modelo Estándar de la física de altas energías, formulado a la luz de los complejos patrones de resonancias observados en los aceleradores entre los años cincuenta y setenta. A través de su simetría $SU(3) \times SU(2)_L \times U(1)$, el Modelo Estándar proporciona así un entorno unificado para las interacciones fuerte, débil y electromagnética, habiendo sido probado en aceleradores hasta energías de varios cientos de GeV's sin que se haya encontrado desviación significativa alguna.

Todavía hay sin embargo todavía algunos puntos débiles que constituyen el paradigma actual de la física teórica. Entre éstos se encuentran la exclusión de la interacción gravitatoria de este marco unificado y el denominado problema de las jerarquías, en el que la masa del Higgs recibe correcciones cuadráticas provenientes de la escala ultravioleta, convirtiendo la teoría en innatural frente a los experimentos de precisión electrodébil que revelan una masa del orden de $m_H \sim 100$ GeV.

En este contexto, simultáneamente al desarrollo del Modelo Estándar, se consideró la posibilidad de combinar las populares simetrías internas con el grupo de Poincaré. Así, Coleman y Mandula [1] mostraron que cualquier grupo de Lie que contenga el grupo de Poincaré y una simetría interna es siempre producto directo de ambos grupos y por tanto da pie a una física trivial. Este teorema fue pronto superado con la formulación de teorías supersimétricas, en las que un subconjunto $\{Q_\alpha\}$ de los generadores del álgebra satisface relaciones de anticonmutación.

Sorprendentemente, las propiedades de renormalizabilidad de las teorías supersimétricas resultaron excepcionales, dando pie al espectro necesario para mantener bajo control las correcciones cuánticas. En particular, constituyó una solución potencial al problema de las jerarquías, pues las correcciones cuadráticas a la masa del Higgs eran canceladas gracias a contribuciones de bucles de squarks y sleptones.

Por otro lado, la otra característica atractiva de las teorías supersimétricas resultó ser la interrelación entre geometría y simetrías internas. Las álgebras extendidas de supersimetría fueron interpretadas en términos de la reducción dimensional de teorías supersimétricas en dimensiones extra, de modo que el grupo de isometrías de la variedad compacta pasaba a formar parte del grupo de simetría R de la teoría reducida. Además, las teorías supersimétricas locales dan pie a teorías supersimétricas con gravedad, unificando de este modo las cuatro interacciones de la Naturaleza.

Sin embargo, lejos de ser el último paso, este esquema sufre todavía de importantes problemas. La supersimetría ha de ser rota espontáneamente en un sector oculto de la teoría que se comunique con el visible a través de interacciones mensajeras, p.ej. interacciones supergravitatorias. Aquí existe gran arbitrariedad en la configuración específica del sector oculto, habiendo muy pocas restricciones experimentales, a parte de la ausencia de corrientes neutras en el sector visible que violen sabor (FCNC). Por otro lado, las teorías de supergravedad no son teorías renormalizables, pues el acoplo gravitatorio tiene dimensiones de $(\text{masa})^{-2}$. De este modo, éstas han de ser consideradas simplemente como teorías efectivas en el infrarrojo.

1.2 Teoría de Supercuerdas.

Hoy en día, la Teoría de Supercuerdas es el mejor candidato que tenemos a Teoría del Todo, representando una complección natural en el ultravioleta de las teorías de supergravedad. Las partículas ya no son puntuales, sino unidimensionales, de modo que cuando se propagan en el tiempo describen una sábana bidimensional, generalizando de este modo el concepto clásico de 'línea de mundo'. La manera en que esta sábana queda embebida en el espacio-tiempo ordinario puede ser descrita mediante una teoría conforme (CFT) $\mathcal{N}=1$ supersimétrica en dos dimensiones, cuya acción efectiva para los modos no masivos viene dada por un modelo sigma no lineal

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma d\sigma d\tau\{(g^{ab}G_{\mu\nu} + \epsilon^{ab}(B_{\mu\nu}) - 2\pi\alpha' F_{\mu\nu})\partial_a X^\mu \partial_b X^\nu + \alpha'\phi R + (\text{fermiones})\}, \quad (1.1)$$

valido en el régimen en que $\alpha'^{1/2}R_c^{-1} \ll 1$, con α' la constante de la cuerda y R_c el radio de curvatura del espacio-tiempo ordinario.

En efecto, cuando uno requiere la ausencia de anomalías de traza de modo que la teoría tenga buenas propiedades de cuantización, o lo que es equivalente, cuando uno requiere que las funciones beta de la teoría sean nulas, las ecuaciones de movimiento de supergravedad son sorprendentemente recuperadas [2]

$$\beta^G_{\mu\nu} = \alpha' \left(R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_{\mu\kappa\sigma} H_\nu{}^{\kappa\sigma} \right) + \mathcal{O}(\alpha'^2) = 0, \tag{1.2}$$

$$\beta^B_{\mu\nu} = \alpha' \left(-\frac{1}{2} \nabla^\kappa H_{\kappa\mu\nu} + \nabla^\kappa \phi H_{\kappa\mu\nu} \right) + \mathcal{O}(\alpha'^2) = 0, \tag{1.3}$$

$$\beta^\phi = \alpha' \left(-\frac{1}{2} \nabla^2 \phi + \nabla_\kappa \phi \nabla^\kappa \phi - \frac{1}{24} H_{\kappa\mu\nu} H^{\kappa\mu\nu} \right) + \mathcal{O}(\alpha'^2) = 0. \tag{1.4}$$

Por otro lado, la ausencia de anomalías conformes, es decir, la ausencia de cargas centrales en la CFT, fija el contenido de supercampos de la sábana, de manera análoga a cómo la ausencia de anomalías gauge fija el espectro de materia en el Modelo Estándar. De este modo, el número de dimensiones del espacio-tiempo ordinario queda fijado. Las teorías de supercuerdas vivirán en 10 dimensiones, requiriendo de este modo la compactificación de seis dimensiones extra y dando pie a una interpretación geométrica (o más bien, topológica) de la teoría efectiva a bajas energías.

Dependiendo de la manera concreta en que las anomalías de la teoría son canceladas en diez dimensiones, existen cinco Teorías de Supercuerdas: dos de ellas (Tipo IIA y Tipo IIB) son $\mathcal{N} = 2$ supersimétricas, mientras que las otras tres (Tipo I, Heterótica $E_8 \times E_8$ y Heterótica $SO(32)$) son solamente $\mathcal{N} = 1$ supersimétricas. Todas ellas están sin embargo relacionadas por una abundante red de dualidades, como se observó durante la segunda revolución de supercuerdas, a mediados de los noventa. Éstas consisten en transformaciones de la teoría de acoplo débil a acoplo débil (T-dualidad) o de acoplo débil a acoplo fuerte (S-dualidad). En particular, la presencia de dualidades débil-fuerte permitió acceder a objetos no perturbativos de la teoría tales como D-branas, NS-branas, etc., protegidos frente a correcciones cuánticas gracias a cotas BPS [3].

La compleja red de dualidades de Teoría de Supercuerdas parece indicar por tanto que estas cinco teorías a acoplo débil corresponden simplemente a diferentes límites de una misma teoría no-perturbativa (Teoría M), que todavía no sabemos como formular.

1.3 Camino del Modelo Estándar.

Si la Teoría de Supercuerdas representa una teoría unificada de todas las partículas e interacciones presentes en la Naturaleza, en algún punto debería contener al Modelo Estándar. Los esfuerzos por encontrar modelos reslistas en Teoría de Supercuerdas comenzaron en los ochenta, considerando compactificaciones de la Heterótica $E_8 \times E_8$ y $SO(32)$ sobre variedades de seis dimensiones con holonomía $SU(3)$ (ver p.ej. [4]). La supersimetría era rota a $\mathcal{N} = 1$ en cuatro dimensiones a través de la holonomía de la variedad compacta, mientras que el grupo gauge de la teoría en diez dimensiones era roto a $SU(3) \times SU(2)_L \times U(1)$, o a alguna Teoría de Gran Unificación (GUT), mediante su inclusión en el grupo de holonomía o la incorporación de líneas de Wilson.

Todos estos modelos heteróticos ya revelaron las principales patologías de Teoría de Supercuerdas para reproducir el Modelo Estándar en su límite de bajas energías. En primer lugar, era manifiesta la gran arbitrariedad en la elección de la variedad interna. La estructura perturbativa de Teoría de Supercuerdas permitía una gran diversidad de vacíos consistentes a bajas energías, en lo que hoy en día se denomina el "paisaje de vacíos" de Teoría de Cuerdas. Surgió entonces la cuestión sobre la existencia de un principio de selección de vacíos, con dos tendencias principales: unos creyendo que un principio de selección será dado una vez que la parte no perturbativa de la teoría se conozca; y otros pensando en el principio antrópico como mecanismo que selecciona el vacío de la Naturaleza.

La arbitrariedad en la compactificación de Teoría de Supercuerdas es parametrizada típicamente en términos de un amplio espacio de moduli. Por supuesto, debido a que todavía desconocemos la formulación completa de la teoría, sólo sabemos cómo trabajar en pequeñas regiones del espacio de moduli. Por ejemplo, una vez fijada la topología de la variedad interna, queda todavía cierta libertad en la elección de la estructura compleja y el tamaño de los diferentes ciclos, parametrizada respectivamente por los moduli de estructura compleja y los moduli de Kähler. Estos moduli en principio permanecen sin estabilizar, constituyendo un conjunto no deseado de escalares no masivos en el espectro de la teoría a bajas energías.

El otro gran reto de Teoría de Supercuerdas para reproducir el Modelo Estándar es la consecución de un mecanismo controlable de ruptura de supersimetría. Los primeros intentos de entender la estructura de los términos soft en compactificaciones perturbativas de Teoría de Supercuerdas se realizaron en los tempranos días de la fenomenología de las cuerdas heteróticas [5]. Una aproximación bastante general fue sugerida en [6, 7, 8], donde se asumía que algunos de los campos auxiliares asociados a los moduli de la compactificación adquirían un valor esperado en el vacío (vev), reduciendo de este modo la supersimetría. Se hacía patente por tanto la relación entre ruptura de supersimetría y estabilización de moduli: encontrar una fuente microscópica para la ruptura de supersimetría probablemente correspondía a encontrar un mecanismo para la estabilización de los moduli de la compactificación.

Con el descubrimiento de las D-branas a mediados de los noventa, aparecieron nuevas formas de obtener teorías gauge quirales en el contexto de Teoría de Supercuerdas. Surgieron así modelos de Tipo IIA y Tipo IIB donde el Modelo Estándar vive en el volumen de D-branas, en configuraciones tales como branas intersecantes [9, 10, 11, 12, 13] o branas en singularidades [14, 15, 16]. Estos modelos, sin embargo, presentaban el mismo tipo de patologías que los modelos heteróticos.

Durante los últimos años se han estudiado intensivamente compactificaciones con flujos de los campos antisimétricos (ver p.ej. [17]), proporcionando un nuevo ingrediente para la construcción de modelos realistas. Uno de los aspectos más interesantes de los flujos es que pueden generar acoplos en el superpotencial para los moduli de la compactificación [18], estabilizando al menos parte de ellos. Además, los moduli pueden ser agrupados en supermultipletes quirales $\mathcal{N} = 1$ cuyas componentes auxiliares F están asociadas a los flujos [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. De este modo, la presencia de flujos de cuerda cerrada constituye además una fuente microscópica para la ruptura de supersimetría en Teoría de Supercuerdas.

Las primeras configuraciones estudiadas intensivamente fueron orientifolds de Tipo IIB con flujos constantes de los campos RR y NSNS [59, 37, 101]. Estos dan pie a potenciales sin escala independientes de los moduli de Kähler, en los que el dilatón y parte, o incluso todos los moduli de estructura compleja, son estabilizados por los flujos. Las ecuaciones de movimiento de supergravedad requieren que los flujos de 3-forma sean imaginarios auto-duales (ISD), y en el caso particular de (0,3) formas, la supersimetría se rompe a $\mathcal{N} = 0^*$ en el espacio cuadridimensional.

En [20, 24] calculamos los términos de ruptura de supersimetría que surgían en el volumen de configuraciones de D3 y D7-branas debidos al efecto de los flujos de 3-forma, mostrando cmo los flujos ISD dan pie a términos soft de relevancia fenomenológica en el volumen de las D7-branas. Además, los moduli geométricos de las D7-branas son generalmente estabilizados también por los flujos, permitiendo de este modo la generación de superpotenciales no-perturbativos relevantes en escenarios de KKLT [29].

En el último par de años, los orientifolds de Tipo IIA con flujos han empezado a recibir también atención [30, 31, 32, 33, 34, 35, 36]. Contrariamente a lo que ocurre con los orientifolds de Tipo IIB, en este caso es posible encender flujos tanto de formas RR pares como de formas NSNS impares. Esto da lugar al importante resultado según el cual los superpotenciales de Tipo IIA generalmente dependen de todos los moduli geométricos así como del dilatón. En este caso además resulta natural incorporar flujos métricos [32, 33], correspondientes a reducciones generalizadas de Scherk-Schwarz [38, 39, 40, 41, 42]. Así, en [43] estudiamos los efectos de añadir flujos RR, NSNS y métricos a un orientifold $T^6/\Omega_P(-1)^{F_L}\sigma$, mostrando la existencia de vacíos AdS con todos los moduli estabilizados sin la necesidad de considerar efectos no-perturbativos. Además, en presencia de flujos métricos, la contribución de los flujos a los tadpoles RR pueden tener cualquier signo o incluso desaparecer. Esto representa una novedad frente a las compactificaciones de Tipo IIB con flujos de 3-forma, donde las ecuaciones de movimiento de supergravedad obligan a los flujos a contribuir a los tadpoles RR siempre con el mismo signo que las D3-branas. Así, en orientifolds de Tipo IIB con flujos de 3-forma uno se ve típicamente forzado a considerar variedades con un número de Euler grande de modo que se satisfagan las condiciones de cancelación de tadpoles y al mismo tiempo se estabilizen los moduli en valores grandes, donde las correcciones en α' y g_s permanecen bajo control. De este modo, los orientifolds de Tipo IIA con flujos métricos abren nuevas posibilidades para la construcción de modelos realistas. Como ejemplo de ello, en [43] presentamos por primera vez un modelo $\mathcal{N} = 1$ supersimétrico con espectro quiral cercano al del MSSM, todos los moduli estabilizados en AdS y los tadpoles RR cancelados sin ayuda de una acción orbifold.

Una cuestión lógica es si los superpotenciales de Tipo IIB inducidos por flujos pueden también depender de todos los moduli. Se ha mostrado recientemente [44] que para recuperar invarianza bajo T-dualidad entre las versiones de Tipo IIA y Tipo IIB de una misma compactificación en presencia de flujos RR, NSNS y métricos es necesario introducir una nueva clase no geométrica de flujos. Éstos han sido estudiados en la literatura por varios autores [45, 46, 47, 48, 49, 50, 51, 52, 53, 54]. Además, S-dualidad requiere la introducción de otro conjunto de flujos adicional, dando pie a nuevos términos en el superpotencial. Siguiendo estas líneas, en [55] generalizamos la propuesta de [44] a orientifolds con varios moduli geométricos y calculamos el superpotencial invariante bajo S-dualidad así como las identidades de Bianchi y tadpoles. La riqueza de flujos que surge permite construir nuevas clases de vacíos Minkowski supersimétricos en los que no sólo el dilatón y los moduli de estructura compleja se encuentran fijados sino también los moduli de Kähler.

En esta memoria resumimos los resultados de nuestro trabajo desarrollado a lo largo de los últimos tres años sobre estos temas [20, 24, 56, 43, 57, 55].

1.4 Plan de la tesis.

El plan de la tesis es el siguiente:

- En el Capítulo 3 discutiremos algunas de las características de Teoría de Supercuerdas que permiten introducir teorías gauge quirales. Así, en la Sección 3.1 discutimos la topología y espacio de moduli asociados a orientifolds de Tipo IIA y IIB, y cómo la simetría “mirror” se ve realizada en ausencia de flujos. En las Secciones 3.2 y 3.3 revisaremos dos tipos de configuraciones que dan lugar a fermiones quirales en cuatro dimensiones: configuraciones de D3 y D7-branas situadas en singularidades orbifold y D6-branas intersecándose en el espacio compacto. La cancelación de anomalías en estas configuraciones a través del intercambio de modos de cuerda cerrada se discutirá en la Sección 3.4, mientras que en la Sección 3.5 mostraremos un par de modelos semirealistas concretos.

- El Capítulo 4 está dedicado al límite de supergravedad de orientifolds de Tipo IIA con O6-planos y orientifolds de Tipo IIB con O3 y O7-planos. Así, tras revisar en las

Secciones 4.1 y 4.2 las supergravedades de Tipo IIB y IIA, en la Sección 4.3 discutiremos las soluciones de Tipo B(ecker) de [58, 59, 60, 101] y los monopolos holomórficos de [61], correspondientes a la descripción en el límite de supegravedad de algunos orientifolds de Tipo IIB/O3 y de Tipo IIA/O6 con flujos.

- La estabilización de moduli será el tema del Capítulo 5. En la Sección 5.1.1 revisaremos la estructura de los orientifolds de Tipo IIB con flujos RR y NSNS constantes, mientras que las Secciones 5.1.2 y 5.2 resumiremos los resultados de [43] sobre orientifolds de Tipo IIA con O6-planos y flujos constantes NSNS, RR y métricos. Entonces generalizaremos estas configuraciones para incluir flujos no geométricos y S-duales, siguiendo [55]. Esto será en las Secciones 5.3 y 5.4. Finalmente, en la Sección 5.5 abordaremos la posibilidad de tener superpotenciales generalizados invariantes bajo todo el grupo de dualidad.

- Los resultados de [20, 24] sobre ruptura de supersimetría inducida por flujos en configuraciones de D3 y D7-branas constituirán el contenido del Capítulo 6. Tras calcular en la Sección 6.2 la acción efectiva a bajas energías para las teorías gauge que viven en el volumen de las D3/D7-branas, analizaremos, en la Sección 6.3, los patrones de ruptura de supersimetría que surgen y los compararemos, en la Sección 6.4, con las predicciones de supergravedad efectiva. Finalmente, la viabilidad fenomenológica de estos patrones será discutida en la Sección 6.5.

- En el Capítulo 7 reproduciremos algunos de los modelos de [43] y [20, 24] que sirven para ilustrar las ideas de los capítulos previos.

- Finalmente, algunos últimos comentarios serán realizados en el Capítulo 8.

Chapter 2

Introduction.

2.1 The present paradigm of theoretical physics.

The great technological progress of the last century has allowed to access unusual ranges of energies and lengths, revealing the extreme complexity of Nature. The concept of symmetry has acquired special relevance, giving support to the ancient idea of unification. Product of this has been the Standard Model of high energy physics, formulated in the light of the complex patterns of resonances measured at accelerators between the fifties and the seventies. Through its $SU(3) \times SU(2)_L \times U(1)$ gauge symmetry, the Standard Model thus provides an unifying framework for the strong, weak and electromagnetic interactions, having been tested at accelerators up to energies of several hundreds of GeV's without any significative deviation.

There are however still some weak points which constitute the present paradigm of theoretical physics. Among these are the exclusion of the gravitational interaction from this unified framework and the so called hierarchy problem, on which the Higgs mass receives quadratic loop corrections from the ultraviolet cutoff, rendering the theory unnatural against the electroweak precision tests, which reveal a mass of the order $m_H \sim 100$ GeV.

Within this context, simultaneously to the development of the Standard Model, the possibility of combining the popular internal symmetries with the Poincaré group was considered. Thus, Coleman and Mandula [1] showed that any Lie group which contains the Poincaré group and an internal symmetry is always a direct product of both groups and therefore it leads to trivial physics. This no-go theorem was soon overcome with the formulation of supersymmetry, on which a subset $\{Q_\alpha\}$ of generators of the algebra satisfy anticommutation relations.

Surprisingly, the renormalizable properties of supersymmetry resulted to be exceptional, leading to the necessary spectrum in order to keep the quantum corrections under control. In particular, it constituted a potential solution to the hierarchy problem, cancelling the quadratic corrections to the Higgs mass through loop contributions of the squarks and sleptons.

On the other hand, the interplay between geometry and internal symmetries was the other attractive feature of supersymmetric theories. Extended algebras of supersymmetry were able to be interpreted as the dimensional reduction of supersymmetric theories in extra dimensions, with the isometry group of the compact manifold being part of the R-symmetry group of the reduced theory. Moreover, gauging supersymmetry led to supersymmetric theories with gravity, thus unifying the four interactions in Nature.

However, far from being the last step, this scheme still suffers from important problems. Supersymmetry must be spontaneously broken in a hidden sector which communicates with the visible sector through some messenger interactions, such as supergravity interactions. Here there is great arbitrariness in the specific configuration of the hidden sector, having only few experimental constraints such as the absence of Flavor Changing Neutral Currents (FCNC) in the visible sector. In addition, supergravity theories are not renormalizable theories, as the gravitational coupling has dimensions of $(\text{mass})^{-2}$. Thus, they should be considered as infrared effective theories.

2.2 Superstring Theory.

Nowadays, Superstring Theory is the best candidate that we have to the Theory of Everything, representing a natural ultraviolet completion of the supergravity theories. Particles are no longer point-like but one dimensional, so when they propagate on time they render a two dimensional worldsheet, generalizing the classical concept of worldline. The embedding of the worldsheet into the target space can be described by a two dimensional $\mathcal{N} = 1$ conformal field theory (CFT), whose effective action for the massless modes is given by the non-linear σ-model

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d\sigma d\tau \{ (g^{ab} G_{\mu\nu} + \epsilon^{ab}(B_{\mu\nu}) - 2\pi\alpha' F_{\mu\nu}) \partial_a X^\mu \partial_b X^\nu + \alpha' \phi R + (\text{fermions}) \}, \quad (2.1)$$

valid in the $\alpha'^{1/2} R_c^{-1} \ll 1$ regime, being α' the string constant and R_c the curvature radius of the target space.

Indeed, when one requires the absence of the trace anomaly in order to have for the theory good quantization properties, or what is equivalent, when one requires the beta functions of the theory to vanish, the supergravity equations of motion are surprisingly recovered [2]

$$\beta^G_{\mu\nu} = \alpha' \left(R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_{\mu\kappa\sigma} H_\nu{}^{\kappa\sigma} \right) + \mathcal{O}(\alpha'^2) = 0, \tag{2.2}$$

$$\beta^B_{\mu\nu} = \alpha' \left(-\frac{1}{2} \nabla^\kappa H_{\kappa\mu\nu} + \nabla^\kappa \phi H_{\kappa\mu\nu} \right) + \mathcal{O}(\alpha'^2) = 0, \tag{2.3}$$

$$\beta^\phi = \alpha' \left(-\frac{1}{2} \nabla^2 \phi + \nabla_\kappa \phi \nabla^\kappa \phi - \frac{1}{24} H_{\kappa\mu\nu} H^{\kappa\mu\nu} \right) + \mathcal{O}(\alpha'^2) = 0. \tag{2.4}$$

On the other hand, the absence of conformal anomalies, that is, the absence of central charges in the CFT, fixes the superfield content of the worldsheet, in a similar fashion to what the absence of gauge anomalies fixes the matter spectrum of the Standard Model. Thus, the number of dimensions of the target space is fixed. Superstring theories will live in 10 dimensions, requiring therefore the compactification of the 6 extra dimensions and providing with a geometrical (or rather, topological) interpretation to the low energy effective theory.

Depending on the concrete way on which the anomalies of the theory are cancelled, there are five consistent Superstring Theories in ten dimensions: two of them (Type IIA and Type IIB) are $\mathcal{N} = 2$ supersymmetric, whereas the other three (Type I, $E_8 \times E_8$ Heterotic and $SO(32)$ Heterotic) are just $\mathcal{N} = 1$ supersymmetric. All of them are however related by a rich network of dualities, as it was realized during the second Superstring revolution, in the mid nineties. These consist on weak-weak (T-duality) and weak-strong (S-duality) transformations of the theory. In particular, the presence of weak-strong dualities allowed to access non-perturbative objects, such as D-branes, NS-branes, etc., protected against quantum corrections by BPS bounds [3].

The complex duality network of Superstring Theory therefore seems to be indicating that the above five theories at weak coupling would simply correspond to different limits of a same non-perturbative theory (M-theory), which we still do not know how to formulate.

2.3 On the road to the Standard Model.

If Superstring Theory represents the unified theory of all particle and interactions in Nature, at some point it should contain the Standard Model. The efforts for finding realistic models in Superstring Theory, began in the eighties, by considering compactifications of the $E_8 \times E_8$ and the $SO(32)$ Heterotic in six dimensional manifolds with $SU(3)$ holonomy (see e.g. [4]). Supersymmetry was broken to four dimensional $\mathcal{N} = 1$ through the holonomy of the compact manifold, whereas the gauge group of the ten dimensional theory was broken to $SU(3) \times SU(2)_L \times U(1)$, or to some Grand Unified Theory (GUT), through its embedding into the holonomy group or the inclusion of Wilson lines.

All these heterotic models already revealed the main pathologies of Superstring Theory for reproducing the Standard Model in its low energy limit. First of all, it was manifest the great arbitrariness in the choice of the internal manifold. The structure of perturbative Superstring Theory allowed for a great complexity of consistent vacua with different low energy physics, in what nowadays is called the String Theory landscape of vacua. The question of a vacuum selection rule arose, with two main tendencies: the ones believing that a selection principle will be given once the non-perturbative part of the theory is known; and the ones thinking on the anthropic principle as the mechanism for selecting the vacuum of Nature.

The arbitrariness in the compactification of Superstring Theory is usually parametrized by a broad moduli space. Of course, since a complete description of the theory is still lacking, we only know how to work at small regions of the moduli space. For example, once the topology of the internal manifold is fixed, there is still some freedom in the choice of the complex structure and the size of the different cycles, parametrized respectively by the complex structure moduli and the Kähler moduli. These moduli in principle remain unstabilized, constituting an undesirable set of massless scalars in the low energy spectrum of the theory.

The other big difficulty of Superstring Theory for reproducing the Standard Model is the addressing of a controllable supersymmetry breaking mechanism. Attempts to understand the structure of soft terms in perturbative heterotic compactifications were done since the early days of heterotic string phenomenology [5]. A rather model-independent approach was suggested in [6, 7, 8], where it was assumed that the auxiliary fields associated to the corresponding moduli of the compactification get a vev, thus breaking supersymmetry. Thus it was patent a relationship between supersymmetry breaking and moduli stabilization: finding a microscopic source for supersymmetry breaking would probably correspond to finding a mechanism for stabilizing the moduli of the compactification.

With the discovery of D-branes in the mid nineties, new ways to obtain chiral gauge theories in the context of Superstring Theory arose. Type IIA and Type IIB models where the Standard Model lives in the worldvolume of D-branes appeared on the light of constructions such as intersecting branes [9, 10, 11, 12, 13] or D-branes at singularities [14, 15, 16]. These models, however, present the same above pathologies of the heterotic models.

During the last few years, fluxes of antisymmetric fields in string compactifications have been studied intensively (see e.g. [17]), thus providing us with new ingredients for model building. One of the most interesting aspects of the presence of fluxes is that they may generate superpotential couplings for the moduli of the compactification [18], thus potentially stabilizing some of them. Moreover, the compactification moduli may be arranged into $\mathcal{N} = 1$ chiral supermultiplets with F auxiliary components associated to the background fluxes [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. In this way, the presence of closed string background fluxes constitutes too a microscopical source for supersymmetry breaking in Superstring Theory.

The first setups extensively studied were Type IIB orientifolds with constant RR and NSNS fluxes [59, 37, 101]. These resulted in no-scale potentials independent of the Kähler moduli, on which the dialton and part, or even all, of the complex structure moduli were stabilized by the fluxes. The supergravity equations of motion required the 3-form fluxes to be imaginary self-dual (ISD) forms, and in the particular case of these being a $(0,3)$ form, supersymmetry was broken to $\mathcal{N}=0^*$ in four dimensional Minkowski space.

In [20, 24] we computed the soft supersymmetry breaking terms arising in the worldvolume of setups of D3 and D7-branes due to the effect of the 3-form fluxes, showing that ISD fluxes lead to non-trivial soft terms of phenomenological relevance in the worldvolume of the D7-branes. Moreover, the fluxes generically stabilize the geometric moduli of the D7-branes, thus allowing for the generation of non-perturbative superpotentials, relevant in KKLT [29] scenarios.

In the last couple of years, Type IIA orientifolds with fluxes have started to receive some attention too [30, 31, 32, 33, 34, 35, 36]. Contrary to what occurs in Type IIB orientifolds with 3-form fluxes, in this case it is possible to switch on backgrounds of both even RR and odd NSNS forms. This in turn implies the important result that the Type IIA flux induced superpotentials depend on all the geometrical moduli as well as on the dilaton. In addition, in this case it is natural to incorporate metric fluxes [32, 33] corresponding to generalized Scherk-Schwarz reductions [38, 39, 40, 41, 42]. In this sense, in [43] we studied the effects of adding RR, NSNS and metric fluxes on a $T^6/\Omega_P(-1)^{F_L}\sigma$ orientifold, showing how one may find AdS vacua with all moduli stabilized without considering extra non-perturbative effects. In addition, in presence of metric backgrounds, the flux contribution to the RR tadpoles can have either sign or even vanish. This represents a novelty with respect to the Type IIB compactifications with 3-form fluxes, where the supergravity equations of motion imply that the flux background always contributes to the RR tadpoles with the same sign as D3-branes do. Thus, for Type IIB orientifolds with 3-form fluxes one is usually enforced to take manifolds with large Euler number in order to fulfill the tadpole conditions and at the same time stabilize the moduli at large values, where the α' and g_s corrections are under control. In this way, Type IIA orientifolds with metric fluxes open new possibilities for model building. As an example of it, in [43] we presented for the first time a $\mathcal{N}=1$ supersymmetric model with a chiral spectrum close to the one of the MSSM, all the moduli stabilized in AdS and the RR tadpoles cancelled without the aid of an orbifold twist.

A logical question is whether Type IIB flux induced superpotentials can also depend on all moduli. It has been recently shown [44] that in order to recover T-duality invariance between the Type IIA and Type IIB versions of the same compactification in the presence of RR, NS and metric backgrounds, new non-geometric fluxes have to be introduced. Such fluxes has been already studied by several authors [45, 46, 47, 48, 49, 50, 51, 52, 53, 54]. Moreover, Type IIB S-duality requires the introduction of a new set of fluxes, leading to further superpotential terms. Along these lines, in [55] we generalized the proposal of [44] to orientifolds with several diagonal geometrical moduli and computed the S-duality invariant superpotential, tadpoles and Bianchi identities. The arising richness of fluxes allows to construct new classes of $\mathcal{N} = 1$ supersymmetric Minkowski vacua on which not only the dilaton and complex structure but also the Kähler moduli are fixed.

In this report we will summarize the results of our work in these topics along the last three years [20, 24, 56, 43, 57, 55].

2.4 Outline of the thesis.

The outline of the thesis is as follows:

- In Chapter 3 we will discuss some of the features of Superstring Theory allowing the embedding of semirealistic chiral gauge theories. Thus, in Section 3.1 we will discuss the topology and moduli space of Type IIA and Type IIB Calabi-Yau orientifolds, and how mirror symmetry is accomplished in absence of background. In Sections 3.2 and 3.3 we will review two setups leading to chiral fermions in four dimensions: D3/D7-brane configurations placed at orbifold singularities and D6-branes intersecting at angles in the compact space. The cancellation of anomalies in these setups through the exchange of closed string modes will be discussed in Section 3.4, whereas we will reproduce in Section 3.5 a couple of concrete semirealistic models.
- Chapter 4 is devoted to the supergravity limit of Type IIA orientifolds with O6-planes and Type IIB orientifolds with O3 and O7-planes. Thus, after reviewing Type IIB and Type IIA supergravity in Sections 4.1 and 4.2, we will discuss the Type B(ecker) solutions of [58, 59, 60, 101] and the mirror holomorphic monopole Type IIA solutions of [61], corresponding respectively to the supergravity description of some Type IIB/O3 and Type IIA/O6 orientifolds with background fluxes.
- The issue of moduli stabilization will be the topic of Chapter 5. In Section 5.1.1 we will review the structure of Type IIB orientifolds with constant RR and NSNS fluxes, whereas Sections 5.1.2 and 5.2 will summarize the results of [43] on Type IIA orientifolds with O6-planes and constant NSNS, RR and metric fluxes. Then we will generalize our backgrounds to include non-geometric and S-dual fluxes, following [55]. This will be done in Sections 5.3 and 5.4. Finally, in Section 5.5 we will discuss the possibility of having generalized duality invariant superpotentials.
- The results of [20, 24] on the soft supersymmetry breaking terms induced by fluxes in the worldvolume of D3/D7-branes will constitute the content of Chapter 6. After computing,

in Section 6.2, the low energy effective actions for the gauge theories in the worldvolume of the D3/D7-branes, we will analyze, in Section 6.3, the soft supersymmetry breaking patterns which arise and will compare them, Section 6.4, with the effective supergravity predictions. Finally, the phenomenological viability of these patterns will be discussed in Section 6.5.

- In Chapter 7 we will reproduce some of the models of [43] and [20, 24] which illustrate the ideas of the previous chapters.

- Finally, some last comments will be made in Chapter 8.

Chapter 3

Embedding the Standard Model in String Theory

If String Theory really represents a unified theory of all particles and interactions present in Nature, at some point it should reproduce in its low energy limit the Standard Model or any of its phenomenologically viable extensions. In this sense, String Theory has revealed to possess appealing mathematical properties, containing almost every tool employed to build up the Standard Model or General Relativity.

Along this chapter we will go through some of these features which allow us to embed semi-realistic chiral gauge theories in String Theory. At the end this will lead us to two major problems which constitute the main topic of this thesis: moduli stabilization and supersymmetry breaking.

3.1 Toroidal orientifolds and mirror symmetry.

The first difficulty when trying to embed the Standard Model in String Theory is an excess of supersymmetry. Indeed, the known ten dimensional String Theories have $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetry and therefore, upon naive dimensional reduction, will lead to $\mathcal{N} = 4$ or $\mathcal{N} = 8$ supersymmetric effective theories in four dimensions. As extended supersymmetry do not allow for chiral fermions, it is patent the necessity of a source of supersymmetry breaking.

Generally, the required breaking to $\mathcal{N} = 1$ is achieved through the holonomy of the compactification manifold, typically a Calabi-Yau orientifold with special holonomy [62, 63, 64, 65, 66, 67, 68, 69]. On this section we will introduce the topology, moduli space and symmetries of Calabi-Yau orientifolds, putting special emphasis on the particular case of toroidal orientifolds.

Calabi-Yau orientifolds are constructed by modding out an ordinary Calabi-Yau threefold (CY_3) with the worldsheet parity reversal operator Ω_p, the space-time fermion number projector for the left-movers $(-1)^{F_L}$, and an internal involution σ which is required to be an isometry of the CY_3.

The fixed points of the orientifold involution σ, usually denoted O-planes, are charged under the RR forms of String Theory, so in order to guarantee the cancellation of the global charge one is enforced to include a twisted sector of open strings supported by D-branes. Thus, Type II String Theories in Calabi-Yau orientifolds are theories with open strings[1].

Depending on how the involution σ acts on the Kähler form J and on the holomorphic 3-form Ω, there will be three kinds of CY_3 orientifolds [72]

Type IIB with O3/O7-planes:	$\sigma^* J = J$	$\sigma^*\Omega = -\Omega$
Type IIB with O5/O9-planes:	$\sigma^* J = J$	$\sigma^*\Omega = \Omega$
Type IIA with O6-planes:	$\sigma^* J = -J$	$\sigma^*\Omega = e^{2i\theta}\overline{\Omega}$

Here, σ^* denotes the pullback of σ and θ is a constant phase related to the calibration of the 3-cycles Λ_n supporting the O6-planes. In fact, in the case of Type IIA orientifolds, the fixed points Λ_n correspond to special Lagrangian 3-cycles with calibrating phase θ (see e.g. [73])

$$J|_{\Lambda_n} = 0, \qquad \mathrm{Im}(e^{-i\theta}\Omega)|_{\Lambda_n} = 0. \tag{3.1}$$

In what follows we will take the conventions on which $\theta = 0$.

The cohomological structure of a CY_3 can be summarized in the following hodge diamond

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{(2,2)} & & 0 & \\
1 & & h^{(1,2)} & & h^{(2,1)} & & 1 \\
 & 0 & & h^{(1,1)} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

with $h^{(1,1)} = h^{(2,2)}$ and $h^{(1,2)} = h^{(2,1)}$ thanks to the Poincaré lemma. In particular, note that no isometry is allowed.

[1] Actually we will see that very often in this kind of compactifications the D-brane charge is no longer a conserved quantity and can be traded by closed string backgrounds which contribute to the RR tadpoles as D-branes do [70, 71].

Upon orientifolding the above cohomological structure will split between σ-odd and σ-even forms ($h = h_- + h_+$) [74, 31]. Thus, it is convenient to define a basis of (1,1)-forms ω_i ($\omega_{\hat{\imath}}$) odd (even) under the orientifold involution and a dual basis of (2,2)-forms $\tilde{\omega}^i$ ($\tilde{\omega}^{\hat{\imath}}$) such that

$$\int \omega_i \wedge \tilde{\omega}^j = \delta_i^j, \qquad \int \omega_{\hat{\imath}} \wedge \tilde{\omega}^{\hat{\jmath}} = \delta_{\hat{\imath}}^{\hat{\jmath}}. \tag{3.2}$$

Analogously, it is possible to define a symplectic basis (α_A, β^B) for the 3-forms so that

$$\int \alpha_A \wedge \beta^B = \delta_A^B. \tag{3.3}$$

Under the orientifold involution these again split among even and odd forms. In particular, for Type IIA orientifolds one can take the basis in such a way that α_A are σ-even forms whereas β^B are σ-odd forms [31]. For Type IIB orientifolds however this cannot be done and the basis is split by the internal involution accordingly to [74]

$$(\alpha_A, \beta^B) = (\alpha_a, \beta^b) \oplus (\alpha_{\hat{a}}, \beta^{\hat{b}}), \tag{3.4}$$

with (α_a, β^b) σ-odd and $(\alpha_{\hat{a}}, \beta^{\hat{b}})$ σ-even forms.

On this thesis we mainly center on Type IIA orientifolds with O6-planes and Type IIB orientifolds with O3/O7-planes. The moduli space of these manifolds has been extensively studied in [74, 31]. Determining the structure of the possible allowed geometrical deformations is crucial for establishing a dynamics over the compact manifold.

For Type IIA orientifolds, the moduli space is given by a set of $h_-^{(1,1)}$ Kähler moduli T_i, $h^{(1,2)}$ complex structure moduli U_a and one axiodilaton S. These are concisely defined in terms of the complexified forms [31]

$$J_c = B_2 + iJ = i \sum_{i=1}^{h_-^{(1,1)}} T_i \omega_i, \tag{3.5}$$

$$\Omega_c = C_3 + i\mathrm{Re}(C\Omega) = iS\alpha_0 - i \sum_{i=1}^{h^{(1,2)}} U_a \alpha_a, \tag{3.6}$$

where B_2 is the NSNS 2-form, C_3 is the σ-even RR 3-form and C is a compensator field specified by

$$C = e^{-\phi_4} e^{K_{cs}/2}, \qquad K_{cs} = -\log[-\frac{i}{8} \int_{T^6} \Omega \wedge \Omega^*],$$

with ϕ_4 the four dimensional dilaton given by $e^{\phi_4} = e^{\phi}/\sqrt{\mathrm{Vol(CY_3)}}$. Then one can define a metric in the moduli space through the Kähler potential

$$K_{\mathrm{IIA}} = -\log\left[\frac{4}{3}\int J \wedge J \wedge J\right] - 2\log\left[2\int \mathrm{Re}(C\Omega) \wedge *\mathrm{Re}(C\Omega)\right]. \tag{3.7}$$

On the other hand, for Type IIB orientifolds with O3/O7-planes the moduli space is constituted by $h_+^{(1,1)}$ Kähler moduli T_i, $h^{(1,2)}$ complex structure moduli U_a and one axiodilaton S. These are defined through [74]

$$S = e^{-\phi} + iC_0, \tag{3.8}$$

$$U_a = i\int \Omega \wedge \alpha_a \qquad a = 1, \ldots, h_+^{(1,2)}, \tag{3.9}$$

$$\mathcal{J}_c = C_4 + \frac{i}{2}e^{-\phi} J \wedge J + (C_2 - iSB_2) \wedge B_2 = i \sum_{i=1}^{h^{(1,1)+}} T_i \tilde{\omega}_i, \tag{3.10}$$

with C_0, C_2 and C_4 the RR scalar (σ-even), 2-form (σ-odd) and 4-form (σ-even) respectively. Ω is normalized in such a way that

$$\int \Omega \wedge \beta_0 = 1. \tag{3.11}$$

Then, the corresponding Kähler potential is given by

$$K_{\mathrm{IIB}} = -\log\left[-i\int \Omega \wedge \overline{\Omega}\right] - \log(S + S^*) - 2\log(\mathrm{Vol}[\mathrm{CY}_3]). \tag{3.12}$$

The fact that the metric in the moduli space can be codified through the Kähler potentials (3.7) or (3.12) is reminiscent of the $\mathcal{N} = 2$ special geometry of the CY_3 moduli space. Indeed, before applying the orientifold projection, the moduli space has a local product structure [75, 76, 77, 78]

$$M^K \times M^Q,$$

being M^K a special Kähler manifold spanned by the Kähler moduli in Type IIA or by the complex structure moduli in Type IIB, and M^Q a quaternionic manifold defined by the dilaton and the complex structure moduli in Type IIA, or by the dilaton and the Kähler moduli in Type IIB.

One of the most relevant advances in String Theory since its origins has been the discovery of *mirror symmetry* (see e.g. [79]). Dualities in String Theory very often have a geometrical counterpart, and such is the case of mirror symmetry. Indeed, the submanifold $M^{\tilde{K}} \subset M^Q$ engendered by the complex structure moduli in Type IIA, or by the Kähler moduli in Type IIB, may be dressed as well with a special Kähler structure. This allows for a strict equivalence between Type IIA String Theory compactified on a CY_3 and Type IIB compactified on the *mirror* manifold $\tilde{\mathrm{CY}}_3$ constructed by exchanging $h^{(1,2)} \leftrightarrow h^{(1,1)}$ and $M^{\tilde{K}} \leftrightarrow M^K$. In order to facilitate the visualization of the mirror map, we have summarized in Table 3.1 the structure of M^K and $M^{\tilde{K}}$.

For orientifold compactifications, M^K and $M^{\tilde{K}}$ are truncated to the invariant elements under the orientifold projection, however mirror symmetry is still expected to hold [72]. Indeed, in order to make explicit the mapping between mirror orientifolds it results useful to

	$M^{\tilde{K}} \subset M^Q$	M^K
Type IIA	3-cycles $\to \{U_i\}$	2-cycles $\to \{T_i\}$
Type IIB	4-cycles $\to \{T_i\}$	3-cycles $\to \{U_i\}$

Table 3.1: Structure of M^K and $M^{\tilde{K}}$ for Type IIA and Type IIB CY_3.

expand the holomorphic 3-form in periods $(\tau_i, \mathcal{F}_i)$ as

$$\Omega = \alpha_0 - i\mathcal{F}_0\beta_0 + i\sum_{j=1}(\tau_j\beta_j + i\mathcal{F}_j\alpha_j), \tag{3.13}$$

where we have already imposed the normalization of eq.(3.11) and the $\mathcal{F}_i$ are holomorphic functions depending on τ_i.

In terms of the periods, then the Type IIA complex structure moduli and axiodilaton defined in eqs.(3.6) read

$$S = 2e^{-\phi}\left(\frac{\prod_j \text{Re } T_j}{\text{Re }[\mathcal{F}_0 + \sum_k \mathcal{F}_k\bar{\tau}_k]}\right)^{1/2} + i\int C_3 \wedge \beta_0, \tag{3.14}$$

$$U_a = 2e^{-\phi}\left(\frac{\prod_j \text{Re } T_j}{\text{Re }[\mathcal{F}_0 + \sum_k \mathcal{F}_k\bar{\tau}_k]}\right)^{1/2} (\text{Re } \mathcal{F}_a) - i\int C_3 \wedge \beta_a, \tag{3.15}$$

whereas the Type IIB complex structure moduli (3.9) reduce to

$$U_i = \tau_i. \tag{3.16}$$

Mirror symmetry between Type IIA orientifolds with O6-planes and Type IIB orientifolds with O3/O7-planes then corresponds to the mapping represented in Table 3.1.

Type IIB with O3/O7-planes		Type IIA with O6-planes
S	$\longleftrightarrow$	S
T_i	$\longleftrightarrow$	U_i
U_i	$\longleftrightarrow$	T_i

Table 3.2: Realization of mirror symmetry between M^K and $M^{\tilde{K}}$.

To illustrate these ideas we will consider the simplest example of CY_3 given by a factorable 6-torus $\otimes_{n=1}^3(T^2)$ modded by a $Z_2 \times Z_2$ orbifold symmetry so the holonomy belongs to a discrete subgroup of $SU(3)$.

The metric of the factorized torus is given by

$$ds^2 = \sum_{j=1}^3 \frac{A_j}{\text{Re } \tau_j}[(dx^j)^2 + |\tau_j|^2(dx^{j+3})^2 - 2(\text{Im } \tau_j)dx^j dx^{j+3}], \tag{3.17}$$

being A_j the area of the j-th 2-torus.

The $Z_2 \times Z_2$ symmetry acts on the internal coordinates according to

$$\begin{aligned} Z_2 :& \quad x^1 \to -x^1 \quad x^2 \to -x^2 \quad x^3 \to x^3 \quad x^4 \to -x^4 \quad x^5 \to -x^5 \quad x^6 \to x^6, \\ Z_2' :& \quad x^1 \to x^1 \quad x^2 \to -x^2 \quad x^3 \to -x^3 \quad x^4 \to x^4 \quad x^5 \to -x^5 \quad x^6 \to -x^6, \end{aligned}$$

whereas the orientifold involution σ is acting as

$$\text{Type IIB with O3/O7-planes:} \qquad \sigma(x^i) = -x^i, \tag{3.18}$$

$$\text{Type IIB with O5/O9-planes:} \qquad \sigma(x^i) = x^i, \tag{3.19}$$

$$\text{Type IIA with O6-planes:} \qquad \sigma(x^i) = \begin{cases} x^i & \text{if } i = 1,2,3 \\ -x^i & \text{if } i = 4,5,6 \end{cases}. \tag{3.20}$$

The complex structure parameters τ_i are given in terms of the holomorphic 3-form by

$$\Omega = (dx^1 + i\tau_1 dx^4) \wedge (dx^2 + i\tau_2 dx^5) \wedge (dx^3 + i\tau_3 dx^6). \tag{3.21}$$

Then, a suitable cohomology basis is given by

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3 & \beta_0 &= dx^4 \wedge dx^5 \wedge dx^6 \\ \alpha_1 &= dx^1 \wedge dx^5 \wedge dx^6 & \beta_1 &= dx^4 \wedge dx^2 \wedge dx^3 \\ \alpha_2 &= dx^4 \wedge dx^2 \wedge dx^6 & \beta_2 &= dx^1 \wedge dx^5 \wedge dx^3 \\ \alpha_3 &= dx^4 \wedge dx^5 \wedge dx^3 & \beta_3 &= dx^1 \wedge dx^2 \wedge dx^6 \\ \omega_1 &= -dx^1 \wedge dx^4 & \tilde{\omega}_1 &= dx^2 \wedge dx^5 \wedge dx^3 \wedge dx^6 \\ \omega_2 &= -dx^2 \wedge dx^5 & \tilde{\omega}_2 &= dx^3 \wedge dx^6 \wedge dx^1 \wedge dx^4 \\ \omega_3 &= -dx^3 \wedge dx^6 & \tilde{\omega}_3 &= dx^2 \wedge dx^5 \wedge dx^1 \wedge dx^4 \end{aligned}$$

and the holomorphic functions $\mathcal{F}_i$ read

$$\mathcal{F}_0 = \tau_1\tau_2\tau_3, \qquad \mathcal{F}_i = \tau_j\tau_k, \tag{3.22}$$

with $i \neq j \neq k$. Note that $\mathcal{F}_0$ acts as a prepotential for the $\mathcal{F}_i$. This is again reminiscent of the $\mathcal{N} = 2$ special geometry.

Note that for Type IIA orientifolds, consistency of the metric (3.17) with the orientifold involution implies

$$(\text{Im }\tau_i)_{\text{IIA}} = 0, \tag{3.23}$$

and each sub-torus has a square lattice with $\tau_j = R_y^j/R_x^j$ and $A_j = R_x^j R_y^j$, being R_x^j and R_y^j the size of the lattice vectors. Thus, in this case eqs. (3.5), (3.14) and (3.15) become

$$S = e^{-\phi} R_y^1 R_y^2 R_y^3 + iC_{123}, \tag{3.24}$$

$$U_i = e^{-\phi} R_x^i R_y^j R_y^k - i \int C_3 \wedge \beta_i \qquad i \neq j \neq k, \tag{3.25}$$

$$T_i = R_x^j R_y^j + i \int B_2 \wedge \tilde{\omega}_i. \tag{3.26}$$

On the other hand, for Type IIB one has that $h_-^{(1,1)} = h_-^{(2,2)} = 0$ and

$$(B_2)_{\text{IIB}} = (C_2)_{\text{IIB}} = 0, \tag{3.27}$$

and eqs.(3.8)-(3.10) become

$$S = -i\tau = e^{-\phi} + iC, \tag{3.28}$$
$$T_i = e^{-\phi} A_j A_k - i \int C_4 \wedge \omega_i \qquad i \neq j \neq k, \tag{3.29}$$
$$U_i = \tau_i. \tag{3.30}$$

With this, the Kähler potentials for Type IIA and Type IIB orientifolds, eqs. (3.7) and (3.12), take exactly the same expression

$$K_{IIA} = K_{IIB} = -\log\,(S + S^*) - \sum_{i=1}^{3} \log\,(U_i + U_i^*) - \sum_{i=1}^{3} \log\,(T_i + T_i^*). \tag{3.31}$$

For toroidal orientifolds, mirror symmetry has a very simple realization through T-duality [80]. Indeed, one can construct the operators $\mathcal{M}_1 \equiv T_1T_2T_3$ and $\mathcal{M}_2 \equiv T_1T_2T_3T_4T_5T_6$ consisting respectively on T-dualizing along the x^1, x^2, x^3 or along the x^1, x^2, x^3, x^4, x^5, x^6 directions[2].

The action of the T-duality operators is given by the Buscher rules [81, 82], which for the metric (3.17) imply

$$C_{x\alpha_1\ldots\alpha_p} \xleftrightarrow{T_x} C_{\alpha_1\ldots\alpha_p}, \tag{3.32}$$
$$\sqrt{\frac{\text{Re } T_j}{\text{Re } \tau_j}}\, e^{-\phi} \xrightarrow{T_{xj}} e^{-\phi}. \tag{3.33}$$

Thus, the mapping of Table 3.1 between M^K and $M^{\tilde{K}}$ is automatically accomplished by $\mathcal{M}_1$. More concretely, the following picture arises

$$\begin{array}{ccc} \text{Type IIB with O3/O7-planes} & \xrightarrow{\mathcal{M}_1} & \text{Type IIA with O6-planes} \\ {\scriptstyle \mathcal{M}_2^{-1}}\big\uparrow & & \big\downarrow{\scriptstyle \mathcal{M}_2} \\ \text{Type IIB with O5/O9-planes} & \xleftarrow{\mathcal{M}_1^{-1}} & \text{Type IIA with O6'-planes} \end{array} \tag{3.34}$$

where we have denoted by 'Type IIA with O6'-planes', the toroidal orientifold with involution σ such that the O6-planes wrap the directions x^4, x^5, x^6 (that is, $\theta = \pm\pi/2$ in eq. (3.1)). The

[2] Since the different T-duality operators do not commute, one has to take some care with the order on which the T-dualities are performed. In particular, performing three T-dualities with a given ordering or with the opposite one exchanges the sign of the RR fields, so the same results are obtained up to a $(-1)^{F_L}$ transformation. Since the action is invariant under $(-1)^{F_L}$ this do not introduce new physics. In other words, one can equally act with $\mathcal{M}_1^{-1}$, instead of $\mathcal{M}_1$, to go from Type IIB with O3/O7-planes to Type IIA with O6-planes, however we find $\mathcal{M}_1$ more suitable for our conventions since it leaves invariant the axiodilaton S.

inverse operators $\mathcal{M}_1^{-1}$ and $\mathcal{M}_2^{-1}$ are defined in the same way than $\mathcal{M}_1$ and $\mathcal{M}_2$ but with the opposite ordering for the T-dualities.

Up to here we have not considered the possibility of having background fluxes for the RR and NSNS field strengths. Indeed, having such backgrounds in general will modify the topology of the original manifold in such a way that some of the original moduli are lifted from the massless spectrum. The precise realization of the moduli stabilization in the effective low energy limit will be described in Chapter 5. However, before moving to this point, let us continue reviewing the necessary tools for embedding the Standard Model in String Theory.

3.2 D3/D7-branes at orbifold singularities.

Once supersymmetry is reduced by the special holonomy of the internal manifold to $\mathcal{N} = 1$, the following step towards embedding the Standard Model into String Theory is to achieve chirality in four dimensions. The literature on the topic is extensive: intersecting D-branes [9, 10, 11, 12, 13], heterotic and Type I compactifications [4, 83], magnetized D-branes [84, 85, 86], D-branes at singularities [14, 15, 16]... all of them related among themselves by a rich network of dualities.

