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# Naturalness of Electroweak Symmetry Breaking on the eve of LHC

Memoria de Tesis Doctoral realizada por

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# Chapter 1

# Introduction

## 1.1 Introduction (English)

In Nature, two kinds of elementary particles can be found: fermions and bosons, differing from their spin (half-integer spin for fermions and integer spin for bosons). Elementary fermions are also classified in quarks and leptons, depending on the dominant interactions - strong interactions for quarks and electromagnetic interactions for leptons.

The Standard Model (SM) [1, 2] of the electroweak and strong interactions describes with extreme good precision the physics of elementary particles from atomic scales down to the shortest currently probed scales, about $10^{-18}$ m. The SM provides a unified framework to describe three of the four forces of Nature: electromagnetic, weak and strong interactions.

The high-precision measurements, performed at the per mille level accuracy and carried out during the last two decades at LEP, SLC and Tevatron, have firmly established the Standard Model as the correct description of the strong and electroweak interactions at present energies. This description is done by gauge theories, meaning that they model the forces between fermions by coupling them to gauge bosons, which mediate the forces. The gauge group of the strong interactions is $SU(3)_C$, and the gauge group of the electroweak interaction is $SU(2)_L \times U(1)_Y$. The couplings of quarks and leptons to the electroweak gauge bosons have been measured precisely and agree with those predicted by the model. However, the electroweak symmetry breaking mechanism that generates particle masses has not been tested yet. And this is one of the main objectives of the next generation of High Energy experiments, like LHC at CERN.

The origin of gauge boson masses and fermion masses is explained in the SM with the electroweak symmetry breaking (EWSB) mechanism. This spontaneous symmetry breaking is implemented by means of the Higgs mechanism [3]. This mechanism introduces a scalar $SU(2)_L$ doublet, $H = (H^+, H^0)$. The interactions of $H$ with the fermions are given by Yukawa couplings, whereas the Higgs potential describes the self interactions of $H$ in such a way that $H^0$ adquires a vacuum expectation value (vev), breaking the gauge symmetry $SU(2)_L \times U(1)_Y$ and giving masses to the fermions and the gauge bosons ($Z$ and $W^\pm$). This leads to just one physical Higgs particle in the spectrum: the SM Higgs boson. The discovery of the Higgs boson is of great importance. However, there is not experimental evidence for the SM Higgs, so the Higgs mechanism has not been tested yet. From direct searches and from precision electroweak data at LEP, the mass of the Higgs boson is restricted to be in the range 114.4 GeV $< m_h <$ 219 GeV at 95 % C.L. [4].

From the experimental perspective, the Standard Model will be completed with the discovery of the Higgs boson. Nevertheless, this model poses a number of unsolved questions and some theoretical problems that indicate that the SM is only a succesful low-energy description of a more fundamental underlying theory. One of the fundamental problems of the Standard Model is the "hierarchy problem" [5]. This is the naturalness problem on the Higgs mass. From experimental data the Higgs mass must be of the order of the electroweak scale ($\sim$ 100 GeV), but from the naturalness perspective this mass should be much larger than the electroweak scale. This is due to the large radiative corrections to the Higgs mass, which imply an unnatural tuning between the tree-level Higgs mass and the radiative corrections in order to stabilize the Higgs mass at $\sim$ 100 GeV. These radiative corrections diverge quadratically, indicating a quadratic sensitivity to the largest scale in the theory.

Solutions to this hierarchy problem imply new physics beyond the SM. New physics must be able to compensate the dangerously large corrections to the Higgs mass, and this can be obtained with the presence of new symmetries and particles. The hierarchy problem is the main motivation for the existence of new physics, but not the only one. In fact, there are some other important open questions within the SM: why three generations of quarks and leptons and the mass hierarchies between them?, what is the explanation of the necessary matter-antimatter asymmetry in the universe?, how can gravity be included within the framework of quantum physics?....

In addition to the above theoretical problems in the SM, there is experimental evidence that suggests the existence of new physics: the observation of neutrino oscillations [6]. This observation attests that neutrinos have masses, although very small. When the SM was formulated, there was no evidence of neutrino masses and, in consequence, it forbids neutrino masses at all orders in perturbation theory. Due to the small neutrino masses compared with the other fermions, the origin of these masses seems to differ from the standard Higgs mechanism. The most natural explanation for this lightness is given in the minimal Seesaw model in which right-handed neutrino singlets are introduced with Majorana masses $M$ much larger than the electroweak scale. However, as we will see in this thesis, this model does not solve the hierarchy problem associated to the Higgs mass, quite the contrary. Now, besides the usual SM radiative corrections to the Higgs mass, there are others coming from the very massive right-handed neutrinos that couple to the Higgs. This represents a new manifestation of the hierarchy problem, suggesting the existence of new physics in addition to the right-handed neutrinos (being supersymmetry the favourite candidate).

The most popular candidate of physics beyond the SM is Supersymmetry (SUSY) [7]. This elegant symmetry relates fermions and bosons in such a way that there are the same number of bosonic and fermionic degrees of freedom, and their coupling are related. With these features, the quadratically divergent contributions from new particles (the sparticles) cancel exactly the contributions from SM particles, technically solving the hierarchy problem. However, no sparticle has been found yet, so Supersymmetry must be a broken symmetry. This SUSY breaking leads to other dangerous logarithmic and finite contributions to the Higgs mass from sparticles, introducing a new fine-tuning problem: the supersymmetric fine-tuning problem.

Among the various candidates of new physics there is a recent proposal that tries to solve the hierarchy problem, the Little Higgs models [8]. This kind of scenarios is based on the idea of making the Higgs a pseudo-Goldstone boson, whose mass is protected at one-loop order from quadratically divergent corrections by a global symmetry. According to their proponents, these models are valid up to a cut-off scale of around 10 TeV with no fine-tuning price. Beyond this cut-off these models need an UV completion.

The present work focusses on these fine-tuning problems in the electroweak symmetry breaking that are present when the Higgs mass suffers from dangerous contributions which tend to destabilize the electroweak scale. First, we have revised the hierarchy problem in the SM, and then we have studied what happens with specific scenarios of physics beyond the SM, mentioned above.

As explained above, in the SM the Higgs mass receives important quadratically divergent contributions, so the requirement of no fine-tuning between these divergent contributions and the tree-level mass sets an upper bound of a few TeV on the scale where new physics should appear. If the scale of new physics is a large scale, e.g the Planck scale ($10^{19}$ GeV), this fine-tuning is huge, leading to the so called Big Hierarchy problem. On the other hand, if the considered scale is $\sim$ 10 TeV (the scale given by the experimental lower bound on the effective scale of some higher order operators), the fine-tuning is around 1%; this is known as the Little Hierarchy problem. In this thesis, these two arguments are re-examined and we also show the limitations of the hierarchy problem argument to estimate the scale of new physics. We will see that, although the estimate of a few TeV for the upper bound given by the SM hierarchy problem could be evaded in the pure SM, quantitatively, this estimate turns out to work reasonably well in most of the cases and it should be considered as a conservative bound. This leads to an optimistic prospect, as it sets the scale of new physics on the possible reach of LHC. This subject is presented in chapter 2.

It could happen, however, that no new physics (apart from the Higgs) is found in LHC, in spite of the naturalness arguments based on the SM Hierarchy Problem. Likewise, it could also happen that a heavy Higgs would be found (out from the range consistent with electroweak precision tests). Both possibilities can be understood in the SM plus higher order operators. But they could also point out to modifications of the ordinary SM Higgs sector. The simplest one is to include a second Higgs doublet. In chapter 3 we examine the naturalness properties in some types of two Higgs doublet models (2HDM) [9]. In that chapter we also analyze the possibility of having a mirror symmetry, in which the entire SM is replicated in a mirror world. This mirror world communicates with our world through a mixing in the Higgs sector. These modified Higgs sectors can be studied with a very particular type of 2HDM and in the literature it has been suggested that they can improve the naturalness SM upper bounds on the scale of new physics.

Chapters 4-6 are devoted to physically relevant examples of new physics. In chapter 4 we study the case of right-handed neutrinos with a seesaw mechanism and show that the case of the non supersymmetric SM plus right-handed seesaw neutrinos suffers from a very important fine-tuning problem, which calls for the existence of *additional* new physics, being a supersymmetric seesaw the optimal candidate to alleviate this problem.

Supersymmetry and some relevant supersymmetric scenarios are considered in chapter 5. In SUSY, the cancellation of quadratic divergent corrections takes place to all orders of pertubation theory. However, as we have seen above, the logarithmic and finite radiative corrections to the Higgs mass lead to another fine-tuning problem: the supersymmetric fine-tuning problem. According to the usual analyses, in the minimal supersymmetric standard model (MSSM), the absence of fine-tuning requires an abnormally stringent upper bound in the masses of the sparticles of $\sim$200 GeV. On the other hand, radiative corrections to the

Higgs mass (needed to have a Higgs mass consistent with the LEP lower bound) together with experimental data, give a lower bound on the sparticle masses of around 300 GeV, which implies that the ordinary MSSM is fine-tuned at the few percent level. The reasons for this abnormally acute tuning of the MSSM are reviewed and updated in this work. Moreover, we discuss how other SUSY scenarios, in particular those with low-scale SUSY breaking, can evade the problematic aspects of the MSSM, saturating the general bound of a few TeV of the Hierarchy Problem of the SM.

Little Higgs models are examined using fine-tuning arguments in chapter 6. It will be shown that the fine-tuning associated to the electroweak breaking in Little Higgs scenarios is much higher than suggested by the rough estimates usually made in the literature. These scenarios have been proposed to solve the Little Hierarchy problem, since they are valid to a cut-off scale of 10 TeV with the Higgs mass protected from quadratically divergent contributions only at the one-loop order. Analyzing the fine-tuning in a rigourous way, we have found that the fine-tuning is essentially comparable to or worse than that of the Little Hierarchy problem of the SM (and higher than in many supersymmetric models). We identify the main sources of potential fine-tuning in this kind of scenarios that should be considered in order to construct a successful Little Higgs model.

## 1.2 Introducción (Castellano)

En la naturaleza existen dos tipos de partículas elementales, los fermiones y los bosones, que se diferencian por el spin (spin entero los bosones y spin semientero los fermiones). Además los fermiones elementales se clasifican en dos tipos: quarks y leptones, dependiendo de la interacción dominante que obedecen (las interacciones fuertes para los quarks y las interacciones electromagnéticas para los leptones).

El Modelo Estándar (MS) de las interacciones electrodébiles y fuertes describe de manera precisa la física de partículas elementales desde las escalas del átomo hasta las escalas más bajas accesibles experimentalmente, alrededor $10^{-18}$ m. El Modelo Estándar proporciona un escenario unificado en el cual se describen tres de las cuatro fuerzas de la Naturaleza: las interacciones electromagnéticas, débiles y fuertes.

Las medidas de alta precisión de las dos últimas decadas llevadas a cabo en los experimentos LEP, SLC y Tevatron han confirmado el Modelo Estándar como la teoría correcta para describir las interacciones electrodébiles y fuertes hasta las energías accesibles actualmente. Esta descripción se realiza a través de teorías gauge, que conforman las interacciones entre fermiones acoplándolos a los bosones de gauge, que son los que median las fuerzas. El grupo gauge de las interacciones fuertes es $SU(3)_C$, y el grupo gauge de las interacciones electrodébiles es $SU(2)_L \times SU(1)_Y$. Los acoplos de leptones y quarks a los bosones de gauge electrodébiles han sido medidos experimentalmente y coinciden con los que predice el modelo de manera teórica. Sin embargo, todavía queda una pieza del Modelo Estándar por comprobar de manera experimental; el mecanismo de ruptura de la simetría electrodébil que genera las masas de las partículas. Este es uno de los objetivos principales de la próxima generación de experimentos de altas energías como el LHC en el CERN.

El origen de las masas de los bosones gauge y los fermiones se explica en el MS a través del mecanismo de ruptura de la simetría electrodébil. Esta ruptura espontánea de simetría se lleva a cabo por medio del mecanismo de Higgs. Este mecanismo introduce un doblete escalar en la teoría, $H = (H^+, H^0)$. Las interacciones de este doblete con los fermiones vienen dadas por los acoplos de Yukawa mientras que el potencial de Higgs describe las interacciones propias del doblete $H$ de tal manera que $H^0$ adquiere un valor esperado en el vacío, rompiendo así la simetría gauge electrodébil $SU(2)_L \times U(1)_Y$ y originando masas para los fermiones y los bosones de gauge $Z$ y $W^\pm$. Este proceso da lugar a una partícula escalar física en el espectro: el bosón de Higgs del Modelo Estándar. El descubrimiento del bosón de Higgs es extremadamente importante en la física de partículas, pero hasta la fecha no hay evidencia experimental de la existencia de este bosón, y por lo tanto el mecanismo de Higgs no ha sido probado. Lo que sí se conoce de las busquedas directas del Higgs y de los datos de precisión electrodébil en LEP es que la masa del Higgs está restringida al rango 114.8 GeV $< m_h <$ 219 GeV al 95% de nivel de confianza.

Desde el punto de vista experimental, el Modelo Estándar se confirmaría con el descubrimiento del bosón de Higgs. Sin embargo, algunos problemas teóricos y varias cuestions sin resolver parecen indicar que el MS es sólo una descripción efectiva de baja energía de una teoría subyacente. Uno de los problemas fundamentales del MS es el "problema de las jerarquías", esto es el problema de naturalidad de la masa del Higgs. De los datos experimentales sabemos que la masa del Higgs debe ser de orden la escala electrodébil ($\sim$ 100 GeV), pero desde la perspectiva de la naturalidad esta masa debería ser mucho mayor. Esto se debe a las grandes correciones radiativas a la masa del Higgs, que nos conducen a un ajuste artificial entre la masa del Higgs a nivel árbol y las correciones radiativas si queremos que la masa se estabilice alrededor de los 100 GeV. Estas correciones radiativas divergen cuadráticamente con la escala, indicando una sensibilidad cuádratica a la escala mayor de la teoría.

Las soluciones propuestas a este problema de las jerarquías implican física más allá del MS. La nueva física debe ser capaz de compensar las correciones peligrosas a la masa del Higgs. Esto puede conseguirse con la presencia de nuevas simetrías y partículas. Una de las motivaciones principales para suponer la existencia de nueva física es el problema de las jerarquías, pero no la única. En realidad, hay otras cuestiones sin respuesta dentro del MS: el porqué de tres generaciones y la jerarquía de masa entre ellas, la explicación de la asimetría de matería-antimatería en el universo, la incorporación de la gravedad en un escenario de física cuántica,.....

Además de los problemas teóricos anteriormente nombrados, hay una evidencia experimental que sugiere la existencia de nueva física: la observación de las oscilaciones de neutrinos. Esta observación demuestra que los neutrinos tienen masa, aunque muy pequeña ($<$ 1 eV). Cuando el Modelo Estándar se formuló, no había evidencia experimental de las masas de los neutrinos, por lo que se prohibían las masas para los neutrinos a todo orden en teoría de pertubaciones. Debido a que la masas de los neutrinos son muy pequeñas comparadas con las masas de los otros fermiones, el origen de las masas de los neutrinos parece diferir del mecanismo de Higgs ordinario. La explicación más natural a estas pequeñas masas se da posiblemente en el modelo mínimo de "seesaw", en el cual se introducen neutrinos singletes dextrógiros con masas de tipo Majorana, $M$, mucho mayores que la escala electrodébil. Sin embargo, como veremos en esta tesis, este modelo "seesaw" no da una solución al problema de las jerarquía asociado a la masa del Higgs, más bien al contrario. Ahora, además de las correciones radiativas usuales a la masa del Higgs, tenemos las originadas por los neutrinos dextrógiros muy masivos que también se acoplan al Higgs. Esto nos da una nueva manifestación del problema de las jerarquías, y nos sugiere la existencia de nueva física aparte de los neutrinos dextrógiros, siendo supersimetría el candidato favoratorio.

El candidato más común de física más allá del MS es supersimetría. Supersimetría es una elegante simetría que relaciona fermiones y bosones de tal manera que hay el mismo número de grados de libertad bosónicos y fermiónicos con acoplos relacionados entre sí. De este modo, las contribuciones cuadráticamente divergentes debidas a la nuevas partículas supersimétricas cancelan exactamente las contribuciones de las partículas del MS, resolviendo así el problema de las jerarquías. Sin embargo, hasta el momento no se ha encontrado ninguna partícula supersimétrica, indicándonos que si existe, la supersimetría debe ser una simetría rota. La ruptura de la supersimetría da lugar a nuevas contribuciones peligrosas a la masa del Higgs, esta vez contribuciones finitas y logarítmicas, introduciendo un nuevo problema de ajuste fino o fine-tuning: el problema del fine-tuning supersimétrico.

Entre los candidatos a nueva física, existe una propuesta reciente que intenta resolver el problema de las jerarquías: los modelos de Little Higgs. Estos escenarios están basados en la idea de hacer el Higgs un boson pseudo-Goldstone, cuya masa esté protegida de correcciones cuadráticamente divergentes al nivel de un loop. Según sus partidarios, estos modelos son válidos hasta una escala de cut-off de alrededor 10 TeV, hasta la cual no habría problema de fine-tuning. A partir de esta escala se necesitaría una teoría ultravioleta que los completara.

El presente trabajo se centra en estos problemas de ajuste fino o fine-tuning en la ruptura electrodébil que son debidos a las contribuciones peligrosas a la masa del Higgs que tienden a desestabilizar la escala electrodébil. En primer lugar, se revisará el problema de las jerarquías del Modelo Estándar, y a continuación se estudiará lo que ocurre con escenarios concretos de física más allá del Modelo Estándar, como los mencionados.

Como se comentó anteriormente, en el Modelo Estándar la masa del Higgs recibe contribuciones cuadráticamente divergentes, de tal modo que el requisito de que no exista fine-tuning entre estas contribuciones divergentes y la masa a nivel árbol, establece un límite superior para la escala de nueva física de unos pocos TeV. Si como escala de nueva física consideramos escalas fundamentales, como la escala de Planck ($\sim 10^{19}$ GeV), el fine-tuning que se tiene en la masa del Higgs es enorme: esto se conoce como el "gran" problema de las jerarquías. Por otro lado, si la escala considerada es $\sim 10$ TeV (el límite experimental inferior en la escala efectiva de algunos operadores de orden superior), el fine-tuning que se tiene es del

1% y este fine-tuning es el llamado "pequeño" problema de las jerarquías. En esta tesis, estos dos argumentos se volverán a examinar y se mostrarán la limitaciones del uso del problema de las jerarquías para dar una estimación sobre la escala de nueva física. Veremos que, aunque la estimación de unos pocos de TeV para el límite superior dada por el problema de las jerarquías podría ser evadido dentro del MS, se tiene que, de manera cuantitativa, esta estimación funciona razonablemente bien en la mayoría de los casos y debe ser considerado como un límite conservador. Esto nos lleva a una perspectiva optimista, ya que establece la escala de nueva física dentro del alcance del LHC. Este tema será presentado en el capítulo 2.

A pesar de estas buenas perspectivas, puede ocurrir que llegado el momento y a pesar del argumento de naturalidad en el que se basa el problema de las jerarquías del Modelo Estándar, en el LHC no se encuentre nueva física aparte del Higgs. Asimismo, puede ocurrir que se encuentre un Higgs pesado que no concuerde con el rango dado por los test de precisión electrodébil. Ambas posibilidades pueden ser explicadas en el MS más operadores de orden superior. Pero también podrían sugerir una modificación al sector de Higgs usual del Modelo Estándar. La modificación más simple es incluir un segundo doblete de Higgs. En el capítulo 3 se examinan las propiedades de naturalidad de algunas clases de modelos con dos dobletes de Higgs. En este capítulo también se analizará la posibilidad de la existencia de un mundo "espejo" replica del MS. La manera de comunicar ambos mundos sería a través de una simetría que permitiera la mezcla entre los bosones de Higgs. Estas modificaciones en el sector de Higgs pueden dar lugar a mejoras en los límites superiores de naturalidad en la escala de nueva física, y esto es lo que se estudiará dentro del capítulo 3.

Los capítulos del 4 al 6 están dedicados a ejemplos relevantes de nueva física. En el capítulo 4 se estudiará el caso de neutrinos dextrógiros, mostrando como el mecanismo de seesaw da lugar a un problema de fine-tuning importante que reclama la existencia de nueva física adicional a los neutrinos dextrógiros.

La supersimetría y algunos escenarios supersimétricos relevantes son considerados en el capítulo 5. En el caso supersimétrico la cancelación de correciones cuadráticamente divergentes tiene lugar a todo orden de teoría de perturbaciones. Sin embargo, como se ha comentado, en el modelo supersimétrico mínimo la correciones finitas y logarítmicas a la masa del Higgs dan lugar a un nuevo problema de fine-tuning: el problema de fine-tuning supersimétrico. Siguiendo los análisis usuales de fine-tuning, en el modelo supersimétrico mínimo (MSSM) la ausencia de fine-tuning requiere una cota superior muy estricta para las masas supersimétricas de aproximadamente 200 GeV. Por otro lado, las correciones radiativas a la masa del Higgs (necesarias para hacerla consistente con la cota inferior experimental) junto a los límites experimentales directos, nos dan una cota inferior para las masas supersimétricas de aproximadamente 300 GeV, con lo que el MSSM está ajustado al nivel de aproximadamente el 4 %. Las razones de este ajuste o tuning acentuado para el MSSM serán revisadas y actualizadas en este trabajo. Y se tratarán otros escenarios supersimétricos que pueden eludir los aspectos problemáticos del MSSM. En particular, se mostrará como modelos con escala baja de ruptura de supersimetría (no muy lejos del TeV) pueden saturar el límite general de unos pocos TeV dado por el problema de las jerarquías del Modelo Estándar.

Los modelos de Little Higgs, son examinados en el capítulo 6. Se mostrará cómo el fine-tuning asociado a la ruptura electrodébil en escenarios de Little Higgs es mucho mayor que lo que sugieren las primeras estimaciones realizadas en la literatura. Estos escenarios se crearon para resolver el “pequeño” problema de las jerarquías, ya que sólo son válidos hasta una escala de cut-off de 10 TeV y la masa del Higgs está protegida de correciones cuadráticamente divergentes hasta esta escala. Si se analiza el fine-tuning de manera rigurosa, se encuentra que éste es similar o peor que el fine-tuning del “pequeño” problema de las jerarquías del MS y mayor que en muchos modelos supersimétricos. Además, en este capítulo se identificarán las principales fuentes de fine-tuning en este tipo de escenarios, para que puedan ser consideradas a la hora de construir un modelo de Little Higgs satisfactorio.

# Chapter 2

# Hierarchy Problem

The "Hierarchy problem" [5] of the Standard Model is a theoretical problem that arises when one assumes that the SM remains valid beyond the scales at which it has been directly tested. What happens is that the SM is not a stable theory if a second energy scale (Planck scale, Grand Unified Theories scale,...) is introduced. This is due to the large radiative corrections to the Higgs mass. These corrections diverge quadratically with the largest scale in the theory, indicating a quadratic sensitivity to this scale. But from the classic constraints of unitarity, triviality and vacuum stability and from precision electroweak constrains, the Higgs mass in the SM should be of the order of the electroweak scale ($\sim 100$ GeV). Therefore, this requires unnatural adjustments between the tree-level Higgs mass and the radiative corrections. These adjustments are known as a "fine-tuning" problem of the SM. This is rather unsatisfactory, unless it may be cured by means of some new physics able to compensate the dangerously radiative corrections to the Higgs mass.

## 2.1 Big and Little Hierarchy Problem

The Hierarchy Problem of the SM motivates the existence of new physics beyond the SM at scale $\Lambda \lesssim$ few TeV. What is the argument?.

If the SM is considered as an effective theory valid below a scale $\Lambda$, the Higgs mass in the lagrangian, $m^2$, receives quadratically-divergent contributions. The most significant of these divergences come from the one-loop diagrams of fig. 2.1 involving the top quark, the gauge bosons and the Higgs itself. These quadratic divergences at one-loop are given by:

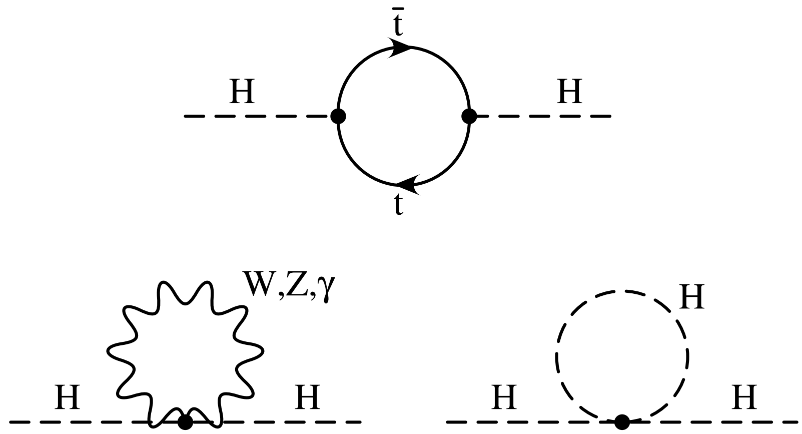



Figure 2.1: The most significant quadratically divergent contributions to the Higgs mass in the Standard Model.

$$\delta_{\rm q} m^2 = \frac{3}{64\pi^2}(3g^2 + g'^2 + 8\lambda - 8\lambda_t^2)\Lambda^2 \ , \tag{2.1}$$

where $g, g', \lambda$ and $\lambda_t$ are the $SU(2) \times U(1)_Y$ gauge couplings, the quartic Higgs coupling and the top Yukawa coupling respectively. In terms of masses

$$\delta_{\rm q} m^2 = \frac{3}{16\pi^2 v^2}(2m_W^2 + m_Z^2 + m_h^2 - 4m_t^2)\Lambda^2 \ , \tag{2.2}$$

where $m_W^2 = g^2 v^2/4$, $m_Z^2 = (g^2 + g'^2)v^2/4$, $m_h^2 = 2\lambda v^2$ and $m_t^2 = \lambda_t^2 v^2/2$, with $v = 246$ GeV. These radiative corrections to the Higgs vacuum expectation value (vev) tend to destabilize the electroweak scale. The requirement of no fine-tuning between the tree-level and the one-loop contribution to $m^2$ sets an upper bound on $\Lambda$. E.g for a Higgs mass $m_h = 115 - 200$ GeV, and imposing that one-loop contributions are not bigger than 10 times the value of $m_h^2$ [1],

$$\frac{\delta^{\rm quad} m^2}{m^2} \leq 10 \ \Rightarrow \ \Lambda \lesssim 2 - 3 \text{ TeV}, \tag{2.3}$$

where we have implicitly used $v^2 = -\frac{m^2}{\lambda}$ and $m_h^2 = 2m^2$. With this argument, new physics should appear to modify the ultraviolet behaviour of the SM. This is known as the "Big Hierarchy" problem [10], since other fundamental scales ($M_{Planck}$, $M_{GUT}$) are larger than this upper bound on $\Lambda$.

[1] Obviously, if one is stricter about the maximum acceptable size of $\delta_q m^2$, then $\Lambda^2$ decreases in the same proportion.

This "Big Hierarchy" argument gives an optimistic point of view for physics beyond the SM, as it predicts that New Physics (NP) will be detected at future high energy colliders, such as the LHC in CERN.

The previous upper bound, $\Lambda \lesssim$ few TeV, is in some tension with the experimental lower bounds on the suppression scale of dimension 6 operators [11]. Below $\Lambda$, the effective theory is the SM plus non-renormalizable operators that parametrize the effects of new physics beyond $\Lambda$. Adding only operators that preserve all local and global SM symmetries, the typical experimental limits on $\Lambda_{\rm LH}$ (defined as the effective scale of the new dimension six operators) are $\Lambda_{\rm LH} \gtrsim 10$ TeV. Then, if we set the scale of NP to be around 10 TeV, there is still a difference of one order of magnitude between this scale, $\Lambda_{\rm LH}$, and the no-fine-tuning upper bound ($\Lambda \lesssim$ few TeV), resulting in a fine-tuning of order 1%. This is known as the "Little Hierarchy" problem [10, 11], and it implies that NP at $\Lambda$ should be "clever" enough to be consistent with the constraints of the dimension 6 operators: non-strongly-interacting, flavour-blind NP is favoured.

As we have seen above, the Hierarchy argument sets an upper bound on the scale of new physics, that would be on the reach of LHC. However, following Veltman [12], one can note that if the Higgs mass lies ("by accident") close to the value that cancels $\delta_q m^2$ in eq. (2.2), this scale could be much larger without a fine-tuning price. At one-loop, Veltman's condition is,

$$3g^2 + g'^2 + 8\lambda - 8\lambda_t^2 \simeq 0 \ , \tag{2.4}$$

which is satisfied for $m_h \simeq 313$ GeV. If Veltman's condition is fulfilled, this will tell us that there is no (one-loop) fine-tuning problem in the SM. At higher order this condition becomes cut-off dependent [13, 14],

$$3g^2(\Lambda) + g'^2(\Lambda) + 8\lambda(\Lambda) - 8\lambda_t^2(\Lambda) \simeq 0 \ . \tag{2.5}$$

This last condition resums leading-log corrections to all orders. Hence, we write:

$$m^2(\Lambda) = m_0^2 + \delta_q m^2\big|_\Lambda \ , \tag{2.6}$$

where $m_0^2$ is the tree-level value of the mass parameter at the scale $\Lambda$, and $\delta_q m^2$ is as in eq. (2.1) but with couplings evaluated at $\Lambda$.

## 2.2 Quantifying the fine-tuning

In this chapter, we re-examine the use of the Hierarchy problem to extract information about the size of $\Lambda$ from naturalness arguments and we also consider the possibility of living near

Veltman's condition [15]. In order to do this, one needs a sensible criterion to quantify the degree of fine-tuning, which should be applied to all the models (SM and models of NP), to allow a fair comparison. Here we follow the standard Barbieri & Giudice criterion [16]. This method follows the next steps:

The Higgs vev, $v^2$, is written as a function of the fundamental parameters of the model, $p_i$. Then, one defines $\Delta_{p_i}$, the fine-tuning parameters associated to $p_i$, by

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta v^2}{v^2} = \Delta_{p_i} \frac{\delta p_i}{p_i} \ , \tag{2.7}$$

where $\delta v^2$ is the change induced in $v^2$ by a change $\delta p_i$ in $p_i$. Therefore, $|\Delta_{p_i}^{-1}|$ measures the probability of a cancellation among terms of a given size to obtain a result which is $|\Delta_{p_i}|$ times smaller [2]. Absence of fine-tuning requires that $\Delta_{p_i}$ should not be larger that $\mathcal{O}(10)$.

In cases where there are dangerous quadratic divergences to the mass parameter, $m$, as it happens in the SM, we can approximate the tuning in $v^2$ by the tuning in $m^2$

$$\frac{\delta v^2}{v^2} \simeq \frac{\delta m^2}{m^2} \simeq \left. \frac{\delta\{\delta_q m^2\}}{m^2} \right|_\Lambda \ , \tag{2.8}$$

where we have evaluated $\delta m^2/m^2$ at the $\Lambda$ scale and used eq. (2.6). This choice simplifies the computation and quantitatively the differences are negligible. Furthermore, it makes sense since the actual cancellation between the tree-level and the radiative contributions to $m^2$ occurs at that scale, as shown in eq. (2.6).

In order not to have unnatural cancellations among parameters to obtain the correct electroweak scale ($v = 246$ GeV), it is imposed that the total amount of fine-tuning, $\Delta$, is less than a certain quantity, $c$. Due to the statistical meaning of $\Delta_{p_i}$, it is sensible to define the total fine-tuning as:

$$\Delta = \sqrt{\sum\nolimits_i \Delta_{p_i}^2} \ . \tag{2.9}$$

If $\Delta \leq c$, a percentage variation of any of the parameteres $p_i$ corresponds to a percentage variation of $v^2$ less than $c$-times larger. For example, $c = 10$ tolerates cancellations among parameteres of at most one order of magnitude, so this corresponds to a 10% fine-tuning. If $c = 100$, the total fine-tuning is of 1%.

[2]Strictly speaking, $\Delta_{p_i}$ measures the sensitivity of $v^2$ against variations of $p_i$, rather than the degree of fine-tuning [17, 18]. However, for the EW breaking it is a perfectly reasonable fine-tuning indicator [17, 19]: when $p_i$ is a mass parameter, $\Delta_{p_i}$ is large only around a cancellation point.

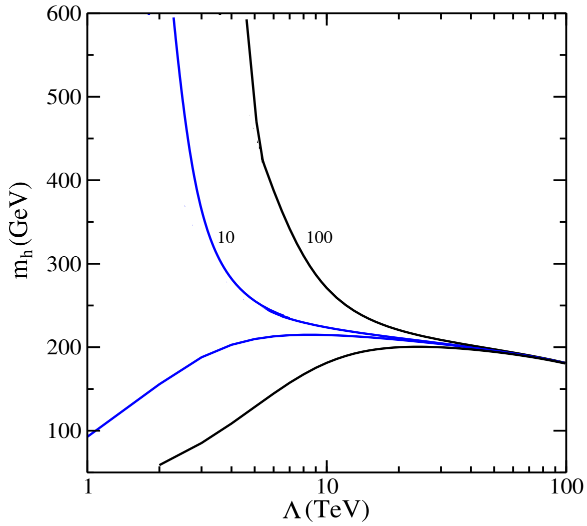



Figure 2.2: Fine-tuning contours corresponding to $\Delta = 10$ and 100.

Alternative definitions of $\Delta$, such as $\Delta = \text{Max}\{\Delta_{p_i}\}$, are possible and have been used in the literature. Although in many cases both definitions give very similar results (typically one single $\Delta_{p_i}$ dominates $\Delta$) we believe that definition (2.9) is more satisfactory conceptually. As an (extreme) example we can consider the case of an observable $O$ that depends on a large number $N$ of input parameters, say $O = \sum_i \alpha_i p_i$ with the $\alpha_i$ of $\mathcal{O}(1)$ and random signs, and with the measured value of $O$ and the natural values of the $p_i$ being of the same order. In such case all $\Delta^2_{p_i} \sim 1$ but the fine-tuning is $\mathcal{O}(\sqrt{N})$ (this example would correspond to a random walk where one expects such wandering of $O$ away from 1).

Now, we go back to the SM. The "Big Hierarchy" problem and Veltman's condition concern the dependence of $v^2$ on the scale $\Lambda$. We can plot $\Delta_\Lambda$ in the $(m_h, \Lambda)$ plane, as shown in fig. 2.2. In this plot the lines of $\Delta_\Lambda = 10$, 100 correspond to lines of 10% and 1% fine-tuning, respectively. This plot shows a throat, sometimes called the "Veltman's throat", that suggests that if $m_h \sim 195 - 215$ GeV, $\Lambda$ could be larger than 10 TeV without fine-tuning (and no "Little Hierarchy" problem). On the negative side, this means that if $m_h$ is in this range, new physics could escape detection at LHC. But this is not the actual situation if we evaluate the fine-tuning in a complete way taking into account all the parameters in the SM, besides $\Lambda$, that are not known or not yet measured with good precision.

Take for instance the top mass. According to the most recent experimental data [20], this mass is

$$M_t = 172.5 \pm 2.3 \text{ GeV}. \tag{2.10}$$

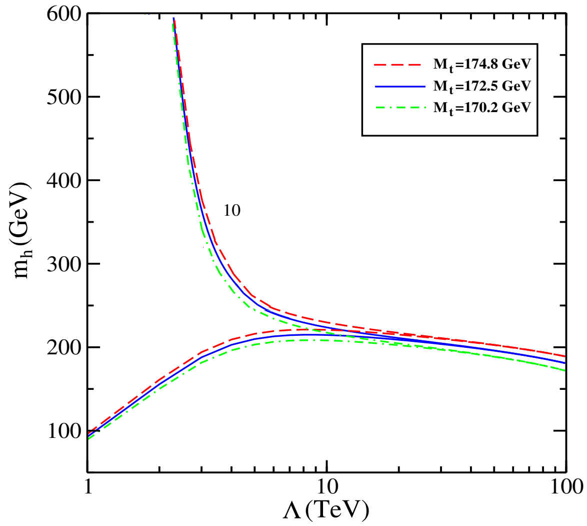



Figure 2.3: Same as fig. 2.2 for $\Delta = 10$ and three different values of the top mass.

This uncertainty in the top mass is remarkably small, but it should not be ignored for fine-tuning analyses. In fig. 2.3 we see three curves with $\Delta_\Lambda = 10$, corresponding to $M_t = 170.2$, 172.5 and 174.8 GeV. Because fine-tuning arguments are based on statistical considerations, the conclusions depend on our partial knowledge of the relevant parameters of the theory. In this case, we have to average our ignorance of the top mass, and this leads to a cut on the throat at $\Lambda \simeq 10$ TeV.