Here we will briefly overview the natural setup arising on Type IIB orientifolds with O3/O7-planes, i.e. configurations of D3 and D7-branes placed at orbifold singularities; whereas in the next section we will describe the corresponding mirror configurations of intersecting D6-branes arising in Type IIA orientifolds.

We will consider stacks of N_3 D3-branes and N_7 D7-branes in flat space filling respectively the directions 0 to 3 and 0 to 7. From the point of view of the worldvolume, open strings whose ends rest in the D3-branes will induce a $\mathcal{N} = 4$ $U(N_3)$ Super Yang Mills theory, whereas strings placed between the D7-branes will induce a $\mathcal{N} = 2$ $U(N_7)$ Super Yang Mills in the eight dimensional worldvolume. There will be in addition a crossed sector made of open strings laying between the D3 and the D7-branes and preserving $\mathcal{N} = 2$ in four dimensions. Thus the complete setup in flat space constitutes a 1/4 BPS configuration with several 1/2 BPS subsectors.

An easy way to check the field content of the worldvolume theory is by solving the open string quantization equations derived from the non-linear σ-model. In fact, varying eq. (2.1) with respect X^μ gives rise to[3]

$$\nabla^2 X^\mu = 0 \tag{3.35}$$

[3] Analogous equations for the fermions could be obtained by the same procedure or by supersymmetry arguments.

and, corresponding to the surface term of the variation, the boundary condition

$$[G_{\mu\nu}\partial_\sigma X^\mu + (F_{\mu\nu} - B_{\mu\nu})\partial_\tau X^\mu]|_{\sigma=0}^{\sigma=\pi} = 0, \tag{3.36}$$

with $F_{\mu\nu} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$ the gauge field strength.

Note that D-branes are non-perturbative BPS configurations so its classical mass spectrum do not receive quantum corrections [3]. Thus, although we will solve the spectrum in the $\alpha'^{1/2} R_c^{-1} \ll 1$ limit, this will remain untouched when one takes into account the complete String Theory.

For the particular case of open strings whose extremes are laying in the D3-branes, the boundary conditions (3.36) become

$$\sigma = 0, \pi \qquad \begin{cases} X^\mu = 0 & \mu > 3 \\ \partial_\sigma X^\mu = 0 & \mu = 0 \ldots 3 \end{cases}, \tag{3.37}$$

and the solution is given by the mode expansion (at zero momentum) [87]

$$\begin{aligned} X^\mu &= \sum_{n=-\infty}^{\infty} n^{-1/2} \alpha_n^\mu e^{-in\tau} \cos n\sigma & \mu = 0 \ldots 3, \\ X^m &= \sum_{n=-\infty}^{\infty} n^{-1/2} \alpha_n^m e^{-in\tau} \sin n\sigma + Y^m \frac{\sigma}{\pi} & m = 4 \ldots 9, \end{aligned}$$

with Y^m parametrizing a possible separation between the branes in the transverse direction.

By $\mathcal{N} = 1$ worldsheet supersymmetry we expect for the fermionic fields the expansion

$$\begin{aligned} \Psi_\pm^\mu &= \sum_{n=-\infty}^{\infty} d_n^\mu e^{-in(\tau\pm\sigma)} & \mu = 0 \ldots 3, \\ \Psi_\pm^m &= \pm \sum_{n=-\infty}^{\infty} d_n^m e^{-in(\tau\pm\sigma)} & m = 4 \ldots 9, \end{aligned}$$

for the R sector, and an analogous expansion for the NS sector with the modding shifted by 1/2.

Acting with the ladder operators α and d, we can construct the different target space fields, with masses given by the mass operator

$$\begin{aligned} \alpha' M^2 &= \frac{Y^2}{4\pi^2\alpha'} + N - \nu, \\ N &= \sum_{n>0} n\alpha_{-n}\alpha_n + \sum_{n+\nu>0} (n+\nu) d_{-n-\nu} d_{n+\nu}. \end{aligned}$$

Here $\nu = 0$ for the R sector, whereas $\nu = 1/2$ for the NS sector.

After applying the GSO projection, the massless states in the worldvolume of a stack of D3-branes are hence given by a $U(N)$ gauge boson $A^\mu = d^\mu_{-1/2}|0>_{NS}$ with R-symmetry charges $(0,0,0)$, six adjoint real scalars $\phi^m = d^i_{-1/2}|0>_{NS}$ with R-symmetry charges $(\pm 1,0,0)$, $(0,\pm 1,0)$ and $(0,0,\pm 1)$ and four adjoint fermions ψ^i, $i = 1\ldots 4$ with charges $\frac{1}{2}(-,-,-)$, $\frac{1}{2}(-,+,+)$, $\frac{1}{2}(+,-,+)$ and $\frac{1}{2}(+,+,-)$, which conform an $\mathcal{N}=4$ $U(N)$ SYM vector multiplet in 4 dimensions, as previously advanced. Note the nice interpretation of the 6 scalars of $\mathcal{N}=4$ SYM as moduli fields with vevs parametrizing the position of the D3-branes in the transverse space. For further reading about the moduli structure of $\mathcal{N}=4$ SYM, we refer the reader to Appendix B.

Similar arguments can be applied in order to find the rest of the low energy spectrum, being given by a $\mathcal{N}=2$ $U(N_7)$ SYM vector multiplet in the eight dimensional worldvolume of the D7-branes and a $\mathcal{N}=2$ hypermultiplet transforming in the $(N_2,\overline{N}_7)+(\overline{N}_3,N_7)$ of $U(N_3)\times U(N_7)$ for the crossed sector. Here, however, a new ingredient emerges in the scene. Since we are only requiring $SO(3,1)$ Poincaré invariance, in principle there is no reason to avoid the presence of a non trivial background for the antisymmetric tensor $B_{\mu\nu}-F_{\mu\nu}$ in the worldvolume coordinates transverse to Minkowski[4]. Having a background for B_2 or F_2 in the worldvolume of a D-brane modifies the boundary conditions for the open strings accordingly to eq. (3.36), inducing in this way a non trivial potential for the fermions and scalars living in the worldvolume of the branes. Details about the shape of the flux induced effective potentials will be found in Chapter 6

To illustrate the effect of the B_2-F_2 background in the low energy spectrum for the crossed sector, let us consider a factorable background with components $(B-F)_{45}$ and $(B-F)_{56}$.[5] The boundary conditions for the D3-D7 strings are then given by

$$\sigma = 0 \qquad \begin{cases} X^\mu = 0 & \mu > 3 \\ \partial_\sigma X^\mu = 0 & \mu = 0\ldots 3 \end{cases},$$

$$\sigma = \pi \qquad \begin{cases} X^\mu = Y^\mu & \mu = 8,9 \\ \partial_\sigma X_a + iB^a\partial_\tau X_a = 0 & a = \tilde{1},\tilde{2} \\ \partial_\sigma X^\mu = 0 & \mu = 0\ldots 3 \end{cases},$$

where $B^{\tilde{1}} \equiv (B-F)_{45}$, $B^{\tilde{2}} \equiv (B-F)_{67}$ and the complex structure is fixed through $X_{\tilde{1}} \equiv X^4 + iX^5$ and $X_{\tilde{2}} \equiv X^6 + iX^7$.

[4]Actually something similar happens for the metric, being the most general allowed background given by a warped metric.

[5]Note that the T-dual picture of this configuration is analogous to a configuration of D7-branes with an additional 1/2 twist corresponding to the D3-branes, which can be interpreted as coming from an infinite magnetic background in the directions 4 to 7 of the worldvolume of some of the D7-branes.

Proceeding as before, we construct a mass operator

$$\alpha' M^2 = \frac{Y^2}{4\pi^2\alpha'} + N + \frac{\nu}{\pi}(\arctan B^{\tilde{1}} + \arctan B^{\tilde{2}}), \tag{3.38}$$

with

$$\begin{aligned} N = & \sum_{n>0} n\alpha_{-n}\alpha_n + \sum_{n+\nu>0}(n+\nu)d_{-n-\nu}d_{n+\nu} + \\ & + \sum_{a=\tilde{1},\tilde{2}}\left[\sum_{r>0}(r^a_+\alpha^a_{-r_+}\alpha^a_{r_+} + r^a_-\alpha^a_{-r_-}\alpha^a_{r_-}) + \frac{1}{\pi}\arctan B^a \alpha^a_{-(1/\pi)\arctan B^a}\alpha^a_{(1/\pi)\arctan B^a}\right] + \\ & + \sum_{a=\tilde{1},\tilde{2}}\left[\sum_{r>0}((r^a_+ + \nu)d^a_{-r_+-\nu}d^a_{r_++\nu} + (r^a_- + \nu)d^a_{-r_--\nu}d^a_{r_-+\nu}) + \right. \\ & \left. + \delta_{\nu,0}(\frac{1}{\pi}\arctan B^a)d^a_{-(1/\pi)\arctan B^a}d^a_{(1/\pi)\arctan B^a}\right] \end{aligned}$$

and

$$r^a_\pm = n \pm \left(-\frac{1}{\pi}\arctan B^a + \frac{1}{2}\right).$$

Hence, in absence of flux there is a degenerate ground state generated by the ladder operators $d^{\tilde{1}}_0$ and $d^{\tilde{2}}_0$. The degeneration then can be broken by the background fluxes. In that case the lightest excitations of the NS sector are given by the massive complex scalars

$$\begin{aligned} (--) \qquad & M^2 = \frac{1}{2\alpha'\pi}(\arctan iB_{2\bar{2}} + \arctan iB_{1\bar{1}}) + \frac{Y^2}{4\pi^2(\alpha')^2}, \\ (+-) \qquad & M^2 = \frac{1}{2\alpha'\pi}(\arctan iB_{2\bar{2}} - \arctan iB_{1\bar{1}}) + \frac{Y^2}{4\pi^2(\alpha')^2}, \\ (-+) \qquad & M^2 = \frac{1}{2\alpha'\pi}(\arctan iB_{1\bar{1}} - \arctan iB_{2\bar{2}}) + \frac{Y^2}{4\pi^2(\alpha')^2}, \\ (++) \qquad & M^2 = -\frac{1}{2\alpha'\pi}(\arctan iB_{1\bar{1}} + \arctan iB_{2\bar{2}}) + \frac{Y^2}{4\pi^2(\alpha')^2}, \end{aligned} \tag{3.39}$$

although only the states $(++)$ and $(--)$ survive to the GSO projection[6]. Note that for $Y = 0$ and non-supersymmetric fluxes one of the NS scalars is always tachyonic. Finally, in the R sector two non-chiral fermions of mass $M^2 = \frac{Y^2}{4\pi^2(\alpha')^2}$ are found.

Up to here we have performed an analysis in flat space. Now one would like to impose the orbifold and orientifold projections in order to reduce the supersymmetry to four dimensional $\mathcal{N} = 1$ and to obtain chiral fermions. With this aim, we will consider the above setup to be placed at a $\mathbb{R}^6/\Gamma$ singularity in a toroidal orientifold, with $\Gamma \subset SU(3)$ acting as a discrete group of holonomy. For easiness here we will restrict ourselves to the case $\Gamma = Z_N$. The generalization to more involved orbifold groups will be immediate [14, 15].

[6] In the same way, one could select the states $(-+)$ and $(+-)$ by considering $\overline{D3}$ or $\overline{D7}$-branes.

The Z_N action can be fixed through its effect on the fermions living inside the D3-branes

$$\psi^k \underset{Z_N}{\longrightarrow} e^{2\pi i a_k/N}\psi^k,$$

with $k = 1\ldots 3$ and $\sum_k a_k = 0 \mod N$ so $\Gamma \subset SU(3)$ and the setup is $\mathcal{N} = 1$ supersymmetric.

By worldsheet supersymmetry one can obtain the induced transformations in the remaining worldvolume fields. In particular, for the three complex scalars living in the D3-branes one finds

$$\Phi^m \underset{Z_N}{\longrightarrow} e^{2\pi i b_m/N}\Phi^m,$$

with $m = 1\ldots 3$ and $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$, $b_3 = a_1 + a_2$. One could proceed analogously for the rest of the worldvolume fields.

The Z_N group will act as well on the Chan-Paton indices. The embedding is usually fixed by the matrices

$$\begin{aligned}
&\gamma_{\theta,3} = diag(I_{n_0}, e^{2\pi i/N} I_{n_1}, \ldots, e^{2\pi i(N-1)/N} I_{n_{N-1}}), \\
&\gamma_{\theta,7} = diag(I_{u_0}, e^{2\pi i/N} I_{u_1}, \ldots, e^{2\pi i(N-1)/N} I_{u_{N-1}}) && \text{for } b_3 = \text{even}, \\
&\gamma_{\theta,7} = diag(e^{\pi i/N} I_{u_0}, e^{6\pi i/N} I_{u_1}, \ldots, e^{2\pi i(2N-1)/N} I_{u_{N-1}}) && \text{for } b_3 = \text{odd},
\end{aligned}$$

with θ the Z_N generator. Thus, the Chan-Paton factors in the different sectors of the setup transform as

$$\begin{aligned}
\lambda_{33} &\underset{Z_N}{\longrightarrow} \gamma_{\theta,3}\lambda_{33}\gamma_{\theta,3}^{-1}, \\
\lambda_{77} &\underset{Z_N}{\longrightarrow} \gamma_{\theta,7}\lambda_{77}\gamma_{\theta,7}^{-1}, \\
\lambda_{37} &\underset{Z_N}{\longrightarrow} \gamma_{\theta,3}\lambda_{37}\gamma_{\theta,7}^{-1}, \\
\lambda_{73} &\underset{Z_N}{\longrightarrow} \gamma_{\theta,7}\lambda_{73}\gamma_{\theta,3}^{-1},
\end{aligned}$$

and the gauge groups of the field theories in the worldvolume of the D3 and the D7-branes are broken respectively to $\prod_{i=0}^{N_3-1} U(n_i)$ and $\prod_{i=0}^{N_7-1} U(u_i)$. The invariant states of the spectrum are given in Table 3.2. This provides us with a basic local setup to build up chiral $\mathcal{N} = 1$ gauge theories in the context of Type IIB String Theory.

It only rests to apply the orientifold projection. This can be naively done for setups of branes placed outside the locus of the O-planes. In that case, the orientifold projection simply imposes a Z_2 identification of the fields due to σ, and for each brane there will be a set of mirror branes obtained under the action of σ and $\sigma\theta$.

Concerning the setups of branes coincident with the O-planes, the projected states in addition have to be invariant under the action $\gamma_{\Omega_P,3}$ and $\gamma_{\Omega_P,7}$ of the orientifold on the Chan-Paton

Sector	Multiplet	Representation
33	Vector mult. Chiral mult.	$\prod_i U(n_i)$ $\sum_i \sum_{r=1}^{3} (n_i, \bar{n}_{i+a_r})$
77	Vector mult. Chiral mult.	$\prod_i U(u_i)$ $\sum_i (u_i, \bar{u}_{i+a_3})$
37, **73**	Chiral mult.	$\sum_i [(n_i, \bar{u}_{i-\frac{1}{2}a_3}) + (u_i, \bar{n}_{i-\frac{1}{2}a_3})]$ a_3 even $\sum_i [(n_i, \bar{u}_{i-\frac{1}{2}(a_3+1)}) + (u_i, \bar{n}_{i-\frac{1}{2}(a_3+1)})]$ a_3 odd

Table 3.3: Spectrum of the D3-D7 setup at a Z_N singularity.

factors. The $U(N)$ gauge groups are then projected down to $SO(N)$ with two-index antisymmetric representations, or to $USp(N)$ with two-index symmetric representations, depending on the orientifold prescription. We refer the reader to [88, 89] for further details.

3.3 Intersecting D6-branes.

The second construction giving rise to four dimensional $\mathcal{N} = 1$ chiral field theories which we would like to review here is the case of intersecting D6$_a$-branes wrapping 3-cycles Π_a on a factorized 6-torus. The 3-cycles are specified in terms of the wrapping numbers as

$$\Pi_a = (n_a^1, m_a^1) \otimes (n_a^2, m_a^2) \otimes (n_a^3, m_a^3). \tag{3.40}$$

Alternatively, one may use the angles of the branes with respect to the x^i axis $(i = 1, 2, 3)$

$$\tan \theta_a^i = \frac{m_a^i R_y^i}{n_a^i R_x^i}. \tag{3.41}$$

Note in particular that the cycles $\Pi_{D3} = (1, 0) \otimes (1, 0) \otimes (1, 0)$ and/or $\Pi_{D7} = (0, 1) \otimes (0, -1) \otimes (1, 0)$ are associated to the Type IIB mirror configurations of D3 and D7-branes discussed in the previous section.

The massless spectrum can be computed in the same way as we did for the D3/D7-brane system [90]. Each stack of D6$_a$-branes will engender a $\mathcal{N} = 4$ $U(N_a)$ SYM theory in four dimensions. In addition, at the intersection between two stacks of D6$_a$-branes and D6$_b$-branes there will be open strings satisfying the boundary conditions

$$\begin{aligned} \sin \theta_a^i \partial_\sigma X^i - \cos \partial_\sigma X^{i+3} &= 0, \\ \sin \theta_a^i \partial_t X^{i+3} - \cos \partial_t X^i &= 0, \end{aligned}$$

for $\sigma = 0$, and similar for $\sigma = \pi$ with $a \to b$. Such conditions lead to twisted states analogous to the ones encountered in the previous section for the D3-D7 strings in presence of $B_2 - F_2$ flux. In particular, the mass of the GSO projected states can be recast as [90, 91, 92]

$$\alpha' M_{ab}^2 = N_{bos}(\theta) + \frac{(r+v)^2}{2} - \frac{1}{2} + \frac{1}{2}\sum_{i=1}^{3} |\theta_i|(1-|\theta_i|), \tag{3.42}$$

being $N_{bos}(\theta)$ the contribution from the bosonic oscillators, irrelevant for our discussion here, and v a 4-dimensional vector whose i-th entry corresponds to $\theta_{ab}^i = (\theta_b^i - \theta_a^i)/\pi$ (and the fourth one equals to 0). The components of the vector r then takes values in $\mathbb{Z}$ or $\mathbb{Z} + \frac{1}{2}$, labelling the states of the NS sector or the R sector respectively.

The lowest states of the spectrum are thus given by a chiral left-handed fermion in the bifundamental representation $(N_a, \overline{N}_b)$ of $U(N_a) \times U(N_b)$, and four real scalars with masses given in Table 3.4.

$v + r$	$\alpha' M^2$
$(\theta_1 - 1, \theta_2, \theta_3, 0)$	$\frac{1}{2}(-\theta_1 + \theta_2 + \theta^3)$
$(\theta_1, \theta_2 - 1, \theta_3, 0)$	$\frac{1}{2}(\theta_1 - \theta_2 + \theta^3)$
$(\theta_1, \theta_2, \theta_3 - 1, 0)$	$\frac{1}{2}(\theta_1 + \theta_2 - \theta^3)$
$(\theta_1 - 1, \theta_2 - 1, \theta_3 - 1, 0)$	$1 - \frac{1}{2}(\theta_1 + \theta_2 + \theta^3)$

Table 3.4: Lowest scalar states for the D6$_a$-D6$_b$ intersection.

Note that for certain angles the scalars may become tachyonic, signaling an instability against recombination of the branes into a single one. In other cases, the angles may be such that some of the scalars become massless and part of the supersymmetry is preserved by the intersections. More concretely, when just one of the scalars is massless the brane intersection will preserve $\mathcal{N} = 1$ supersymmetry in four dimensions and the scalar will be arranged together with the chiral fermion to form a $\mathcal{N} = 1$ chiral multiplet. Thus, a priori no extra orbifold twist is required in this case to reduce the supersymmetry.

In general, the cycles Π_a and Π_b will intersect several times, as given by the intersection number

$$I_{ab} = \prod_i (n_a^i m_b^i - m_a^i n_b^i), \tag{3.43}$$

and hence there will be I_{ab} replicas of the spectrum.

With respect to the orientifold projection, one has something similar to what we saw in the previous section for the setups of D3/D7-branes. Thus, for D6-branes outside the locus of the O6-planes the effect of the orientifold is exclusively due to the σ involution and for each brane wrapping a cycle Π_a, there will be a mirror brane wrapping the cycle

$$\Pi_a^* = (n_a^1, -m_a^1) \otimes (n_a^2, -m_a^2) \otimes (n_a^3, -m_a^3). \tag{3.44}$$

Open strings in the ab^* and aa^* intersections will lead respectively to I_{ab^*} chiral fermions in the (N_a, N_b) of $U(N_a) \times U(N_b)$ and to I_{aa^*} fermions in symmetric or antisymmetric representations [86, 84].

Concerning the branes at the locus of the O6-planes one has in addition to impose the orientifold projection on the Chan-Paton factors, as occurred in the previous section. In that case, the gauge groups again get reduced to $SO(N)$ or $USp(N)$, depending on the prescription for the O-planes.

3.4 Anomalies and Tadpole Cancellation.

One of the very attractive features of String Theory for describing our real world is that consistency of the theory in ten dimensions usually guarantees the absence of pathologies for the field theories living inside the D-branes. More concretely, the cancellation of the non-abelian anomalies is automatically ensured once the tadpole cancellation conditions for the RR closed string modes are satisfied [89, 92]. On the other hand, the mixed $U(1)$ anomalies are cancelled thanks to a combination of the RR tadpole cancellation conditions and the Green-Schwarz mechanism [93]. The explicit realization of these mechanisms is model dependent. Here we will overview how the cancellation is achieved in the setups of the previous sections.

In a quiver theory, the cancellation of cubic non-abelian anomalies for the a-th node reads

$$\sum_b I_{ab} N_b = 0, \tag{3.45}$$

with I_{ab} the number of bifundamentals $(N_a, \overline{N}_b)$ of $U(N_a) \times U(N_b)$.

For configurations of D6_a-branes this condition has a direct interpretation, being I_{ab} the intersection number defined in eq. (3.43). In that case, eq. (3.45) is automatically satisfied once cancellation of the global charge associated to the RR 7-form in the compact manifold is imposed. This is equivalent to require the condition

$$\sum_a N_a [\Pi_a] = 0, \tag{3.46}$$

with $[\Pi_a]$ the homology class corresponding to the 3-cycle Π_a. Then, one may check that in terms of the wrapping numbers eq. (3.46) reads

$$
\begin{aligned}
&\sum_a N_a n_a^1 n_a^2 n_a^3 = 0, && \sum_a N_a n_a^1 m_a^2 m_a^3 = 0,\\
&\sum_a N_a m_a^1 n_a^2 n_a^3 = 0, && \sum_a N_a m_a^1 n_a^2 m_a^3 = 0,\\
&\sum_a N_a n_a^1 m_a^2 n_a^3 = 0, && \sum_a N_a m_a^1 m_a^2 n_a^3 = 0,\\
&\sum_a N_a n_a^1 n_a^2 m_a^3 = 0, && \sum_a N_a m_a^1 m_a^2 m_a^3 = 0,
\end{aligned}
$$

and the left hand side of eq. (3.45) automatically vanishes.

For D3/D7-branes placed on top of orbifold singularities the mechanism is analogous, but the role played by the untwisted RR 7-form is now played by twisted fields. Indeed, for the chiral spectrum of a D3/D7 configuration, eq. (3.45) reads

$$
\sum_\alpha (n_{i+a_\alpha} + n_{i-a_\alpha}) + \sum_r (u_{i+\frac{1}{2}b_r} - u_{i-\frac{1}{2}b_r}) = 0, \tag{3.47}
$$

where we have generalized the configurations of Section 3.2 to consider all the three possible kinds of $D7_r$-branes, filling everything but the r-th complex plane transverse to Minkowski.

The twisted tadpole cancellation conditions for D3/D7-brane configurations are given by [89]

$$
[\prod_{r=1}^{3} 2\sin(\pi k b_r/N)\mathrm{Tr}\gamma_{\theta^k,3}] + 2\sum_{r=1}^{3} \sin(\pi k b_r/N)\mathrm{Tr}\gamma_{\theta^k,7_r} = 0. \tag{3.48}
$$

Then, substituting

$$
\begin{aligned}
n_j &= \frac{1}{N}\sum_{k=1}^{N} e^{-2\pi i k j/N}\mathrm{Tr}\gamma_{\theta^k,3},\\
u_j &= \frac{1}{N}\sum_{k=1}^{N} e^{-2\pi i k j/N}\mathrm{Tr}\gamma_{\theta^k,7},
\end{aligned}
$$

into (3.48) and performing some algebra, the cancellation condition (3.47) is trivially satisfied.

Concerning the cancellation of $U(1)$ mixed anomalies, there are some subtleties. Indeed, the amplitude for the $U(1)_a - [SU(N_b)]^2$ triangle diagram is given by

$$
\mathcal{A}_{ab} = \frac{1}{2}\delta_{ab}\sum_c N_c I_{bc} + \frac{1}{2}N_b I_{ab}, \tag{3.49}
$$

so just the first piece will vanish due to the tadpole conditions. In order to ensure a complete cancellation of the mixed gauge anomaly a generalized Green-Schwarz mechanism is required in the game. Diagrams not cancelled through the tadpole conditions are cancelled by stringy diagrams on which there is an exchange of closed string untwisted modes for D6-branes, or twisted modes for D3/D7-branes. For the sake of clarity, here we will describe only the exchange of untwisted closed string modes by intersecting D6-branes. Something analogous should be expected for configurations of D3 and D7-branes.

The D6-brane action contains the following relevant piece [92]

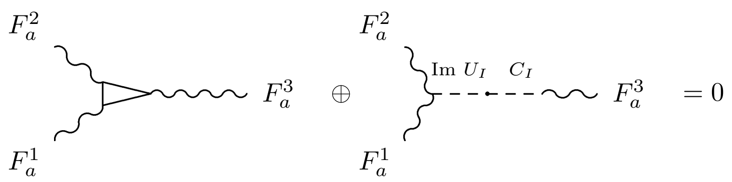
$$\int_{D6} (C_3 \wedge F_a \wedge F_a + C_5 \wedge F_a) = \sum_I \int_{M_4} \left[p_I^a (\text{Im } U_I) F_a \wedge F_a + N_a c_I^a C_I^{(2)} \wedge F^a \right], \tag{3.50}$$

where $U_0 = -S$ corresponds to the axiodilaton, U_i are the complex structure moduli and $C_I^{(2)} = \int_{[\Pi_I]} C_5$. The coefficients (c_I^a, p_I^a) are defined as

$$c_0^a = m_a^1 m_a^2 m_a^3, \qquad c_1^a = m_a^1 n_a^2 n_a^3, \qquad c_2^a = n_a^1 m_a^2 n_a^3, \qquad c_3^a = n_a^1 n_a^2 m_a^3, \tag{3.51}$$

$$p_0^a = n_a^1 n_a^2 n_a^3, \qquad p_1^a = n_a^1 m_a^2 m_a^3, \qquad p_2^a = m_a^1 n_a^2 m_a^3, \qquad p_3^a = m_a^1 m_a^2 n_a^3. \tag{3.52}$$

The Green-Schwarz mechanism guarantees then that the residual $U(1)$ mixed anomaly is cancelled through the exchange of closed string geometric moduli due to the couplings (3.50), i.e.



Indeed, the amplitude for the second diagram is

$$N_a \sum_I p_I^a c_I^a = N_a I_{ab}, \tag{3.53}$$

which precisely cancels the residual mixed $U(1)$ anomaly of eq. (3.49).

The potentially anomalous $U(1)$ becomes massive and remains as a global symmetry of the low energy effective theory [94, 95, 96]. In fact, rewriting the coupling $C_I^{(2)} \wedge F^a$ as

$$-\frac{N_a c_I^a}{6} \epsilon^{\mu\nu\rho\sigma} (dC_I^{(2)})_{\mu\nu\rho} A_\sigma^a - \frac{N_a c_I^a}{6} (\text{Im } U_I) \epsilon^{\mu\nu\rho\sigma} \partial_\mu (dC_I^{(2)})_{\nu\rho\sigma}, \tag{3.54}$$

and making use of the equation of motion for $dC_I^{(2)}$

$$(dC_I^{(2)})^{\mu\nu\rho} = -N_a c_I^a \epsilon^{\mu\nu\rho\sigma} (A_\sigma + \partial_\sigma (\text{Im } U_I)),$$

one arrives to a Stückelberg's mass term for the vector boson A^a

$$-\frac{(N_a c_I^a)^2}{2} (A_\sigma^a + \partial_\sigma (\text{Im } U_I))^2.$$

Taking a basis $\{Q_a/N_a\}$ for the generators of the different $U(1)$'s, then the linear combinations of gauge bosons which remain massless after the Green-Schwarz mechanism are given by the solutions to the equation [92]

$$\sum_a \frac{I_{ab}Q_a}{N_a} = 0. \tag{3.55}$$

3.5 Model building without fluxes.

Based on the above ideas, and on other possibilities not discussed here, a lot of semirealistic models with chiral content and gauge group similar to the ones of the MSSM have been presented in the last years. Although the models are clearly on the road of the Standard Model, they usually involve some extra $U(1)$'s or hidden sectors, and in particular a fully realistic model still has not been achieved.

Here we will present a couple of examples based on the ideas of this chapter and which are representative of the effort done during the *'pre-fluxed'* era of the model building. All these models have in common two major pathologies: the lack of a non trivial scalar potential for the closed string moduli, giving rise to a whole set of massless scalars in the spectrum; and the absence of a non-trivial scalar potential for the open string moduli, in particular for the Higgs(es), thus lacking a (controllable) supersymmetry breaking mechanism. The addressing of these two problems by considering backgrounds for the closed string modes will constitute the bulk of this thesis.

3.5.1 Model 1: D3/D7-branes on a Z_3 singularity.

We will reproduce here one of the models of [15].The basic idea to get the Standard Model group is to consider a stack of D3-branes placed on a Z_3 singularity plus three stacks of D7-branes, each one wrapping two different complex planes in the transverse directions to Minkowski, as illustrated in Figure 3.1.

The twisted tadpole conditions (3.48) for this configuration read

$$\mathrm{Tr}\gamma_{\theta,7_3} - \mathrm{Tr}\gamma_{\theta,7_1} - \mathrm{Tr}\gamma_{\theta,7_2} + 3\mathrm{Tr}\gamma_{\theta,3} = 0, \tag{3.56}$$

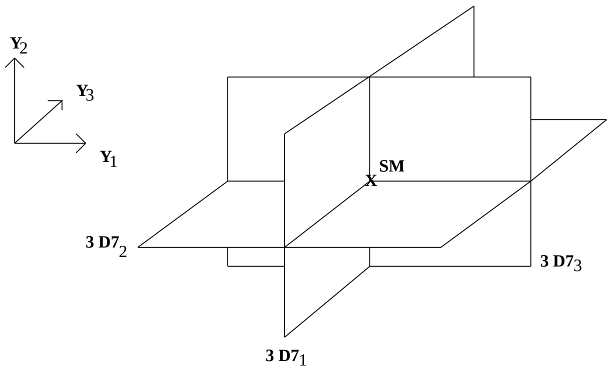


Figure 3.1: Local Type IIB Z_3 singularity yielding the SM spectrum. Six D3-branes sit on top of the Z_3 singularity. Twisted tadpoles are cancelled by intersecting D7-branes with their worldvolume transverse to different complex planes.

which can be solved through the Chan-Paton embedding

$$\gamma_{\theta,3} = \begin{pmatrix} I_3 & & \\ & \alpha I_2 & \\ & & \alpha^2 I_1 \end{pmatrix},$$

$$\gamma_{\theta,7_3} = -\gamma_{\theta,7_1} = -\gamma_{\theta,7_2} = \begin{pmatrix} 0 & & \\ & \alpha I_1 & \\ & & \alpha^2 I_2 \end{pmatrix},$$

with $\alpha = e^{2\pi i/3}$.

This gives rise to a $SU(3) \times SU(2) \times U(1)$ gauge theory living in the worldvolume of the D3-branes, plus three $U(1) \times U(2)$ hidden sectors in the D7-branes. The matter content is depicted in Table 3.5. Note the remarkable similarity to the Standard Model spectrum.

The only anomaly free combination of $U(1)'s$

$$Y = -(\frac{1}{3}Q_3 + \frac{1}{2}Q_2 + Q_1) \tag{3.57}$$

corresponds to the hypercharge, which arises from the **33** sector.

One may embed this local setup into a $T^6/\Omega_p(-1)^{F_L}\sigma Z_3$ toroidal orientifold. This has 27 orbifold fixed points labelled by (m, n, p), $m, n, p = 0, \pm 1$. Among these, only the origin $(0, 0, 0)$ corresponds to a fixed point of the orientifold involution σ and holds 64 O3-planes. Thus, in order to cancel the untwisted tadpoles, it is required the presence of 32 D3-branes accommodated among the different orbifold and orientifold points. Some of these branes in the orbifold points will correspond to the D3-branes of the above local setup containing the

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	1/6
$3(\bar{3},1)$	-1	0	1	0	0	-2/3
$3(1,2)$	0	1	-1	0	0	1/2
$\mathbf{37}_r$ sector						
$(3,1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;2')$	-1	0	0	0	1	1/3
$(1,2;2')$	0	1	0	0	-1	-1/2
$(1,1;1')$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1;2)'$	0	0	0	1	-1	0

Table 3.5: Spectrum of the $SU(3) \times SU(2) \times U(1)$ model.

Standard Model sector. On the other hand, to cancel the global charge of D7, one has to add the same number of $\overline{D7}$-branes as D7-branes. Unfortunately, the presence of $\overline{D7}$-branes leads to instabilities such as non vanishing NS tadpoles or partial annihilation against the D7-branes. These are always difficult to manage and rest attractiveness to the model. We refer the reader to [15] for a concrete example of global embedding along these lines.

3.5.2 Model 2: Four-stacks model of intersecting D6-branes.

The simplest models of intersecting D6-branes containing the same chiral content than the Standard Model are made of four stacks of branes. These may be conveniently labelled as *baryonic, leptonic, left* and *right* stacks, with respective multiplicities $N_a = 3$, $N_d = 1$, $N_b = 2$ and $N_c = 1$ [95, 97, 98]. The disposition of the Standard Model fields at the different intersections is depicted in Figure 3.2. In some cases, as in the model presented here, the baryonic and leptonic branes have the same wrapping numbers, thus giving rise to a very symmetrical configuration.

We will consider here the setup of D6-branes of Table 3.6 placed in a simple Type IIA toroidal orientifold $T^6/\Omega_P(-1)^{F_L}\sigma$. In addition, there will be some mirror branes under the σ involution accordingly to eq. (3.44). The left brane will be considered to be coincident with the orientifold plane so, due to the orientifold projection of the Chan-Paton factors, it will engender a $SU(2)$ group instead of the usual $U(1)$.[7] This model was first presented in [98].

[7] Another flat direction of this model consists on making the right brane to overlap as well with the orientifold plane thus giving rise to a left-right $SU(2)_L \times SU(2)_R$ supersymmetric model.

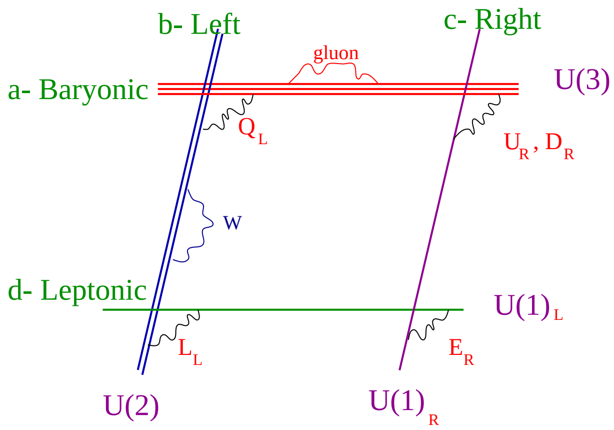


Figure 3.2: Schematics of the SM fields at the intersections in a model of four stacks of D6-branes.

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^2, m_i^2)
$N_a + N_d = 4$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_b = 1$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 1$	$(0, 1)$	$(0, -1)$	$(1, 0)$

Table 3.6: Wrapping numbers of a four stack model of D6-branes holding the same chiral spectrum than the MSSM.

Each intersection preserves a $\mathcal{N} = 1$ supersymmetry, and thus one of the scalars of Table 3.4 remains massless at each intersection. The condition for all the intersections to preserve the same $\mathcal{N} = 1$ supersymmetry than the orientifold planes is then

$$\sum_i \arctan\left(\frac{m_a^i R_y^i}{n_a^i R_x^i}\right) = 0, \tag{3.58}$$

which leads to $\tau_1 = \tau_2 = \tau_3$.

The gauge group, after splitting the baryonic and leptonic branes, is given by $U(3) \times SU(2)_L \times [U(1)]^2$. Now, by making use of eq. (3.55), one observes that one of the $U(1)$'s becomes massive through the Green-Schwarz mechanism and the final group at low energies becomes by $SU(3) \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$.

The chiral content of the model can be easily computed, resulting to be the same than in the MSSM: three generations of quarks and leptons with right-handed neutrinos and two doublets of Higgsses living between the left and right branes.

One may embed this local setup into a global Type IIA toroidal orientifold. Naively, in order to cancel the D6-brane tadpoles along the nmm directions without introducing $\overline{D6}$-branes, one is enforced to consider an extra orbifold symmetry, such as in the $Z_2 \times Z_2$ example discussed in Section 3.1. We will see however in Section 7.2.3 a different approach which makes use of closed string backgrounds in order to cancel the tadpoles and at the same time stabilize all the moduli of the compactification.

Chapter 4

Type II Supergravity and Flux Compactifications.

A good understanding of the low energy limit is required in order to gain some insight into the structure of String Theory. Thus, on this chapter we turn into the task of describing the two ten dimensional $\mathcal{N} = 2$ supergravities which arise in the low energy limit of Type II String Theory. These are usually denoted as Type IIA and Type IIB Supergravity, depending on whether the two supercharges have different or the same chirality.

New degrees of freedom in the compactifications of Chapter 3 will appear, consisting on non-trivial backgrounds for the RR and NSNS field strengths along the cycles of the internal manifold. These background fluxes will determine the low energy dynamics of the theory through changes in the topology of the internal manifold, thus revealing interesting properties for addressing some of the pathologies described in the previous chapter.

4.1 Type IIB Supergravity.

As mentioned, in this case the two supercharges have the same chirality. The field content can be obtained by decomposing the product of them in terms of irreducible representations of the little group $SO(8)$

$$(\mathbf{8_v} + \mathbf{8_c}) \otimes (\mathbf{8_v} + \mathbf{8_c}) = (\mathbf{1} + \mathbf{28} + \mathbf{35_v} + \mathbf{28} + \mathbf{35_c})_B + (\mathbf{8_s} + \mathbf{8_s} + \mathbf{56_s} + \mathbf{56_s})_F.$$

This corresponds to a graviton $h_{\mu\nu}$, a complex axiodilaton $\tau = C + ie^{-\phi}$, a complex three-form $F_3 = dA_2$, a self-dual 5-form $F_5 = dA_4 - (\kappa/8)\mathrm{Im}(A_2 \wedge F_3^*)$, a complex Weyl gravitino ψ_M ($\gamma\psi_M = -\psi_M$) and a complex Weyl dilatino λ ($\gamma\lambda = \lambda$). Note that we have taken a different definition for the axiodilaton than in eq. (3.8). Both conventions may be related through

$S = -i\tau$ plus an extra $(-1)^{F_L}$ transformation. On this thesis we will use τ when talking about the ten dimensional supergravity, and S when doing about the moduli space of the compactification.

The equations of motion and supersymmetry variations of Type IIB supergravity were deduced by Schwarz et al. in [99, 100] by a different method to the Noether's procedure described in Appendix A. Basically the method consists on deriving one of the equations of motion making use of the $SU(1,1)$ non-perturbative symmetry of the theory. Then, applying the supersymmetry transformations and imposing the closure of the algebra, the rest of the equations can be obtained.

The supersymmetric variations for the fermions are given by

$$\delta\lambda^* = -\frac{i}{\kappa}\gamma^m P_m^*\epsilon + \frac{i}{4}G^*\epsilon^*, \tag{4.1}$$

$$\delta\psi_r = \frac{1}{\kappa}\left(D_r - \frac{i}{2}Q_r\right)\epsilon + \frac{i}{480}\gamma^{mnopq}F_{mnopq}\gamma_r\epsilon - \frac{1}{16}\gamma_r G\epsilon^* - \frac{1}{8}G\gamma_m\epsilon^*, \tag{4.2}$$

with $G = \frac{1}{6}G_{mnp}\gamma^{mnp}$, $G_3 = f(F_3 - BF_3^*)$, $\epsilon = \epsilon_1 + i\epsilon_2$ the complex Weyl supersymmetry parameter ($\Gamma\epsilon = -\epsilon$) and

$$P_m = f^2\partial_m B, \qquad Q_m = f^2 Im(B\partial_m B^*),$$
$$B = \frac{1+i\tau}{1-i\tau}, \qquad f^{-2} = 1 - BB*.$$

In order to relate these quantities with the ones which usually appear in String Theory it is common to make the redefinitions [82, 101]

$$G_3 \to i\frac{g_s}{\kappa\sqrt{Im\ \tau}}\left(\frac{1+i\tau^*}{1-i\tau}\right)^{1/2}G_3, \tag{4.3}$$

$$4\kappa F_5 \to gF_5, \tag{4.4}$$

$$2\kappa^2 = (2\pi)^7 g_s^2\alpha'^4, \tag{4.5}$$

and to work in $SL(2,\mathbb{R}) \simeq SU(1,1)$ covariant variables. In that case $\epsilon = \begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix}$. The $SL(2,\mathbb{R})$ group is generated by

$$\mathcal{S}_1 = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} \quad , \quad \mathcal{S}_2 = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \quad , \quad \mathcal{S}_3 = \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}.$$

Thus, in general an element $\Lambda = \begin{pmatrix}a & b\\ c & d\end{pmatrix} \in SL(2,\mathbb{R})$ with $ad - bc = 1$ will act non-perturbatively on the Type IIB fields as

$$S \to \frac{aS - ib}{icS + d} \quad , \quad B_2 \to dB_2 + cC_2 \quad , \quad C_2 \to bB_2 + aC_2 \quad , \quad C_4 \to C_4. \tag{4.6}$$

In particular, $\mathcal{S}_1$ will correspond to Peccei-Quinn symmetries $S \to S - i$, $\mathcal{S}_2$ to S-duality transformations

$$S \to \frac{1}{S} \quad , \quad C_2 \to -B_2 \quad , \quad B_2 \to C_2 \quad , \quad C_4 \to C_4, \tag{4.7}$$

and $\mathcal{S}_3$ will be associated to the generator of the R-symmetry group $U(1) \subset SL(2,\mathbb{R})$. At the quantum level, the fields in general will be quantized over the cycles of the internal manifold, and only a subgroup $SL(2,\mathbb{Z}) \subset SL(2,\mathbb{R})$ will survive [102].

Making use of the redefinitions (4.3)-(4.5) and working in the Einstein's frame[1] on which the Einstein's term has the canonical normalization, it is possible to rewrite the equations of motion of [100] as[2]

$$\begin{aligned}
dF_5 &= \frac{i}{2} g_s G_3 \wedge G_3^*, && (4.8)\\
\nabla^m \left(\frac{i}{2} e^{\phi} \partial_m \tau\right) &- \frac{1}{2} e^{2\phi} \partial^m C \partial_m \tau = \frac{g_s}{24} G_{mnp} G^{mnp}, && (4.9)\\
R_{mn} &= \frac{1}{4} e^{2\phi} \partial_m \tau \partial_n \tau^* + \frac{1}{4} e^{2\phi} \partial_m \tau^* \partial_n \tau + \frac{g_s^2}{96} F_{mqrst} F_n{}^{qrst} \\
&+ \frac{g_s}{8} \left(G_m{}^{pq} G^*_{npq} + G_n{}^{pq} G^*_{mpq} - \frac{1}{6} g_{mn} G^*_{pqr} G^{pqr} \right), && (4.10)\\
d * ReG_3 &= F_5 \wedge H_3, && (4.11)
\end{aligned}$$

with

$$\begin{aligned}
F_3 &= dC_2, & H_3 &= dB_2,\\
F_5 &= dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3, & G_3 &= F_3 - \tau H_3,
\end{aligned}$$

and where the prescription for the hodge dual operation * is the common one used in supergravity

$$*A_{\mu_1 \dots \mu_{d-p}} \equiv \frac{1}{p!} \epsilon_{\mu_1 \dots \mu_{d-p}}{}^{\nu_1 \dots \nu_p} A_{\nu_1 \dots \nu_p}. \tag{4.12}$$

Although a manifest Lorentz covariant action for these equations cannot be written, one can always write an action supplemented by the on-shell condition $F_5 = *F_5$ for the five-form field strength [103, 104]

$$\begin{aligned}
S_{IIB} &= \frac{1}{2\kappa^2} \int d^{10}x \, (-G)^{1/2} \, [\, R - \frac{e^{2\phi}}{2} \partial_m \tau \partial^m \tau^* - \frac{1}{12} e^{\phi} G_3 \cdot G_3^* - \frac{1}{480} F_5^2 \,] + \\
&+ \frac{1}{8i\kappa^2} \int e^{\phi} C_4 \wedge G_3 \wedge G_3^*.
\end{aligned} \tag{4.13}$$

[1] The Einstein and the string frames are related by a re-scaling of the metric as $(g_{st})_{mn} = g_s^{1/2} (g_E)_{mn}$. This induces as well a re-scaling of the Dirac algebra $\gamma^m_{st} = g_s^{-1/4} \gamma^m_E$ through the relation $\{\gamma^m, \gamma^n\} = 2g^{mn}$.

[2] We only present here the equations more involved with our work.

Then one can apply the usual Euler-Lagrange procedure[3] to recover the equations (4.8)-(4.11).

The theory as well can be formulated in terms of the dual field strengths F_5, $*F_3$ and $*F_1$, exchanging in this way the equations of motion and the Bianchi identities with respect to the ordinary formulation. In that case, one defines the potentials C_6 and C_8 through the relations

$$-*\,Re\; G_3 = dC_6 - H_3 \wedge \left(C_4 + \frac{1}{2} B_2 \wedge C_2\right), \tag{4.14}$$

$$-*\,Re\; d\tau = dC_8 - H_3 \wedge C_6, \tag{4.15}$$

which result useful in the context of D-brane actions, as we will see in Chapter 6. In addition, in order to recover the $SL(2,\mathbb{R})$ symmetry in the dual formulation one has to include new potentials C_8' and $\tilde{C}_8$ [105, 106, 107]. Indeed, it happens that C_8 comes in a $SL(2,\mathbb{R})$ triplet of 8-forms $(C_8, C_8', \tilde{C}_8)$ transforming as

$$\begin{pmatrix} C_8' & \tilde{C}_8 \\ C_8 & -C_8' \end{pmatrix} \rightarrow \Lambda \cdot \begin{pmatrix} C_8' & \tilde{C}_8 \\ C_8 & -C_8' \end{pmatrix} \cdot \Lambda^{-1}, \tag{4.16}$$

with $\Lambda \in SL(2,\mathbb{R})$. Since these potentials correspond to the dual of the axiodilaton τ in th ordinary formulation of the theory, it will be required a constraint among the field strengths so that there are only two propagating degrees of freedom.

4.2 Type IIA Supergravity and Romans Supergravity

We can proceed as in the previous section with Type IIA supergravity. On this case the two supercharges have different chirality and the product of them in terms of irreducible representations of the little group becomes

$$(\mathbf{8_v} + \mathbf{8_c}) \otimes (\mathbf{8_v} + \mathbf{8_c}') = (\mathbf{1} + \mathbf{28} + \mathbf{35_v} + \mathbf{8_v} + \mathbf{56_v})_B + (\mathbf{8_s} + \mathbf{8_s}' + \mathbf{56_s} + \mathbf{56_s}')_F.$$

This corresponds to a graviton $h_{\mu\nu}$, a real scalar ϕ, a real three-form $H_3 = dB_2$, a real two-form $F_2 = dA_1$, a real four-form $F_4 = dC_3 - H_3 \wedge A_1$, a complex Weyl gravitino ψ_M $(\gamma\psi_M = -\psi_M^*)$

[3] It results useful to derive a symbolic procedure in order to take the Euler-Lagrange equations of a expression written in language of forms. The rule consists in taking

$$\frac{\partial \mathcal{L}}{\partial C_n} = d \frac{\partial \mathcal{L}}{\partial (dC_n)},$$

where $\mathcal{L}$ is expressed in such a way that on each wedge product of forms C_n and dC_n are on the left hand side of the product. The partial derivatives are then symbolically defined through

$$\begin{aligned} \frac{\partial}{\partial C_n}(C_n \wedge A_p) &= A_p, \\ d\frac{\partial}{\partial (dC_n)}(dC_n \wedge A_p) &= (-1)^n dA_p. \end{aligned}$$

and a complex Weyl dilatino λ ($\gamma\lambda = \lambda^*$). The decomposition admits as well a cosmological constant term m. In that case, the field B_2 eats the vector field A_1 and acquires a mass, in a similar fashion to the Higgs mechanism. The arising supergravity is usually called massive Type IIA supergravity or Romans supergravity [108].

Contrary to what happens in Type IIB supergravity, it is possible to write a complete manifest Lorentz covariant action for Type IIA supergravity. This can be done by different methods. The simplest one is through Kaluza-Klein reduction of the maximal supergravity in eleven dimensions. However, doing so it is not possible to obtain the massive version of Type IIA supergravity.

Here we will start with the results of Romans [108], written in the Einstein frame

$$\begin{aligned} S_{IIA} = \frac{1}{2\kappa^2}\int d^{10}x(-G)^{1/2}[R - \frac{1}{2}\partial_m\phi\partial^m\phi - \frac{e^{-\phi}}{12}(H_3)^2 - \frac{e^{3\phi/2}m^2}{4}(B_2)^2 - \\ - \frac{e^{\phi/2}}{48}(F_4)^2 - \frac{e^{5\phi/2}}{2}m^2] + \frac{1}{4\kappa^2}\int[B_2\wedge dC_3\wedge dC_3 + \frac{m}{3}B_2\wedge B_2\wedge B_2\wedge dC_3 + \\ + \frac{m^2}{20}B_2\wedge B_2\wedge B_2\wedge B_2\wedge B_2], \end{aligned} \tag{4.17}$$

with $H_3 = dB_2$ and $F_4 = dC_3 + \frac{m}{2}B_2\wedge B_2$. Note that in this picture B_2 is massive. Then, one can jump to the frame on which the degrees of freedom associated to the vector field A_1 decouple from the ones of B_2 by performing the gauge transformation

$$B_2 \to B_2 + \frac{1}{m}dA_1 \equiv \frac{F_2}{m}, \tag{4.18}$$

$$C_3 \to C_3 + B_2\wedge A_1 - \frac{1}{2m}A_1\wedge A_1. \tag{4.19}$$

With this, the action (4.17) becomes, up to a total derivative,

$$\begin{aligned} S_{IIA} = \frac{1}{2\kappa^2}\int d^{10}x(-G)^{1/2}[R - \frac{1}{2}\partial_m\phi\partial^m\phi - \frac{e^{-\phi}}{12}(H_3)^2 - \frac{e^{3\phi/2}}{4}(F_2)^2 - \\ - \frac{e^{\phi/2}}{48}(F_4)^2 - \frac{e^{5\phi/2}}{2}m^2] + \frac{1}{4\kappa^2}\int[B_2\wedge dC_3\wedge dC_3 + \frac{m}{3}B_2\wedge B_2\wedge B_2\wedge dC_3 + \\ + \frac{m^2}{20}B_2\wedge B_2\wedge B_2\wedge B_2\wedge B_2], \end{aligned} \tag{4.20}$$

where

$$H_3 = dB_2, \tag{4.21}$$

$$F_2 = dA_1 + mB_2, \tag{4.22}$$

$$F_4 = dC_3 - H_3\wedge A_1 + \frac{m}{2}B_2\wedge B_2. \tag{4.23}$$

Ordinary Type IIA supergravity then corresponds to setting $m = 0$.

From (4.21)-(4.23) one obtains the sourceless Bianchi identities

$$dH_3 = dm = 0 \quad , \quad dF_2 = mH_3 \quad , \quad dF_4 = F_2 \wedge H_3, \tag{4.24}$$

whereas the rest of the equations of motion may be obtained by applying the usual Euler-Lagrange procedure to eq. (4.20)

$$\begin{aligned} 0 = R_{mn} &- \frac{1}{2}\partial_m\phi\partial_n\phi - \frac{1}{12}e^{\phi/2}F_{mpqr}F_n{}^{pqr} + \frac{1}{128}e^{\phi/2}g_{mn}(F_4)^2 \\ &- \frac{1}{4}e^{-\phi}H_{mpq}H_n{}^{pq} + \frac{1}{48}e^{-\phi}g_{mn}(H_3)^2 \\ &- \frac{1}{2}e^{3\phi/2}F_{mp}F_n{}^p + \frac{1}{32}e^{3\phi/2}g_{mn}(F_2)^2 - \frac{m^2}{16}e^{5\phi/2}g_{mn}, \end{aligned} \tag{4.25}$$

$$0 = \nabla^2\phi - \frac{1}{96}e^{\phi/2}(F_4)^2 + \frac{1}{12}e^{-\phi}(H_3)^2 - \frac{3}{8}e^{3\phi/2}(F_2)^2 - \frac{5}{4}m^2e^{5\phi/2}, \tag{4.26}$$

$$0 = d(e^{-\phi} * H_3) - \frac{1}{2}F_4 \wedge F_4 + e^{\phi/2}F_2 \wedge *F_4 + me^{3\phi/2} * F_2, \tag{4.27}$$

$$0 = d(e^{\phi/2} * F_4) - H_3 \wedge F_4, \tag{4.28}$$

$$0 = e^{\phi/2}H_3 \wedge *F_4 + d(e^{3\phi/2} * F_2). \tag{4.29}$$

Finally, as for Type IIB supergravity, it may result useful to reformulate the theory in terms of the dual field-strengths $*F_4$, $*F_2$ and $*m$. Then, one defines the dual potentials C_5, C_7 and C_9 appearing in the D-brane effective actions through the relations

$$e^{\phi/2} * F_4 = dC_5 + H_3 \wedge C_3 - \frac{m}{6}B_2 \wedge B_2 \wedge B_2, \tag{4.30}$$

$$-e^{3\phi/2} * F_2 = dC_7 + H_3 \wedge C_5 + \frac{m}{24}B_2 \wedge B_2 \wedge B_2 \wedge B_2, \tag{4.31}$$

$$-e^{5\phi/2} * m = dC_9 + H_3 \wedge C_7 - \frac{m}{120}B_2 \wedge B_2 \wedge B_2 \wedge B_2 \wedge B_2. \tag{4.32}$$

4.3 Some representative solutions.

Numerous solutions to the equations of motion of Type IIA and Type IIB supergravity have been worked out in the literature. On this thesis we will be mainly interested in the so called Type B(ecker) solutions of Type IIB supergravity [58, 59, 60, 101] and the corresponding holomorphic monopole mirror solutions of Type IIA supergravity [61]. These represent respectively the low energy behavior of the supersymmetric Type IIB orientifolds with O3/O7-planes and Type IIA orientifolds with O6-planes described in Chapter 3.1. From the analysis of the supergravity limit we will realize about the existence of new degrees of freedom on these orientifolds, consisting on backgrounds for the RR and NS field-strengths along the cycles of the compact manifold. These deform the geometry of the internal Calabi-Yau into more general manifolds with $SU(3)$ structure. Some of these deformations will break spontaneously the supersymmetry to $\mathcal{N} = 0^*$, thus providing us with a controllable source for supersymmetry breaking.

4.3.1 Type B(ecker) solutions.

A general Type IIB background respecting $SO(3,1)$ Poincaré invariance can be written as

$$ds^2 = Z(x^m)^{-1/2}\eta_{\mu\nu}\, dx^\mu dx^\nu + Z(x^m)^{1/2}\, ds^2{}_{CY}, \tag{4.33}$$

$$\tau = \tau(x^m), \tag{4.34}$$

$$G_3 = \frac{1}{3!}G_{lmn}(x^m)\, dx^l dx^m dx^n, \tag{4.35}$$

$$\chi_4 = \chi(x^m)\, dx^0 dx^1 dx^2 dx^3, \tag{4.36}$$

$$F_5 = d\chi_4 + *_{10}\, d\chi_4. \tag{4.37}$$

The ten dimensional complex supersymmetry parameter is then decomposed accordingly to $SO(9,1) \to SO(3,1) \otimes SO(6)$ as

$$\epsilon = \xi \otimes \chi_1 + \xi^* \otimes \chi_2^*, \tag{4.38}$$

so the $\mathcal{N}=1$ supersymmetry preserved by a Type IIB orientifold with O3/O7-planes is given by

$$\epsilon = \xi \otimes \chi_1. \tag{4.39}$$

Note that through the breaking of $\mathcal{N}=4$ to $\mathcal{N}=1$ the orientifold is implicitly selecting a preferred complex structure for the internal manifold.

Due to eq. (4.39), the terms proportional to ϵ and ϵ^* in eqs. (4.1) and (4.2) will be linearly independent and in particular, will vanish separately for SUSY preserving solutions. Moreover, the absence of a fermionic background, which would spoil Poincaré invariance, guarantees the vanishing of the supersymmetric variations for the bosonic fields.

Let us start analyzing the conditions derived from the vanishing of the terms proportional to ϵ in (4.1) and (4.2) [101]. From $\delta\lambda = 0$ one obtains

$$\gamma^m P^*_m \chi_1 = 0, \tag{4.40}$$

and thus the axiodilaton τ is an holomorphic function in the complex structure selected by the orientifold projection.

On the other hand, the vanishing of $\delta\psi_\mu$ implies

$$\partial_\mu \epsilon = 0, \qquad\qquad \chi_4 = Z^{-1}. \tag{4.41}$$

The first of these conditions is trivially satisfied since Poincaré supersymmetries are always independent of x^μ, whereas the second equation establishes a relation between the warping and the 5-form field strength.

Finally, from the vanishing of $\delta\psi_m$ one gets

$$\left(\tilde{D}_m - \frac{i}{2}Q_m\right)(Z^{1/8}\chi_1) = 0, \tag{4.42}$$

where $\tilde{D}_m$ is the covariant derivative for ds^2_{CY}. This is indeed a generalization of the condition for existing a covariantly constant spinor χ_1 in the internal Calabi-Yau manifold. Now the covariant derivative contains an extra piece proportional to Q_m, which is not other but the gauge field associated to the compact $U(1)$ in the $SL(2,\mathbb{R})$ non-perturbative group. With this, the covariantly constant spinor is now given by $Z^{1/8}\chi_1$ and thus the deformed manifold is no longer Calabi-Yau but rather an $SU(3)$ structure manifold with non-trivial torsion.

In fact, due to the effect of the warping, the complex structure J and the holomorphic 3-form Ω are no longer closed forms

$$dJ = \frac{1}{2}Z^{-1}dZ \wedge J, \tag{4.43}$$

$$d\Omega = \frac{3}{4}Z^{-1}dZ \wedge \Omega. \tag{4.44}$$

One may then read the torsion classes from (see e.g. [109, 110, 111])

$$dJ = \frac{3}{2}\mathrm{Im}\ (\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\ , \tag{4.45}$$

$$d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega\ , \tag{4.46}$$

with $\mathcal{W}_1$ a complex 0-form, $\mathcal{W}_2$ a primitive ($\mathcal{W}_2 \wedge J \wedge J = 0$) complex 2-form, $\mathcal{W}_3$ a primitive ($\mathcal{W}_3 \wedge J = 0$) real $(2,1) \oplus (1,2)$-form, $\mathcal{W}_4$ a real 1-form and $\mathcal{W}_5$ a complex (1,0)-form.

Comparing with (4.43) and (4.44) it is easy to see that $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 0$ and $2\mathcal{W}_5^* = 3\mathcal{W}_4 = (3/2)Z^{-1}dZ$ so the backreacted manifold actually corresponds to a conformal Calabi-Yau. The amount of torsion is proportional to the fluxes and, in particular, making use of the relation (4.41) it is possible to re-express F_5 in terms of the torsion as

$$F_5 = 2g_s^{-1}Z^{-1}(1+*)\mathcal{W}_4 \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4, \tag{4.47}$$

in agreement with the results of [112, 113].

Concerning the vanishing of the terms proportional to ϵ^*, one has the conditions

$$G\chi_1 = G_{ijk} = G^j{}_{ij} = 0, \tag{4.48}$$

$$G\chi_1^* = G_{\bar{i}\bar{j}\bar{k}} = G^{\bar{j}}{}_{\bar{i}\bar{j}} = 0, \tag{4.49}$$

$$G\gamma^{\bar{i}}\chi_1^* = G_{\bar{i}\bar{j}k} = 0, \tag{4.50}$$

implying that G_3 is a primitive (2,1)-form ($G_3 \wedge J = 0$).

Up to here, we have analyzed the vanishing of (4.1) and (4.2). This guarantees that the conditions (4.40)-(4.42) and (4.48)-(4.50) correspond to a $\mathcal{N}=1$ solution of the Type IIB supergravity equations of motion. Note that in particular the backreaction of a system of D3 and D7-branes satisfies these conditions. Indeed, this is given by a background of the form (4.33)-(4.37) with [114, 115]

$$ds^2_{CY} = 2(dz^1 d\bar{z}^1 + dz^2 d\bar{z}^2 + e^{-\phi} dz d\bar{z}), \tag{4.51}$$

$$\tau = \frac{i}{g_s} + \frac{1}{2\pi i}\sum_i^{N_7} \ln(z - z_i), \tag{4.52}$$

$$Z = 1 - \frac{1}{2\pi^2}\sum_i^{N_3} \frac{1}{|\vec{x} - \vec{x}_i|^4}, \tag{4.53}$$

$$G_3 = 0, \tag{4.54}$$

and F_5 determined through eq. (4.41). Here, $\vec{x}_i$ are the position vectors for the D3-branes and z_i the positions of the D7-branes. As one would expect, the D7-branes source the axiodilaton τ, whereas the D3-branes source F_5 and Z giving rise to an $AdS_5 \times S^5$ geometry near the $D3$-branes.

It may happen that the complete family of solutions is still more general than the conditions (4.40)-(4.42) and (4.48)-(4.50) and thus involves some possible non-supersymmetric deformations. With this aim, we will analyze in what follows the Type IIB equations of motion [37].

Inserting the metric background (4.33) into the sourced version of eq. (4.10)

$$\tilde{\nabla}^2 Z^{-1} = Z^{-1/2}\frac{G_{mnp}\bar{G}^{mnp}}{12\, Im\,\tau} + Z^{3/2}\Big[\partial_m\chi\partial^m\chi + \partial_m Z^{-1}\partial^m Z^{-1}\Big] + \frac{\kappa^2}{2}Z^{-1/2}(T^m_m - T^\mu_\mu)^{\rm loc}, \tag{4.55}$$

and integrating over the compact Calabi-Yau, we arrive to a no-go theorem: the left hand side vanishes whereas the flux and warp factor terms on the right hand side are positive definite, so there cannot be consistent solutions with fluxes unless there are localized sources with adequate tension.

Indeed, one can compute the stress tensor for a Dp-brane from its Dirac-Born-Infeld (DBI) action. To leading order in α' it results to be

$$(T^m_m - T^\mu_\mu)^{\rm loc} = (7-p)T_p\delta(\Sigma), \tag{4.56}$$

with T_p the tension of the brane and $\delta(\Sigma)$ a delta function Poicaré dual to the $(p-3)$-cycle Σ which the brane wraps. Note how for $p<7$, in order to avoid the above no-go theorem, it is necessary to include localized sources with negative tensions such as orientifold planes. For D7-branes the condition is apparently satisfied without negative sources, however when one computes higher orders in α', one realizes that there are actually instantonic $D3$-brane charges induced in the worldvolume of the $D7$-branes by the fluxes. As it will be shown in Section 6.2.2, these charges are precisely the ones which determine the dynamics of the D7-branes on these orientifold compactifications.

Let us work out now the eq. (4.8). Plugging in our background ansatz one gets

$$\tilde{\nabla}^2\chi = iZ^{-1/2}\frac{G_{mnp}(*_6 G_3^*)^{mnp}}{12\,\mathrm{Im}\,\tau} + 2Z^{3/2}\partial_m\chi\partial^m Z^{-1} + 2\kappa^2 Z^{-1/2}T_3\rho_3^{\mathrm{loc}}, \tag{4.57}$$

and subtracting from eq. (4.55),

$$\tilde{\nabla}^2(Z^{-1}-\chi) = \frac{Z^{-1/2}}{6\,\mathrm{Im}\,\tau}\left|iG_{(3)} - *_6 G_{(3)}\right|^2 + Z^{3/2}|\partial(Z^{-1}-\chi)|^2 + \\ + 2\kappa^2 Z^{-1/2}\left[\frac{1}{4}(T^m_m - T^\mu_\mu)^{\mathrm{loc}} - T_3\rho_3^{\mathrm{loc}}\right]. \tag{4.58}$$

Therefore, making use of eqs. (4.41) and (4.56), we see that G_3 is required to be an imaginary self-dual (ISD) 3-form

$$*_6 G_3 = iG_3. \tag{4.59}$$

This allows for the primitive (2,1)-form of eqs. (4.48)-(4.50), plus a (3,0)-form and a non-primitive (1,2)-form

$$G_{ISD} = G_{(2,1)_P} \oplus G_{(3,0)} \oplus G_{(1,2)_{NP}}. \tag{4.60}$$

The $G_{(1,2)_{NP}}$ piece is actually of the form $J \wedge \eta$, with η a non-trivial closed (0,1)-form. This component thus will be absent in a compact Calabi-Yau orientifold since, as we saw in Section 3.1, there are not such (0,1)-forms in its cohomology. The $G_{(3,0)}$ component on the other hand will be proportional to Ω and in general will be present. Therefore, apart from the $\mathcal{N}=1$ supersymmetric deformations due to $G_{(2,1)_P}$, Type IIB orientifolds with O3/O7-planes always admit a supersymmetry breaking deformation through $G_{(3,0)}$.

Let us finally remark that the 3-form flux carries charge of $D3$-brane which enters in the untwisted tadpole conditions. Indeed, integrating the sourced Bianchi identity for F_5, we obtain

$$dF_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa^2 T_3\rho_3^{\mathrm{loc}} \Rightarrow \frac{1}{2\kappa^2 T_3}\int_{\mathcal{M}_6} H_{(3)} \wedge F_{(3)} \; + \; Q_3^{\mathrm{loc}} = 0. \tag{4.61}$$

Since Q_3^{loc} is an integer, the integral of the fluxes over the 3-cycles of the internal manifold must be integer as well [116]. This is consistent with the Dirac quantization conditions

$$\frac{1}{2\pi\alpha'}\int F_{(3)} \in 2\pi\mathbf{Z}\ , \quad \frac{1}{2\pi\alpha'}\int H_{(3)} \in 2\pi\mathbf{Z}\ . \tag{4.62}$$

Here the flux density depends on the characteristic radius R of compactification as

$$G_3 \propto \frac{\alpha'}{R^3}, \tag{4.63}$$

and thus, for sufficiently large radius or diluted fluxes, the tower of Kaluza-Klein states, which goes as $1/R$, will be much higher than the scale of G_3 and it will be possible to discuss the physics of fluxes by looking at the four dimensional effective theory. This will be the approach of Chapter 5 for getting a deeper understanding of the structure of Type II toroidal orientifolds with non vanishing fluxes.

4.3.2 Monopole solutions.

We have described in Section 3.1 mirror symmetry between Calabi-Yau orientifolds. Now we have seen that the inclusion of background fluxes deforms the internal manifold and the Calabi-Yau condition no longer holds. However, as we will see in next chapter, mirror symmetry is still expected to hold.

On this section we would like to study the mirror Type IIA supergravity configurations dual to the Type B(ecker) solutions described in the previous section. These are related to holomorphic monopole configurations describing the low energy behavior of Type IIA orientifolds with O6-planes and D6-branes. This kind of solutions has been much less studied than their Type IIB counterpart and in particular, a detailed study of its non-supersymmetric deformations is still lacking.

Here we will describe the supersymmetric monopole solutions related to purely geometric backgrounds of M-theory on manifolds with G_2 holonomy [61]. From the point of view of Type IIA String Theory these correspond to non-trivial backgrounds for the metric, the dilaton and the RR 2-form.

A 7-dimensional manifold with G_2 holonomy can be characterized by an invariant 3-form [117]

$$\Phi = \frac{1}{6}\phi_{ABC}\eta^A \wedge \eta^B \wedge \eta^C, \tag{4.64}$$

with $\{\eta^A\}$ the set of tangent 1-forms. From the ten dimensional perspective, this decomposes into the Kähler 2-form J and the holomorphic 3-form Ω as

$$\Omega = \psi_3 - i *_6 \psi_3 \quad , \quad J_{ab} = \phi_{ab7}, \tag{4.65}$$

with $\psi_{abc} = \phi_{abc}$, and a, b, c running from 1 to 6.

Requiring $\mathcal{N} = 1$ supersymmetry in four dimensions imposes the existence of a covariantly constant spinor ϵ on the seven dimensional internal manifold, which can be shown to be equivalent in ten dimensions to the conditions [61]

$$\frac{1}{4}e^{\phi}F_{ab}J^{ab}\gamma\epsilon + \left(\frac{1}{4}e^{\phi}F_{ab}\psi^{ab}{}_{c} - \frac{2}{3}(\partial_a\phi)J^{a}{}_{c}\right)\gamma^{c}\epsilon = 0, \tag{4.66}$$

$$\left(D_a + \frac{i}{6}(\partial_b\phi)J^{b}{}_{a}\gamma\right)\epsilon + i\left(\frac{1}{6}(\partial_b\phi)\psi^{b}{}_{ac} - \frac{1}{4}e^{\phi}F_{ab}J^{b}{}_{c}\right)\gamma^{c}\epsilon = 0, \tag{4.67}$$

with D_a the covariant derivative in the corresponding Type IIA internal manifold.

The first of these conditions gives rise to

$$F^{ab}J_{ab} = 0 \quad , \quad (\partial_a\phi)J^{a}{}_{c} = \frac{3}{8}e^{\phi}F^{ab}(\mathrm{Re}\ \Omega)_{abc}, \tag{4.68}$$

which correspond to some generalized monopole equations. The condition (4.67) then reduces to the condition for the existence of a generalized covariantly constant spinor in the 6-dimensional backreacted manifold

$$\left(D_a + \frac{i}{6}(\partial_b\phi)J^{b}{}_{a}\gamma - \frac{i}{8}e^{\phi}(F_{ab}J^{b}{}_{c} + F_{cb}J^{b}{}_{a})\gamma^{c}\right)\epsilon = 0. \tag{4.69}$$

More generally, it is possible to show that

$$dJ = 0, \qquad d\Omega = \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega, \tag{4.70}$$

with

$$\mathcal{W}_5 = \frac{d\phi}{3}, \qquad \mathcal{W}_2 = -e^{\phi}F^{(1,1)}, \tag{4.71}$$

so the backreacted manifold is a symplectic manifold, and in the particular case on which $F^{(1,1)} = 0$, Kähler.

The simplest example of solution to the constraints (4.68) and (4.69) is given by the backreaction of a stack of D6-branes. This can be written as [114, 115]

$$\begin{aligned} ds^2 &= Z^{-1/2}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + Z^{1/2}dx^{q}dx^{q}, \\ e^{\phi} &= Z^{3/4}, \\ F_{jk} &= -\epsilon_{ijk}\partial^{i}Z, \end{aligned} \tag{4.72}$$

with μ, ν along the worldvolume coordinates and q along the transverse directions.

Note that these are in general configurations with non constant dilaton and therefore there can appear additional dependencies on the transverse coordinates when moving to the Einstein's frame. Thus for example, in terms of the dual potential C_7, the background (4.72) becomes in the Einstein's frame

$$\begin{aligned} ds^2 &= Z^{-1/8}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + Z^{7/8}dx^{q}dx^{q}, \\ e^{\phi} &= Z^{3/4}, \\ C_7 &= -g_s^{3/4}Z^{-1}d\mathrm{Vol}_7, \end{aligned} \tag{4.73}$$

so the powers of the warp factor in the metric are indeed very different to the ones appearing in the string frame.

In the next chapter we will study the low energy effective theory of Type II orientifold compactifications, performing a more systematic analysis of the structure of these solutions and how mirror symmetry is realized among them. As commented in view of eq. (4.63), such approach will be justified in the limit of large compactification radius, on which the topological information of the deformed internal manifold is captured by the effective superpotential.

Chapter 5

Moduli stabilization and $\mathcal{N}=1$ Superpotentials.

We have revealed along the previous chapter how new degrees of freedom consisting on background fluxes for the RR and NSNS field strengths arise in Type II orientifold compactifications. The fluxes deform the geometry and topology of the internal manifold stabilizing some of the moduli of the compactification and, in some cases, breaking the supersymmetry to $\mathcal{N}=0^*$. In this way, flux compactifications address some of the major pathologies described in Chapter 3.

A systematic analysis of the solutions to the supergravity equations of motion associated to these orientifold compactifications is not viable due to its complexity. On this chapter however we will adopt a different approach by performing a systematic analysis of the four dimensional effective theory. Indeed, in the limit of diluted fluxes the Kaluza-Klein replicas are much heavier than the flux induced masses, and thus it is expected that the four dimensional $\mathcal{N}=1$ effective superpotentials will properly describe the low energy physics in terms of the topology of the internal manifold. This will allow us in particular to get some insight into how mirror symmetry is realized in presence of background fluxes.

5.1 Type II toroidal orientifolds with fluxes.

We will start describing the four dimensional effective theories associated to Type IIA and Type IIB String Theory compactified in the toroidal orientifolds of Section 3.1, with constant RR and NSNS background fluxes. The analysis of the induced superpotentials will reveal that new backgrounds have to be taken into account in order to match the vacuum structure of mirror Type IIA and Type IIB orientifolds. By studying this vacuum structure we will be able to get a deeper insight into the nature of the supergravity solutions of Section 4.3 and into the shape of the moduli spaces associated to the backreacted non Calabi-Yau manifolds.

5.1.1 Type IIB orientifolds with O3/O7-planes.

For Type IIB orientifolds with O3/O7-planes, the RR and NSNS 3-forms are odd under the orientifold involution σ. Thus, the most general *constant* background for H_3 and F_3 in the internal manifold is given by

$$\overline{F}_3 = -m\alpha_0 - e_0\beta_0 + \sum_{i=1}^{3}(e_i\alpha_i - q_i\beta_i), \tag{5.1}$$

$$\overline{H}_3 = h_0\beta_0 - \sum_{i=1}^{3} a_i\alpha_i + \overline{h}_0\alpha_0 - \sum_{i=1}^{3}\overline{a}_i\beta_i. \tag{5.2}$$

As already mentioned in Section 4.3.1, this kind of backgrounds induce C_4 tadpoles through the coupling [37]

$$\int C_4 \wedge \overline{H}_3 \wedge \overline{F}_3. \tag{5.3}$$

In terms of the background parameters, then the tadpole cancellation conditions (4.61) read

$$N_{D3} + \frac{1}{2}[mh_0 - e_0\bar{h}_0 + \sum_i (q_i a_i + e_i\bar{a}_i)] = 16, \tag{5.4}$$

where we have assumed the possibility of having N_{D3} D3-branes filling the non compact directions.

The effective four dimensional $\mathcal{N}=1$ physics can be suitably described in terms of the Gukov-Vafa-Witten (GVW) superpotential [18]

$$W = \int_{T^6} (\overline{F}_3 - iS\overline{H}_3) \wedge \Omega, \tag{5.5}$$

whose vacua correspond to the dimensional reduction of the Type B(ecker) solutions discussed in Section 4.3.1. Indeed, requiring $D_{U_i}W = D_SW = 0$ is equivalent to imposing G_3 to be an ISD 3-form, as in eq. (4.59), whereas taking in addition $W=0$ restricts G_3 to a (2,1) primitive form, as depicted in eqs. (4.48) and (4.50) from the vanishing of the supersymmetry variations.

Substituting (5.1) and (5.2) into (5.5) we obtain

$$\begin{aligned} W = e_0 + i\sum_{i=1}^{3} e_iU_i - q_1U_2U_3 - q_2U_1U_3 - q_3U_1U_2 + imU_1U_2U_3 + \\ + S[ih_0 - \sum_{i=1}^{3} a_iU_i + i\bar{a}_1U_2U_3 + i\bar{a}_2U_1U_3 + i\bar{a}_3U_1U_2 - \bar{h}_0U_1U_2U_3]. \end{aligned} \tag{5.6}$$

Note that the superpotential is independent of the Kähler moduli T_i, so these remain undetermined unless additional contributions such as non-perturbative effects are considered (see Appendix D). This particular structure gives rise to no-scale scalar potentials [118]

$$V = e^K \left(\sum_{\Phi=S,T_i,U_i} (\Phi+\Phi^*)^2 |D_\Phi W|^2 - 3|W|^2 \right) = e^K \sum_{\Phi=S,U_i} (\Phi+\Phi^*)^2 |D_\Phi W|^2. \tag{5.7}$$

Indeed, due to the particular form of the Kähler potential (3.31), the F-terms associated to the T_i moduli exactly cancel the negative contribution in the scalar potential and hence the cosmological constant is zero at tree level, even for non-supersymmetric vacua with $\langle F_{T_i}\rangle \neq 0$.

The vacuum structure of the GVW superpotential has been extensively studied in [119] (see as well [120, 121, 122, 123, 124]). There, it was shown that finding supersymmetric vacua for this superpotential is equivalent to solving the attractor equations [125]

$$e_0 + im\mathcal{F}_0 - \sum_j (q_j \mathcal{F}_j - ie_j\tau_j) = 0, \tag{5.8}$$

$$h_0 + i\bar{h}_0\mathcal{F}_0 + \sum_j (\bar{a}_j \mathcal{F}_j + ia_i\tau_j) = 0, \tag{5.9}$$

$$(m+\phi\bar{h}_0)\mathcal{F}_j + i\sum_{i,k} \epsilon_{ijk}(q_i + \phi\bar{a}_i)\tau_k + (e_j + \phi a_j) = 0. \tag{5.10}$$

Here we will present just a simple example given by [43]

$$W = e_0 + ih_0 S + i\sum_i e_i T_i - S(a_2T_2 + a_3T_3) - T_1(q_2T_3 + q_3T_2). \tag{5.11}$$

Then, the existence of supersymmetric Minkowski vacua requires $a_2q_2 - a_3q_3 = 0$, $e_2a_3 - e_3a_2 = 0$, $h_0q_2 - a_3e_1 = 0$ and $h_0e_2 - e_0a_2 = 0$. And neither the imaginary nor the real parts of the moduli are fully determined, only

$$ih_0 = a_2U_2 + a_3U_3, \qquad ie_2 = a_2S + a_3U_1. \tag{5.12}$$

For Re $S > 0$ and Re $U_i > 0$ one must have $q_2q_3 < 0$ and $a_2q_2 > 0$ so the contribution to the RR tadpoles is positive. Indeed, this is a general feature for all these vacua.

The other kind of vacua in (5.7) are non-supersymmetric vacua with vanishing cosmological constant. In that case G_3 has at the minimum a non trivial (3,0) component, as outlined in Section 4.3.1. Let us consider for example (5.6) with $\bar{h}_0 = \bar{a}_i = a_1 = m = 0$. Then Re $U_2 \neq 0$ and Re $U_3 \neq 0$ requires $q_1 = 0$ and from $D_SW = D_{U_i}W = 0$ one finds that the axions are fully determined to be

$$\text{Im } S = \frac{e_2q_2 - e_3q_3}{q_2a_2 - q_3a_3}, \qquad \text{Im } U_1 = \frac{e_3a_2 - e_2a_3}{q_2a_2 - q_3a_3}, \tag{5.13}$$
$$\text{Im } U_2 = \frac{h_0q_2 - e_1a_3}{q_2a_2 - q_3a_3}, \qquad \text{Im } U_3 = \frac{e_1a_3 - h_0q_3}{q_2a_2 - q_3a_3}.$$

There is a further relation

$$e_0 = h_0 \text{Im } S + e_1 \text{Im } U_1 . \tag{5.14}$$

Concerning the real parts, these verify $q_2 u_1 u_3 = a_2 s u_2$ and $q_3 u_1 u_2 = a_3 s u_3$, with $s \equiv \text{Re } S$ and $u_i \equiv \text{Re } U_i$. Thus we must have $a_2 q_2 > 0$ and $a_3 q_3 > 0$, indicating again a positive contribution to the RR tadpoles. On the other hand, only pairwise ratios of real moduli are fixed, namely

$$s^2 = \frac{q_2 q_3}{a_2 a_3}(u_1)^2, \qquad (u_3)^2 = \frac{a_2 q_3}{a_3 q_2}(u_2)^2 \ . \tag{5.15}$$

In a variant of this model one can further set $a_2 = 0$ and for consistency $q_2 = 0$. The imaginary parts are obtained substituting these values in (5.13). For the real parts it follows that $a_3 s u_3 = q_3 u_1 u_2$.

A second example of no-scale vacuum can be given by eq. (5.6) with $m \neq 0$, but still $\bar{h}_0 = \bar{a}_i = a_1 = 0$. In that case one finds that in order to have a solution the fluxes must verify

$$\gamma_2 = \frac{a_2 \gamma_3}{a_3}, \qquad h_0 \gamma_3 = a_3(e_1 q_1 + m e_0), \tag{5.16}$$

with $\gamma_i \equiv m e_i + q_j q_k$ $(i \neq j \neq k)$.

For the imaginary parts we obtain

$$\begin{aligned} \text{Im } S &= \frac{m e_0 + q_1 e_1}{m h_0}, &\qquad \text{Im } U_1 &= -\frac{q_1}{m}, \\ \text{Im } U_2 &= -\frac{q_2}{m} + \frac{a_2 s u_2}{m u_1 u_3}, &\qquad \text{Im } U_3 &= -\frac{q_3}{m} + \frac{a_3 s u_3}{m u_1 u_2}. \end{aligned} \tag{5.17}$$

The real parts instead satisfy

$$a_2 a_3 s^2 = \gamma_1 (u_1)^2, \qquad (m u_1 u_2 u_3)^2 + (a_2 s u_2)^2 + (a_3 s u_3)^2 = (h_0 m + a_2 q_2 + a_3 q_3) s u_1 u_2 u_3 . \tag{5.18}$$

This shows that $(h_0 m + a_2 q_2 + a_3 q_3) > 0$ and hence the flux contribution to tadpoles is again positive. Notice that the above solution simplifies upon taking $a_2 = 0$ which is consistent if $\gamma_1 = \gamma_2 = 0$. In this case

$$u_3 = \frac{(h_0 m + q_3 a_3) s u_1 u_2}{(a_3 s)^2 + (m u_1 u_2)^2} . \tag{5.19}$$

We also find that at the minimum

$$W_0 = -\frac{2(h_0 m + q_3 a_3) s u_1 u_2}{a_3 s + i m u_1 u_2}, \tag{5.20}$$

and the gravitino mass turns out to be

$$m_{3/2}^2 = \frac{(h_0 m + q_3 a_3)}{32 t_1 t_2 t_3} \tag{5.21}$$

which, as expected for a no-scale model, only depends on $t_i \equiv \text{Re } T_i$.

5.1.2 Type IIA orientifolds with O6-planes.

Type IIA orientifolds in presence of fluxes has not been considered until recently [30, 31, 32, 33, 34, 43, 57, 36]. In this case H_3, F_2 and F_6 are odd under the orientifold involution σ, whereas m and F_4 are even. Thus, the most general constant background in the internal manifold will be

$$
\begin{aligned}
\overline{H}_3 &= \sum_{L=0}^{3} h_L \beta_L \ ; \\
\overline{F}_0 &= -m \quad ; \quad \overline{F}_2 = \sum_{i=1}^{3} q_i \omega_i \quad ; \quad \overline{F}_4 = \sum_{i=1}^{4} e_i \tilde{\omega}_i \quad ; \quad \overline{F}_6 = e_0 \alpha_0 \wedge \beta_0 \ .
\end{aligned}
\tag{5.22}
$$

The integrals of the fluxes over the corresponding p-cycles are quantized [116], in a similar fashion to (4.62) for Type IIB fluxes. More concretely,

$$
\frac{\ell^3 \mu_1}{2\pi} \int_{\Pi_3} \overline{H}_3 \in \mathbb{Z}, \qquad \frac{\ell^p \mu_{p-2}}{2\pi} \int_{\Pi_p} \overline{F}_p \in \mathbb{Z}, \tag{5.23}
$$

with $\ell = 2\pi\sqrt{\alpha'}$ and $\mu_p = 1/(2\pi)^p \alpha'^{(p+1)/2}$ [68, 126]. Thus, we will take the cohomology basis to be an integer basis, so that in units of $2\pi/\mu_{p-2}\ell^p = 1/\ell$ the coefficients in the above expansions are integers. With this, all the forms have dimensions of (length)$^{-1}$ and the moduli fields are dimensionless. To avoid subtleties with exotic orientifold planes [37, 119], in addition we will take the flux integers to be even.

Due to the piece of the Type IIA action

$$
\int C_7 \wedge m\overline{H}_3, \tag{5.24}
$$

the $\overline{H}_3$ fluxes will induce a non-trivial tadpole for C_7 in massive supergravity [127, 30, 33, 34]. Allowing for the presence of D6-branes wrapping 3-cycles Π_a, the cancellation conditions read

$$
\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2} m h_0 = 16, \tag{5.25}
$$

$$
\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2} m h_1 = 0, \tag{5.26}
$$

$$
\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2} m h_2 = 0, \tag{5.27}
$$

$$
\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2} m h_3 = 0. \tag{5.28}
$$

In an orbifold setup there would be additional O6-planes contributing negatively to the right hand side of (5.26)-(5.28).

The four dimensional effective action has been computed in [31] by dimensional reduction of Type IIA supergravity. More concretely, for setups of D6-branes preserving $\mathcal{N}=1$

supersymmetry in four dimensions, it can be recast in terms of the effective superpotential

$$W = \int_{T^6} (\Omega_c \wedge \overline{H}_3 + e^{J_c} \wedge \overline{F}_{RR}), \tag{5.29}$$

where $\overline{F}_{RR}$ represents a formal sum of the even RR fluxes. In terms of the flux parameters, this reads

$$W = e_0 + ih_0 S + i\sum_{i=1}^{3}(e_i T_i - h_i U_i) - q_1 T_2 T_3 - q_2 T_1 T_3 - q_3 T_1 T_2 + imT_1T_2T_3. \tag{5.30}$$

Note that there is dependence on both the Kähler and the complex structure moduli. This is actually not very surprising, since for this case the background involves both the even and the odd dimensional cycles of the internal manifold.

In [43] we extensively explored the vacuum structure of (5.30). For simplicity, here we will consider isotropic fluxes with $q_1 = q_2 = q_3 \equiv -c_2$ and $e_1 = e_2 = e_3 \equiv c_1$, so $T_1 = T_2 = T_3 = T$ is a solution to the supergravity equations of motion.

We will look first for supersymmetric vacua with $D_i W = 0$. Consistent vacua with Re $U_i \neq 0$, Re $T \neq 0$ and Re $S \neq 0$ requires $W \neq 0$. Thus, the only supersymmetric vacua for this superpotential are AdS$_4$ vacua. These were recently addressed in [34]. We obtain similar results. Indeed, in that case, the extremum only fixes one linear combination of the imaginary parts of the dilaton and complex structure moduli, given by

$$h_0 \mathrm{Im}\ S - \sum_{k=1}^{3} h_k \mathrm{Im}\ U_k = e_0 - 3c_1 v - 3c_2 v^2 + mv^3, \tag{5.31}$$

with $v \equiv \mathrm{Im}\ T$. This can be taken as part of a theorem: if a superpotential has only a linear dependence in a subset of moduli, then the supersymmetric minima are such that in AdS the corresponding axions remain undetermined but a linear combination, whereas in Minkowski both the real and imaginary parts are undetermined but a linear combination. The proof is immediate once one considers the dependence of W, K and D_i on the moduli. In Section 7.1 we will see that the presence of flat directions for the axions is related with the absence of flux induced anomalies in the worldvolume of the D6-branes.

Concerning v, one finds $v = c_2/m$ and the real parts are fixed to

$$m^2 t^2 = -\frac{5}{3}\gamma, \qquad h_0 s = -h_k u_k = \frac{2}{5} m t^3 \qquad k = 1, 2, 3. \tag{5.32}$$

To go beyond supersymmetric minima, let us look for solutions to $\partial_i V$ [43]. Then one finds two branches for v

$$v_s = \frac{c_2}{m}, \qquad v_{ns} = \frac{c_2 \pm \sqrt{\gamma - m^2 t^2/2}}{m}, \tag{5.33}$$

with $t \equiv \text{Re } T$ and $\gamma \equiv mc_1 + c_2^2$. For each value of v there are various sub-branches according to the relation among the real parts of S and U_k. We will concentrate on the simplest case with $h_1u_1 = h_2u_2 = h_3u_3$. In this case there are two sub-branches characterized by

$$\begin{aligned} (I) : h_k u_k &= -h_0 s \qquad k = 1, 2, 3, \\ (II) : h_k u_k &= h_0 - mt^3. \end{aligned} \tag{5.34}$$

In the v_s sub-branch I,

$$m^2t^2 = \pm\frac{5}{3}\gamma \quad , \quad h_0 s = \frac{2}{5}mt^3, \tag{5.35}$$

and the cosmological constant is

$$\Lambda_s = -\frac{\gamma^2}{24m^2 s u_1 u_2 u_3 t}. \tag{5.36}$$

For $\gamma < 0$ this is the AdS supersymmetric vacuum, whereas for $\gamma > 0$ it is a non-supersymmetric AdS extremum with same data for the moduli and the cosmological constant. The flux contribution to the tadpoles has opposite sign to the one of the orientifold planes, since $mh_0 > 0$ and $mh_k < 0$.

In the v_s sub-branch II

$$m^2t^2 = \pm\frac{5}{\sqrt{6}}\gamma \quad ; \quad h_0 s = \frac{4}{5}mt^3 \quad ; \quad \frac{\Lambda}{\Lambda_s} = \frac{32}{27}\left(\frac{6}{9}\right)^{1/4} \sim 1.071.$$

Both $\gamma < 0$ and $\gamma > 0$ are allowed. In either case it is a non-supersymmetric AdS extremum.

The v_{ns} branch can occur only if $\gamma > 0$. In that case there are two non-supersymmetric AdS sub-branches according to (5.34). Their data are

$$\begin{aligned} (I) : m^2t^2 &= \frac{4}{3}\gamma, \quad h_0 s = \frac{2\gamma t}{3m}, \quad \frac{\Lambda}{\Lambda_s} = \frac{25\sqrt{5}}{48} \sim 1.165 \\ (II) : m^2t^2 &= \frac{196}{99}\gamma, \quad h_0 s = \frac{14\gamma t}{9m}, \quad \frac{\Lambda}{\Lambda_s} = \frac{11^4 5^2 3^2\sqrt{55}}{2^4 7^7\sqrt{3}} \sim 1.070 \end{aligned}$$

Note that in all these examples the fixed moduli scale with respect to the RR 4-form and 2-form fluxes c_1 and c_2 as

$$t \;\simeq\; s^{1/3} \;\simeq\; u_k^{1/3} \;\simeq\; \gamma^{1/2} \;\simeq\; c_1^{1/2} \;,\; c_2 \;, \tag{5.37}$$

for large fluxes. Thus the compactification volume can be made arbitrarily large for large c_1 and/or c_2. Concerning the four- and ten-dimensional dilatons, one has

$$e^{\phi_4} \;\simeq\; c_1^{-3/2} \;,\; c_2^{-3} \;\; ; \;\; e^{\phi} \;\simeq\; c_1^{-3/4} \;,\; c_2^{-3/2} \tag{5.38}$$

and the vacua lie in a perturbative regime for sufficiently large RR 4-form and/or 2-form fluxes. Finally, for the cosmological constant one can check that for large c_1 and c_2 it scales as

$$\Lambda \simeq \; - \, \gamma^{-9/2} \;\simeq\; -c_1^{-9/2} \;,\; -c_2^{-9} \;. \tag{5.39}$$

Thus, for large fluxes the cosmological constant goes with the string dilaton like $e^{6\phi}$. The density of RR fluxes is also suppressed. As pointed out in [34], to compute this density a factor of $g_s = e^{\phi}$ must be included. Then, the flux density of $\overline{F}_4$ ($\overline{F}_2$) behaves like $c_1^{-3/2}$ (c_2^{-3}) for large c_1 (c_2) fluxes.

Apart from the supersymmetric AdS vacua, the superpotential (5.30) as well contains some no-scale models dual to the ones discussed in the previous section for the Gukov-Vafa-Witten superpotential. Indeed, taking $h_i = 0$ the superpotential (5.30) becomes independent of the complex structure moduli U_i. In that case, the axions are determined by

$$\text{Im } T = \frac{c_2}{m}, \qquad h_0 \text{Im S} = e_0 + \frac{c_2^3}{m^2}, \tag{5.40}$$

whereas for the real parts

$$h_0 s = m t^3.$$

Moreover, it is required the conditions $\gamma = 0$ and $h_0 m > 0$, so the flux contribution to the tadpoles is positive.

Note that the superpotentials (5.6) and (5.30) do not completely match under mirror symmetry, finding in this way a different landscape of vacua for each one and revealing the existence of new possible deformations of the Type B(ecker) and monopole solutions discussed in Section 4.3. The underlying reason is that up to here we have only considered *constant* backgrounds for the vielbeins and the RR and NSNS forms. In the following section we will slightly generalize this to include some new geometric deformations consisting on non constant backgrounds for the metric. This will be a first step into matching the Type IIA and Type IIB superpotentials.

5.2 Metric fluxes and twisted tori.

5.2.1 Geometry and topology of the twisted torus.

Let us consider here the possible metric deformations of the factorized torus giving rise to parallelizable manifolds with a globally well defined basis of tangent 1-forms η^i

$$\eta^i = N_n{}^m(x) dx^n \quad ; \quad dx^n = N^n{}_m(x)\eta^m. \tag{5.41}$$

Such kind of backgrounds appear naturally in the context of Scherk-Schwarz dimensional reductions [38] and can be shown to be equivalent to compactification on a twisted torus [39] with

$$d\eta^p = -\frac{1}{2}\omega^p_{mn}\eta^m \wedge \eta^n, \tag{5.42}$$

where ω^p_{mn} are the metric fluxes we are interested in. These are constant coefficients antisymmetric in the lower indices.

Defining the isometry generators as

$$Z_m = N^n{}_m \partial_n, \tag{5.43}$$

the metric fluxes can be visualized as the gauging of some of the isometries of the original torus. Indeed, the Lie algebra generated by $\{Z_m\}$ is given by

$$[Z_m, Z_n] = \omega^p_{mn} Z_p, \tag{5.44}$$

so eq. (5.42) corresponds to the Maurer-Cartan equation of the algebra.

Either from the Jacobi identity of the algebra or from the Bianchi identity of (5.42) one then finds the constraint

$$\omega^p_{[mn}\omega^s_{r]p} = 0, \tag{5.45}$$

which guarantees the nilpotency of $d^2 = 0$ in the cohomology of the twisted torus. In addition, it can be further shown that $\omega^p_{pn} = 0$ [38] and Poincaré lemma continues to be valid [128].

Twisted tori are manifolds with torsion. Indeed, defining a general p-form (a p-cochain) as

$$A^{[p]} = \frac{1}{p!} A_{i_1 i_2 \ldots i_p} \eta^{i_1} \wedge \eta^{i_2} \wedge \ldots \wedge \eta^{i_p}, \tag{5.46}$$

one has that

$$dA^{[p]} = \frac{1}{(p+1)!}[(p+1)\partial_{[i_i} A_{i_2 \ldots i_{p+1}]} + \omega^k_{[i_1 i_2} A_{i_3 \ldots i_{p+1}]k}]\eta^{i_1} \wedge \eta^{i_2} \wedge \ldots \wedge \eta^{i_{p+1}}. \tag{5.47}$$

This can be expressed as

$$dA^{[p]} \equiv \tilde{d}A^{[p]} + \omega A^{[p]}, \tag{5.48}$$

where $\tilde{d}A^{[p]}$ is the exterior derivative of $A^{[p]}$ in an ordinary torus, without metric fluxes, and $\omega A^{[p]}$ is a torsional piece coming from the gauging of the isometries. Due to this, formally one can work in a twisted torus as being in an ordinary torus with some extra fluxes ω^i_{jk}.

In general, the (co)homology of the twisted torus will be smaller than the one of the ordinary torus, since some of the original cycles become homologically trivial in order the tangent 1-forms to continue being globally well defined [47, 128]. More concretely, the space C^p of $A^{[p]}$ forms with constant coefficients can be split into[1]

$$C^p = \Gamma^p \oplus \partial \Xi^{p-1} \oplus \Xi^p, \tag{5.49}$$

[1] The cohomological classification of the harmonic forms on a twisted torus is known by the mathematicians as Chevalley's cohomology.

where Γ^p are closed p-forms which are not exact, Ξ^p are non-closed p-forms and $\partial\Xi^{p-1}$ are closed p-forms which are exact and which are associated to the torsional cycles of the twisted torus. Due to eq. (5.47) there will exist a one to one mapping between the elements of C^p and the harmonic p-forms in an ordinary torus. Thus,

$$\begin{aligned}
\dim\, C^p &= \dim\, \Gamma^p + \dim\, \partial\Xi^{p-1} + \dim\, \Xi^p = b^p,\\
\dim\, \partial\Xi^p &= \dim\, \Xi^p,\\
\dim\, \Gamma^{6-p} &= \dim\, \Gamma^p,\\
\dim\, \Gamma^p &\le b^p,
\end{aligned}$$

with b^p the Betti numbers for an ordinary torus and the last equation being saturated in the case of vanishing metric fluxes.

In what follows, let us consider the factorized toroidal orientifold of Section 3.1. For Type IIB orientifolds with O3/O7-planes, the metric fluxes do not survive to the orientifold projection so we will concentrate exclusively on the Type IIA picture with O6-planes. In that case, the ω^i_{jk} parameters are even under the orientifold projection and there are twelve parameters surviving and respecting the factorability of the torus

$$\mathcal{M} \equiv \begin{pmatrix} -a_1 & -a_2 & -a_3 \\ b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{pmatrix} = \begin{pmatrix} -\omega^1_{56} & -\omega^2_{64} & -\omega^3_{45} \\ -\omega^1_{23} & \omega^5_{34} & \omega^6_{42} \\ \omega^4_{53} & -\omega^2_{31} & \omega^6_{15} \\ \omega^4_{26} & \omega^5_{61} & -\omega^3_{12} \end{pmatrix}. \tag{5.50}$$

The Jacobi identities (5.45) then imply the constraints

$$b_{ij}a_j + b_{jj}a_i = 0 \qquad i \neq j, \tag{5.51}$$

$$b_{ik}b_{kj} + b_{kk}b_{ij} = 0 \qquad i \neq j \neq k. \tag{5.52}$$

There are some obvious solutions to these constraints. For instance [43], (1): $b_{ij} = 0,\ \forall i, j$; (2): $a_i = 0,\ b_{ij} = b_i\delta_{ij}$; (3): $a_i = a,\ b_{ij} = b,\ i \neq j,\ b_{ii} = -b$.

This kind of geometric deformations very often survive to the action of an orbifold group, such as the $Z_2 \times Z_2$ example of Section 3.1, even though this kills all the original isometries of the torus. Thus, it is not surprising to have similar deformations in more generic manifolds.

The cohomology of the twisted torus can be easily computed by acting with the exterior derivative (5.48). Thus, a (1,1)-form $\Theta = \sum_i^3 \Theta_i\omega_i$ belonging to Γ^2 will satisfy

$$(d\Theta)_{lmn} = \omega^p_{[lm}\Theta_{n]p} = 0. \tag{5.53}$$

Substituting eq. (5.50) and operating one has

$$\mathcal{M}\cdot \begin{pmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{pmatrix} = 0. \tag{5.54}$$

Similarly, for $C^3 = \Gamma^3 \oplus \Xi^3 \oplus \partial\Xi^2$ one has that a generic 3-form $\Pi = \sum_{i=0}^3 (\Pi_i \alpha_i + \tilde{\Pi}_i \beta_i)$ in $\Gamma^3 \oplus \partial\Xi^2$ satisfies

$$(d\Pi)_{lmno} = \omega^p_{[lm} \Pi_{no]p} = 0, \tag{5.55}$$

which can be shown to be equivalent to

$$\mathcal{M} \cdot \begin{pmatrix} \Pi_0 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{pmatrix} = 0, \tag{5.56}$$

with $\tilde{\Pi}_i$ unconstrained. Now, since we have computed Ξ^2 in the previous paragraph, it is immediate to distinguish Γ^3 from $\partial\Xi^2$. In particular, it happens that Poincaré duality always relate elements of Ξ^p with the corresponding symplectic partners in $\partial\Xi^{5-p}$. This reveals the importance of having the geometric deformations preserving the symplectic structure of the 3-cycles.

	Γ^p	Ξ^p	$\partial\Xi^{p-1}$
$p=2$	$(\text{Ker } \mathcal{M})_{\{\omega_i\}}$	$(\text{Ker } \mathcal{M})^\perp_{\{\omega_i\}}$	0
$p=3$	$(\text{Ker } \mathcal{M}^T)_{\{\alpha_i,\beta_i\}}$	$(\text{Ker } \mathcal{M}^T)^\perp_{\{\alpha_i\}}$	$(\text{Ker } \mathcal{M}^T)^\perp_{\{\beta_i\}}$
$p=4$	$(\text{Ker } \mathcal{M})_{\{\tilde{\omega}_i\}}$	0	$(\text{Ker } \mathcal{M})^\perp_{\{\tilde{\omega}_i\}}$

Table 5.1: Reduced cohomology of a factorized twisted 6-torus.

We have summarized in Table 5.1 the reduced cohomology of a factorized twisted torus. For the dimensions of the cohomology groups one has

$$\dim\, \Xi^2 = \dim\, \Xi^3 = \dim\, \partial\Xi^2 = \dim\, \partial\Xi^3 = 3 - \dim\, \Gamma^2 = 4 - (\dim\, \Gamma^3/2) = \text{rank}\, \mathcal{M}. \tag{5.57}$$

The cohomology of the twisted tori will play a crucial role in the understanding of which are the moduli stabilized by the metric fluxes and the consistency conditions for D6-branes in presence of fluxes, as revealed in [129].

The torsion of the factorized twisted torus can be suitably described in terms of the exterior derivative of the holomorphic and the Kähler forms [43]

$$d\Omega = \frac{1}{\text{Re } S} \sum_i (a_i \text{Re } S + \sum_j b_{ij} \text{Re } U_j) \tilde{\omega}_i, \tag{5.58}$$

$$dJ = \sum_i (a_i \beta_0 - \sum_j b_{ij} \beta_j) \text{Re } T_i. \tag{5.59}$$

Clearly, $d(J \wedge J) = d(\text{Im } \Omega) = 0$ so the twisted torus is a particular case of half-flat manifold. Moreover, comparing with eqs. (4.45) and (4.46) one easily reads $\mathcal{W}_4 = \mathcal{W}_5 = 0$.

It is enlightening to see how the twisted torus structure arises by mirror symmetry of Type IIB O3/O7 orientifolds with non vanishing NSNS 3-form flux [109, 47]. Concretely, one can start from a Type IIB background

$$ds^2 = \sum_{i=1}^{6}(dx^i)^2, \qquad \overline{H}_3 = -\sum_{i=1}^{3} a_i \alpha_i, \tag{5.60}$$

and perform three T-dualities in x^1, x^2, x^3, accordingly to what we saw in Chapter 3.1. Taking

$$B_2 = -a_1 x^6 dx^1 \wedge dx^5 - a_2 x^4 dx^2 \wedge dx^6 - a_3 x^5 dx^3 \wedge dx^4, \tag{5.61}$$

and using the Buscher rules [81, 82], gives the Type IIA metric

$$ds^2 = (dx^1 + a_1 x^6 dx^5)^2 + (dx^2 + a_2 x^4 dx^6)^2 + (dx^3 + a_3 x^5 dx^4)^2 + (dx^4)^2 + (dx^5)^2 + (dx^6)^2,$$

from where we read off the following tangent 1-forms

$$\begin{aligned} \eta^1 &= dx^1 + a_1 x^6 dx^5, & \eta^4 &= dx^4, \\ \eta^2 &= dx^2 + a_2 x^4 dx^6, & \eta^5 &= dx^5, \\ \eta^3 &= dx^3 + a_3 x^5 dx^4, & \eta^6 &= dx^6, \end{aligned}$$

and $\omega^1_{56} = a_1, \omega^2_{64} = a_2$ and $\omega^3_{45} = a_3$. Thus, we can formally write the following T-duality rule for the integrated fluxes

$$\overline{H}_{mnp} \xleftrightarrow{T_m} -\omega^m_{np}, \tag{5.62}$$

which complement eqs. (3.32) and (3.33).

Note that metric fluxes are also quantized. For the a_i fluxes this is obvious from the mirror symmetry argument we have just seen. For a more general argument, we refer the reader to [130].

5.2.2 Effective superpotential and vacuum structure.

Having described the geometry and topology of the twisted torus, now let us turn into the analysis of the corresponding four dimensional $\mathcal{N}=1$ effective superpotential. This will provide us with the low energy dynamics associated to the deformations of the ordinary torus into a twisted torus. Along this section, we will follow our work in [43].

The superpotential has been computed by different methods in [33] and [32], resulting to be

$$W = \int (e^{J_c} \wedge \overline{F}_{RR} + \Omega_c \wedge (\overline{H}_3 + dJ_c)), \tag{5.63}$$

which in terms of the background parameters reads

$$W = e_0 + ih_0S + i\sum_{i=1}^{3}(e_iT_i - h_iU_i) - q_1T_2T_3 - q_2T_1T_3 - q_3T_1T_2 + imT_1T_2T_3 - \\ - \sum_{i=1}^{3}(a_iS + \sum_{j=1}^{3} b_{ij}U_j)T_i, \quad (5.64)$$

where the last row corresponds to the terms induced by the metric fluxes. Note that the a_i terms were already present in the Type IIB superpotential (5.6), as expected from mirror symmetry.

Integrability of the H_3 Bianchi identity (4.24) gives rise to the additional constraint [33]

$$\omega \overline{H}_3 = 0, \quad (5.65)$$

which is trivially satisfied by the fluxes (5.22) and (5.50).

The metric fluxes will contribute to the C_7 tadpoles through [33, 43]

$$\int C_7 \wedge (m\overline{H}_3 + d\overline{F}_2), \quad (5.66)$$

so the tadpole cancellation conditions (5.25)-(5.28) in the presence of metric fluxes will become

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}(mh_0 + a_1q_1 + a_2q_2 + a_3q_3) = 16, \quad (5.67)$$

$$\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}(mh_1 - q_1b_{11} - q_2b_{21} - q_3b_{31}) = 0, \quad (5.68)$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}(mh_2 - q_1b_{12} - q_2b_{22} - q_3b_{32}) = 0, \quad (5.69)$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}(mh_3 - q_1b_{13} - q_2b_{23} - q_3b_{33}) = 0. \quad (5.70)$$

We extensively studied the vacuum structure of the low energy scalar potential induced by (5.64) in [43], revealing the existence of additional branches in the mirror Type B(ecker) solutions of Section 4.3.1.