This effect can be reproduced by adding the two fine-tuning parameters $\Delta_\Lambda$ and $\Delta_{\lambda_t}$ in quadrature:

$$\Delta = \left(\Delta_\Lambda^2 + \Delta_{\lambda_t}^2\right)^{1/2} \quad . \tag{2.11}$$

Since $\lambda_t$ should only vary within the experimentally allowed range, the definition of $\Delta_{\lambda_t}$ must be modified as [19] (see Appendix C):

$$\Delta_{\lambda_t} = \frac{\partial v^2}{\partial \lambda_t}\frac{\lambda_t}{v^2} \times \frac{\delta^{\rm exp}\lambda_t}{\lambda_t}. \tag{2.12}$$

With this new $\Delta$ we can repeat the plot of fig. 2.2, obtaining the plot in the left side of fig. 2.4. We indeed see a cut at $\Lambda \simeq 10$ TeV for $\Delta = 10$. All this means that fine-tuning arguments will always set an upper limit on the acceptable value of $\Lambda$, and living near Veltman's condition does not raise significantly this upper bound.

The other remark concerns the fact that $m_h$ itself (and thus the quartic Higgs coupling, $\lambda$) is not known at present. Therefore the previous results, in particular the left plot in fig. 2.4, correspond to a future time when $m_h$ will be known. For instance, if LEP's inconclusive

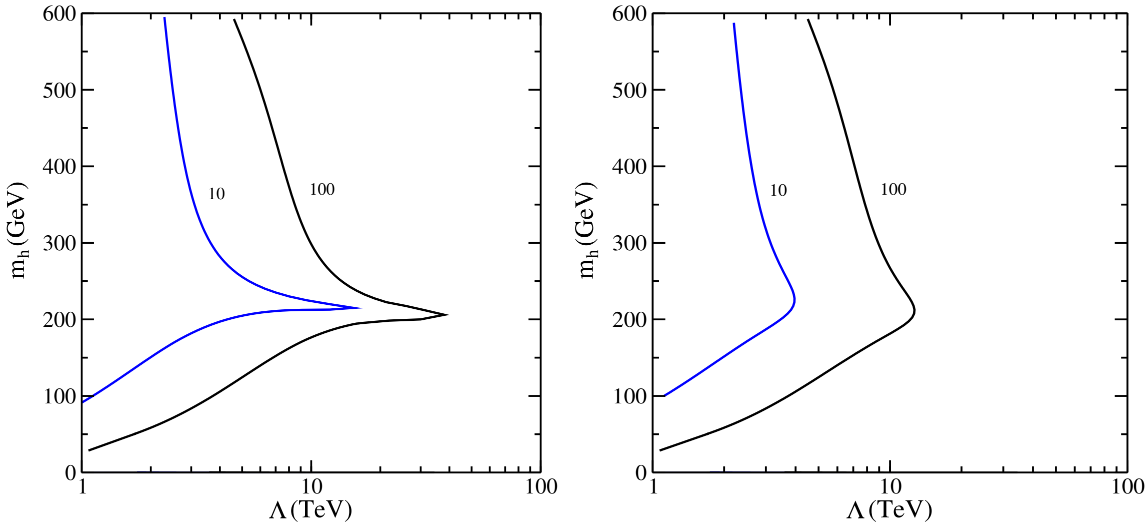



Figure 2.4: Contour plots of $\Delta = 10, 100$. Left: with $\Delta$ as defined in eqs. (2.11, 2.12). Right: with $\Delta$ as defined in eq. (2.13).

evidence for $m_h \simeq 115$ GeV [4] gets confirmed, then one expects $\Lambda \lesssim 1.4$ TeV. But at present one should consider the uncertainty in the Higgs mass and average over all the possible values of $m_h$, say in the range 115 GeV $\leq m_h \leq$ 600 GeV. This gives the result of an upper bound of $\Lambda \lesssim 2.5$ TeV, shown in the right side of fig. 2.4. This average can be implemented by adding in quadrature $\Delta_\lambda$ to obtain the global fine-tuning,

$$\Delta = \left(\Delta_\Lambda^2 + \Delta_{\lambda_t}^2 + \Delta_\lambda^2\right)^{1/2} \ . \tag{2.13}$$

The corresponding curve for $\Delta = 10$ is shown in the right plot of fig. 2.4, which corresponds to the present status of the problem. $\Lambda$ depends slightly on $m_h$, being always below 4 TeV. In average we can conclude that $\Lambda^{\rm av} \lesssim 2.5$ TeV from fine-tuning arguments.

As some authors have pointed out [23], it could happen that the New Physics (NP) that cancels the quadratically divergent corrections is different for the loops involving the top, the Higgs, etc. In that case, the coefficient $\Lambda$ in front of eq. (2.1) would be different for each term inside the parenthesis:

$$\delta_{\rm q} m^2 = \frac{3}{64\pi^2}\left[(3g^2 + g'^2)\Lambda_g^2 + 8\lambda\Lambda_h^2 - 8\lambda_t^2\Lambda_t^2\right] \ . \tag{2.14}$$

Then one should consider $\Delta_{\Lambda_t}$ and $\Delta_{\Lambda_h}$ (the most relevant fine-tuning parameters) separately. Fig. 2.5 shows the contour plots $\Delta_{\Lambda_t}, \Delta_{\Lambda_h} = 10$ in the $\{m_h, \Lambda\}$ plane (red and blue dashed lines). Notice that $\Delta_{\Lambda_t}$ decreases with increasing $m_h$ (or, equivalently, for fixed $\Delta_{\Lambda_t}$, the larger $m_h$, the larger may $\Lambda_t$ be). This follows trivially from $v^2 = -m^2/\lambda$ and the one-loop

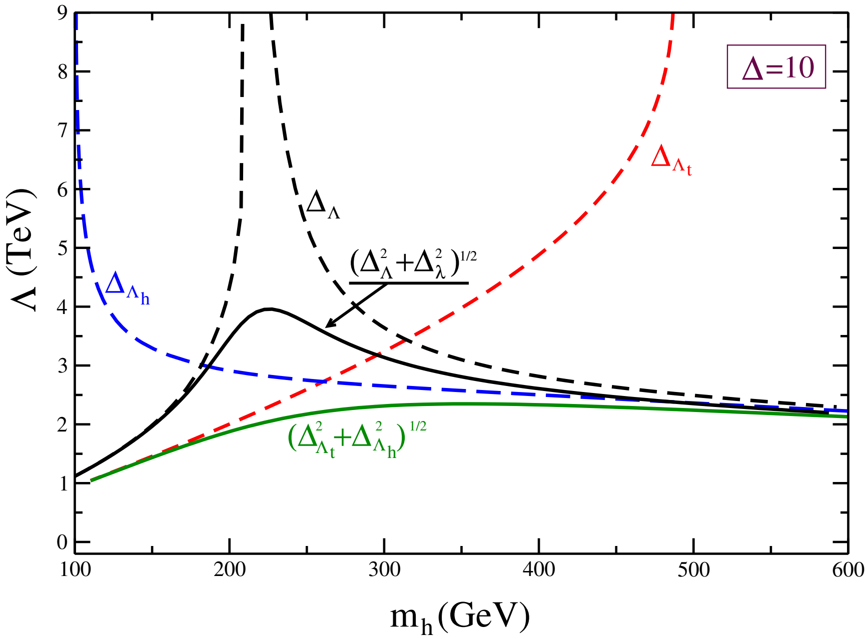



Figure 2.5: SM contour plots of $\Delta = 10$ in the $\{m_h, \Lambda\}$ plane with $\Delta = \Delta_{\Lambda_t}$ (red dashed), $\Delta_{\Lambda_h}$ (blue dashed), $(\Delta^2_{\Lambda_t} + \Delta^2_{\Lambda_h})^{1/2}$ (solid green). The black lines show the corresponding contour plots when a single cut-off, $\Lambda$, is used: $\Delta = \Delta_\Lambda$ (black dashed), $(\Delta^2_\Lambda + \Delta^2_{\Lambda_\lambda})^{1/2}$ (black solid).

expression for $\delta_q m^2$, eq. (2.1). Then,

$$\Delta_{\Lambda_t} \simeq \frac{3\lambda_t^2}{4\pi^2}\frac{\Lambda_t^2}{\lambda v^2} = \frac{3\lambda_t^2}{2\pi^2}\frac{\Lambda_t^2}{m_h^2}. \tag{2.15}$$

This fact has been used sometimes to suggest that a heavy Higgs behaves better for naturalness than a light one. A similar reasoning would indicate that $\Delta_{\Lambda_h}$ is independent of $m_h$. However, the actual behaviour is not that. We can see from fig. 2.6 that for larger $m_h$, $\Delta_{\Lambda_h}$ increases (especially if $\Lambda$ is large). This is a consequence of the increase of $\lambda$ when ran from $m_h$ to the $\Lambda$ scale (where $\delta_q m^2$ is evaluated). Due to the RGE of $\lambda$, this increase is more important for larger $\lambda$ (and thus $m_h^2$). This important feature [24] (sometimes not recognized in the literature) is lost if $\delta_q m^2$ is used at the lowest order, as given by eq. (2.1), without leading log corrections. The combined $\Delta = (\Delta^2_{\Lambda_t} + \Delta^2_{\Lambda_h})^{1/2}$ is dominated, at large $m_h$, by the $\Delta_{\Lambda_h}$ contribution (see the solid green line $\Delta = 10$ in fig. 2.5). Notice that, although $\Lambda_t$ and $\Lambda_h$ are independent parameters, they have been taken to be numerically equal in this figure.

On the other hand, it is perfectly possible that the new physics that cancels the quadratic divergences associated to the top loops is the same that cancels the Higgs loops. In that case one should take $\Lambda_t \sim \Lambda_h$. Then, one has to evaluate a single $\Delta_\Lambda$, as in fig. 2.2; the corresponding contour plot $\Delta_\Lambda = 10$ is shown by the black dashed line in fig. 2.5, and the

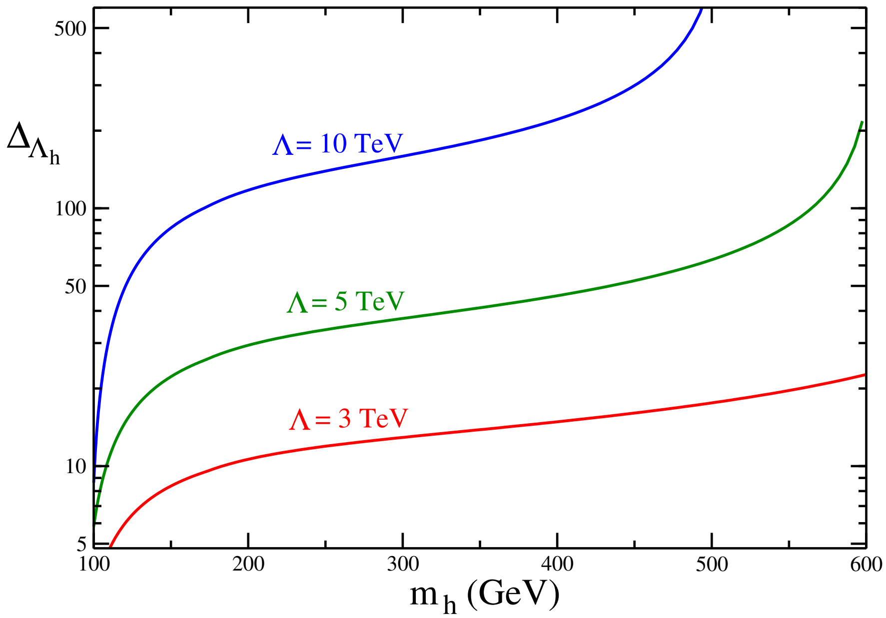



Figure 2.6: Fine-tuning in $\Lambda_h$ as a function of the Higgs mass in the SM with three different cut-offs $\Lambda =$ 3,5 and 10 TeV.

combined $\Delta = (\Delta^2_\Lambda + \Delta^2_\lambda + \Delta^2_{\lambda_t})^{1/2} = 10$ contour plot is shown by the solid black line in fig. 2.5, where Veltman's throat has become much less deep, as we have seen in fig. 2.4 (right). In summary, the solid lines of fig. 2.5 show the degree of fine-tuning of the SM for given $\{\Lambda, m_h\}$ under the assumption of independent[3] or correlated $\Lambda_t$ and $\Lambda_h$ cut-offs.

Another possibility is that the cut-offs of all quadratically divergent contributions are correlated, but not all equal. In that case, $\Lambda_t^2 = \zeta\Lambda_h^2$ with some $\zeta = \mathcal{O}(1)$ factor of proportionality, which depends on the unknown NP. This should be plugged in eq. (2.1) to re-evaluate $\delta_q m^2$ and the fine-tuning $\Delta$. Obviously, varying $\zeta$, even slightly, shifts the value of $\lambda$ (and thus of $m_h$) where the approximate cancellation of the quadratic contributions takes place. Consequently, the position of Veltman's throat changes, as shown in fig. 2.7, where the plotted cut-off is always the smallest one. Interestingly, taking a modest $\zeta = 1/2$ the throat is around $m_h = 150$ GeV, with $\Lambda \sim 3$ TeV. In this situation NP could escape LHC detection with no fine-tuning. It is quite remarkable the way in which correlated, but slightly different, cut-offs change in a physically significant form the ordinary expectations about the (approximate) cancellation of the quadratically divergent contributions [which can

[3]In the case of independent cut-offs the total fine-tuning should also include the $\Delta_\lambda$ contribution, but this does not modify substantially the result.

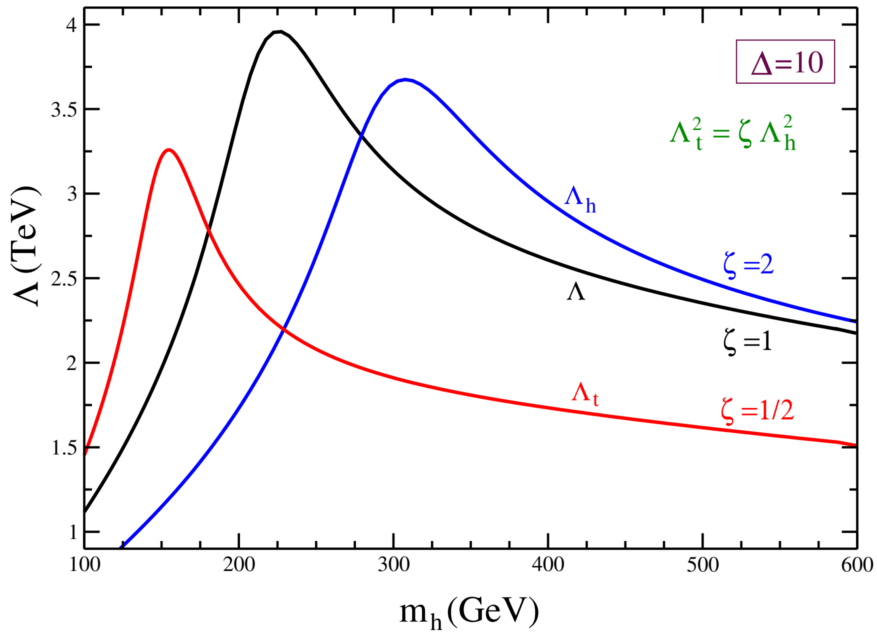



Figure 2.7: SM contour plots of $\Delta = 10$ with correlated cut-offs, $\Lambda_t^2 = \zeta \Lambda_h^2$. The plotted cut-off is the smallest one in each case.

mean detection or non-detection of NP at LHC]. In that sense, and as we will see in the next section, fine-tuning arguments can only be performed reliably when a particular scenario of NP is assumed.

Finally, let us look at the fine-tuning associated to the Little Hierarchy problem in the SM: i.e. the fine-tuning for $\Lambda = 10$ TeV. Fig. 2.8 shows this fine-tuning, with the value of $\Delta_\Lambda$ vs. $m_h$ given by the (bottom) black line. As we have seen in fig. 2.2 for $\Lambda = 10$ TeV, "Veltman's throath" is at $m_h \sim 220$ GeV. This results in the deep throath of the black line of fig. 2.8. This throat is cut when the fine-tuning parameter associated to the top mass ($\Delta_{\lambda_t}$) is added in quadrature as explained above, giving the (middle) red line. Finally, once the fine-tuning parameter associated to the Higgs mass itself ($\Delta_\lambda$) is included as well, the value of $\Delta$ is given by the (top) blue line, which thus represents the fine-tuning associated to the Little Hierarchy problem: 0.4-1 % tuning. Scenarios of new physics that attempt to solve the Little Hierarchy problem, such as the Little Higgs models, should be able to improve this situation.

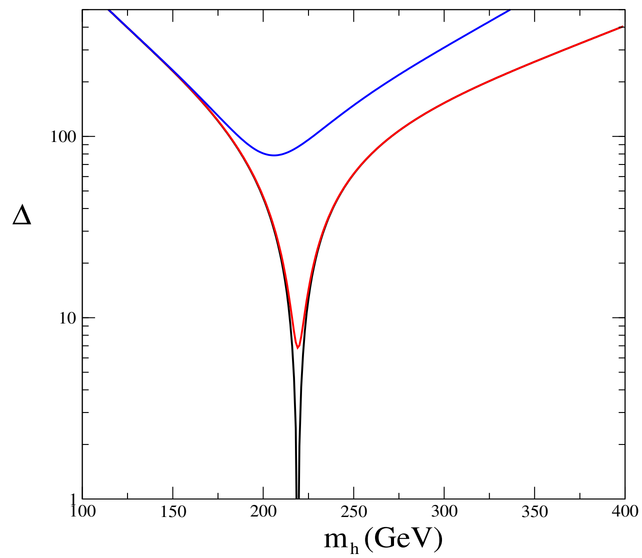



Figure 2.8: Total fine-tuning as a function of the Higgs mass in the SM with a cut-off $\Lambda = 10$ TeV. This can be considered as the fine-tuning of the Little Hierarchy problem in the SM. Different curves correspond to progressively more sophisticated definitions of $\Delta$ (from black [bottom line] to red to blue [top line], see text for details).

## 2.3 Limitations of the use of the hierarchy problem to estimate the scale of New Physics

Besides the previous remarks about the shape of Veltman's throat, there are more general caveats about using the hierarchy problem to estimate the scale of New Physics (NP) [15].

The "Hierarchy problem" argument implicitly assumes that the quadratically divergent contributions evaluated in the effective theory below $\Lambda$ (i.e. the SM in this case) remain uncancelled (except for artificial tunings or fortunate accidents) and the job of new physics is to cancel the dangerous contributions of the SM diagrams for momenta above $\Lambda$. However, the effects of new physics do not enter in such an abrupt and sharp way, and the new diagrams give non-negligible contribution already below $\Lambda$, and do not cancel exactly the SM contributions above $\Lambda$ [25]. This is what happens with the finite and logarithmic contributions from NP, which are not simply given by the SM divergent part cut off at the scale $\Lambda$.

We can just look at the Minimal Supersymmetric Standard Model (MSSM), where the cancellation of quadratically divergent contributions between the SM particles and their superpartners takes place at all scales. Due to this fact, Veltman's condition in this case is irrelevant for establishing the value of $\Lambda$. In SUSY the quadratically-divergent contributions to $m^2$ are cancelled anyway, with or without Veltman's condition, but there are dangerous

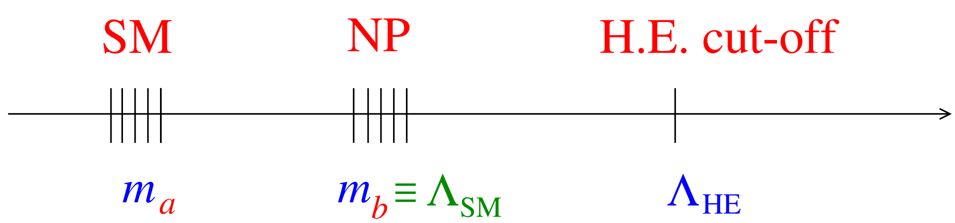



Figure 2.9: Schematic representation of the spectra associated to the various scales of the theory: Standard Model (SM), New Physics (NP) beyond the Standard Model and the high energy cut-off.

logarithmic and finite contributions from new physics, which do not cancel (in principle), and are totally unrelated to Veltman's condition.

We can generalized this argument in a straightforward way. For this we write the general one-loop effective potential using a momentum cut-off regularization, $V = V_0 + V_1 + \cdots$, with

$$V_0 = \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 \ , \tag{2.16}$$

$$V_1 = \frac{1}{64\pi^2}\mathrm{Str}\left[2\Lambda^2\mathcal{M}^2 + \mathcal{M}^4\left(\log\frac{\mathcal{M}^2}{\Lambda^2} - \frac{1}{2}\right)\right] + \mathcal{O}\left(\frac{\mathcal{M}^6}{\Lambda^2}\right) \ , \tag{2.17}$$

where $h$ is the (real and neutral) Higgs field, the supertrace Str counts degrees of freedom with a minus sign for fermions, and $\mathcal{M}^2$ is the (tree-level, $h-$dependent) mass-squared matrix. The one-loop contribution to the Higgs mass parameter is

$$\begin{aligned} \delta m^2 &= \left.\frac{\partial^2 V_1}{\partial h^2}\right|_{h=0} \\ &= \frac{1}{32\pi^2}\mathrm{Str}\left[\frac{\partial^2\mathcal{M}^2}{\partial h^2}\left(\Lambda^2 + \mathcal{M}^2\log\frac{\mathcal{M}^2}{\Lambda^2}\right) + \left(\frac{\partial\mathcal{M}^2}{\partial h}\right)^2\left(\log\frac{\mathcal{M}^2}{\Lambda^2} + 1\right)\right]_{h=0} . \end{aligned} \tag{2.18}$$

The supertraces can be written as sums over the SM states and the new physics states separately, with the masses of the lightest new physics states acting as the effective SM cut-off, $\Lambda$. However, since the new physics may not be the fundamental theory, we should consider the possible existence of a High-Energy cut-off, $\Lambda_{\rm HE}$ [4]. This is schematically represented in fig. 2.9. And the separation in the supertraces is given by

$$\begin{aligned}\delta m^2 &= \frac{1}{32\pi^2}\sum_a^{\rm SM} N_a \left[\frac{\partial^2 m_a^2}{\partial h^2}\left(\Lambda^2 + m_a^2 \log\frac{m_a^2}{\Lambda^2}\right)\right]_{h=0} \\ &+ \frac{1}{32\pi^2}\sum_b^{\rm NP} N_b \left[\frac{\partial^2 m_b^2}{\partial h^2}\left(\Lambda^2 + m_b^2 \log\frac{m_b^2}{\Lambda^2}\right) + \left(\frac{\partial m_b^2}{\partial h}\right)^2\left(\log\frac{m_b^2}{\Lambda^2}+1\right)\right]_{h=0}, \end{aligned} \quad (2.19)$$

where $m_a, N_a$ $(m_b, N_b)$ represent the mass and multiplicity, with negative sign for fermions, of the SM (NP) states, and $\Lambda \equiv \Lambda_{\rm HE}$. From the SM contributions only the quadratically divergent ones are dangerous. The other terms are vanishing, except for the contribution of the Higgs field itself, which is not large. On the other hand, all NP contributions (quadratic, logarithmic and finite) are potentially dangerous. Now, we can consider several situations that might take place:

**i)** There are no special cancellations among the different contributions in eq. (2.19). In that case the Hierarchy argument, based on the size of the quadratic contributions, and the corresponding bound $\Lambda \lesssim 2-3$ TeV, apply. The argument is clearly a conservative one due to the presence of extra contributions, which are discussed below.

**ii)** We are close to the Veltman's condition., i.e. the SM quadratically divergent contributions cancel (maybe approximately) by themselves. This situation was discussed in the previous section and there we concluded that, in the absence of a fundamental reason for the exact cancellation, one expects NP not far from the TeV scale from fine-tuning arguments. However, the new states of the second line of eq. (2.19) re-introduce a new fine-tuning problem, which is the situation discussed in the next points.

[4] In this argument we are assuming that the four-dimensional scalar Higgs field continues to be a fundamental degree of freedom up to the High-Energy cut-off. This set-up can change if, above $\Lambda$, the Higgs shows up as a composite field and/or new space-time dimensions open up. However one might expect that the new degrees of freedom would play a similar role as the NP states considered here, so that the conclusions might not differ substantially.

**iii)** The SM and NP quadratically divergent contributions cancel each other, i.e.

$$\sum_a^{\rm SM} N_a \left.\frac{\partial^2 m_a^2}{\partial h^2}\right|_{h=0} + \sum_b^{\rm NP} N_b \left.\frac{\partial^2 m_b^2}{\partial h^2}\right|_{h=0} = 0 \ . \qquad (2.20)$$

This is the case of SUSY and Little-Higgs models. But, again, the logarithmic and finite NP contributions in eq. (2.19) remain uncancelled and give another fine-tuning problem. Notice also that these contributions (unlike the quadratic ones) show up in any regularization scheme, differing only in the value of the finite pieces. In the $\overline{\rm MS}$ scheme these contributions are

$$\delta_{\rm NP}^{\overline{\rm MS}}\ m^2 = \sum_b^{\rm NP} \frac{N_b}{32\pi^2}\left[\frac{\partial^2 m_b^2}{\partial h^2} m_b^2 \left(\log\frac{m_b^2}{Q^2} - 1\right) + \left(\frac{\partial m_b^2}{\partial h}\right)^2 \log\frac{m_b^2}{Q^2}\right]_{h=0} \ , \qquad (2.21)$$

where $Q$ is the renormalization scale, to be identified with the cut-off scale, $\Lambda$. Quantitatively, these contributions are of the same magnitude as the SM quadratically-divergent one, replacing $\Lambda \to m_b \equiv \Lambda_{\rm SM}$. This gives the basis for the estimate of the "naive" Hierarchy argument discussed previously. However, new parameters not present in the SM might enter through the $m_b$ masses and, moreover, the presence of the logarithmically-enhanced terms makes the new contributions typically more dangerous than the SM estimate (as happens for instance in the supersymmetric case commented above). Hence, from fine-tuning arguments we can keep $m_b \equiv \Lambda_{\rm SM} \lesssim 2 - 3$ TeV, as a conservative bound.

Of course, if the new physics is itself an effective theory derived from a more fundamental one, further extra states [which could have $\mathcal{O}(M_p)$ masses] would be even more dangerous, unless their contributions are under control for some reason. The only clear example of this desirable property occurs when the theory is supersymmetric.

**iv)** It may happen that, besides the cancellation of quadratic contributions, the other dangerous contributions also cancel or are absent. In the $\overline{\rm MS}$ scheme this means that eq. (2.21) vanishes. This could happen by accident or for some fundamental reason, allowing the scale of new physics to be much larger than a few TeV. But, unluckily, not such fundamental reason has been formulated yet. In its absence, one has to average over the possible ranges of variation of the parameters defining the new physics (e.g. the soft masses for the MSSM). In this way the usual result $\Lambda \lesssim 2-3$ TeV is generically recovered.

However, one should not disregard completely the possibility that unknown fundamental physics is smart enough to implement naturally a cancellation like that of eq. (2.21), as SUSY does with quadratic divergences. In that case $\Lambda$ could be very large and any fine-tuning argument would be inappropriate.

In sum, what we can conclude is that the consideration of the SM quadratically-divergent contribution in order to estimate the scale of new physics works reasonably well in most cases. Because this procedure neglects unknown contributions, it is a conservative one but, because of the same reason, we cannot make more detailed statements. In order to derive more accurate implications for new physics from fine-tuning arguments, one should consider specific examples of physics beyond the SM.

## 2.4 Conclusions

In this chapter we have re-examined the use of the Hierarchy Problem of the SM to estimate the scale of New Physics (NP). The common argument is based on the size of the quadratically-divergent contributions to the squared Higgs mass parameter, $m^2$. Treating the SM as an effective theory valid below $\Lambda$, and imposing that those contributions are not much larger than $m^2$ itself, one obtains $\Lambda \lesssim 2-3$ TeV.

Because this argument predicts the scale of NP to be in the range of detection of future high energy colliders (such as the LHC at CERN), the future prospects for particle physics, both experimental and theoretical, are optimistic about the possibility of finding NP in the next years.

It has been argued in the literature [14] that, if $m_h$ lies (presumably by accident) close to the value that cancels the quadratic contributions (i.e. the famous Veltman's condition), $\Lambda$ could be much larger. However, we have shown with a rigorous fine-tuning evaluation (which should include the sensitivity to the top Yukawa coupling and the Higgs self-coupling) that this is not the case at present and, in average, the upper bound of $\Lambda \lesssim 2-3$ TeV is still kept [15].

We have also studied the case where the scale of NP that cancels the SM quadratic divergences is different for the loops involving the top, the Higgs, etc. Then, when the cut-offs are considered separately and analyzing the fine-tuning in the most relevant parametes (i.e., the cut-off on the momenta of virtual top quarks, $\Lambda_t$, and the one for virtual Higgses, $\Lambda_h$), we have seen that the total fine-tuning is typically a bit stronger than in the case of an unique cut-off. It could also happen that the cut-offs are not equal but correlated. In that situation, the Veltman's condition is satisfied for different $m_h$ depending on the correlation factor, and this shows that the use of the hierarchy problem to estimate the scale of New Physics should be done carefully.

As pointed above, the use of "Hierarchy problem" to estimate the scale of NP has its limitations. Its reasoning is arguably too naive, as it implicitly assumes that the SM quadratically divergent contributions, cut off at $\Lambda$, remain uncancelled by the effect of NP, except for accidents or artificial tunings. However, the NP diagrams give a non-negligible contribution already below $\Lambda$, and do not cancel exactly the SM contributions above $\Lambda$. Remnants of this imperfect cancellation are finite and logarithmic contributions from NP, which are not simply given by the SM divergent part cut off at $\Lambda$. The general analysis presented here, based on a model-independent study of the one-loop effective potential, shows that, quantitatively, these contributions are typically more dangerous than the estimate of the "naive" Hierarchy argument, where these contributions were neglected. Therefore, $\Lambda \lesssim 2 - 3$ TeV is indeed a *conservative* bound. In conclusion, the consideration of the SM quadratically-divergent contribution in order to estimate the scale of new physics works reasonably well in most cases.

# Chapter 3

# Modified Higgs Sectors

There are a number of reasons to consider possible modifications of the minimal Higgs sector of the Standard Model. The first one is the "Little Hierarchy" problem. Second, one should be prepared to interpret the possible situation in which no new physics is found at LHC, apart from the Higgs boson, in spite of the naturalness "Big Hierarchy" argument based on the SM Higgs sector. Finally, it could happen that the Higgs found at LHC is beyond the range consistent with electroweak precision test (EWPT), i.e. $m_h \geq 220$ GeV. Both situations can be interpreted in the SM as an effective theory, where the effects of New Physics are parametrized by higher order operators, but below the effective scale the spectrum of the theory is the SM one [24]. But they can also suggest some modification to the ordinary SM Higgs sector.

One of the simplest modifications of the SM Higgs sector one can think is to add a extra Higgs doublet, giving us a two Higgs doublet model (2HDM) [9, 26]. These 2HDMs, in a suitable setting, could raise the expected scale of new physics above the LHC reach or they could give a heavy Higgs consistent with EWPT [21, 22, 23]. We consider in this chapter three specific settings proposed in the literature and their properties from fine-tuning arguments.

## 3.1 Two Higgs Doublet Model (2HDM) description

Here we consider a generic scenario where the Higgs sector consists of two $SU(2)_L$ doublets of opposite hypercharge, $H_1$ and $H_2$ [9, 26]. The most general Higgs potential for such two Higgs doublet models (2HDM) is, at tree-level:

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - \left[m_3^2 H_1\cdot H_2 + \text{h.c.}\right] \\ &+ \frac{1}{2}\lambda_1|H_1|^4 + \frac{1}{2}\lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1\cdot H_2|^2 \\ &+ \left[\frac{1}{2}\lambda_5(H_1\cdot H_2)^2 + \lambda_6|H_1|^2 H_1\cdot H_2 + \lambda_7|H_2|^2 H_1\cdot H_2 + \text{h.c.}\right]\,. \end{aligned} \tag{3.1}$$

In most discussions of 2HDMs, the terms proportional to $\lambda_6$ and $\lambda_7$ are absent. This can be implemented by imposing a discrete symmetry $H_1 \to -H_1$ on the model. Such a symmetry would also require $m_3 = 0$ unless we allow a soft violation of this discrete symmetry by dimension-two terms. The fields will develop non-zero vacuum expectation values (vevs) if the mass matrix $m_{ij}^2$ has at the origin at least one negative eigenvalue. Assuming that CP invariance and $U(1)_{\text{EM}}$ gauge symmetry are not spontaneously broken, the minimum of the potential is

$$\langle H_1\rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}, \tag{3.2}$$

where the $v_i$ can be chosen real. The vevs have been normalized so that $M_W^2 = \frac{1}{4}g^2(v_1^2 + v_2^2)$. We introduce the following notation:

$$v^2 = v_1^2 + v_2^2, \qquad \tan\beta = v_2/v_1, \tag{3.3}$$

with $v = 246$ GeV. From the original eight scalar degrees of freedom, three Goldstone bosons are “eaten” by the $W^\pm$ and $Z$. The remaining five physical Higgs particles are two CP-even scalars ($h^0$ and $H^0$ with $m_{h^0} \le m_{H^0}$), one CP-odd scalar ($A^0$) and a charged Higgs pair ($H^\pm$). The mass parameters $m_1^2$ and $m_2^2$ can be eliminated by the minimization of the scalar potential,

$$\begin{aligned} m_1^2 - m_3^2\,\frac{v_2}{v_1} + \frac{1}{2}(\lambda_1 v_1^2 + \tilde\lambda v_2^2 + 3\lambda_6 v_1 v_2 + \lambda_7\frac{v_2^3}{v_1}) = 0, \\ m_2^2 - m_3^2\,\frac{v_1}{v_2} + \frac{1}{2}(\lambda_2 v_2^2 + \tilde\lambda v_1^2 + 3\lambda_7 v_1 v_2 + \lambda_6\frac{v_1^3}{v_3}) = 0, \end{aligned} \tag{3.4}$$

where $\tilde\lambda = \lambda_3 + \lambda_4 + \lambda_5$. After minimization, the resulting squared masses for the CP-odd and the charged Higgs states are

$$\begin{aligned} m_{A^0}^2 &= \frac{m_3^2}{\sin\beta\cos\beta} - \frac{1}{2}v^2(2\lambda_5 + \lambda_6\tan\beta^{-1} + \lambda_7\tan\beta), \\ m_{H^\pm}^2 &= \frac{m_3^2}{\sin\beta\cos\beta} - \frac{1}{2}v^2(\lambda_4 + \lambda_5 + \lambda_6\tan\beta^{-1} + \lambda_7\tan\beta)\,. \end{aligned} \tag{3.5}$$

The two CP-even Higgs states mix according to the following squared mass matrix:

$$\begin{aligned}\mathcal{M}^2_{H^0} &= m_3^2 \begin{pmatrix} t_\beta & -1 \\ -1 & t_\beta^{-1} \end{pmatrix} \\ &+ \frac{1}{2} v^2 s_\beta c_\beta \begin{pmatrix} 2\lambda_1 t_\beta^{-1} + 3\lambda_6 - \lambda_7 t_\beta^2 & 2\tilde{\lambda} + 3(\lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) \\ 2\tilde{\lambda} + 3(\lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) & 2\lambda_2 t_\beta + 3\lambda_7 - \lambda_6 t_\beta^{-2} \end{pmatrix}, \end{aligned} \tag{3.6}$$

where $s_\beta = \sin\beta$, $c_\beta = \cos\beta$ and $t_\beta = \tan\beta$. The corresponding mass eigenvalues are

$$m^2_{H^0,h^0} = \frac{1}{2}[\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}], \tag{3.7}$$

and the mixing angle $\alpha$ is obtained from

$$\cos 2\alpha = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}, \tag{3.8}$$

$$\sin 2\alpha = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}. \tag{3.9}$$

In the case of a non-supersymmetric 2HDM, the mass parameters $m_1^2$ and $m_2^2$ receive quadratically divergent contributions. We obtain these corrections in the same way as in the SM case, but now the Higgs sector is slighty more complicated. The quadratically divergent radiative corrections in the effective potential at one-loop are:

$$\delta_q V = \frac{\Lambda^2}{32\pi^2} \mathrm{Str}[\mathcal{M}^2], \tag{3.10}$$

where the supertrace Str counts degrees of freedom with a minus sign for fermions and $\mathcal{M}^2$ is the (tree-level, higgs-dependent) mass-squared matrix (note that in the 2HDM all the mass matrices for the Higgs sector must be taken in account). We must be careful to compute the corrections before the minimization. Assuming that $H_1$ ($H_2$) couples exclusively to down-type (up-type) fermions [1], then, the quadratically divergent corrections to $m_1^2$ and $m_2^2$ in 2HDM are given by:

$$\begin{aligned}\delta_\mathrm{q} m_1^2 &= \frac{3\Lambda^2}{64\pi^2} \left\{ (3g^2 + g'^2) + 4\lambda_1 + \frac{4}{3}(2\lambda_3 + \lambda_4) \right\} , \\ \delta_\mathrm{q} m_2^2 &= \frac{3\Lambda^2}{64\pi^2} \left\{ (3g^2 + g'^2) + 4\lambda_2 + \frac{4}{3}(2\lambda_3 + \lambda_4) - 8\lambda_t^2 \right\} . \end{aligned} \tag{3.11}$$

In the next non-supersymmetric 2HDMs, the way to evaluate the fine-tuning is as in (2.8), but in this case, because there are two mass parameters that receive quadratically divergent

[1] In the specific settings we are going to study, the Higgs-fermion interactions are different in each case, giving rise to different quadratically divergent corrections from the top-loop contribution.

corrections, the two fine-tunings associated to the two mass parameters must be considered:

$$\left|\Delta_{p_i}^{(m_1)}\right| \simeq \left|\frac{p_i}{\delta p_i}\right| \left|\frac{\delta\{\delta_q m_1^2\}}{m_1^2}\right|_\Lambda , \qquad \left|\Delta_{p_i}^{(m_2)}\right| \simeq \left|\frac{p_i}{\delta p_i}\right| \left|\frac{\delta\{\delta_q m_2^2\}}{m_2^2}\right|_\Lambda . \tag{3.12}$$

The Renormalization Group Equations (RGEs) in this 2HDM are given in Appendix A.