Supersymmetric Minkowski vacua.

Considering the presence of non trivial metric fluxes, it turns out that it is possible to find consistent supersymmetric Minkowski vacua for Type IIA orientifolds with O6-planes. In that case, the superpotential is required to be dependent on four or more moduli. With four fields, it is enough to study in detail $W = W(S, T_1, T_2, T_3)$ independent of the U_i. Note that this is exactly the piece of the superpotential dual to the Gukov-Vafa-Witten superpotential discussed in Section 5.1.1, and thus the vacuum structure will be the same, after the exchanging $U_i \leftrightarrow T_i$. In this context, the vacua will correspond to the holomorphic monopole solutions of Section 4.3.2. Thus for example, for the case of (5.11) one can easily compute the vacuum expectation value for H_3 and dJ at the minimum in the mirror Type IIA picture, resulting to be

$$\langle H_3 \rangle = \overline{H}_3 - (a_2 \text{Im } T_2 + a_3 \text{Im } T_3)\beta_0 = 0, \tag{5.71}$$

$$\langle dJ \rangle = (a_2 \text{Re } T_2 + a_3 \text{Re } T_3)\beta_0 = 0, \tag{5.72}$$

in agreement with the results of [112, 113] for holomorphic monopole solutions. Moreover, part of the moduli stabilization process can be understood in terms of the topology of the induced twisted torus. Thus, for this particular example a suitable cohomology basis for the twisted torus engendered by a_2 and a_3 is given by

	Γ^p	Ξ^p	$\partial\Xi^{p-1}$
$p=2$	$\{\omega_1, a_2\omega_3 - a_3\omega_2\}$	$\{a_3\omega_2 + a_2\omega_3\}$	0
$p=3$	$\{\alpha_1, \alpha_2, \alpha_3, \beta_1\beta_2, \beta_3\}$	$\{\alpha_0\}$	$\{\beta_0\}$
$p=4$	$\{\tilde{\omega}_1, a_2\tilde{\omega}_3 - a_3\tilde{\omega}_2\}$	0	$\{a_3\tilde{\omega}_2 + a_2\tilde{\omega}_3\}$

and therefore, $\mathcal{T} \equiv a_3T_3 + a_2T_2$ and S are no longer moduli fields in presence of the metric fluxes a_2, a_3, in agreement with the Type IIB results (c.f. eq. (5.12)).

However, not every supersymmetric Minkowski vacua of (5.64) corresponds to the dual of a Minkowski vacua engendered by the Gukov-Vafa-Witten superpotential, signaling in this way the existence of additional branches in the Type B(ecker) solutions of Section 4.3.1. For example, one can consider the case by

$$W = -T_2(a_2S + b_{21}U_1) - T_3(a_3S + b_{31}U_1) + e_0 + ih_0S - ih_1U_1 + ie_2T_2 + ie_3T_3. \tag{5.73}$$

This is clearly equivalent to (5.11) after renaming $U_2 \to T_2$, $U_3 \to T_3$, $e_1 \to -h_1$, $q_2 \to b_{31}$ and $q_3 \to b_{21}$. The physics is however different. In particular, since all $q_i = 0$ and $m = 0$, the fluxes do not contribute at all to the RR tadpoles. Therefore, this is an example in which one can fix moduli without affecting tadpoles.

Finally, one can consider cases on which the superpotential depends on five moduli. For example, consider the superpotential

$$W = -(q_3T_2 + q_2T_3)T_1 - (b_{22}T_2 + b_{32}T_3)U_2 - (b_{23}T_2 + b_{33}T_3)U_3. \tag{5.74}$$

Observe that the non-zero b_{ij} trivially satisfy the constraints (5.52). If the fluxes satisfy $q_2b_{22} = q_3b_{32}$, $q_2b_{23} = q_3b_{33}$ and $q_2q_3 < 0$, there is a solution with $|q_2|t_3 = |q_3|t_2$. There is also a relation $-q_3t_1 = b_{22}u_2 + b_{23}u_3$. To have $t_1 > 0$ for any $u_2, u_3 > 0$, we need $q_2b_{22} > 0$ and $q_2b_{23} > 0$. Hence, the flux piece in (5.69) and (5.70) is negative (same as D6-branes).

As already commented, there cannot be supersymmetric Minkowski solutions when W depends on more than five fields. To see this, first observe that without loss of generality we can always take the three T_i to be among the fields in W. Next, $\partial_{U_i}W = \partial_S W = 0$ implies $\{\text{Re } T_i\} = \text{Ker } \mathcal{M}$, in agreement with Table 5.1. Thus, to have solutions with Re $T_i \neq 0$ it is required rank $\mathcal{M} \leq 2$. After using the Jacobi identities one is left with rank $\mathcal{M} = 1$. One can check then that the number of fields in W is at most five.

No-scale models.

Something similar occurs for the no-scale vacua. Some of these correspond to the mirrors of no-scale vacua engendered by the Gukov-Vafa-Witten superpotential. For example, the no-scale models discussed in Section 5.1.1 are as well contained in (5.64) upon simply taking $U_i \to T_i$. Thus, there are non-supersymmetric branches for the holomorphic monopole solutions of Section 4.3.2 consisting on the inclusion of geometric backgrounds.

Apart from the no-scale vacua dual to the ones engendered by the GVW superpotential, there are additional no-scale vacua for superpotentials depending on the moduli sets $\{U_i, T_1, T_2, T_3\}$, $\{S, U_i, T_k, T_l\}$ or $\{U_i, U_j, T_k, T_l\}$. These can be directly computed from the no-scale examples of Section 5.1.1 by replacing S or one or two U_i's by T_i's and exchanging the appropriate fluxes. As for Minkowski vacua, this reveals the existence of additional branches in the Type B(ecker) solutions. The nature of these deformations will be studied in the next section.

AdS vacua.

There are two classes of models depending on whether $m = 0$ or not. In the former case one finds that fluxes in general contribute to all RR tadpole directions with a sign which is *opposite* to that of D6-branes. This is important since it offers an alternative to orientifold planes to cancel RR tadpoles. In the second case with $m \neq 0$ one finds the interesting result that, depending on different flux choices the sign of the contribution to the RR tadpoles may be arbitrary and the net contribution may vanish. In the latter case one has a cancellation of a positive RR-NS contribution $h_0 m$ with a RR-metric flux contribution of type $a_i q_i$. This is interesting because in this class of backgrounds all real moduli are determined but the fluxes are unconstrained by RR tadpole cancellation conditions.

We will examine the case $T_k = T$ and look for supersymmetric minima for any m. The superpotential then becomes

$$W = e_0 + 3ic_1T + 3c_2T^2 + imT^3 + ih_0S - 3aST - \sum_{k=1}^{3}(ih_k + b_kT)U_k. \tag{5.75}$$

From $D_{U_k}W = 0$ and $D_S W = 0$, we find

$$3as = b_k u_k \ . \tag{5.76}$$

Hence, a and b_k must be both non-zero and of the same sign. Moreover, there are consistency conditions

$$3h_k a + h_0 b_k = 0 \quad ; \quad k = 1, 2, 3 \ . \tag{5.77}$$

Therefore, either both h_0 and h_k vanish or both are non-zero and of opposite sign. These conditions do not involve the moduli so at most we will have five equations for six unknowns, i.e. there will be at least one flat direction for the supersymmetric minima. In fact, only a combination of complex structure axions is fixed as

$$3a\text{Im } S + \sum_{k=1}^{3} b_k \text{Im } U_k = 3c_1 + \frac{3c_2}{a}(3h_0 - 7av) - \frac{3m}{a} v(3h_0 - 8av) \ , \tag{5.78}$$

with $v = \text{Im } T$.

If $h_0, h_k \neq 0$, using (5.77) we can write the fixed axion combination as $h_0\text{Im } S - \sum_k h_k \text{Im } U_k$. We also find

$$a\text{Re } S = 2(c_2 - mv)\text{Re } T \ . \tag{5.79}$$

Except for some axion directions, we have thus determined all the moduli in terms of T which is found from the remaining equations. The solution depends on whether m is different from zero or not. We now specify to these two possibilities.

i) $m = 0$.

When $m = 0$ one finds the simple results

$$v = \frac{h_0}{3a} \quad ; \quad 9c_2(\text{Re } T)^2 = e_0 - \frac{h_0 c_1}{a} - \frac{h_0^2 c_2}{3a^2} \ . \tag{5.80}$$

At the minimum, $W_0 = -12c_2(\text{Re } T)^2$ and the cosmological constant turns out to be

$$\Lambda = -\frac{ab_1 b_2 b_3}{128\, c_2^2\, (\text{Re } T)^3} \ , \tag{5.81}$$

where Re T is given in (5.80). It is important to notice that (5.79) fixes $c_2 a > 0$ so that in the supersymmetric minima with $m = 0$ the metric fluxes give a negative contribution to the nnn RR tadpoles. Similarly, $c_2 b_k > 0$ and the flux contribution to the nmm tadpoles (5.68)-(5.70) is positive.

Let us now check whether we have enough freedom to locate all moduli at large volume and small dilaton so that one can trust the effective 4-dimensional field theory approximation being used. The fluxes of type a, b_k and c_2 are constrained by the RR tadpole cancellation conditions and by the extra conditions (5.77). The values of h_0 and h_k are constrained by the latter but in principle both may be large as long as $h_0/h_k = -3a/b_k$. On the other hand, the fluxes of the RR 6-form e_0 and 4-form c_1 are unconstrained and may be arbitrarily large. Note then that for large e_0 and c_1 the moduli fields behave all like

$$\text{Re } T \;\simeq\; \text{Re } S \;\simeq\; \text{Re } U_k \;\simeq e_0^{1/2} \;,\; c_1^{1/2}. \tag{5.82}$$

In order our vacuum to remain in a perturbative regime we would like to have small values for the 4-dimensional coupling e^{ϕ_4} and the 10-dimensional string coupling e^{ϕ}. They are found to be

$$\begin{aligned} e^{\phi_4} &= (su_1u_2u_3)^{-1/4} \;=\; (\text{Re } T)^{-1}\frac{(ab_1b_2b_3)^{1/4}}{2\cdot 3^{3/4}c_2}, \\ e^{\phi} &= e^{\phi_4}(\text{Re } T)^{3/2} \;=\; (\text{Re } T)^{1/2}\frac{(ab_1b_2b_3)^{1/4}}{2\cdot 3^{3/4}c_2}. \end{aligned} \tag{5.83}$$

We thus see that for large Re T (which may be obtained e.g. with a large 6-form flux e_0) the 4-dimensional dilaton is small. However the string dilaton grows with Re T. Only by appropriately choosing the fluxes, i.e. with large c_2 one can perhaps maintain it under control. On the other hand such fluxes are in general very much constrained by the RR tadpole conditions so it seems difficult having small string dilaton and large volume at the same time. We will see however that in the case with $m \neq 0$ one can easily stabilize the moduli in the perturbative regime.

ii) $m \neq 0$.

To deal with $m \neq 0$ it is convenient to introduce a new variable for Im T. If $h_0 \neq 0$ we use

$$v = (\lambda + \lambda_0)\frac{h_0}{3a} \quad ; \quad \lambda_0 = \frac{3c_2a}{mh_0}. \tag{5.84}$$

The value of λ follows from the cubic equation

$$160\lambda^3 + 186(\lambda_0-1)\lambda^2 + 27(\lambda_0-1)^2\lambda + \lambda_0^2(\lambda_0-3) + \frac{27a^2}{mh_0^3}(e_0a - c_1h_0) = 0. \tag{5.85}$$

Clearly, we need a real solution for λ such that $\lambda(\lambda+\lambda_0-1) > 0$ since now Re T is determined from

$$3a^2(\text{Re } T)^2 = 5h_0^2\lambda(\lambda+\lambda_0-1). \tag{5.86}$$

Notice also that (5.79) takes the form $3a^2\text{Re } S = -2h_0m\lambda\text{Re } T$. For the cosmological constant we find

$$\Lambda = -\frac{ab_1b_2b_3\lambda_0^2(16\lambda+\lambda_0-1)}{1920\, c_2^2\, (\text{Re } T)^3\lambda^3} \;, \tag{5.87}$$

where λ is the appropriate solution of (5.85) and Re T is given in (5.86).

There is a variety of cases depending on the values of the different fluxes. One of the interesting features when $m \neq 0$ is that the contribution to the RR tadpoles may have either sign and even vanish. In fact, the flux-induced nnn and nmm tadpoles in (5.67)-(5.70) are respectively $\frac{1}{2}h_0 m(1-\lambda_0)$ and $\frac{1}{2}h_k m(1-\lambda_0)$. Thus, the flux tadpoles vanish at the special value $\lambda_0 = 1$. This is important, as we mentioned above, since it allows to fix the moduli without any constraint from RR tadpole cancellations.

To analyze the equations that determine λ and Re T we can proceed in various ways. We could choose for example e_0 and c_1 and study the allowed values of λ_0. For instance, with $e_0 = c_1 = 0$, one finds that to have solutions for λ with acceptable Re T necessarily $\frac{1}{3} < \lambda_0 < 3$. We also need $mh_0 < 0$ and $mh_k > 0$ so that Re $S > 0$ and Re $U_k > 0$. The special value $\lambda_0 = 1$ at which tadpoles vanish is allowed and leads in turn to

$$\mathrm{Re}\, T = \sqrt{\frac{5}{3}}\lambda\left|\frac{h_0}{a}\right|, \qquad \mathrm{Re}\, S = -\frac{2mh_0\lambda}{3a^2}\mathrm{Re}\, T, \qquad \mathrm{Re}\, U_k = -\frac{2mh_0\lambda}{ab_k}\mathrm{Re}\, T. \tag{5.88}$$

From the cubic equation we find the value $\lambda = (10)^{2/3}/20$.

With $m \neq 0$ one can locate the minima in perturbative regions. Consider for instance the case $e_0 = c_1 = 0$ and $\lambda_0 = 1$ so that the real moduli are given in (5.88). Note that one can have h_0, h_k and c_2 arbitrarily large as long as $\lambda_0 = 1$ and eq.(5.77) is respected. Then one can check that

$$e^{\phi_4} \simeq h_0^{-2} \quad ; \quad e^{\phi} \simeq h_0^{-1/2} , \tag{5.89}$$

so that for sufficiently large h_0 the minima will be perturbative. Note also that the NS flux density is diluted for large fluxes since it goes like $h_0/(\mathrm{Re}\, T)^{3/2} \simeq h_0^{-1/2}$. Concerning the RR flux $\overline{F}_2$, one has that taking the factor of g_s into account [34], its density also goes like $h_0^{-1/2}$ for large fluxes. The cosmological constant eq. (5.87) scales like

$$\Lambda \simeq -\, h_0^{-5} \simeq -(e^{\phi})^{10} \tag{5.90}$$

and hence is substantially suppressed for large h_0. Similar results may be obtained for values of λ_0 sufficiently close to 1, which would allow for contributions to RR tadpoles with either sign and of arbitrary size. In Section 7.2.3 we will consider this possibility to construct a semi-realistic intersecting D6-brane model with all diagonal closed string moduli stabilized.

Finally, let us mention for completeness other solutions within this class of AdS minima with $m \neq 0$. We may start by choosing a preferred value for some of the moduli. For example, we can set $v = 0$, and $h_0\mathrm{Im}\, S - \sum_k h_k \mathrm{Im}\, U_k = 0$. Then, necessarily $c_1 = -3h_0c_2/a$ and, from the cubic equation with $\lambda = -\lambda_0$, $e_0 = 45h_0c_2^2/ma$. This is the solution found in [33].

Another way to proceed with the analysis is to fix λ_0. For example, we can take $c_2 = 0$ so that $\lambda_0 = 0$. Obviously, $(\mathrm{Re}\, T)^2 > 0$ then requires either $\lambda < 0$ or $\lambda > 1$. If $\lambda < 0$, then Re $S > 0$ and Re $U_k > 0$ demand $h_0 m > 0$ and $h_k m < 0$, and the flux contribution to the tadpoles is like that of D6-branes. It is more interesting to consider $\lambda > 1$ so that $h_0 m < 0$ and $h_k m > 0$. Furthermore, to satisfy (5.85), it must be $(e_0 a - c_1 h_0) > 0$. Were it not for the fact that the fluxes are integers, we could always find solutions for some chosen λ. But still there is room to adjust the fluxes. For example, for $\lambda = 3/2$ we just need $(e_0 a - c_1 h_0) = -6h_0^3 m/a^2$ and this could be verified say for $e_0 = 0$, $c_1 = 24m$ and $h_0 = 2a$.

We can also set $h_0 = h_k = 0$, but to this end a different parametrization of v, amounting to $\lambda \to \lambda_0 \hat{\lambda}$, must be employed. Now the interesting case is $\hat{\lambda} < -1$ because Re $S > 0$ and Re $U_k > 0$ require $c_2 a > 0$ and $c_2 b_k < 0$ so that the flux contribution to tadpoles could cancel that of D6-branes. Again we can choose some $\hat{\lambda}$ and find values of e_0 to satisfy the cubic equation for v. For instance, for $\hat{\lambda} = -3/2$ we need $e_0 m^2 = -161 c_2^3$. One can check however that in this and the previous solution it is again hard to achieve at the same time a large value for the volume and a small value for the 10-dimensional dilaton, the reason being that now the value of fluxes h_0, c_2 and h_k will be constrained by RR tadpole cancellation conditions.

One can also easily find non-susy AdS vacua. We will just show a particularly simple example. In general, there are solutions in which (5.76) and (5.77) are still satisfied. To go further let us set $m = 0$. Then there are solutions with $a(\text{Re } S) = 2c_2(\text{Re } T)$ and $v = h_0/3a$, but with the novelty that

$$\pm 9c_2(\text{Re } T)^2 = e_0 - \frac{h_0 c_1}{a} - \frac{h_0^2 c_2}{3a^2} \ . \tag{5.91}$$

With plus sign this is the supersymmetric minimum, but we can also choose the minus sign depending on the fluxes. For instance, if $e_0 = c_1 = 0$, only the non-supersymmetric choice is available. In this case the minimum is AdS and it is typically stable because the eigenvalues of the Hessian are positive or negative but above the Breitenlohner-Freedman bound [131].

5.3 Non-geometric fluxes.

We have seen in Section 5.1 that the superpotentials induced by constant NSNS and RR backgrounds do not completely match for Type IIA/O6 and Type IIB/O3 orientifolds. Considering metric fluxes in Type IIA orientifolds slightly improves the situation, but as commented in the previous section, there are still branches of vacua in (5.64) which are not contained in (5.6) and viceversa, thus signaling the existence of new kinds of backgrounds.

If mirror symmetry is truly a symmetry of String Theory, the complete four dimensional effective theory should be universal and independent of whether it comes from a Type IIA or a Type IIB compactification. The low energy four dimensional physics should be thus described by a duality invariant superpotential. In this section we will analyze the degrees of freedom necessary to completely match the landscape of vacua for Type IIA orientifolds with O6-planes and the one for Type IIB orientifolds with O3/O7-planes. We will follow our work in [55].

5.3.1 T-duality and non-geometric fluxes.

We want to analyze how a generic Type IIB/O3 $\overline{H}_3$ background as the one given in eq. (5.2) transforms under mirror symmetry. We have seen that the h_0 component remains invariant, whereas the a_i components become metric fluxes in the Type IIA picture. Thus, let us concentrate in the $\bar{a}_i$ and $\bar{h}_0$ components.

Let us start considering the following background

$$ds^2 = \sum_{i=1}^{6}(dx^i)^2 \quad , \quad \overline{H}_3 = -\sum_{i=1}^{3}\bar{a}_i\beta_i. \tag{5.92}$$

A suitable gauge for $\overline{H}_3$ is then

$$B_2 = -\bar{a}_1 x^4 dx^2 \wedge dx^3 + \bar{a}_2 x^5 dx^1 \wedge dx^3 - \bar{a}_3 x^6 dx^1 \wedge dx^2, \tag{5.93}$$

and implementing mirror symmetry by performing three T-dualities along x^1, x^2, x^3, we arrive to the non-geometric background

$$ds^2 = \frac{C}{2}(\sum_{i,j,l,k=1}^{3}\epsilon_{ijl}Q^{ij}_{k+3}x^{k+3}dx^l)^2 + \sum_{k=4}^{6}(dx^k)^2, \tag{5.94}$$

$$B_2 = C\sum_{i,j,k=1}^{3}Q^{ij}_{k+3}x^{k+3}dx^i \wedge dx^j, \tag{5.95}$$

with

$$C = (1 + Q^{23}_4 x^4 + Q^{31}_5 x^5 + Q^{12}_6 x^6)^{-1} \tag{5.96}$$

and $Q^{23}_4 = \bar{a}_1$, $Q^{31}_5 = \bar{a}_2$ and $Q^{12}_6 = \bar{a}_3$. Therefore, this kind of backgrounds can be still parametrized by a finite set of constant parameters $\{Q^{ij}_k\}$, extending (5.62) to

$$\overline{H}_{mnp} \overset{T_m}{\longleftrightarrow} -\omega^m_{np} \overset{T_n}{\longleftrightarrow} -Q^{mn}_p. \tag{5.97}$$

Non-geometric fluxes were first introduced in [44].

Note that the monodromies for these backgrounds mix the metric and the B_2 field

$$\frac{1}{T_i} \to \frac{1}{T_i} - Q^{jk}_{i+3} \qquad \text{as } x^{i+3} \to x^{i+3} + 1 \qquad i \neq j \neq k, \tag{5.98}$$

with T_i the Kähler moduli, thus signaling the non-geometrical character of the induced manifold.

Concerning the $\bar{h}_0$ component, one has that Buscher rules are no longer applicable to this case. Indeed, now B_2 depends linearly on x^1, x^2 and/or x^3 and therefore the directions on which we T-dualize are not isometries of the background. However, although we do not know the concrete expression of the ten dimensional background, by counting the degrees of freedom in the four dimensional effective theory, one can introduce a new set of fluxes R^{mnp} and complete the formal rule (5.97) to [44]

$$\overline{H}_{mnp} \xleftrightarrow{T_m} -\omega^m_{np} \xleftrightarrow{T_n} -Q^{mn}_p \xleftrightarrow{T_p} R^{mnp}. \tag{5.99}$$

We have summarized in Table 5.2 the transformation of the Type IIB/O3 3-form $\overline{H}_3$ under mirror symmetry. For completeness we have included as well the corresponding dictionary for Type I String Theory, i.e. Type IIB with O9/O5-planes.

IIB/O3	IIA/O6	IIB/O9	flux
$\overline{H}_{123}$	R^{123}	R^{123}	$\bar{h}_0$
$\overline{H}_{423}$	$-Q^{23}_4$	R^{423}	$-\bar{a}_1$
$\overline{H}_{153}$	$-Q^{31}_5$	R^{153}	$-\bar{a}_2$
$\overline{H}_{126}$	$-Q^{12}_6$	R^{126}	$-\bar{a}_3$
$\overline{H}_{156}$	$-\omega^1_{56}$	R^{156}	$-a_1$
$\overline{H}_{426}$	$-\omega^2_{64}$	R^{426}	$-a_2$
$\overline{H}_{453}$	$-\omega^3_{45}$	R^{453}	$-a_3$
$\overline{H}_{456}$	$\overline{H}_{456}$	R^{456}	h_0

Table 5.2: NS IIB/O3 fluxes and their T-duals.

A similar procedure could be followed starting with the NSNS 3-form $\overline{H}_3$ and metric fluxes ω^i_{jk} in the Type IIA picture. The results are summarized in Table 5.3, where again the corresponding Type I fluxes are represented. Note that the two last sets of non-geometric fluxes are not obtained by mirror symmetry, but by $SO(6)$ rotations of the other fluxes. For completeness, in Table 5.4 we have represented the corresponding dictionary for the RR flux parameters in the different Type II orientifolds.

IIB/O3	IIA/O6	IIB/O9	flux
$\begin{pmatrix} Q^{23}_4 & Q^{31}_5 & Q^{12}_6 \end{pmatrix}$	$-\begin{pmatrix} \overline{H}_{423} & \overline{H}_{153} & \overline{H}_{126} \end{pmatrix}$	$\begin{pmatrix} \omega^4_{23} & \omega^5_{31} & \omega^6_{12} \end{pmatrix}$	$-\begin{pmatrix} h_1 & h_2 & h_3 \end{pmatrix}$
$\begin{pmatrix} -Q^{23}_1 & Q^{34}_5 & Q^{42}_6 \\ Q^{53}_4 & -Q^{31}_2 & Q^{15}_6 \\ Q^{26}_4 & Q^{61}_5 & -Q^{12}_3 \end{pmatrix}$	$\begin{pmatrix} -\omega^1_{23} & \omega^4_{53} & \omega^4_{26} \\ \omega^5_{34} & -\omega^2_{31} & \omega^5_{61} \\ \omega^6_{42} & \omega^6_{15} & -\omega^3_{12} \end{pmatrix}$	$\begin{pmatrix} -\omega^1_{23} & \omega^5_{34} & \omega^6_{42} \\ \omega^4_{53} & -\omega^2_{31} & \omega^6_{15} \\ \omega^4_{26} & \omega^5_{61} & -\omega^3_{12} \end{pmatrix}$	$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$
$\begin{pmatrix} Q^{56}_1 & Q^{64}_2 & Q^{45}_3 \end{pmatrix}$	$-\begin{pmatrix} R^{156} & R^{426} & R^{453} \end{pmatrix}$	$\begin{pmatrix} \omega^1_{56} & \omega^2_{64} & \omega^3_{45} \end{pmatrix}$	$-\begin{pmatrix} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 \end{pmatrix}$
$\begin{pmatrix} -Q^{56}_4 & Q^{61}_2 & Q^{15}_3 \\ Q^{26}_1 & -Q^{64}_5 & Q^{42}_3 \\ Q^{53}_1 & Q^{34}_2 & -Q^{45}_6 \end{pmatrix}$	$\begin{pmatrix} -Q^{56}_4 & Q^{26}_1 & Q^{53}_1 \\ Q^{61}_2 & -Q^{64}_5 & Q^{34}_2 \\ Q^{15}_3 & Q^{42}_3 & -Q^{45}_6 \end{pmatrix}$	$\begin{pmatrix} -\omega^4_{56} & \omega^2_{61} & \omega^3_{15} \\ \omega^1_{26} & -\omega^5_{64} & \omega^3_{42} \\ \omega^1_{53} & \omega^2_{34} & -\omega^6_{45} \end{pmatrix}$	$\begin{pmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \end{pmatrix}$

Table 5.3: Non-geometric IIB/O3 fluxes and their T-duals.

IIB/O3	IIA/O6	IIB/O9	flux
$\overline{F}_{123}$	$\overline{F}_0$	$-\overline{F}_{456}$	$-m$
$\overline{F}_{423}$	$\overline{F}_{14}$	$\overline{F}_{156}$	$-q_1$
$\overline{F}_{153}$	$\overline{F}_{25}$	$\overline{F}_{426}$	$-q_2$
$\overline{F}_{126}$	$\overline{F}_{36}$	$\overline{F}_{453}$	$-q_3$
$\overline{F}_{156}$	$\overline{F}_{2536}$	$-\overline{F}_{423}$	e_1
$\overline{F}_{426}$	$\overline{F}_{1436}$	$-\overline{F}_{153}$	e_2
$\overline{F}_{453}$	$\overline{F}_{1425}$	$-\overline{F}_{126}$	e_3
$\overline{F}_{456}$	$\overline{F}_{142536}$	$\overline{F}_{123}$	$-e_0$

Table 5.4: RR IIB/O3 fluxes and their T-duals.

An alternative view of the non-geometric fluxes can be given in terms of the algebra of isometries [44, 53]. We saw in Section 5.2.1 how the metric fluxes can be interpreted as the gauging of some of the isometries of the corresponding Lie algebra. Extending the algebra (5.44) to include the generators X^m, $m = 1, \ldots, 6$, corresponding to the gauge symmetries arising from the reduction of the B-field [39], one has

$$[Z_m, Z_n] = -\overline{H}_{mnp} X^p + \omega^p_{mn} Z_p, \tag{5.100}$$

$$[Z_m, X^p] = -\omega^p_{mn} X^n + Q^{pr}_m Z_r, \tag{5.101}$$

$$[X^m, X^n] = Q^{mn}_p X^p - R^{mnp} Z_p. \tag{5.102}$$

Using eq. (5.99) and

$$Z_m \xleftrightarrow{T_m} X^m,$$

we see that the proposed algebra is invariant under T-duality. Thus, it actually applies to any of the IIA and IIB orientifolds, provided that all the fluxes allowed by the orientifold projection are kept in each case.

The Jacobi identities of the algebra give constraints on the fluxes, analogously to what we saw in the case with only metric fluxes. Written in terms of the various tensors, the identities take a different form in each case. However, in terms of the individual flux parameters appearing in the T-dual superpotential there is just one set of constraints valid on all orientifolds.

It is convenient to work in the Type IIB with O3-planes picture in which only NSNS $\overline{H}_3$ and non-geometric Q^{ij}_k fluxes appear [55]. The ZZZ Jacobi identity leads to

$$Q^{rp}_{[l} \overline{H}_{mn]p} = 0 \ . \tag{5.103}$$

Substituting the fluxes of Tables 5.3 and 5.2 then yields

$$\bar{h}_0 h_j + \bar{a}_i b_{ij} + \bar{a}_j b_{jj} - a_k \bar{b}_{kj} = 0, \tag{5.104}$$

$$h_0 \bar{h}_j + a_i \bar{b}_{ij} + a_j \bar{b}_{jj} - \bar{a}_k b_{kj} = 0, \tag{5.105}$$

$$\bar{h}_0 b_{kj} + \bar{a}_i \bar{b}_{jj} + \bar{a}_j \bar{b}_{ij} - a_k \bar{h}_j = 0, \tag{5.106}$$

$$h_0 \bar{b}_{kj} + a_i b_{jj} + a_j b_{ij} - \bar{a}_k h_j = 0. \tag{5.107}$$

In all cases $i \neq j \neq k$. The XXX Jacobi identity simply gives

$$Q_p^{[mn} Q_r^{l]p} = 0, \tag{5.108}$$

and in terms of the explicit fluxes

$$-b_{ii} b_{jk} + \bar{b}_{ki} h_k + h_i \bar{b}_{kk} - b_{ji} b_{ik} = 0, \tag{5.109}$$

$$-\bar{b}_{ii} \bar{b}_{jk} + b_{ki} \bar{h}_k + \bar{h}_i b_{kk} - \bar{b}_{ji} \bar{b}_{ik} = 0, \tag{5.110}$$

$$-b_{ii} \bar{b}_{ij} + \bar{b}_{ji} b_{jj} + h_i \bar{h}_j - b_{ki} \bar{b}_{kj} = 0, \tag{5.111}$$

$$\bar{b}_{ii} b_{ij} - b_{ji} \bar{b}_{jj} + h_i \bar{h}_j - b_{ki} \bar{b}_{kj} = 0. \tag{5.112}$$

There are no further constraints from other Jacobi identities.

It is easy to work out some simple solutions to these constraints in the case of isotropic fluxes. Indeed, taking

$$\begin{array}{l} e_i = e \quad , \quad q_i = q \quad , \quad a_i = a \quad , \quad \bar{a}_i = \bar{a} \quad , \quad h_i = h \quad , \quad \bar{h}_i = \bar{h}_i, \\ b_{ij} = b \ (i \neq j) \quad , \quad b_{ii} = \beta \quad , \quad \bar{b}_{ij} = \bar{b} \ (i \neq j) \quad , \quad \bar{b}_{ii} = \bar{\beta}, \end{array} \tag{5.113}$$

one has that the following configurations solve the constraints

$$1.\ h = \bar{h} = b = \beta = \bar{b} = \bar{\beta} = 0 \ (Q_k^{ij} = 0) \quad ; \quad a, \bar{a}, h_0, \bar{h}_0 \neq 0 \ (\overline{H}_3 \neq 0), \tag{5.114}$$

$$2.\ a = \bar{a} = h_0 = \bar{h}_0 = 0 \ (\overline{H}_3 = 0) \quad ; \quad h\bar{h} = b\bar{b} \quad ; \quad h(\bar{b} + \bar{\beta}) = b(b + \beta), \tag{5.115}$$

$$3.\ a = \bar{a} = h = \bar{h} = b = \bar{b} = 0 \quad ; \quad h_0, \bar{h}_0, \beta, \bar{\beta} \neq 0, \tag{5.116}$$

$$4.\ \beta = -b \quad ; \quad \bar{\beta} = -\bar{b} \quad ; \quad h\bar{h} = b\bar{b} \quad ; \quad \bar{h}_0 h = a\bar{b} \quad ; \quad h_0 \bar{h} = \bar{a} b. \tag{5.117}$$

5.3.2 Superpotential and tadpoles in IIB with O3-planes.

We want to determine the superpotential and tadpoles induced by the Q_c^{ab} fluxes [55]. An useful fact is that we can contract a p-form X with Q to obtain a $(p-1)$-form QX with components

$$(QX)_{lm_1 \cdots m_{p-2}} = \frac{1}{2} Q_{[l}^{ab} X_{m_1 \cdots m_{p-2}]ab} \ . \tag{5.118}$$

This is analogous to the ω contraction defined in eq. (5.48).

Observing the IIA superpotential (5.64) it is clear that the Q_c^{ab} fluxes must induce new terms linear in the T_i and up to cubic order in the U_i. Such terms can be generated by adding to W a piece $\int Q\mathcal{J}_c \wedge \Omega$, where $\mathcal{J}_c$ is the 4-form encoding the Type IIB Kähler moduli, c.f. (3.10), and $Q\mathcal{J}_c$ is a 3-form according to (5.118). The complete IIB superpotential is then

$$W = \int_{T^6} (\overline{F}_3 - iS\overline{H}_3 + Q\mathcal{J}_c) \wedge \Omega \ . \tag{5.119}$$

Substituting the fluxes yields

$$\begin{aligned} W \ &= \ e_0 + i\sum_{i=1}^{3} e_i U_i - q_1 U_2 U_3 - q_2 U_1 U_3 - q_3 U_1 U_2 + imU_1U_2U_3 \\ &+ \ S\big[ih_0 - \sum_{i=1}^{3} a_i U_i + i\bar{a}_1 U_2 U_3 + i\bar{a}_2 U_1 U_3 + i\bar{a}_3 U_1 U_2 - \bar{h}_0 U_1 U_2 U_3\big] \\ &+ \ \sum_{i=1}^{3} T_i\big[-ih_i - \sum_{j=1}^{3} U_j b_{ji} + iU_2U_3\bar{b}_{1i} + iU_1U_3\bar{b}_{2i} + iU_1U_2\bar{b}_{3i} + U_1U_2U_3\bar{h}_i\big], \end{aligned} \tag{5.120}$$

where the Q-induced terms are in the last row. This general superpotential agrees with the proposal of [44] if we assume a symmetry under exchange of the three sub-tori.

As already discussed in Section 5.1.1, there is a C_4 tadpole in this orientifold. From T-duality, then we also expect C_8 tadpoles that can receive contributions from D7-branes and O7-planes. A natural candidate is $Q\overline{F}_3$, where the 2-form is computed according to (5.118). The proposal then is just

$$\int_{M_4\times T^6} (C_4 \wedge \overline{H}_3 \wedge \overline{F}_3 - C_8 \wedge Q\overline{F}_3) \ . \tag{5.121}$$

The minus sign in front of the second term is needed to match the known IIA results when only NS and metric fluxes are present. There are three different tadpoles according to the components of C_8 that can couple to the D7_i-branes. Taking into account a number N_{D7_i} of D7_i-branes and the flux tadpoles arising from (5.121), one has the cancellation conditions

$$N_{D3} + \frac{1}{2}[mh_0 - e_0\bar{h}_0 + \sum_i (q_i a_i + e_i \bar{a}_i)] = 16, \tag{5.122}$$

$$-N_{D7_i} + \frac{1}{2}[mh_i - e_0\bar{h}_i - \sum_j (q_j b_{ji} + e_j \bar{b}_{ji})] = 0 \ . \tag{5.123}$$

A new interesting feature is the dependence of the tadpoles on all RR fluxes.

5.3.3 Superpotential and tadpoles in IIA with O6-planes.

In this case there are non-geometric Q and R fluxes. As in (5.118), we can contract Q with a p-form X to obtain a $(p-1)$-form QX. Analogously, contracting with R we obtain a $(p-3)$-form with components

$$(RX)_{m_1\cdots m_{p-3}} = \frac{1}{6}R^{abc}X_{[m_1\cdots m_{p-3}]abc} \ . \tag{5.124}$$

The Q and R fluxes are expected to induce superpotential terms quadratic and cubic in the IIA Kähler moduli. There are appropriate 2 and 3-forms that encode the required combination of the T_i, namely

$$\begin{aligned} J_c^{(2)} &\equiv \frac{1}{2}J_c\wedge J_c = -T_2T_3\,\tilde{\omega}_1 - T_1T_3\,\tilde{\omega}_2 - T_1T_2\,\tilde{\omega}_3, \\ J_c^{(3)} &\equiv \frac{1}{6}J_c\wedge J_c\wedge J_c = -iT_1T_2T_3\,\alpha_0\wedge\beta_0 \ . \end{aligned} \tag{5.125}$$

Then, the Type IIA superpotential mirror to (5.119) can be written as [55]

$$W = \int_{T^6}\left[e^{J_c}\wedge \overline{F}_{RR} + \Omega_c\wedge(\overline{H}_3 + \omega J_c + QJ_c^{(2)} + RJ_c^{(3)})\right] \ . \tag{5.126}$$

Substituting the fluxes precisely reproduces (5.120) upon exchanging $T_i \leftrightarrow U_i$.

The idea behind the general formula for W is to wedge Ω_c with all available 3-forms. An analogous reasoning suggests that the C_7 tadpoles induced by the fluxes follow from

$$\int_{M_4\times T^6} C_7\wedge(-\overline{H}_3\overline{F}_0 + \omega\overline{F}_2 - Q\overline{F}_4 + R\overline{F}_6) \ . \tag{5.127}$$

The signs have been chosen to match the results in Type IIB. Including tadpoles due to O6-planes and stacks of intersecting D6-branes leads to the general cancellation conditions

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}[mh_0 - e_0\bar{h}_0 + \sum_i(q_ia_i + e_i\bar{a}_i)] \quad = \quad 16, \tag{5.128}$$

$$\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}[mh_1 - e_0\bar{h}_1 - \sum_i(q_ib_{i1} + e_i\bar{b}_{i1})] \quad = \quad 0, \tag{5.129}$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}[mh_2 - e_0\bar{h}_2 - \sum_i(q_ib_{i2} + e_i\bar{b}_{i2})] \quad = \quad 0, \tag{5.130}$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}[mh_3 - e_0\bar{h}_3 - \sum_i(q_ib_{i3} + e_i\bar{b}_{i3})] \quad = \quad 0. \tag{5.131}$$

These agree with (5.122) and (5.123).

5.3.4 Superpotential and tadpoles in IIB with O9-planes.

For completeness we will include as well the effective superpotential for the case of Type IIB orientifolds with O9-planes. Since on this case the orientifold involution is the identity, only even fluxes are allowed. There are eight RR $\overline{F}_{lmn}$, twenty-four metric ω^l_{mn}, and eight non-geometric R^{lmn}. The components are displayed in Tables 5.2, 5.3 and 5.4.

The superpotential can be derived from the Type IIB/O3 results by implementing T-dualities in each of the six internal coordinates [55]. The moduli then transform as $S \leftrightarrow S$, $T_i \leftrightarrow T_i$, but $U_i \leftrightarrow 1/U_i$. The Kähler potential transforms as

$$K \to K + \log |U_1 U_2 U_3|^2. \tag{5.132}$$

Invariance of the Kähler function, $\mathcal{G} = K + \log |W|^2$, then requires

$$W_{\rm O9} = \frac{-iW}{U_1 U_2 U_3}, \tag{5.133}$$

where we have chosen a convenient phase. Therefore, in terms of the Type IIB/O9 moduli, the superpotential reads

$$\begin{aligned} W_{\rm O9} \;\; = \;\; & m + i\sum_{i=1}^{3} q_i U_i + e_1 U_2 U_3 + e_2 U_1 U_3 + e_3 U_1 U_2 - i e_0 U_1 U_2 U_3 \\ + \;\; & S\big[i\bar{h}_0 + \sum_{i=1}^{3} \bar{a}_i U_i + i a_1 U_2 U_3 + i a_2 U_1 U_3 + i a_3 U_1 U_2 + h_0 U_1 U_2 U_3\big] \\ + \;\; & \sum_{i=1}^{3} T_i\big[-i\bar{h}_i + \sum_{j=1}^{3} \bar{b}_{ji} U_j + i b_{1i} U_2 U_3 + i b_{2i} U_1 U_3 + i b_{3i} U_1 U_2 - h_i U_1 U_2 U_3\big] \ . \end{aligned} \tag{5.134}$$

In absence of metric ω and non-geometric R fluxes $W_{\rm O9}$ depends only on the complex structure moduli. Linear terms in T_i and S are induced by ω and R respectively.

Fluxes contribute to C_{10} and C_6 tadpoles. Indeed, there is a candidate tadpole term

$$\int_{M_4\times T^6} C_6 \wedge \omega\overline{F}_3 + C_{10} \wedge R\overline{F}_3 \ , \tag{5.135}$$

where $R\overline{F}_3$ is a 0-form according to (5.124) and $\omega\overline{F}_3$ is a 4-form according to (5.48). Substituting the fluxes and including the corresponding sources gives

$$N_{D9} + \frac{1}{2}[m h_0 - e_0 \bar{h}_0 + \sum_i (q_i a_i + e_i \bar{a}_i)] = 16 \ , \tag{5.136}$$

$$N_{D5_i} + \frac{1}{2}[m h_i - e_0 \bar{h}_i - \sum_j (q_j b_{ji} + e_j \bar{b}_{ji})] = 0 \ . \tag{5.137}$$

5.4 Type IIB S-duality and fluxes

By considering some new non-geometric backgrounds, we have built up in the previous section a T-duality invariant superpotential. However, in the language of Type IIB String Theory, this new kind of background fluxes spoils the Type IIB $SL(2,\mathbb{Z})$ symmetry described in Section 4.1. As we saw, this is a symmetry of the full Type IIB theory, and therefore it should be inherited by the four dimensional effective superpotential. In fact, when only standard RR and NS backgrounds are switched on, the effective potential is invariant under S-duality transformations.

In order to recover a S-dual superpotential in presence of non-geometric fluxes, one is then advocated to consider new kinds of non-geometric deformations, parametrized by a set of constants $\{P_o^{mn}\}$ defined by

$$\begin{pmatrix} Q \\ P \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix} , \tag{5.138}$$

for $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$. These new objects are thus some sort of RR non-geometric fluxes with the same tensor structure and number of components than the Q fluxes.

We conjecture then that the effective superpotential for Type IIB orientifolds with O3/O7-planes is given by

$$W = \int_{T^6} [(\overline{F}_3 - iS\overline{H}_3) + (Q - iSP)\mathcal{J}_c] \wedge \Omega , \tag{5.139}$$

or in terms of the background parameters

$$\begin{aligned} W &= e_0 + i\sum_{i=1}^{3} e_i U_i - q_1 U_2 U_3 - q_2 U_1 U_3 - q_3 U_1 U_2 + imU_1U_2U_3 \\ &+ S\big[ih_0 - \sum_{i=1}^{3} a_i U_i + i\bar{a}_1 U_2 U_3 + i\bar{a}_2 U_1 U_3 + i\bar{a}_3 U_1 U_2 - \bar{h}_0 U_1 U_2 U_3\big] \\ &+ \sum_{i=1}^{3} T_i\big[-ih_i - \sum_{j=1}^{3} U_j b_{ji} + iU_2U_3\bar{b}_{1i} + iU_1U_3\bar{b}_{2i} + iU_1U_2\bar{b}_{3i} + U_1U_2U_3\bar{h}_i\big] - \\ &- S\sum_{i=1}^{3} f_i T_i + iS\sum_{i,j=1}^{3} U_j g_{ji} T_i + SU_2U_3\sum_{i=1}^{3} \bar{g}_{1i} T_i + SU_1U_3 \sum_{i=1}^{3} \bar{g}_{2i} T_i \\ &+ SU_1U_2\sum_{i=1}^{3} \bar{g}_{3i} T_i - iSU_1U_2U_3\sum_{i=1}^{3} \bar{f}_i T_i , \end{aligned} \tag{5.140}$$

being the last two rows the generated by the P fluxes. Here we have defined

$$\begin{pmatrix} P_4^{23} \\ P_5^{31} \\ P_6^{12} \end{pmatrix} = \begin{pmatrix} -f_1 \\ -f_2 \\ -f_3 \end{pmatrix} , \quad \begin{pmatrix} -P_1^{23} & P_5^{34} & P_6^{42} \\ P_4^{53} & -P_2^{31} & P_6^{15} \\ P_4^{26} & P_5^{61} & -P_3^{12} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} , \tag{5.141}$$

$$\begin{pmatrix} P_1^{56} \\ P_2^{64} \\ P_3^{45} \end{pmatrix} = \begin{pmatrix} -\bar{f}_1 \\ -\bar{f}_2 \\ -\bar{f}_3 \end{pmatrix} , \quad \begin{pmatrix} -P_4^{56} & P_2^{61} & P_3^{15} \\ P_1^{26} & -P_5^{64} & P_3^{42} \\ P_1^{53} & P_2^{34} & -P_6^{45} \end{pmatrix} = \begin{pmatrix} \bar{g}_{11} & \bar{g}_{12} & \bar{g}_{13} \\ \bar{g}_{21} & \bar{g}_{22} & \bar{g}_{23} \\ \bar{g}_{31} & \bar{g}_{32} & \bar{g}_{33} \end{pmatrix} . \tag{5.142}$$

Imposing closure under $SL(2,\mathbb{Z})_S$ for the Bianchi identities (5.103) and (5.108) now requires the conditions

$$Q_p^{[mn}Q_r^{l]p} = 0, \tag{5.143}$$
$$P_p^{[mn}P_r^{l]p} = 0, \tag{5.144}$$
$$Q_p^{[mn}P_r^{l]p} + P_p^{[mn}Q_r^{l]p} = 0, \tag{5.145}$$
$$Q_{[l}^{rp}\overline{H}_{mn]p} - P_{[l}^{rp}\overline{F}_{mn]p} = 0. \tag{5.146}$$

In terms of the background parameters, the new constraints (5.144) and (5.145) read respectively

$$-g_{ii}g_{jk} + \bar{g}_{ki}f_k + f_i\bar{g}_{kk} - g_{ji}g_{ik} \quad = \quad 0, \tag{5.147}$$
$$-\bar{g}_{ii}\bar{g}_{jk} + g_{ki}\bar{f}_k + \bar{f}_i g_{kk} - \bar{g}_{ji}\bar{g}_{ik} \quad = \quad 0, \tag{5.148}$$
$$-g_{ii}\bar{g}_{ij} + \bar{g}_{ji}g_{jj} + f_i\bar{f}_j - g_{ki}\bar{g}_{kj} \quad = \quad 0, \tag{5.149}$$
$$\bar{g}_{ii}g_{ij} - g_{ji}\bar{g}_{jj} + f_i\bar{f}_j - g_{ki}\bar{g}_{kj} \quad = \quad 0, \tag{5.150}$$

and

$$b_{kk}\bar{g}_{kj} - h_k\bar{f}_j - \bar{b}_{jk}g_{jj} + b_{ik}\bar{g}_{ij} + g_{kk}\bar{b}_{kj} - f_k\bar{h}_j - \bar{g}_{jk}b_{jj} + g_{ik}\bar{b}_{ij} \quad = \quad 0, \tag{5.151}$$
$$b_{kk}g_{ij} - h_k\bar{g}_{jj} - \bar{b}_{jk}f_j + b_{ik}g_{kj} + g_{kk}b_{ij} - f_k\bar{b}_{jj} - \bar{g}_{jk}h_j + g_{ik}b_{kj} \quad = \quad 0, \tag{5.152}$$
$$\bar{b}_{kk}\bar{g}_{ij} - \bar{h}_k g_{jj} - b_{jk}\bar{f}_j + \bar{b}_{ik}\bar{g}_{kj} + \bar{g}_{kk}\bar{b}_{ij} - \bar{f}_k b_{jj} - g_{jk}\bar{h}_j + \bar{g}_{ik}\bar{b}_{kj} \quad = \quad 0, \tag{5.153}$$
$$\bar{b}_{kk}g_{kj} - \bar{h}_k f_j - b_{jk}\bar{g}_{jj} + \bar{b}_{ik}g_{ij} + \bar{g}_{kk}b_{kj} - \bar{f}_k h_j - g_{jk}\bar{b}_{jj} + \bar{g}_{ik}b_{ij} \quad = \quad 0, \tag{5.154}$$

whereas eq. (5.146) reads

$$\bar{h}_0 h_j + \bar{a}_i b_{ij} + \bar{a}_j b_{jj} - a_k\bar{b}_{kj} + mf_j - q_i g_{ij} - q_j g_{jj} - e_k\bar{g}_{kj} \quad = \quad 0, \tag{5.155}$$
$$h_0\bar{h}_j + a_i\bar{b}_{ij} + a_j\bar{b}_{jj} - \bar{a}_k b_{kj} - e_0\bar{f}_j - e_i\bar{g}_{ij} - e_j\bar{g}_{jj} - q_k g_{kj} \quad = \quad 0, \tag{5.156}$$
$$\bar{h}_0 b_{kj} + \bar{a}_i\bar{b}_{jj} + \bar{a}_j\bar{b}_{ij} - a_k\bar{h}_j + mg_{kj} - q_i\bar{g}_{jj} - q_j\bar{g}_{ij} - e_k\bar{f}_j \quad = \quad 0, \tag{5.157}$$
$$h_0\bar{b}_{kj} + a_i b_{jj} + a_j b_{ij} - \bar{a}_k h_j - e_0\bar{g}_{kj} - e_i g_{jj} - e_j g_{ij} - q_k f_j \quad = \quad 0. \tag{5.158}$$

Concerning the flux induced tadpoles, we already mentioned in Section 4.1 that C_8 is part of a $SL(2,\mathbb{Z})$ triplet. Thus, starting from eq. (5.121) and imposing $SL(2,\mathbb{Z})_S$ invariance, one gets

$$\int_{M_4\times T^6} C_4\wedge\overline{H}_3\wedge\overline{F}_3 - C_8\wedge Q\overline{F}_3 + \tilde{C}_8\wedge P\overline{H}_3 + C_8'\wedge\left(Q\overline{H}_3 + P\overline{F}_3\right)\,, \tag{5.159}$$

which gives rise to the cancellation conditions

$$N_{D3}+\frac{1}{2}[mh_0-e_0\bar{h}_0+\sum_i(q_ia_i+e_i\overline{a}_i)]=16, \tag{5.160}$$

$$-N_{D7_i}+\frac{1}{2}[mh_i-e_0\bar{h}_i-\sum_j(q_jb_{ji}+e_j\bar{b}_{ji})]=0, \tag{5.161}$$

$$-N_{NS7_i}+\frac{1}{2}[h_0\bar{f}_i-\bar{h}_0f_i-\sum_j(\bar{a}g_{ji}-a_j\bar{g}_{ji})=0, \tag{5.162}$$

$$N_{I7_i}+\frac{1}{2}[e_0\bar{f}_i-mf_i+\sum_j(q_jg_{ji}+e_j\bar{g}_{ji})]=0. \tag{5.163}$$

We have made use of eq. (5.146) in order to simplify the last equation. Moreover, we have considered the possibility of having NS7_i and I7_i-branes [132] sourcing respectively $\tilde{C}_8$ and C_8'.

To analyze systematically the vacuum structure of the scalar potential induced by (5.140) is a difficult task due to its great complexity. Here we will concentrate on a couple of examples of $\mathcal{N}=1$ Minkowski vacua on which the dilaton, the complex structure and the overall Kähler moduli are fixed [55]. This is a novelty with respect to the Minkowski vacua discussed up to here, since in those cases there were always numerous flat directions.

Thus, let us consider the isotropic background of eq. (5.113) together with

$$\begin{aligned}
&f_i=f \quad , \quad \bar{f}_i=\bar{f}, \\
&g_{ij}=g \quad (i\neq j) \quad , \quad g_{ii}=\gamma \quad , \quad \bar{g}_{ij}=\bar{g} \quad (i\neq j) \quad , \quad \bar{g}_{ii}=\bar{\gamma}.
\end{aligned} \tag{5.164}$$

Then the superpotential (5.140) becomes

$$W=E_1+\sigma E_2+\tau E_3+\sigma\tau E_4 \ , \tag{5.165}$$

with

$$U=-i\rho \quad , \quad S=-i\sigma \quad , \quad T=-i\tau, \tag{5.166}$$

and

$$E_1=E_0+3e\rho+3q\rho^2-m\rho^3 \ , \tag{5.167}$$

$$E_2=h_0+3a\rho-3\bar{a}\rho^2-\bar{h}_0\rho^3 \ , \tag{5.168}$$

$$E_3=3[-h+(2b+\beta)\rho-(2\bar{b}+\bar{\beta})\rho^2+\bar{h}\rho^3] \ , \tag{5.169}$$

$$E_4=3[f-(2g\gamma)\rho+(2\bar{g}+\bar{\gamma})\rho^2-\bar{f}\rho^3] \ . \tag{5.170}$$

The problem consists on finding solutions of

$$W=\frac{\partial W}{\partial\rho}=\frac{\partial W}{\partial\sigma}=\frac{\partial W}{\partial\tau}=0. \tag{5.171}$$

From $\partial W/\partial\sigma=0$ and $\partial W/\partial\tau$ one gets

$$\tau=-\frac{E_2}{E_4} \quad , \quad \sigma=-\frac{E_3}{E_4} \ , \tag{5.172}$$

and substituting back in $W=0$ and $\partial W/\partial\rho=0$ then gives rise to

$$E = E_1E_4 - E_2E_3 = 0, \qquad E' = 0. \tag{5.173}$$

Thus, E must have a double root ρ_0 necessarily complex

$$E = 3(\rho-\rho_0)^2(\rho-\rho_0^*)^2(\alpha\rho^2+\delta\rho+\epsilon) \tag{5.174}$$

with α, δ, ϵ and ρ_0 depending on the fluxes.