## 3.2 The Inert Doublet Model

This model has been presented in [23] with the general aim of improving the naturalness of the SM by raising the Higgs mass, $m_h$. This must be done preserving perturbativity and consistency with electroweak precision test (EWPT), and, in this particular case, it has been done in a way as economical as possible. The model, called the "Inert Doublet Model" (IDM), is a particular 2HDM with parity

$$H_2 \to -H_2 , \tag{3.13}$$

while $H_1$ and the other fields are invariant. This imposes natural flavor conservation and makes $H_1$ the only Higgs field coupled to matter [2]. The scalar potential is

$$\begin{aligned} V \;=\;& m_1^2|H_1|^2 \;+\; m_2^2|H_2|^2 \;+\; \frac{\lambda_1}{2}|H_1|^4 \;+\; \frac{\lambda_2}{2}|H_2|^4 \;+\; \lambda_3|H_1|^2|H_2|^2 \\ +\;& \lambda_4|H_1^\dagger H_2|^2 \;+\; \frac{\lambda_5}{2}[(H_1^\dagger H_2)^2 \;+\; \text{h.c.}] . \end{aligned} \tag{3.14}$$

The next assumption is that the parameters of the potential are such that only $H_1$ acquires a vev. The doublet $H_1$ is identified as the SM Higgs doublet and $H_2$ does not couple to fermions and does not get a vev (thus the name "inert doublet") but it couples through weak and quartic interactions, playing an active role for EWPT. The Higgs vev, $v = \sqrt{2}\langle H_1\rangle$ is given by

$$v^2 = -\frac{2m_1^2}{\lambda_1} . \tag{3.15}$$

Parametrizing

$$H_1 = \begin{pmatrix} \phi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ S+(h+iA)/\sqrt{2} \end{pmatrix}, \tag{3.16}$$

where $\phi^+, \chi$ are Goldstones, $h$ is the SM-like Higgs and $H^+, S$ and $A$ are a charged, scalar and pseudoscalar extra Higgs states, the corresponding masses are

$$\begin{aligned} m_h^2 &= \lambda_1 v^2 \\ m_I^2 &= m_2^2 + \lambda_I v^2, \quad I = \{H, S, A\}, \end{aligned} \tag{3.17}$$

[2] We can still use the formulas of previous section and appendix A by reversing the usual roles of $H_1$ and $H_2$.

with

$$\begin{aligned}\lambda_H &= \lambda_3 \\ \lambda_S &= \lambda_3 + \lambda_4 + \lambda_5 \\ \lambda_A &= \lambda_3 + \lambda_4 - \lambda_5 .\end{aligned} \tag{3.18}$$

As mentioned above, this model is conceived to accommodate a heavy SM-like Higgs, $m_h \simeq 400 - 600$ GeV, consistent with the EWPT, particularly with the experimental $T$ parameter ($T^{\rm exp} \simeq 0.1 \pm 0.15$). The Higgs mass induces a contribution to $T$:

$$T \simeq -\frac{3}{8\pi c^2} \ln \frac{m_h}{M_Z}, \tag{3.19}$$

which for $m_h = 400$ GeV is excluded at 99.9% C.L. However, the inert doublet induces an additional contribution [23]

$$\Delta T \simeq \frac{1}{12\pi^2 \alpha v^2} (m_H - m_A)(m_H - m_S), \tag{3.20}$$

which compensates the too negative contribution of eq. (3.19) if $\Delta T \simeq 0.25 \pm 0.1$. This typically requires $m_H > m_A, m_S$, with

$$(m_H - m_A)(m_H - m_S) = M^2, \quad M = 120^{+20}_{-30} \text{ GeV} . \tag{3.21}$$

Besides this, there are additional constraints to ensure the stability of the vacuum and the perturbativity of the $\lambda_i$ couplings [23]. First, we assume that the potential of eq. (3.14) is bounded from below, which happens if and only if

$$\lambda_{1,2} > 0; \qquad \lambda_{\rm L} \equiv \lambda_3 + \lambda_4 - |\lambda_5| > -(\lambda_1 \lambda_2)^{1/2}. \tag{3.22}$$

Under this assumption, the minimum is stable and global, as long as all masses squared are positive. Since the inert parity, eq. (3.13), is unbroken, the lightest inert particle (LIP) will be stable.

Of the two dimensionless couplings, $\lambda_2$ only affects the self-interactions between the inert particles. To avoid additional problems with perturbativity, we assume that it is quite small,

$$\lambda_2 \lesssim 1 . \tag{3.23}$$

The perturbativity of the other couplings can be derived from their RGEs [3], in such a way that no coupling becomes stronger (i.e., it does not grow by 30% from its value in the IR).

[3] We can use the RGEs of Appendix A, taking into account that now it is $H_1$ the one that couples to the top.

From these requirements, we get the constraint [23]

$$|2\lambda_3(\lambda_3+\lambda_4)+\lambda_4^2+\lambda_5^2| \lesssim 50. \tag{3.24}$$

Between the two possibilities that satisfy this contraint, the one that lead to $\Delta T > 0$ (as needed to compensate for the heavy Higgs) is chosen. This choice is that $|\lambda_4|$ becomes large, while $\lambda_5$ stays relatively small. Then, from vacuum stability, eq. (3.22), $\lambda_3$ must also become large, $\lambda_3 \gtrsim |\lambda_4|$. This way we get

$$|\lambda_4| \lesssim \lambda_3 \lesssim 7 \quad (\lambda_4 < 0, \lambda_5 \quad \text{small}). \tag{3.25}$$

This model represents an interesting example of how to interpret the possible detection of a heavy Higgs in LHC. On the other hand, the authors of the model [23] stress that the heavy Higgs mass relaxes the SM Hierarchy Problem, since it allows a larger $\Lambda_t$. The price would be that the New Physics (NP) responsible for the cancellation of the SM quadratic divergences could escape LHC (though we could observe a modified Higgs sector). We would like to discuss this last aspect here. As shown in sect. 2 of chapter 2 (fig. 2.6), a large $m_h$ does not necessarily imply less fine-tuning. In the SM context, although $\Delta_{\Lambda_t}$ decreases with $m_h$, $\Delta_{\Lambda_h}$ increases (as a result of the increasing of the Higgs self-coupling at $\Lambda$) and eventually dominates the total fine-tuning. This happened because of the RG increasing of $\lambda$ from $m_h$ to $\Lambda$, where the quadratic radiative correction $\delta_q m^2$ to the Higgs mass parameter is to be computed. In the present case something similar is likely to take place. From eq. (3.15) the relevant mass parameter for EW breaking is $m_1^2$, which receives the quadratically-divergent radiative correction

$$\delta_{\rm q} m_1^2 = \frac{3}{64\pi^2}\left\{-8\lambda_t^2\Lambda_t^2+(3g^2+g'^2)\Lambda_g^2+4\lambda_1\Lambda_{H_1}^2+\frac{4}{3}(2\lambda_3+\lambda_4)\Lambda_{H_2}^2\right\}\ , \tag{3.26}$$

where, following the assumption of the authors of ref.[23], we have allowed independent cut-offs for the various contributions. Eq. (3.26) has a structure very similar to the SM equation (2.1), with the role of $\lambda$ played by $\lambda_1$. The two main differences are the presence of the $\propto \Lambda_{H_2}^2$ term and the RGE for $\lambda_1$:

$$\frac{d\lambda_1}{d\ln\mu^2} = \frac{1}{16\pi^2}\left(6\lambda_1^2+2\lambda_3^2+2\lambda_3\lambda_4+\lambda_4^2+\lambda_5^2+\ \ldots\right). \tag{3.27}$$

The first term in this RGE is as in the SM, but the additional terms cause $\lambda_1$ to grow with the scale more quickly than $\lambda$ in the SM. Both differences contribute to a larger fine-tuning than in the SM for a given $m_h$ (although, of course, the EWPT are now under control). The best case for both effects will occur for small $\lambda_{3,4,5}$. On the other hand, the $\lambda_i$ couplings cannot

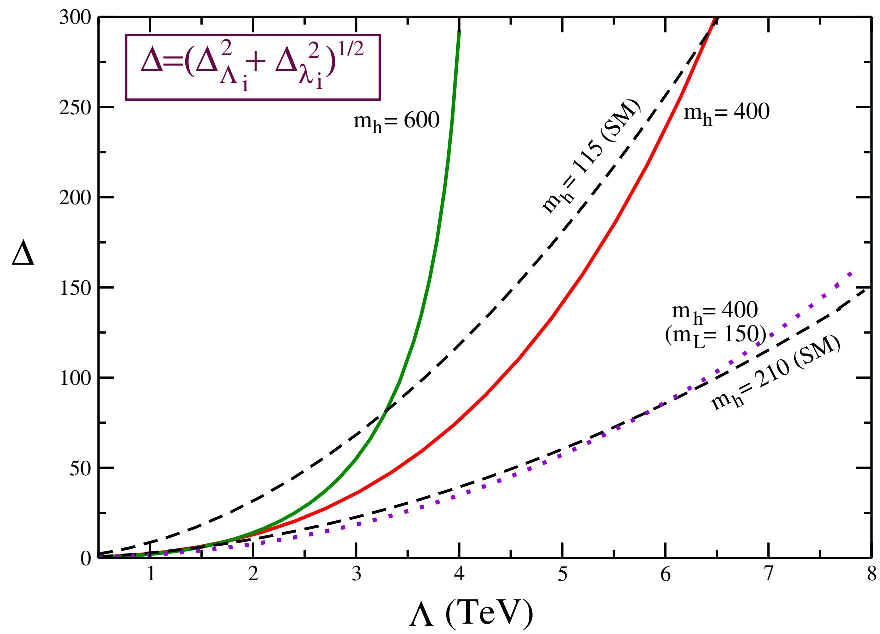



Figure 3.1: Total fine-tuning, $\Delta$ versus the cut-off $\Lambda$ in the Inert Doublet Model (see text) for $m_h = 400$ and 600 GeV using $m_L = m_h$ GeV, $\Delta m = 50$ GeV (red and green solid lines); and $m_h = 400$ GeV using $m_L = 150$ GeV, $\Delta m = 50$ GeV (purple dotted). The dashed black lines show the total SM fine-tuning for $m_h = 115$ GeV, 210 GeV, using uncorrelated cut-offs.

be chosen at will. They have to be consistent with the desired $\Delta T$ and the perturbativity constraints explained above. It is useful to have a parametrization in terms of the masses of the two neutral inert particles, $m_L$ for the lightest and $m_{NL}$ for the next-to-lightest. We consider the general case when $\Delta m = m_{NL} - m_L$ can be sizeable. The charged scalar is always heavier than both neutrals, and using eq. (3.21), we have

$$m_H - m_{NL} = \sqrt{M^2 + \frac{(\Delta m)^2}{4}} - \frac{\Delta m}{2}. \tag{3.28}$$

The couplings $\lambda_{4,5}$ can be also expressed via $m_L$,$m_{NL}$ using eq. (3.28) and eq. (3.17), giving

$$\begin{aligned} \lambda_4 &= -\frac{2}{v^2}\left(M^2 + (m_L + m_{NL})\sqrt{M^2 + \frac{(\Delta m)^2}{4}}\right) < 0 \\ |\lambda_5| &= \frac{m_{NL}^2 - m_L^2}{v^2} < |\lambda_4|. \end{aligned} \tag{3.29}$$

The sign of $\lambda_5$ depends on whether it is the scalar $S$ or the pseudoscalar $A$ which is the heavier. The coupling $\lambda_3$ (or $\lambda_L \equiv \lambda_3 + \lambda_4 - |\lambda_5|$) is the only free parameter and it should be chosen in agreement with the perturbativity, naturalness and vacuum stability constraints.

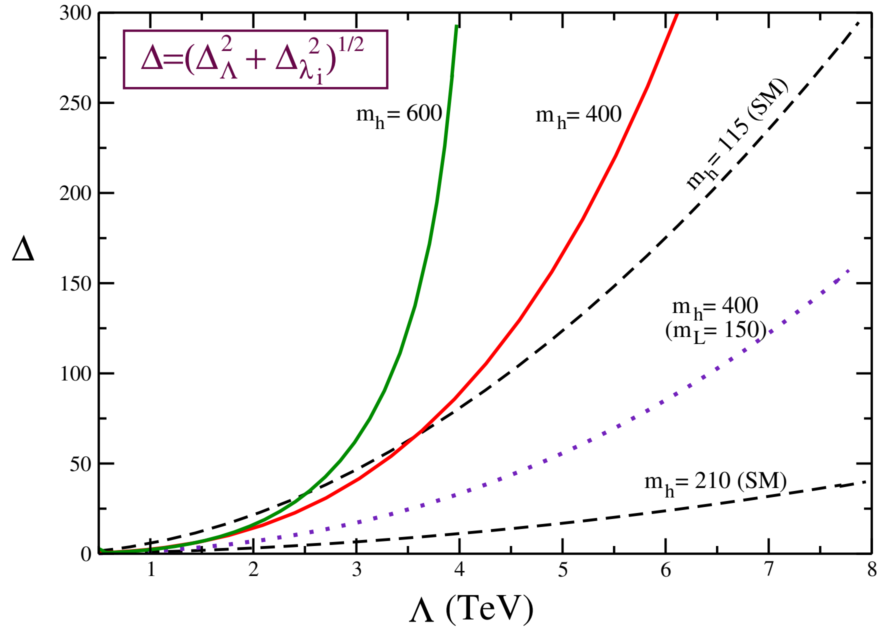



Figure 3.2: Same as fig. 3.1, but now using a unique cut-off, Λ, in both cases, the IDM and the SM.

Being more rigourous in the fine-tuning analysis of this model, we must compare with the one of the SM following the steps of sect. 2.2. Fig. 3.1 shows the total fine-tuning $\Delta = (\sum_{a=t,H_1,H_2} \Delta^2_{\Lambda_a} + \sum_{i=1,3,4} \Delta^2_{\lambda_i})^{1/2}$ versus the cut-off $\Lambda$ (for simplicity we show the results when the cut-offs are numerically equal) for $m_h = 400$ and 600 GeV (red and green solid lines) with $m_L = m_h$ GeV, $\Delta m = 50$ GeV and $\lambda_{\rm L} = -0.5$. Notice that the $m_h = 400$ GeV behaves better, as we expected from our previous arguments (but not in [23]). The purple dotted line corresponds to $m_h = 400$ GeV, $m_L = 150$ GeV, $\Delta m = 50$ GeV and $\lambda_{\rm L} = -0.5$. This is a case which leads to small $\lambda_{3,4,5}$, and the fine-tuning is sensibly smaller, in agreement with the above discussion. Hence, this line is close to the optimal situation in this scenario. In any case, it is clear that to reach $\Lambda > 2$ TeV requires a substantial ($\Delta > 10$) fine-tuning. To see if this situation improves the SM one, we have plotted the SM fine-tuning $\Delta = (\Delta^2_{\Lambda_t} + \Delta^2_{\Lambda_\lambda} + \Delta^2_\lambda)^{1/2}$ for the lower and upper bounds $m_h = 115$ GeV, 210 GeV (dashed black lines). Clearly, the situation of the IDM can be hardly considered as an improvement in naturalness, especially if the SM Higgs is close to its experimental upper bound. This conclusion is strengthened if one assumes a universal cut-off for all the contributions in $\delta_q m^2$ and $\delta_q m_1^2$ [4]. The corresponding fine-tuning curves are shown in fig. 3.2, where the SM (for $m_h$ not far from 200 GeV) is in a much better position, even if the optimal choice ($m_L = 150$

[4]As discussed in sect.2.2 this is a perfectly reasonable situation.

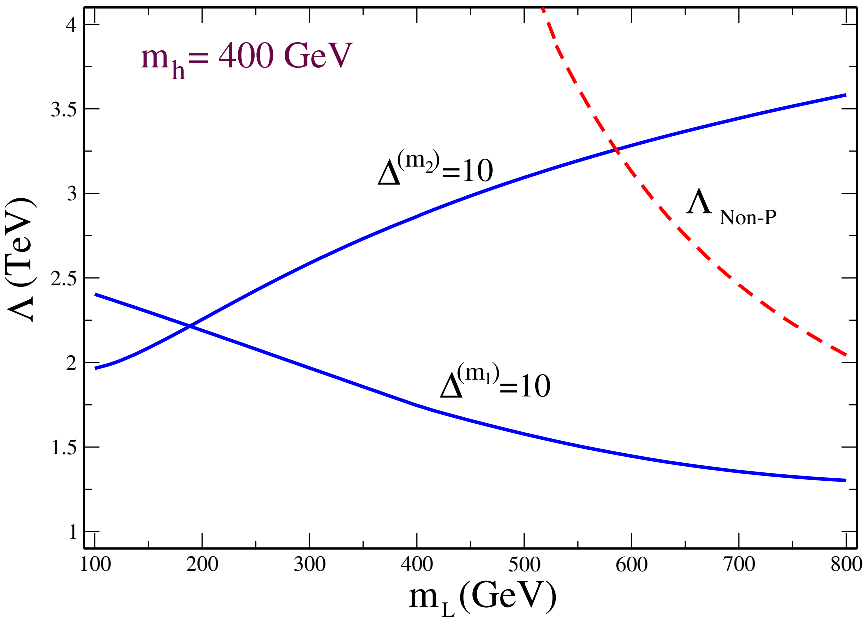



Figure 3.3: IDM contour plots of $\Delta = 10$ in the $\{m_L, \Lambda\}$ plane with $\Delta = \Delta^{(m_1)}$, $\Delta^{(m_2)}$ (solid blue). The red dashed line shows the limit of perturbativity ($\lambda_1 = 4\pi$).

GeV, $\Delta m = 50$ GeV) is used for the IDM.

Our conclusion at this point is therefore that the IDM offers an appealing explanation for an hypothetical detection of a heavy Higgs. However, it does not improve the naturalness of the SM with respect to quadratic radiative corrections. This conclusion is softened if i) one assumes independent cut-offs, or ii) if one requires $\Delta$ to be $\mathcal{O}(1)$. Notice that in the latter case $\Lambda$ is smaller, and thus the effect of the RG running on $\Delta_{\Lambda_H}$ (which is harmful for the fine-tuning) is less important. Both assumptions were taken in ref.[23], which also diminishes the error introduced in their estimate by ignoring the RG running of $\delta_q m_1^2$. However, we find that requiring $\Delta = 1$ is excessively severe, especially taking into account that a small $\Lambda$ generically leads to problems with EWPT.

The $\delta_q m_1^2$ contribution is not the only source of potential fine-tuning in the IDM: $\delta_q m_2^2$ can be also the source of an (independent) fine-tuning , being

$$\delta_{\rm q} m_2^2 = \frac{3\Lambda^2}{64\pi^2}\left\{(3g^2 + g'^2) + 4\lambda_2 + \frac{4}{3}(2\lambda_3 + \lambda_4)\right\} \ . \tag{3.30}$$

Let us call the corresponding fine-tunings $\Delta^{(m_1)}$ and $\Delta^{(m_2)}$. Notice from eq. (3.17) that a small $m_L$ (the optimal situation for $\Delta^{(m_1)}$) typically requires small $m_2^2$. Then a large $\delta_q m_2^2$ would make this possibility somewhat unnatural. Fig. 3.3 shows $\Delta^{(m_2)}$ (evaluated in a fashion similar to $\Delta^{(m_1)}$) as a function of $m_L$ for a representative case ($m_h = 400$ GeV, $\Delta m = 50$ GeV, $\lambda_{\rm L} = -0.5$). We have also plotted the perturbativity limit, defined as the scale, $\Lambda_{\rm Non-P}$, beyond which $\lambda_1 \geq 4\pi$. As expected, $\Delta^{(m_2)}$ increases for smaller $m_L$ (i.e. the cut-off of $\Delta$=10 is smaller for smaller $m_L$ ), which balances the behaviour of $\Delta^{(m_1)}$. In fact one should multiply both fine-tunings, as they correspond to different quantities, but, even if one does not, it is clear that the improvement gained in $\Delta^{(m_1)}$ by going to small $m_L$ is lost by this additional source of fine-tuning in $m_2$.

From the previous discussion we finally conclude that i) although the IDM model is very interesting, it does not improve the naturalness of the SM; ii) the structure of the model requires additional fine-tunings in $m_2^2$ of a size similar to that required for a correct EW breaking (i.e. the tuning associated to $m_1^2$). This fact is ordinarily present in models with a structure more complicated than the SM. Normally intricacy is penalized in naturalness estimates (which is somehow satisfactory). This happens e.g. in Little Higgs Models [27] and also here, though in a less dramatic way. Therefore the possibility that the NP responsible for the cancellation of the dangerous quadratic divergences could escape LHC detection is similar in the IDM and in the SM. In the IDM case, however, a Higgs sector different from the SM one would be observed.

## 3.3 The Barbieri-Hall Model

This model, presented in [22], consists of another particular version of the 2HDM aimed at improving the naturalness of the SM. The idea here was to maintain the lightest (SM-like) Higgs within the SM experimental range ($m_h \lesssim 220$ GeV), but coupling the top mainly to the heaviest Higgs, so that $\Delta_{\Lambda_t}$ can be much smaller than in the SM.

The Higgs potential has the form of eq. (3.14), but now the various parameters are chosen so that both $H_1$ and $H_2$ get vevs: $\langle H_i \rangle = v_i/\sqrt{2}$, with $v_1^2 + v_2^2 = (246 \text{ GeV})^2$. More precisely, the minimization conditions read

$$\begin{aligned} m_1^2 + \frac{\lambda_1}{2} v_1^2 + \frac{\tilde{\lambda}}{2} v_2^2 &= 0, \\ m_2^2 + \frac{\lambda_2}{2} v_2^2 + \frac{\tilde{\lambda}}{2} v_1^2 &= 0, \end{aligned} \tag{3.31}$$

where $\tilde{\lambda} = \lambda_3 + \lambda_4 + \lambda_5$. In addition a discrete symmetry is imposed so that only $H_2$ couples to the up-quarks, in particular to the top quark. The squared mass matrix for the two neutral

Higgs bosons is

$$\begin{pmatrix} \lambda_1 v_1^2 & \tilde{\lambda} v_1 v_2 \\ \tilde{\lambda} v_1 v_2 & \lambda_2 v_2^2 \end{pmatrix} \tag{3.32}$$

Assuming that the 22 entry is the largest and the off-diagonal entry is small, the two mass eigenvalues $m_\pm^2$ are

$$m_+^2 \simeq \lambda_2 v_2^2, \quad m_-^2 \simeq \left(\lambda_1 + \frac{\tilde{\lambda}}{\lambda_2}\right) v_1^2 \ . \tag{3.33}$$

The important point is that, in this case, the lightest and the heaviest neutral Higgs bosons are mainly along the $h_1$ and $h_2$ directions respectively: $h_- = \cos\alpha\ h_1 - \sin\alpha\ h_2$, $h_+ = \cos\alpha\ h_2 + \sin\alpha\ h_1$, with a small mixing angle:

$$\alpha \simeq \frac{\tilde{\lambda}}{\lambda_2 \tan\beta} \ , \tag{3.34}$$

where $\tan\beta = v_2/v_1$. Therefore the lightest Higgs, $h_-$, has almost no coupling to the up-quarks, and in particular to the top. The quadratically divergent corrections to the $m_i^2$ mass parameters are

$$\begin{aligned} \delta_q m_1^2 &= \frac{3\Lambda^2}{64\pi^2}\left\{(3g^2 + g'^2) + 4\lambda_1 + \frac{4}{3}(2\lambda_3 + \lambda_4)\right\} \ , \\ \delta_q m_2^2 &= \frac{3\Lambda^2}{64\pi^2}\left\{(3g^2 + g'^2) + 4\lambda_2 + \frac{4}{3}(2\lambda_3 + \lambda_4) - 8\lambda_t^2\right\} \ , \end{aligned} \tag{3.35}$$

where we have taken a universal cut-off for simplicity, but each term can be multiplied by a different cut-off if desired. It is explicit from eq. (3.35) that the $\lambda_t$ coupling only introduces corrections to $m_2^2$.

Before analyzing the naturalness of this model, we see how large can be $m_+$ from EWPT. We must consider the contributions of the scalars to the $S$ and $T$ parameters [28]. Approximating $\beta - \alpha$ by $\beta$ because $\alpha$ is small, EWPT give the following constraint:

$$m_+ < m_- \left(\frac{m_{EW}}{m_-}\right)^{\frac{1}{\sin^2\beta}} \ , \tag{3.36}$$

where $m_{EW}$ is the present upper bound on the SM Higgs mass from EWPT for the SM (i.e. 186-219 GeV [4]). Then, if $m_- < m_{EW}$, the bound on $m_+$ gets exponentially relaxed as $\sin\beta$ is reduced, but $v_2$ cannot be reduced too much as we have assumed that $\lambda_2 v_2^2$ is the largest term in the Higgs boson mass matrix, so that reducing $v_2$ leads to a large $\lambda_2$ beyond its perturbativity limit ($\lambda_2 \lesssim 4\pi$). With these considerations, for $m_-$ close to the direct search limit of 115 GeV, a value of $\sin\beta = 0.6$—$0.7$ is sufficient to raise the bound on $m_+$ to near a TeV.

Let us focus for the moment on the impact of $\delta_{\rm q} m_2^2$ on the fine-tuning issue, as the authors of [22] do. From eqs. (3.31) and the smallness of $\alpha$, we see that $v_2^2 = v^2 \sin^2\beta \simeq (-2m_2^2/\lambda_2)$. So, the fine-tuning associated to $\Lambda_t$ is given by

$$\Delta_{\Lambda_t} \simeq \frac{3\lambda_t^2}{2\pi^2}\frac{\Lambda_t^2}{\lambda_2 v_2^2} = \frac{3\lambda_t^2}{2\pi^2}\frac{\Lambda_t^2}{m_+^2}\ . \tag{3.37}$$

Hence, $\Delta_{\Lambda_t}$ is suppressed for large $m_+$, even if the SM-like Higgs, $h_-$, is light. This trick was used in [22] to push $\Lambda_t$ to quite high values: taking $m_+ \simeq 500 - 1000$ GeV (consistent with EWPT as we have seen in the last paragraph), $\Lambda_t$ can be as large as 2 TeV, even if one demands $\Delta_{\Lambda_t} \leq 1$.

The previous analysis, however, does not take into account the running of $\lambda_2$ (which is very important due to the large size assumed for $m_+$) and the other couplings. As we have seen in the SM [see sect.2.2], the RG increase of $\lambda_2$ from $m_+$ to $\Lambda$ enhances the corresponding contribution to $\delta_q m_2^2$, and thus the value of $\Delta_{\Lambda_{H_2}}$ (not considered in [22]). And it is this fine-tuning which puts the strongest constraint on the scale of NP. Actually, $\lambda_2$ eventually reaches a Landau Pole and, at a slightly lower scale, $\Lambda_{\rm Non-P}$, it gets non-perturbative values, say $\lambda_2 \geq 4\pi$. Beyond $\Lambda_{\rm Non-P}$ the model enters a strong-coupling regime. This sets a peturbativity limit on $\Lambda$ that, for some choices of the parameters, is even below the previous estimates of $\Lambda_t$. In any case, the cut-off scale can be determined as the one which produces a total fine-tuning $\Delta \leq 10$ (or any other sensible value).

Of course, the effect of the RGE (see App.A) for $\lambda_2$ depends on the values of other couplings, especially on $\lambda_{3,4,5}$. Explicitly,

$$\frac{d\lambda_2}{d\ln\mu^2} = \frac{1}{16\pi^2}\left[6\lambda_2^2 + \lambda_3^2 + (\lambda_3+\lambda_4)^2 + \lambda_5^2 + \ ...\right]. \tag{3.38}$$

Notice that the extra couplings always contribute to strengthen the RG increase of $\lambda_2$, so a good choice for fine-tuning purposes is to minimize their effect by taking their values as small as possible, which also minimizes the quadratically-divergent corrections, $\delta_{\rm q} m_{1,2}^2$, as given by eqs.(3.35). Taking into account that the masses of the charged and the pseudoscalar Higgs [see eq. (3.5)], given in this case by

$$\begin{aligned} m_{H^+}^2 &= -(\lambda_4+\lambda_5)\frac{v^2}{2} \\ m_{A^0}^2 &= -\lambda_5 v^2, \end{aligned} \tag{3.39}$$

it seems that $\lambda_3 = \lambda_4 = 0$ can be an optimal choice. Then $\lambda_5$ must be negative, with a lower bound (in absolute size) given by the lower bound on $m_{H^+}$. For $\tan\beta = 0.8 - 1$ (which is

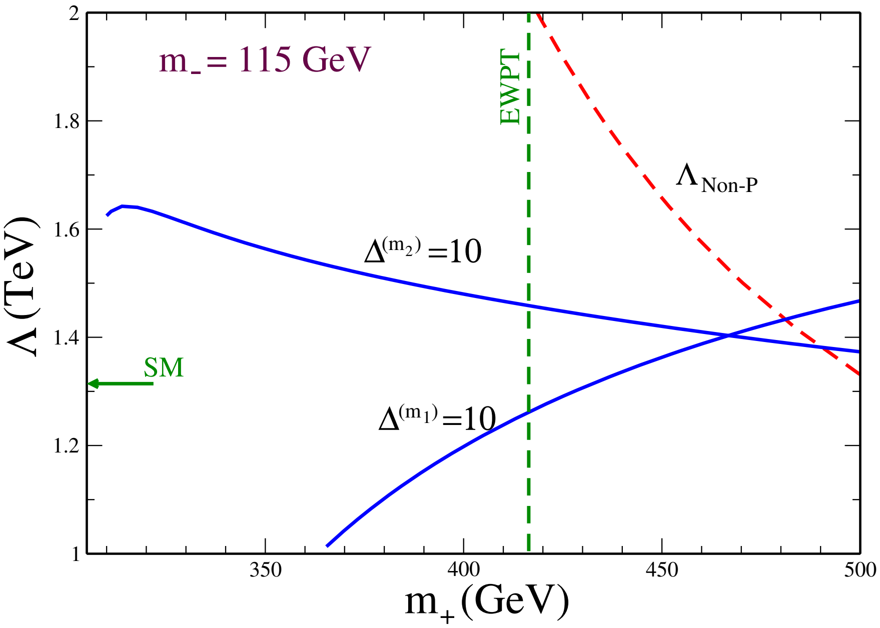



Figure 3.4: Contour plots of $\Delta = 10$ (with $\Delta = \Delta^{(m_1)},\ \Delta^{(m_2)}$) in the $\{m_L, \Lambda\}$ plane for the BH Model with $\lambda_3 = \lambda_4 = 0$ (solid blue). The $\Lambda_{\rm Non-P}$ (red dashed) line gives the perturbativity limit ($\lambda_2 = 4\pi$). The vertical (pink dashed) line corresponds to the upper limit on $m_+$ (at 95% C.L.) from EWPT.

the preferred range to be consistent with EWPT as we have seen above [22]), this bound is about $200 - 250$ GeV.

Using a unique cut-off, $\Lambda_t \equiv \Lambda_{H_i} \equiv \Lambda$, the corresponding fine-tuning for $m_- = 115$ GeV is shown in fig. 3.4 as the contour plot $\Delta^{(m_2)} = \left[(\Delta_\Lambda^{(m_2)})^2 + \sum_{i=1}^5 (\Delta_{\lambda_i}^{(m_2)})^2\right]^{1/2} = 10$ in the $\{m_+, \Lambda\}$ plane. The perturbativity limit, $\Lambda_{\rm Non-P}$, is also represented. The origin of the lower bound $m_+ > 310$ GeV visible in fig. 3.4 is the following. For $\tan\beta = 1$ and a given value of $m_-^2$ [i.e. $(115\ {\rm GeV})^2$ in this figure], the minimal value of $m_+^2$ occurs for $\lambda_2 = \lambda_1$, and $|\tilde\lambda|$ as small as possible. More precisely

$$(m_+^2)_{\rm min} = m_-^2 + |\tilde\lambda| v^2 = m_-^2 + 2 m_{H^+}^2 \ , \tag{3.40}$$

where we have used $\lambda_3 = \lambda_4 = 0$ and eq. (3.39). Then, using $m_{H^+} \geq 200 - 250$ GeV, we get the above lower bound on $m_+^2$. Besides this lower bound, there is the upper bound from EWPT [see eq. (3.36)], represented as well in the figure. For comparative purposes we show in the same figure the naturalness cut-off of the SM for the same Higgs mass (see solid green line of fig. 2.5).

Clearly, no substantial improvement is gained with respect to the SM. The only improvement of this model is that $\Lambda_t$ can be much higher than usual, as it is clear from eq. (3.37).

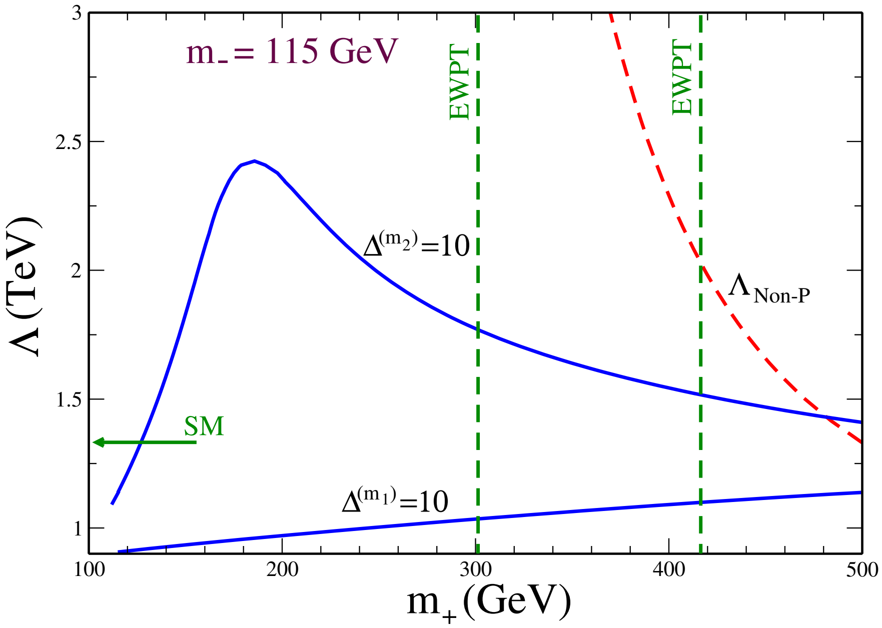



Figure 3.5: The same as fig. 3.4, but using $\lambda_4 = 0$, $\tilde{\lambda} = 0$. The two pink dashed lines correspond to the $m_+$ upper bounds from EWPT.

Actually, it is clear from fig. 3.4 that, although $\Delta^{(m_2)}$ increases with $m_+$, the fine-tuning associated to $m_1^2$, i.e. $\Delta^{(m_1)}$, goes the opposite way. The situation is similar to that in the IDM (see fig. 3.3) reversing the behaviours of $\Delta^{(m_1)}$ and $\Delta^{(m_2)}$ : the improvement gained in $\Delta^{(m_2)}$ by going to small $m_+$ is counterbalanced by the $\Delta^{(m_1)}$ fine-tuning. Again, we should multiply both fine-tunings, but even if we do not (i.e. if we are conservative) it is clear again how models with more structure are penalized in naturalness considerations.