We will consider two particular solutions to the Bianchi identities (5.143)-(5.146), summarized in Table 5.5.

Case	$P\cdot P=0$	$Q\cdot Q=0$	$Q\cdot P+P\cdot Q=0$	$Q\cdot\overline{H}_3-P\cdot\overline{F}_3=0$
1	$f=\bar f=g=\bar g=0$	$\beta=-b$, $\bar\beta=-\bar b$ $h\bar h=b\bar b$	$b\gamma=h\bar\gamma$	$h_0\bar b=e\gamma+\bar a h$ $\bar h_0 h=q\gamma+a\bar b$
2	$\gamma=-g$, $\bar\gamma=-\bar g$ $f\bar f=g\bar g$	$h=\bar h=b=\bar b=0$	$f\bar\beta=g\beta$	$mg=e\bar f-\bar a\bar\beta$ $qg=a\bar\beta-e_0\bar f$

Table 5.5: Some solutions to the identities (5.143)-(5.146) for isotropic fluxes.

Case 1

We find a class of minima with fluxes satisfying the relations

$$q=0 \quad , \quad e_0\gamma=4ah \quad , \quad mh_0h\gamma=(e\gamma+4h\bar a)(e\gamma+h\bar a). \tag{5.175}$$

Besides, $q=0$ implies $h\bar h_0=a\bar b$, with $\bar b$ given in Table 5.5. As free parameters we can then take a, $\bar a$, b, h_0, h, γ and e. They must be such that the remaining dependent fluxes come out integers as well. Furthermore, there are sign relations required for consistency. For example, one finds

$$\rho_0=i|\rho_0| \quad , \quad |\rho_0|^2=-\frac{h_0h}{e\gamma+h\bar a}\ . \tag{5.176}$$

This needs $h_0h(e\gamma+h\bar a)<0$, then $U=\sqrt{-h_0h/(e\gamma+h\bar a)}$.

The remaining moduli turn out to be

$$S=\frac{2h}{\gamma U} \quad , \quad T=\frac{h\big[h_0(2\bar a h-e\gamma)-2iaU(e\gamma+\bar a h)\big]}{3\gamma U(e\gamma+\bar a h)(h-ibU)}. \tag{5.177}$$

To guarantee Re $S>0$ and Re $T>0$ we need

$$h\gamma>0 \quad , \quad (2ab-2\bar a h+e\gamma)>0. \tag{5.178}$$

For example, choosing

$$a=-8\ ,\ b=-4\ ,\ h=-4\ ,\ \gamma=-4\ ,\ \bar a=12\ ,\ h_0=8\ ,\ e=-16\ , \tag{5.179}$$

we find that all dependent fluxes are also even integers. The moduli are determined to be

$$U = S = \sqrt{2} \quad , \quad T = \frac{1}{9}(14\sqrt{2} + 16i). \tag{5.180}$$

It is also interesting to compute the tadpoles. In particular one finds

$$N_{D3} - 288 = 16 \quad , \quad -N_{D7} + 80 = 0 \quad , \quad -N_{\rm NS7} + 40 = 0 \quad , \quad N_{\rm I7} + 32 = 0. \tag{5.181}$$

Observe the peculiar result that fluxes contribute to the C_4 tadpole as O3-planes instead of D3-branes. This is again a novelty with respect to the vacua involving just Q-fluxes. However, this is not generic. One can easily find other examples on which the C_4 flux tadpole comes out positive. The 8-form tadpoles can have either sign, or even cancel, depending on the parameters.

Within the above class of vacua we can set $e = 0$, implying $h_0 \bar{b} = \bar{a} h$. The I7 tadpole then cancels, since now $q = e = 0$. On the other hand, from the condition Re $S > 0$ we conclude that in this case fluxes always contribute to the C_4 tadpole as D3-branes. Concerning the flux contributions to the C_8 and $\tilde{C}_8$ tadpoles, one has that both are positive (opposite sign as D7/NS7-branes) and proportional to Re T. To give a numerical example, we can take $\bar{a} = -h_0 = 2$, $U = 1$, and

$$a = -8 \quad , \quad b = -2 \quad , \quad h = -2 \quad , \quad \gamma = -2 \quad , \quad S = 2 \quad , \quad T = \frac{5}{3} - i. \tag{5.182}$$

It is easy to verify that the dependent fluxes are all even integers.

Case 2

The second example we would like to present consists on a set of solutions with free parameters β, $\bar{\beta}$, e_0, e, $\bar{f}$, g and a, and remaining fluxes determined by

$$\bar{a} = 0 \quad , \quad h_0 \bar{\beta} = 4eg \quad , \quad \bar{h}_0 \bar{\beta} = \frac{\bar{f}(4e_0\bar{f} - 3a\bar{\beta})}{g} \ . \tag{5.183}$$

Notice that then $m = e\bar{f}/g$ and q is given in Table 5.5. Then

$$U = \sqrt{-\frac{g}{\bar{f}}} \quad , \quad S = \frac{\bar{\beta}}{2\bar{f}U} \quad , \quad T = \frac{U}{3g} \frac{\left[2e\bar{f}U + i(2e_0\bar{f} - 3a\bar{\beta})\right]}{(\bar{\beta}U + i\beta)}. \tag{5.184}$$

It is easy to check that Re $S > 0$, Re $U > 0$ and Re $T > 0$ as long as

$$g\bar{f} < 0 \quad , \quad g\bar{\beta} < 0 \quad , \quad \left(2eg + 3a\beta - \frac{2e_0\beta\bar{f}}{\bar{\beta}}\right) > 0. \tag{5.185}$$

For an illustrative example, consider the parameters

$$\bar{f} = 2 \ , \ \beta = 8 \ , \ \bar{\beta} = 8 \ , \ g = -2 \ , \ a = 28 \ , \ e_0 = 16 \ , \ e = 96. \tag{5.186}$$

The moduli are then fixed as

$$U = 1 \quad , \quad S = 2 \quad , \quad T = \frac{1}{3}(7 + 31i), \tag{5.187}$$

whereas for the flux-induced tadpoles we obtain

$$N_{D3} + 32 = 16 \quad , \quad N_{\rm I7} + 112 = 0. \tag{5.188}$$

The flux contribution to C_8 and $\tilde{C}_8$ tadpoles is zero.

The I7 tadpoles cancel when $a = 0$. Then it is simple to show that the free parameters can be chosen so that all other fluxes are integers while Re S and Re T are large and positive. The sign relations among the parameters imply that the C_4, C_8 and $\tilde{C}_8$ flux tadpoles are positive. The latter two are proportional to Re T.

Other solutions of the Bianchi identities can be obtained by combining the building blocks of Table 5.5. For example, $P \cdot P = 0$ can be fulfilled as in case 2, and $Q \cdot Q = 0$ as in case 1. Then the solution of $Q{\cdot}P + P{\cdot}Q = 0$ can be written as $(bf - gh)(h\bar{f} - g\bar{b}) = 0$. Now, if $h\bar{f} = g\bar{b}$, E_3 and E_4 have a common quadratic factor but the polynomial E cannot be factorized as needed. When $bf = gh$, to avoid Re $S = 0$ it must be that U is necessarily complex. In this more complicated case we were not able to find supersymmetric Minkowski minima.

In summary, some differences compared to the type IIB results [119] without non-geometric nor S-dual fluxes are evident. The situation now is rather more involved but still we have found some concrete results. For simplicity we have analyzed the case with isotropic fluxes and moduli $T_i = T$, $U_i = U$. We find Minkowski $\mathcal{N} = 1$ vacua in which not only the dilaton and complex structure fields are fixed but also the Kähler modulus T is fixed. However, if we analyze the more general case with independent T_i and U_i fields, generically only one linear combination of the Kähler moduli T_i is fixed. This is due to the fact that the superpotential is only linear in the T_i and essentially only depends on a linear combination of these moduli.

When S-dual backgrounds are switched on, the contribution from fluxes to the tadpole of the RR C_4 form can have either sign depending on the flux values. This is a surprising result. We know that in absence of S-dual fluxes the C_4 tadpole due to $\overline{H}_3$ and $\overline{F}_3$ fluxes consistent with the imaginary self-dual condition needed for supersymmetry is always positive [37, 119]. Concerning the C_8 tadpole, if only non-geometric fluxes are present the flux tadpole is negative (same sign as D7-branes). However, in presence of S-dual backgrounds the flux contribution can be positive (same sign as O7-planes), negative, or even vanish. This is similar to what occurs in Type IIA AdS vacua with metric fluxes [43]. The fact that fluxes may contribute to tadpoles as orientifold planes may be useful for model-building as we will show in Section 7.2.3.

The value of the real parts for the dilaton S and the overall Kähler modulus T may be made large by appropriately choosing the fluxes. This is in general required to maintain perturbative values for the couplings and the validity of the supergravity approximation. On the other hand, in our supersymmetric Minkowski vacua Re S and Re T cannot be made arbitrarily large because in general they are tied to RR tadpoles induced by the fluxes. We assume that localized sources of different kinds may be added to the theory rendering it tadpole free. In this connection, notice that if we want to add D3 and/or D7-branes to vacua like these, the existence of undetermined Kähler T_i moduli may in fact be necessary, as it will be emphasized in Section 7.1.

5.5 Generalized duality invariant superpotentials.

The approach followed up to here has been to implement the ten dimensional dualities of Type II orientifolds in the low energy effective theory. Before the orbifold truncation, this low energy effective theory corresponds to four dimensional $\mathcal{N} = 4$ supergravity, spontaneously broken to $\mathcal{N} \leq 4$ by the background fluxes. In terms of this, the dualities of the ten dimensional theory appear as elements of the duality group of $\mathcal{N} = 4$ supergravity, given by $O(6, 6; \mathbb{Z}) \times SL(2, \mathbb{Z})$, although only a subgroup $SL(2, \mathbb{Z})^7$ is realized for factorized toroidal orientifolds.

The kind backgrounds considered along the previous chapters are invariant under a subgroup $SL(2, \mathbb{Z})^4 \subset SL(2, \mathbb{Z})^7$, corresponding to T-duality and Type IIB S-duality. The remaining elements in $SL(2, \mathbb{Z})^7$, correspond in ten dimensions to Type I/Heterotic S-duality and Heterotic T-duality. Here we would like to generalize the superpotentials of previous chapters to a fully invariant superpotential under the whole $SL(2, \mathbb{Z})^7$ duality group of the effective theory. This will require the addition of new background fluxes. In order to give support and to show the nature of these new degrees of freedom, we will first present some results from M-theory and Heterotic compactifications revealing the necessity of new terms in the superpotential. Then we will systematize the action of the duality group on the background parameters and will derive a fully invariant superpotential.

5.5.1 M-theory on a twisted 7-tori.

We will consider here the G_2-holonomy manifolds X_7 obtained as certain $Z_2 \times Z_2 \times Z_2$ orbifolds of the 7-torus, $X_7 = \mathrm{T}^7/Z_2 \times Z_2 \times Z_2$ [117]. We will follow the results and notation used in ref.[133]. One has seven complex moduli fields $M_I(x)$, $I = 1, ..., 7$. They may be defined in terms of the complexified G_2-form

$$\mathbf{\Phi_c} = C_3 \; + \; i\,\mathbf{\Phi} \; = \; iM_I(x)\; \phi^I(y), \tag{5.189}$$

where $\phi^I \in H^3(X_7)$, C_3 is the M-theory 3-form and $\mathbf{\Phi} = \mathrm{Re}\; M_I(x)\phi^I(y)$ is given by eq. (4.64), with Re $M_I(x)$ parameterizing the volume of the 7 invariant 3-cycles in $X_7 = T^7/Z_2 \times Z_2 \times Z_2$. We will consider the addition of metric fluxes in this toroidal model. This is a Scherk-Schwarz reduction which proceeds in an analogous way to that described for Type IIA orientifold compactifications. In particular we replace the differentials dy^P, $P = 1, \cdots, 7$, by twisted forms η^P satisfying

$$d\eta^P = -\frac{1}{2}\omega^P_{MN}\eta^M \wedge \eta^N \quad , \quad \omega^P_{[MN}\omega^S_{R]P} = 0 \ , \tag{5.190}$$

where one also has $\omega^P_{PN} = 0$ [38]. Among these metric fluxes ω^P_{MN}, only twenty-one are invariant under the twists. In addition we consider the presence of seven 4-form backgrounds g_{IJKL} corresponding to fluxes of the M-theory 3-form. The presence of these two types of fluxes gives rise to a superpotential [134, 135, 133]

$$W_7 \ = \ \frac{1}{4}\int_{X_7}(C+i\mathbf{\Phi})\wedge[g+\frac{1}{2}d(C+i\mathbf{\Phi})] \ + \ \frac{1}{4}\int_{X_7}G_7. \tag{5.191}$$

Here G_7 is the flux of the 3-form dual. Expanding this superpotential in terms of the seven moduli in Type IIA notation [133] one obtains:

$$\begin{aligned} W_7 \ &= \ g_{567891011} \ + \ i(g_{78910}T_1 + g_{56910}T_2 + g_{5678}T_3) \ + \\ &+ \ i(g_{57911}S - g_{581011}U_1 - g_{671011}U_2 - g_{68911}U_3) \\ &+ \ (\omega^{11}_{910}T_1T_2 + \omega^{11}_{56}T_2T_3 + \omega^{11}_{78}T_1T_3) - S(\omega^6_{79}T_1 + \omega^8_{95}T_2 + \omega^{10}_{57}T_3) \\ &+ \ (\omega^6_{810}T_1U_1 + \omega^8_{106}T_2U_2 + \omega^{10}_{68}T_3U_3) \ - \ (\omega^5_{710}T_1U_2 + \omega^5_{89}T_1U_3 + \omega^7_{105}T_2U_1) \\ &+ \ (\omega^7_{96}T_2U_3 + \omega^9_{58}T_3U_1 + \omega^9_{67}T_3U_2) \\ &- \ S(\omega^6_{511}U_1 + \omega^8_{711}U_2 + \omega^{10}_{911}U_3) \ + \ \omega^9_{1011}U_1U_2 + \omega^5_{611}U_2U_3 + \omega^7_{811}U_1U_3. \end{aligned} \tag{5.192}$$

All terms in this superpotential, except for those in the last line, may be understood in terms of ordinary RR and NS backgrounds in the Type IIA orientifold supplemented by metric fluxes. Indeed, all those terms correspond to the fluxes e_0, e_i, h_0, h_i, q_i, a_i, and b_{ij}, described in Section 5.2.2. The absence of a $T_1T_2T_3$ term (type IIA mass parameter m) is expected since in the M-theory scheme considered massive IIA supergravity does not arise.

The new terms appearing in the last line are interesting. The first three correspond to the S-dual fluxes f_i introduced before in order to maintain S-duality in the IIB orientifold version of this model. Thus one has the interesting result that the f_i fluxes introduced before may be understood as certain ordinary metric fluxes

$$f_i \ = \ \omega^{K+1}_{K11} \quad , \quad K=5,7,9, \tag{5.193}$$

in an M-theory version of the same model. On the other hand the last three terms, bilinear in the U_i (T_i) in the IIA (IIB) version, are new and are absent even in the extended set of flux-induced superpotential terms discussed in Section 5.4. This suggests, as commented before, that there is an even bigger set of flux degrees of freedom to be considered. We will see now that the presence of new terms bilinear and cubic in the U_i's are also expected if we consider fluxes in the heterotic version of the same class of models.

5.5.2 Heterotic fluxes.

Type IIA orientifolds with O6-planes are mirror to Type IIB orientifolds with O9-planes, i.e. Type I String Theory, through the $\mathcal{M}_2$ operator in the diagram 3.34. On the other hand we know that Type I is related by S-duality to the $SO(32)$ heterotic string. Therefore, it is interesting to compare the induced superpotentials in both theories. Flux-induced heterotic superpotentials have been analyzed in [136, 137, 138, 139]. It has been argued that heterotic H-flux forces the internal manifold X_6 to be non-Kähler with $dJ \neq 0$. Both effects produce a superpotential

$$W_{\text{het}} \;=\; \int_{X_6} \Omega \wedge (\overline{H}_{\text{het}} \;+\; dJ_c). \tag{5.194}$$

It is interesting to evaluate W_{het} in the case of compactification on a factorized T^6 with arbitrary metric fluxes on top. The H-flux is a generic 3-form, namely

$$\overline{H}_{\text{het}} = -e_0\alpha_0 + m\beta_0 - \sum_{i=1}^{3}(q_i\alpha_i + e_i\beta_i)\ . \tag{5.195}$$

Our choice of parameters is dictated by the fact that by S-duality $\overline{H}_{\text{het}}$ is equal to the RR flux, given in Table 5.4, of IIB with O9-planes, alias Type I. Moreover, the heterotic metric fluxes are the same as those as in IIB/O9 shown in Table 5.3. We also need to use that in the toroidal compactification the heterotic complex structure moduli coincide with the geometric parameters, i.e. $U_i = \tau_i$. The Kähler moduli arise from $J_c = i\sum_j T_j\omega_j$. Putting all pieces together we find

$$\begin{aligned} W_{\text{het}} \;&=\; m + i\sum_{i=1}^{3} q_i U_i + e_1 U_2 U_3 + e_2 U_1 U_3 + e_3 U_1 U_2 - ie_0 U_1 U_2 U_3 \\ &+\; \sum_{i=1}^{3} T_i\big[-i\bar{h}_i + \sum_{j=1}^{3}\bar{b}_{ji}U_j + ib_{1i}U_2U_3 + ib_{2i}U_1U_3 + ib_{3i}U_1U_2 - h_iU_1U_2U_3\big]\ . \end{aligned} \tag{5.196}$$

Superpotentials of this kind have been recently considered in [140]. With isotropic choice of fluxes W_{het} agrees with results of [32].

Comparing with (5.134) shows that W_{het} matches W_{O9} except for the terms linear in S that are due to non-geometric fluxes R^{mno} in IIB/O9. Additional S-dependent terms in W_{O9} will appear if S-dual fluxes are included (T-dual to the P_o^{mn}). Thus, we conjecture that analogous dilaton-dependent superpotential terms will emerge in the heterotic side from new flux degrees of freedom R_{het} and P_{het}.

Moreover, we know that the 4-dimensional compactified heterotic strings are self T-duality invariant [141]. As a consequence, the complete Kähler function $\mathcal{G} = K + \log|W|^2$ should be invariant under the $SL(2,\mathbb{Z})^3$ heterotic T-duality symmetries. It is easy to convince oneself that this demands additional terms quadratic and cubic in the Kähler moduli T_i [142], thus realizing the complete $SL(2,\mathbb{Z})^7$ duality group. In the following section we will construct systematically the $SL(2,\mathbb{Z})^7$ invariant superpotential, along the lines of our work in [55].

5.5.3 Fluxes and $SL(2,\mathbb{Z})^7$ invariance.

The general flux superpotential will be a polynomial of degree up to seven on the moduli $M_I = (S, T_1, T_2, T_3, U_1, U_2, U_3)$ and at most linear on any of them. One can write this superpotential in the form

$$W \;=\; \sum_{n=0}^{7} D^{(n)}_{i_1...i_n}\; M_{i_1}...M_{i_n}, \tag{5.197}$$

where the $D^{(n)}$ are integer coefficients associated to generalized fluxes[2].

Each of the seven $SL(2,\mathbb{Z})_X$ factors consists of two generators

$$S_{X,1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad ; \quad S_{X,2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{5.198}$$

The action of a general element $\Lambda_X = \begin{pmatrix} k_X & \ell_X \\ m_X & n_x \end{pmatrix} \in SL(2,\mathbb{Z})$ on the modulus M_X is given by

$$M_X \rightarrow \frac{(k_X M_X - i\ell_X M_X)}{(im_X M_X + n_X)} \quad ; \quad k_X n_X - \ell_X m_X = 1 \quad ; \quad k_X, \ell_X, m_X, n_X \in \mathbb{Z}. \tag{5.199}$$

The toroidal Kähler potential transforms like

$$K \;\rightarrow K \;+\; \log|im_X M_X + n_X|^2, \tag{5.200}$$

and the complete Kähler function is invariant as long as the fluxes $D^{(n)}$ transform like

$$(D^{(n)}_{ijk..}, D^{(n+1)}_{xijk..}) \longrightarrow (D^{(n)}_{ijk..}, D^{(n+1)}_{xijk..}) \begin{pmatrix} n_X & m_X \\ \ell_X & k_X \end{pmatrix}. \tag{5.201}$$

The fluxes $D^{(n)}$ may be viewed as symmetric tensors of n indices, with all diagonal components vanishing, thus with binomial coefficient $\binom{7}{n}$ independent components. Hence, the total number of generalized fluxes is $\sum_{n=0}^{7}\binom{7}{n} = 2^7 = 2^{(h_{21}+h_{11}+1)}$. They provide the 128 components of a representation $(2,2,2,2,2,2,2)$ under $SL(2,\mathbb{Z})^7$. As explained in Appendix C, this in turn may be embedded into the spinorial $\mathbf{128}$ of $SO(7,7;\mathbb{Z})$. One can decompose the two Weyl spinors of fluxes accordingly to its $SU(7)$ tensorial structure

$$\begin{aligned} \mathbf{64} &= \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{21} \oplus \mathbf{35} \\ \mathbf{64'} &= \mathbf{1'} \oplus \mathbf{7'} \oplus \mathbf{21'} \oplus \mathbf{35'} \end{aligned} \tag{5.202}$$

The components of each representation are then given by

[2] General superpotentials of this type were considered previously in [32] from the point of view of gauged $\mathcal{N}=4$ supergravity.

Rep.				Flux Components			
1				e_0			
7′			e_i	h_0	h_i		
21		q_i	a_i	b_{ij}	f_i	h'_i	
35′	m	$\bar{a}_i$	$\bar{b}_{ij}$	g_{ij}	b'_{ij}	f'_i	e'_0
35	$\bar{h}_0$	$\bar{h}_i$	$\bar{g}_{ij}$	$\bar{b}'_{ij}$	g'_{ij}	e'_i	h'_0
21′		$\bar{f}_i$	$\bar{h}'_i$	$\bar{g}'_{ij}$	q'_i	a'_i	
7			$\bar{f}'_i$	m'	$\bar{a}'_i$		
1′				$\bar{h}'_0$			

Note that in the M-theory setting described above, only the representations **1**, **7′** and **21** appear explicitly [133].

In terms of component fluxes the full duality covariant superpotential (5.197) may be written as

$$
\begin{aligned}
W \;=\;& e_0 - i\sum_{i=1}^{3} h_i T_i + \frac{1}{2}\sum_{l\neq m\neq n} h'_l T_m T_n + i e'_0 T_1 T_2 T_3 \\
+\;& \Big(i h_0 - \sum_{i=1}^{3} f_i T_i - \frac{i}{2}\sum_{l\neq m\neq n} f'_l T_m T_n - h'_0 T_1 T_2 T_3 \Big) S \\
+\;& \sum_{i=1}^{3}\Bigg[\Big(-a_i + i\sum_{j=1}^{3} g_{ij} T_j - \frac{1}{2}\sum_{l\neq m\neq n} g'_{il} T_m T_n + i a'_i T_1 T_2 T_3 \Big) S \\
+\;& i e_i - \sum_{j=1}^{3} b_{ij} T_j - \frac{i}{2}\sum_{l\neq m\neq n} b'_{il} T_m T_n - e'_i T_1 T_2 T_3 \Bigg] U_i \\
+\;& \frac{1}{2}\sum_{r\neq s\neq t}\Bigg[\Big(i\bar{a}_r + \sum_{j=1}^{3} \bar{g}_{rj} T_j + \frac{i}{2}\sum_{l\neq m\neq n} \bar{g}'_{rl} T_m T_n - \bar{a}'_r T_1 T_2 T_3 \Big) S \\
-\;& q_r + i\sum_{j=1}^{3} \bar{b}_{rj} T_j - \frac{1}{2}\sum_{l\neq m\neq n} \bar{b}'_{rl} T_m T_n + i q'_r T_1 T_2 T_3 \Bigg] U_s U_t \\
+\;& \Bigg[-\Big(\bar{h}_0 + i\sum_{j=1}^{3} \bar{f}_j T_j - \frac{1}{2}\sum_{l\neq m\neq n} \bar{f}'_l T_m T_n + i\bar{h}'_0 T_1 T_2 T_3 \Big) S \\
+\;& i m + \sum_{j=1}^{3} \bar{h}_j T_j + \frac{i}{2}\sum_{l\neq m\neq n} \bar{h}'_l T_m T_n - m' T_1 T_2 T_3 \Bigg] U_1 U_2 U_3 .
\end{aligned}
\tag{5.203}
$$

The complexity of this superpotential makes its analysis difficult, except in particular cases like those we have discussed in previous sections. In any event it is clear that there are many parameters which should allow for new possibilities in fixing moduli. It is important to remark that these 128 flux degrees of freedom are not independent. We already saw how Bianchi identities and RR tadpoles strongly restrict the possible fluxes in the simpler case with 64 degrees of freedom. In the most general case analogous constraints should be fulfilled. It would be interesting to have close expressions for these constraints in the more general case.

Note that the above discussion does not imply that the effective action has full $SL(2,\mathbb{Z})^7$ duality invariance. Rather, the above discussion shows how the presence of each particular flux explicitly breaks the duality symmetries. As we have seen, some of these flux degrees of freedom have a simple interpretation as metric fluxes or explicit RR or NS backgrounds in some particular version (Type IIA or IIB orientifolds, heterotic, M-theory orbifold, ...) of compactified string theory. Some other fluxes do not admit a simple geometric interpretation and yet others are implied by Type IIB S-duality and/or Heterotic self-T-dualities. Yet all of the 128 fluxes may in general be present in the complete underlying theory.

Chapter 6

Flux induced SUSY-breaking in D-brane configurations.

We have described in Chapter 5 how compactifications of String Theory in presence of background fluxes give rise to non-trivial superpotentials in the four dimensional $\mathcal{N} = 1$ effective theory. These lead to a large landscape of vacua on which some, or even all, of the closed string moduli are stabilized. From the ten dimensional point of view, the lifting of some of the moduli from the massless spectrum is signaling changes in the topology of the internal manifold due to the backreaction of the fluxes. It is therefore expected that the dynamics of the D-branes, required to cancel the RR tadpoles and to embed the Standard Model, will be as well affected by these changes.

On this chapter we would like study how the background fluxes affect the D-branes, or more concretely, the gauge theories living inside them. In particular, we will compute the flux induced superpotentials for the open string moduli parametrizing the location and Wilson lines of the D-branes. In terms of the MSSM, this is equivalent to compute the potentials for the squarks, sleptons and Higgsses, or in other words, the soft supersymmetry breaking lagrangian. This has been our task in [20, 24].

6.1 Supersymmetry breaking and Soft-terms.

The algebra of $\mathcal{N} = 1$ supersymmetry appears as a natural mathematical construction providing us with a solution to the hierarchy problem. Moreover, having the setup of branes preserving $\mathcal{N} = 1$ in four dimensions avoids inconsistencies such as non-vanishing NSNS tadpoles. However, the present constraints coming from LEP situates the lowest masses for the sleptons, stop and sbottom above the 100 GeVs, so definitively we live in a non-supersymmetric vacuum.

The vacuum expectation value for the supersymmetry variations are of the form (see e.g. [143])

$$\langle \delta\psi \rangle = \sqrt{2}\epsilon \langle F \rangle, \tag{6.1}$$

$$\langle \delta\lambda \rangle = \epsilon \langle D \rangle. \tag{6.2}$$

Thus, there are two different ways to break spontaneously $\mathcal{N} = 1$ supersymmetry while preserving Lorentz invariance: by means of a F-term vev in the scalar potential (O'Raifeartaigh mechanism) or by means of a D-term vev (Fayet-Illiopoulos mechanism).

The F and D auxiliary fields can be determined from their own equations of motion. More concretely, one has

$$F_i = e^{G/2}(G^{-1})_i^j G_j - \frac{1}{4} f_{abk}(G^{-1})_i^k \lambda^a \lambda^b - (G^{-1})_i^k G_k^{jl} \psi_j \psi_l + \frac{1}{2}\psi_i G_j \psi^j, \tag{6.3}$$

$$D^a = i(Re\ f_{ab}^{-1})(gG^i T_i^{bj} \phi_j) + \frac{i}{2} f_{cb}^i \psi_i \lambda^c - \frac{i}{2} f_i^{*\ bc} \psi^i \lambda_c + \frac{1}{2}\lambda_a G^i \psi_i, \tag{6.4}$$

so non-vanishing vevs for $\delta\psi$ and/or $\delta\lambda$ can be generated by both, perturbative or non-perturbative effects.

F-breaking nicely does not spoil the cancellation of quadratic divergences, giving rise to a soft breaking of the supersymmetry. This is related with the so called non-renormalization theorems, which ensure that the superpotential does not receive radiative corrections, i.e. to each order in perturbation theory only D-terms are generated.

On the other hand, supersymmetry breaking through D-terms in principle produces quadratic divergences. However, imposing the additional constraint Tr $Y = 0$, Y being the generator of the $U(1)$ gauge group under which ϕ transforms, it is possible to guarantee that only logarithmic divergences are generated. In that case, Fayet-Illiopoulos terms [144] $\langle D \rangle$ appear in the scalar potential of the theory, e.g.

$$V = \frac{1}{2}|\langle D \rangle + e\phi^*\phi|^2. \tag{6.5}$$

Then, if $(1/e)\langle D \rangle < 0$ the effect of the D-term can be compensated by a vev of the mass term $\langle \phi^*\phi \rangle > 0$ so $V = 0$ and supersymmetry remains unbroken, although the gauge symmetry is being broken. Contrary, if $(1/e)\langle D \rangle > 0$ the effect cannot be compensated, $\langle \phi \rangle = 0$, $V \neq 0$ and supersymmetry is being broken. In the MSSM some of the sleptons and squarks do not have superpotential mass terms. Thus, D-breaking in the MSSM must be clearly subdominant or absent, as it could induce a disastrous color breaking, electromagnetism breaking or violation of the lepton number.

In any case, when local supersymmetry is broken, the super-Higgs effect takes place and a spin 1/2 Goldstino is generated

$$\eta = \langle e^{G/2} G^i \rangle \psi_i. \tag{6.6}$$

This gets combined with the gravitino so it becomes massive

$$m_{3/2} = \frac{1}{\kappa} \langle e^{G/2} \rangle. \tag{6.7}$$

From the point of view of the effective theory, there is a limited number of supersymmetry breaking couplings which can be added to the low-energy effective action without spoiling the cancellation of quadratic divergences. In particular,

$$\mathcal{L}_{soft} = -(m^2)_{ij} \phi_i \phi_j^* - \frac{1}{3!} A^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B^{ij} \phi_i \phi_j - \frac{1}{2} M^a \lambda_a \lambda_a + h.c. \tag{6.8}$$

In addition, one can consider the following extra piece [145]

$$\mathcal{L}_{soft}^{(2)} = -\frac{1}{2} \mu^{ij} \psi_i \psi_j + \frac{1}{2} C^{ijk} \phi_i \phi_j^* \phi_k^* + M_g^{ia} \psi_i \lambda_a + h.c. \tag{6.9}$$

The terms of $\mathcal{L}_{soft}^{(2)}$ in principle may lead to quadratic divergences. However, in the case on which the chiral multiplets are not singlet under the gauge group these are absent and one should consider them.

Note that there are still regions of the parameter space for which supersymmetry remains unbroken. Indeed, it is possible to prove that when

$$\begin{aligned} (m^2)_{ij} &= |\mu^{ij}|^2, \\ C^{ijk} &= -(h^{jkl})^* \mu_{il}, \end{aligned} \tag{6.10}$$

with the rest of the soft terms vanishing, the theory becomes supersymmetric and is described by a trilinear superpotential plus a supersymmetric mass term $\frac{1}{2}\mu_{ij}\phi_i\phi_j$.

Other interesting situations are given by

$$\begin{aligned} A^{ijk} &= -M h^{ijk}, \\ Tr\ (m^2) &= |M|^2, \end{aligned} \tag{6.11}$$

with the other soft terms vanishing and a superpotential of the form

$$W(\phi) = \frac{1}{3!} h^{ijk} \phi_i \phi_j \phi_k. \tag{6.12}$$

These conditions in a $\mathcal{N} = 4$ theory ensure that the theory remains ultraviolet finite to all orders in perturbation theory [146, 147, 148, 161] and in particular the soft parameters do not run under the renormalization group flow, although supersymmetry is being broken to $\mathcal{N} = 0^*$.

In superstring and supergravity theories the patterns (6.8) and (6.9) arise naturally when supersymmetry is being broken spontaneously in a hidden sector communicating with the observable sector through some messenger interactions, such as gravity. In fact, supersymmetry cannot be broken spontaneously in the observable sector since a D-term vev for the hypercharge group $U(1)_Y$ does not lead to an acceptable spectrum, and on the other hand, there is no gauge singlet whose F-term could develop a vev.

Thus, it results convenient to expand the superpotential and the Kähler potential in terms of the chiral superfields of the observable sector as

$$W = \hat{W}(h_m) + \frac{1}{2}\mu_{ij}(h_m)\Phi^i\Phi^j + \frac{1}{3!}Y_{ijk}(h_m)\Phi^i\Phi^j\Phi^k \ldots, \tag{6.13}$$

$$K = \hat{K}(h_m, h_m^*) + \tilde{K}_{\bar{i}j}(h_m, h_m^*)\overline{\Phi}^{\bar{i}}\Phi^j + \frac{1}{2}Z_{ij}(h_m, h_m^*)\Phi^i\Phi^j + h.c. + \ldots, \tag{6.14}$$

where h_m are the chiral superfields of the hidden sector. In some specific models the μ and Z terms can be forbidden by gauge invariance. This is not the case of the MSSM, where the two Higgs doublets have opposite hypercharges.

When supersymmetry is broken in the hidden sector, the auxiliary fields of some h_m superfields get a vev through eq. (6.3) and soft-susy breaking terms are generated in the observable sector due to the gravitational couplings between the h_m fields and the Φ fields. These can be explicitly computed in the so called flat-limit, on which $M_P \to \infty$ with $m_{3/2}$ fixed. In that case the non-renormalizable gravity corrections decouple and one has a globally supersymmetric lagrangian with soft susy-breaking terms which are functions of the above F^m, $\hat{K}$, $\tilde{K}$, etc. Explicit expressions can be found in [149].

In the particular case of String Theory compactifications, the role of the hidden sector is played by the closed string moduli of Section 3.1. More concretely, these are grouped into $\mathcal{N}=1$ chiral multiplets with F auxiliary components associated to the background fluxes [74]. A non trivial background flux in the internal manifold will correspond to a non-vanishing vev for the F auxiliary fields associated to the axiodilaton, complex structure and/or Kähler moduli. In this sense, the $\mathcal{N}=1$ chiral multiplets are acting as spurion superfields [22, 23], i.e. chiral fields to which the coupling constants of the theory are promoted. For example, promoting the gauge coupling to a superfield

$$S = g + M\epsilon\epsilon, \tag{6.15}$$

one has

$$\int d^2\epsilon S F^\alpha F_\alpha = \int d^2\epsilon g F^\alpha F_\alpha + M\lambda\lambda, \tag{6.16}$$

so the gaugino masses are given by the vacuum expectation value of the auxiliary field M, which is related to closed string backgrounds of String Theory.

6.2 D-brane low energy effective actions.

The idea is to compute the low energy effective actions for the field theories living inside the D-branes in the flux compactifications described in Chapter 5. Unfortunately, a complete analysis of the soft-supersymmetry breaking parameters induced by the above whole set of backgrounds is still missing. Here we will instead concentrate in the Type B(ecker) scenarios described in Section 4.3.1, i.e. on Type IIB configurations of D3 and D7 branes with constant NSNS and RR fluxes and vanishing magnetic fluxes. This was done in [20, 24]. Alternative approaches can be found in [19, 21, 25, 26, 27, 28].

We will adopt a microscopic local point of view by expanding directly the non-abelian Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions associated to the D3 and D7-branes. Our results will be thus generic in the sense that the only input is the local supergravity configuration around the brane. All the global information will be in some sense contained there by consistency with the supergravity equations of motion. The difficulty is then, given a global configuration which solves the supergravity equations of motion, to compute the local background around the brane.

6.2.1 Low energy effective action for D3-branes.

The non-abelian extension of the DBI and CS actions for a Dp-brane was worked by Myers [150] based on T-duality arguments. For the particular case of a D3-brane it is given by

$$\begin{aligned} S = \quad & -\mu_3 \int d^4\xi \, Tr \left[e^{-\phi} \sqrt{-\det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^{ij} E_{j\nu}] \, + \, \sigma F_{\mu\nu}) \, \det(Q_{ij})} \right] \\ & + \mu_3 \int Tr \left(P \left[e^{i\sigma \mathbf{i}_\phi \mathbf{i}_\phi} \left(\textstyle\sum_n C_n + \frac{1}{2} B_2 \wedge C_2 \right) e^{-B} \right] \, e^{\sigma F} \right) \ , \end{aligned} \tag{6.17}$$

where $P[M]$ denotes the pullback of the ten dimensional background field M onto the D3-brane worldvolume and

$$\begin{aligned} E_{mn} &= G_{mn} - B_{mn} \ , \\ Q^m{}_n &= \delta^m{}_n + i\sigma \, [\phi^m, \phi^p] \, E_{pn} \ , \\ \sigma &= 2\pi\alpha' \ . \end{aligned} \tag{6.18}$$

We will perform a local expansion around the position of the D3-brane, parametrized by the vevs of the six transverse scalars ϕ^m of $\mathcal{N} = 4$ Super Yang-Mills. Then, we will make the identification

$$x^m = 2\pi\alpha' \phi^m \ , \tag{6.19}$$

where ϕ^m, from now on, makes reference to $\langle \phi^m \rangle$.

Thus, let us expand (4.33)-(4.37) as[1]

$$
\begin{aligned}
(Z_1(x^m))^{-1/2} &= Z_1^{-1/2} + \frac{1}{2} K_{mn}\, x^m x^n + \dots\ , \\
(Z_2(x^m))^{1/2} &= Z_2^{1/2} + \dots\ , \\
\tau &= \tau_0 + \frac{1}{2}\tau_{mn}\, x^m x^n\ , \\
\chi_4 &= (\text{const.} + \frac{1}{2}\chi_{mn}\, x^m x^n + \dots)\, dx^0 dx^1 dx^2 dx^3\ , \\
G_{lmn}(x^m) &= G_{lmn} + \dots\ ,
\end{aligned}
\tag{6.20}
$$

where the coefficients in the right hand side are constant. Plugging this into eq. (6.17) and expanding will give us the low energy effective action of the gauge theory living in the world-volume of the D3-branes. This is a generalization of the expansion carried out in [19]. Note that, for stability reasons, we have not included linear terms in the above expansion, as they usually lead to linear contributions in the scalar potential.

In order to plug (6.20) into (6.17) we need first to integrate the RR and NSNS field-strengths

$$
\begin{aligned}
B_{mn} &= \frac{1}{6}\sigma i g_s (G_3 - G_3^*)_{lmn}\phi^l\ , \\
(C_4)_{0123} &= \frac{\sigma^2}{2}\chi_{mn}\phi^m\phi^n\ , \\
(C_6)_{0123mn} &= -\frac{\sigma}{6} Z_1^{-1}[*_6(G_3 + G_3^*)]_{mnp}\phi^p\ ,
\end{aligned}
\tag{6.21}
$$

where we have made use of eq. (4.14) and $*_6$ is taken with respect to the metric without taking into account the warp factor. The gauge choice actually contains global information about the internal manifold. We have taken here a symmetric gauge which puts on equal footing the six transverse coordinates. This is somehow natural for a toroidal reduction, since in that case there is no difference among the six internal coordinates.

In principle, one could think that the gauge choice should not affect the physics, and in some sense this is true, but there are some subtleties about it. On one hand, the CS action is a topological action and thus directly involves the gauge potentials. However, here we are expanding it perturbatively. This gives some unexpected behavior. One has to consider all the terms in the series expansion in order to have a gauge invariant physics. Since we are taking a truncation to the lowest orders in the expansion, i.e. to the soft terms, apparently our low energy effective action will depend on the gauge choice of the RR fields. On the other hand, B_2 is not invariant under ordinary gauge transformations $\delta B_2 = d\Lambda$ but rather under modified gauge transformation of the form [151]

$$
\delta B_2 = d\Lambda \quad , \quad \delta A = 2\pi\Lambda. \tag{6.22}
$$

[1] Actually, we have considered a slightly more general background than in (4.33)-(4.37), with a metric of the form $ds^2 = Z_1^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z_2^{1/2}ds^2_{CY}$.

Thus, the DBI-CS action is completely gauge invariant only after one considers the possibility of having non trivial magnetic backgrounds. Different gauges for B_2 will differ in non-trivial backgrounds for F_2.

Making use of eqs. (6.21) and plugging

$$\mathrm{Im}\,\tau = \mathrm{Im}\,\tau_0 + \frac{\sigma^2}{2}\mathrm{Im}\,\tau_{mn}\,\phi^m\phi^n \ , \tag{6.23}$$

$$\det(Q)^{1/2} = 1 - \frac{i\sigma}{2}\,B_{mn}\,[\phi^n,\phi^m] - \frac{\sigma^2}{4}Z_2\,[\phi^m,\phi^n]\,[\phi^m,\phi^n] \ , \tag{6.24}$$

$$[-\det(P[E_{\mu\nu}])]^{1/2} = Z_1^{-1}(1 - \frac{\sigma^2}{2}Z_1^{1/2}Z_2^{1/2}\,\partial_\mu\phi^m\,\partial_\mu\phi^m \, + \sigma^2 Z_1^{1/2}\,K_{mn}\,\phi^m\phi^n) \ , \tag{6.25}$$

$$(i_\phi i_\phi C_6)_{0123} = \frac{\sigma}{2}(C_6)_{0123mn}[\phi^n,\phi^m] \ , \tag{6.26}$$

into (6.17) we get the relevant terms of the bosonic low energy action for the gauge theory inside the D3-branes

$$\begin{aligned}
\mathcal{L}_{\mathrm{DBI}} = & \frac{\mu_3\sigma^2}{g_s}Z_1^{-1}\mathrm{Tr}\,[\,\frac{1}{2}Z_1^{1/2}Z_2^{1/2}\,\partial_\mu\,\phi^m\partial_\mu\phi^m - \frac{Z_2}{4}\,[\phi^m,\phi^n]\,[\phi^n,\phi^m] - \\
& - (Z_1^{1/2}K_{mn} + \frac{g_s}{2}\mathrm{Im}\,\tau_{mn})\,\phi^m\phi^n - \frac{g_s}{12}\,(G_3 - G_3^*)_{lmn}\,\phi^l\,[\phi^n,\phi^m]\ ] \ , \\
\mathcal{L}_{CS} \ = & \ \mu_3\sigma^2\,\mathrm{Tr}\left[\frac{1}{2}(\mathrm{Re}\ \tau)\,F_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{1}{2}\chi_{mn}\,\phi^m\phi^n - \frac{iZ_1^{-1}}{12} *_6(G_3 + G_3^*)_{lmn}\,\phi^l[\phi^n,\phi^m]\right] \ .
\end{aligned}$$

A similar expansion can be carried out for the fermionic action by considering the fermionic completion of the ten dimensional supersymmetric DBI-CS action [152]. This expansion in powers of the fermionic fields has been worked out in the literature by different methods (see e.g. [153, 154]). In particular, here we are interested in the piece giving rise to fermionic masses and kinetic terms

$$\mathcal{L}_{\mathrm{ferm.}} \ = \ \frac{\sigma^2\mu_3}{g_s}\left(-\frac{1}{2}\,\bar\Theta\,\Gamma^\mu\,D_\mu\,\Theta + \frac{1}{48}g_s\,\bar\Theta\,\Gamma^{pqr}\,\Theta\mathrm{Re}(*_6G - iG)_{pqr}\right) \ , \tag{6.27}$$

where the different terms are understood in the superspace formalism.

The coordinates of the $\mathcal{N}=2$ IIB superspace are given by (x^μ,θ^α), with θ^α a pair of real 16 component Majorana-Weyl spinors which parametrize the supersymmetry transformations. Arranging the open string Ramond states into a ten dimensional Majorana-Weyl spinor Θ, in the same way as we can arrange the NS states into a 10 dimensional vector field A^μ, and decomposing it accordingly to $SO(9,1)\to SO(3,1)\times SO(6)$, we get the fermionic analogous to eq. (6.19)

$$\begin{pmatrix}\theta^1\\ \theta^2\end{pmatrix} = 2\pi\alpha'\begin{pmatrix}a\\ b\end{pmatrix}\Theta \tag{6.28}$$

with $a^2+b^2=1$. The $SO(2)$ vector is used to fix the embedding of the D3-brane supersymmetry in the 10d $\mathcal{N}=2$ IIB supersymmetry, reflecting the κ-symmetry freedom. We will work in the choice $(a,b)=(1,0)$.

Performing the dimensional reduction of (6.27) accordingly to

$$\Gamma^\mu = Z_1^{1/4}\gamma^\mu \otimes 1 \qquad , \qquad \Gamma^m = Z_2^{-1/4}\gamma_{(5)} \otimes \gamma^m \ , \tag{6.29}$$

we get the fermionic soft term lagrangian

$$\mathcal{L}_{\text{ferm.}} = \frac{\mu_3\sigma^2 Z_1^{1/4}}{g_s}\left(-\frac{1}{2}\bar{\psi}^a\,\gamma^\mu\,D_\mu\psi^a + \frac{g_s}{96}Z_1^{-1/4}Z_2^{-3/4}(*_6G_3 - iG_3)_{mnp}\,\psi^a\,(\gamma^{mnp})_{ab}\,\psi^b + \text{h.c.}\right) .$$

Thus, putting everything together, going to the Einstein frame and rescaling all the fields in order to get rid of the warping and the global $\mu_3\sigma^2 = \frac{1}{2\pi}$ factors, we get the complete soft term lagrangian

$$\begin{aligned}\mathcal{L}_{\text{soft}} = \text{Tr}\Bigg[&-(\,Z_2^{-1/2}K_{mn} - \frac{1}{2}Z_1^{1/2}Z_2^{-1/2}\chi_{mn} + \frac{g_s}{2}Z_1^{-1/2}Z_2^{-1/2}\text{Im}\tau_{mn}\,)\phi^m\phi^n + \\ &+ \frac{ig_s\sqrt{2\pi}}{6}Z_1^{-1/4}Z_2^{-3/4}\,(*_6G_3 - iG_3)_{lmn}\,\phi^l\phi^m\phi^n + \\ &+ \frac{ig_s^{1/2}}{96}Z_1^{-1/4}Z_2^{-3/4}\,(*_6G_3 - iG_3)_{lmn}\,\psi\,\gamma^{lmn}\,\psi\Bigg] .\end{aligned} \tag{6.30}$$

It is interesting to express this result in terms of the local $SO(6) \to SU(3)\times U(1)$ local symmetry irreducible representations. Thus, taking

$$z^l = \frac{1}{\sqrt{2}}(x^{2l+2} + ix^{2l+3}) \quad , \quad \bar{z}^l = \frac{1}{\sqrt{2}}(x^{2l+2} - ix^{2l+3}) \ , \tag{6.31}$$

and decomposing the $\mathcal{N}=4$ vector supermultiplet in terms of $\mathcal{N}=1$ supermultiplets accordingly to eq. (B.3), we obtain three complex chiral $\mathcal{N}=1$ multiplets (Φ^i, Ψ^i) and a vector multiplet (A^μ, λ). Note that although we are assuming a concrete complex structure, the expressions thus obtained will remain valid for whatever other complex structure selected by the supergravity equations of motion.

The flux tensor G_{mnp} transforms as a reducible $\mathbf{20} = \mathbf{10} + \overline{\mathbf{10}}$ representation of $SO(6)$. The $\overline{\mathbf{10}}$ and $\mathbf{10}$ correspond respectively to the imaginary self-dual (ISD) G_3^+ and imaginary anti self-dual (IASD) G_3^- parts, defined as

$$G_3^\pm \ = \ \frac{1}{2}(G_3 \mp i *_6 G_3) \quad , \quad *_6G_3^\pm \ = \ \pm iG_3^\pm \ .$$

Under the $SO(6) \to SU(3)\times U(1)$ decomposition the $\mathbf{10}$ decomposes in $SU(3)$ irreducible representations as $\mathbf{10} = \mathbf{6} + \mathbf{3} + \mathbf{1}$. Thus, it is convenient to introduce the tensors [59]

$$\begin{aligned} S_{ij} &= \frac{1}{2}(\epsilon_{ikl}G_{j\bar{k}\bar{l}} + \epsilon_{jkl}G_{i\bar{k}\bar{l}}) \ , \\ A_{\bar{i}\bar{j}} &= \frac{1}{2}(\epsilon_{\bar{i}\bar{k}\bar{l}}G_{kl\bar{j}} - \epsilon_{\bar{j}\bar{k}\bar{l}}G_{kl\bar{i}}) \ , \end{aligned}$$

	ISD			IASD	
$SU(3)$ rep.	Form	Tensor	$SU(3)$ rep.	Form	Tensor
$\overline{1}$	$(0,3)$	$G_{\bar{1}\bar{2}\bar{3}}$	1	$(3,0)$	G_{123}
$\overline{6}$	$(2,1)_P$	$S_{\bar{i}\bar{j}}$	6	$(1,2)_P$	S_{ij}
$\overline{3}$	$(1,2)_{NP}$	A_{ij}	3	$(2,1)_{NP}$	$A_{\bar{i}\bar{j}}$

Table 6.1: $SU(3)$ decomposition of antisymmetric $G_{(3)}$ fluxes.

corresponding respectively to the **6** and the **3** of $SU(3)$. An analogous decomposition could be done for the ISD fluxes. We summarize the different flux representations in Table 6.1.

Using these definitions, the above soft term lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} = \mathrm{Tr}\Big[&- \left(2Z_2^{-1/2}K_{i\bar{j}} - Z_1^{1/2}Z_2^{-1/2}\chi_{i\bar{j}} + g_s Z_1^{-1/2}Z_2^{-1/2}(\mathrm{Im}\tau)_{i\bar{j}} \right)\Phi^i\Phi^{\bar{j}} - \\ &- \frac{1}{2}\left(2Z_2^{-1/2}K_{ij} - Z_1^{1/2}Z_2^{-1/2}\chi_{ij} + g_s Z_1^{-1/2}Z_2^{-1/2}(\mathrm{Im}\tau)_{ij} \right)\Phi^i\Phi^j \ + \mathrm{h.c.} + \\ &+ g_s\sqrt{2\pi}Z_1^{-1/4}Z_2^{-3/4}\left[\frac{1}{3}G_{123}\,\epsilon_{ijk}\,\Phi^i\Phi^j\Phi^k + \frac{1}{2}\epsilon_{\bar{i}\bar{j}\bar{l}}\,(S_{lk}-(A_{\bar{l}\bar{k}})^*)\,\Phi^{\bar{i}}\Phi^{\bar{j}}\Phi^k \ + \mathrm{h.c.}\right] + \\ &+ \frac{g_s^{1/2}}{2\sqrt{2}}Z_1^{-1/4}Z_2^{-3/4}\left[G_{123}\,\lambda\lambda + \frac{1}{2}\epsilon_{ijk}A_{\bar{j}\bar{k}}\,\Psi^i\lambda + \frac{1}{2}S_{ij}\,\Psi^i\Psi^j \ + \mathrm{h.c.}\right]\Big] , \end{aligned} \tag{6.32}$$

where we have defined

$$(\mathrm{Im}\tau)_{i\bar{j}} = \frac{1}{2i}(\tau_{i\bar{j}} - (\tau_{j\bar{i}})^*) \ , \tag{6.33}$$

$$(\mathrm{Im}\tau)_{ij} = \frac{1}{2i}(\tau_{ij} - (\tau_{\bar{j}\bar{i}})^*) \ . \tag{6.34}$$

Comparing this with equations (6.8) and (6.9) we identify the soft parameters as

$$\begin{aligned} m_{ij}^2 &= 2Z_2^{-1/2}K_{i\bar{j}} - Z_1^{1/2}Z_2^{-1/2}\chi_{i\bar{j}} + g_s Z_1^{-1/2}Z_2^{-1/2}(\mathrm{Im}\tau)_{i\bar{j}} \ , \\ B_{ij} &= 2Z_2^{-1/2}K_{ij} - Z_1^{1/2}Z_2^{-1/2}\chi_{ij} + g_s Z_1^{-1/2}Z_2^{-1/2}(\mathrm{Im}\tau)_{ij} \ , \\ A^{ijk} &= -h^{ijk}Z_1^{-1/4}Z_2^{-3/4}\frac{g_s^{1/2}}{\sqrt{2}}\ G_{123} \ , \\ C^{ijk} &= +h^{ijl}Z_1^{-1/4}Z_2^{-3/4}\frac{g_s^{1/2}}{2\sqrt{2}}(S_{lk}-(A_{\bar{l}\bar{k}})^*) \ , \\ M^a &= \frac{g_s^{1/2}}{\sqrt{2}}\ Z_1^{-1/4}Z_2^{-3/4}G_{123} \\ \mu_{ij} &= -\frac{g_s^{1/2}}{2\sqrt{2}}Z_1^{-1/4}Z_2^{-3/4}S_{ij} \ , \\ M_g^{ia} &= \frac{g_s^{1/2}}{4\sqrt{2}}Z_1^{-1/4}Z_2^{-3/4}\epsilon_{ijk}A_{\bar{j}\bar{k}} \ , \end{aligned} \tag{6.35}$$

and the Yukawa coupling h_{ijk}

$$h_{ijk} = 2\sqrt{2}g_{YM,33}\epsilon_{ijk} \ , \tag{6.36}$$

$$g_{YM,33} = \sqrt{2\pi g_s} \ . \tag{6.37}$$

It only rests to check the constraints imposed by the supergravity equations of motion. As we saw in Section 4.3.1, these establish relations between the warping, the five-form flux, the complex dilaton and the three form fluxes. We should expect something similar from the local point of view. Indeed, plugging (6.20) into the eqs. (4.8)-(4.10), one gets, to lowest order, the following constraints

$$\begin{aligned}
i\sum \tau_{l\bar{l}} &= \frac{Z_2^{-1}}{2}\left(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{lk}S_{\overline{lk}} + \frac{1}{4}A_{lk}A_{\overline{lk}}\right) ,\\
4Z_1^{1/2}\sum K_{l\bar{l}} &= \frac{g_s}{2}Z_2^{-1}\left(|G_{123}|^2 + |G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 + |S_{\bar{i}\bar{j}}|^2 + |A_{ij}|^2 + |A_{\bar{i}\bar{j}}|^2)\right) ,\\
-2Z_1\sum \chi_{l\bar{l}} &= \frac{g_s}{2}Z_2^{-1}\left(|G_{123}|^2 - |G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 - |S_{\bar{i}\bar{j}}|^2 - |A_{ij}|^2 + |A_{\bar{i}\bar{j}}|^2)\right) ,
\end{aligned}$$

so the 3-form fluxes determine through the equations of motion the trace of the scalar mass matrix

$$\begin{aligned}
m_1^2 + m_2^2 + m_3^2 = \frac{g_s}{2}Z_2^{-3/2}Z_1^{-1/2}\Bigg[&|G_{123}|^2 + \frac{1}{4}\sum_{ij}(|S_{ij}|^2 + |A_{\bar{i}\bar{j}}|^2) - \\
&- \mathrm{Re}\,(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{lk}S_{\overline{lk}} + \frac{1}{4}A_{lk}A_{\overline{lk}})\Bigg] . \qquad (6.38)
\end{aligned}$$

Hence, the masses are not fully determined by the fluxes and one is enforced to consider the complete configuration in order to fully determine them. This was already observed in [155].