In order to relax the lower bound on $m_+$, one can prove to change the $\lambda_3 = \lambda_4 = 0$ assumption. A convenient procedure is to keep $\lambda_4 = 0$ and adjust $\lambda_3$ to get $\tilde{\lambda} = 0$. The result is shown in fig. 3.5, where a kind of Veltman's cancellation is found around $m_+ \sim 180$ GeV. But once more is clear that the improvement gained in $\Delta^{(m_2)}$ by going to small $m_+$ is counterbalanced by the $\Delta^{(m_1)}$ fine-tuning.

Finally, one can calculate the fine-tuning using uncorrelated $\Lambda_t$ and $\Lambda_{H_i}$ cut-offs. This does not improve the naturalness since the accidental cancellation in $\Delta^{(m_2)}$ around $m_+ \sim 180$ GeV is now absent. This is illustrated in fig. 3.6, where the parameters of the model have been taken with the same values as in fig. 3.5. In fig. 3.6 $\Lambda_t = \Lambda_{H_i}$ are numerically equal, but one could take different values for the different cut-offs. Then $\Lambda_t$ can be much higher than usual, as already discussed around eq. (3.37). So the NP coupled to the Higgs in a fashion

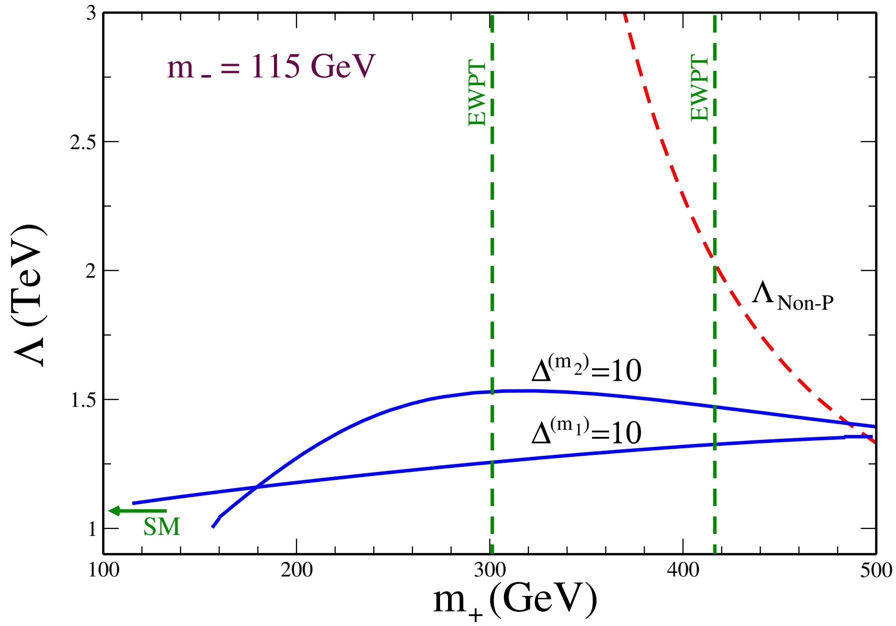



Figure 3.6: Same as fig.3.5, but now using uncorrelated cut-offs, $\Lambda_t$ and $\Lambda_{H_i}$.

similar to the top could be beyond LHC reach, as stressed in ref.[22]. However, the NP that compensates the large quadratic corrections associated to the Higgs sector itself, should show up at much lower scales[5].

In summary, this 2HDM model shows how $\Lambda_t$ could be much larger than $\Lambda_H$ even if the light Higgs is within the experimentally preferred range. However, the global fine-tuning is not improved and we generically expect NP to be on the LHC reach.

## 3.4 Twin Higgs Model

This model, originally proposed by Chacko, Goh and Harnik in ref. [29], postulates the existence of a mirror world: a $Z_2$ replica of the SM. Calling $H_1$ the SM Higgs and $H_2$ its mirror copy, the Higgs sector of this model is a very particular kind of 2HDM with potential

$$V = \mu^2(|H_1|^2 + |H_2|^2) + \lambda(|H_1|^2 + |H_2|^2)^2 + \gamma(|H_1|^4 + |H_2|^4) \ , \tag{3.41}$$

[5]In that circumstance, there exists the caveat of how rigorous is to work with the SM as the effective theory (for top couplings) below $\Lambda_t$. But even if one decides to be conservative (i.e. one does not consider the effective theory beyond the smallest cut-off scale), these results give at least an indication in the sense that strongly coupled NP might appear at higher scales than other kinds of NP.

that respects the $Z_2$ parity but allows communication with the mirror world through a mixed term $|H_1|^2|H_2|^2$. The naturalness of electroweak symmetry breaking in this model was discussed already by ref. [29] and later on by [21] in more detail. For $\gamma > 0$ and $\mu^2 > 0$ this potential has a minimum that breaks the electroweak symmetry with $\langle H_1^0\rangle = \langle H_2^0\rangle = v/\sqrt{2}$ [21]. Three of the four degrees of freedom of each doublet are eaten by the longitudinal components of the $SU(2)\times U(1)$ gauge bosons of our world and the mirror world. Two scalar degrees of freedom remain as physical Higgses with a squared mass matrix that reads

$$\left(\begin{array}{cc} 2(\lambda+\gamma)v^2 & 2\lambda v^2 \\ 2\lambda v^2 & 2(\lambda+\gamma)v^2 \end{array}\right) , \tag{3.42}$$

with eigenvalues

$$\begin{aligned} m_-^2 &= 2\gamma v^2 , \\ m_+^2 &= 2(2\lambda+\gamma)v^2 . \end{aligned} \tag{3.43}$$

and eigenvectors $h_\pm^0 = \mathrm{Re}(H_1^0 \pm H_2^0)$. We see that the Higgs mass eigenstates are mixtures with 50% $H_1$ component and 50% $H_2$ component so that they have reduced couplings to matter and gauge bosons in our world. The mass $m_+$ corresponds to the Higgs excitations along the breaking direction and therefore is the parameter that plays a similar role to $m_h$ in the SM (in fact, $m_+^2 = -2\mu^2$). With $\gamma = 0$, the global $SU(4)$ symmetry of the $(|H_1|^2 + |H_2|^2)$ terms would result in a massless Goldstone boson in the direction transverse to the breaking, *i.e.* $m_- = 0$. Having $\gamma \ll \lambda$ gives naturally a light Higgs in the spectrum [29].

Let us examine the structure of quadratically divergent corrections to the Higgs mass parameters in this model, $\mu$. As the $Z_2$ symmetry is not broken both $H_1$ and $H_2$ receive the same corrections, given by

$$\delta_q\mu^2 = \frac{\Lambda^2}{8\pi^2}\left[\frac{3}{8}(3g^2+g'^2)+5\lambda+3\gamma-3\lambda_t^2\right] . \tag{3.44}$$

Notice that this formula assumes all the couplings in the mirror world take exactly the same values as in our world. We can write eq. (3.44) in terms of particle masses by writing $5\lambda+3\gamma = (5m_+^2+m_-^2)/(4v^2)$. Then we find a result very similar to the SM case with the replacement $m_h^2 \to (5m_+^2+m_-^2)/6$. If we fix $m_-$ to a low value, say $m_- = 115$ GeV, the quadratic correction in (3.44) as a function of $m_+^2$ behaves a bit better than the SM quadratic correction as a function of $m_h^2$.

A second difference with respect to the SM behaviour comes from a different RG evolution of the couplings in this case. We should compare the RGE for $(5\lambda+3\gamma)$ in this model with

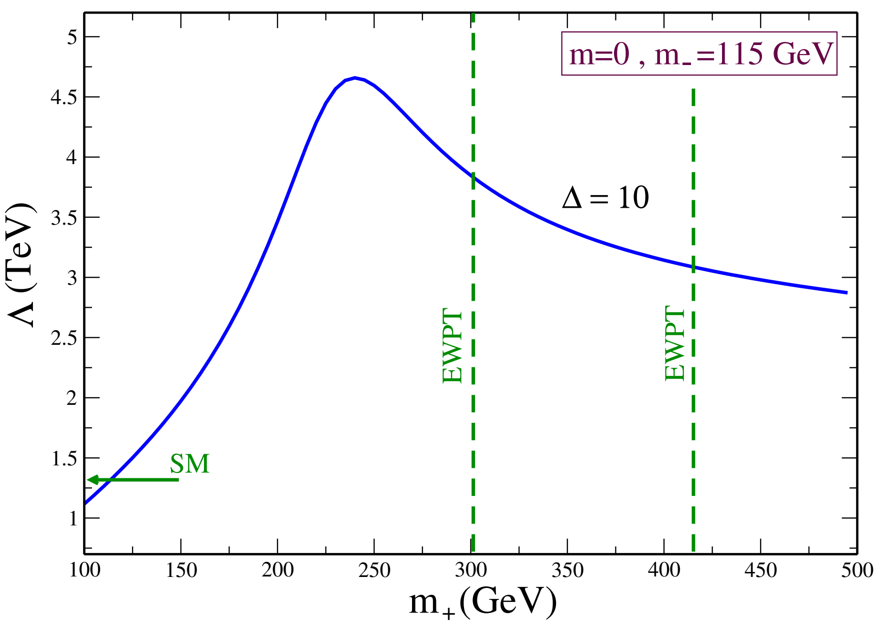



Figure 3.7: Upper bound on the scale $\Lambda$ of New Physics from the requirement of less than 10% tuning of EWSB in the Twin Higgs model with $m = 0$ and $m_- = 115$ GeV. Shown by the dashed lines are the EWPT upper bounds on $m_+$. For comparison, the arrow marks the SM naturalness upper bound on $\Lambda$ for $m_h = m_-$

that for $(3\lambda)$ in the SM. Again we find a very similar result except for the replacement $4(3\lambda)^2 \to (16/5)(5\lambda + 3\gamma)^2 + 4(3\gamma)^2$. We therefore conclude that RG effects in the quadratic corrections of this model are a bit softer than in the SM.

Finally, as happened in the models discussed before, the constraints on the Higgs masses derived from EWPT are modified with respect to the SM ones. Now one has [21]

$$m_+ m_- < m_{EW}^2 \ , \tag{3.45}$$

where $m_{EW} = \{186, 219\}$ GeV is the EWPT indirect upper bound on the SM Higgs mass.

As a result of all the effects just discussed, this model is able to improve over the naturalness of the SM Higgs sector. Fig. 3.7 shows the upper bound on the scale of New Physics, $\Lambda$, imposing that the fine-tuning in $\mu^2$ (calculated as discussed in previous sections) is smaller than 10 and choosing $m_- = 115$ GeV. For comparison, the SM bound $\Lambda < 1.32$ TeV (for $m_h = 115$ GeV) is also indicated. However it is more instructive to compare the bound on $\Lambda$ as function of $m_+$ with the SM curve as a function of $m_h$ (black solid line of fig. 2.5). Then we see that the current curve has a very similar shape to the SM one, but it is a slightly bit higher. Moreover, the range of $m_+$ compatible with EWPT is also wider, fully including the maximum of the curve. Nevertheless, the improvement with respect to the SM situation is not dramatic.

Let us finally discuss the case in which one introduces a small breaking of the $Z_2$ symmetry by considering different masses for $H_1$ and $H_2$. More explicitly we add to the potential (3.41) a term [21]

$$\delta V = m^2(|H_1|^2 - |H_2|^2)\ . \tag{3.46}$$

With such modification, the minimum of the potential moves away from $\tan\theta \equiv \langle H_1^0\rangle/\langle H_2^0\rangle \equiv v_1/v_2 = 1$ (although we still have to keep $v_1 = 246$ GeV), with

$$\cos 2\theta = -\frac{m^2}{\mu^2}\frac{2\lambda+\gamma}{\gamma}\ . \tag{3.47}$$

The squared mass matrix for the two Higgses takes now the form

$$\begin{pmatrix} 2(\lambda+\gamma)v_1^2 & 2\lambda v_1 v_2 \\ 2\lambda v_1 v_2 & 2(\lambda+\gamma)v_2^2 \end{pmatrix}\ , \tag{3.48}$$

with eigenvalues

$$\begin{aligned} m_-^2 &\simeq 2\gamma(v_1^2 c_\theta^2 + v_2^2 s_\theta^2)\ , \\ m_+^2 &\simeq 2\lambda(v_1^2+v_2^2) + 2\gamma(v_1^2 s_\theta^2 + v_2^2 c_\theta^2)\ , \end{aligned} \tag{3.49}$$

where we have expanded in $\gamma/\lambda$. The eigenvectors are defined as $h_+ = \sqrt{2}\mathrm{Re}(s_\alpha H_1^0 + c_\alpha H_2^0)$ and $h_- = \sqrt{2}\mathrm{Re}(c_\alpha H_1^0 - s_\alpha H_2^0)$. From (3.48)

$$\tan 2\alpha = \frac{\lambda}{\lambda+\gamma}\tan 2\theta\ . \tag{3.50}$$

For $\gamma \ll \lambda$, one has $\alpha \simeq \theta$, so that $h_+$ is still aligned with the breaking direction and its mass is still of direct relevance for the naturalness of electroweak breaking (again $m_+^2 \simeq -2\mu^2$).

Before presenting the results for the fine-tuning in the case $m \neq 0$, notice that $\lambda$ and $\gamma$ in (3.48) can be obtained in terms of $m_+$ and $m_-$ as

$$\begin{aligned} \lambda &= \pm\frac{1}{2v_1v_2}\sqrt{(m_+^2 s_\theta^2 - m_-^2 c_\theta^2)(m_+^2 c_\theta^2 - m_-^2 s_\theta^2)}\ , \\ \gamma &= \frac{m_+^2 + m_-^2}{2(v_1^2+v_2^2)} - \lambda\ . \end{aligned} \tag{3.51}$$

It follows that, for fixed $m_+$, $m_-$ and $\theta$, there are two different solutions for $\lambda$ and $\gamma$ with different signs for $\lambda$ (the region $\lambda < 0$ is accesible provided $|\lambda| < \gamma/2$ to avoid an instability in the scalar potential). It can be shown that the best case for naturalness corresponds to $\lambda > 0$ and we restrict our analysis to that case.

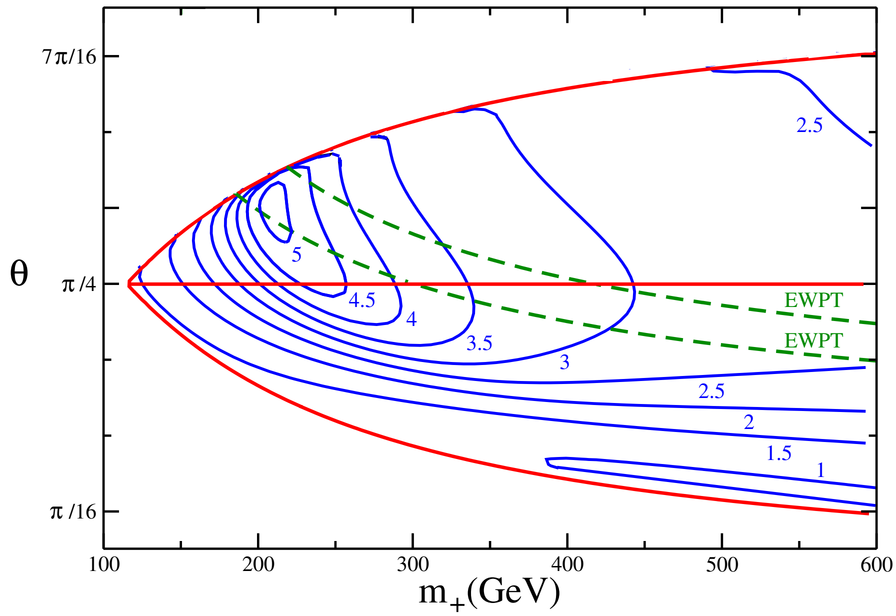



Figure 3.8: Contour lines of the 10% naturalness upper bounds on the scale of New Physics $\Lambda$ (in TeV) in the Twin Higgs model with $m \neq 0$ and $m_- = 115$ GeV. The dashed lines show the EWPT upper bounds on $m_+$.

We can also see from eqs. (3.51) that the parameter space is limited to the region $m_+ \geq \text{Max}\{m_- \tan\theta, m_-/\tan\theta\}$. This is shown in fig. 3.8 which corresponds to fixing $m_- = 115$ GeV: the accesible parameter space lies inside the "fish" profile. The minimal value of $m_+$ corresponds to taking $\lambda = 0$ in the matrix (3.48). In this limit, $H_1$ is the SM Higgs with mass $m_h^2 = 2\gamma v_1^2$ and $H_2$ is the Higgs boson of the mirror world, with mass $m_{h'}^2 = 2\gamma v_2^2$. In fig. 3.8 the last limit corresponds to the boundary of the allowed region of parameter space. Along the upper limit, with $\tan\theta > 1$ one has $m_+ \equiv m_h > m_- \equiv m_{h'}$. Therefore, along that line $m_+$ plays the role of the mass of the SM Higgs boson. For the lower limit of parameter space, with $\tan\theta < 1$, one has instead $m_+ \equiv m_{h'} > m_- \equiv m_h$ and therefore, along that line $m_+$ is simply the mass of the mirror Higgs, totally decoupled from our world, which has a Higgs mass fixed to $m_h = 115$ GeV. We have also marked in fig. 3.8 the line $\theta = \pi/4$, which corresponds to $m = 0$.

The above comments are very useful to understand the behaviour of the fine-tuning associated to EWSB in the general case with $m \neq 0$. Before discussing them, let us remark that, in the case with $v_1 \neq v_2$ we are really interested in the tuning associated with electroweak breaking in our world, and therefore in the tuning necessary to get right $v_1$, which is fixed by the minimization condition

$$v_1^2 = -\frac{\mu^2}{2\lambda + \gamma} - \frac{m^2}{\gamma} \ . \tag{3.52}$$

The upper bounds on $\Lambda$ coming from requiring less than 10% tuning are shown by fig. 3.8 in the parameter space $\{m_+, \theta\}$ for $m_- = 115$ GeV. We can recognize the SM numbers along the upper limit of the allowed region of parameter space, along which $m_+$ is precisely the SM Higgs mass. Along the line $\theta = \pi/4$ we can recognize the numbers corresponding to the $m = 0$ case shown by fig. 3.7. Finally, along the lower limit of the allowed parameter space we recover the upper bound $\Lambda \simeq 1.3$ TeV, corresponding to the SM case with $m_- = 115$ GeV. The plot also shows the constraint on $m_+$ and $m_-$ from EWPT, which reads now [21]

$$m_+ < m_- \left(\frac{m_{EW}}{m_-}\right)^{1+1/\tan^2\alpha} \ . \tag{3.53}$$

Eq. (3.53) generalizes to (3.45) for the case of $m \neq 0$. The region above these lines (corresponding to the two cases $m_{EW} = 186$ GeV and 219 GeV) is disfavoured. From this plot we again conclude that, even though the upper bound on the scale of New Physics can be higher than in the SM, the global effect is never dramatic.

## 3.5 Conclusions

Solutions to the Hierarchy problem imply New Physics (NP) capable to implement a cancellation of the dangerous quadratic divergences to the Higgs mass. From the general naturalness arguments based on the SM Higgs physics, the NP should be at a scale around a few TeV, hopefully at the reach of LHC. However, it could happen that LHC would not find NP apart from the Higgs, in spite of the previous naturalness arguments. Likewise, it could happen that the Higgs found at LHC is beyond the range consistent with EWPT ($m_h \lesssim 219$ GeV). Both situations could be understood by modifying the ordinary SM Higgs sector [6].

In this chapter, we have considered three specific settings of two Higgs doublet models (2HDMs), that have been proposed in the literature with the general aim of improving the naturalness of the SM.

The first case is the Inert Doublet Model, where the SM is extended to include a second Higgs doublet that has neither a vev nor couplings to fermions (thus the name "inert doublet"), but couples through weak and quartic couplings. This model allows a heavy SM-like

[6]But this is not mandatory: if the SM is considered as an effective theory, the two situations can be accomodated by the effects of higher order operators.

Higgs while keeping consistency with EWPT, thanks to the contributions of the inert doublet to the $T$ parameter. However, contrary to what their authors have claimed, we have found that this model does not improve the naturalness of the SM with respect to quadratic radiative corrections. On the other hand, the structure of this model is more complicated than the SM, which introduces new sources of fine-tuning. Therefore, the usual (SM based) estimate for the scale of NP is still valid in this model [24].

The second model is another particular version of the 2HDM. Here there are two Higgses, a light one ($\sim$ 115 GeV) and a heavy one ($\sim$ 500 GeV), and the top couples mainly to the heavy one. Due to this fact, the new physics that cancels the top quadratic divergence of the SM-like Higgs could be at a scale larger than other cut-off scales (if these cut-offs could be different), and then outside the LCH reach. Nevertheless, the global fine-tuning is again not improved with respect to the SM.

In the third model the entire SM is replicated in a mirror world, and the SM and its mirror world communicate through a mixing term between the Higgses. We have found that the naturalness upper bound on the scale of NP can be a bit larger than the SM, but the improvement with respect to the SM is not significant.

We conclude that for these 2HDMs the estimate of the NP scale coming from naturalness arguments is inside the LHC reach, similar to that in the SM [24]. The claims of improved naturalness are not justified after a detail fine-tuning analysis.

# Chapter 4

# Neutrinos

When the Standard Model was formulated, there was no indication of neutrino masses. Then, $B-L$ is an exact symmetry of the SM, forbidding neutrino masses at all orders in pertubation theory. However, the evidence of neutrino oscillations provided by solar, atmospheric, accelerator and reactor neutrino experiments proved the existence of small but non-zero neutrino masses [6]. Therefore, the Standard Model must be extended to incorporate neutrino masses. The lightness of neutrinos compared to the other fermions strongly suggests an origin for their masses different from the standard Higgs mechanism. The simplest extension of the SM is obtained by adding right-handed neutrino singlets (one per family). Neutrino masses can be due to conventional Yukawa couplings or to a seesaw mechanism. The latter is probably the most elegant mechanism to explain the smallness of neutrino masses. However, the new contributions of right-handed neutrinos to the Higgs mass give a clear manifestation of the hierarchy problem of the SM, as we are going to analyze in this chapter

## 4.1 Neutrinos masses and Physics beyond the Standard Model

At present, experimental data on neutrino oscillation provide a firm evidence for neutrino masses. The neutrino oscillation experiments together with cosmological bounds indicate that at least one type of neutrino has mass 0.05 eV $\leq m_\nu \leq$ 1.0 eV. This indication of very small neutrino masses implies that the SM should be enlarged to incorporate neutrino masses. Usually this requires a new physics scale, $M$, where $B-L$ is broken. Because $B-L$ is an accidental global symmetry of the SM and not a fundamental piece, it can be broken in order to fit the new data on neutrino masses. Indeed, not only neutrino masses result in the breaking of this symmetry, but also the generation of the matter-antimatter asymmetry of the universe at scales higher than the electroweak scale.

Below $M$ one can typically has the $B-L$ dimension $d=5$ operator

$$-\delta\mathcal{L} = \frac{c}{M}\,\left(\overline{L}H\right)^2 + \text{h.c}\ , \qquad (4.1)$$

where $c$ is a constant, $L$ is the $SU(2)_L$ leptonic doublet and $H$ the Higgs doublet. When the Higgs takes a vev, $v$, this operator gives a Majorana neutrino mass:

$$m_\nu = \frac{v^2}{2}\,\frac{c}{M}\,. \qquad (4.2)$$

These masses are suppressed by a factor $\frac{v}{M}$ with respect to the charged fermions of the SM.

Small neutrino masses of the right order of magnitude ($m_\nu \leq 1$ eV) can be obtained in a natural way ($c \sim \mathcal{O}(1)$) if $M \gg v$, suggesting a new physics scale of order

$$10^{13}\ \text{GeV} \leq M \leq 10^{15}\ \text{GeV}. \qquad (4.3)$$

This points to the scale of Grand Unified Theories (GUT's) [30]. Despite the fact that this scale arises from naturalness arguments for small neutrino masses, it is not a natural scale for new physics taking into account the hierarchy argument discussed in chapter 2.

There are other mechanisms to obtain renormalizable neutrino masses, such as adding to the SM a triplet scalar or via the Higgs mechanism with Dirac neutrino masses. The last one implies neutrino Yukawa couplings of $\mathcal{O}(10^{-11})$, i.e. many orders of magnitude smaller than those of the charged leptons of the SM, which, unless a fundamental underlying theory explains this difference, seems unplausible. There is another way to obtain small neutrino masses, the seesaw mechanism, which is probably the most elegant way. The next section will be devoted to it.

## 4.2 Seesaw mechanism

The simplest extension of the SM that incorporates neutrino masses is obtained by adding right-handed neutrinos, $\nu_R$ (one per family). The SM lagrangian is enlarged with

$$\mathcal{L}_\nu = i\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \left(\lambda_\nu\,\overline{L}\,\widetilde{H}\,\nu_R - \frac{1}{2}M_R\,\overline{\nu_R{}^c}\,\nu_R\right) + \text{h.c.}\,, \qquad (4.4)$$

where $\mathcal{L}_\nu$ contains a kinetic term and a Majorana mass term $M_R$ for the right-handed neutrinos as well as neutrino Yukawa couplings $\lambda_\nu$. The Majorana mass, $M_R$, for right-handed neutrinos is not protected by any symmetry, and, then, it could be much larger than the electroweak scale. When EWSB occurs and the Higgs takes a vev, $v$, we can write the mass term in eq. (4.4) in this way:

$$-\mathcal{L}^{\text{mass}}_{\text{neutrino}} = \frac{1}{2}\,\overline{n_L{}^c}\mathcal{M}^* n_L + \text{h.c.}\ , \tag{4.5}$$

where

$$n_L = \begin{pmatrix} \nu_L \\ \nu_R{}^c \end{pmatrix}\ , \tag{4.6}$$

and the Majorana mass matrix $M$ for 3 leptonic generations is:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\ . \tag{4.7}$$

Here $m_D$ is the 3×3 Dirac neutrino mass matrix, related to the 3×3 neutrino Yukawa coupling matrix $\lambda_\nu$ by $m_D = \lambda_\nu v$ and $M_R$ is a 3×3 diagonal Majorana matrix for the right-handed neutrinos. Assuming that the matrix elements of $M_R$ are much larger than $v$, the neutrino mass matrices are approximately given by:

$$\begin{aligned} m_\nu &\simeq -m_D M_R^{-1} m_D^T\ , \\ m_N &\simeq M_R\,. \end{aligned} \tag{4.8}$$

After diagonalization, one gets 3 light Majorana neutrino mass eigenstates $\nu_i$ (with small masses $m_{\nu_i}$) and 3 heavy ones $N_i$ (with large masses $m_{N_i}$). This is known as the seesaw mechanism [31].

The dimension 5 operator of eq. (4.1) results from the above seesaw mechanism by integrating out the heavy Majorana neutrinos $N_i$, giving rise to the same masses as in (4.8). Therefore, if the Yukawa couplings are of $\mathcal{O}(1)$, the estimate on the scale $M_R$ given in (4.3) applies to seesaw neutrinos.

## 4.3 The fine-tuning problem with seesaw neutrinos

The seesaw mechanism results in 3 light mass eigenvalues and 3 heavy ones given by eq. (4.8). The lightest eigenstates are part of the effective low-energy theory and do not contribute radiatively to the Higgs mass parameter, as it is apparent from eq. (2.19). The heaviest ones, however, contribute not only to the quadratic divergence, but also to the finite and logarithmic parts of $\delta m^2$:

$$\delta_\nu m^2 = -\frac{\lambda_\nu^2}{16\pi^2}\left[2\Lambda^2 + 2M_R^2 \log\frac{M_R^2}{\Lambda^2}\right]\ . \tag{4.9}$$

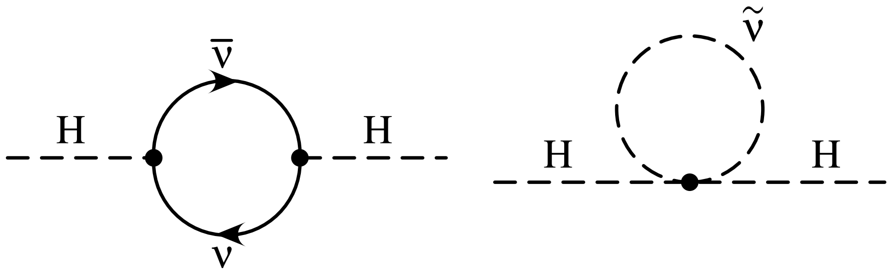



Figure 4.1: In Supersymmetry the quadratically divergent diagrams coming from heavy neutrinos loops cancel exactly with those of sneutrinos loops.

This equation is a particularly simple example of the second line of eq. (2.19) and illustrates the general discussion of sect.3 of chapter 2. In particular, even if the quadratically divergent contributions of the SM cancel the one of eq. (4.9) (which, incidentally, means that Veltman's condition is modified by undetectable physics), there are other dangerous contributions which do not cancel. In other words, this represents a new manifestation of the hierarchy problem [15, 32]. A cancellation between the quadratic and the logarithmic and finite contributions would be completely artificial and depends on the choice of renormalization scheme.

The logarithmic and finite contributions are especially disturbing as their size is associated to $M_R$ (which is expected to be very large) and show up in any renormalization scheme. In the $\overline{\rm MS}$ scheme [see eq. (2.21)]

$$\delta_\nu^{\overline{\rm MS}} m^2 = -\frac{\lambda_\nu^2}{8\pi^2} M_R^2 \left[\log\frac{M_R^2}{\Lambda^2} - 1\right] \ , \tag{4.10}$$

where we have already set $Q = \Lambda$. Now it is easy to obtain a lower bound on the size of $M_R$ from the request of no fine-tuning. Demanding

$$\left|\frac{\delta_\nu m^2}{m^2}\right| \leq \Delta \ , \tag{4.11}$$

requires

$$M_R < 10^7 \ {\rm GeV} \ \Delta^{1/3} \left(\frac{m_h}{200 \ {\rm GeV}}\right)^{2/3} \left(\frac{m_\nu}{5\times 10^{-2} \ {\rm eV}}\right)^{-1/3} \left[\log\frac{\Lambda^2}{M_R^2} + 1\right]^{-1/3} \ . \tag{4.12}$$

Hence, for any sensible value of $\Delta$, we obtain a quite robust bound [32]

$$M_R \lesssim 10^7 \ {\rm GeV} \ , \tag{4.13}$$

which can only be satisfied if the neutrino Yukawa coupling is very small, $\lambda_\nu \lesssim 10^{-4}$. This is possible, but undermines the plausibility of the seesaw mechanism as an explanation of the smallness of $m_\nu$.

The previous bound has been obtained under the assumption that right-handed neutrinos are the only new physics beyond the SM. In a supersymmetric scenario, this electroweak fine-tuning problem could be much softened. The large contributions from heavy right-handed neutrinos are cancelled by those of their scalar partners (sneutrinos, $\tilde{\nu}$) [see fig. 4.1], rendering $\delta_\nu m^2$ small and not dangerous. With the previous arguments we can conclude that the seesaw mechanism in the SM suffers from a very important fine-tuning problem which cannot be evaded by invoking e.g. a Veltman-like cancellation of the quadratically divergent contributions to $m^2$, but can be softened by the introduction of new physics, such as supersymmetry.

## 4.4 Conclusions

The evidence of neutrino oscillations in neutrino experiments have proved the existence of small neutrino masses. The simplest extension of the SM to incorporate neutrino masses is obtained by adding right-handed neutrinos, $\nu_R$, with a large Majorana mass, $M_R$. The smallness of neutrino masses can be explained with a seesaw mechanism, where for each generation there are two neutrino eigenstates, one very light ($m_\nu < 1$ eV) and the other very heavy (say $M_R \sim 10^{13}$ GeV). The heaviest eigenstate contributes to the quadratic divergence to the Higgs mass, and also to the finite and logarithmic contributions. These finite and logarithmic contributions are specially dangerous as their size is associated to $M_R$ (expected to be very large). Demanding that these finite and logarithmic corrections are not much larger than $m^2$ itself translates into the upper bound $M_R \lesssim 10^7$ GeV, which spoils the naturalness of the seesaw mechanism to explain the smallness of $m_\nu$. Therefore, in the context of the SM, the seesaw mechanism suffers a very important fine-tuning problem which calls for the existence of *additional* NP, besides right-handed neutrinos. In this sense, Supersymmetry is the favourite framework to accommodate the seesaw mechanism.

# Chapter 5

# Supersymmetry

The most familiar example of physics beyond the SM and many theorists' favorite candidate for new physics is Supersymmetry (SUSY) [7]. Supersymmetry provides a solution to the Big Hierarchy problem, since in SUSY the quadratically divergent radiative corrections to the Higgs mass are cancelled. Unlike for many of its competitors, perturbativity is maintained to an arbitrary high-energy scale because the cancellation of quadratic divergences takes places at all orders of pertubation theory, i.e. at any scale, even after the breaking of SUSY.

To ensure the desired cancellations, SUSY relates bosons and fermions, in such a way that in the spectrum of a supersymmetric theory there are the same number of bosonic and fermionic degrees of freedom, and it also relates couplings (e.g. quartic couplings depend on gauge and Yukawa couplings). In the supersymmetric extension of the SM, we have new scalar fields, which are superpartners of the SM fermions, and new fermionic fields, which are superpartners of the SM gauge bosons and the Higgs boson. This is done by organizing the particles into a *superfield* , with contains fields differing by one-half unit of spin. Chirality is attached to the bosons by their association with the fermions.

These new superparticles (sparticles) would have the same quantum numbers and the same masses as their SM partners. Thus, now there are two types of contributions to the Higgs mass, one from the fermionic loops and other from the bosonic loops, and because the scalar and fermionic interactions have the same couplings, the cancellation of quadratic divergence occurs automatically in SUSY.

The supersymmetric SM connects particles of differing spin, but with all other characteristics remaining the same. Then it is clear that SUSY must be a broken symmetry, since there is no observed sparticle with the same mass as the correspondending particle of the

SM. This non-degeneracy of masses between the particles of a superfield is a signal for SUSY breaking. Even though the mechanism of SUSY breaking is still unknown to date, it has to be implemented without introducing new quadratic divergences that would spoil the solution provided by SUSY to the hierarchy problem. This breaking is done by a set of specific SUSY breaking terms, called *soft-supersymmetry-breaking-terms* [33]. These soft terms provide both the masses of the sparticles (heavier than their corresponding SM particle) and the required spontaneous Electroweak symmetry breaking (EWSB) at low energies.

In this chapter, after describing briefly the minimal supersymmetric extension of the SM (MSSM) we analyze the supersymmetric fine-tuning problem. We then discuss other supersymmetric scenarios, such as those with low-scale SUSY breaking, which have the potential of improving significantly the naturalness of EWSB.

## 5.1 Minimal Supersymmetric Standard Model (MSSM)

### 5.1.1 MSSM description

The MSSM [34] respects the same $SU(3)\times SU(2)_L\times U(1)_Y$ gauge symmetries as the SM. As in any supersymmetric theory there is a superpartner for each SM particle, with the same mass and the same quantum numbers, but differing by one-half unit of spin. The superpartners of quarks and leptons are called squarks and sleptons (of spin zero). The superparners of the gauge bosons are the gauginos (fermions of spin 1/2), and the ones of the higgses are the higgsinos (spin 1/2).

In the MSSM the choice of the Higgs sector is crucial. One needs at least two scalar fields, $H_1$ with hypercharge $Y = -1$ and $H_2$ with $Y = 1$. The reasons for this choice are mainly two. The first one is due to anomaly cancellation. A model with one single Higgs doublet superfield has non-vanishing gauge anomalies associated with fermion triangle diagrams, since the contribution from the fermionic partner of the Higgs doublet remains uncancelled. This is solved with a second Higgs doublet of opposite hypercharge and its fermionic partner. The second reason is related to the chirality of fermions. Because the masses of chiral fermions must be supersymmetric, they have their origin from terms in the superpotential. As the superpotential has to be analytic, it is not allowed to introduce the hermitian conjugate of Higgs superfields. It is then not possible to introduce $U(1)_Y$ invariant terms that give masses to both up and down-type quarks without introducing a second Higgs doublet superfield.

Contrary to what happens in the SM, the baryon and lepton number, $B$ and $L$, are not conserved quantum numbers in the MSSM. In order to get them preserved, a discrete symmetry is imposed. This symmetry, called $R$-parity, is a multiplicative quantum number defined such that $R = 1$ for SM and Higgs particles and $R = -1$ for their spartners. The main phenomenological implication of the assumption of $R$-parity conservation is that spartners can only be produced in pairs from SM particles and, therefore, the lightest supersymmetric particle is stable.