6.2.2 Low energy effective action for D7-branes.

The case of $D7$-branes is rather more complicated than the one of $D3$-branes. Thus, we will take some additional simplifying assumptions. The $D7$-branes in general will be wrapping a 4-cycle Σ_4 on the compact space. Therefore an expansion on all the coordinates transverse to Minkowski has no longer any sense. Instead, we will consider a tubular neighborhood around the 4-cycle and will expand the background in the normal coordinate around this neighborhood. For simplicity we will center on the case on which the fibration is trivial. This allows only for the cases on which the 4-cycle is Calabi-Yau, i.e. $T^4 \times \mathbb{C}$ and $K3 \times \mathbb{C}$.

The geometric symmetry is now reduced to $SU(2) \times SU(2)' \times U(1)$, in agreement with the considerations of Appendix B. Therefore, it will result useful to decompose the flux under the breaking $SU(3) \times U(1) \to SU(2) \times SU(2)' \times U(1)$, i.e.

$$\mathbf{10} = (3,1)_- + (1,3')_+ + (2,2')_0 \ , \quad \overline{\mathbf{10}} = (3,1)_+ + (1,3')_- + (2,2')_0 \ ,$$

where the subindex refers to the ± 1 $U(1)$ charge. Choosing our 4-cycle to be parametrized by z^1 and z^2, and localized in the transverse direction z^3, the triplets of $SU(2)$ and $SU(2)'$ are related with the fluxes in $SU(3)$ notation by

$$\begin{aligned}
(1,3')_+ &= \{\mathcal{G}'_0 = \frac{-1}{\sqrt{2}}A_{\bar{1}\bar{2}},\ \mathcal{G}'_x = \frac{-1}{\sqrt{2}}(\frac{1}{2}S_{33} - G_{123}),\ \mathcal{G}'_y = \frac{-i}{\sqrt{2}}(\frac{1}{2}S_{33} + G_{123})\}\ , \\
(3,1)_- &= \{\mathcal{G}_0 = \frac{1}{\sqrt{2}}S_{12},\ \mathcal{G}_x = \frac{-1}{\sqrt{2}}(\frac{1}{2}S_{11} - \frac{1}{2}S_{22}),\ \mathcal{G}_y = \frac{-i}{\sqrt{2}}(\frac{1}{2}S_{11} + \frac{1}{2}S_{22})\}\ , \\
(1,3')_- &= \{G'_0 = \frac{-1}{\sqrt{2}}A_{12},\ G'_x = \frac{-1}{\sqrt{2}}(\frac{1}{2}S_{\bar{3}\bar{3}} - G_{\bar{1}\bar{2}\bar{3}}),\ G'_y = \frac{-i}{\sqrt{2}}(\frac{1}{2}S_{\bar{3}\bar{3}} + G_{\bar{1}\bar{2}\bar{3}})\}\ , \\
(3,1)_+ &= \{G_0 = \frac{1}{\sqrt{2}}S_{\bar{1}\bar{2}},\ G_x = \frac{-1}{\sqrt{2}}(\frac{1}{2}S_{\bar{1}\bar{1}} - \frac{1}{2}S_{\bar{2}\bar{2}}),\ G_y = \frac{-i}{2}(\frac{1}{2}S_{\bar{1}\bar{1}} + \frac{1}{2}S_{\bar{2}\bar{2}})\}\ .
\end{aligned}$$

Regarding the triplets of $SU(2)$ as vectors of $SO(3)$, one can identify then the $SU(2)$ invariant scalar product as

$$A \cdot B = A_0 B_0 + A_x B_x + A_y B_y. \tag{6.39}$$

The G_{mnp} components transforming like $(2,2)_0$ correspond to the SU(3) components $S_{i3}, A_{i3}, S_{\bar{i}\bar{3}}, A_{\bar{i}\bar{3}}$ with $i = 1, 2$. These fluxes are special in several aspects and will not be considered here. In particular, if Σ_4 contains 3-cycles C_3 , the $(2,2)_0$ multiplet contains fluxes such that

$$\int_{C_3} F_3 \neq 0 \quad ; \quad \int_{C_3} H_3 \neq 0. \tag{6.40}$$

This is for instance the case for T^4, on which much of our analysis centers. This is problematic because, as we will see in Section 7.1, non-zero integrals of H_3 on a D-brane cycle generate a world-volume tadpole for the gauge potential $\int_{D7} H_3 \wedge A_5$, rendering the configuration inconsistent [156] [2]. Moreover, in general, such fluxes along world-volume directions are quantized, and cannot be diluted away keeping the D7-brane physics four-dimensional. Thus, their presence can lead to qualitatively large changes in the four dimensional physics, which may not be well described with our perturbative techniques.

Still, the remaining fluxes transforming like $(3,1) + (1,3')$ contains the most interesting cases, and will lead to non-trivial effects. Those fluxes have always two legs in Σ_4, allowing us to associate to each of the above $SO(4) \times SO(2)$ representations a different 2-form in Σ_4. Thus, G_3 can be decomposed as

$$G_3 = \beta \wedge dz^3 + \beta' \wedge d\bar{z}^3 + \gamma \wedge dz^3 + \gamma' \wedge d\bar{z}^3\ , \tag{6.41}$$

where β, β' and γ, γ' are self-dual and anti self-dual 2-forms in Σ_4, namely

$$*_4\beta = \beta \quad ; \quad *_4\gamma = -\gamma \tag{6.42}$$

(and similarly for the primed forms), corresponding respectively to the $(1,3')_+$, $(1,3')_-$, $(3,1)_+$ and $(3,1)_-$ pieces of G_3. Then, the scalar product (6.39) between SU(2) triplets will induce a positive definite product in Σ_4 given by

$$\omega_1 \cdot \omega_2 = \int_{\Sigma_4} \omega_1 \wedge *_{\Sigma_4}\omega_2\ . \tag{6.43}$$

[2] This can be solved by introducing additional branes ending on the D7-branes [157], but this is outside the kind of configurations here considered.

For clarity we summarize in Table 6.2 the main properties of the induced 2-forms. These will be associated with the charge of D3-brane induced in the worldvolume of the D7-branes through Chern-Simons couplings and it will be on the root of the D7-brane soft terms.

Form	SD/ASD in Σ_4	Corresponding G_3 rep.	ISD/IASD flux
β	SD	$(1,3)_+$	IASD
β'	SD	$(1,3)_-$	ISD
γ	ASD	$(3,1)_+$	ISD
γ'	ASD	$(3,1)_-$	IASD

Table 6.2: Properties of the 2-forms induced by the flux in Σ_4.

With all of this, and assuming that our background is independent of the coordinates on the 4-cycle, the part of B_2 laying completely in Σ_4 is given by

$$B_2|_{\Sigma_4} = -\frac{g_s}{6i}\left((\beta-\beta^*)z^3 + (\beta'-\beta'^{\,*})z^{\bar{3}} + (\gamma-\gamma^*)z^3 + (\gamma'-\gamma'^{\,*})z^{\bar{3}}\right) \ , \tag{6.44}$$

with $(\beta^*)_{mn} = (\beta'_{\bar{m}\bar{n}})^*$, etc. In particular, notice the explicit dependence of the components of B_2 on the transverse coordinates. Actually, these will be the only components relevant in the computation of the soft terms.

The Myers' action for a $D7$-brane is given by[3]

$$\begin{aligned} S = \quad & -\mu_7 \int d^8\xi\, STr\left[e^{-\phi}\sqrt{-\det(P[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] + \sigma F_{\mu\nu})\det(Q_{ij})}\right] \\ & + \mu_7 g_s \int STr\,(\,P\,[\sigma C_6 F_2 + C_8 - C_6 B_2]) \ , \end{aligned}$$

with the definitions of the above section and where we will assume that the flux background is purely ISD or AISD, guaranteeing in this way that the dilaton background remains constant over the internal manifold[4] (c.f. eq. (4.9)).

For our particular background $P[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}]$ and $Q^i_{\ j}$ are given by

$$\begin{aligned} P[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] &= P[g_{\mu\nu}g_s^{1/2} - B_{ab}\delta^a_\mu\delta^b_\nu + \delta^a_\mu B_{am}(Q^{-1}-\delta)^{mn}B_{nb}\delta^b_\nu] \ , \\ Q^i{}_j &= \delta^i{}_j + i\sigma[\Phi^i,\Phi^k](G_{kj}g_s^{1/2} - B_{kj}) \ . \end{aligned}$$

We will start by computing the first determinant in the DBI piece of the action. Neglecting derivative couplings, it can be factorized between the Minkowski and the 4-cycle pieces as

$$\begin{aligned} & \det(P[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] + F_{\mu\nu}) = \\ = \quad & \det(g_{\mu\nu}g_s^{1/2} + 2g_s^{1/2}\sigma^2 Z_2^{1/2} D_\mu\Phi^3 D_\nu\Phi^{\bar{3}} + \sigma F_{\mu\nu})\cdot\det(g_s^{1/2}Z_2^{1/2} + \sigma F_{ab} - B_{ab} + \\ & + 2g_s^{1/2}\sigma^2 Z_2^{1/2} D_a\Phi^3 D_b\Phi^{\bar{3}} + B_{am}(Q^{-1}-\delta)^{mn}B_{nb} - \sigma B_{3(a}D_{b)}\Phi^3 - \sigma B_{\bar{3}(a}D_{b)}\Phi^{\bar{3}}) \ , \end{aligned}$$

[3] On this case we will work directly in the Einstein's frame. In the CS action we only show the pieces giving possible contributions to the soft terms.

[4] We saw in eq. (4.52) the dilaton background generated by a D7-brane is not constant. However, at lowest order in our expansion the supergravity equations of motion becomes linear and the backreaction decouples from the effect of the fluxes. This is the probe limit.

with a, b running over the internal coordinates of Σ_4 and

$$D_a\Phi^m = \partial_a\Phi^m + i[A_a, \Phi^m] = i[A_a, \Phi^m] \tag{6.45}$$

so there can be still possible contributions to the soft-terms coming from the covariant derivative in the non-abelian case.

We can expand now the determinants with the aid of the formula

$$\det(1+M) = 1 + Tr\ M - \frac{1}{2}Tr\ M^2 + \frac{1}{2}(Tr\ M)^2 + \dots\ , \tag{6.46}$$

where the dots in this and in further expressions refer to contributions giving rise to derivative couplings or couplings with dimension higher than four in the low energy effective action. Indeed, in the two cases we are centering on, namely $T^4 \times \mathbb{C}$ and K3$\times\mathbb{C}$, the derivative couplings do not give rise to contributions to the soft-terms, as the relevant fields have constant profiles in the 4-cycle.

Therefore,

$$\begin{aligned}\det(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}] + \sigma F_{\mu\nu}) = \\ = -g_s^4[Z_1^{-1}(\Phi^3, \Phi^{\bar{3}})Z_2(\Phi^3, \Phi^{\bar{3}})]^2 + 2g_s^4\sigma^2 Z_1^{-3/2}Z_2^{5/2}\partial_\mu\Phi^3\partial_\mu\Phi^{\bar{3}} - \\ - \frac{g_s^3}{2}Z_1^{-2}Z_2(B-\sigma F)_{ab}(B-\sigma F)_{\bar{a}\bar{b}} - g_s^4 Z_1^{-3/2}Z_2^{3/2}\sigma^2 F_{\mu a}F^{\mu a} + \\ + \sigma^2 g_s^{9/2}Z_1^{-2}Z_2^2[A_a, \Phi^3][A^a, \Phi^{\bar{3}}] + ig_s^4 Z_1^{-2}Z_2\sigma(B_{3a}[A^a, \Phi^3] + B_{\bar{3}a}[A^a, \Phi^{\bar{3}}]) + \dots\end{aligned} \tag{6.47}$$

The last term of this expression will survive to the KK reduction only in cases on which the Wilson line moduli and Φ^3 have profiles with different parity. This is not the case of compactifications on 4-cycles trivially fibered in the normal direction. Therefore, in what follows, we will include as well these terms in the final dots.

The second determinant of the DBI piece is much simpler to compute

$$\det(Q^i_j) = 1 + \sigma^2 Z_2([\Phi^3, \Phi^{\bar{3}}])^2 g_s\ . \tag{6.48}$$

Putting everything together and Taylor expanding the square root, we have the following eight dimensional action

$$\begin{aligned}\mathcal{L} = \mu_7 g_s STr\, Z_1^{-1}(\Phi^3, \Phi^{\bar{3}})Z_2(\Phi^3, \Phi^{\bar{3}})\Big(1 + \sigma^2 Z_1^{1/2}Z_2^{1/2}\partial_\mu\Phi^3\partial_\mu\Phi^{\bar{3}} - \\ - \frac{Z_2^{-1}}{2}(B_2|_{\Sigma_4} - \sigma F_2)\wedge *_4(B_2|_{\Sigma_4} - \sigma F_2) + \frac{\sigma^2 g_s^{-1}}{2}Z_1^{1/2}Z_2^{-1/2}F_{\mu a}F_{\mu a} - \\ - \frac{g_s}{2}Z_2\sigma^2([\Phi^3, \Phi^{\bar{3}}])^2 - \frac{g_s}{2}Z_2^{-1}\sigma^2[A^a, \Phi^3][\Phi^{\bar{3}}, A^{\bar{a}}] + \\ + Z_1 Z_2^{-1}(\sigma C_6\wedge F_2 + C_8 - C_6\wedge B_2)|_{\Sigma_4} + \dots\Big)\ .\end{aligned} \tag{6.49}$$

Concerning the CS piece, we can make use of eqs. (4.14), (4.15) and (6.41) to integrate the relevant RR and NS field strengths

$$C_6|_{M_4\times\Sigma_4} = -\frac{Z_1^{-1}}{6i}((\beta-\beta^*)z^3 - (\beta'-\beta'^*)\bar{z}^3 - (\gamma-\gamma^*)z^3 + (\gamma'-\gamma'^*)\bar{z}^3)\wedge dVol_{4d} + \dots \ ,$$
$$C_8|_{M_4\times\Sigma_4} = -\frac{g_s Z_1^{-1}}{36}\left[(\beta z^3 + \gamma'\bar{z}^3 - \beta'^*\bar{z}^3 - \gamma^* z^3)\wedge(\beta z^3 + \gamma'\bar{z}^3 - \beta'^*\bar{z}^3 - \gamma^* z^3) - \right.$$
$$\left. -(\beta'\bar{z}^3 + \gamma z^3 - \beta^* z^3 - \gamma'^*\bar{z}^3)\wedge(\beta'\bar{z}^3 + \gamma z^3 - \beta^* z^3 - \gamma'^*\bar{z}^3)\right]\wedge dVol_{4d} \ .$$

Plugging these expressions into (6.49) we finally get

$$\begin{aligned}\mathcal{L} = \mu_7 g_s\sigma^2 STr\, Z_1^{-1}(\Phi^3,\Phi^{\bar 3})Z_2(\Phi^3,\Phi^{\bar 3})\Big(\frac{1}{\sigma^2} + Z_1^{1/2}Z_2^{1/2}\partial_\mu\Phi^3\partial_\mu\Phi^{\bar 3} + \\ -\frac{g_s}{36}Z_2^{-1}(2\gamma^*\wedge *_4\gamma'\Phi^{\bar 3}\Phi^3 + 2\beta^*\wedge *_4\beta'\Phi^{\bar 3}\Phi^3 - \gamma'\wedge *_4\gamma'\Phi^{\bar 3}\Phi^{\bar 3} - \\ -\beta'\wedge *_4\beta'\Phi^{\bar 3}\Phi^{\bar 3} + h.c.)|_{\Sigma_4} - \frac{g_s}{3}(\beta'_{ab}\Phi^{\bar 3}A^aA^b + \gamma'_{ab}\Phi^{\bar 3}A^aA^b + h.c.) - \\ \frac{g_s^{-1}Z_2^{-1}}{4}F_{ab}F_{\bar a\bar b} + \frac{1}{2}g_s^{-1}Z_2^{-1/2}Z_1^{1/2}F_{\mu a}F_{\mu\bar a} - \frac{1}{2}g_s Z_2([\Phi^3,\Phi^{\bar 3}])^2 - \\ -\frac{1}{2}g_s Z_2^{-1}[A^a,\Phi^3][\Phi^{\bar 3},A^{\bar a}] + \dots\Big) \ ,\end{aligned} \tag{6.50}$$

which in terms of SU(3) irreducible representations can be rewritten as

$$\begin{aligned}\mathcal{L} = \mu_7 g_s\sigma^2 STr Z_1^{-1}(\Phi^3,\Phi^{\bar 3})Z_2(\Phi^3,\Phi^{\bar 3})\big(\frac{1}{\sigma^2} + Z_1^{1/2}Z_2^{1/2}\partial_\mu\Phi^3\partial_\mu\Phi^{\bar 3} - \\ -\frac{g_s}{18}Z_2^{-1}\big(\frac{1}{4}(S_{12})^2 + \frac{1}{4}(A_{12})^2 - \frac{1}{2}G_{\bar 1\bar 2\bar 3}S_{\bar 3\bar 3} - \frac{1}{4}S_{22}S_{11}\big)\Phi^{\bar 3}\Phi^{\bar 3} + h.c. - \\ -\frac{g_s}{18}Z_2^{-1}\big(|G_{\bar 1\bar 2\bar 3}|^2 + \frac{1}{4}|S_{\bar 3\bar 3}|^2 + \frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2 + |A_{ij}|^2)\big)\Phi^{\bar 3}\Phi^3 + \\ +\sum_{j,k,p=1,2}\frac{g_s}{3}\epsilon_{3jk}((S_{kp})^* + (A_{kp})^*)\Phi^3A^{[j}A^{\bar p]} + h.c. - \\ -\frac{g_s}{6}\epsilon_{ij3}S_{\bar 3\bar 3}A^iA^j\Phi^{\bar 3} + h.c. - \frac{2g_s}{3}G_{\bar 1\bar 2\bar 3}\Phi^{\bar 3}A^{[\bar 1}A^{\bar 2]} + h.c. + Z_2^{1/2}Z_1^{-1/2}\partial_\mu A^a\partial_\mu A^{\bar a} - \\ -g_s Z_2[A^a,A^{(b}][A^{\bar b)},A^{\bar a}] - \frac{g_s}{2}Z_2([\Phi^3,\Phi^{\bar 3}])^2 - \\ -\frac{g_s}{2}Z_2[A^a,\Phi^3][A^{\bar a},\Phi^{\bar 3}]\big) \ + \ \dots\end{aligned} \tag{6.51}$$

Note the cancellation of the contributions coming from the warping for backgrounds of black-brane type [114], where $Z_1 = Z_2$. Since, we are restricting here to Type B(ecker) solutions, from now on we will assume for simplicity $Z_1 = Z_2 \equiv Z$.

It is interesting as well to express (6.50) in $SU(2)\times SU(2)'\times U(1)$ manifest invariant variables. Then one has

$$\begin{aligned}\mathcal{L} = \mu_7\, g_s\, \sigma^2 \mathrm{STr}\,\Bigg[\, & \frac{1}{\sigma^2} + Z\partial_\mu\Phi^3\partial_\mu\Phi^{\bar{3}} - \\ & - \frac{g_s}{36} Z^{-1}\, \left[\mathcal{G}^*\cdot\mathcal{G}^* + (G')^*\cdot(G')^*\right]\, \Phi^3\Phi^3 + h.c. - \frac{g_s}{18} Z^{-1}\, \left(\mathcal{G}\cdot\mathcal{G}^* + G'\cdot(G')^*\right)\, \Phi^{\bar{3}}\Phi^3 - \\ & - \frac{g_s\sqrt{2}}{3}\Phi^3\left[\begin{pmatrix} A^{\bar{1}} & A^{\bar{2}}\end{pmatrix}(\mathcal{G}^*\cdot\vec{\sigma})\begin{pmatrix}A^1\\ A^2\end{pmatrix} + \begin{pmatrix} A^{\bar{1}} & A^2\end{pmatrix}(G'^*\cdot\vec{\sigma})\begin{pmatrix}A^1\\ A^{\bar{2}}\end{pmatrix} + h.c.\right] - \\ & - g_s Z\,[A^a, A^{(b}][A^{\bar{b})}, A^{\bar{a}}] + Z\partial_\mu A^a\partial_\mu A^{\bar{a}} - \frac{g_s}{2}Z([\Phi^3,\Phi^{\bar{3}}])^2 - \frac{g_s}{2}Z[A^a,\Phi^3][\Phi^{\bar{3}},A^{\bar{a}}]\ \Bigg]\ ,\end{aligned} \tag{6.52}$$

where $\vec{\sigma}$ is the vector of Pauli matrices given by

$$\vec{\sigma} = \{\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\}\ . \tag{6.53}$$

In this language, the effective action only depends on backgrounds of type $\mathcal{G}$ and G', and not on $\mathcal{G}'$ and G. In terms of the local R-symmetries this means that only backgrounds with negative $U(1)$ charge (i.e. $(3,1)_- + (1,3)_-$) appear in the effective action. In other words, only fluxes with a particular correlation between their self-duality properties in the threefold and their duality properties on the 4-cycle give rise to soft terms, while fluxes with the opposite correlation lead to cancellation between the DBI and the CS contributions. This is not surprising. As already commented, the fluxes, apart from the $D3/\overline{D3}$-brane charge induced on the bulk, lead to instantonic $D3/\overline{D3}$-brane charges on the worldvolume of the D7-branes accordingly to the duality properties of the induced 2-form in Σ_4. It is exclusively the confrontation between the charge in the bulk and the charge in the brane worldvolume what dictates the dynamics of the D7-branes, and thus the vanishing or non-vanishing of the soft-terms.

The action we have obtained is an eight dimensional action, and therefore it is still necessary to perform its reduction over Σ_4. This requires an exact knowledge of the topology of Σ_4. As usual, the number of scalars in four dimensions is determined by the number of zero modes of the eight dimensional scalars and the number of internal components of the gauge fields, counted by $h^{2,0}(\Sigma_4)$ and $h^{1,0}(\Sigma_4)$ respectively. Thus, in cases where the normal bundle is trivial, there will be only a four dimensional complex scalar arising from Φ^3. We will explicitly solve the simplest case of $\Sigma_4 = T^4$. In that case, $h^{2,0}(\Sigma_4) = 2$. The results however can be easily extended to more involved compactifications.

To perform the dimensional reduction over the 4-torus is easy, as the derivative couplings will not give contributions to the four dimensional action and the integration over T^4 is trivial.

The resulting four dimensional action is given by[5]

$$
\begin{aligned}
\mathcal{L} = \mathrm{Tr}\,\Big\{ & \partial_\mu \Phi^m \partial_\mu \Phi^{\bar m} - \\
& -\frac{g_s}{18} Z^{-2}\left(\frac{1}{4}[(S_{12})^*]^2 + \frac{1}{4}[(A_{12})^*]^2 - \frac{1}{2}(G_{\bar 1\bar 2\bar 3})^*(S_{\bar 3\bar 3})^* - \frac{1}{4}(S_{22})^*(S_{11})^*\right)\Phi^3\Phi^3 + \\
& + h.c. - \frac{g_s}{18} Z^{-2}\left(|G_{\bar 1\bar 2\bar 3}|^2 + \frac{1}{4}|S_{\bar 3\bar 3}|^2 + \frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2 + |A_{ij}|^2)\right)\Phi^{\bar 3}\Phi^3 + \\
& + \sum_{k,p=1,2} \frac{g_s^{1/2} g_{YM}}{6} Z^{-1}\epsilon_{ijk}((S_{kp})^* + (A_{kp})^*)\Phi^i\Phi^j\Phi^{\bar p} + h.c. - \\
& - \frac{g_s^{1/2} g_{YM}}{6} Z^{-1}\epsilon_{ij3} S_{\bar 3\bar 3}\Phi^i\Phi^j\Phi^{\bar 3} + h.c. - \frac{g_s^{1/2} g_{YM}}{9} Z^{-1} G_{\bar 1\bar 2\bar 3}\epsilon_{\bar i\bar j\bar k}\Phi^{\bar i}\Phi^{\bar j}\Phi^{\bar k} + h.c. - \\
& - g_{YM}^2[\Phi^i, \Phi^{(j}][\Phi^{\bar j)}, \Phi^{\bar i}]\ \Big\}\ , \qquad (6.54)
\end{aligned}
$$

with $g_{YM,77}$ defined by

$$
g_{YM,77}^2 \;=\; g_s(2\pi)^5(\alpha')^2\mathcal{V}^{-1}Z^{-1} \qquad (6.55)
$$

and $\mathcal{V}$ the volume of the 4-torus. Note that on this case the gauge coupling depends inversely on the warp factor. And thus, very warp-suppressed soft-terms will give rise to too small gauge couplings.

The fermionic action can be obtained from dimensional reduction of the supersymmetric extension of the DBI-CS action [152]. The $SO(3,1)\times SO(4)\times SO(2)$ invariant bilinears involving the 3-form flux are given by

$$
\begin{aligned}
& C\bar\Theta\Gamma^{mnp}\Theta[2ab(\mathrm{Re}G_3)_{mnp} - (a^2-b^2)(\mathrm{Im}G_3)_{mnp}] + \\
& \qquad + C'\bar\Theta\Gamma_{(7)}\Gamma^{mnp}\Theta[(a^2-b^2)(\mathrm{Re}G_3)_{mnp} - 2ab(\mathrm{Im}G_3)_{mnp}]\ , \qquad (6.56)
\end{aligned}
$$

where C and C' are two constants, and $\Gamma_{(7)} = -(i/8!)\epsilon_{ijklmnpq}\Gamma^i\Gamma^j\Gamma^k\Gamma^l\Gamma^m\Gamma^n\Gamma^p\Gamma^q$, with $i\ldots p$ running from 0 to 7 and a, b fixing the embedding of the D7-brane supersymmetry in the 10d $\mathcal{N}=2$ IIB supersymmetry through eq. (6.28). We will work in the choice $(a,b)=(1,0)$ as in the last section. The first term in (6.56) arises from contributions from the DBI piece of the eight dimensional supersymmetric action, whereas the second term comes from CS contributions.

Then, the fermionic masses in the four-dimensional action are given in terms of the four adjoint $\mathcal{N}=1$ fermions λ, Ψ^i, $i=1,2,3$, by

$$
\begin{aligned}
\mathcal{L}_{\text{ferm.}} = & 6\sqrt{2}i(C'+C)[(G_{\bar 1\bar 2\bar 3})^*\lambda\lambda + \frac{1}{2}(S_{\bar 3\bar 3})^*\Psi^3\Psi^3 + (A_{12})^*\Psi^3\lambda + \frac{1}{2}\sum_{ij=1,2} S_{ij}\Psi^i\Psi^j] + \\
& + 6\sqrt{2}i(C'-C)[G_{123}\lambda\lambda + \frac{1}{2}S_{33}\Psi^3\Psi^3 + A_{\bar 1\bar 2}\Psi^3\lambda + \frac{1}{2}\sum_{\bar i\bar j=\bar 1,\bar 2}(S_{\bar i\bar j})^*\Psi^i\Psi^j] + \mathrm{h.c.} \qquad (6.57)
\end{aligned}
$$

[5] We have rescaled all the fields by $Z^{-1/2}(2\pi)^{5/2}\alpha'\mathcal{V}^{-1/2}$ in order to have canonically normalized kinetic terms.

The coefficients C and C' can be determined easily by supersymmetry arguments. Indeed, an $S_{\bar{3}\bar{3}}$ background is ISD and primitive, and thus preserves an unbroken $\mathcal{N}=1$ supersymmetry. On the other hand, from (6.54), the scalar Φ^3 gets a mass term for such a background and hence the fermion Ψ^3 should get an equal mass. From this one has that $C+C' = -ig_s^{1/2}Z^{-1}/72$. Moreover, one may show from an analogous argument applied to $S_{\bar{1}\bar{2}}$, etc. that the second term in (6.57) is absent and hence $C=C'$. Thus, the final four dimensional fermion masses are

$$\mathcal{L}_{\text{ferm.}} = \frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}} Tr[(G_{\bar{1}\bar{2}\bar{3}})^*\lambda\lambda + \frac{1}{2}(S_{\bar{3}\bar{3}})^*\Psi^3\Psi^3 + (A_{12})^*\Psi^3\lambda + \frac{1}{2}\sum_{ij=1,2} S_{ij}\Psi^i\Psi^j] + \text{h.c.}$$

or in terms of $SO(4)\times SO(2)$ irreducible representations

$$\mathcal{L}_{\text{ferm.}} = \frac{g_s^{1/2}Z^{-1}}{12} Tr[\begin{pmatrix}\lambda\,,\,\Psi^3\end{pmatrix} i\sigma_y(G'^*\cdot\vec{\sigma})\begin{pmatrix}\lambda\\ \Psi^3\end{pmatrix} + \begin{pmatrix}\Psi^1\,\Psi^2\end{pmatrix} i\sigma_y(\mathcal{G}^*\cdot\vec{\sigma})\begin{pmatrix}\Psi^1\\ \Psi^2\end{pmatrix}] + \text{h.c.}$$

Again, note that only fluxes transforming like $(3,1)_-$ and $(1,3')_-$ appear, with the $SU(2)\times SU(2)'\times U(1)$ R-symmetry relating independently λ with Ψ^3, and Ψ^1 with Ψ^2.

Comparing to eqs. (6.8) and (6.9) we extract the soft parameters

$$\begin{aligned}
m^2_{1\bar{1}} &= m^2_{2\bar{2}} = 0 \;\;;\;\; B_{ij} \;=\; 0 \;\;,\; i,j\neq 3\;,\\
m^2_{3\bar{3}} &= \frac{g_sZ^{-2}}{18}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{4}\sum_{i,j=1,2}(|S_{ij}|^2+|A_{ij}|^2)\right)\;,\\
B_{33} &= \frac{g_sZ^{-2}}{9}\left(\frac{1}{4}(S_{12})^{*2} + \frac{1}{4}(A_{12})^{*2} - \frac{1}{2}(G_{\bar{1}\bar{2}\bar{3}})^*(S_{\bar{3}\bar{3}})^* - \frac{1}{4}(S_{22})^*(S_{11})^*\right)\;,\\
A^{ijk} &= -h^{ijk}\frac{g_s^{1/2}Z^{-1}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^*\;,\\
C^{ijk} &= -\frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}\left[\sum_{l=1,2}h^{jkl}(S_{li}+A_{li}) - h^{jk3}(S_{\bar{3}\bar{3}})^*\right]\;,\\
M^a &= \frac{g_s^{1/2}Z^{-1}}{3\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^*\;,\\
\mu_{33} &= -\frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}(S_{\bar{3}\bar{3}})^*\\
\mu_{ij} &= -\frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}S_{ij}\;,\; i,j=1,2\;,\\
M_g^{3a} &= \frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}(A_{12})^*\;,
\end{aligned} \tag{6.58}$$

with

$$h_{ijk} = 2\epsilon_{ijk}\sqrt{2}g_{YM,77} \tag{6.59}$$

the Yukawa coupling.

It only would rest to compute the constraints from the supergravity equations of motion (4.8)-(4.11). However, since the action does not depend explicitly on the warping or the 5-form field strength, one can easily check that in this case the equations of motion do not give any extra condition.

Note that there is no soft masses for the Wilson line scalars. This is expected as they are forbidden by gauge invariance in the eight dimensional action. One could think of more involved situations where they arise from derivative couplings. However, the number of four dimensional zero modes is a topological quantity corresponding to the first Betti number of the internal 4-cycle and thus it cannot be changed by topologically trivial backgrounds.

As commented, these results can be easily extended to more general situations. In particular, in a realistic compactification, due to the backreaction of distant D-branes and O-planes, one would expect the background to vary along the 4-cycle. The computation is however very similar, differing just in the appearance of new derivative couplings. The KK reduction would proceed as well in the same way, but now the backgrounds are not pulled out from the integrals over the 4-cycle and the soft parameters have the same structure than in (6.58) with coefficients being integral convolutions of the background with the internal wavefunctions of the four dimensional zero modes. Thus, in the particular case of a T^4, as the internal profiles are constant, the structure of the soft parameters would be exactly the same but with the fluxes replaced by its average values over the 4-torus.

The case $K3 \times \mathbb{C}$ can be solved as well with very little modification. In fact, the expression (6.50) is still valid, but now the 2-homology is different. There is one (2,0)-form, one (0,2)-form and 20 (1,1)-forms. The internal product (6.43) endows a 22-dimensional space with signature (3,19), namely there are 3 selfdual and 19 anti-selfdual 2-forms. So the above forms β, β', γ and γ' are linear combinations of these. The other difference concerns the KK reduction. Since K3 has no harmonic 1-forms, the KK reduction of the internal components of the eight dimensional gauge field leads to no massless scalars. Therefore, this system provides the simplest realization of a D7-brane system where all the D7-brane moduli are fixed, being specially relevant for KKLT scenarios [29], as discussed in Appendix D. The stabilization of all D7-brane moduli for this kind of system was already discussed in [158]. For an alternative approach based on the formalism of calibrations, the reader may as well consult [159].

The stabilization of the D7-brane moduli by the flux background is a generic feature which can be extended to situations where the normal bundle is no longer trivial and there are multiple geometric moduli coming from the KK reduction of Φ^3. In that case, one would expect additional terms in the eight dimensional action coming from the metric background. However, the mass term for Φ^3 would be still present thus eliminating all the possible zero modes of the KK reduction.

A similar situation occurs when there are several stacks of D7-branes warping different 4-cycles in the transverse space. The fields of the 7_i7_j sector can be understood as parametrizing a possible brane recombination and, in particular, they are mapped to the geometric moduli of the recombined brane. Thus, one expects as well the 7_i7_j moduli to be stabilized by the flux.

6.2.3 Low energy effective action for the 37 sector.

In this case there is no analogue of the DBI+CS action, so one is enforced to take a different approach. In order to seed some light, one may look at the quantization of the open strings laying between the D3 and the D7-branes, as we did in Section 3.2. There we saw (c.f. eq. (3.39)) that the effect of the flux is to give non vanishing masses to the complex scalars of the twisted sector

$$\frac{i}{2\pi\alpha'}(\, B_{1\bar{1}} + B_{2\bar{2}}\,)\,(\Phi_{73}\,\Phi_{73}^* - \Phi_{37}\Phi_{37}^*) \,+\, \text{h.c.} \tag{6.60}$$

Looking at eq. (6.21), one has that

$$B_{1\bar{1}} \;+\; B_{2\bar{2}} \;=\; -\frac{g_s}{6i}\left(A_{12}\bar{z}^3 \;-\; A_{\bar{1}\bar{2}}z^3 \;+\; (A_{12})^*z^3 \;-\; (A_{\bar{1}\bar{2}})^*\bar{z}^3\right)\;, \tag{6.61}$$

and the above terms lead to the trilinear couplings

$$\frac{g_s}{6}[(A_{12})^* - A_{\bar{1}\bar{2}}]\Phi_{77}^3\Phi_{73}\Phi_{73}^* \;+\; \text{h.c} \tag{6.62}$$

All the terms obtained in this way correspond exclusively to a would be DBI action. However, there should be still additional contributions coming from the RR fields. In order to obtain the full soft lagrangian for the twisted sector one may instead exploit the symmetries of the system. In fact, we saw in eq. (6.58) that a $\mathcal{N}=1$ SUSY preserving $S_{\bar{3}\bar{3}}$ flux gives rise to a superpotential mass term of the form

$$W_\mu^{(77)} \;=\; -\frac{1}{2}\mu_{(77)}\Phi_{77}^3\Phi_{77}^3 \;=\; \frac{g_s^{1/2}Z^{-1}}{12\sqrt{2}}(S_{\bar{3}\bar{3}})^*\Phi_{77}^3\Phi_{77}^3\;. \tag{6.63}$$

This, together with the superpotential of the 37 sector,

$$W_{37} \;=\; \Phi_{77}^3\Phi_{73}\Phi_{37} \;-\; \Phi_{33}^3\Phi_{37}\Phi_{73}\;, \tag{6.64}$$

yields the scalar potential

$$V \;=\; |\mu_{(77)}\Phi_{77}^3 \;-\; \Phi_{73}\Phi_{37}|^2\;, \tag{6.65}$$

whose crossed term leads to the coupling

$$\frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}(S_{\bar{3}\bar{3}})^*\Phi_{77}^3\Phi_{73}^*\Phi_{37}^* \;+\; \text{h.c.} \tag{6.66}$$

Covariantizing now by the geometric $SU(2)\times SU(2)'\times U(1)$ symmetry, then gives rise to

$$\begin{aligned}\frac{g_s^{1/2}Z^{-1}}{6}(\Phi_{37}^* \quad \Phi_{73})(G'^*\cdot\vec{\sigma})\begin{pmatrix}\Phi_{37}\\ \Phi_{73}^*\end{pmatrix}\Phi_{77}^3 = \frac{g_s^{1/2}Z^{-1}}{6\sqrt{2}}\big[(S_{\bar{3}\bar{3}})^*\Phi_{77}^3\Phi_{73}^*\Phi_{37}^* + \\ + 2(G_{\bar{1}\bar{2}\bar{3}})^*\Phi_{77}^3\Phi_{73}\Phi_{37} \;+ (A_{12})^*\Phi_{77}^3\Phi_{37}\Phi_{37}^* \;+\; (A_{12})^*\Phi_{77}^3\Phi_{73}\Phi_{73}^* \;+\; h.c.\big]\;.\end{aligned} \tag{6.67}$$

Note how indeed the first trilinear scalar coupling of (6.62) reappears here from a completely different argumentation, while the second has been cancelled presumably by RR flux contributions.

Analogously, one can apply the same argument to backgrounds in the $(1,3')_+$ representation. By looking at (6.35), one observes that there is a superpotential mass term in of the form

$$W_\mu^{(33)} \;=\; -\frac{1}{2}\mu_{(33)}\Phi^3_{33}\Phi^3_{33} \;=\; \frac{g_s^{1/2}Z^{-1}}{4\sqrt{2}}S_{33}\Phi^3_{33}\Phi^3_{33}\ . \tag{6.68}$$

Thus, proceeding as before one obtains

$$\begin{aligned}\frac{g_s^{1/2}Z^{-1}}{2}(\Phi_{37}\quad \Phi^*_{73})(\mathcal{G}'\cdot\vec{\sigma})\begin{pmatrix}\Phi^*_{37}\\ \Phi_{73}\end{pmatrix}\Phi^3_{33} = -\frac{g_s^{1/2}Z^{-1}}{2\sqrt{2}}\big[S_{33}\Phi^3_{33}\Phi^*_{37}\Phi^*_{73} +\\ + 2G_{123}\Phi^3_{33}\Phi_{37}\Phi_{73} \;+\; A_{\bar{1}\bar{2}}\Phi^3_{33}\Phi_{73}\Phi^*_{73} \;+\; A_{\bar{1}\bar{2}}\Phi^3_{33}\Phi_{37}\Phi^*_{37} \;+\; h.c.\big]\ .\end{aligned} \tag{6.69}$$

6.3 Soft supersymmetry breaking patterns in D3/D7 configurations.

Now that we have computed the low energy effective action for the setups of D3 and D7-branes in Type IIB/O3 orientifolds without non-geometric fluxes, let us discuss the different supersymmetry breaking patterns arising. We will not discuss the case of non-primitive fluxes, since as commented in Section 4.3.1, the corresponding forms do not exist in the cohomology of a Calabi-Yau orientifold.

ISD backgrounds.

As we saw in Section 4.3.1, these are the only allowed 3-form backgrounds by the supergravity equations of motion for Type IIB orientifolds with O3/O7-planes. Thus, in that case the soft supersymmetry breaking pattern depends on the two possible kinds of ISD fluxes allowed by the cohomology of the orientifold, i.e. (0,3) forms and (2,1) primitive forms.

i) ISD (0,3) backgrounds.

They correspond to the $SU(3)$ singlet $G_{\bar{1}\bar{2}\bar{3}}$. As we saw in Section 4.3.1, ISD (0,3) fluxes break supersymmetry to $\mathcal{N}=0^*$, giving rise to no-scale models. A consequence of the no-scale structure of the scalar potential is that, as occurs with the contributions of the Kähler moduli, the contributions of the D3-brane geometric moduli to the potential are as well cancelled. Due to this, it does not appear soft supersymmetry breaking terms in the worldvolume of the D3-branes at tree level, even though supersymmetry is being broken. The situation resembles the sequestered scenarios of [160], where the soft-masses are acquired through quantum effects: radiative loop corrections of the order of the warp suppressed Planck scale, due to the exchange of T and S fields, and Weyl-anomaly contributions of the order of the gravitino mass $m_{3/2}$. However, in this case the dynamics is clearly dominated by the exchange of moduli fields and thus the sequestering actually does not take place, at least in the sense of [160].

Things are different in the worldvolume of D7-branes. The confrontation between the instantonic D3-brane charges in the worldvolume of the D7-branes and the bulk D3-brane charges, gives rise to a non-trivial potential for the geometric moduli of the D7-branes. In this way, there appear non-trivial soft-terms given by

$$m^2_{\Phi^3_{77}} = \frac{g_s}{18} Z^{-2} |G_{\bar{1}\bar{2}\bar{3}}|^2 \ ; \ M^{(77)} = \frac{g_s^{1/2} Z^{-1}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^* \ ; \ A^{ijk\ (77)} = -h^{ijk} \frac{g_s^{1/2} Z^{-1}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^* \tag{6.70}$$

and

$$A_{\Phi^3_{77}(73)(37)} = -\frac{g_s^{1/2} Z^{-1}}{3\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^* \ . \tag{6.71}$$

Note that in particular the relations (6.11) are satisfied and the theory remains finite to all orders in perturbation theory. This can be understood from the point of view of holography. Indeed, the dependence of the complex dilaton τ on the internal coordinates corresponds in the CFT side to the running of the gauge coupling with the renormalization group scale. Since here we are considering constant dilaton solutions, the beta function vanishes and the theory remains finite. Actually, it was shown in [161, 148] that the finiteness conditions for a softly broken $\mathcal{N} = 4$ theory are in fact more general, allowing for

$$m_1^2 + m_2^2 + m_3^2 = M^2 \ . \tag{6.72}$$

This is indeed what we obtained in eq. (6.38) for the D3-branes: only the trace of the scalar mass matrix is determined through the supergravity equations of motion.

ii) ISD (2,1) primitive backgrounds.

As discussed in Section 4.3.1, these components of the flux preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. For the rest, the situation is analogous to the one for (0,3) fluxes: vanishing soft-terms in the worldvolume of the D3-branes and non-trivial ones in the worldvolume of the D7-branes, although $\mathcal{N} = 1$ supersymmetric. Indeed, by looking at (6.58) one observes

$$\mu_{(77)} = -\frac{g_s^{1/2} Z^{-1}}{6\sqrt{2}} (S_{\bar{3}\bar{3}})^* \quad , \quad C^{ijk\ (77)} = \frac{g_s^{1/2} Z^{-1}}{6\sqrt{2}} h^{jk3} (S_{\bar{3}\bar{3}})^* \ , \tag{6.73}$$

so the relation (6.10) is satisfied. In addition, there is a trilinear coupling in the twisted sector

$$A_{\Phi^3_{77}(73)^*(37)^*} = -\frac{g_s^{1/2} Z^{-1}}{6\sqrt{2}} (S_{\bar{3}\bar{3}})^* \ . \tag{6.74}$$

When a $G_{\bar{1}\bar{2}\bar{3}}$ flux is switched on, a non-supersymmetric B-term is as well generated for the scalars Φ^3_{77}

$$B_{33} = -\frac{g_s}{18} Z^{-2} \ (G_{\bar{1}\bar{2}\bar{3}})^* (S_{\bar{3}\bar{3}})^* = 2M\mu_{(77)} \ . \tag{6.75}$$

IASD backgrounds.

The reader my wonder why we have computed the soft supersymmetry breaking patterns induced by IASD fluxes, even though these do not satisfy the supergravity equations of motion for the Type B(ecker) solutions discussed in Section 4.3.1. The reason is twofold. First of all, our local approach is in some sense more general than Type B(ecker) solutions, and remains valid for other possible globally consistent backgrounds. For example, non-perturbative effects such as Euclidean D3-branes or gaugino condensation in distant D7-branes could backreact the geometry around our local setup, inducing non vanishing IASD components of the flux. In other cases, the supergravity equations of motion allow for IASD components. We refer the reader to [162] for a concrete example.

The second reason is to take into account the possibility of having the MSSM in the worldvolume of antibranes. Indeed, to compute the worldvolume action for an antibrane one has to take the opposite GSO projection than for a brane. This selects the opposite DBI-CS cancellation, and thus exchanges ISD and IASD fluxes. I.e. the soft supersymmetry breaking pattern induced in an $\overline{D3}$-brane by ISD fluxes is exactly the same than the soft-supersymmetry breaking induced in a $D3$-branes upon exchanging ISD by IASD. Concerning the $\overline{3}7$ sector, one can directly obtain from eqs. (6.67) and (6.69) the corresponding trilinear couplings by making the replacements

$$\begin{aligned} (A_{12}, S_{\bar{3}\bar{3}}, 2G_{\bar{1}\bar{2}\bar{3}}) &\longrightarrow (-S_{12}, S_{11}, S_{22}) \;, \\ (A_{\bar{1}\bar{2}}, S_{33}, 2G_{123}) &\longrightarrow (-S_{\bar{1}\bar{2}}, S_{\bar{1}\bar{1}}, S_{\bar{2}\bar{2}}) \;. \end{aligned} \tag{6.76}$$

i) IASD (3,0) backgrounds.

These somehow gives rise to the opposite DBI-CS cancellation than the (0,3) ISD fluxes. Indeed, these backgrounds do not lead to soft terms in the worldvolume of the D7-branes, however, they produce a dilaton dominated SUSY breaking pattern in the worldvolume of the D3-branes

$$m^2_{\Phi^3_{33}} = \frac{g_s}{2} Z^{-2} |G_{123}|^2 \; ; \; M^{(33)} = \frac{g_s^{1/2} Z^{-1}}{\sqrt{2}} G_{123} \; ; \; A^{ijk\;(33)} = -h^{ijk} \frac{g_s^{1/2} Z^{-1}}{\sqrt{2}} G_{123} \tag{6.77}$$

with the corresponding trilinear coupling in the 37 sector

$$A_{\Phi^3_{33}(37)(73)} = \frac{g_s^{1/2} Z^{-1}}{\sqrt{2}} G_{123} \; . \tag{6.78}$$

ii) IASD (1,2) primitive backgrounds.

These backgrounds produce a great variety of soft terms. On one hand in the D7-branes we have

$$B_{33} = \frac{g_s Z^{-2}}{36}\left((S_{12})^{*2} - (S_{22})^*(S_{11})^*\right) \ , \tag{6.79}$$

$$C^{ijk}_{(77)} = -\frac{g_s^{1/2} Z^{-1}}{6\sqrt{2}} \sum_{l=1,2} h^{jkl} S_{li} \ , \tag{6.80}$$

$$\mu_{ij} = -\frac{g_s^{1/2} Z^{-1}}{6\sqrt{2}} S_{ij} \quad i,j=1,2 \ . \tag{6.81}$$

Excepting S_{33} which does not appear in the action, these are all non-supersymmetric soft terms. On the other hand, in the D3-branes

$$m_1^2 + m_2^2 + m_3^2 \ = \ \frac{g_s}{8} Z^{-2} \sum_{ij} |S_{ij}|^2 \ ; \ C^{ijk} \ = \ -h^{ijl}\,\mu_{kl} \ ; \ \mu_{ij} \ = \ -\frac{g_s^{1/2} Z^{-1}}{2\sqrt{2}}\, S_{ij} \ . \tag{6.82}$$

which are compatible with $\mathcal{N}=1$ supersymmetry[6].

And finally, for the twisted sector we have the trilinear coupling

$$A_{\Phi^3_{33}(37)^*(73)^*} = \frac{g_s^{1/2} Z^{-1}}{2\sqrt{2}} S_{33} \ . \tag{6.83}$$

6.4 Comparison to effective supergravity predictions.

The above scheme of soft-terms can be nicely interpreted in terms of $\mathcal{N}=1$ effective supergravity [163, 74, 27]. In this sense, the background fluxes induce non-trivial vacuum expectation values for the F auxiliary fields in (6.1) associated to the axiodilaton and the Kähler moduli of the compactification. Indeed, the perturbative piece of eq. (6.3) can be recast as

$$F_m = e^{G/2} K_{m\bar{n}} G^{\bar{n}} \ . \tag{6.84}$$

Making use of eqs. (3.31) and (5.5), then one has

$$F_S \ = \ \frac{1}{M_p^2}\,(S+S^*)^{1/2}\,(T+T^*)^{-3/2}\,\kappa^{-2} \int \overline{G}_3 \wedge \Omega \ , \tag{6.85}$$

$$F_T \ = \ -\frac{1}{M_p^2}\,(S+S^*)^{-1/2}\,(T+T^*)^{-1/2}\,\kappa^{-2} \int G_3 \wedge \Omega \ , \tag{6.86}$$

where, for simplicity, we have considered just a single overall Kähler modulus T. Thus, we observe that ISD $(0,3)$ backgrounds actually corresponds to $\langle F_S\rangle = 0$ and $\langle F_T\rangle \neq 0$, whereas AISD $(3,0)$ backgrounds lead to dilaton dominated scenarios with $\langle F_S\rangle \neq 0$ and $\langle F_T\rangle = 0$.

On the other hand, the gravitino mass (c.f. eq. (6.7)) is given by

$$m^2_{3/2} \ = \ \frac{1}{M_p^4}\,(S+S^*)^{-1}\,(T+T^*)^{-3}\,\kappa^{-4}\,|\int G_3 \wedge \Omega\,|^2 \ , \tag{6.87}$$

[6] The underlying reason to this $\mathcal{N}=1$ structure is presumably the relation of these backgrounds with the $\mathcal{N}=1$ supersymmetric backgrounds of Polchinski-Strassler [155].

and the cosmological constant becomes

$$\Lambda = \frac{\kappa^{-4}|\int \overline{G}_3 \wedge \Omega|^2}{M_p^2(S+S^*)(T+T^*)^3} \ , \tag{6.88}$$

in analogy with eq. (5.21).