We can now look at the MSSM lagrangian, and in particular at the scalar potential, that we will need for our fine-tuning analysis.

The Higgs sector of the MSSM is a CP-conserving two-Higgs-doublet model (2HDM) [see section 1 of chapter 3], with a Higgs potential whose dimension-four terms respect supersymmetry and with restricted Higgs-fermion couplings in which $H_1$ couples exclusively to down-type fermions while $H_2$ couples exclusively to up-type fermions. Then, the SUSY-preserving part of the MSSM Higgs potential is given by,

$$V_{SUSY} = |\mu^2|(|H_1|^2 + |H_2|^2) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2|H_1^* H_2|^2, \tag{5.1}$$

where the mass parameter $\mu$ is a supersymmetric Higgs mass. The self-couplings in the Higgs sector are given in terms of the gauge couplings $g$ and $g'$. The potential (5.1) is positive and presents a trivial minimum, then it can not produce the correct EWSB. The introduction of appropiate explicit SUSY breaking terms in the Higgs potential is mandatory in order to implement both SUSY breaking and EWSB.

Because the SUSY breaking mechanism is not yet known, the SUSY breaking lagrangian is not completely determined and one usually assumes a set of breaking terms of the most general form that are fixed by demanding $SU(3) \times SU(2) \times U(1)$ invariance and by requiring them to be soft in order that the cancellation of quadratic divergences is maintained. These soft SUSY breaking terms are classified into four different types [33]: Majorana mass terms for gauginos, scalar mass terms for sfermions and Higgs particles, interaction terms among three scalar particles with trilinear couplings and scalar-scalar bilinear terms. In the MSSM this soft susy breaking potential for the Higgs sector is:

$$V_{\rm soft} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - m_3^2 \left( H_1 \cdot H_2 + {\rm h.c.} \right) , \tag{5.2}$$

where $m_3^2 \equiv B\mu$, with $\mu$ the mass parameter introduced in (5.1) and $B$ is a bilinear soft SUSY breaking parameter.

Once the $m_{H_1}$, $m_{H_2}$ and $m_3$ mass terms are included in the Higgs potential, the tree-level scalar potential for the neutral components, $H_{1,2}^0$, of the Higgs doublets is:

$$V^{\rm MSSM}(H_1^0, H_2^0) = m_1^2|H_1^0|^2 + m_2^2|H_2^0|^2 - 2m_3^2 H_1^0 H_2^0 + \frac{1}{8}(g^2+g'^2)(|H_1^0|^2 - |H_2^0|^2)^2, \quad (5.3)$$

with $m_{1,2}^2 = \mu^2 + m_{H_{1,2}}^2$. Minimization of $V^{\rm MSSM}$ leads to a vacuum expectation value (vev) $v^2 \equiv 2(\langle H_1^0\rangle^2 + \langle H_2^0\rangle^2)$ and thus to a mass for the $Z^0$ gauge boson, $M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$, given by

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} \ . \quad (5.4)$$

### 5.1.2 Analyzing the fine-tuning in the MSSM

The minimization equation relevant to us in order to analyze the fine-tuning is the one of eq. (5.4). This condition is the only equation that quantitatively relates some soft breaking masses at the electroweak scale to a measured value, $M_Z$. The quantities in the right hand side of eq. (5.4) are quantities evaluated at low energy. These quantities are related to more fundamental parameters at a higher scale, i.e. the initial UV parameters, via the renormalizations group equations (RGEs).

The MSSM RGEs for the mass parameters in eq. (5.4) are coupled to those of other soft terms, e.g. gaugino masses, stop masses, trilinear terms, etc., so $M_Z$ can be expressed as a linear combination of the initial UV mass-squared parameters with coefficients that can be calculated by integrating the RGEs. For example, for large $\tan\beta$ and $\Lambda_{UV} = M_{GUT} = 1.4\times 10^{16}$ GeV [35]:

$$M_Z^2 \simeq -2.02\mu^2 + 3.57M^2 + 0.07m^2 + 0.22A^2 + 0.75AM \ , \quad (5.5)$$

where $M, m, A$ are the gaugino mass, scalar soft mass and trilinear soft term respectively. We have used universality at the GUT scale for simplicity. In eq. (5.5) we can see that even for soft masses smaller than 1 TeV, some terms in the sum can be much larger than $M_Z^2$, so a non-trivial cancellation among terms in the sum would be needed in order to obtain the correct value for $M_Z^2$. Hence, this leads to a fine-tuning problem [16, 36, 17, 18, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48] : the supersymmetric fine-tuning problem.

There are two ways to escape from this fine-tuning: 1) the cancellation among terms occurs naturally in a fundamental theory underlying the MSSM ; 2) each term in the sum of the right hand side of eq. (5.5) is not larger than a few times $M_Z^2$. The first way is difficult to imagine, because the cancellation involves the sizes of all the soft breaking terms and the $\mu$-parameter, and also the different magnitudes of the coefficients in front of the soft masses, which have to do with the RG running between the initial and the low energy scale. These quantities have such a different physical origin that it is difficult to imagine a fundamental reason why they should be correlated in the correct way to enforce a cancellation. As a matter of fact, the analyses in the literature [41, 46] of many superstring, superstring-inspired and supergravity models do not find such correlations.

The second way is ruled out by the experimental lower bounds for the sparticles masses. The problem is especially acute for the LEP bound on the Higgs mass, $m_h \geq 115$ GeV [4] as has been stressed by a number of authors [39, 40, 41, 42]. This can be seen if we consider the tree-level and the dominant one-loop correction [49] to the theoretical upper bound on $m_h$ in the MSSM:

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \log \frac{M_{\rm SUSY}^2}{m_t^2} + ... \tag{5.6}$$

where $m_t$ is the (running) top mass ($\simeq$ 165 GeV for $M_t = 173$ GeV) and $M_{\rm SUSY}$ is an average of stop masses. Since the experimental lower bound on $m_h$ exceeds the tree-level contribution ($M_Z^2 \cos^2 2\beta$), the radiative corrections must be responsible for the difference, and this translates into a lower bound on $M_{\rm SUSY}$:

$$M_{\rm SUSY} \ \gtrsim \ e^{-2.2\cos^2 2\beta} e^{(m_h/61 \ {\rm GeV})^2} m_t \ \gtrsim \ 3.7 \ m_t \ , \tag{5.7}$$

where the last figure corresponds to $m_h = 115$ GeV and large $\tan\beta$. What eq. (5.7) shows us is that $M_{\rm SUSY}^2$ must be more than 40 times bigger than $M_Z^2$ and this number increases exponentially for larger (smaller) $m_h$ ($\tan\beta$). On the other hand $M_{\rm SUSY}$ is itself a low energy quantity that has a dependence on the initial soft masses analogous[1] to eq. (5.5):

$$M_{\rm SUSY}^2 \simeq 3.36M^2 + 0.49m^2 - 0.05A^2 - 0.19AM + m_t^2 + ({\rm D-terms}) \ . \tag{5.8}$$

Roughly, $M_{\rm SUSY}^2$ has a magnitude similar to the main positive contribution in the r.h.s. of (5.5), which then implies that some of the terms in that sum are at least $\sim 45$ times larger than $M_Z^2$, showing up the fine-tuning.

In the previous discussion, we have used dimensional arguments to explain the fine-tuning in the MSSM. Now, we want to quantify this fine-tuning in a more detailed way, following Barbieri and Giudice [16] (as explained in chapter 2 and Appendix A.2). We can remember here the definition of the fine-tuning parameters, $\Delta_{p_i}$,

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta v^2}{v^2} = \Delta_{p_i} \frac{\delta p_i}{p_i} \ . \tag{5.9}$$

[1] We have approximated in eq. (5.8) the geometric average of the stop masses by the arithmetic one, which is sufficiently precise for the argument.

We focus here on the fine-tuning on $\mu^2$, $\Delta_{\mu^2}$, since it is the parameter that usually requires the largest fine-tuning. This happens because, due to the negative sign of its contribution in eqs. (5.4, 5.5), the $\mu^2$ term has to compensate the (globally positive and large) remaining contributions.[2]

As an example, we can evaluate $\Delta_{\mu^2}$ for large $\tan\beta$ and $m_h = 115$ GeV. In this case, eqs.(5.6, 5.8) imply that the universal soft masses must be $m = M = A \simeq 325$ GeV. With this requirement and using the tree-level potential of eq.(5.3), we get $\Delta_{\mu^2} \sim 55$. If one includes the dominant logarithmic corrections at one-loop from the top-stop sector (see Appendix A.2), $\Delta_{\mu^2}$ gets down [17, 50] to $\sim 35$. This fine-tuning could be lower if more radiative corrections are added. In the optimal case it can brought down to 20 [41, 40, 51]. This gives an abnormally large fine-tuning for the MSSM, since one could naively expect that if the soft parameters had a size $m^2_{\rm soft} \sim av^2$, the fine-tuning would be $\Delta \sim a$, but what one gets is $\Delta \gtrsim 20a$.

Why is this fine-tuning larger than expected?. To understand the reasons for this, let us write the generic Higgs potential along the breaking direction as

$$V = \frac{1}{2}m^2v^2 + \frac{1}{4}\lambda v^4 \ , \tag{5.10}$$

where $\lambda$ and $m^2$ are functions of the $p_\alpha$ parameters and $\tan\beta$, in particular

$$m^2 = c^2_\beta\, m^2_1(p_\alpha) + s^2_\beta\, m^2_2(p_\alpha) - s_{2\beta}\, m^2_3(p_\alpha) \ . \tag{5.11}$$

Minimization of (5.10) leads to

$$v^2 = \frac{-m^2}{\lambda} \ . \tag{5.12}$$

Then, the fine-tuning is $\Delta \sim \mathtt{m}^2_{\mathtt{i}}/(\lambda v^2)$, where $\mathtt{m}^2_{\mathtt{i}}$ are the individual contributions to $m^2$. Clearly from this, the fine-tuning will increase if the size of the individual $\mathtt{m}^2_{\mathtt{i}}$ are large and $\lambda$ is small. In the MSSM, $\lambda$ is at tree-level:

$$\lambda_{\rm MSSM} = \frac{1}{8}(g^2 + g'^2)\cos^2 2\beta \ \simeq \ \frac{1}{15}\cos^2 2\beta \ , \tag{5.13}$$

which already implies a fine-tuning $\sim 15$ times larger (for the most favorable case of large $\tan\beta$) than the above naive expectations. The previous $\lambda_{\rm MSSM}$ was evaluated at tree-level but radiative corrections can make $\lambda$ larger, thus reducing the fine-tuning. The ratio $\lambda_{\rm tree}/\lambda_{1-{\rm loop}}$ is basically the ratio $(m^2_h)_{\rm tree}/(m^2_h)_{1-{\rm loop}}$, so for large $\tan\beta$ and $m_h = 115$ GeV the previous

[2]As pointed out in ref. [19], it is more sensible to use $\mu^2$ rather than $\mu$ for the fine-tuning parameter, since this is the form in which it appears in the sum.

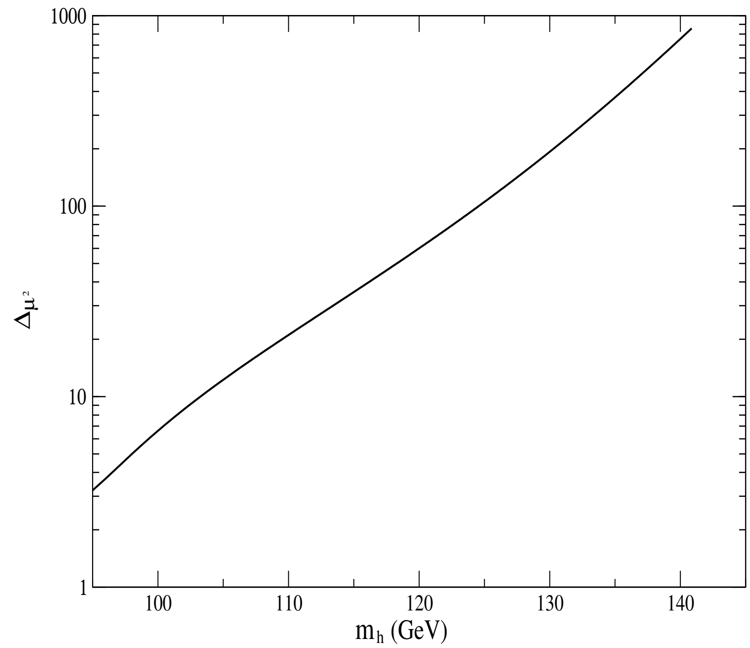



Figure 5.1: Fine-tuning in the MSSM (measured by $\Delta_{\mu^2}$) as a function of the Higgs mass (in GeV) for $\tan\beta = 10$.

factor 15 is reduced by a factor $M_Z^2/m_h^2$ down to $\sim 9$. Finally, for the MSSM (with large $\tan\beta$ and $\Lambda_{UV} = M_{GUT}$), $m^2$ is:

$$m^2 = m_1^2 c_\beta^2 + m_2^2 s_\beta^2 - m_3^2 s_{2\beta} \simeq 1.01\mu^2 - 2.31\tilde{m}^2 \ , \tag{5.14}$$

where we have set $A = M = m = \tilde{m}$ for simplicity. For a given value of $\tilde{m}^2$, the large renormalization group (RG) coefficient in front of $\tilde{m}^2$ implies that the required cancellation must be (in this case) $\sim 2.31$ times more accurate than naive expectations so, finally the factor 9 is enhanced to $\sim 20$. Notice that those large RG coefficients are a consequence of the radiative mechanism of EW breaking.

From the above discussion it is important to notice that, although for a given size of the soft terms the radiative corrections reduce the fine-tuning, the requirement of sizeable radiative corrections implies itself large soft terms, which in turn worsens the fine-tuning. More precisely, for the MSSM $\delta_{\rm rad}\lambda \propto \log(M_{\rm SUSY}/m_t)$, so $\lambda$ can only be radiatively enhanced by increasing $M_{\rm SUSY}$, and thus the individual $\mathtt{m}_\mathtt{i}^2$. A given increase in $M_{\rm SUSY}$ reflects linearly in $\mathtt{m}_\mathtt{i}^2$ and only logarithmically in $\lambda$, so the fine-tuning $\Delta \sim \mathtt{m}_\mathtt{i}^2/(\lambda v^2)$ gets usually worse. As discussed previously, for the MSSM $(m_h)_{\rm tree} < (m_h)_{\rm exp}$, hence sizeable radiative corrections are in fact mandatory and the fine-tuning is consequently aggravated. As a consequence, the

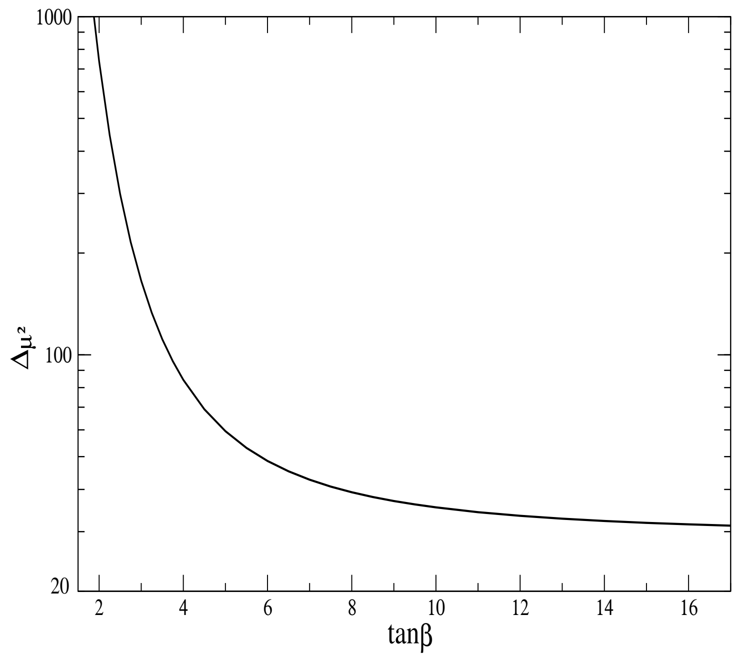



Figure 5.2: Lower bound on the MSSM fine-tuning ($\Delta_{\mu^2}$) as a function of $\tan\beta$ from the LEP bound $m_h \geq 115$ GeV.

fine-tuning increases exponentially for increasing (decreasing) $m_h$ ($\cos^2 2\beta$) as indicated by eq. (5.7).

Now we can illustrate the MSSM fine-tuning with some figures. In fig. 5.1 $\Delta_{\mu^2}$ at one-loop is plotted as a function of the Higgs boson mass, $m_h$, for $\tan\beta = 10$. A large value of $\tan\beta$ has been chosen, because the fine-tuning in this case is smaller, as we have seen before. Only the dominant one-loop correction to $m_h$ is included, as in eq.(5.6), and the soft parameters are assumed to be universal at the GUT scale. Although the fine-tuning can be made smaller in non-universal cases, figure 5.1 shows the typical size of $\Delta_{\mu^2}$ in the MSSM. As expected, $\Delta_{\mu^2}$ grows exponentially for increasing $m_h$.

The dependence of $\Delta_{\mu^2}$ with $\tan\beta$ is shown in fig. 5.2 for $m_h$ at the LEP bound, $m_h = 115$ GeV (the optimal choice for the fine-tuning). The curve for $\Delta_{\mu^2}$ increases exponentially for decreasing $\cos^2 2\beta$, again as expected. This curve can be interpreted as a LEP lower bound on the MSSM fine-tuning.

Finally, fig. 5.3 shows contour lines of constant $\Delta_{\mu^2}$ in the $(\tilde{m}, \tan\beta)$ plane, where $\tilde{m}$ is the universal soft mass at $\Lambda_{UV}$. We also plot dashed contour lines of constant $m_h$ and the LEP lower bound on $m_h$. Again, it is clear how the fine-tuning is greater for smaller

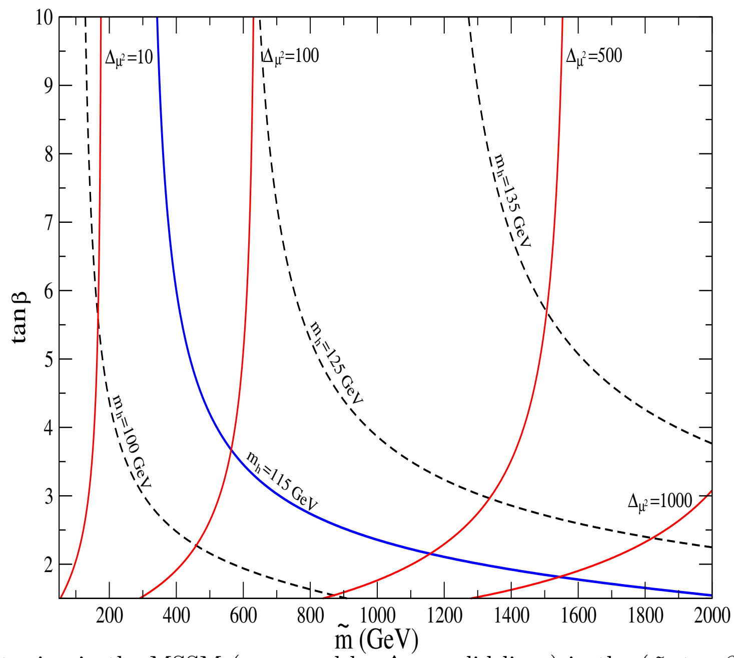



Figure 5.3: Fine-tuning in the MSSM (measured by $\Delta_{\mu^2}$, solid lines) in the $(\tilde{m}, \tan\beta)$ plane. Dashed lines are contour lines of constant Higgs mass.

$\tan\beta$ and how it grows, together with $m_h$, for larger $\tilde{m}$. The upper horizontal line and the $m_h = 115$ GeV contour line correspond to figs. 5.1 and 5.2 respectively. As an example, if we follow the $\Delta_{\mu^2} = 10$ line (there is no fine-tuning below this region), we will conclude that one cannot obtain $\tilde{m}$ larger than $\sim 175$ GeV (which translates into upper bounds on superpartner masses) nor Higgs masses larger than $\sim 103$ GeV, already ruled out by LEP.

However, in general, trying to have a very accurate determination of the fine-tuning does not make much sense, as we have seen in sects. 2.2-2.3. There is also the experimental uncercainty on the top quark mass, eq.(2.10), which translates into a significant uncercainty on the fine-tuning parameters. For these reasons, in our numerical one-loop estimates of $\Delta_{\mu^2}$ we have just included the logarithmic correction to $m_h^2$ given in eq.(5.6). And also because our purpose is to compare the fine-tuning of the MSSM with that of other scenarios of physics beyond the SM.

How can we reduce the MSSM fine-tuning in other SUSY models?. As we have seen, the fine-tuning problem of the MSSM is due to the smallness of the tree-level Higgs quartic coupling, $\lambda_{\rm tree}$. The problem is worsened by the fact that sizeable radiative corrections (and thus sizeable soft terms) are needed to satisfy the experimental bound on $m_h$. This is also a consequence of the smallness of $\lambda_{\rm tree}$: if it were bigger, radiative corrections would not be necessary. Therefore, the most efficient way of reducing the fine-tuning is to consider supersymmetric models where $\lambda_{\rm tree}$ is larger than in the MSSM. Let us compute the corresponding improvement in the fine-tuning.

The value of $\Delta_p$ for a generic parameter $p$ of a given model has the form [see Appendix A.2]

$$\Delta_p = \frac{p}{m^2}\left[\frac{\partial m^2}{\partial p} + \frac{v^2}{2}\frac{\partial\lambda}{\partial\beta}\frac{d\beta}{dp} + v^2\frac{\partial\lambda}{\partial p}\right] \ . \tag{5.15}$$

If we focus on the $\mu^2$ parameter, we can write

$$\Delta_{\mu^2} \simeq \frac{\mu^2}{m^2}\frac{\partial m^2}{\partial\mu^2} \simeq -\frac{\mu^2}{\lambda v^2} \simeq -2\frac{\mu^2}{m_h^2} \ , \tag{5.16}$$

where the last two terms of eq.(5.15) are neglected to get (5.16), because they are suppressed by a factor $\mathcal{O}(v^2/\mu^2)$. In eq.(5.16) it is used that the dependence of the low-energy $m^2$ on the inital (UV) $\mu$ parameter is dominated by the tree-level contribution and $m_h^2$ is the Higgs mass matrix element along the breaking direction, but in many cases it is very close to one of the mass eigenvalues. Therefore,

$$\Delta_{\mu^2} \simeq \Delta_{\mu^2}^{\rm MSSM}\left[\frac{m_h^{\rm MSSM}}{m_h}\right]^2\left[\frac{\mu}{\mu^{\rm MSSM}}\right]^2 \ . \tag{5.17}$$

In this equation we can see how a theory can improve the MSSM fine-tuning: increasing $m_h$ and/or decreasing $\mu$. The first way corresponds to increasing $\lambda$. The second, for a given $m_h$, corresponds to reducing the size of the soft terms, which is only allowed if radiative contributions are not essential to raise $m_h$. Both improvements indeed concur for larger $\lambda_{\rm tree}$.

It should be said that it is not difficult to come up with alternative SUSY models which perform better than the MSSM concerning the fine-tuning of the EWSB. One popular model is the Next-to-Minimal Supersymmetric SM (NMSSM) which adds a singlet chiral multiplet to the MSSM [52]. In this models the Higgs quartic coupling gets larger thanks to additional $F$-term contributions from the singlet. One can also consider scenarios in which the breaking of SUSY takes place at a low-scale [53, 54, 55, 56] (not far from the TeV scale), where a tree-level quartic Higgs couplings larger than in the MSSM occurs in a natural way. These low-scale SUSY breaking scenarios are developed in detail in the next section.

## 5.2 Low-scale SUSY breaking

### 5.2.1 Low-scale SUSY breaking description

As we have seen for the MSSM, phenomenological studies of supersymmetric models are done with a supersymmetric lagrangian, adding a set of supersymmetry breaking ($\not{\text{SUSY}}$) terms that parameterize the effect of the breaking, without making any assumption about the nature of the breaking itself. These terms must be *soft* terms (they do not generate quadratic divergences in the renormalization of scalar masses), and in many SUSY breaking approaches, this breaking takes place at a very high energy scale.

In any realistic breaking of SUSY, there are two scales involved: the $\not{\text{SUSY}}$ scale, say $\sqrt{F}$, which corresponds to the vevs of the relevant auxiliary fields in the $\not{\text{SUSY}}$ sector; and the messenger scale, $M$, associated to the interactions that transmit the breaking (through effective operators suppressed by powers of $M$) to the observable sector. These operators give rise to soft terms (such as scalar soft masses), but also hard terms (such as quartic scalar couplings):

$$m_{\text{soft}}^2 \sim \frac{F^2}{M^2}\,, \quad \lambda_{\not{\text{SUSY}}} \sim \frac{F^2}{M^4} \sim \frac{m_{\text{soft}}^2}{M^2}\,. \tag{5.18}$$

Phenomenology requires $m_{\text{soft}} = \mathcal{O}(1\,\text{TeV})$, but this does not fix the scales $\sqrt{F}$ and $M$ separately. The MSSM assumption is that there is a hierarchy of scales: $m_{\text{soft}} \ll \sqrt{F} \ll M$, so that the hard terms are negligible and the soft ones are the only observable trace of $\not{\text{SUSY}}$. However, there is no real need for such a strong hierarchy, so the scales $\sqrt{F}$ and $M$ could well be of similar order (thus not far from the TeV scale). This happens in the so-called low-scale $\not{\text{SUSY}}$ scenarios. In this framework, the hard terms of eq. (5.18), are not negligible anymore and hence the $\not{\text{SUSY}}$ contributions to the Higgs quartic couplings can be easily larger than the ordinary MSSM value (5.13). As discussed in the previous section, this is exactly the optimal situation to ameliorate the fine-tuning problem.

It is convenient to describe these models using an effective field theory approach [54, 56]. If one tries to break supersymmetry in a renormalizable model, one finds that the supertrace of the mass matrix is identically zero even when SUSY is broken. This means that the sum of the masses of fermions is equal to the sum of the masses of bosons. So if we try to break supersymmetry using only the MSSM fields and a renormalizable lagrangian we will always find sparticles lighter than some ordinary particles. These difficulties are overcome in models in which the transmission of $\not{\text{SUSY}}$ to MSSM particles can be described using an effective non-renormalizable lagrangian (that is valid only up to some high energy scale

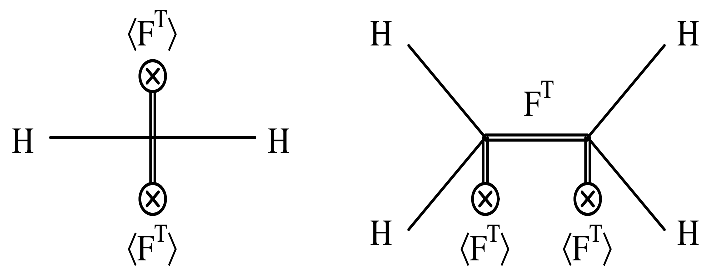



Figure 5.4: Higgs soft masses and hard quartic couplings that arise from the Kähler operator (5.19).

$M$). In this spirit, the approach that we will follow is to describe the transmission of $\not{\text{SUSY}}$ using effective interactions, without relying on any specific microscopic dynamics that can generate it. This messenger scale $M$ should be now close to the electroweak scale. E.g. there could be some massive fields responsible for the $\not{\text{SUSY}}$ mediation (like in gauge mediation) with masses $\sim M$; or there could be a more fundamental reason, as in models with large extra-dimensions or in supersymmetric Randall-Sundrum models. This scale of new physics, although close to the electroweak scale, still has to be somewhat larger than it, so it makes sense to consider the effective theory [54, 56] below it (but above the electroweak scale) instead of sticking to one of these particular examples. Denoting by $T$ the superfield responsible for $\not{\text{SUSY}}$, $\langle F_T \rangle \neq 0$, and assuming that, apart from the $T$ field, the spectrum is minimal (i.e. the same as in the MSSM), the effective theory is like the SUSY part of the MSSM, plus some effective interactions which include couplings between $T$ and the observable fields, suppressed by powers of $M$. These effective interactions can appear in the superpotential, $W$, as well as in the Kähler potential, $K$, or the gauge kinetic function.

As a simple example, suppose that the Kähler potential contains the operator

$$K \supset -\frac{1}{M^2}|T|^2|H|^2 + \cdots \tag{5.19}$$

where $H$ denotes any Higgs superfield. Once $F_T$ takes a vev, the above non-renormalizable interaction produces soft terms as well as hard terms, as schematically represented in the diagrams of fig. 5.4. Notice that $m^2_{\rm soft} \sim |F_T|^2/M^2$, $\lambda_{\not{\rm SUSY}} \sim |F_T|^2/M^4 \sim m^2_{\rm soft}/M^2$, in agreement with (5.18).

The Higgs potential has the structure of a generic two Higgs doublet model (2HDM),

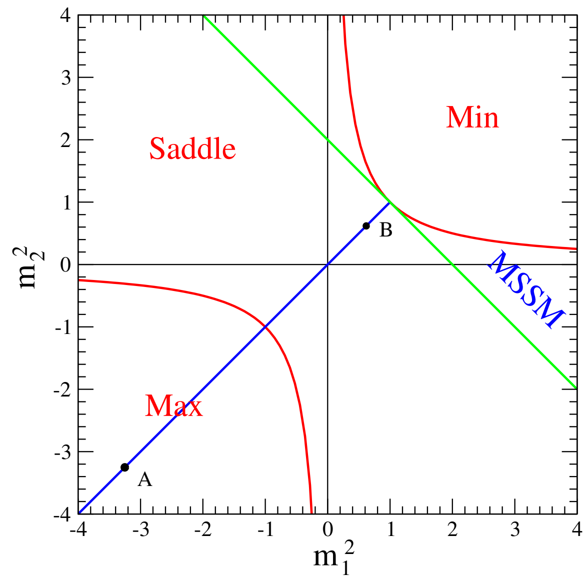



Figure 5.5: Depending on the values of $m_1^2$ and $m_2^2$ the origin of the Higgs potential can be destabilized or not. Absence of UFB directions impose additional constraints on the Higgs masses for the MSSM.

with $T$-dependent coefficients [56],

$$\begin{aligned} V &= V_0(\bar{T},T) + m_1^2(\bar{T},T)|H_1|^2 + m_2^2(\bar{T},T)|H_2|^2 - \left[m_3^2(\bar{T},T)H_1\cdot H_2 + \text{h.c.}\right] \\ &+ \frac{1}{2}\lambda_1(\bar{T},T)|H_1|^4 + \frac{1}{2}\lambda_2(\bar{T},T)|H_2|^4 + \lambda_3(\bar{T},T)|H_1|^2|H_2|^2 + \lambda_4(\bar{T},T)|H_1\cdot H_2|^2 \\ &+ \left[\frac{1}{2}\lambda_5(\bar{T},T)(H_1\cdot H_2)^2 + \lambda_6(\bar{T},T)|H_1|^2 H_1\cdot H_2 + \lambda_7(\bar{T},T)|H_2|^2 H_1\cdot H_2 + \text{h.c.}\right] \\ &+ \ldots \end{aligned} \tag{5.20}$$

where we have truncated at $\mathcal{O}(H^4)$, which makes sense whenever $v^2/M^2$ is small. The quantities $m_i^2$, $\lambda_i$ can be expressed in terms of the parameters appearing in $W$ and $K$ (explicit expressions can be found in ref. [56]). If the $T$ field is heavy enough it can be integrated out and one ends up with a truly 2HDM. The previous potential is to be compared with the MSSM one [eq. (5.3)] with $\lambda_{1,2} = \frac{1}{4}(g^2 + g'^2)$, $\lambda_3 = \frac{1}{4}(g^2 - g'^2)$, $\lambda_4 = -\frac{1}{2}g^2$, $\lambda_{5,6,7} = 0$.

With the form of the Higgs potential in eq. 5.20 we can make some general observations on the possible patterns of EWSB. There are two neccesary conditions for EWSB in a general 2HDM. The first one is that the Higgs-field space must be a minimum, a saddle point or a maximum, depending on the mass parameters $m_i^2$:

$$m_1^2 m_2^2 - |m_3^2|^2 > 0\ ,\ m_1^2 + m_2^2 > 0\ , \qquad \text{[Minimum]} \tag{5.21}$$

$$m_1^2 m_2^2 - |m_3^2|^2 \ < \ 0 \ , \qquad \text{[Saddle Point]} \qquad (5.22)$$
$$m_1^2 m_2^2 - |m_3^2|^2 \ > \ 0 \ , \ m_1^2 + m_2^2 \ < \ 0 \ . \qquad \text{[Maximum]} \qquad (5.23)$$

These equations define three regions in the $\{m_1^2, m_2^2\}$-plane, labelled by 'Min', 'Saddle' and 'Max' in fig. 5.5. Such regions are separated by the upper and lower branches of the hyperbola $m_1^2 m_2^2 - |m_3^2|^2 = 0$. Electroweak breaking can take place in the regions 'Saddle' or 'Max', while the region 'Min' is excluded. The second condition is the abscence of unbounded from below (UFB) directions along which the quartic part of the potential gets destabilized. In the MSSM the quartic couplings receive only contributions from D-terms, namely $\lambda_{1,2} = \frac{1}{4}(g^2 + g'^2)$, $\lambda_3 = \frac{1}{4}(g^2 - g'^2)$, $\lambda_4 = -\frac{1}{2}g^2$, $\lambda_{5,6,7} = 0$. Then the potential is indeed stabilized by the quartic terms, except along the D-flat directions $|H_1| = |H_2|$. It is then required that the quadratic part of $V$ be positive along these directions:

$$m_1^2 + m_2^2 - 2|m_3^2| > 0 \ . \qquad \text{[Potential bounded from below]} \qquad (5.24)$$

This condition applies *only* to the MSSM and corresponds to the region of fig. 5.5 above the straight line tangent to the upper branch of the hyperbola, that it is a subset of the region "Saddle" and is labelled by "MSSM".

However, when SUSY is broken at a moderate low scale, the $\lambda_i$ couplings in the potential (5.20) also receive sizeable $\mathcal{O}(F^2/M^4)$ contributions, besides the $\mathcal{O}(g^2)$ ones. Hence, the condition (5.24) is no longer mandatory to avoid UFB directions, since the boundedness of the potential can be ensured by imposing appropriate conditions on the $\lambda_i$ parameters. This turns out in new possibilities for EWSB. In particular, both alternatives (5.22, 5.23) are now possible. This means that most of the $\{m_1^2, m_2^2\}$ plane can in principle be explored: only the region 'Min' is excluded. This gives rise to important applications to the phenomenology of this scenario. For example, the universal case $m_1^2 = m_2^2$ is now allowed, as well as the possibility of having both $m_1^2$ and $m_2^2$ negative (with $m_3^2$ playing a minor role). In addition, and unlike in the MSSM, there is no need of radiative corrections to destabilize the origin, and EW breaking generically takes places already at tree-level, which is just fine since the effects of the RG running are small as the cut-off scale is $M$.

Finally, the fact that quartic couplings are very different from those of the MSSM changes dramatically the Higgs spectrum and properties. In particular, the MSSM upper bound on the mass of the lightest Higgs field no longer applies, which has also an important and positive impact on the fine-tuning problem, as is clear from the discussion in the previous section.

### 5.2.2 Fine-tuning in a low-scale SUSY breaking example.

In this section a particular example with low-scale $\not{\text{SUSY}}$ is considered, evaluating numerically the fine-tuning involved in the EWSB and comparing it with that of the MSSM [36]. We choose a model first introduced in [56] and analyzed there for its own sake. We show now that the fine-tuning problem is greatly softened in this model even if it was not constructed with that goal in mind.