Now we would like to marginally deform this picture to include the open string moduli. It was found in [163] (see also [27]) that the gauge kinetic function f_7 (f_3) and the Kähler potential K for the D7 (D3) worldvolume action are

$$\begin{aligned}
f_3 \ &= \ S \quad , \quad f_7 \ = \ T_3 \ , \\
K \ &= \ -\log(S+S^*-|\Phi_{77}^3|^2) - \log(T_3+T_3^*-|\Phi_{33}^3|^2) \\
&- \ \log(T_2+T_2^*-|\Phi_{33}^2|^2-|\Phi_{77}^1|^2) - \log(T_1+T_1^*-|\Phi_{33}^1|^2-|\Phi_{77}^2|^2) \\
&+ \ \frac{|\Phi_{37}|^2+|\Phi_{73}|^2}{(T_1+T_1^*)^{1/2}(T_2+T_2^*)^{1/2}} \ ,
\end{aligned}$$

which for large overall Kähler modulus T becomes

$$\begin{aligned}
f_3 \ &= \ S \quad , \quad f_7 \ = \ T \ , \\
K \ &= \ -\log(S+S^*) - 3\log(T+T^*) \\
&+ \ \frac{|\Phi_{77}^3|^2}{(S+S^*)} + \frac{1}{(T+T^*)}\left[\ (\sum_{a=1}^{3}|\Phi_{33}^a|^2) \ + \ (\sum_{b=1}^{2}|\Phi_{77}^b|^2) \ + \ (|\Phi_{37}|^2+|\Phi_{73}|^2) \ \right] \ .
\end{aligned} \tag{6.89}$$

The soft terms then can be extracted from the scalar potential generated by (5.5) and (6.89). Thus, for ISD 3-form fluxes one correctly recovers the expressions (6.70), (6.71) and (6.73)-(6.75). In fact, in that case the scalar potential becomes positive definite

$$V_{ISD} = e^K \left(g^{3\bar{3}}(D_3W)(\overline{D}_{\bar{3}}\overline{W}) \right) \ , \tag{6.90}$$

and, after rescaling the matter fields to its canonical value, one has

$$\begin{aligned}
V_{ISD} \ &= \ | \ -M_{77}^*{\Phi_{77}^3}^* \ + \ \partial_{\phi^3}W \ |^2 = \\
&= -M_{77}^*{\Phi_{77}^3}^* \ - \ \mu_{(77)}\Phi_{77}^3 \ + \ \Phi_{77}^1\Phi_{77}^2 \ + \ \Phi_{73}\Phi_{37} \ |^2 \ .
\end{aligned} \tag{6.91}$$

Thus, for Type B(ecker) solutions, the soft supersymmetry breaking pattern in the D7-branes depends on just two parameters, which correspond to the two possible ISD components of the 3-form flux in the neighborhood of the D7-brane.

Concerning the IASD fluxes, one can proceed in the same way. However, on this case on finds some differences with respect to the results of the previous section. Thus for example, for the dilaton dominated scenarios engendered by the IASD (3,0) components one has

$$m^2_{\Phi^3_{33}} \ = \ \frac{g_s}{2}\,|G_{123}|^2, \quad M^{(33)} = \frac{g_s^{1/2}}{\sqrt{2}}G_{123}, \quad A^{ijk}_{(33)} = -h^{ijk}\frac{g_s^{1/2}}{\sqrt{2}}G_{123}, \tag{6.92}$$

so the values for the scalar masses do not agree with the ones obtained in eq. (6.77). This is not surprising, as the local background configuration (4.33)-(4.37) assumed Poicaré invariance in the four longitudinal directions. However, we are seeing now that AISD fluxes generate through eq. (6.88) a non-vanishing vacuum energy, incompatible with Poincaré invariance. Indeed, eqs. (6.92) describe the soft-terms obtained for D3-branes in a modified background which allows for deviations from Poincaré invariance. In this sense, it is interesting to note the symmetry of the soft-terms under the exchange $D3 \leftrightarrow D7$ and $G_{\bar{1}\bar{2}\bar{3}} \leftrightarrow G_{123}$. This can be understood from the point of view of T-duality along Σ_4. Under this, one has

$$\begin{array}{ccc} D7_3 & \longleftrightarrow & D3\ , \\ S & \longleftrightarrow & T_3\ . \end{array} \tag{6.93}$$

Thus, T-duality is on the root of this apparent symmetry.

Finally, let us comment on the effective supergravity description of various stacks of $D7_r$-branes wrapping 4-cycles transverse to different r-th complex planes. The four dimensional superpotential for the open string moduli on this case has the form

$$W_{D7_i D7_j} \ = \ \phi_{ij}\phi_{ji}\phi^k_{ii} \ - \ \phi_{ij}\phi_{ji}\phi^k_{jj} \ + \ \phi_{ij}\phi_{37_i}\phi_{37_j}\ , \tag{6.94}$$

where $k \neq i, j$ and there is no sum over the i, j indices. For ISD fluxes, from the general no-scale property described above, only the auxiliary fields of matter fields on the D7-branes will contribute to the scalar potential and therefore

$$V_{D7_i D7_j} \ = \ |\ -M^*_{77}\Phi_{ij}{}^* + \partial_{\phi_{ij}} W\ |^2 \ = \ |-M^*_{77}\Phi_{ij}{}^* + \phi_{ji}\phi^k_{ii} + \phi_{ji}\phi^k_{jj} + \phi_{37_i}\phi_{37_j}|^2\ . \tag{6.95}$$

Thus, the scalars at the $7_i 7_j$ intersections become massive, as was advanced at the end of Section 6.2.2 from brane recombination arguments.

6.5 Phenomenological issues.

The soft supersymmetry breaking lagrangian for the MSSM is highly unconstrained, representing the addition of more than one hundred extra parameters in the model. Few empirical constraints exist, mainly coming from CP violation experiments, the absence of Flavour Changing Neutral Currents (FCNC) or cosmological arguments. On the other hand, we have seen that the supersymmetry breaking patterns which naturally arise in the worldvolume of the D3/D7-brane configurations depend only on the very few free components of the fluxes around the branes. On this section we would like to go a step further and to wonder about the phenomenological viability of these induced patterns. For a more detailed study than the one presented here, the reader may consult [164, 165].

First of all, we would like to analyze the naturalness of the different scales involved in the compactification [20]. Although supersymmetry guarantees the cancellation of the quadratic divergences in the worldvolume of the branes, it does not explain the large ratio M_{Planck}/M_{weak}. In other words, since the Higgs mass still receives quadratic corrections from the soft scalar masses m_{soft}, these cannot be too large compared with M_{weak} in order supersymmetry to be an efficient mechanism. Here we took the flux as the only source of supersymmetry breaking to $\mathcal{N} = 0^*$, as it does not lead to NSNS tadpoles, in the theory. As pointed out in eq. (4.63), the flux density goes like α'/R^3, so the typical scale for the soft terms will be given by

$$m_{\text{soft}} = \frac{g_s^{1/2} Z^{-1}}{\sqrt{2}} G_3 = \frac{f g_s^{1/2}}{\sqrt{2}} \frac{\alpha'}{R^3} = \frac{f M_s^2}{M_P} \ . \tag{6.96}$$

The suppression of the localized masses by powers of the warping in scenarios with highly warped extra dimensions was already noted by Randall and Sundrum in [166]. Basically, the idea consists in generating an exponential hierarchy of masses by means of the gravitational redshift generated by a warped metric in the extra dimensions. Indeed, having a metric background as the one of eq. (4.33), with exponential warping

$$Z^{1/2}(x^m) = e^{-2A(x^m)} \tag{6.97}$$

induced by the backreaction of the fluxes through eq. (4.55), imply the four dimensional Planck mass M_4 and the Higgs mass scale M_0 of a Standard Model embedded in a Dp-brane are [166, 167]

$$M_4^2 = M_p^8 (2\pi)^{-6} \int_{\mathcal{M}_6} \sqrt{g_6} e^{2A} \ , \tag{6.98}$$

$$M_0^2 \sim M_p^2 \int d^{p-3} z \sqrt{g_{Dp}} e^{2A} \ , \tag{6.99}$$

with g_{Dp} the determinant of the induced metric in the worldvolume of the brane. Thus, if we place the branes containing the observable sector at the bottom of the AdS throat, where $e^A \ll 1$, the four dimensional masses will be suppressed relative to the fundamental Planck scale M_p.

In this sense, flux compactifications provide a microscopical implementation of the Randall-Sundrum scenario [168, 37]. Moreover, they put on an equal footing both supersymmetry and warped compactifications: on one hand the fluxes backreact the spacetime to $AdS_5 \times S^5$, whereas on the other they provide a mechanism to softly break the supersymmetry.

For the case of D7-branes there are however some subtleties. In fact, we saw in (6.55) that the gauge coupling in the worldvolume of the D7-brane is suppressed by the same power of the warping than the soft masses. Thus, placing a D7-brane in a highly warped region could lead to too small gauge couplings.

The other relevant scale in Type II orientifold compactifications, apart from m_{soft} and M_{Planck}, is the Kaluza Klein scale. In order the effective supergravity approach to be valid, we must ensure that the tower of Kaluza Klein replicas decouples from the masses induced by the fluxes. This can be easily done in the large radius limit. Indeed, in that case

$$g_s^{1/2} Z^{-1} \frac{\alpha'}{R^3}(\text{fluxes}) \ll \frac{1}{R}(\text{KK}) \ll M_s \ , \tag{6.100}$$

and there appear another hierarchy of masses from the radius of the compactification. Thus, neglecting the warping, if one wants to identify m_{soft} with the electroweak scale, the string scale $M_s = \alpha'^{-1/2}$ must be at an intermediate scale $\sim 10^{10}$ GeV and the compact radii be of the order $R = 10^3 \alpha'^{1/2}$. This is consistent with an F auxiliary field getting a vacuum expectation value of order M_s^2. However, one could consider still non-homogeneous situations on which the D-branes are located in regions of the internal space with hierarchically diluted fluxes compared to other regions.

Let us analyze now how the flux induced soft parameters fit into the region of parameter space allowed by the experiments. We will restrict to the Type B(ecker) compactifications discussed in Section 4.3.1. In this context, the four dimensional theory is described by a positive definite scalar potential

$$V_{SB} = \sum_i |\partial_i W - M_i^* \Phi_i^*|^2 \ , \tag{6.101}$$

with i running over the chiral multiplets of the theory. Thus, if the ultraviolet completion of the MSSM corresponds to a D3/D7-configuration in Type IIB String Theory without non-geometric fluxes, one must replace the globally $\mathcal{N} = 1$ SUSY scalar potential

$$V_{SUSY} = \sum_i |\partial_i W|^2 \tag{6.102}$$

by the soft SUSY breaking potential (6.101). Here W is the MSSM superpotential

$$W = -h_U^{ij} H_u Q_i U_j + h_D^{ij} H_d Q_i D_j + h_L^{ij} H_d L_i E_j - \mu H_d H_u \ , \tag{6.103}$$

with Q the left-handed quark supermultiplets, U and D the right handed quark supermultiplets, L and E the left and right handed leptons, and i, j running over the three generations. Hence, the 105 extra parameters describing the soft supersymmetry breaking pattern of the MSSM are reduced to a set of gaugino masses M_i and μ_i-parameters corresponding, through eqs. (6.70) and (6.73), to the amount of flux $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{r}\bar{r}}$ in the neighborhood of the D7_r-branes in the observable sector. Thus, even if we do not know the explicit ultraviolet setup describing the embedding of the MSSM, this is an important improvement of the situation.

The phenomenology of (6.101) is such that could address some of the empirical constraints for the soft breaking lagrangian. Indeed, these can be summarized as [143]:

1. **Lack of flavor universality.** Experiments at accelerators reveal that FCNC are strongly suppressed. The prediction of this phenomena is one of the biggest successes of the Standard Model. Accordingly to this, FCNC does not exist at tree-level due to

the particular form of the couplings of the fermions to the neutral currents. In addition, higher order charged contributions to the FCNC processes are strongly suppressed through the GIM mechanism, associated to 1-loop box diagrams.

In the MSSM however there appear many extra contributions to the FCNC, both at tree-level, through explicit flavour changing couplings, and at 1-loop level, through the failure of the super-GIM mechanism. In particular, a trivial flavor structure of the scalar mass matrix is required, being the off-diagonal components highly suppressed by experimental FCNC constrains such as Δm_K, Δm_B, Δm_D, $b \to s\gamma$, $\mu \to e\gamma$, etc.

In this sense, universality is somehow natural in (6.101). Indeed, all the gaugino masses, scalar masses and holomorphic trilinear couplings come from the same component $G_{\bar{1}\bar{2}\bar{3}}$ of the flux, so a homogeneous background in the neighborhood of the visible sector will lead to $M_i \simeq M$, $A_{ijk} \simeq h_{ijk}A$, etc.

2. **CP-violating electric dipole moments.** Among the great number of parameters involved in the soft supersymmetry breaking pattern, there are some CP violating phases. As in the Standard Model, some of them are associated directly to the flavor changing couplings, i.e. to the off-diagonal entries of the trilinear parameters and the scalar mass matrix, and thus do not represent a problem in universal scenarios. However, additional flavor conserving phases may appear through the complex gaugino masses M, the μ-parameters, the B-terms and the trilinears. Of these, actually only a subset constitutes the physical phases. Indeed, the MSSM lagrangian in the limit $\mu, \mathcal{L}_{\text{soft}} \to 0$ has two global U(1) symmetries: the $\mathcal{N}=1$ R-symmetry and an additional Peccei-Quinn symmetry, under which the above soft parameters transform non-trivially. From the ten dimensional perspective, this is simply due to the fact that the background fluxes are charged under them. Due to this, the phases invariant under reparametrization by these symmetries are given by the linear combinations

$$\begin{aligned} \phi_1 &= \phi_\mu + \phi_A - \phi_B \ , \\ \phi_2 &= \phi_\mu + \phi_M - \phi_B \ , \end{aligned} \tag{6.104}$$

with $\mu = |\mu| e^{i\phi_\mu}$, $M = |M| e^{i\phi_M}$, $A = |A| e^{i\phi_A}$ and $B = |B| e^{i\phi_B}$. The present empirical upper bound for the phases situates in $\phi_{1,2} \lesssim 10^{-2}$, from electric dipole moment experiments.

For the particular case of (6.101) however, the relations (6.70) and (6.75), imply

$$\phi_A = \phi_M \ , \tag{6.105}$$

$$\phi_B = \phi_\mu + \phi_M \ , \tag{6.106}$$

thus guaranteeing the vanishing of the physical phases (6.104).

3. **Electroweak breaking constrains and the μ problem.** The scalar potential of the two Higgs doublets is completely determined by the soft supersymmetry breaking lagrangian. A potential leading to the correct electroweak symmetry breaking will involve thus a constrained range of the soft parameters.

   The analysis can be simplified by making use of the $SU(2)_L$ gauge freedom to rotate away a possible vev of one of the weak isospin components in one of the Higgs doublets $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$. We will take for example $H_u^+ = 0$. Then, it is possible to show that the minimizing condition $\partial V/\partial H_u^+ = 0$ implies also $H_d^- = 0$, and the potential gets simplified to

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (BH_u^0 H_d^0 + \text{c.c.}) + \\ + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2 \ , \quad (6.107)$$

   with g and g' respectively the weak and hypercharge couplings.

   Since the minimum of the potential must break $SU(2)_L \times U(1)_Y$, it should not occur for $\langle H_{u,d}^0 \rangle = 0$. This leads to

$$(2|\mu|^2 + m_{H_d}^2)(2|\mu|^2 + m_{H_u}^2) < B^2 \ . \quad (6.108)$$

   On the other hand, the quartic terms of the scalar potential come from D-terms. Thus, for D-flat directions ($\langle H_u^0 \rangle = \langle H_d^0 \rangle$) the potential could be unbounded from below, leading to potential stability problems[7]. This requires

$$2|\mu|^2 + m_{H_d}^2 + m_{H_u}^2 > 2B \ . \quad (6.109)$$

   This condition is automatically satisfied by (6.101), since on this case $B = 2M\mu$. Thus, the scalar potential (6.101) is positively definite even for D-flat directions and does not represent a problem for the electroweak symmetry breaking mechanism.

   Note however that, since $m_{H_u}^2 = m_{H_d}^2 = |M|^2$, the constrains (6.108) and (6.109) cannot be both satisfied. One is enforced then to consider the renormalization group equations and expect that at the electroweak scale the condition $m_{H_u}^2 < m_{H_d}^2$ is verified, so radiative electroweak symmetry breaking takes place. For further details on the behavior of (6.101) under the renormalization group one may consult [165].

   Finally, observing eqs. (6.108) and (6.109), we expect the μ-parameter to be of the same order than the soft parameters. This however does not represent a problem for the kind of scenarios we are discussing here. As the M and μ parameters are associated to different components of the 3-form flux it is therefore natural to have $\mu \sim m_{soft} \sim m_Z$. Thus,

[7] Actually, radiative corrections could make the potential bounded from below, getting a large unphysical minima, with possible exotic phenomena such as charge or color breaking.

the μ-problem finds a natural solution in the context of String Theory compactifications with background fluxes.

Chapter 7

Applications to model building.

On this chapter we present some examples which serve to illustrate how the ideas discussed along the previous chapters are applied the building of semirealistic models. In particular, we present in Section 7.2 two $\mathcal{N} = 1$ models and a no-scale one with chiral spectrum very close to the one of the MSSM and part or all the closed string moduli stabilized in dS or AdS. In Section 7.3, on the other hand, we show some examples of the flux induced soft supersymmetry breaking patterns arising in Type IIB orientifolds with O3-planes.

None of these models pretend to be The Model of all particles and interactions, but rather to illustrate how such a model could be realized in the context of String Theory. Of course, a lot of work still have to be done in order to obtain a fully realistic setup. In particular, much effort is needed to address the long-standing problem of the cosmological constant or to find selection rules for the huge landscape of vacua.

7.1 D-branes in presence of fluxes and the Freed-Witten anomaly.

Before discussing the models, we would like to give some words about the consistency conditions for D-branes in presence of fluxes. We saw in Section 6.2.2 that fluxes in the $(2,2)_0$ representation of the geometric $SU(2)\times SU(2)'\times U(1)$ symmetry induce Freed-Witten anomalies [169, 70, 71] in the worldvolume of the D7-branes, i.e. non-trivial tadpoles for the gauge fields living inside them. Something similar is expected to occur for the mirror Type IIA orientifolds. Indeed, consider a D3-brane wrapping a 3-cycle Π_a through which there is some quantized NS flux $\overline{H}_3$. On the world-volume of the D3-brane there is a CS coupling of the form $\int C_2 \wedge F_2$, C_2 being the RR 2-form and F_2 the open string gauge field strength. After performing a IIB S-duality transformation one gets a coupling [71]

$$\int_{\Pi_a \times \mathbf{R}} H_3 \wedge \tilde{A}_1 \ , \tag{7.1}$$

where $\tilde{A}_1$ is the gauge field dual to the A_1 form living on the D3-brane. This shows that a background for H_3 gives rise to a tadpole for $\tilde{A}_1$ and hence to an inconsistency. Now, performing three T-dualities one expects for D6-branes the analogous term

$$\int_{D6} H_3 \wedge \tilde{A}_4 \tag{7.2}$$

and thus, the resulting tadpole is avoided only if

$$\int_{\Pi_a} \overline{H}_3 = 0 \ . \tag{7.3}$$

Moreover, under mirror symmetry part of the Type IIB H_3 flux transforms to Type IIA metric and non-geometric fluxes. As we saw in Section 5.2.1, these induce changes in the topology of the original torus, so that some of the original cycles are no longer cycles in presence of metric fluxes. Thus, it is expected that some of the Freed-Witten anomalies of Type IIB orientifolds with O3-planes will map to topological inconsistencies induced by the metric fluxes in Type IIA orientifolds with O6-planes [129]. On this section we would like to explore these inconsistencies from the point of view of the gauge theory living in the worldvolume of the D6-branes. We will mainly follow our work in [43].

As described in Section 3.4, for setups of intersecting D6-branes wrapping 3-cycles Π_a of the internal manifold, there is an exchange of complex structure and dilaton axions between the corresponding gauge bosons and the potentially anomalous $U(1)$'s become massive through a generalized Green-Schwarz mechanism. In this sense, the linear combinations

$$\Phi^a \equiv c_0^a \text{Im } S - \sum_i c_i^a \text{Im } U_i \ , \tag{7.4}$$

with c_I^a given by eq. (3.51), act as Goldstone bosons, transforming with a shift under $U(1)_a$ gauge transformations. However, we have seen on the other hand that NSNS, metric and non-geometric fluxes induce terms in the superpotential linear in Im U_i and Im S, and therefore not invariant under the shifts induced by $U(1)_a$ gauge transformations. The condition to restore consistency and gauge invariance is then to impose the constraint

$$\int_{\Pi_a} (\overline{H}_3 + \omega J_c + Q J_c^{(2)} + R J_c^{(3)}) = 0 \ . \tag{7.5}$$

This is a strong constraint on the possible 3-cycles which the D6-branes may wrap. Note that in absence of metric and non-geometric fluxes, this equation can be understood in terms of the Freed-Witten anomaly (7.3). In that case, the condition (7.5) becomes

$$\sum_{I=0}^{3} c_I^a \, h_I \ = \ 0 \ , \tag{7.6}$$

guaranteing that the combinations of axions getting masses by mixing with vector bosons are orthogonal to those becoming massive from fluxes, the latter being typically of the form $h_0\text{Im } S - \sum_k h_k \text{Im } U_k$.

An interesting particular case is given by the $\mathcal{N}=1$ AdS vacua of Section 5.2.2, with all the moduli stabilized but some linear combinations of axions. There, one finds that at the minima $h_i/h_0 = -\text{Re } S/\text{Re } U_i$. Substituting this into eq. (7.6) and multiplying by the torus volume one arrives at

$$m_a^1 m_a^2 m_a^3 (R_y^1 R_y^2 R_y^3) \;-\; m_a^1 n_a^2 n_a^3 (R_y^1 R_x^2 R_x^3) \;-\; n_a^1 m_a^2 n_a^3 (R_x^1 R_y^2 R_x^3) \;-\; n_a^1 n_a^2 m_a^3 (R_x^1 R_x^2 R_y^3) \;=\; 0 \tag{7.7}$$

This condition means that the D6-brane wraps a special Lagrangian cycle (sLag). From the point of view of the low energy effective action, this is proportional to a Fayet-Iliopoulos (FI) term [170] and hence it dynamically imposes that the D6-brane configuration should be supersymmetric (i.e. all FI terms should vanish). Notice that, due to the relations (5.77), including metric fluxes in this class of minima does not add any extra constraint to be satisfied.

In turn, this is also found in a more general analysis of Type IIA supersymmetric AdS vacua [171, 172], for which

$$\overline{H}_3 + dJ_c \propto \text{Im } \Omega \ , \tag{7.8}$$

so, in absence of non-geometric fluxes, eq. (7.5) becomes the special lagrangian condition

$$\text{Im } \Omega|_{\Pi_a} = 0 \ . \tag{7.9}$$

Therefore, even if the NS fluxes vanish, in these models there is still a Freed-Witten constraint of the form

$$3ac_0^a - b_1 c_1^a - b_2 c_2^a - b_3 c_3^a = 0 \ . \tag{7.10}$$

For instance, in the models of Section 5.2.2 this can be deduced using (5.76).

The vanishing of the FI-terms in the supersymmetric vacua has been observed as well in [173] from the point of view of the $\mathcal{N}=1$ effective supergravity. Indeed, due to the relation

$$D_a = iG_i N^i{}_a \ , \tag{7.11}$$

with $N^i{}_a$ given in terms of the isometry generators by eq. (5.43), the D-terms at the minimum of the scalar potential are proportional to the F-terms, $F_i = e^{G/2}G_i$. Since in a supersymmetric minimum the F-terms vanish, it is thus expected for consistency that as well the D-terms will be zero. More concretely, the constraint (7.5) can be shown in this language to be equivalent to the integrability condition for the Bianchi identity of F_2.

Analogous conditions could be obtained for other vacua. In particular, for Minkowski vacua with vanishing non-geometric fluxes one can check that the condition (7.5) is equivalent to impose the 3-cycle Π_a to be an element of $\Gamma^3 \oplus \partial\Xi^2$ (c.f. Section 5.2.1), being the Freed-Witten anomalies associated to the 3-cycles in Ξ^3. For further details, we refer the reader to [129].

7.2 MSSM-like vacua.

Now that we know more about the consistency conditions for D-branes in presence of fluxes, let us describe some explicit examples. We will consider here three Type IIA orientifold models with semirealistic spectrum and vanishing non-geometric fluxes, all of them based on the intersecting D6-brane model of Section 3.5.2. The first two examples correspond to Minkowski vacua and thus present several flat directions, as commented in Section 5.2.2. The third one is a $\mathcal{N}=1$ AdS vacua with all closed string moduli stabilized and tadpoles cancelled without the aid of an orbifold symmetry. These models were first presented in [43].

Of course, one could think of more elaborated models with non-geometric and S-dual fluxes. In particular, the results we saw in Section 5.4 seem to indicate that considering S-dual fluxes it is possible to build models with all the moduli stabilized in Minkowski. However, a better understanding of these fluxes is still needed.

7.2.1 Model 1: $\mathcal{N}=1$ Minkowski model.

Consider a $Z_2 \times Z_2$ extension [174, 175] of the model of Section 3.5.2 ([98, 176]), consisting on six stacks of branes with wrapping numbers given as in Table 7.1. This corresponds to the same brane content than the models of [177].

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
$N_a = 8$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 2$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_{h_1} = 2$	$(-2, 1)$	$(-3, 1)$	$(-4, 1)$
$N_{h_2} = 2$	$(-2, 1)$	$(-4, 1)$	$(-3, 1)$
$8N_f$	$(1, 0)$	$(1, 0)$	$(1, 0)$

Table 7.1: Wrapping numbers giving rise to a MSSM-like spectrum. Branes h_1, h_2 and o are added in order to cancel RR tadpoles.

We will assume [176] that the branes b and c sit on top of the orientifold plane so the corresponding gauge symmetries are enhanced to $SU(2)_L$ and $SU(2)_R$ respectively. Thus, the full initial gauge group is $U(4) \times SU(2)_L \times SU(2)_R \times [U(1)_1 \times U(1)_2]$. Separating one of the a-branes from the other three produces the breaking $U(4) \to U(3) \times U(1)$. Furthermore, two out of the three $U(1)$'s get a Stückelberg mass by combining with RR axion fields. We are thus left with a gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times [U(1)]$, which contains the left-right symmetric extension of the Standard Model plus an extra $U(1)$. The branes a, b, c, give rise to a 3-generation MSSM-like spectrum whereas the additional branes $h_{1,2}$ in Table 7.1 are used to help in cancelling the RR tadpoles.

Note that for this $Z_2 \times Z_2$ Type IIA orientifold the RR tadpole cancellation conditions in the presence of fluxes have the form

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}(h_0 m + a_1 q_1 + a_2 q_2 + a_3 q_3) = 16 \ , \tag{7.12}$$

$$\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}(m h_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) = -16 \ , \tag{7.13}$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}(m h_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) = -16 \ , \tag{7.14}$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}(m h_3 - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) = -16 \ , \tag{7.15}$$

where the -16 in the last three conditions is the RR tadpole contribution of the other 3 orientifold planes existing in the $Z_2 \times Z_2$ case.

We will turn on non-vanishing fluxes as in the supersymmetric Minkowski example of eq. (5.73) with non-vanishing b_{31}, b_{21}, a_2, a_3. The addition of NS fluxes h_0 and h_1 and RR fluxes e_0, e_2 and e_3 is optional, but we set all the remaining backgrounds to zero. The superpotential has then the form

$$W = -T_2(a_2 S + b_{21} U_1) - T_3(a_3 S + b_{31} U_1) + e_0 + i h_0 S - i h_1 U_1 + i e_2 T_2 + i e_3 T_3 \ . \tag{7.16}$$

As explained in Section 5.2.2 this leads to a Minkowski supersymmetric minimum with

$$h_0 = a_2 v_2 + a_3 v_3 \quad , \quad e_2 = a_2 \mathrm{Im}\ S + b_{21} \mathrm{Im}\ U_1 \quad , \quad t_3 = -\frac{b_{21} t_2}{b_{31}} \quad , \quad s = -\frac{b_{21} u_1}{a_2} \tag{7.17}$$

as long as $e_2 a_3 = e_3 a_2$, $h_0 b_{31} = -a_3 h_1$, $h_0 b_{21} = -a_2 h_1$ and $h_0 e_2 = e_0 a_2$. Thus in this supersymmetric Minkowski background two complex linear combinations of moduli are stabilized at the minimum.

Note that, since $m = q_i = 0$, the fluxes do not contribute to the RR tadpoles. Hence one can consider the addition of D6-branes as in Table 7.1 with $N_f = 5$. As pointed out in [177], with this choice all the RR tadpoles cancel without the addition of fluxes in Type IIB theory. In the present Type IIA case we can rather add the background considered here and the RR tadpoles are not modified and thus cancel. However the moduli are partially fixed through eq. (7.17).

It is easy to check that the a, b and c branes, where the Standard Model lives, trivially satisfy the Freed-Witten constraint. However the branes of type $h_{1,2}$ may be problematic unless

$$a_2(m_a^1 m_a^2 m_a^3) - b_{21}(m_a^1 n_a^2 n_a^3) \;=\; a_2 - 12 b_{21} \;=\; 0 \;, \tag{7.18}$$

which, on the other hand, may be easily satisfied by appropriately choosing a_2, b_{21}. Note that this condition guarantees that the linear combination of Im S and Im U_1 getting masses through fluxes (c.f. eq. (7.17)) is orthogonal to the linear combination which becomes massive by mixing with the $U(1)$'s of the branes $h_{1,2}$.

7.2.2 Model 2: No-scale model.

One can also consider one of the no-scale backgrounds discussed in Section 5.2.2 and include a set of D6-branes as in Table 7.1. A simple example is as follows. Take non-vanishing a_3 and q_3 with the remaining $q_i = a_i = 0$. In addition, one may include non-vanishing h_0, e_0 and e_i but set the remaining backgrounds to zero. The superpotential has then the form

$$W(S, T_i) = -a_3 S T_3 - q_3 T_1 T_2 + e_0 + i h_0 S + i \sum_i e_i T_i \;. \tag{7.19}$$

The imaginary part of S and the T_i are fixed as in eq. (5.13) whereas for the real parts one has the relationship $a_3 s t_3 = q_3 t_1 t_2$. In addition one has the constraint $e_0 = h_0 \mathrm{Im}\ S + e_1 v_1$. There is only a contribution equal to $\frac{1}{2} a_3 q_3$ to the first RR tadpole. We consider fluxes quantized in units of 8 to avoid problems with flux quantization [178, 71]. One can then cancel the tadpoles in a $Z_2 \times Z_2$ orientifold with branes as in Table 7.1 with $N_f = 1$ and $a_3 = q_3 = 8$.

One may instead consider a no-scale model with non-vanishing mass parameter m and with no metric fluxes, as described in Section 5.1.2 . Indeed, taking non-vanishing m and h_0 and e_i and q_j verifying $\gamma_i = m e_i + q_j q_k = 0$ $(i \neq j \neq k)$ one has a non-supersymmetric vacua with no-scale structure. Then setting $h_0 = m = 8$ and $N_f = 1$ one cancels all tadpoles. Note that this model, which has no metric fluxes, is the Type IIA mirror of a similar no-scale model considered in [177].

Both these no-scale models however have FW anomalies. The danger comes from the $h_{1,2}$ branes which have a non-vanishing product $m_a^1 m_a^2 m_a^3 \neq 0$. One possibility which might cure the problem is that, as suggested in [177], the brane h_1 recombines with the mirror of h_2 into a single non-factorizable D6-brane $h_1 + h_2'$. In this case, since h_1 and h_2' have equal and opposite $m_a^1 m_a^2 m_a^3$, the FW would cancel on the recombined brane. However it is not clear whether after the addition of fluxes a flat direction in the effective potential exists corresponding to the recombination of those branes.

7.2.3 Model 3: $\mathcal{N}=1$ AdS model with all closed string moduli stabilized.

The previous intersecting brane models were able to combine a semi-realistic spectrum with a partial determination of some of the closed string moduli. We now present an AdS model with all such moduli stabilized, thus providing a semi-realistic string model with all closed string moduli stabilized at weak coupling. Note first that in the past it has been argued that it is impossible to construct semi-realistic $\mathcal{N}=1$ supersymmetric intersecting D6-brane models wrapping the IIA orientifold $T^6/(\Omega(-1)^{F_L}\sigma)$. The reason for this was essentially the impossibility to cancel the four RR tadpole conditions simultaneously while maintaining supersymmetry. To obtain $\mathcal{N}=1$ supersymmetric models, extra orbifold twisting (e.g. $Z_2\times Z_2$, as in previous examples) had to be added, giving rise to additional orientifold planes which help in the cancellation of RR tadpoles [174]. We will show here that one can build $\mathcal{N}=1$ supersymmetric configurations in the purely toroidal orientifold in which the RR tadpoles are cancelled by the addition of NS/RR and metric fluxes. The role played by the additional orientifold planes in orbifold models is here played by the fluxes contributing like orientifold planes. At the same time, those fluxes stabilize all closed string moduli in AdS space. Moreover the complex structure moduli are fixed at values which render the D6-brane configuration supersymmetric. Notice that in the $\mathcal{N}=1$ supersymmetric models previously considered in the literature those moduli where not determined by the dynamics.

We will consider the set of D6-branes wrapping factorizable cycles in the orientifold as in Table 7.2. Note that this set only differs from the previous examples in the branes h_1 and h_2.

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
$N_a=4$	$(1,0)$	$(3,1)$	$(3,-1)$
$N_b=1$	$(0,1)$	$(1,0)$	$(0,-1)$
$N_c=1$	$(0,1)$	$(0,-1)$	$(1,0)$
$N_{h_1}=3$	$(2,1)$	$(1,0)$	$(2,-1)$
$N_{h_2}=3$	$(2,1)$	$(2,-1)$	$(1,0)$
$N_o=4$	$(1,0)$	$(1,0)$	$(1,0)$

Table 7.2: A MSSM-like model with tadpoles cancelled by fluxes. Branes h_1, h_2 and o are added in order to cancel RR tadpoles.

The other difference is that now we have a purely toroidal (no $Z_2\times Z_2$) orientifold without further twisting. The corresponding chiral spectrum at the intersections is given in Table 7.3. The gauge group after separating branes and after two of the $U(1)$'s get Stückelberg masses is $SU(3)\times SU(2)_L\times U(1)_R\times U(1)_{B-L}\times[U(1)\times SU(3)^2]$. Note that, unlike the case of the $Z_2\times Z_2$ models above, one can make the breaking $SU(2)_R\to U(1)_R$ by brane splitting, and hence the gauge group is that of the MSSM supplemented by some extra $U(1)$'s. We have three generations of quarks and leptons, one Higgs multiplet H and extra matter fields

Intersection	Matter fields	Rep.	Q_{3B+L}	Q_1	Q_2
$a-b$	F_L	$3(4,2_L)$	1	0	0
$a-c$	F_R	$3(\bar{4},2_R)$	-1	0	0
$b-c$	H	$(2_L,2_R)$	0	0	0
$a-h_1$	T_1	$(4,\bar{3}_1)$	1	-1	0
$a-h_1'$	T_1'	$5(4,3_1)$	1	1	0
$a-h_2$	T_2	$5(\bar{4},\bar{3}_2)$	-1	0	1
$a-h_2'$	T_2'	$(\bar{4},\bar{3}_2)$	-1	0	-1
$b-h_2$	D_2	$2(2_L,\bar{3}_2)$	0	0	-1
$c-h_1$	D_1	$2(2_R,\bar{3}_1)$	0	-1	0
h_1-h_2'	X	$4(\bar{3}_1,3_2)$	0	-1	1

Table 7.3: Chiral spectrum of the MSSM-like model. A prime indicates the mirror brane.

involving the auxiliary branes h_1, h_2 and o.[1]

With this brane content (plus the mirrors) the RR tadpole cancellation conditions are

$$64 \ + \ \frac{1}{2}(h_0 m + a_1 q_1 + a_2 q_2 + a_3 q_3) = 16 \ , \tag{7.20}$$
$$-4 \ + \ \frac{1}{2}(h_1 m - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) = 0 \ , \tag{7.21}$$
$$-4 \ + \ \frac{1}{2}(h_2 m - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) = 0 \ , \tag{7.22}$$
$$-4 \ + \ \frac{1}{2}(h_3 m - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) = 0 \ . \tag{7.23}$$

We see that to cancel the tadpoles the sign of the flux contribution must be opposite to that of D6-branes. We will now consider the AdS background with metric fluxes and $m \neq 0$ discussed in Section 5.2.2. The reader can check that choosing the fluxes as

$$q_i = q = h_i - 2 \quad , \quad a_i = 16 \quad , \quad m = b_{ij} = -b_{ii} = 4 \ , \tag{7.24}$$

all the RR tadpoles are cancelled. Note that eq. (5.77) fixes $h_0 = -12h_i$, otherwise the values of q, h_0 and h_i may be arbitrarily large, still cancelling the tadpoles.

The above type of flux backgrounds does give rise to supersymmetric AdS vacua with all real moduli fixed. In fact, the fluxes in (7.24) are isotropic so that the superpotential is of the form (5.75). Then, one can easily check that with the above fluxes $\lambda_0 = 1 + (24/h_0)$, which is arbitrarily close to 1 for large h_0. Substituting these fluxes yields for the real moduli

$$s = -\frac{h_0\lambda}{96}t \quad , \quad u_k = -\frac{h_0\lambda}{8}t \quad , \quad t = \sqrt{\frac{5}{3}\frac{|h_0|}{16}}\lambda^{1/2}(\lambda + \frac{24}{h_0})^{1/2} \ , \tag{7.25}$$

[1] One can check that if the branes h_1 and h_2' recombine, most of the extra matter beyond the SM disappears from the massless spectrum, with only additional $SU(2)_{L,R}$ doublets remaining.

where λ is the appropriate solution to eq. (5.85) for the λ_0 indicated above. For large h_0, λ_0 is close to 1 so that, when $e_0 = c_1 = 0$, $\lambda \simeq (10)^{2/3}/20$. In this case one needs $h_0 < 0$. The imaginary part of the Kähler moduli are fixed as in eq. (5.84) whereas only the linear combination of dilaton and complex structure axions $12\text{Im } S + \sum_{k=1}^{3} \text{Im } U_k$ is fixed, as in eq. (5.78). As discussed in Section 5.2.2, for large h_0 (which also implies large h_k and q) all the moduli are stabilized in a regime on which perturbation theory in four dimensions is a good approximation.

Note that the Freed-Witten conditions (7.6) for the branes a, h_1 and h_2 read respectively

$$h_2 = h_3 \quad , \quad h_1 = h_3 \quad , \quad h_1 = h_2 \ , \tag{7.26}$$

which are automatically satisfied, since $h_1 = h_2 = h_3 = -h_0/12$. As we mentioned, this guarantees that the $\mathcal{N} = 1$ conditions at the brane intersections (c.f. eq. (3.58))

$$\text{atan}\left(\frac{\tau_2}{3}\right) - \text{atan}\left(\frac{\tau_3}{3}\right) = 0 \ , \tag{7.27}$$

$$\text{atan}\left(\frac{\tau_1}{2}\right) - \text{atan}\left(\frac{\tau_2}{2}\right) = 0 \ , \tag{7.28}$$

$$\text{atan}\left(\frac{\tau_1}{2}\right) - \text{atan}\left(\frac{\tau_3}{2}\right) = 0 \ , \tag{7.29}$$

are satisfied, since $u_1 = u_2 = u_3$, in agreement with the results of the previous section. Here, $\tau_i = R^i_y/R^i_x$.

It is also interesting to look at the structure of $U(1)$'s and the Im U_I RR fields. One may check that the couplings (3.50) give masses to two linear combinations of $U(1)$'s by combining with certain linear combinations of Im U_I fields. Only the generator $Q_a - \frac{3}{2}(Q_1 - Q_2)$ remains massless at this level. On the other hand, the fields Im S and $\sum_k \text{Im } U_k$ do not mix with the $U(1)$'s at all, as expected, since FW anomalies cancel. Note that the combination $12\text{Im } S + \sum_k \text{Im } U_k$ is the one which gets a mass from fluxes. Thus, the orthogonal linear combination is massless and may be identified with an axion which may be of relevance for the strong CP problem.

Although here we have studied only the diagonal closed string moduli of the orientifold, setting all the off-diagonal moduli to zero solves the extremum conditions. Furthermore, since we are in a $\mathcal{N} = 1$ supersymmetric AdS background, this guarantees that these off-diagonal moduli are also stable. Thus, the closed string background discussed is completely stable. We have succeeded in building a semi-realistic $\mathcal{N} = 1$ supersymmetric model with all closed string moduli stabilized in a consistent perturbative regime.

7.3 Models of SUSY-breaking.

The consistent models presented in the previous section were all $\mathcal{N}=1$ supersymmetric (excepting the $\mathcal{N}=0^*$ no-scale models which suffered from Freed-Witten anomalies). We would like to present now three $\mathcal{N}=0^*$ examples, in order to provide some semi-realistic models of soft supersymmetry breaking patterns. Regarding moduli stabilization and chiral spectrum, these models are however less involved than the ones presented in the previous section. More concretely, for the first two examples we will consider just the local configuration of D3 and D7-branes holding the observable sector, whereas the third example involves the presence of antibranes, which may induce NSNS tadpoles. These models were presented in [20, 24]. Of course, one could think of more elaborated models, by considering setups like the ones of the previous section but placed in non-supersymmetric vacua. However, the computation of the soft-supersymmetry breaking patterns in presence of non-trivial metric fluxes becomes substantially more complicated. We refer the reader to [177, 179, 180] for some more involved $\mathcal{N}=0^*$ models.

7.3.1 Model 1: D7-branes on a Z_3 singularity.

Let us consider a local Z_3 singularity with twist given by

$$\theta \quad : \quad (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-4\pi i/3} z_3) \ , \tag{7.30}$$

and a stack of nine D7-branes transverse to the third complex plane and with Chan-Paton embedding

$$\gamma_{\theta,7} \ = \ \mathrm{diag}\left(\mathrm{I}_3, \alpha \ \mathrm{I}_2, \alpha^2 \ \mathrm{I}_2, \mathrm{I}_2\right) \ , \tag{7.31}$$

with $\alpha \equiv \exp(\mathrm{i}2\pi/3)$. Additionally, we will consider a Wilson line, around the second 2-torus, with

$$\gamma_{W,7} \ = \ \mathrm{diag}\left(I_7, \alpha, \alpha^2\right) \ , \tag{7.32}$$

so the $U(9)$ group in the stack of D7-branes is broken to $U(3)\times U(2)\times U(2)\times U(1)^2$.

The twisted cancellation conditions (3.48) on this case read

$$\mathrm{Tr}\gamma_{\theta,7} \ + \ 3\mathrm{Tr}\gamma_{\theta,3} \ = \ 0 \ , \tag{7.33}$$

where $\gamma_{\theta,3}$ refers to the possible D3-branes at the orbifold singularities. More concretely, there are nine fixed points $(n,0)$, $(n,1)$ and $(n,-1)$ in $\Sigma_4 = T^2\times T^2$, with $n=0,\pm1$, which feel respectively the twists $\gamma_{\theta,7}$, $\gamma_{\theta,7}\gamma_{W,7}$ and $\gamma_{\theta,7}\gamma_{W,7}^{-1}$. Thus, the condition (7.33) is satisfied automatically at the six fixed points $(n,\pm1)$, without the need of adding any D3-brane on them. On the other hand, one has to add two D3-branes at each of the $(n,0)$ fixed points with

$$\gamma_{\theta,3_n} \ = \ \mathrm{diag}\left(\alpha, \alpha^2\right) \quad , \quad n=0,\pm1 \ , \tag{7.34}$$

in order to satisfy the twisted tadpole conditions on them.

Due to the Green-Schwarz mechanism, most of the $U(1)$'s becomes massive and the total gauge group is reduced to the left-right $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetric extension of the Standard Model, plus some additional $U(1)$'s.

From the 77 sector one gets chiral multiplets transforming with respect to $SU(3)\times SU(2)_L\times SU(2)_R \times U(1)_{B-L}$ like

$$Q_L^i = (3,2,1)_{1/3} \quad , \quad Q_R^i = (\bar{3},1,2)_{-1/3} \quad , \quad H^i = (1,2,2)_0 \ , \tag{7.35}$$

with $i = 1,2,3$. We thus get three generations of quarks together with three sets of Higgs multiplets. From each of the 3_n7 sectors one gets chiral multiplets

$$\begin{aligned} L^n &= (1,2,1)_{-1}, & R^n &= (1,1,2)_1 \ , \\ D^n &= (3,1,1)_{-2/3}, & \overline{D}^n &= (\bar{3},1,1)_{2/3} \ , \end{aligned} \tag{7.36}$$

plus four additional gauge singlets. Finally, from each 33 sector one has three more singlets. In total the spectrum is that of a Standard Model with three generations of quarks, lepton and Higgsses, and three sets of vector-like colored particles D^n and $\overline{D}^n$, which may in fact become massive if one of the singlets from the 33 sector gets a vev.

The relevant superpotential terms for this model are

$$W_{77} \ = \ \epsilon_{ijk} Q_L^i Q_R^j H^k \ , \tag{7.37}$$

$$W_{37} \ = \ Q_L^3(\sum_n L^n \bar{D}^n) \ + \ Q_R^3(\sum_n R^n D^n) \ . \tag{7.38}$$

Now, let us assume we add a (0,3) ISD flux background $G_{\bar{1}\bar{2}\bar{3}}$. From the results of Section 6.3, the following soft-terms are obtained for gaugino masses, scalar masses and trilinear terms

$$\begin{aligned} M_3 = M_L = M_R = M_{B-L} = M \quad &= \quad \frac{g_s^{1/2}}{3\sqrt{2}} \, (G_{\bar{1}\bar{2}\bar{3}})^* \ , \\ m_{Q_L^3}^2 \ = \ m_{Q_R^3}^2 \ = \ m_{H^3}^2 \quad &= \quad M^2 \ , \\ A_{77} \ = \ A_{37} \quad &= \quad -h \ M \ . \end{aligned}$$

Note that all soft terms are determined by a single parameter M and that only one generation of quarks gets soft scalar masses. This is not however a serious problem, since, once gaugino masses are present, the rest of the scalars will get a mass from one-loop diagrams with gauginos in the loop.

However, in this particular model, the masses of the squarks are not universal, which in fully realistic models may lead to phenomenological problems with too much FCNC, as we described in Section 6.5. Furthermore, the Z_3 projection does not allow for explicit supersymmetric masses (μ terms), which is one of the ingredients of the MSSM. However, as we will see in the following example, these are not generic properties of the flux-induced soft-terms, but rather peculiarities of this model.

7.3.2 Model 2: D7-branes on a Z_4 singularity.

For the following model we will consider a Z_4 singularity with twist

$$\theta \quad : \quad (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/4} z_1, e^{2\pi i/4} z_2, e^{\pi i} z_3) \ , \tag{7.39}$$

and a stack of twelve D7-branes transverse to the third complex plane with

$$\gamma_{\theta,7} \ = \ \mathrm{diag}\left(\mathrm{I}_4, \alpha^3 \ \mathrm{I}_2, \alpha \ \mathrm{I}_2, \mathrm{I}_2, -\mathrm{I}_2\right) \tag{7.40}$$

and $\alpha = \exp(i2\pi/4)$.

The Z_4 action leaves $2 \times 2 = 4$ fixed points in $\Sigma_4 = T^2 \times T^2$, which we will denote by (n, m), with $n, m = 0, 1$. We add now one Wilson line with Chan-Paton matrix

$$\gamma_{W,7} \ = \mathrm{diag}\left(\mathrm{I}_8, -\mathrm{I}_2, \mathrm{I}_2\right) \tag{7.41}$$

around (say) the second torus. The local tadpole cancellation conditions have now the general form Tr $\gamma_{\theta,7}$ + 2Tr $\gamma_{\theta,3} = 0$ for each θ-twisted sector. It is easy to check that all local tadpoles are cancelled if we locate four D3-branes at each of the two fixed points $(n, 0)$ with $\gamma_{\theta,3_n} \ = \ \mathrm{diag}\left(\alpha, \alpha^3, -\mathrm{I}_2\right)$ for $n = 0, 1$. The D7 gauge group is $U(4) \times U(2)_L \times U(2)_R \times U(2)^2$ and there are chiral multiplets from the 77 sector as follows

$$\begin{aligned} F_L^i = (4, \bar{2}, 1) \quad &, \quad F_R = (\bar{4}, 1, 2) \quad , \quad i = 1, 2 \ , \\ H = (1, \bar{2}, 2) \quad &, \quad \bar{H} = (1, 2, \bar{2}) \ . \end{aligned} \tag{7.42}$$

Thus, there are two standard quark/lepton generations corresponding to the first two complex planes. From the third (transverse) complex plane, we get Higgs doublets able to trigger electroweak symmetry breaking, and with Yukawa couplings $\epsilon_{ij} \bar{H} F_L^i F_R^j$ to quarks and leptons. We will not display the spectrum from the 37 sectors which just give vector-like multiplets with respect to the Pati-Salam symmetry.

Turning on now ISD backgrounds corresponding to $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$, one obtains the following non-vanishing soft terms

$$\begin{aligned} M_4 = M_L = M_R = M \ &= \ \frac{g_s^{1/2}}{3\sqrt{2}} \ (G_{\bar{1}\bar{2}\bar{3}})^* \ , \\ m^2_{H,\bar{H}} \ = \ M^2 \quad , \quad \mu \ &= \ -\frac{g_s^{1/2}}{6\sqrt{2}} (S_{\bar{3}\bar{3}})^* \ , \\ A_{77} = -h \ M \quad , \quad B \ &= \ M \ \mu \ , \end{aligned} \tag{7.43}$$

where μ is a SUSY mass for $H, \bar{H}$. Although this Z_4 model has only two generations, the above set of soft terms is quite simple and predictive and have a number of interesting properties. All soft terms are determined by the fluxes $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$ or, alternative by the parameters M and μ. The squark/slepton masses are universal and equal to zero. This poses no phenomenological problem since they all get large masses at the one-loop level, as has been abundantly analyzed in the SUSY literature. On the other hand, due to its universality, FCNC transitions are suppressed. As we said, one of the interesting aspects is that both a μ and a B-term are obtained, which are important ingredients in the MSSM, with the simple prediction $B = M\mu$.

7.3.3 Model 3: $\overline{D3}$-branes on a Z_3 singularity.

Let us illustrate finally a slightly different case on which the MSSM is embedded in the worldvolume $\overline{D3}$-branes, giving rise to a dilaton dominated soft supersymmetry breaking scenario.

We will consider a Type IIB Z_3 orientifold, where the orbifold action is generated by the twist

$$\theta \quad : \quad (z_1, z_2, z_3) \longrightarrow (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-4\pi i/3} z_3) \ . \tag{7.44}$$

Let us mod out by the orientifold action $\Omega_P \sigma(-1)^{F_L}$, where $\sigma : z_i \to -z_i$, and introduce a G_3 flux of the form

$$G_3 = A\, d\bar{z}_1 d\bar{z}_2 d\bar{z}_3 \ , \tag{7.45}$$

which fixes the dilaton vev to $\tau = iS = e^{2\pi i/3}$. The coefficient A is an even number to ensure proper quantization over toroidal 3-cycles. The flux is purely $(0,3)$ and breaks supersymmetry, but is ISD and obeys the equations of motion. Its contribution to the 4-form tadpole is $N_{\text{flux}} = 3|A|^2$.

There are 27 fixed points, which we label by (m, n, p), where $m, n, p = 0, \pm 1$, as shorthand for the three possible positions of the fixed points in each complex plane. At the fixed point $(0, 1, 0)$ (and its orientifold mirror $(0, -1, 0)$) we locate 7 $\overline{\text{D3}}$'s (see Figure 7.1) with Chan-Paton matrices

$$\gamma_{\theta,\bar{3}} \ = \text{diag}\left(\text{I}_3, \alpha\ \text{I}_2, \alpha^2\ \text{I}_2\right) \ . \tag{7.46}$$

In order to cancel twisted tadpoles at the fixed points $(0, n, p)$ we add 6 anti-D7-branes passing through them, with Chan-Paton matrix

$$\gamma_{\theta,\bar{7}} \ = \ \text{diag}\left(\text{I}_2, \alpha\ \text{I}_2, \alpha^2\ \text{I}_2\right) \ . \tag{7.47}$$

We furthermore add a Wilson line γ_W on the second complex plane, given by

$$\gamma_{W,\bar{7}} \ = \ \text{diag}\left(\alpha, \alpha^2, \text{I}_2, \text{I}_2\right) \ . \tag{7.48}$$

In this way the gauge group coming from the $\overline{\text{D7}}$'s is broken to $U(2) \times U(1)$ and the different fixed points have now different $\overline{\text{D7}}$-brane CP matrices. We complete a consistent configuration,

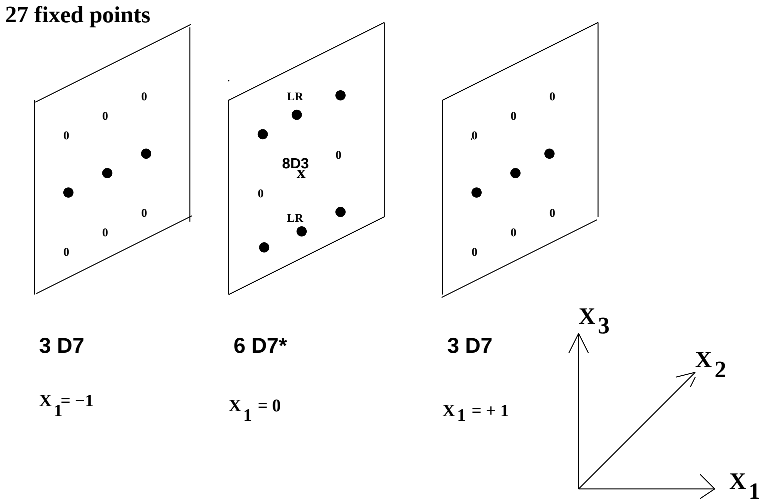


Figure 7.1: A compact Type IIB $T^6/\sigma(-1)^{F_L}\Omega_P Z_3$ orientifold model with a three generation $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge theory. Here X_i represent the three complex compact dimensions. The gauge theory lives on the worldvolume of 7 $\overline{\mathrm{D3}}$'s located at the fixed point marked LR (and its orientifold mirror). The rest of the black dots represent one $\overline{\mathrm{D3}}$. Upon switching on a self-dual $(0,3)$ flux RR-tadpoles cancel and SUSY-breaking soft terms appear on the worldvolume of the LR branes.

cancelling all the RR twisted tadpoles, as follows (see Figure 7.1). We locate one $\overline{\mathrm{D3}}$-brane at each of the four fixed points $(0,\pm1,\pm1)$ with CP matrix $\gamma_{\theta,\bar{3}}=1$, and 8 D3-branes at the origin, with CP matrix $\gamma_{\theta,3}=\mathrm{diag}\left(\alpha \mathrm{I}_4,\alpha^2\mathrm{I}_4\right)$. Finally, in order to cancel global RR tadpoles, we add 3 $\overline{\mathrm{D3}}$'s as well as 3 parallel $D7$-branes (and their orientifold mirrors) passing respectively through the fixed points of type $(1,n,p)$ and $(-1,n,p)$ (see [15] for details). One can easily check that all twisted tadpoles cancel in this configuration.

The total untwisted RR 4-form charge in the configuration is

$$Q_{RR} \;=\; -32(O3)-24(D3^*)+8(D3) \;=\; -48\ . \tag{7.49}$$

This charge is neatly cancelled if we add ISD $(0,3)$ flux with $A=4$, which contributes to the RR-charge $3\times(4)^2=48$ units.

This brane configuration is (meta)stable. The $D7$ and $\overline{\mathrm{D7}}$'s are stabilized on the planes passing through the orbifold points. They are forced to remain there in order to maintain twisted RR tadpole cancellation. Equivalently, the orbifold projection removes the scalar associated to flat directions describing brane motion. This will avoid $D7$- $\overline{\mathrm{D7}}$ annihilation. The addition of fluxes will not destabilize them, since their only effect would be to generate a potential for those scalars, if they were present.

The same happens for the $\overline{\rm D3}$'s whose scalars get masses of the dilaton dominated type. The D3-branes at the origin do not get masses from fluxes, since they feel the ISD flux which gives no soft terms (to leading order) for them. They are however stuck at the origin again by the twisted RR tadpole conditions, which would be violated if any of those branes travelled to the bulk (equivalently, their worldvolume field theory does not contain scalars parametrizing the possibility of moving away the D3-branes). In fact the four $\overline{D3}$'s at the four $(0, \pm 1, \pm 1)$ fixed points are also stuck by twisted tadpole conditions. All in all, the whole brane configuration is (meta)stable due to a combination of trapping at the fixed points and flux-induced scalar potentials.

The gauge group is $U(3)\times U(2)\times U(2)$, with two anomalous $U(1)$'s being actually massive, and the diagonal combination giving $B-L$. In the $\overline{\mathbf{33}}$ sector, we obtain matter fields

$$\overline{\mathbf{33}}\text{ sector}: \qquad 3\ \left[(3,\bar{2},1)+(\bar{3},1,2)+(1,2,\bar{2})\right]\ , \tag{7.50}$$

corresponding to three left(right)-handed quarks $Q_L^a(Q_R^a)$, $a=1,2,3$, and three sets of standard Higgs multiplets H^a. From the $\overline{\mathbf{37}}$ and $\overline{\mathbf{73}}$ sectors, one gets

$$\begin{aligned}
&\overline{\mathbf{37}}\text{ sector}: \qquad (3,1,1;1)_{-1}+(3,1,1;2)_0+(1,2,1;1)_1+(1,2,1;2)_0\ ,\\
&\overline{\mathbf{73}}\text{ sector}: \qquad (\bar{3},1,1;1)_1+(\bar{3},1,1;2)_0+(1,1,2;1)_1+(1,1,2;2)_0\ .
\end{aligned} \tag{7.51}$$

These contains three left(right)-handed leptons $L^a(R^a)$. There are also some extra vector-like pairs of color triplets which in general become massive once some $7_i 7_i$ states get vevs (see [15]). The orientifold projection map the sets of branes at $(0,1,0)$ and $(0,-1,0)$ fixed points to each other, so only one copy of the LR model is obtained.

The quarks in this model have a superpotential

$$W_Y\ =\ g\sqrt{2}\epsilon_{abc}Q_L^a Q_R^b H^c\ . \tag{7.52}$$

On the other hand there are no renormalizable lepton Yukawas which may only appear after a blowing up of the singularity [15]. We will thus concentrate here on the quarks. The flux background is of ISD $(0,3)$ type, hence it leads to dilaton dominated soft terms on the worldvolume of the anti-D3-branes. As discussed in Section 6.3, these are

$$\begin{aligned}
m^2_{Q_L^a}=m^2_{Q_R^a}=m^2_{H^a}=m_a^2\ \ a=1,2,3\ ,\\
m_1^2+m_2^2+m_3^2=\frac{g_s}{6}|G_{\bar{1}\bar{2}\bar{3}}|^2\ ,\\
M_3=M_L=M_R=M_{B-L}=M=\frac{g_s^{1/2}}{\sqrt{2}}\ G_{\bar{1}\bar{2}\bar{3}}\ ,\\
A^{abc}=-h^{abc}M\ ,
\end{aligned} \tag{7.53}$$

with the rest of the soft terms vanishing. This kind of SUSY-breaking soft terms applied to the MSSM have been abundantly studied in the literature, and provides a phenomenologically interesting and viable soft term pattern [181, 8, 182, 183, 184].

Chapter 8

Final comments.

Flux compactifications introduce a canonical mechanism to address the long-standing problem of moduli stabilization in Superstring Theory compactifications. Besides this, they constitute a tractable source for supersymmetry breaking in a computable regime. Thus, fluxes are advocated to play a prominent role in embedding the Standard Model into Superstring Theory.

In this thesis we have reviewed some of our contributions to these topics. In particular, we have analyzed the vacuum structure of the flux induced potential for a simple Type IIA $T^6/\Omega_P(-1)^{F_L}\sigma$ orientifold with constant NSNS, RR and metric fluxes. Compared to Type IIB orientifolds with ordinary fluxes, Type IIA orientifolds possess a richness of flux options, leading to vacua with all the closed string moduli stabilized in AdS without the need of non-perturbative effects.

Moreover, Type IIA metric fluxes lead to new possibilities for cancelling the RR tadpoles. Indeed, one may find situations on which the fluxes do not contribute to the RR tadpoles or contribute negatively, i.e. as O-planes do, thereby providing the interesting possibility of disposing of orientifold planes in some cases. This represents a breakthrough with respect to the Type IIB compactifications, where the ISD condition enforces the flux to contribute positively to the RR tadpoles, so in order to fulfill the tadpole conditions and at the same time stabilize the moduli at large regions, one usually has to take manifolds with large Euler numbers.

In this sense, our approach has been to consider the metric fluxes as a deformation added to the original torus. The resulting twisted torus is a half-flat manifold with reduced cohomology. Thus, metric fluxes are somehow enlarging the region of the moduli space to which we may access, taking into account not only the possible geometrical deformations of the torus, but as well some topological deformations.

It is enlightening to view the twisted torus structure as arising from mirror symmetry of Type IIB orientifolds with non-vanishing NSNS 3-form flux. Then, one may identify among the different vacua of Type IIA orientifolds, the mirrors of some of the Type IIB no-scale vacua. Inspired by this fact, one may wonder whether Type IIB flux induced potentials can also depend on all moduli. The addition of non-geometric fluxes in Type IIB orientifolds restores T-duality between Type IIA and Type IIB theories, but on this case monodromies in the internal manifold mix the metric with the B field, therefore removing its geometrical character.

The inclusion of non-geometric fluxes spoils the Type IIB S-duality inherited by the effective potential in presence of standard RR and NS backgrounds. In order to recover the symmetry in presence of non-geometric fluxes one has to introduce an extra set of S-dual fluxes. The new and old fluxes are subject to a number of Bianchi and RR tadpole cancellation conditions. These can be obtained by making use of $SL(2,\mathbb{Z})_S$ transformations. We have seen that the arising structure is rich enough to hold some Minkowski vacua with not only the dilaton and complex structure moduli stabilized, but as well the Kähler moduli.

Generalizing this picture, one may impose in the superpotential invariance under the full duality group. This idea finds support in the results from heterotic and M-theory compactifications. In our toroidal examples we have seen this requires the presence of 2^7 parameters (fluxes) describing the complete $SL(2,\mathbb{Z})^7$-invariant set of backgrounds. Some of these flux degrees of freedom have a simple interpretation as metric fluxes or explicit RR or NS backgrounds in some particular version of Superstring Theory. Some others do not admit a simple interpretation and their origin is still to be understood. Yet, all of these fluxes may in general be present in the complete underlying theory.

A natural question in all these models, is the interplay between D-branes and fluxes. We have seen that NSNS 3-form fluxes may induce tadpoles for the gauge potentials in the D6 and in the D7-branes. On the other hand, and related to this, metric and non-geometric fluxes parametrize topological changes in the internal manifold so that some of the original cycles disappear from the homology and the corresponding moduli are lifted from the massless spectrum. Wrapping D-branes on these submanifolds may induce inconsistencies in the theory inside the brane. Here we have analyzed the example of D6-branes in Type IIA orientifolds. In this case, fluxes may induce masses for the RR axions mediating the cancellation of $U(1)$ anomalies through the Green-Schwarz mechanism. Such terms do not respect the $U(1)$ transformation properties for the axions. Imposing consistency, then requires some constraints on the 3-cycles which the D6-brane may wrap. These conditions can be viewed as a generalization of the Freed-Witten anomaly cancellation. In particular, in the case of $\mathcal{N}=1$ supersymmetric AdS vacua, they force the different sets of D6-branes to be calibrated, thus preserving $\mathcal{N}=1$ supersymmetry in four dimensions. In this way, one may construct explicit models with chiral spectrum close to that of the MSSM with three generations and with all the closed string moduli stabilized in AdS, where the $\mathcal{N}=1$ supersymmetry is dynamically imposed by the background fluxes. We have constructed an example of such a model on which we make use of the fluxes in order to cancel the RR tadpoles, without the aid of an extra orbifold twist.

From the point of view of the four dimensional effective action, the dynamics of the D-branes in presence of closed string fluxes is determined through the flux induced potential for the open string moduli. In terms of the MSSM, this one may be identified with the soft supersymmetry breaking potential, with the open string moduli corresponding to the squarks/sleptons and Higgsses. We have computed from a microscopical local point of view the soft supersymmetry breaking lagrangian induced by RR and NSNS 3-form fluxes in the worldvolume of D3 and D7-brane configurations. For ISD fluxes, the soft terms in the worldvolume of D3-branes vanish, in agreement with the no-scale structure of the potential. On D7-branes however, there appear non-null soft supersymmetry breaking patterns even for $\mathcal{N} = 1$ supersymmetric fluxes. This is interesting, since such fluxes lead to consistent string compactifications to four dimensional Minkowski space without any runway potential for the Kähler moduli. Hence, these models constitute four dimensional string vacua with zero cosmological constant and non-trivial soft terms for the gauge sector on D7-branes. Moreover, the D7 geometric moduli are generically stabilized, being this fact particularly relevant for KKLT scenarios.

The richness of the flux induced soft supersymmetry breaking patterns is such that allows for a variety of phenomenological scenarios. In particular, ISD fluxes induce a positive definite potential in the worldvolume of the D7-branes, parametrized by the gaugino masses and the μ-terms. The interesting properties for this potential could address some of the present empirical constraints for the soft supersymmetry breaking lagrangian. On the other hand, in the worldvolume of $\overline{D3}$-branes, ISD fluxes induce dilaton domination patterns, on which the breaking of the supersymmetry can be understood as coming from the vev of the F auxiliary field associated to the axiodilaton S.

Thus, based on all these ideas, we can affirm that the prospects of flux compactifications are promising. Still a lot of work has to be done, both in the direction of analyzing the flux induced soft SUSY breaking patterns as of understanding the possible flux degrees of freedom in general orientifold compactifications and constructing the corresponding ten dimensional supergravity solutions behind them. We hope to come back to all these issues in the near future.

Chapter 9

Comentarios finales.

Las compactificaciones en presencia de flujos introducen un mecanismo canónico para la resolución del problema de la estabilización de moduli en compactificaciones de Teoría de Supercuerdas. Por otro lado, constituyen una fuente tratable para la ruptura de supersimetría en un régimen computacional. De este modo, los flujos juegan necesariamente un papel prominente en la inclusión del Modelo Estándar en Teoría de Supercuerdas.

Aquí hemos revisado algunas de nuestras contribuciones a estos temas. En particular, hemos analizado la estructura de vacíos del potencial inducido por los flujos en un sencillo orientifold $T^6/\Omega_P(-1)^{F_L}\sigma$ de Tipo IIA con flujos constantes NSNS, RR y métricos. Comparado a los orientifolds de Tipo IIB, los orientifolds de Tipo IIA poseen una riqueza de flujos que da pie a vacíos con todos los moduli de cuerda cerrada estabilizados en AdS sin la necesidad de efectos no perturbativos.

Además, los flujos métricos abren nuevas posibilidades para la cancelación de los tadpoles RR. Así, uno puede encontrar situaciones en las que los flujos no contribuyen a los tadpoles RR o contribuyen negativamente, es decir, del mismo modo que los O-planos, proporcionando en algunos casos la interesante posibilidad de suprimir O-planos. Esto constituye un notable avance respecto a las compactificaciones de Tipo IIB, donde la condición de ISD fuerza al flujo a contribuir positivamente a los tadpoles RR de modo que, para satisfacer las condiciones de tadpole y al mismo tiempo estabilizar los moduli en valores grandes, uno típicamente se ve obligado a considerar variedades con números de Euler grandes.

Nuestra aproximación al problema ha sido considerar los flujos métricos como una deformación añadida al toro original. El toro con torsión resultante es una variedad semi-plana con cohomología reducida. De este manera, los flujos métricos en cierto modo están extendiendo la región del espacio de moduli a la cual podemos acceder, tomando en consideración no solo las posibles deformaciones geométricas del toro, sino también algunas deformaciones topológicas.

Resulta iluminador el visualizar la estructura del toro con torsión como la resultante de hacer T-dualidad en orientifolds de Tipo IIB con flujos de 3-forma NSNS no nulos. En este caso, uno puede identificar entre los diferentes vacíos de los orientifolds de Tipo IIA, los vacíos especulares de algunos de los modelos sin escala de Tipo IIB. Inspirados por este hecho, cabe preguntarse si los potenciales de Tipo IIB inducidos por los flujos pueden también depender de todos los moduli. La inclusión de flujos no geométricos en orientifolds de Tipo IIB restaura la T-dualidad entre teorías efectivas de Tipo IIA y de Tipo IIB, pero en este caso las monodromías en la variedad interna mezclan la métrica con el campo B, desapareciendo de este modo el carácter geométrico de la compactificación.

La inclusión de flujos no geométricos elimina la S-dualidad heredada por el potencial efectivo en presencia de flujos RR y NSNS ordinarios. Para recuperar esta simetría uno ha de introducir un nuevo conjunto de flujos S-duales. Tanto los flujos nuevos como los antiguos están sujetos a una serie de condiciones de Bianchi y de cancelación de tadpoles. Éstas pueden ser obtenidas mediante uso de transformaciones de $SL(2,\mathbb{Z})_S$. Hemos visto que la estructura resultante es suficientemente rica como para contener vacíos Minkowski con no solo el dilatón y los moduli de estructura compleja estabilizados, sino también los moduli de Kähler.

Generalizando este esquema, uno puede imponer en el superpotencial invarianza bajo el grupo de dualidad completo. La idea se sustenta en los resultados provenientes de compactificaciones de la heterótica y de teoría M. En nuestros ejemplos toroidales hemos visto que esto requiere la presencia de 2^7 parámetros (flujos) para describir el conjunto completo de configuraciones invariantes bajo $SL(2,\mathbb{Z})^7$. Algunos de estos grados de libertad poseen una interpretación simple como flujos métricos, RR o NSNS explícitos en alguna de las versiones particulares de Teoría de Supercuerdas. Otras por contra no admiten tal interpretación y su origen todavía está a expensas de ser comprendido. A pesar de todo, es muy posible que todos estos flujos estén de alguna manera presentes en una formulación completa de la teoría.

Una cuestión natural en todos estos modelos, es la interrelación entre D-branas y flujos. Hemos visto que los flujos de 3-forma NSNS pueden inducir tadpoles para los potenciales gauge que viven en las D6 y en las D7-branas. Por otro lado, y directamente relacionado con esto, los flujos métricos y no geométricos parametrizan cambios topológicos en la variedad interna de modo que algunos de los ciclos originales desaparecen de la homología y los moduli correspondientes son desplazados del espectro no masivo. El enrollar D-branas en estas sub-variedades podría inducir inconsistencias en la teoría dentro de la brana. Aquí hemos analizado el caso particular de las D6-branas en orientifolds de Tipo IIA. En este caso, los flujos pueden inducir masas para los axiones que median la cancelación de anomalías $U(1)$ a través del mecanismo de Green-Schwarz. Tales términos no respetan las propiedades de transformación de los axiones bajo rotaciones $U(1)$. El imponer consistencia requiere entonces ciertas restricciones en los 3-ciclos que las D6-branas pueden enrollar. Estas condiciones pueden ser visualizadas como una generalización de la cancelación de anomalías de Freed-Witten. En particular, en el caso de vacíos AdS $\mathcal{N}=1$ supersimétricos, estas condiciones fuerzan a los diferentes conjuntos de D6-branas a estar calibrados, es decir, a preservar la misma supersimetría $\mathcal{N}=1$ en cuatro dimensiones. De este modo, uno puede construir modelos explícitos con espectro quiral cercano al del MSSM con tres generaciones y con todos los moduli de cuerda cerrada estabilizados en AdS, dónde además la supersimetría $\mathcal{N}=1$ es impuesta dinámicamente por los flujos. Aquí hemos mostrado un ejemplo concreto en el que además se hace uso de los flujos para cancelar los tadpoles RR sin la ayuda de una acción orbifold.

Desde el punto de vista de la teoría efectiva en cuatro dimensiones, la dinámica de las D-branas en presencia de flujos de cuerda cerrada está determinada a través del potencial para los moduli de cuerda abierta inducido por los flujos. En términos del MSSM, éste puede ser identificado con el potencial de ruptura de supersimetría, estando los moduli de cuerda abierta asociados a los squarks/sleptones y Higgsses. Aquí hemos calculado, desde un punto de vista microscópico, el lagrangiano de ruptura de supersimetría inducido por los flujos de las 3-formas NSNS y RR en el volumen de las D3 y D7-branas. Para flujos ISD, los términos soft en el volumen de las D3-branas se hacen nulos, en concordancia con la estructura sin escalas del potencial. En las D7-branas sin embargo aparecen patrones de ruptura de supersimetría no nulos incluso para flujos $\mathcal{N}=1$ supersimétricos. Esto resulta interesante puesto que tales flujos dan pie a compactificaciones de cuerdas consistentes con espacios cuadridimensionales de tipo Minkowski. De este modo, estos modelos constituyen vacíos de cuerdas con constante cosmológica nula y términos soft no nulos en el sector gauge de las D7-branas. Además, los moduli geométricos de las D7-branas son generalmente estabilizados, siendo este hecho particularmente relevante para escenarios KKLT.

La riqueza de los patrones de ruptura de supersimetría inducidos por los flujos es tal que permite una gran variedad de escenarios fenomenológicamente viables. En particular, los flujos ISD inducen un potencial definido positivo en el volumen de las D7-branas, que puede ser parametrizado simplemente por las masas de los gauginos y los términos μ. Las interesantes propiedades de este potencial podrían arrojar luz sobre algunas de las restricciones experimentales observadas para el lagrangiano de ruptura de supersimetría. Por otro lado, en el volumen de las $\overline{D3}$-branas, los flujos ISD inducen patrones dominados por el dilatón, en los cuales la ruptura de la supersimetría se puede entender como proveniente del vev adquirido por el campo auxiliar F asociado al axiodilatón S.

Basándonos en todas estas ideas, podemos afirmar por tanto que los prospectos de las compactificaciones con flujos son prometedores. Todavía mucho trabajo ha de ser realizado, tanto en la dirección del análisis de los patrones de ruptura de supersimetría inducidos por los flujos, como de la comprensión de los posibles grados de libertad asociados a los flujos en compactificaciones orientifold generales y de la construcción de soluciones de supergravedad en diez dimensiones. Esperamos volver sobre todos estos temas en un futuro cercano.

Appendix A

$\mathcal{N} = 1$ Supersymmetry in 4 dimensions revisited.

Most of the content of this thesis is related to four dimensional $\mathcal{N} = 1$ supersymmetry. Supersymmetry has played indeed a crucial role, addressing stability issues for the D-brane configurations and the low energy limit of Type II orientifolds. In this appendix we would like thus to summarize some of the main features of four dimensional $\mathcal{N} = 1$ supersymmetry.

As it is well known, supersymmetry provides an efficient solution to the hierarchy problem by associating to every fermion a scalar super-partner whose contribution to the quadratic divergences is the same but with opposite sign. From the mathematical point of view, the algebra of supersymmetry arises as a natural extension of the Poincaré group. In fact, during the late 60's, Coleman and Mandula [1] showed that any Lie group which contains the Poincaré group and an internal symmetry group is always a direct product of both groups and therefore it leads to trivial physics. Therefore, it was necessary to consider more general structures. This was done by Gol'fand and Likhtman [185] who considered a subset $\{Q_\alpha\}$ of generators of the algebra as satisfying anti-commutation relations, in what is called a 'Z_2 graded structure'. Taking a minimal set of these anticommuting generators in the spinorial representation[1] of the Lorentz group, starting with the Poincaré group and applying the generalized Jacobi

[1] Actually, this is the only choice giving rise to the generators of the translations in the vectorial representation.

identities for graded algebras one obtains the $\mathcal{N}=1$ supersymmetry algebra (see e.g. [186])[2]

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot\beta} \ , \tag{A.1}$$

$$[Q_\alpha, P^\mu] = 0 \ , \tag{A.2}$$

$$[Q_\alpha, M_{\mu\nu}] = i(\sigma_{\mu\nu})^\beta_\alpha Q_\beta \ , \tag{A.3}$$

$$[Q_\alpha, R] = -iQ_\alpha \ , \tag{A.4}$$

where we have decomposed the Majorana supercharge Q into its two Weyl components Q_α, so the indices α and $\dot\alpha$ belong respectively to the first and the second $SU(2)$ of $SO(3,1) \simeq SU(2)_L \times SU(2)_R$, with metrics $\epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} = i\sigma^2$ and related among themselves by the conjugation operation $\overline{Q}_{\dot\alpha} = (Q_\alpha)^*$. P_μ is the generator of translations, i.e. the space-time momentum, $M^{\mu\nu}$ the Lorentz generators given by

$$M^{\mu\nu} = -\frac{i}{4}[\gamma^\mu, \gamma^\nu] \tag{A.5}$$

and $\sigma_{\mu\nu} = \sigma_{[\mu}\sigma_{\nu]}$. R is the only allowed internal generator for this case, the so called R-symmetry, which here represents a chiral rotation.

We observe thus how the internal $U(1)$ symmetry gets combined in a non trivial fashion with the supersymmetry generators, avoiding on this way the no-go theorem of Coleman and Mandula pointed out above.

Let us review briefly the construction of $\mathcal{N}=1$ supersymmetric theories [187]. The possible particle content of the theory is given by the different irreducible representations of the algebra. The easiest way to construct them is by means of the superspace formalism on which the supercharges are considered as generators of translations in the fermionic coordinates ϵ and $\bar\epsilon$, which now are Grassmann numbers. The action of the SUSY generators on a given superfield $S(x^\mu, \epsilon, \bar\epsilon)$ is then given by

$$-iP_\mu = \partial_\mu \ , \tag{A.6}$$

$$iQ_\alpha = \partial_{\epsilon^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial_\mu \equiv D_\alpha \ , \tag{A.7}$$

$$-i\bar{Q}_{\dot\alpha} = \partial_{\bar\epsilon^{\dot\alpha}} - i\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \equiv \bar{D}_{\dot\alpha} \ . \tag{A.8}$$

Hence, the left chiral irreducible representations will correspond to superfields Φ satisfying the condition $\bar{D}_{\dot\alpha}\Phi = 0$. Expanding now in the fermionic coordinates we get the following components

$$\Phi(y,\epsilon) = \phi(y) + \sqrt{2}\epsilon\psi(y) + \epsilon\epsilon F(y) \ , \tag{A.9}$$

with $y^\mu = x^\mu + i\epsilon\sigma^\mu\bar\epsilon$. So a left chiral supermultiplet is a $(0,1/2)$ multiplet, whereas its CPT conjugate, a right chiral supermultiplet, is a $(0,-1/2)$ multiplet.

[2] Here we will adopt the mostly minus signature for the metric in four dimensional $\mathcal{N}=1$, as it is usual in field theory. Therefore, there will be an additional minus sign in the contractions of the Minkowski indices when these come from a higher dimensional metrics, as noticed in the Appendix of [20].

The F-field, as usual, is not a physical field but an auxiliary field. In fact, the closure of the supersymmetry algebra is given by

$$[\delta_\epsilon, \delta_{\epsilon'}]\psi = -2(\epsilon\sigma^\mu\bar{\epsilon}' - \epsilon'\sigma^\mu\bar{\epsilon})i\partial_\mu\psi = 0 \ , \tag{A.10}$$

which is equivalent to impose the system to be on-shell

$$\overline{\sigma}^\mu \partial_\mu \sigma = 0 \ , \tag{A.11}$$

with $\bar{\sigma}^\mu = (I_2, -\vec{\sigma})$. The F-field extends this relation to off-shell situations, introducing additional bosonic degrees of freedom which vanish on-shell, so we have in general the same amount of fermionic and bosonic components.

The vector supermultiplet is constructed imposing the reality condition

$$S(x,\epsilon,\bar{\epsilon})^+ = S(x,\epsilon,\bar{\epsilon}) \ , \tag{A.12}$$

which gives rise to the expansion

$$V(x,\epsilon,\bar{\epsilon}) = c(x) + i\epsilon\chi(x) - i\bar{\epsilon}\bar{\chi}(x) + (i\epsilon\epsilon\bar{\epsilon}[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi] + h.c.) + \\ + \frac{1}{2}\epsilon\epsilon\bar{\epsilon}\bar{\epsilon}[D(x) - \frac{1}{2}\partial_\mu\partial^\mu] \tag{A.13}$$

with C, χ and D auxiliary fields. But since a U(1) gauge transformation is given now by

$$V \to V + i(\Lambda - \Lambda^+) \ , \tag{A.14}$$

with Λ a chiral superfield, we have extra degrees of freedom in the gauge parameter that we can fix. The usual choice is the so called Wess-Zumino gauge on which we use this to eliminate C, M, N and χ, so the gauge transformation becomes the usual one

$$V_\mu \to V_\mu - \partial_\mu(\eta + \eta^+) \ , \tag{A.15}$$

η being the lowest component of the chiral superfield, and the superfield expansion reduces to

$$V^a = \epsilon\sigma^\mu\bar{\epsilon}V^a_\mu(x) + (i\epsilon\epsilon\bar{\epsilon}\bar{\lambda}^a(x) - i\bar{\epsilon}\bar{\epsilon}\epsilon\lambda^a(x)) + \frac{1}{2}\epsilon\epsilon\bar{\epsilon}\bar{\epsilon}D^a(x) \ . \tag{A.16}$$

Therefore, the vector multiplet is a $(1/2, 1)$ multiplet.

Up to now we have been talking about global supersymmetry. However, by making it local, it is possible to couple supersymmetry to gravity and set up $\mathcal{N}=1$ supergravity. The way to do it is by means of the usual Noether procedure [187, 186]. Let us illustrate it for the simplest case, the Wess-Zumino model, with just one free chiral multiplet. The globally invariant action is given by[3]

$$\mathcal{L} = (\partial^\mu\phi)^+(\partial_\mu\phi) + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi \ , \tag{A.17}$$

[3] Here ψ is in the Majorana form.

with supersymmetry transformations

$$\delta_\epsilon(Re\ \phi) = \bar{\epsilon}\psi\ , \tag{A.18}$$

$$\delta_\epsilon(Im\ \phi) = -i\bar{\epsilon}\gamma_{(5)}\psi\ , \tag{A.19}$$

$$\delta_\epsilon\psi = -i\gamma^\mu[\partial_\mu((Re\ \phi) - i\gamma_{(5)}(Im\ \phi))]\epsilon\ . \tag{A.20}$$

Now, making $\epsilon = \epsilon(x)$ we have

$$\delta_\epsilon\mathcal{L} = \partial_\mu\bar{\epsilon}J^\mu(x)\ , \tag{A.21}$$

with

$$J^\mu(x) = \gamma^\mu[\gamma^{nu}\partial_\nu((Re\ \phi) - i\gamma_{(5)}(Im\ \phi))]\psi\ . \tag{A.22}$$

So we introduce a new superfield, the supergravity multiplet, a (3/2,2) multiplet composed by the gravitino Ψ^μ_α and the graviton $h_{\mu\nu}$, and we add the following terms in the lagrangian density

$$\mathcal{L}_N = -\frac{\kappa}{2}\overline{\Psi}_\mu J^\mu - \kappa h_{\mu\nu}T^{\mu\nu} + \mathcal{O}(\kappa^2)\ , \tag{A.23}$$

with $T^{\mu\nu}$ the energy-momentum tensor. The supergravity multiplet therefore plays the role of a vector superfield for the local supersymmetry. Now, $\mathcal{L} + \mathcal{L}_N$ is invariant under local supersymmetry transformations, although the action is not renormalizable anymore. It only rests to add the kinetic terms for the new superfield. These are given by the Hilbert action for the metric, and the Rarita-Schwinger action for the gravitino

$$\mathcal{L}_{kin} = -\frac{1}{2\kappa^2}eR - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_{(5)}\gamma_\nu D_\rho\psi_\sigma\ , \tag{A.24}$$

with e^m_μ the vielbein and D_μ the covariant derivative with respect to gravity

$$D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu{}^{mn}\sigma_{mn}\ . \tag{A.25}$$

It is possible to show that the most general $\mathcal{N}=1$ SUSY invariant non-renormalizable action can be written as [188, 189]

$$\mathcal{L} = \int d^2\epsilon d^2\bar{\epsilon}K(\Phi^+e^{2gV},\Phi) + \int d^2\epsilon W(\Phi) + h.c. + \int d^2\epsilon f_{ab}(\Phi)F^\alpha_a F_{\alpha b} + h.c.\ , \tag{A.26}$$

where K is the Kähler potential, which determines the kinetic terms and derivative couplings (D-terms); W is the superpotential, an holomorphic function which contains the mass terms and interaction terms (F-terms); f_{ab} is the gauge kinetic holomorphic function, which determines the gauge couplings; and F^a_α is the gauge field strength superfield given by

$$F^a_\alpha = \bar{D}^2D_\alpha V^a + igf^{abc}\bar{D}^2(D_\alpha V^b)V^c\ . \tag{A.27}$$

In the particular case of renormalizable theories these functions take the form

$$\begin{aligned} K(\Phi^+e^{2gV},\Phi) &= \Phi^+e^{2gV}\Phi\ , \\ W(\Phi) &= \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{3}\lambda_{ijk}\Phi^i\Phi^j\Phi^k\ , \\ f_{ab} &= g_a^{-2}\delta_{ab}\ , \end{aligned} \tag{A.28}$$

which holds the bosonic lagrangian

$$e^{-1}\mathcal{L}_B = -e^G(G^i(G^{-1})^j{}_i G_j - 3) - \frac{g^2}{2} Re\ f^{-1}_{ab} G^i (T^a)^j_i \phi_j G^k (T^b)^l_k \phi_l - \\ - \frac{1}{4}(Re\ f_{ab}) F^a_{\mu\nu} F^{\mu\nu b} + \frac{i}{4}(Im\ f_{ab}) F^a_{\mu\nu} \tilde{F}^{\mu\nu b} + G^i_j D_\mu \phi_i D^\mu \phi^{j*} - \frac{1}{2} R \quad \text{(A.29)}$$

with the Kähler metric G defined by

$$G = K(\phi, \bar{\phi}) + \log|W(\phi)|^2 \ , \quad \text{(A.30)}$$

$$G^i = \frac{\partial G}{\partial \phi_i}, \text{ etc.} \quad \text{(A.31)}$$

The fermionic piece is however much more complicated. Here we will show just the mass terms

$$e^{-1}\mathcal{L}_F = \frac{1}{2} e^{G/2} [-G^{ij} - G^i G^j + G^l (G^{-1})_l{}^k G_k{}^{ij}] \bar{\psi}_i \psi_j + \frac{1}{4} e^{G/2} G^l (G^{-1})^k_l f^*_{ab,k} \lambda^a \lambda^b \ . \quad \text{(A.32)}$$

The first line of eq. (A.29) corresponds to the scalar potential $V(\phi, \bar{\phi})$. The first term contains the F-terms whereas the second the D-terms. Very often it results useful to rewrite the F-term of the scalar potential as

$$V_F = e^K \left(g^{i\bar{j}} D_i W \overline{D}_{\bar{j}} \overline{W} - 3|W|^2 \right) \ , \quad \text{(A.33)}$$

with the Kähler derivative given by

$$D_i W = \partial_i W + W \partial_i K \ . \quad \text{(A.34)}$$

With all of this, the Minimal Supersymmetric extension of the Standard Model (MSSM) is built by taking the following steps

1. Promote every field of the Standard Model to its corresponding superfield.
2. Consider two Higgs doublets, instead of just one, with opposite supercharges, so the theory remains anomaly free and the superpotential is holomorphic.
3. Add a supersymmetry breaking pattern.

Appendix B

$\mathcal{N} = 4$ Super Yang-Mills and its decompositions.

Apart from the $\mathcal{N} = 1$ supersymmetry algebra summarized in Appendix A, it is possible to construct algebras with extended supersymmetry. These naturally arise in the context of String Theory, as we have seen. In particular, we will devote this appendix to the algebra of four dimensional $\mathcal{N} = 4$ supersymmetry, or what is the same, $\mathcal{N} = 1$ in ten dimensions. This is the maximal supersymmetry algebra that can be built inside a D-brane in String Theory and thus, the other supersymmetric theories appearing in the worldvolume of the D-branes can be understood as truncations of this theory.

We can build the $\mathcal{N} = 4$ algebra by direct procedures, like the ones we schematically mentioned in Appendix A, or by dimensional reduction of the ten dimensional $\mathcal{N} = 1$ supersymmetry [190]. From this last point of view, the R-symmetry acquires a nice geometrical interpretation as the product of the R-symmetry of the original theory before performing the dimensional reduction and the isometry group of the manifold on which we reduce. On this case, we have a single Majorana-Weyl supercharge in the **16** of $SO(9,1)$, so there is no R-symmetry.[1] Hence, the whole internal symmetry of $\mathcal{N} = 4$ in four dimensions can be understood as entirely coming from the isometry group of a 6 torus, i.e. $SO(6)$.

The four dimensional $\mathcal{N} = 4$ algebra is given by

$$\{Q^A_\alpha, \bar{Q}^b_\beta\} = -2P_\mu \delta^{AB}\Gamma^\mu_{\alpha\beta} - 2P_{Rm}\Gamma^{mAB}\delta\alpha\beta \ , \tag{B.1}$$

and the only irreducible representation of the global algebra is the vector supermultiplet, given by a non-chiral multiplet $(-1, -(1/2)^4, 0^6, (1/2)^4, 1)$. Hence, we have a pure Super Yang-Mills

[1] In $\mathcal{N} = 1$ in four dimensions we have as well a single supercharge in the **4** of $SO(3,1)$ but, as it is in the Weyl (or the Majorana) representation, chiral rotations are allowed.

theory with lagrangian density

$$\mathcal{L} = -\frac{1}{4g^2}Tr(F_{\mu\nu}F^{\mu\nu} + 2D_\mu\phi_m D^\mu\phi_m - [\phi_m, \phi_n]^2) - \frac{i}{2g^2}Tr(\bar{\psi}\Gamma^\mu D_\mu\psi + i\bar{\psi}\Gamma_m[\phi_m, \psi]). \quad \text{(B.2)}$$

The effect of the $SO(6)$ R-symmetry is to rotate components of equal spin inside the vector multiplet. In particular, the six scalars transform in a vectorial $\mathbf{6_v}$ of $SO(6)$ whereas the four spinors transform in the spinorial representation $\mathbf{4_s}$. Its CPT conjugates transform therefore in the $\mathbf{4_c}$.

It is interesting to note that the four-dimensional $\mathcal{N}=4$ theory can be rearranged in terms of $\mathcal{N}=1$ supermultiplets. Obviously, since the symmetry of $\mathcal{N}=4$ is much bigger than the one of $\mathcal{N}=1$, there are multiple ways to choose an $\mathcal{N}=1$ subset inside the $\mathcal{N}=4$ theory. This freedom is parametrized by the choice of the $\mathcal{N}=1$ R-symmetry group. Indeed, for a given decomposition (see e.g. [87])

$$\begin{aligned} SO(6) &\to SU(3) \times U(1) \ , \\ \mathbf{4_s} &\to \mathbf{1}_3 + \mathbf{3}_{-1} \ , \\ \mathbf{4_c} &\to \mathbf{1}_{-3} + \bar{\mathbf{3}}_1 \ , \\ \mathbf{6_v} &\to \mathbf{3}_{-1} + \bar{\mathbf{3}}_1 \ , \end{aligned} \quad \text{(B.3)}$$

the $U(1)$ corresponds to the R-symmetry of $\mathcal{N}=1$ whereas the $SU(3)$ remains as an extra global symmetry of the theory. It is possible now to rearrange the gaugino $\mathbf{1}_3$ together with the gauge boson in a $\mathcal{N}=1$ vector supermultiplet and its CPT conjugate; and the six different scalars $\mathbf{3}_{\pm 1}$ in three chiral supermultiplets and its CPT conjugates.

Other interesting decomposition is in terms of $\mathcal{N}=2$ multiplets. This supersymmetry usually appears in 1/4 BPS systems in String Theory, such as in the twisted sector of a D3/D7 system in flat space. From the geometrical point of view it can be understood as coming from dimensional reduction of the six dimensional $\mathcal{N}=1$ supersymmetry algebra, where there is a single self-conjugated Weyl supercharge. The R-symmetry group of $\mathcal{N}=2$ in four dimensions is $SU(2)$ whereas the possible irreducible representations of the global algebra are then the hypermultiplet, given by the non-chiral multiplet $(-1/2, 0^2, 1/2) + (-1/2, 0^2, 1/2)$, and the vector supermultiplet, given by $(-1, (-1/2)^2, 0) + (0, (1/2)^2, 1)$.

The way to decompose $\mathcal{N}=4$ SYM in terms of $\mathcal{N}=2$ representations is through the splitting of the R-symmetry group accordingly to

$$\begin{aligned} SO(6) &\to SO(4) \times SO(2) \simeq SU(2) \times SU(2) \times U(1) \ , \\ \mathbf{4_s} &= (2,1)_+ + (1,2)_- \ , \\ \mathbf{4_c} &= (1,2)_+ + (2,1)_- \ , \\ \mathbf{6_v} &= (2,2) + (1,1)_\pm \ , \end{aligned} \quad \text{(B.4)}$$

so the three possible ways to factorize $SO(4)$ in $SU(2)\times SU(2)$ correspond to the three possible orthogonal choices of the complex structure in the internal hyper-Kähler manifold of holonomy $SO(4)\simeq SU(2)\times SU(2)$. One of the $SU(2)$ of $SO(4)$ will act as the R-symmetry of the $N=2$ theory, whereas the remaining $SU(2)\times U(1)$ will be an extra global symmetry.

Once we fix the complex structure in the internal manifold, it is possible to rearrange the $\mathcal{N}=4$ vector multiplet in a $\mathcal{N}=2$ vector multiplet, composed by the gauge boson A^μ, the gaugino λ, Ψ^3 and Φ^3; and an hypermultiplet, composed by Ψ^1, Φ^2, Φ^3 and Ψ^2. The effect of the R-symmetry will be then to rotate Φ^1 into Φ^2 and Ψ^3 into λ.

In the case of $\mathcal{N}=2$ supersymmetry, the Kähler potential can be derived from an holomorphic prepotential $\mathcal{F}(\Phi)$, Φ representing the hypermultiplets of the theory,

$$K(\Phi,\Phi^*)=\mathrm{Im}\left(\sum_a \Phi^{a*}\partial_a\mathcal{F}(\Phi)\right)\ , \tag{B.5}$$

so the Kähler metric is given by

$$G_{a\bar{b}}=\mathrm{Im}(\partial_a\partial_b F)\ , \tag{B.6}$$

which is also known as 'rigid special Kähler metric'. It is important to notice however that this hyper-Kähler structure of the moduli space of $\mathcal{N}=2$ is spoiled when we couple it to gravity, although this will not affect us since, as we already commented, we will work in the flat-limit.

Appendix C

Spinorial embedding of background fluxes.

We have seen in Section 5.5 how the generalized duality invariant superpotential presents a $SL(2,\mathbb{Z})^7$ symmetry. In this appendix we will describe how the background fluxes are arranged into this structure and their embedding into the spinorial representation of $SO(7,7;\mathbb{Z})$. This was done [55].

Each of the seven $SL(2,\mathbb{Z})_X$ factors consists of two generators $S_{X,1}$ and $S_{X,2}$

$$S_{X,1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad , \quad S_{X,2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ , \tag{C.1}$$

acting on the modulus M_X. From eq. (5.199) one can see that $S_{X,1}$ corresponds to shifts on the corresponding axion and $S_{X,2}$ to M-duality transformations $M_X \to 1/M_X$.

The set of fluxes, denoted $\mathbb{G}$, contains 128 weights of the form $(\pm,\pm,\pm,\pm,\pm,\pm,\pm)$, where $\pm$ stands for $\pm\frac{1}{2}$. The transformation $M_X \to 1/M_X$ is then simply given by

$$S_{X,2}(n_1,\ldots,n_X,\ldots,n_7) = \mathrm{Sign}(n_X)(n_1,\ldots,-n_X,\ldots,n_7) \ . \tag{C.2}$$

Thus, eq. (5.203) transforms in such a way that the full supergravity scalar potential is invariant under the $S_{X,2}$ generators. The resulting map between weights and flux components is presented in Table C.1. We see, for instance, that $\overline{F}_3$ fluxes (Table 5.4) correspond to $(+,+,+,+,\underline{\pm,\pm,\pm})$ while $\overline{H}_3$ fluxes (Table 5.2) are represented by $(-,+,+,+,\underline{\pm,\pm,\pm})$ spinorial weights.

From Table C.1 we can easily read the action of the duality group in the different fluxes. Notice also that the duality transformations can be easily obtained by expressing the $SL(2,\mathbb{Z})$

generators in terms of lowering and raising operators. Namely, $S_{X,2} = S_{X,+} - S_{X,-}$ and $S_{X,1} = I + S_{X,-}$. Thus, for instance, $S_{1,2}\overline{F}_3 = -\overline{H}_3$ corresponds to Type IIB S-duality.

It is interesting to note how half of the degrees of freedom of each of the two Weyl spinors on which $\mathbb{G} = \mathbf{64} \oplus \mathbf{64'}$ can be decomposed correspond to RR fluxes, whereas the other half are generalized NS fluxes. Of these, half are heterotic and half are ordinary fluxes, thus giving a very symmetric structure.

Flux parameter	Weight	Flux parameter	Weight
$\bar{h}'_0$	$(-,-,-,-,-,-,-)$	e_0	$(+,+,+,+,+,+,+)$
h_0	$(-,+,+,+,+,+,+)$	m'	$(+,-,-,-,-,-,-)$
$-h_i$	$(+,\overbrace{-,+,+}^{i},+,+,+)$	$-\bar{f}'_i$	$(-,\overbrace{+,-,-}^{i},-,-,-)$
e_j	$(+,+,+,+,\overbrace{-,+,+}^{j})$	$\bar{a}'_j$	$(-,-,-,-,\overbrace{+,-,-}^{j})$
$\bar{h}'_i$	$(+,\overbrace{+,-,-}^{i},-,-,-)$	f_i	$(-,\overbrace{-,+,+}^{i},+,+,+)$
q'_j	$(+,-,-,-,\overbrace{+,-,-}^{j})$	a_j	$(-,+,+,+,\overbrace{-,+,+}^{j})$
$\bar{g}'_{ji}$	$(-,\overbrace{+,-,-}^{i},\overbrace{+,-,-}^{j})$	b_{ji}	$(+,\overbrace{-,+,+}^{i},\overbrace{-,+,+}^{j})$
a'_j	$(-,-,-,-,\overbrace{-,+,+}^{j})$	q_j	$(+,+,+,+,\overbrace{+,-,-}^{j})$
$-g_{ji}$	$(-,\overbrace{-,+,+}^{i},\overbrace{-,+,+}^{j})$	$-\bar{b}'_{ji}$	$(+,\overbrace{+,-,-}^{i},\overbrace{+,-,-}^{j})$
$-\bar{a}_j$	$(-,+,+,+,\overbrace{+,-,-}^{j})$	$-e'_j$	$(+,-,-,-,\overbrace{-,+,+}^{j})$
$-\bar{b}_{ji}$	$(+,\overbrace{-,+,+}^{i},\overbrace{+,-,-}^{j})$	$-g'_{ji}$	$(-,\overbrace{+,-,-}^{i},\overbrace{-,+,+}^{j})$
$-m$	$(+,+,+,+,-,-,-)$	$-h'_0$	$(-,-,-,-,+,+,+)$
b'_{ji}	$(+,\overbrace{+,-,-}^{i},\overbrace{-,+,+}^{j})$	$\bar{g}_{ji}$	$(-,\overbrace{-,+,+}^{i},\overbrace{+,-,-}^{j})$
f'_i	$(-,\overbrace{+,-,-}^{i},+,+,+)$	$\bar{h}_i$	$(+,\overbrace{-,+,+}^{i},-,-,-)$
$-e'_0$	$(+,-,-,-,+,+,+)$	$-\bar{h}_0$	$(-,+,+,+,-,-,-)$
$-\bar{f}_i$	$(-,\overbrace{-,+,+}^{i},-,-,-)$	$-h'_i$	$(+,\overbrace{+,-,-}^{i},+,+,+)$

Table C.1: Spinorial embedding of the background fluxes. The weights in each column correspond to one of the two Weyl spinors on which the set of fluxes $\mathbb{G}$ can be decomposed.

One can proceed analogously with the set of moduli $\mathbb{T}$. In this case they transform as a vectorial $\mathbf{7}$ of $SL(2,\mathbb{Z})^7$, as shown in Table C.2. Let us define

$$e^{i\mathbb{T}} \equiv 1 + i\mathbb{T} - \mathbb{T}\otimes\mathbb{T} + \dots \; . \tag{C.3}$$

In this language, the superpotential (5.203) then takes the very compact form

$$W = \mathbb{G} \otimes e^{i\mathbb{T}}|_{(+,+,+,+,+,+,+)} \ , \tag{C.4}$$

which is reminiscent of the typical expressions for flux induced superpotentials.

Moduli	Weight
S	$(1,0,0,0,0,0,0)$
T_i	$(0,\overbrace{1,0,0}^{i},0,0,0)$
U_i	$(0,0,0,0,\overbrace{1,0,0}^{i})$

Table C.2: Embedding of the moduli in a **7** of $SL(2,\mathbb{Z})^7$.

Moreover, the Bianchi identities now correspond to constraints in the components of the bispinor of fluxes

$$\mathbb{G} \otimes \mathbb{G} \ = \ \mathbb{G} \cdot \mathbb{G} \ \oplus \ \mathbb{G}\Gamma^{I_1}\mathbb{G} \ \oplus \ \ldots \ \oplus \ \mathbb{G}\Gamma^{I_1 I_2 I_3 I_4 I_5 I_6 I_7}\mathbb{G} \ , \tag{C.5}$$

where $\Gamma^{I_1 \ldots I_n} \equiv \Gamma^{[I_1} \cdot \ldots \cdot \Gamma^{I_n]}$ and Γ^I are the complexified gamma matrices of the relevant Clifford algebra, and $I_a = 1, \bar{1}, \ldots, 7, \bar{7}$.

Appendix D

Supersymmetry breaking in KKLT scenarios.

The bulk of this thesis has dealt with stabilization of the moduli by perturbative background fluxes. However, other effects such as quantum corrections [191, 192, 193] or non-perturbative [194] contributions can play as well a role in the stabilization of the moduli. Indeed, before introducing the non-geometric fluxes, this was the best available mechanism to stabilize the Kähler moduli in Type IIB orientifolds with O3-planes and constant RR and NSNS background fluxes. Indeed, as we saw in Section 5.1.1, these are suitably described at low energies by the Gukov-Vafa-Witten superpotential (5.5), which is independent of the Kähler moduli and thus leads to no-scale models.

In this context, Kachru, Kallosh, Linde and Trivedi (KKLT) [29] provided a construction on which to break the no-scale structure by making use of non-perturbative effects, thus stabilizing the Kähler moduli and at the same time lifting the vacua to dS. The construction consists on the following steps. First of all one chooses a non-supersymmetric vacuum of the Gukov-Vafa-Witten superpotential [18], on which the dilaton and complex structure moduli are fixed. Then, at the minimum, one has $W = W_0$. The second step then consists on adding a non-perturbative superpotential generated by Euclidean D3-branes [194] or gaugino condensation in the worldvolume of D7-branes. In this sense, the stabilization of the open string moduli for the D7-branes is crucial, leaving below the flux induced mass scale, a pure $\mathcal{N} = 1$ Super Yang Mills theory with gauge coupling related to the Kähler moduli through eq. (6.55). Then, for a single overall Kähler modulus T, the total effective superpotential after the second step becomes

$$W = W_0 + Ae^{-aT} , \tag{D.1}$$

which has a non-trivial minimum corresponding to a supersymmetric AdS vacuum.

The last step is then to add a source of D-terms contributing positively to the cosmological constant. In particular, they considered the inclusion of $\overline{D}3$-branes. This induces an additional energy density of the form

$$\Delta V \propto \frac{1}{(\mathrm{Im}\ T^3)} \ , \tag{D.2}$$

which lifts the vacuum to dS, although still some fine-tuning is needed to achieve small values for the cosmological constant. Alternative mechanisms, such as magnetic fluxes in the world-volume of the D7-branes [195], has been proposed. We refer the reader to [196, 197, 198, 202] for some other proposals.

Although the KKLT construction seems very promising at a first sight, it has been revealed that building explicit consistent models is not a simple task. In particular, ideally one should minimize the full scalar potential in one step instead of two. This makes the problem of finding minima far more difficult because, once a T dependent superpotential is considered, the no-scale structure disappears, the potential is no longer positive definite and the minimum does not necessarily correspond to the solution of the supersymmetric conditions $D_iW = 0$. Moreover, in many cases there are actually no minima at all, the potential is unbounded from below and all extrema are saddle points [199, 200, 201]. In these cases, one is enforced to consider additional effects such as gaugino condensation in the worldvolume of D3-branes or corrections to the gauge coupling constant on D7-branes due to magnetic fluxes.

In any case, the quantum and non-perturbative effects modify the general picture that we found in Chapter 6 for the flux induced soft supersymmetry breaking. A microscopical analysis now becomes almost impossible to be carried out, and one is enforced to take an effective supergravity approach to the problem.

Here we will consider a Kähler potential and gauge kinetic function slightly more general than (6.89), given by

$$K = -log\ (S+S^*) - 3log\ (T+T^*) + \frac{|\phi|^2}{(S+S^*)^{\xi_S}(T+T^*)^{\xi_T}} \quad , \quad f = \alpha S + \beta T \ . \tag{D.3}$$

Note that this potential includes the above kind of open string deformations, corresponding to $(\xi_S, \xi_T) = (1,0)$ and $(\xi_S, \xi_T) = (0,1)$, plus new cases such as the scalars arising at brane intersections or branes wrapping different two torus ($(\xi_S, \xi_T) = (1/2, 1/2)$).

Then one can compute the scalar potential in terms of the superpotential W and extract the following soft supersymmetry breaking terms [201, 56]

$$m_\phi^2 = m_{3/2}^2 \ - \ \xi_S \frac{|F_S|^2}{(S+S^*)^2} \ - \ \xi_T \frac{|F_T|^2}{3(T+T^*)^2} + V_0 \ , \tag{D.4}$$

$$M_{1/2} = \frac{F_T}{\alpha(S+S^*) + \beta(T+T^*)} \ , \tag{D.5}$$

$$A_{123} = \frac{-F_S}{(S+S^*)}\,(1-\xi_S^1-\xi_S^2-\xi_S^3) \;+\; \frac{-F_T}{(T+T^*)}\,(1-\xi_T^1-\xi_T^2-\xi_T^3)\;, \tag{D.6}$$

$$B = F_S(\frac{2\xi_S-1}{(S+S^*)} + \partial_S log\mu(S)) \;+\; \frac{F_T}{(T+T^*)}(2\xi_T-3) \;-\; m_{3/2}\;, \tag{D.7}$$

with $m_{3/2} = W\exp(K/2)$ and

$$F_S = e^{K/2}(S+S^*)^2(W_S \;-\; \frac{W}{(S+S^*)})\;, \tag{D.8}$$

$$F_T = e^{K/2}\frac{(T+T^*)^2}{3}(W_T \;-\; \frac{3W}{(T+T^*)})\;. \tag{D.9}$$

Now W include non-perturbative effects, the Gukov-Vafa-Witten piece and the open string perturbative terms discussed in Chapter 6.

Note that the relations (6.10) and (6.11) are in general no longer satisfied for arbitrary F_S, F_T, ξ_S and ξ_T. Thus, the presence of non-perturbative effects or $\overline{D3}$-branes lead to very model dependent soft supersymmetry breaking patterns, determined by the concrete values of both F_S and F_T and allowing in principle for a rich structure of soft supersymmetry breaking scenarios. We refer the reader to [201] for further details.

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