The superpotential is given by

$$W = \Lambda_S^2 T + \mu H_1 \cdot H_2 + \frac{\ell}{2M}(H_1 \cdot H_2)^2 \ , \tag{5.25}$$

and the Kähler potential is

$$\begin{aligned} K &= |T|^2 + |H_1|^2 + |H_2|^2 \\ &- \frac{\alpha_t}{4M^2}|T|^4 + \frac{\alpha_1}{M^2}|T|^2\left(|H_1|^2 + |H_2|^2\right) + \frac{e_1}{2M^2}\left(|H_1|^4 + |H_2|^4\right) \ . \end{aligned} \tag{5.26}$$

All parameters are real with $\alpha_t > 0$. Here $T$ is the singlet field responsible for the breaking of supersymmetry, $\Lambda_S$ is the $\not{\text{SUSY}}$ scale and $M$ the 'messenger' scale. The typical soft masses are $\sim \tilde{m} \equiv \Lambda_S^2/M$. In particular, the mass of the scalar component of $T$ is $\mathcal{O}(\tilde{m})$ and, after integrating this field out, the effective potential for $H_1$ and $H_2$ is a 2HDM, like (5.20), with very particular Higgs mass terms:

$$m_1^2 = m_2^2 = \mu^2 - \alpha_1 \tilde{m}^2 \ , \quad m_3^2 = 0 \ , \tag{5.27}$$

and Higgs quartic couplings like those of the MSSM plus contributions of order $\mu/M$ and $\tilde{m}^2/M^2$:

$$\begin{aligned} \lambda_1 = \lambda_2 &= \frac{1}{4}(g^2 + g'^2) + 2\alpha_1^2\frac{\tilde{m}^2}{M^2} \ , \\ \lambda_3 &= \frac{1}{4}(g^2 - g'^2) + \frac{2}{M^2}(\alpha_1^2\tilde{m}^2 - e_1\mu^2) \ , \\ \lambda_4 &= -\frac{1}{2}g^2 - 2\left(e_1 + 2\frac{\alpha_1^2}{\alpha_t}\right)\frac{\mu^2}{M^2} \ , \\ \lambda_5 &= 0 \ , \\ \lambda_6 = \lambda_7 &= \frac{\ell\mu}{M} \ . \end{aligned} \tag{5.28}$$

The symmetry of the potential under $H_1 \leftrightarrow H_2$ allows to solve the minimization conditions explicitly not only for $v$ but also for $\tan\beta$. Depending on the value of the parameter $l$, one gets either[3] $\tan\beta = 1$ or $\tan\beta > 1$. The explicit expressions for $v$ and $\sin 2\beta$, and the spectrum of Higgs masses, can be found in [56], but they are written here for completeness.

Concerning the value of $\tan\beta$, we have the two possible solutions

$$|\tan\beta| = 1 \ , \tag{5.29}$$

and

$$\sin 2\beta = \frac{\ell\mu/M}{(g^2+g'^2)/4 + 2\hat{e}_1\mu^2/M^2} \ , \tag{5.30}$$

where we use $\hat{e}_1 \equiv e_1 + \alpha_1^2/\alpha_t$

The explicit expressions for $v$ and the Higgs masses are the following.

$\underline{\tan\beta = 1}$:

$$\begin{aligned}
v^2 &= \frac{-2(m_1^2+m_3^2)}{(1/2)(\lambda_1+\lambda_3+\lambda_4+\lambda_5)+2\lambda_6} \ , \\
m_h^2 &= 2\left(\alpha_1^2\frac{\tilde{m}^2}{M^2} - \hat{e}_1\frac{\mu^2}{M^2} + \frac{\ell\mu}{M}\right)v^2 \ , \\
m_H^2 &= \left[\frac{1}{4}(g^2+g'^2) + 2\hat{e}_1\frac{\mu^2}{M^2} - \frac{\ell\mu}{M}\right]v^2 \ , \\
m_A^2 &= -\frac{\ell\mu}{M}v^2 \ , \\
m_{H^\pm}^2 &= \left[\frac{1}{4}g^2 + (2\hat{e}_1 - e_1)\frac{\mu^2}{M^2} - \frac{\ell\mu}{M}\right]v^2 \ ,
\end{aligned}$$

$\underline{\tan\beta \neq 1}$:

$$\begin{aligned}
v^2 &= \frac{2(\lambda_6 m_3^2 - (1/2)(-\lambda_1+\lambda_3+\lambda_4+\lambda_5)m_1^2}{(1/2)(-\lambda_1+\lambda_3+\lambda_4+\lambda_5)\lambda_1 - \lambda_6^2} \ , \\
m_h^2 &= \left[\frac{1}{4}(g^2+g'^2) + 2\alpha_1^2\frac{\tilde{m}^2}{M^2} + \frac{\ell\mu}{M}s_{2\beta}\right]v^2 \ , \\
m_H^2 &= -\left[\frac{1}{4}(g^2+g'^2) + 2\hat{e}_1\frac{\mu^2}{M^2}\right]v^2 c_{2\beta}^2 \ , \\
m_A^2 &= -\left[\frac{1}{4}(g^2+g'^2) + 2\hat{e}_1\frac{\mu^2}{M^2}\right]v^2 \ , \\
m_{H^\pm}^2 &= -\left(\frac{1}{4}g'^2 + e_1\frac{\mu^2}{M^2}\right)v^2 \ .
\end{aligned} \tag{5.31}$$

One important difference with respect to the MSSM spectrum is that all Higgs masses are now of order $v$. The CP-even scalars $h, H$ can be in the region accessible to LEP searches. Although the charged Higgs, $H^\pm$, and the pseudoscalar, $A^0$, are usually too heavy for detection at LEP, in some regions of parameter space they might also be light and their possible production must be considered too. Limits on the parameter space of this model that result from Higgs searches at LEP are discussed in Appendix B and will be explicitly shown later in the fine-tuning analysis.

[3]One has $\mathrm{sgn}(\tan\beta) = -\mathrm{sgn}(l\mu/M)$. We are implicitly taking parameters such that $\tan\beta > 0$.

To evaluate the fine-tuning in this model we simply plug (5.27) and (5.28) in the general expression for $\Delta_{\mu^2}$ given in Appendix A:2 [eq. (A.14)] to obtain

$$\Delta_{\mu^2} = -\frac{\mu^2}{\lambda v^2}\left[1 + v^2\left(\frac{l s_{2\beta}}{2\mu M} - \frac{1}{M^2}\hat{e}_1 s^2_{2\beta}\right)\right] \ , \tag{5.32}$$

where $\lambda$ is the quartic scalar coupling along the breaking direction, explicitly given in eq. (A.9) and $\hat{e}_1 \equiv e_1 + \alpha_1^2/\alpha_t$. This expression is valid both for $\tan\beta = 1$ and $\tan\beta > 1$ and is dominated by the first term. Equation (5.32) is a tree-level result, useful for understanding most of the parametric dependence of $\Delta_{\mu^2}$, but we use for the numerical comparison with the MSSM a one-loop-refined evaluation of $\Delta_{\mu^2}$ (both in the MSSM and the present model), computed following the procedure explained in Appendix A.2.

We should also comment on the relation between the coupling $\lambda$ along the breaking direction [which is the coupling relevant for (5.32)] and the Higgs mass. At tree-level one of the CP-even Higgses lies along the breaking direction and therefore has mass-squared $2\lambda v^2$, but this is no longer the case at one loop: radiative corrections induce a deviation in the direction of the mass eigenstates, the effect being larger for light tree level masses. We will use the notation $m^2_{\|} = 2\lambda v^2$ for the mass matrix element that controls the fine-tuning (5.32) keeping in mind that it does not always correspond to the mass of a physical state. Explicitly, in the region $\tan\beta > 1$ on which we focus here,

$$m^2_{\|} = \left[\frac{1}{4}(g^2 + g'^2) + 2\alpha_1^2\frac{\tilde{m}^2}{M^2} + \frac{l\mu}{M}s_{2\beta}\right]v^2 \ + \frac{3m_t^4}{2\pi^2 v^2}\log\frac{M^2_{\rm SUSY}}{m_t^2} + ... \tag{5.33}$$

where we have added the dominant one-loop stop correction, as in the MSSM.

For the values of the parameters of the unconventional model we take as a first example (set A) those used in [56]: $\mu/M = 0.6$, $e_1 = -1.3$, $\tilde{m}/M = 0.5$ and $\alpha_t = 3$. The exact value of $\alpha_1$ is fixed by the minimization condition for $v$: it is always $\alpha_1 \gtrsim \mu^2/\tilde{m}^2 = 1.44$, and gets closer and closer to $\mu^2/\tilde{m}^2$ for increasing $\tilde{m}$. The parameter $l$ is free and can be traded by $\tan\beta$. To understand some of the numerical results that follow, it is important to study the dependence of $m^2_{\|}$ on $\tilde{m}$ (for fixed $\tilde{m}/M$). Its tree-level part decreases monotonically with increasing $\tilde{m}$ due to the behaviour of $\alpha_1$, while the one-loop correction increases logarithmically with $\tilde{m}$ (it enters through $M^2_{\rm SUSY}$ which we take to be $M^2_{\rm SUSY} \simeq \tilde{m}^2 + m_t^2$). The combination of these two opposite effects results in a mass $m_{\|}$ that decreases with $\tilde{m}$ for small $\tilde{m}$ (where the tree-level dependence dominates), reaches a minimum, and then starts increasing again for larger $\tilde{m}$ (when the one-loop dependence takes over). For this reason, every value of $m_{\|}$ corresponds to two values of $\tilde{m}$: a low value, associated to a large tree level Higgs coupling and a small radiative effect, which has small fine-tuning; and a high value, associated to

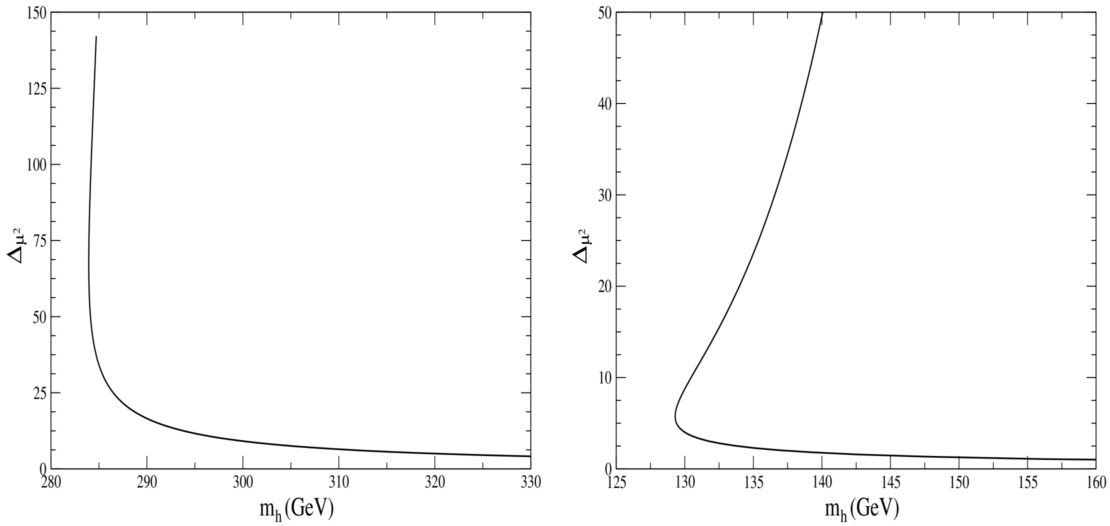



Figure 5.6: Fine-tuning in the unconventional SUSY scenario of section 5.2 as a function of the Higgs mass (in GeV) for $\tan\beta = 10$ and the rest of parameters as in set A (left) or as in set B (right).

a larger radiative effect, which has larger fine-tuning. This causes $\Delta_{\mu^2}$ to be a bi-valued function of $m_\parallel$. Moreover, for this set of parameters $m_\parallel$ is a good approximation to $m_h$.

This behaviour is shown in Figure 5.6, left plot, which is the equivalent of fig. 5.1, but for the unconventional scenario just introduced, with $\tan\beta = 10$. We can use the soft mass $\tilde{m}$ as a parameter along the curve plotted, with $\Delta_{\mu^2}$ growing for increasing $\tilde{m}$. In the large-$\tilde{m}$ range of this curve (its steep upper branch) radiative corrections dominate the Higgs mass and the behaviour of the fine-tuning is similar to that in the MSSM (i.e. it grows with increasing $m_h$). If we restrict our attention to the more interesting low-$\tilde{m}$ range (the lower branch of the curve), the contrast with the MSSM result is evident: now, the larger $m_h$ is, the smaller the tuning becomes and for $m_h \gtrsim 300$ one gets $\Delta_{\mu^2} < 10$. All this is the straightforward result of having a larger tree-level contribution to the Higgs mass. For the choice of parameters considered here the resulting Higgs mass is somewhat large, but we can easily choose different parameters in order to lower the Higgs mass without loosing the dramatic improvement in $\Delta_{\mu^2}$. This is shown on the right plot of fig. 5.6, which has (set B): $\mu/M = 0.3$, $e_1 = -2$, $\tilde{m}/M = 0.5$, $\alpha_t = 1$ and $\alpha_1 \gtrsim 0.36$. The bi-valuedness of $\Delta_{\mu^2}$ is more evident in this case.

We plot $\Delta_{\mu^2}$ vs. $m_h$ in fig. 5.7 to make even clearer the difference of behaviour with respect to the MSSM (see fig. 5.1). We take $\mu = 330$ GeV, $\tilde{m} = 550$ GeV, $e_1 = -2$, $\alpha_t = 1$, $l$ chosen to give $\tan\beta = 10$ and instead of fixing $\tilde{m}/M$ we vary it from 0.05 to 0.8. In this way we can study the effect on the fine-tuning of varying $\lambda$ when the low energy mass scales ($\mu$ and $\tilde{m}$) are kept fixed. When $\tilde{m}/M$ is small (and this implies that $\mu/M$ is also small), the

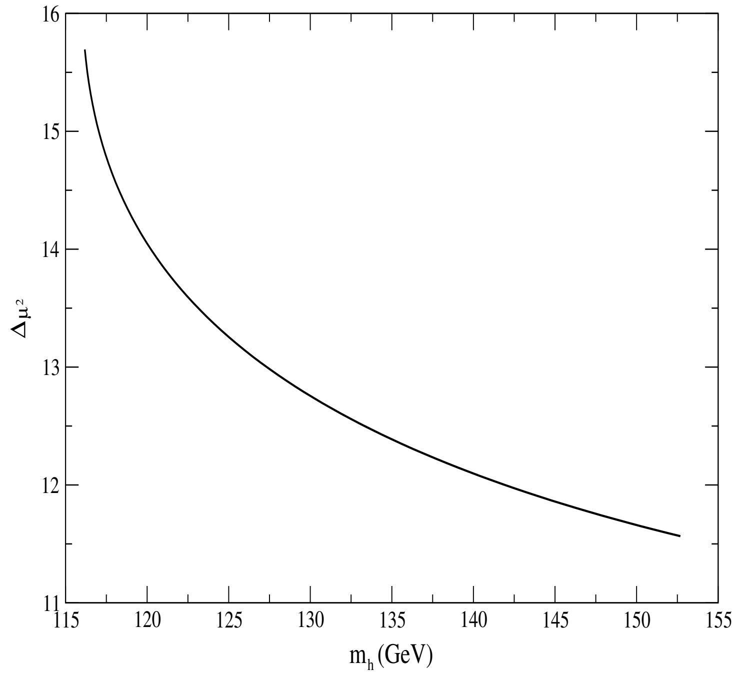
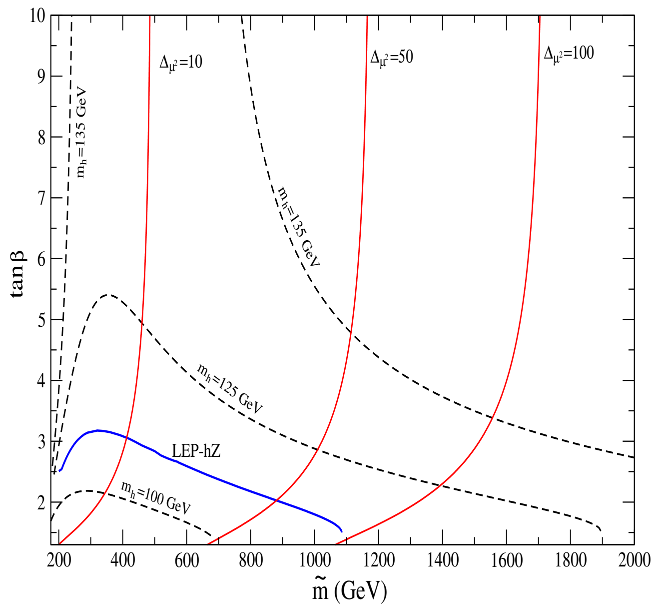



Figure 5.7: Fine-tuning in a low-scale SUSY breaking scenario as a function of the Higgs mass (in GeV) for $\tan\beta = 10$.

unconventional corrections to quartic couplings are not very important and the Higgs mass tends to its MSSM value[4]. As $\tilde{m}/M$ increases, the tree level Higgs mass (or $\lambda$) also grows and this makes $\Delta_{\mu^2}$ decrease with $m_h$, just the opposite of the MSSM behaviour.

Finally, fig. 5.8 is the version of fig. 5.3 for this unconventional model. The values of the parameters are those of set B. We show lines of constant $\Delta_{\mu^2}$ in the $(\tilde{m}, \tan\beta)$ plane, together with lines of constant $m_h$ (upper plot) and $m_H$ (lower plot). In each plot we also draw the experimental lower bound on the corresponding Higgs mass coming from LEP, either for Higgs-strahlung or associated production as indicated (see Appendix B). We find that the fine-tuning is larger for smaller $\tan\beta$ and larger $\tilde{m}$, as in the MSSM, but now the overall value of $\Delta_{\mu^2}$ is significantly smaller. From the figure we can estimate that for soft masses $\tilde{m}^2 \sim av^2$, the fine-tuning in this model (say near $m_h = 115$ GeV and $\tan\beta = 3$) is $\Delta \sim 3.5a$ instead of the $\sim 20a$ we found for the MSSM. The pattern of Higgs masses is also different and restricting the fine-tuning to be less than 10 does not impose an upper bound on the Higgs masses, in contrast with the MSSM case. As a result, the LEP bounds do not imply

[4]For the model at hand this limit is not realistic, as it implies too small (or even negative) values of $m_A^2$, $m_H^2$ and $m_{H^\pm}^2$. However, we are interested in the opposite limit, of sizeable $\tilde{m}/M$.



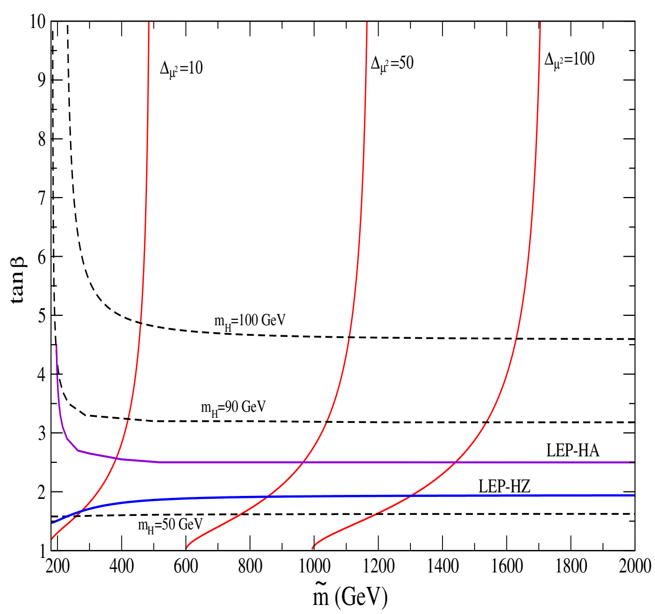




Figure 5.8: Fine-tuning in the $(\tilde{m}, \tan\beta)$ plane in a low-scale SUSY breaking scenario with parameters as in set B. Dashed lines are contour lines of constant $m_h$ (upper plot) or $m_H$ (lower plot). The LEP bound for each case is also shown.

a large fine-tuning: in the region with small $\tilde{m}$ and $\tan\beta$ not too close to 1,[5] we can get simultaneously Higgs masses large enough to escape LEP detection and small fine-tunings. In any case, following the line of $\Delta_{\mu^2} = 10$ we do find an upper bound $\tilde{m} \lesssim 500$ GeV, so that LHC would either find superpartners or revive a (LHC) fine-tuning problem for these scenarios (although the problem would be much softer than in the MSSM).

We can conclude in this section that we can achieve a dramatic improvement in the fine-tuning problem in models of low-scale $\not{\text{SUSY}}$ , and in this explicit model (studied in previous works by its own sake) this improvement occurs for any range of $\tan\beta$ and the Higgs mass (which can be as large as several hundred GeV if desired, but this is not necessary).

## 5.3 A peculiar SUSY scenario

We have seen how in generic SUSY models the usual fine-tuning bound $\Lambda \lesssim 2-3$ TeV holds, although in many cases the bound is more stringent due to finite and logarithmic contributions to $m^2$, which have no reason to cancel. However, it can be amusing to think of a scenario where, besides the cancellation of quadratic contributions, the other dangerous contributions also cancel or are absent, i.e. where eq. (2.21) vanishes.

If a non-accidental cancellation occurs in eq. (2.21), a plausible possibility seems to require universality, *i.e.* $m_b(h=0) \equiv \tilde{m}$, for all particles beyond the SM ones. This universality does not exactly coincide with the universality of the soft breaking terms usually invoked in MSSM analyses. The differences occur in the Higgs/higgsino sector: degeneracy of higgsinos with the other states requires adjusting also the $\mu$ parameter. Moreover, now the Higgs soft masses are not equal to the other soft masses; instead, they have to be adjusted so that one Higgs doublet (some combination of $H_1$ and $H_2$) is heavy and degenerate with the other states while the orthogonal combination is kept light and plays the role of the SM Higgs.

We will be interested in considering the possibility of $\tilde{m} \gg m_W$ even if this is against the usual naturalness argument for bounding the soft masses in the MSSM. In this universal case, eq. (2.21) reads

$$\delta_{\rm NP}^{\overline{\rm MS}}\, m^2 = \frac{1}{32\pi^2}\sum_b^{\rm NP} N_b \left[\tilde{m}^2\left(\log\frac{\tilde{m}^2}{Q^2}-1\right)\left.\frac{\partial^2 m_b^2}{\partial h^2}\right|_{h=0} + \log\frac{\tilde{m}^2}{Q^2}\left.\left(\frac{\partial m_b^2}{\partial h}\right)^2\right|_{h=0}\right] . \qquad (5.34)$$

The logarithmic and finite contributions in this expression have a clear interpretation in

[5]Besides the $\tan\beta > 1$ region we have explored, there is a wide region of parameter space with $\tan\beta = 1$ which is also experimentally allowed [56].

the language of effective field theories. The logarithms can be interpreted as coming from RG running beyond the scale $\tilde{m}$, while the finite contributions can be interpreted as threshold corrections coming from integrating out the physics at $\tilde{m}$. In fact this threshold correction is just $\delta_{\rm NP}^{\overline{\rm MS}}\ m^2$ evaluated at the scale $Q=\tilde{m}$. In this section we will require only that this threshold correction vanishes and therefore we disregard the possible effects from running beyond the scale $\tilde{m}$. (In other words, we are imposing the universality condition at $Q=\tilde{m}$.) Needless to say, such effects might spoil the electroweak hierarchy (unless $\tilde{m}$ is the cut-off scale in the fundamental theory, see below) but we are being conservative and only require that the NP particles at $\tilde{m}$ do not destabilize that hierarchy themselves. After setting then $Q=\tilde{m}$ we get

$$\delta_{\rm NP}^{\overline{\rm MS}}\ m^2(Q=\tilde{m}) = -\frac{\tilde{m}^2}{32\pi^2}\sum_b^{\rm NP} N_b\left.\frac{\partial^2 m_b^2}{\partial h^2}\right|_{h=0}\ . \tag{5.35}$$

Using now the fact that the theory is supersymmetric and there is a cancellation of quadratic divergences, we can use (2.20) to rewrite (5.35) in the form

$$\delta_{\rm NP}^{\overline{\rm MS}}\ m^2(Q=\tilde{m}) = \frac{\tilde{m}^2}{32\pi^2}\sum_a^{\rm SM} N_a\left.\frac{\partial^2 m_a^2}{\partial h^2}\right|_{h=0}\ . \tag{5.36}$$

This result is interesting because it involves only SM particles[6] and therefore the potentially large quantity $\delta_{\rm NP}^{\overline{\rm MS}}\ m^2(Q=\tilde{m})$ is proportional to the same combination of couplings that appears in Veltman's condition. In this very peculiar scenario then, imposing Veltman's condition would also work for the cancellation of the dangerous contributions of NP particles.

The first concern is that Veltman's condition can be satisfied in the SM by adjusting the unknown Higgs mass, while in the MSSM there are strong bounds on the latter. In fact, Veltman's "prediction", $m_h \simeq 313$ GeV seems to be hopelessly large for the MSSM. However, this not so: as we have seen, RG effects are important to evaluate a refined value of $m_h$ coming from Veltman's condition,

$$3g^2+g'^2+8\lambda-8\lambda_t^2=0\ , \tag{5.37}$$

which is supposed to hold at $\Lambda$. There are two competing effects. First, $\lambda_t$ gets smaller and smaller when the energy scale increases and consequently the predicted $\lambda(\Lambda)$ gets smaller for increasing $\Lambda$. Second, for a given $\lambda(\Lambda)$ a larger interval of running causes $\lambda$, and thus $m_h$, to be bigger at the low scale. The first effect turns out to win and $m_h$ decreases with

[6]It is not difficult to work out explicitly 5.35 to check 5.36. One has to make use of the SUSY relation $\lambda = (1/8)(g^2+g'^2)\cos^2 2\beta$.

increasing $\Lambda$ (see figure 2.2). In the peculiar SUSY scenario we are discussing, one should take these important effects into account to see whether Veltman's condition can be satisfied at some scale. Remarkably, the scale at which Veltman's condition holds [with $\lambda = (1/8)(g^2 + g'^2)\cos^2 2\beta$] turns out to be around the string scale ($\Lambda \simeq 10^{18}$ GeV for $\tan\beta \gg 1$ and $\Lambda \simeq 10^{25}$ GeV for $\tan\beta = 1$). Running $\lambda$ down in energy one obtains $m_h \simeq 140 - 150$ GeV as a further prediction of this model.

We consider this scenario, somewhat reminiscent of Split Supersymmetry [57], as a mere curiosity. The reasons that prevent us from taking it seriously are manifold: first, there is in principle no theoretical reason to expect that Veltman's condition should be satisfied, even though the couplings $g, g', \lambda$ and $\lambda_t$ can all be related to the string coupling, and for a particular string vacuum this could be the case. Second, the fulfillment of Veltman's condition at higher loop order is more difficult to justify or even impossible (which is a problem given the large value of $\tilde{m}$). Third, the condition $m_b^2(h=0) = \tilde{m}$ is equally difficult to justify theoretically, especially in the Higgs sector, which involves both SUSY and soft masses. Even generating $\mu$ through the Giudice-Masiero mechanism [58] requires tuning to achieve the desired universality. Finally, the MSSM relation $\lambda = 1/8(g^2 + g'^2)\cos^2 2\beta$ we have used to evaluate $\tilde{m}$ receives $\not{\text{SUSY}}$ contributions which (as discussed above) are important for $\sqrt{F} \sim M$, which is the case now. Moreover, the appropriate framework to study this problem is SUGRA rather than the conventional MSSM with global SUSY.

## 5.4 Conclusions

SUSY is the paradigmatic example of a theory in which the quadratically divergent radiative corrections of the SM are cancelled by those coming from new physics. The new particles introduced by SUSY (gauginos, sfermions and higgsinos) have masses of the order of the scale of the soft breaking terms, $\mathcal{O}(m_{\rm soft})$. Despite the cancellation of the quadratic corrections, logarithmic and finite contributions due to the new states lead to the usual (but conservative) fine-tuning upper bound $m_{\rm soft} \equiv \Lambda \lesssim 2 - 3$ TeV [remember sect.2.3].

In fact, according to the usual analyses, in the Minimal Supersymmetric Standard Model (MSSM) a successful electroweak breaking requires a substantial fine-tuning, giving a more stringent upper bound, namely $m_{\rm soft} \lesssim$ few hundred GeV [16, 19, 36]. Actually, the available experimental data already imply that typically the ordinary MSSM is fine-tuned at the few percent level.

The main reason for this abnormally acute tuning of the MSSM is the small magnitude of the tree-level Higgs quartic coupling $\lambda_{\rm MSSM} = \frac{1}{8}(g^2 + g'^2)\cos^2 2\beta \;\simeq\; \frac{1}{15}\cos^2 2\beta$. This has two effects:

- The "natural" value for the Higgs vev, $v^2 \sim m^2_{\rm soft}/\lambda$ tends to be much larger than $m^2_{\rm soft}$ due to the $\frac{1}{15}$ factor in $\lambda_{\rm MSSM}$ , specially if $\tan\beta$ is not large.
- Sizeable radiative corrections are needed to satisfy the experimental bound on $m_h$, which worsens the fine-tuning problem because sizeable radiative corrections require large soft terms. Since $m_h$ increases logarithmically with $m_{\rm soft}$, the problem gets exponentially worse for increasing $m_h$.

In addition, the radiative mechanism for EW breaking aggravates the problem, since it induces large coefficients for the individual contributions of certain soft terms to the effective potential.

As a consequence, the most efficient way of reducing the fine-tuning is to consider supersymmetric models where $\lambda_{\rm tree}$ is larger than in the MSSM. This happens in the Next-to-Minimal Supersymmetric SM (NMSSM), and also this can take place naturally in scenarios in which the breaking of SUSY occurs at a low scale (not far from the TeV scale) that we have studied. Then, the quartic couplings get $\not{\rm SUSY}$ corrections, $\delta\lambda \sim m^2_{\rm soft}/M^2$, so that $\lambda + \delta\lambda$ can be easily larger than $\lambda_{\rm MSSM}$, as desired to ameliorate the fine-tuning problem. Moreover, this opens up many new possibilities for EW breaking and for a non-conventional Higgs spectrum.

We demonstrate this in an explicit model of low-scale $\not{\rm SUSY}$ studied in a previous work by its own sake (and not with the goal of solving the fine-tuning problem). This indicates that the improvement in the fine-tuning is indeed a generic feature of these scenarios. By modifying the parameters of the model we achieve a dramatic improvement of the fine-tuning for any range of $\tan\beta$ and the Higgs mass (which can be as large as several hundred GeV if desired, but this is not necessary). It is in fact quite easy to get e.g. $\Delta < 5$ (i.e. no fine-tuning), in contrast with the MSSM values, $\Delta > 20$ (and much larger for $m_h > 115$ GeV and/or small $\tan\beta$).

In scenarios with low-scale $\not{\rm SUSY}$ , the interval of running of the soft parameters is small, which has further consequences:

- EW breaking takes place at tree-level, which, as discussed before, also helps in reducing the fine-tuning.

- The cross-talk (through RG running) between mass parameters in the Higgs sector and those of other sectors (squarks, gluinos, etc.) is drastically reduced. Then, the soft parameters can be (much) heavier than $M_Z$ without upsetting the naturalness of the electroweak scale. In this sense these scenarios represent an alternative to other options which try to reduce the fine-tuning by postulating correlations between different parameters to implement cancellations in $M_Z$: here $M_Z$ does not even depend strongly on those parameters.

Then, this low-scale $\not{\rm SUSY}$ scenario behaves better than MSSM from fine-tuning arguments, and it can saturate the general bound $m_{soft} \equiv \Lambda_{SM} \leq$ 2-3 TeV without fine-tuning problems. Besides the MSSM and the low-scale $\not{\rm SUSY}$ cases, we have considered a supersymmetric scenario where the cancellation of all the dangerous contributions takes places. In this peculiar scenario we have considered universality for all the particles beyond the SM ones with masses $\tilde{m} \gg m_W$. We have also imposed the universality condition at the scale $Q = \tilde{m}$ and $\lambda = (1/8)(g^2 + g'^2)\cos^2 2\beta$. With this ingredients, the SM Veltman's condition would also work for the cancellation of dangerous contributions of the new particles. The scale at which the Veltman's condition holds for this case turns out to be around the string scale ($\Lambda \simeq 10^{20}$ GeV). However, we should not take this scenario seriously, because the assumptions on which it is based are very difficult to justify.

# Chapter 6

# Little Higgs

Little Higgs models [8] are a recent development in the quest to solve the "Little Hierarchy" problem of the SM. The Higgs mass stabilization by making the Higgs a pseudo-Goldstone boson resulting from a spontaneously broken global symmetry is the main achievement of these models (this did not occur in the first scenarios attempting this idea [59]). In Little Higgs models the Higgs mass is protected at one-loop from quadratically divergent contributions. In principle, since the remaining quadratic corrections to the Higgs mass appear at the two-loop level, no fine-tuning should be required to keep the Higgs sufficiently light until a scale of $\sim 10$ TeV, avoiding the "Little Hierarchy" problem. Therefore, the "Big Hierarchy" problem can be postponed above 10 TeV, the cut-off scale beyond which these models need an ultraviolet (UV) completion.

In order to achive the one-loop cancellation, the mechanism of "collective breaking" is used. Little Higgs models are based on a non-linear sigma model, where the Higgs boson transforms as a Goldstone boson when a global symmetry, $G$, is broken to a subset, $H$, at a scale $f$ around 1 TeV. The group $G$ contains in turn a gauged subgroup $G_1 \times ... \times G_n$ (with $n \geq 2$). All the $G_i$ factors commute with a different subgroup of $G$. If only one of the $G_i$ factors is gauged, the Higgs transforms as a massless Goldstone boson because there is a remaining unbroken global symmetry. It is when the full $G_1 \times ... \times G_n$ group is gauged that the Higgs acquires a mass, with divergent contributions generated at the $n$-th loop:

$$\delta m_H^2 = \frac{g_1^2}{(4\pi)^2} ... \frac{g_n^2}{(4\pi)^2} \Lambda^2. \tag{6.1}$$

In this case, the SM appears when the extended $G_1 \times ... \times G_n$ group is broken at the scale $f$. These leads to new degrees of freedom at this scale $f \sim 1$ TeV. The divergent one-loop diagrams for the Higgs mass from each SM particle are cancelled by one-loop diagrams involving

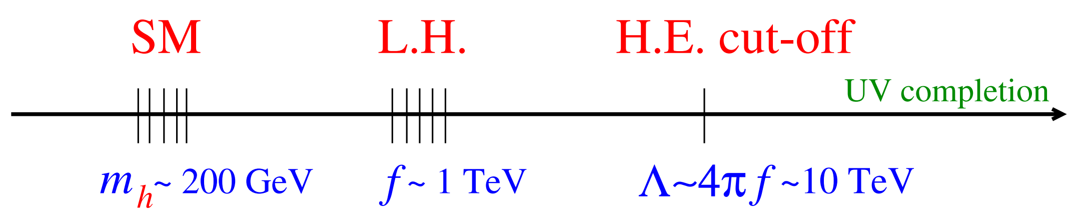



Figure 6.1: An illustration of the scales in the Little Higgs picture.

the new particles, with this cancellation taking place between particles of the same spin (the opposite of supersymmetric models). The interactions of these extra states can be described within perturbation theory, and detailed predictions of their properties can be made. They provide distinct signatures that can be searched for at LHC and induce calculable corrections to precision electroweak observables (often under control). This effective description is only valid up to an energy scale of order 10 TeV, beyond which Little Higgs models need to be replaced by a more fundamental theory, its "ultraviolet (UV) completion". Fig. 6.1 shows the generic spectrum of Little Higgs models.

Despite the good prospects, the absence of fine-tuning in a particular LH scenario should be checked in practice. More precisely, the fine-tuning must be computed for the different LH models with the same level of rigor employed for the supersymmetric models. We will focus on the naturalness of electroweak symmetry breaking (EWSB) in four popular and representative LH scenarios [61, 62, 63, 64]. We devote a section of this chapter to each of them.

## 6.1 The Littlest Higgs

### 6.1.1 Structure of the Littlest Higgs model

The first and most representative LH scenario is the "Littlest Higgs" [61]. It is one of the most economical and attractive implementations of the collective symmetry breaking mechanism. Most of the phenomenological studies up to date have been performed in the context of this model or its modifications. This section contains a review of the structure of this model and the fine-tuning analysis.

The Littlest Higgs model is a non-linear sigma model based on a global $SU(5)$ symmetry which is spontaneously broken to $SO(5)$ at a scale $f \sim 1$ TeV, and explicitly broken by the gauging of an $[SU(2) \times U(1)]^2$ subgroup. After the spontaneous breaking, the latter gets broken to its diagonal subgroup, identified with the SM electroweak gauge group. The spontaneous breaking of $SU(5)$ down to $SO(5)$ is produced by the vacuum expectation value of a $5 \times 5$ symmetric matrix field $\Phi$,

$$\langle \Phi \rangle = \Sigma_0 = \begin{pmatrix} 0 & 0 & I_2 \\ 0 & 1 & 0 \\ I_2 & 0 & 0 \end{pmatrix} . \tag{6.2}$$

This breaking of the global $SU(5)$ symmetry produces 14 Goldstone bosons that include the Higgs doublet field. These Goldstone bosons can be parametrized through the nonlinear sigma model field

$$\Sigma = e^{i\Pi/f} \Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f} \Sigma_0, \tag{6.3}$$

where $\Pi = \sum_a \Pi^a X^a$, with $X^a$ the broken $SU(5)$ generators and $\Pi_a$ the Goldstone boson fields. The model assumes a gauged $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ subgroup of $SU(5)$ with generators ($\sigma^a$ are the Pauli matrices)

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & 0_{2\times 3} \\ 0_{3\times 2} & 0_{3\times 3} \end{pmatrix} , \quad Q_2^a = \begin{pmatrix} 0_{3\times 3} & 0_{3\times 2} \\ 0_{2\times 3} & -\sigma^{a^*}/2 \end{pmatrix} , \tag{6.4}$$

and

$$Y_1 = \frac{1}{10}\mathrm{diag}(-3, -3, 2, 2, 2), \quad Y_2 = \frac{1}{10}\mathrm{diag}(-2, -2, -2, 3, 3). \tag{6.5}$$

The vacuum expectation value in eq. (6.2) breaks $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ down to the diagonal $SU(2) \times U(1)$, identified with the SM group.

The Goldstone and (pseudo)-Goldstone bosons in the hermitian matrix $\Pi$ in $\Sigma = e^{2i\Pi/f}\Sigma_0$ fall in representations of the SM group as

$$\Pi = \begin{pmatrix} \xi & \frac{H^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{H}{\sqrt{2}} & 0 & \frac{H^*}{\sqrt{2}} \\ \phi & \frac{H^T}{\sqrt{2}} & \xi^T \end{pmatrix} + \frac{1}{\sqrt{20}}\zeta^0 \mathrm{diag}(1,1,-4,1,1) \ , \tag{6.6}$$

where $H = (h^0, h^+)$ is the Higgs doublet; $\phi$ is a complex $SU(2)$ triplet given by the symmetric $2\times 2$ matrix:

$$\phi = \begin{bmatrix} \phi^0 & \frac{1}{\sqrt{2}}\phi^+ \\ \frac{1}{\sqrt{2}}\phi^+ & \phi^{++} \end{bmatrix} \ , \tag{6.7}$$

the field $\zeta^0$ is a singlet which is the Goldstone associated to the $U(1)_1 \times U(1)_2 \to U(1)_Y$ breaking and finally, $\xi$ is the real triplet of Goldstone bosons associated to $SU(2)_1 \times SU(2)_2 \to SU(2)$ breaking:

$$\xi = \frac{1}{2}\sigma^a\xi^a = \begin{bmatrix} \frac{1}{2}\xi^0 & \frac{1}{\sqrt{2}}\xi^+ \\ \frac{1}{\sqrt{2}}\xi^- & -\frac{1}{2}\xi^0 \end{bmatrix} \ . \tag{6.8}$$

All the fields in $\Pi$ as written above are canonically normalized.

The $[SU(2)\times U(1)]^2$ gauge interactions give a radiative mass to the SM Higgs, but only when the couplings of both groups are simultaneously present. Hence, the quadratically divergent contributions only appear at two-loop order, and the high-energy cut-off can be pushed up to a scale $\Lambda \sim 4\pi f \sim 10$ TeV. For the potentially dangerous fermions interactions (in particular top-Yukawa interactions), things work in a similar way: the spectrum is enlarged with two extra (left-handed and right-handed) fermions, and the conventional top-Yukawa coupling, $\lambda_t$, is not an input parameter but results from two independent couplings $\lambda_1, \lambda_2$. Both must be present in order to generate a radiative correction to the Higgs mass, and this again forbids quadratically divergent corrections to $m_h^2$ at one-loop. We are going to see how this happens by examining the lagrangian.

The relevant part of the lagrangian consists of two pieces

$$\mathcal{L} = \mathcal{L}_{\rm kin}(g_1, g_2, g_1', g_2') \ + \ \mathcal{L}_{\rm f}(\lambda_1, \lambda_2) \ , \tag{6.9}$$

where $g_2, g_1$ $(g_2', g_1')$ are the gauge couplings of the first (second) $SU(2)\times U(1)$ factor, and $\lambda_1, \lambda_2$ are the two independent fermionic couplings.

The kinetic part is

$$\mathcal{L}_{kin} = \frac{f^2}{8}\mathrm{T}r[(D_\mu\Sigma)(D^\mu\Sigma)^\dagger] \ , \tag{6.10}$$

where

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1}^{2} g_j W_j^a (Q_j^a\Sigma + \Sigma Q_j^{aT}) - i\sum_{j=1}^{2} g'_j B_y (Y_j\Sigma + \Sigma Y_j^T). \tag{6.11}$$

In this model, additional fermions are introduced in a vector-like coloured pair $t', t'^c$ to cancel the Higgs mass quadratic divergence from top loops (other Yukawa couplings are neglected). The relevant part of the lagrangian containing the top Yukawa coupling is given by

$$\mathcal{L}_\mathrm{f} = \frac{1}{2}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\chi_i\Sigma_{jx}\Sigma_{ky}u_3'^c + \lambda_2 f t' t'^c + h.c., \tag{6.12}$$

where $\chi_i = (t, b, t')$, indices $i, j, k$ run from 1 to 3 and $x, y$ from 4 to 5, and $\epsilon_{ijk}$ and $\epsilon_{xy}$ are the completely antisymmetric tensors of dimension 3 and 2, respectively.

We can consider now the symmetry breaking $[SU(2)\times U(1)]^2 \longrightarrow SU(2)_L \times U(1)_Y$. In the gauge sector, the $\Sigma_0$ vev gives mass to one linear combination of the two $SU(2)$ gauge bosons $W_1$ and $W_2$ and to one linear combination of the two $U(1)$ gauge bosons $B_1$ and $B_2$ as follows:

$$\begin{aligned} W'^a &= -\cos\theta W_1^a + \sin\theta W_2^a, & M_{W'} &= \frac{gf}{\sin 2\theta}, \\ B' &= -\cos\theta' B_1 + \sin\theta' B_2, & M_{B'} &= \frac{g' f}{\sqrt{5}\sin 2\theta'}, \end{aligned} \tag{6.13}$$

where we define the mixing angles in terms of the gauge couplings by

$$g = g_1\sin\theta = g_2\cos\theta, \qquad g' = g'_1\sin\theta' = g'_2\cos\theta'. \tag{6.14}$$

In the fermionic sector, inserting the $\Sigma_0$ vev, $t'$ marries a linear combination of $t'^c$ and $u_3'^c$ and gets a mass of order $f$,

$$M_T = f\sqrt{\lambda_1^2 + \lambda_2^2}. \tag{6.15}$$

The orthogonal linear combination becomes the right-handed top quark, and remains massless at this level. When EWSB takes places, the top quark will get a mass given by:

$$m_t = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}v. \tag{6.16}$$

Then, as a result of the $[SU(2)\times U(1)]^2 \longrightarrow SU(2)_L \times U(1)_Y$ breaking, the couplings are constrained by the relations with the SM couplings,

$$\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}\,, \qquad \frac{1}{g'^2} = \frac{1}{g_1'^2} + \frac{1}{g_2'^2}\,, \qquad \frac{2}{\lambda_t^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}\,, \tag{6.17}$$

where $g$ and $g'$ are the $SU(2)$ and $U(1)_Y$ gauge couplings, respectively, and $\lambda_t$ is the top Yukawa coupling.

As we have seen above, the lagrangian (6.9) gives $\mathcal{O}(f)$ masses to $W', B'$ and $T$. After EWSB these heavy masses have a non-trivial and involved dependence on the full non-linear field $\Sigma$, which contains the $H$ and $\phi$ fields. In particular, retaining only the dependence on $h \simeq \mathrm{Re}(h^0)\sqrt{2}$ we get

$$\begin{aligned} m^2_{W'}(h) &= M^2_{W'} + \mathcal{O}(h^2) = \frac{1}{4}(g_1^2+g_2^2)f^2 - \frac{1}{4}g^2h^2 + \mathcal{O}(h^4/f^2) \; , \\ m^2_{B'}(h) &= M^2_{B'} + \mathcal{O}(h^2) = \frac{1}{20}(g_1'^2+g_2'^2)f^2 - \frac{1}{4}g'^2h^2 + \mathcal{O}(h^4/f^2) \; , \\ m^2_{T}(h) &= M^2_{T} + \mathcal{O}(h^2) = (\lambda_1^2+\lambda_2^2)f^2 - \frac{1}{2}\lambda_t^2h^2 + \mathcal{O}(h^4/f^2) \; . \end{aligned} \tag{6.18}$$

At this level, $H$ and $\phi$ are massless, but they get massive radiatively. The simplest way to see this is by using the effective potential. Let us consider first the quadratically divergent contribution to the one-loop scalar potential, given by

$$V_1^{\mathrm{quad}} = \frac{1}{32\pi^2}\Lambda^2 \; \mathrm{Str}\mathcal{M}^2 \; , \tag{6.19}$$

where the supertrace Str counts degrees of freedom with a minus sign for fermions, and $\mathcal{M}^2$ is the (tree-level, field-dependent) mass-squared matrix. In our case, the previous formula reads

$$V_1^{\mathrm{quad}} = \frac{1}{32\pi^2}\Lambda^2 \; \left[6m_W^2 + 9m_{W'}^2 + 3m_Z^2 + 3m_{B'}^2 - 12(m_t^2+m_T^2)\right] \; . \tag{6.20}$$

By looking at the $h$-dependence of the masses above, it is easy to check that $V_1^{\mathrm{quad}}$ does not contain a mass term for $h$ (this will be generated by the logarithmic and finite contributions to the potential, to be discussed shortly). The reason for this result is the following. If $\lambda_1 = g_2 = g_2' = 0$, the lagrangian (6.9) recovers a global $SU(3)$ [$SU(3)_1$, living in the upper corner of $SU(5)$] that protects the mass of the Higgs (which transforms by a shift under that symmetry). On the other hand, if $\lambda_2 = g_1 = g_1' = 0$, then a different $SU(3)$ symmetry [$SU(3)_2$, living in the lower corner of $SU(5)$] is recovered that also protects the Higgs mass. A non-zero value for the Higgs mass can only be generated by breaking both $SU(3)$'s and therefore both type-1 and type-2 couplings should be present. Quadratically divergent diagrams involve only one type of coupling and therefore cannot contribute to the Higgs mass. This is the so-called collective breaking (discussed in the introduction of this chapter) of the original $SU(5)$ symmetry and is one of the main ingredients of Little Higgs models.

These symmetries do not protect the mass of the triplet. In fact, if we include the full dependence of the bosonic $(W', B')$ and fermionic $(T)$ masses on the $\Sigma$ field, $V_1^{\rm quad}$ contains operators, $\mathcal{O}_V(\Sigma)$ and $\mathcal{O}_F(\Sigma)$ respectively, that produce a mass term for the triplet $\phi$ of order $\Lambda^2/(16\pi^2) \sim f^2$ and also a quartic coupling for $h$. These operators are:

$$\mathcal{O}_V(\Sigma) = f^4 \sum_{i=1,2} g_i^2 \sum_a {\rm Tr}[(Q_i^a\Sigma)(Q_i^a\Sigma)^*] + f^4 \sum_{i=1,2} g_i'^2 {\rm Tr}[(Y_i\Sigma)(Y_i\Sigma)^*], \qquad (6.21)$$

$$\mathcal{O}_F(\Sigma) = -f^4 \frac{\lambda_1^2}{8} \epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz} \, . \qquad (6.22)$$

These operators respect all the symmetries of the theory. Then, following [61], it is reasonable to assume that $\mathcal{O}_V(\Sigma)$ and $\mathcal{O}_F(\Sigma)$ are already present at tree-level, as a remnant of the heavy physics integrated out at $\Lambda$ (a threshold effect). Hence, these effects can be accounted for by adding an extra piece to the lagrangian,

$$-\Delta\mathcal{L} = c\ \mathcal{O}_V(\Sigma)\ +\ c'\ \mathcal{O}_F(\Sigma)\ , \qquad (6.23)$$

where $c$ and $c'$ are unknown coefficients. For future use, it is convenient to discuss here what is the natural size of $c$ and $c'$. Naive dimensional analysis [66] leads one to expect $c, c' \sim \mathcal{O}(1)$. However, we can make a more precise estimate by evaluating the one-loop contributions to $c$ and $c'$ coming from (6.20) keeping the full dependence on $\Sigma$. Then we get

$$\begin{aligned} c &= c_0 + c_1 = c_0 + 3/4 \, , \\ c' &= c'_0 + c'_1 = c'_0 + 24 \, . \end{aligned} \qquad (6.24)$$

where the subindex 0 labels the unknown threshold contributions from the physics beyond $\Lambda$.

Besides giving a mass to $\phi$, the operators in eq. (6.23) produce a coupling $\sim h^2\phi$ and a quartic coupling for $h$. The $h^2\phi$ coupling induces a tadpole for $\phi$ after EWSB. Keeping the vev of $\phi$ small enough is a necessary requirement to obtain an acceptable model and we ensure that this is the case in our numerical analysis. Then, it is a good approximation to neglect the effect of that small vev. Another effect of the $h^2\phi$ coupling is the modification of the quartic coupling once the heavy triplet is integrated out. After that is done, the Higgs quartic coupling $\lambda$ can be written in the simplest manner as

$$\frac{1}{\lambda} = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} \, , \qquad (6.25)$$

with $\lambda_a \equiv c(g_2^2 + g_2'^2) - c'\lambda_1^2$ and $\lambda_b \equiv c(g_1^2 + g_1'^2)$. We see that the structure of these scalar couplings is very similar to that in the fermion and gauge boson sectors, with $\lambda_a$ ($\lambda_b$) being a type-1 (type-2) coupling.

In order to write the one-loop Higgs potential, we need explicit expressions for the $h$-dependent masses. In the scalar sector, we decompose $h^0 \equiv (h^{0r} + ih^{0i})/\sqrt{2}$ and $\phi^0 \equiv i(\phi^{0r} + i\phi^{0i})/\sqrt{2}$. In the $CP$-even sector we write the relevant part of the mass matrix in the basis $\{h^{0r}, \phi^{0r}\}$; in the $CP$-odd sector we use the basis $\{h^{0i}, \phi^{0i}\}$ and finally, in the charged sector the basis $\{h^+, \phi^+\}$. The three mass matrices are very similar in structure and can be written simultaneously as[1]

$$M_\kappa^2(h) = \begin{bmatrix} \frac{1}{4}a_\kappa\lambda_+ h^2 + \frac{1}{\sqrt{2}}s_\kappa\lambda_- ft + \mathcal{O}(h^4/f^2) & b_\kappa\lambda_- fh + \mathcal{O}(h^2) \\ b_\kappa^*\lambda_- fh + \mathcal{O}(h^2) & \lambda_+(f^2 - c_\kappa h^2) + \mathcal{O}(h^4/f^2) \end{bmatrix}, \tag{6.26}$$

where the index $\kappa = \{0r, 0i, +\}$ labels the different sectors, and $a_\kappa = \{3, 1, 1\}$, $s_\kappa = \{1, -1, 0\}$, $b_\kappa = \{1/\sqrt{2}, 1/\sqrt{2}, i/2\}$ and $c_\kappa = |b_\kappa|^2$, and we have defined $\lambda_+ \equiv \lambda_a + \lambda_b$, $\lambda_- \equiv \lambda_a - \lambda_b$. We have also included the contribution of the triplet vev, $t \equiv \langle \phi^{0r} \rangle$, with

$$t \simeq -\frac{1}{2\sqrt{2}}\frac{\lambda_- h^2}{\lambda_+ f}, \tag{6.27}$$

to the mass matrices. The off-diagonal entries in (6.26) are due to the $h^2\phi$ coupling and they cause mixing between $h$ and $\phi$. Concerning the masses, the effect of this mixing is negligible for the triplet [at order $h^2$ in the masses, the components $\phi^{0r}$ and $\phi^{0i}$ can still be combined in a complex field $\phi^0$]. We will call $h'^{0r}$, $h'^{0i}$ and $h'^+$ the light mass eigenstates of (6.26) in the different sectors.

The explicit masses for the different components of the triplet field are then

$$\begin{bmatrix} m^2_{\phi^0}(h) \\ m^2_{\phi^+}(h) \\ m^2_{\phi^{++}}(h) \end{bmatrix} = M_\phi^2 + \mathcal{O}(h^2) = (\lambda_a + \lambda_b)f^2 - \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \lambda h^2 + \mathcal{O}(h^4/f^2), \tag{6.28}$$

while for the light states we get

$$\begin{bmatrix} m^2_{h'^{0r}}(h) \\ m^2_{h'^{0i}}(h) \\ m^2_{h'^+}(h) \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \lambda h^2 + \mathcal{O}(h^4/f^2). \tag{6.29}$$

From the previous expressions it is straightforward to check that, in the contribution of scalars to $V_1^{quad}$,

$$\frac{\Lambda^2}{32\pi^2}\left(m^2_{h'^{0r}} + m^2_{h'^{0i}} + 2m^2_{h'^+} + 2m^2_{\phi^0} + 2m^2_{\phi^+} + 2m^2_{\phi^{++}}\right), \tag{6.30}$$

[1] At this point there is no tree-level mass term for the Higgs field but the presence of a quartic coupling gives it a nonzero mass in a background $h$.

there is also a cancellation of $h^2$ terms. This is due to the fact that the operators (6.21) and (6.22) still respect the same $SU(3)_i$ symmetries as the original lagrangian (6.9).

Finally, a non-vanishing mass parameter for $h$ arises from the logarithmic and finite contributions to the effective potential. In the $\overline{\rm MS}$ scheme, setting the renormalization scale $Q = \Lambda$,

$$\begin{aligned} m^2 &= \frac{3}{64\pi^2}\left\{3g^2 M_{W'}^2\left[\log\frac{\Lambda^2}{M_{W'}^2}+\frac{1}{3}\right]+g'^2 M_{B'}^2\left[\log\frac{\Lambda^2}{M_{B'}^2}+\frac{1}{3}\right]\right\} \\ &+ \frac{3\lambda}{8\pi^2}M_\phi^2\left[\log\frac{\Lambda^2}{M_\phi^2}+1\right]-\frac{3\lambda_t^2}{8\pi^2}M_T^2\left[\log\frac{\Lambda^2}{M_T^2}+1\right] , \end{aligned} \tag{6.31}$$

where we have included the contribution from the $\phi$ masses.

It is convenient to write the effective potential of the Higgs field in the SM-like form

$$V = \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 \tag{6.32}$$

where $\lambda$ and $m^2$ are given by eqs. (6.25) and (6.31). The Higgs vev is simply:

$$v^2 \simeq \frac{-m^2}{\lambda} . \tag{6.33}$$

### 6.1.2 Fine-tuning analysis in the Littlest Higgs

A rough estimate of the fine-tuning associated to electroweak breaking in the Littlest Higgs model can be obtained from eq. (6.31). The contribution of the heavy top, $T$, to the Higgs mass parameter is

$$\delta_T m^2 = -\frac{3\lambda_t^2}{8\pi^2}M_T^2\left[\log\frac{\Lambda^2}{M_T^2}+1\right] . \tag{6.34}$$

Using eqs. (6.17, 6.18), it turns out that $M_T^2 \geq 2\lambda_t^2 f^2$, and thus $\delta_T m^2 \geq 0.37 f^2$ (the minimum corresponds to $\lambda_1 = \lambda_2 = \lambda_t$). Thus the ratio $\delta_T m^2/m^2$, tends to be quite large: *e.g.* for $f = 1$ TeV and $m_h = 150$ (250) GeV, $\delta_T m^2/m^2 \geq 33$ (12). Since there are other potential sources of fine-tuning, this should be considered as a lower bound on the total fine-tuning. Actually, the overall fine-tuning is usually much larger than this estimate, as we show below.

In order to perform a complete fine-tuning analysis [27] we determine first the independent parameters, $p_i$, and then calculate the associated fine-tuning parameters, $\Delta_{p_i}$, according to eq. (2.7), i.e. $\Delta_{p_i} = (p_i/v^2)(\partial v^2/\partial p_i)$. For the Littlest Higgs model the input parameters of the lagrangian [eqs. (6.9, 6.23)] are

$$p_i = \{g_1,\ g_2,\ g_1',\ g_2',\ \lambda_1,\ \lambda_2,\ c,\ c',\ f\} . \tag{6.35}$$

We have not included $\Lambda$ among these parameters since we are assuming $\Lambda \simeq 4\pi f$. In any case, this assumption reduces the amount of fine-tuning, so it is a conservative one. On the other hand, the parameter $f$ basically appears as a multiplicative factor in the mass parameter, $m^2$, so $\Delta_f$ is always $\mathcal{O}(1)$, and can be ignored. Finally, the above parameters are constrained by the measured values of the top mass and the gauge couplings $g, g'$, according to eq. (6.17). The procedure to estimate the fine-tuning in the presence of constraints is discussed in Appendix C. The net effect is a reduction of the "unconstrained" total fine-tuning, $\Delta = (\sum_i \Delta_{p_i}^2)^{1/2}$, according to eq. (C.7). In this particular case, that equation gives

$$\Delta = \left[ \sum_i \Delta_{p_i}^2 - \sum_\alpha \frac{1}{N_\alpha^2} \left( \sum_i p_i \frac{\partial G_\alpha^{(0)}}{\partial p_i} \Delta_{p_i} \right)^2 \right]^{1/2} , \tag{6.36}$$

where $G_\alpha^{(0)} = g^2, g'^2, \lambda_t^2$ are functions of the $p_i$ as given in eq. (6.17), and

$$N_\alpha^2 \equiv \sum_i p_i^2 \left( \frac{\partial G_\alpha^{(0)}}{\partial p_i} \right)^2 , \tag{6.37}$$

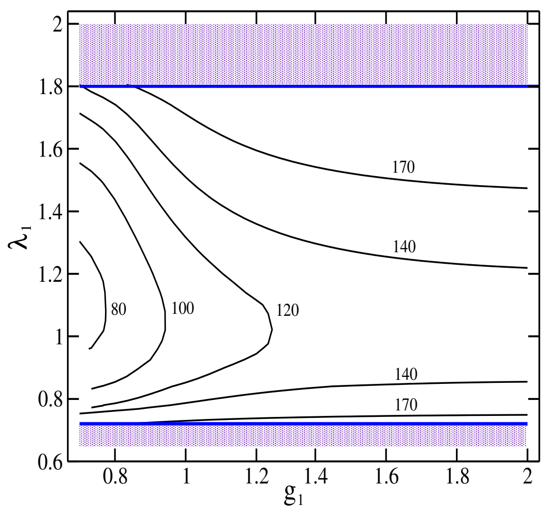
are normalization constants.

As announced before, $\Delta$ is in general much larger than our initial rough estimate, although the precise magnitude depends strongly on the region of the parameter space considered and decreases significantly as $m_h$ increases. Let us discuss how this comes about. The negative contribution from $M_T^2$ to $m^2$ in eq. (6.31) must be compensated by the other positive contributions. Typically, this implies a large value of the triplet mass, $M_\phi^2 = (\lambda_a + \lambda_b) f^2$, which implies a large value of $(\lambda_a + \lambda_b)$, but keeping $1/\lambda = 1/\lambda_a + 1/\lambda_b$ in the phenomenological range. There are two ways of achieving this[2]:

$$\begin{aligned} &\text{a)} \qquad \lambda \simeq \lambda_b \ll \lambda_a \simeq M_\phi^2/f^2 , \\ &\text{b)} \qquad \lambda \simeq \lambda_a \ll \lambda_b \simeq M_\phi^2/f^2 . \end{aligned} \tag{6.38}$$

Notice that the one-loop $m^2$ is a symmetric function of $\lambda_a$ and $\lambda_b$, so cases a) and b) are simply related by $\lambda_a \leftrightarrow \lambda_b$. This means that the triplet and Higgs masses are exactly the same in both cases although the fine-tuning may be different (since the dependence of $\lambda_{a,b}$ on $p_i$ is not the same), and indeed it is, as we discuss next.

[2]The existence of two separate regions of solutions can be also noticed from the fact that, for given values of $\lambda, \lambda_1, g_1, g_1'$, and in the approximation $\log(\Lambda^2/M_\phi^2) \simeq$ const., the minimization condition (6.33) becomes quadratic in $c$.



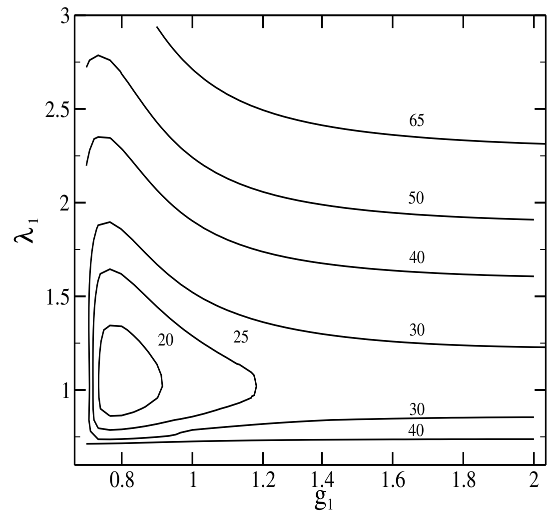




Figure 6.2: Preliminary fine-tuning contours for the Littlest Higgs model, case a) of eq. (6.38), for two different values of the Higgs mass: $m_h = 115$ GeV (left) and $m_h = 250$ GeV (right).

For case a), the value of $\Delta$ is shown by the contour plots of fig. 6.2 which correspond to two different values of the Higgs mass. We present our results in the plane $\{g_1,\lambda_1\}$. In each point of this plane $g_2$ and $\lambda_2$ are fixed by eq. (6.17), and the values of $c$ and $c'$ are fixed by the minimization condition for electroweak breaking and the choice of the Higgs mass. The value of $g'_1$ has been taken at $g'^2_1 = g'^2_2 = g'^2/2$, which nearly minimizes the fine-tuning. (Note also that $g_1 \geq g$ and thus smaller values of $\Delta$ cannot be reached by lowering $g_1$ in fig. 6.2). The shaded areas correspond to regions where there is not a correct electroweak symmetry breaking (in these regions, $M_\Phi \geq \Lambda$, which besides being beyond the range of validity of the effective theory, makes negative the triplet contribution to $m^2$. These plots illustrate the large size of $\Delta$, which is significantly larger than the previous rough estimate. This is not surprising since, as stated before, besides the heavy top contribution to $m^2$ (on which the estimate was based), there are other contributions that depend in various ways on the different independent parameters. This gives additional contributions to the total fine-tuning, increasing its value. The plots also show how $\Delta$ decreases for increasing $m_h$. This is due to the fact that the larger $m_h$, and thus $\lambda$, the larger the required value of $m^2$ in (6.33), which reduces the level of cancellation needed between the various contributions to $m^2$ [36, 15]. Although the fine-tuning is substantial, it could be considered as tolerable [i.e.

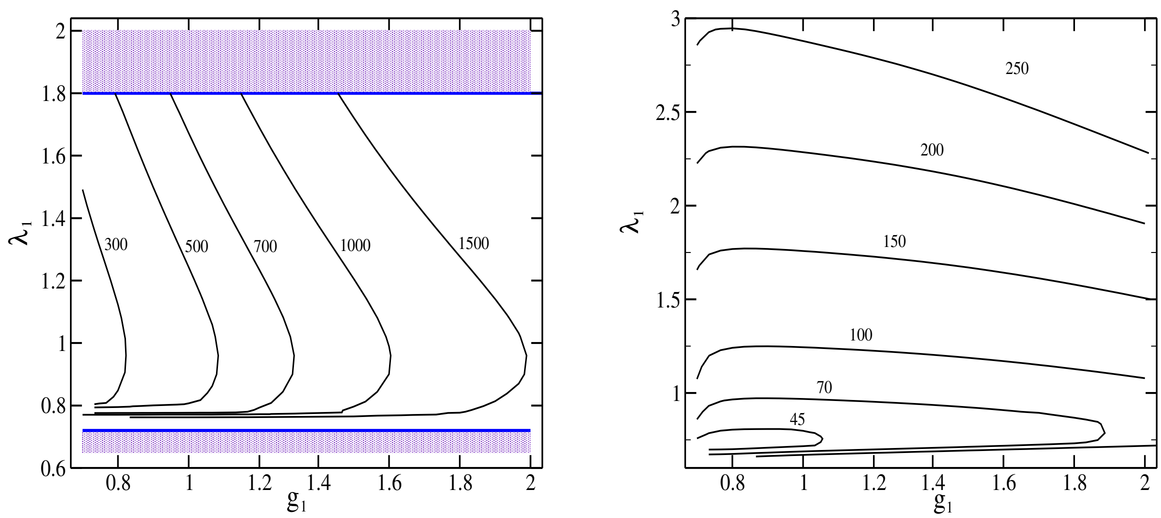



Figure 6.3: Same as fig. 6.2 but using $c_0$ and $c_0'$ of eq. (6.24) as unknown parameters.

$\mathcal{O}(10)$], for some (small) regions of the parameter space (at least for large $m_h$). However, on closer examination the fine-tuning turns out to be larger than shown by fig. 6.2. From the condition a) in (6.38),

$$\lambda \simeq c(g_1^2 + g_1'^2) = \lambda_b \ll \lambda_a = c(g_2^2 + g_2'^2) - c'\lambda_1^2, \tag{6.39}$$

it is clear that in this case $c'$ is large (and negative), while $c$ is small. But then, eq. (6.24) shows that there is an implicit tuning between $c_0$ and $c_1$ to get the small value of $c$. This effect can be taken into account by using $\{c_0,\ c_0'\}$, rather than $\{c,\ c'\}$ as the independent unknown parameters appearing in (6.35). Since $\Delta_{c_0} = |(c_0/c)\Delta_c|$ (and similarly for $\Delta_{c_0'}$), the global fine-tuning becomes much larger. This is illustrated in fig. 6.3, where $\Delta$ is systematically above $\mathcal{O}(10)$, even for large $m_h$.

There is a simple way of understanding the order of magnitude of $\Delta$. We can repeat the rough argument at the beginning of this subsection, but considering now the contribution of the triplet to the Higgs mass parameter in (6.31). More precisely, since $M_\phi^2 = (\lambda_a + \lambda_b)f^2 = [c(g_1^2 + g_1'^2 + g_2^2 + g_2'^2) - c'\lambda_1^2]f^2$, we can focus on the contribution proportional to $c'$:

$$\delta_{c'} m^2 = -\frac{3\lambda}{8\pi^2} c'\lambda_1^2 f^2 \left[\log\frac{\Lambda^2}{M_\phi^2} + 1\right] . \tag{6.40}$$

Now, $c'$ itself contains a radiative piece $c_1' = -24$ [see eq. (6.24)], whose relative contribution

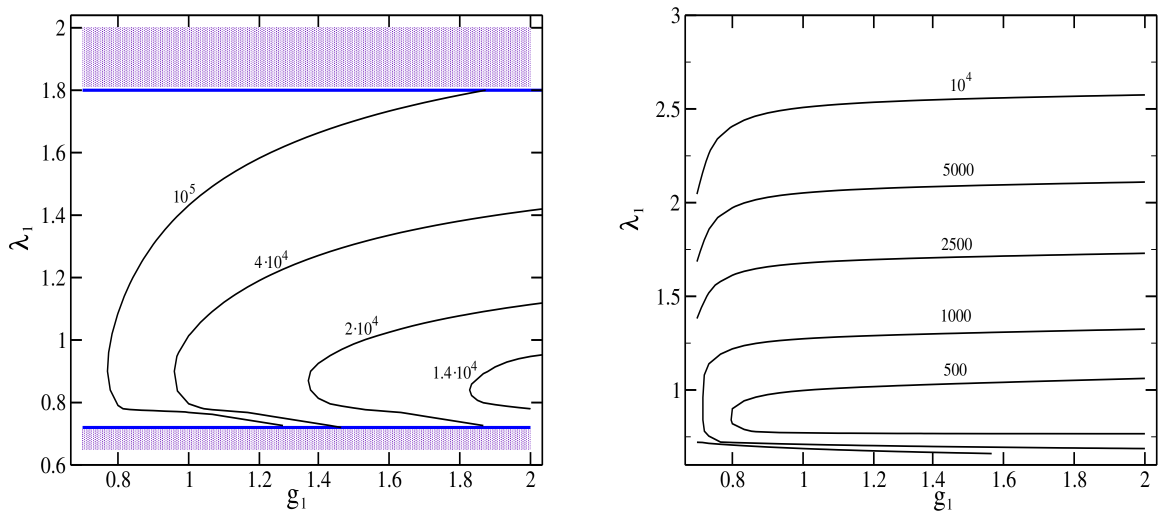



Figure 6.4: Fine-tuning contours, using $c_0$ and $c_0'$ of eq. (6.24) as unknown parameters, for the Littlest Higgs model, case b) of eq. (6.38), for $m_h = 115$ GeV (left) and $m_h = 250$ GeV (right).

to $m^2$ is then given by

$$\left|\frac{\delta_{c_1'} m^2}{m^2}\right| \geq \frac{9}{2\pi^2}\lambda_t^2 \frac{f^2}{v^2}\left[\log\frac{\Lambda^2}{M_\phi^2}+1\right] \simeq 45 \ , \tag{6.41}$$

where we have first used $\lambda_1^2 \geq \lambda_t^2/2$ and then $M_\phi \sim f$. Hence we easily expect $\mathcal{O}(100)$ contributions to $\Delta$, as reflected in fig. 6.3.

It is interesting to note that this rough argument holds even if there are additional contributions to $m^2$, since it is based on the size of contributions that are present anyway. In particular, two-loop corrections or 'tree-level' (i.e. threshold) corrections to $m^2$ are not likely to help in improving the fine-tuning. Of course, it might happen that they have just the right size to cancel the known large contributions, such as those of eqs. (6.34) and (6.41). However, in the absence of a theoretical argument for that cancellation, this possibility can only be understood a priori as a fortunate accident. The chances for the latter are precisely what the fine-tuning analysis evaluates.

For case b) in eq. (6.38) things are much worse, as illustrated in the contour plots of fig. 6.4, that show huge values of $\Delta$. The reason is the following: in case b), both $c$ and $c'$ are sizeable, so there is no implicit tuning between $c_0$ ($c_0'$) and $c_1$ ($c_1'$), but this implies a cancellation to get $\lambda_a \simeq \lambda$, which requires a delicate tuning. This "hidden fine-tuning"

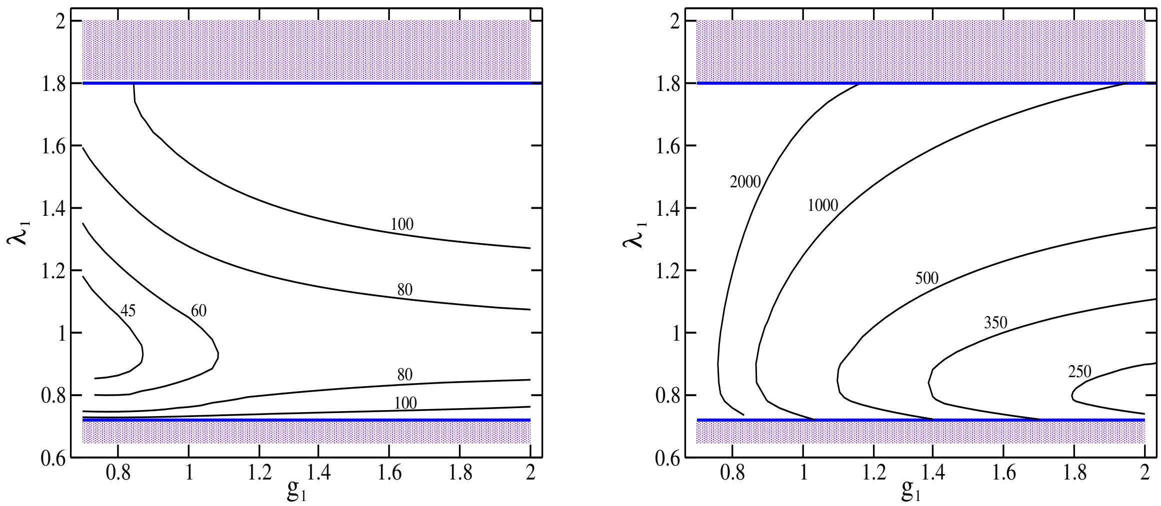



Figure 6.5: Fine-tuning contours for the Littlest Higgs model, case b) in eq. (6.38), with fixed $\lambda$ (left); and fine-tuning associated to $\lambda$ itself (right). The Higgs mass is $m_h = 115$ GeV.

is responsible for the unexpectedly large values of $\Delta$. In other words, small changes in the independent parameters of the model produce large changes in the value of $\lambda$, and thus in the value of $v^2$.

Now, imagine some future time after the Higgs mass has already been measured so that the parameter $\lambda$ takes a particular value and the other parameters of the model can only be varied in such a way that $\lambda$ remains constant. Then, according to the above discussion, the fine-tuning for case b) should be dramatically reduced and, apparently, this is exactly what happens. The condition of constant $\lambda$ can be incorporated in the computation of $\Delta$ using eq. (C.7) with an additional constraint $G_4^{(0)} = \lambda$ [3]. The new "constrained" fine-tuning in case b) (for $m_h = 115$ GeV), is shown in the left plot of fig. 6.5, to be compared with the left plot of fig. 6.4. Although still sizeable, the fine-tuning is now much smaller.

However, this behaviour does not alleviate the fine-tuning problems. If the Higgs mass is measured, one can also consider what is the fine-tuning between the independent parameters of the model to produce such value of $m_h$, in the same way that one examines the fine-tuning to produce the measured value of $v^2$. Let us denote the fine-tuning in $m_h^2$ (or equivalently in

[3]The constraint $G_4^{(0)} = \lambda$ is not independent of the others (for $g^2$, $g'^2$ and $\lambda_t$). A Gramm-Schmidt orthonormalization of the different constraints is enough to deal with this complication (see Appendix C).

$\lambda$) associated to a parameter $p_i$ by $\Delta_{p_i}^{(\lambda)}$. It is given by

$$\frac{\delta m_h^2}{m_h^2} = \frac{\delta\lambda}{\lambda} = \Delta_{p_i}^{(\lambda)} \frac{\delta p_i}{p_i} \ . \tag{6.42}$$

If $\Delta^{(\lambda)} > \mathcal{O}(1)$, this fine-tuning must be taken into account. Then, since $\Delta$ and $\Delta^{(\lambda)}$ represent independent inverse probabilities, they should be multiplied to estimate the total fine-tuning in the model. The right plot in fig. 6.5 shows that the values of $\Delta^{(\lambda)}$ are certainly quite large, as expected. Hence the product $\Delta \cdot \Delta^{(\lambda)}$ is very large, comparable to the values of $\Delta$ before the measurement of $m_h$.

The final conclusion is that the "standard" Littlest Higgs model has built-in a significant fine-tuning problem, especially for $m_h < 250$ GeV, even if other problems with electroweak observables are ignored. In this range the fine-tuning is typically $\Delta \gtrsim \mathcal{O}(100)$, i.e. essentially of the same order (or higher) than that of the Little Hierarchy problem of the SM [see fig. 2.8] and more severe than the MSSM one. For larger values of $m_h$, which is not so attractive from the point of view of fits to electroweak observables [67], the situation is better, although still $\Delta > 10$.

Let us finish this subsection with two additional comments. First, notice that the plots presented correspond to $f = 1$ TeV, which is a desirable and standard value in Little Higgs models. For other values of $f$, the parametric dependence of the fine-tuning is $\Delta \propto f^2$. In fact, precision electroweak observables in the Littlest Higgs model require larger values of the masses of the new particles and therefore of $f$ [68], which makes the fine-tuning even more severe. The second comment concerns perturbativity. We have just seen that a large value of $c'$ [and also $c$ for region b) in eq. (6.38)] is generically required for a correct electroweak breaking. Actually, from eq. (6.24), it seems indeed natural to expect large values of $c'$, which might be a problem for perturbativity. One way of obtaining a smaller value of $c'$ would be to lower $\Lambda$, making it smaller than $4\pi f$, which reduces the low-energy radiative contribution to $c'$. In fact, it is known [69] that chiral perturbation theory as a low energy description of technicolor theories with a large number of technifermions, $N$, breaks down at the scale $4\pi f/\sqrt{N}$. In the Littlest Higgs model we do have a large number of degrees of freedom, so, the low-energy effective theory would not be reliable all the way up to $4\pi f$. Conversely, if one insists in keeping $\Lambda \simeq 4\pi f/\sqrt{N} \simeq 10$ TeV to solve the Little Hierarchy problem, one would need $f$ larger than 1 TeV. This would help with the fits to precision electroweak measurements but would worsen significantly the fine-tuning.

## 6.2 A Modified Littlest Higgs Model [62]

### 6.2.1 Structure of the Modified Littlest Higgs

This model is also based on the $SU(5)/SO(5)$ Littlest Higgs [61], but modified [62] in such a way that only one abelian $U(1)$ factor (identified with hypercharge) is gauged. The $SU(2)_1 \times SU(2)_2$ generators are as in the Littlest model [eq. (6.4)] and the hypercharge generator is $Y = \mathrm{diag}(1,1,0,-1,-1)/2$. The field content of the hermitian matrix $\Pi$ in $\Sigma$ is the same as in the Littlest Higgs model but now the field $\zeta^0$ [Goldstone associated to the breaking of the $U(1)$ symmetry left ungauged] is not absorbed by the Higgs mechanism and remains in the physical spectrum, so there is no heavy $B'$ gauge bosons what helps with precision electroweak fits [62].

The kinetic part of the lagrangian is as in the Littlest Higgs, eq. (6.10) model but now with

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1}^{2} g_j W_j^a (Q_j^a\Sigma + \Sigma Q_j^{aT}) - ig' B_Y(Y\Sigma + \Sigma Y^T). \tag{6.43}$$

The fermionic couplings in the lagrangian can be kept as in the Littlest Higgs model also. Then the scalar operators $\mathcal{O}_F(\Sigma)$ and $\mathcal{O}_V(\Sigma)$, induced by fermion and gauge boson loops have the same form of (6.21), (6.22) but with the $U(1)$ part limited to $U(1)_Y$ only. The main difference with respect to the Littlest Higgs case is that now the Higgs mass is not protected from quadratically divergent radiative corrections involving $U(1)_Y$ interactions even at one-loop level, this mass is of order $g'^2 f^2$. Therefore, those corrections are not especially dangerous, due to the smallness of the $g'$ coupling.

The $h$-dependent field masses, needed for the calculation of the one-loop Higgs potential, are the following. In the gauge boson sector we have

$$m_{W'}^2(h) = \frac{1}{4}(g_1^2 + g_2^2) f^2 - \frac{1}{4} g^2 h^2 + \mathcal{O}(h^4/f^2)\ , \tag{6.44}$$

with no $B'$ gauge boson. In the fermion sector, the heavy Top has mass:

$$m_T^2(h) = M_T^2 + \mathcal{O}(h^2) = (\lambda_1^2 + \lambda_2^2) f^2 - \frac{1}{2}\lambda_t^2 h^2 + \mathcal{O}(h^4/f^2)\ . \tag{6.45}$$

In the scalar sector, decomposing $h^0 \equiv (h^{0r} + ih^{0i})/\sqrt{2}$ and $\phi^0 \equiv i(\phi^{0r} + i\phi^{0i})/\sqrt{2}$ and using $\lambda'_a \equiv cg_2^2 - c'\lambda_1^2$ and $\lambda'_b \equiv cg_1^2$, combined in $\lambda'_+ \equiv \lambda'_a + \lambda'_b$ and $\lambda'_- \equiv \lambda'_a - \lambda'_b$, the masses are as follows. Writing simultaneously the relevant part of the mass matrices in the $CP$-even sector (using the basis $\{h^{0r}, \phi^{0r}\}$), the $CP$-odd sector (in the basis $\{h^{0i}, \phi^{0i}\}$) and the charged sector (in the basis $\{h^+, \phi^+\}$), we get

$$
\begin{aligned}
M^2_\kappa(h) &= \begin{bmatrix} \frac{1}{4}a_\kappa\lambda'_+ h^2 + \frac{1}{\sqrt{2}}s_\kappa\lambda'_- ft + \mathcal{O}(h^4/f^2) & b_\kappa\lambda'_- fh + \mathcal{O}(h^2) \\ b^*_\kappa\lambda'_- fh + \mathcal{O}(h^2) & \lambda'_+\left(f^2 - c_\kappa h^2\right) + \mathcal{O}(h^4/f^2) \end{bmatrix} \\
&+ cg'^2 \begin{bmatrix} f^2 - d_\kappa h^2 + \mathcal{O}(h^4/f^2) & \mathcal{O}(h^2) \\ \mathcal{O}(h^2) & 4f^2 - e_\kappa h^2 + \mathcal{O}(h^4/f^2) \end{bmatrix} , \qquad (6.46)
\end{aligned}
$$

where the index $\kappa = \{0r, 0i, +\}$ labels the different sectors. The numbers $a_\kappa$, $b_\kappa$, $c_\kappa$ and $s_\kappa$ are as in (6.26) while $d_\kappa = \{1, 1/6, 1/6\}$ and $e_\kappa = 13|b_\kappa|^2/3$. We have also included in these mass matrices the contribution of the triplet vev, $t \equiv \langle\phi^{0r}\rangle$, with

$$
t \simeq -\frac{1}{2\sqrt{2}}\frac{\lambda'_- h^2}{(\lambda'_+ + 4cg'^2)f} \; . \qquad (6.47)
$$

As in the Littlest Higgs model, the off-diagonal entries in (6.46) are due to the $h^2\phi$ coupling which causes mixing between $h$ and $\phi$ after electroweak symmetry breaking. This effect is negligible for the heavy triplet [at order $h^2$ in the masses, the components $\phi^{0r}$ and $\phi^{0i}$ can still be combined in a complex field $\phi^0$]. We call $h'^{0r}$, $h'^{0i}$ and $h'^+$ the light mass eigenvalues of (6.26) in the different sectors. The explicit masses for the different components of the triplet field are then[4]

$$
\begin{bmatrix} m^2_{\phi^0}(h) \\ m^2_{\phi^+}(h) \\ m^2_{\phi^{++}}(h) \end{bmatrix} = M^2_\phi + \mathcal{O}(h^2) = (\lambda'_+ + 4cg'^2)f^2 - \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \left(\lambda + \frac{17}{12}cg'^2\right) h^2 + \mathcal{O}(h^4/f^2) \; . \qquad (6.48)
$$

For $h'^{0r}$, $h'^{0i}$ and $h'^+$ we get

$$
\begin{bmatrix} m^2_{h'^{0r}}(h) \\ m^2_{h'^{0i}}(h) \\ m^2_{h^+}(h) \end{bmatrix} = M^2_s + \mathcal{O}(h^2) = cg'^2 f^2 + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \lambda h^2 + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \frac{1}{6} cg'^2 h^2 + \mathcal{O}(h^4/f^2) \; . \qquad (6.49)
$$

From the previous expressions for the masses one can check that the cancellation of $h^2$ terms in Str$M^2$ works except for the $g'$-dependent terms, as expected. The presence of the coupling $g'$, which does not respect the $SU(3)_{1,2}$ symmetries, complicates the structure of

[4] In writing the expansions for these masses we are assuming $cg'^2 f^2 \sim \lambda h^2 \ll \lambda'_+ f^2$.

couplings in the Higgs sector. For instance, the Higgs quartic coupling after integrating out the heavy triplet is given by

$$\lambda = \frac{1}{4}\left[\lambda'_a + \lambda'_b - \frac{4}{3}cg'^2 - \frac{(\lambda'_a - \lambda'_b)^2}{(\lambda'_a + \lambda'_b + 4cg'^2)}\right] , \tag{6.50}$$

to be compared with the theoretically cleaner formula (6.25) that holds in the Littlest Higgs case. All mass formulas and couplings written above reproduce those of the Littlest Higgs model in the limit $\lambda'_{a,b} \to \lambda_{a,b}$ and $g' \to 0$. After electroweak symmetry breaking some kinetic terms are non-canonical due to $\mathcal{O}(h^2/f^2)$ corrections from non-renormalizable operators. The masses above include effects from field redefinitions necessary to render canonical all fields.

### 6.2.2 Fine-tuning analysis in the Modified Littlest Higgs

The input parameters of the model are now

$$p_i \;=\; \{g_1,\ g_2,\ \lambda_1,\ \lambda_2,\ c,\ c',\ f\} , \tag{6.51}$$

to be compared with (6.35) for the Littlest Higgs model [27]. As in that model, $f$ can be ignored for the fine-tuning analysis.

For the fine-tuning analysis we need the $h$-dependent masses, which enter the one-loop effective potential, and they are given by the above eqs. (6.44), (6.45), (6.48), (6.49). Besides the absence of $g'_1$ and $g'_2$, the main difference with the original Littlest Higgs model is that the Higgs mass parameter $m^2$ gets an additional positive contribution from the operator $c\,\mathcal{O}_V(\Sigma)$,

$$\delta m^2 = cg'^2\frac{\Lambda^2}{16\pi^2} = cg'^2 f^2 . \tag{6.52}$$

This contribution involves $g'$ as anticipated. Adding the one-loop logarithmic corrections we get

$$\begin{aligned} m^2 \;=\;& cg'^2 f^2 + \frac{9g^2}{64\pi^2}M^2_{W'}\left[\log\frac{\Lambda^2}{M^2_{W'}} + \frac{1}{3}\right] - \frac{3\lambda_t^2}{8\pi^2}M_T^2\left[\log\frac{\Lambda^2}{M_T^2} + 1\right] \\ +\;& \frac{3}{8\pi^2}\left\{\left(\lambda + \frac{17}{12}cg'^2\right)M^2_\phi\left[\log\frac{\Lambda^2}{M^2_\phi} + 1\right] - \left(\lambda + \frac{1}{12}cg'^2\right)M_s^2\left[\log\frac{\Lambda^2}{M_s^2} + 1\right]\right\} , \end{aligned} \tag{6.53}$$

where the Higgs quartic coupling is the one of eq. (6.50). The expression for $M_T$ is as for the Littlest Higgs, the triplet mass is $M_\phi^2 = (\lambda'_a + \lambda'_b + 4cg'^2)f^2$ and $M_s^2 = cg'^2 f^2$ is the squared mass associated to the light Higgses [see eqs.( 6.48), (6.49)]. Eqs. (6.53) and (6.50) have to be compared with (6.31) and (6.25) for the Littlest Higgs.

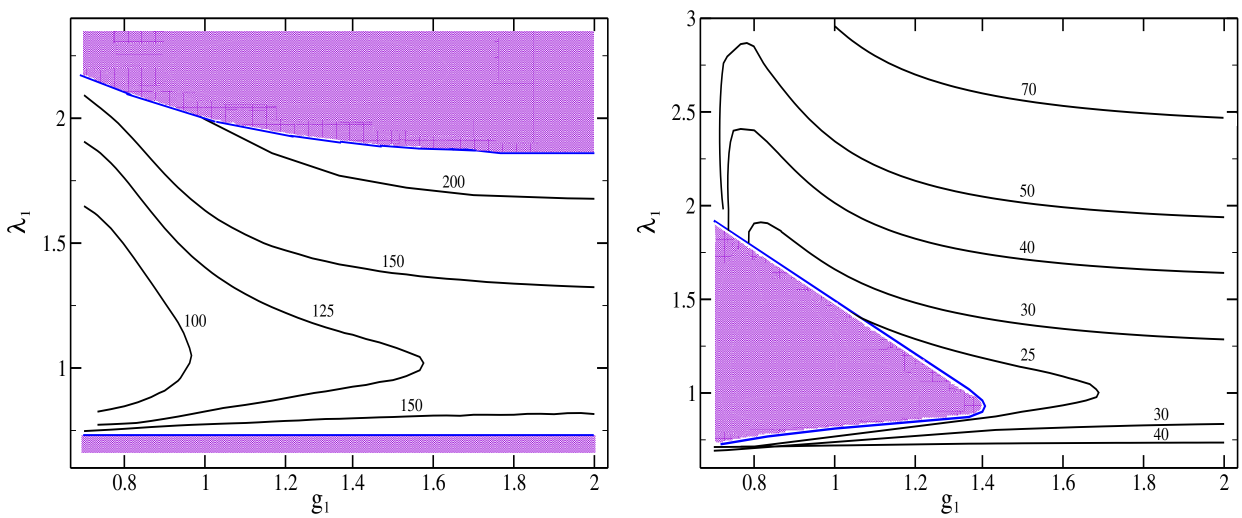



Figure 6.6: Preliminary fine-tuning contours, using $c$ and $c'$ as unknown parameters, for the Little Higgs model of [62], case a), with $m_h = 115$ GeV (left plot) and $m_h = 250$ GeV (right plot).

The presence of the $g'$ terms in $m^2$ complicates the parameter dependence of the minimization condition for electroweak breaking: $c$ and $c'$ do no longer enter in $m^2$ just through $\lambda'_a$ and $\lambda'_b$. Nevertheless, there are still two separate regions of solutions, which are the respective heirs of the two regions named a) and b) for the Littlest Higgs model [eq. (6.38)][5]; thus we keep the same notation.

The fine-tuning $\Delta$ for the region a), using $c$ and $c'$ as input parameters, is shown in fig. 6.6. The magnitude of $\Delta$ is similar to that in the Littlest Higgs model, fig. 6.2. In the present case the tree-level contribution $cg'^2 f^2$ in (6.53), which is positive [6], helps in compensating the negative correction from the heavy Top, so that the contribution from the triplet, and thus the triplet mass $M_\phi^2$, is not required to be as large as before. Consequently, the values of $c$ and $c'$ will be smaller, as it happens for $c$ in the region a) of the Littlest Higgs model. However, as discussed in the previous section, small $c$ and $c'$ cause additional fine-tuning[7], which can be taken into account by using $c_0$ and $c'_0$, rather than $c$ and $c'$ as the input parameters appearing

[5] Again, the existence of these two regions can be understood here using the approximation explained in footnote 2 of this chapter.

[6] For $c < 0$ one breaks the electroweak symmetry at tree-level. However, this possibility leads to a large vev for the triplet and therefore we focus on $c > 0$.

[7] Note that eq. (6.24) holds also in this model.

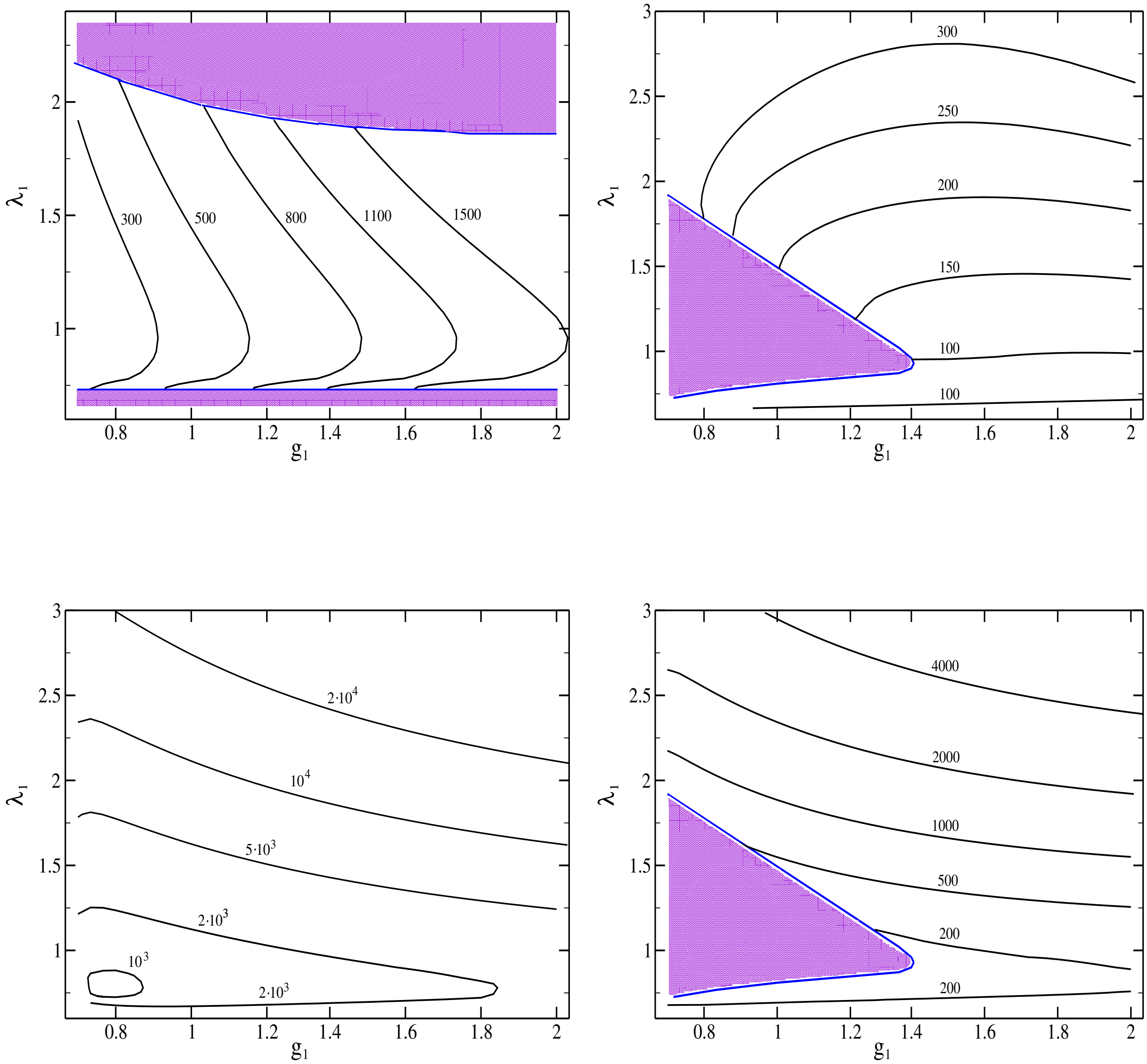



Figure 6.7: Final fine-tuning contours for the Little Higgs model of [62], using $c_0$ and $c_0'$ as unknown parameters, for the two regions of solutions: a) (top) and b) (bottom), and two different values of the Higgs mass: $m_h = 115$ GeV (left) and $m_h = 250$ GeV (right).

in (6.51). This enhancement of the fine-tuning can be appreciated in the corresponding plots [both for a) and b) regions] in fig. 6.7.

Fig. 6.7 represents our final results for the model analyzed in this section. The fine-tuning is quite similar to that for the Littlest Higgs model (fig. 6.3 and fig. 6.4). Therefore, the same comments apply here: the fine-tuning is always substantial ($\Delta > 10$) and for $m_h < 250$ GeV is essentially of the same order as (or higher than) that of the Little Hierarchy problem [$\Delta \gtrsim \mathcal{O}(100)$] and worse than in the MSSM. As in the Littlest Higgs, two-loop or 'tree-level' contributions to $m^2$ are not likely to improve the situation [note in particular that eqs. (6.34) and (6.41) remain the same in this scenario].

## 6.3 The Littlest Higgs with $T$-parity [63]

### 6.3.1 Structure of the Littlest Higgs with $T$-parity

This model, proposed in [63], is also based on the $SU(5)/SO(5)$ structure of the Littlest Higgs model, with the same gauge and scalar field content. However, the lagrangian is different: a $T$-parity is imposed such that the triplet and the heavy gauge bosons are $T$-odd while the Higgs doublet is $T$-even. This $T$-parity plays a role similar to $R$-parity in SUSY: it has the welcome effect of forbidding a number of dangerous couplings (like the $h^2\phi$ one responsible for the triplet vev, as discussed in previous sections; or direct couplings of the SM fields to the new gauge bosons) improving dramatically the fit to electroweak data.

The gauge kinetic part of the lagrangian is as in eq. (6.10) with $T$-parity and imposes the equalities

$$g_1 = g_2 = \sqrt{2}g \ , \qquad g_1' = g_2' = \sqrt{2}g' \ , \tag{6.54}$$

where $g$ and $g'$ are the gauge coupling constants of the SM. Imposing $T$-invariance on the fermionic sector requires the introduction of several new degrees of freedom. Those relevant for making the fermionic lagrangian of eq. (6.12) $T$-symmetric are a new vector-like pair of coloured doublets $\tilde{q}_3, \tilde{q}_3^c$ ($T$-even) plus two new coloured singlets $u'^c_T$ (the $T$-image of $u'^c$) and $U$ (which is $T$-odd). The fermionic lagrangian reads [63]

$$\mathcal{L}_f = \frac{1}{4}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\left[(\xi Q)_i \Sigma_{jx}\Sigma_{ky} u'^c + (\tilde{\xi}Q)_i \tilde{\Sigma}_{jx}\tilde{\Sigma}_{ky} u'^c_T\right] + \lambda_2 f t' t'^c + \frac{1}{\sqrt{2}}\lambda_3 f U (u'^c - u'^c_T) + h.c., \tag{6.55}$$

plus (heavy) mass terms for $\tilde{q}_3$. Here we have used $Q \equiv (q_3, t', \tilde{q}_3)^T$, $\xi \equiv \exp[i\Pi/f]$, $\tilde{\xi} \equiv \Omega \exp[i\Pi/f]\Omega$ [with $\Omega \equiv \mathrm{diag}(1,1,-1,1,1)$] and $\tilde{\Sigma} \equiv \tilde{\xi}^2\Sigma_0$. The index convention is as in

(6.12). Finally, the scalar operators (6.21),(6.22) turn out to be given by

$$\begin{aligned} -\Delta\mathcal{L} \;=\; V &= 2cg^2 f^4 \sum_{i=1,2}\sum_a \mathrm{Tr}[(Q_i^a\Sigma)(Q_i^a\Sigma)^*] + 2cg'^2 f^4 \sum_{i=1,2}\mathrm{Tr}[(Y_i\Sigma)(Y_i\Sigma)^*] \\ &- \frac{1}{16}c' f^4\lambda_1^2\epsilon^{wx}\epsilon_{yz}\left(\Sigma_{iw}\Sigma_{jx}\Sigma^{iy*}\Sigma^{jz*} + \tilde\Sigma_{iw}\tilde\Sigma_{jx}\tilde\Sigma^{iy*}\tilde\Sigma^{jz*}\right) , \end{aligned} \tag{6.56}$$

which is simply a $T$-invariant version of the lagrangian of eq. (6.23).

In this model, the squared masses to $\mathcal{O}(h^2)$, needed for the calculation of the one-loop Higgs potential, are very similar to those in the Littlest Higgs model. In the gauge boson sector they are exactly the same as in (6.18), with gauge couplings related by eq. (6.54):

$$\begin{aligned} m^2_{W'}(h) &= M^2_{W'} + \mathcal{O}(h^2) = g^2f^2 - \frac{1}{4}g^2h^2 + \mathcal{O}(h^4/f^2) , \\ m^2_{B'}(h) &= M^2_{B'} + \mathcal{O}(h^2) = \frac{1}{5}g'^2f^2 - \frac{1}{4}g'^2h^2 + \mathcal{O}(h^4/f^2) . \end{aligned} \tag{6.57}$$

In the fermion sector, the only mass relevant for our purposes is that of the heavy Top which, to order $h^2$, remains the same as in the Littlest Higgs model:

$$m_T^2(h) = M_T^2 + \mathcal{O}(h^2) = (\lambda_1^2+\lambda_2^2)f^2 - \frac{1}{2}\lambda_t^2h^2 + \mathcal{O}(h^4/f^2) . \tag{6.58}$$

The squared masses of the other heavy fermions do not have an $h^2$-dependence.

In the scalar sector, an important difference with respect to the Littlest Higgs model is that now there is no $\phi h^2$ coupling. As a result, the Higgs quartic coupling does not get modified after decoupling the triplet field and is simply given by

$$\lambda = \frac{1}{4}(\lambda_a+\lambda_b) , \tag{6.59}$$

[now $\lambda_a = 2c(g^2+g'^2) - c'\lambda_1^2$ and $\lambda_b = 2c(g^2+g'^2)$] to be compared with eq. (6.25). Another direct consequence of not having a $\phi h^2$ coupling is the absence of the off-diagonal entries in the scalar mass matrices in the $CP$-even, $CP$-odd and charged sectors. Using the same conventions of eq. (6.26), these mass matrices are given by

$$M_\kappa^2(h) = \begin{bmatrix} a_\kappa\lambda h^2 + \mathcal{O}(h^4/f^2) & 0 \\ 0 & 4\lambda(f^2 - c_\kappa h^2) + \mathcal{O}(h^4/f^2) \end{bmatrix} , \tag{6.60}$$

with the constants $a_\kappa$ and $c_\kappa$ exactly as in the Littlest Higgs model, eq. (6.26). The explicit masses for the different components of the heavy triplet field are still given by (6.48), and making use of (6.59) they simply read

$$\begin{bmatrix} m^2_{\phi^0}(h) \\ m^2_{\phi^+}(h) \\ m^2_{\phi^{++}}(h) \end{bmatrix} = M_\phi^2 + \mathcal{O}(h^2) = 4\lambda f^2 - \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \lambda h^2 + \mathcal{O}(h^4/f^2) . \tag{6.61}$$

For the light eigenvalues of (6.60), which now do not mix with the triplet components, we simply get $m^2_{h^{0r}}(h) = 3\lambda h^2$, $m^2_{h^{0i}}(h) = m^2_{h^+}(h) = \lambda h^2$, as in the Standard Model.

The one-loop generated Higgs mass parameter, $m^2$, is given by the same expression as that of the Littlest Higgs model [eq. (6.31)] but, as we have seen, $T$-parity imposes strong relations between the parameters of the model. In particular, we have now

$$M^2_{W'} = g^2 f^2 \ , \quad M^2_{B'} = \frac{1}{5} g'^2 f^2 \ , \quad M^2_\phi = 4\lambda f^2 \ . \tag{6.62}$$

The model is therefore much more constrained than the Littlest Higgs.

### 6.3.2 Fine-tuning analysis in the Littlest Higgs with $T$-parity

For the fine-tuning analysis [27], we start by identifying the input parameters, which are now

$$p_i \ = \ \{\lambda_1,\ \lambda_2,\ c,\ c',\ f\} \ , \tag{6.63}$$

to be compared with (6.35) and (6.51). Again, we can leave $f$ aside as explained after (6.35). The couplings $\lambda_{1,2}$ are related by the usual top-Yukawa constraint in eq. (6.17) while $c$ and $c'$ are related to $\lambda$ through eq. (6.59). For a given value of the Higgs mass (and therefore of the coupling $\lambda$) the minimization condition for electroweak breaking can be solved for $M_T^2$, which fixes $\lambda_1^2 + \lambda_2^2$, but not $\lambda_1$ or $\lambda_2$ separately. From this continuum of solutions, the top mass constraint [eq. (6.17)] leaves only two of them, simply related by $\lambda_1 \leftrightarrow \lambda_2$. We will refer to these two solutions as

$$1)\ \lambda_1 \leq \lambda_2 \ , \qquad 2)\ \lambda_2 \leq \lambda_1 \ . \tag{6.64}$$

If $\lambda$ is small, $M_\phi$ is not large enough to compensate the negative heavy Top contribution to the one-loop Higgs mass and the minimization condition is not satisfied. If, on the other hand, $\lambda$ is too large then the Top contribution, which cannot be arbitrarily large (it grows with $M_T$, but only up to $M_T = \Lambda$), is also unable to satisfy the minimization condition. Thus, we obtain a limited range for $m_h$: 280 GeV $\lesssim m_h \lesssim$ 625 GeV, for $f = 1$ TeV. This result has interest by itself for the phenomenology of the Littlest Higgs model with $T$-parity, with the caveat that possible two-loop (or 'tree-level') contributions to the Higgs mass parameter can change the limits of that interval for $m_h$, as we discuss in more detail below.

The resulting constrained fine-tuning [using $c_0$ and $c_0'$ of eq. (6.24) as unknown parameters] is shown in figure 6.8. As $g_1$ is not a free-parameter anymore, we present our results in the plane $\{c, m_h\}$. The black solid lines correspond to case 1) and the red dashed ones to case 2). At the lower bound for $m_h$, which is determined by the minimal possible value

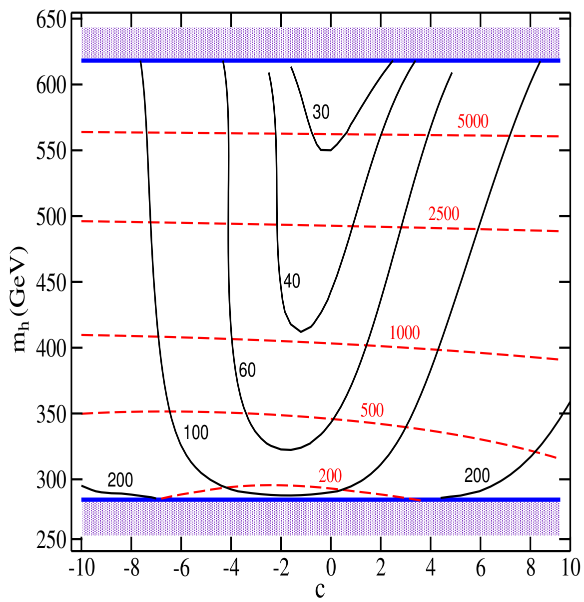



Figure 6.8: Fine-tuning contours in a Little Higgs model with $T$-parity, using $c_0$ and $c_0'$ of eq. (6.24) as unknown parameters. Solid (dashed) lines correspond to case 1 (2) of eq. (6.64).

of $M_T^2 = (\lambda_1^2 + \lambda_2^2) f^2$, one has $\lambda_1 = \lambda_2 = \lambda_t$ and therefore cases 1) and 2) give the same results for the fine-tuning, as it can be seen in the figure. At the upper bound on $m_h$ one has $M_T^2 = \Lambda^2$, which implies $\lambda_i \simeq 4\pi$ for $i = 1$ or 2, at the limit of perturbativity. We see that the fine-tuning is sizeable throughout all parameter space in spite of the large values of the Higgs mass. It is always larger for case 2) because a larger value of $\lambda_1$ affects directly the parameter $\lambda_a$ and therefore the value of $\lambda$. In fact, as it will be clearer shortly, the largest contribution to the fine-tuning comes, in most cases, through the dependence of $\lambda$ on $c, c'$ and $\lambda_1$.

From the previous discussion, it follows that at some future time, after the Higgs mass has already been measured (and thus $\lambda$ gets fixed), the fine-tuning would get dramatically reduced, especially in case 2). This is shown by fig. 6.9, left plot, which presents the fine-tuning when the constraint of fixed $\lambda$ is enforced. The fine-tuning is nearly independent of $c$, and varies only through the values of $\lambda_{1,2}$, getting the smallest values at the boundaries of parameter space. This can be understood from the simple analytical approximation

$$\Delta \simeq \frac{M_T^2}{2\lambda v^2} \frac{|\lambda_1^2 - \lambda_2^2|}{\sqrt{\lambda_1^4 + \lambda_2^4}} \frac{3\lambda_t^2}{2\pi^2} \log \frac{\Lambda^2}{M_T^2} , \tag{6.65}$$

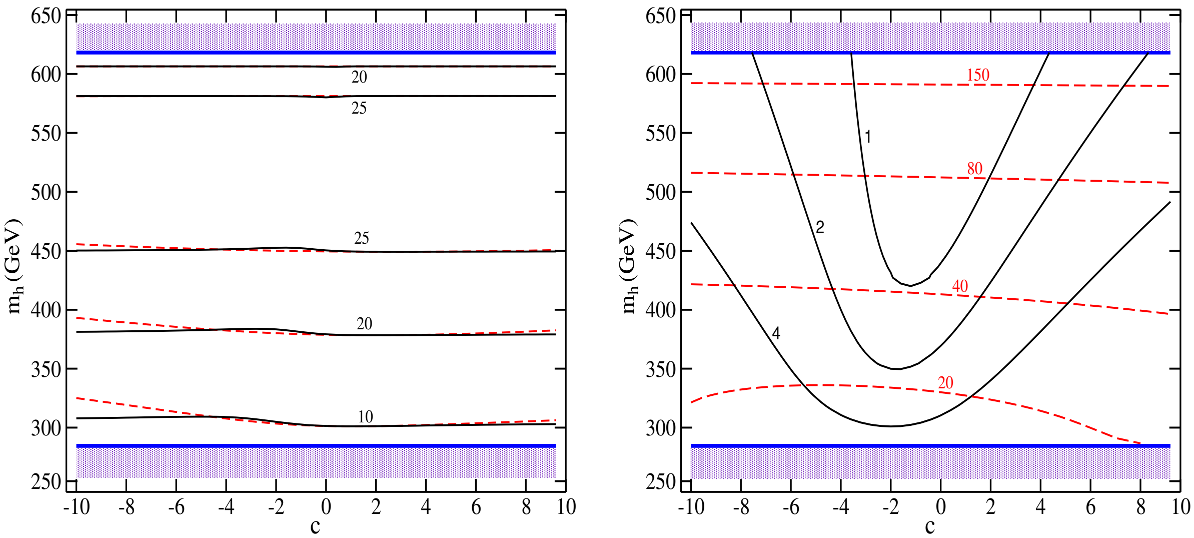



Figure 6.9: Left: Same as in fig. 6.8 but keeping fixed $\lambda$. Right: Fine-tuning associated to $\lambda$ itself. [Solid (dashed) lines correspond to case 1 (2) of eq. (6.64)].

which is easy to derive and explains why cases 1) and 2) give very similar values for the fine-tuning[8]. Although the fine-tuning is moderate, we still have to worry about the tuning in $\lambda$ itself, as we did in the last section for the model of ref. [62]. We show that tuning in the right plot of fig. 6.9. Analytically we find

$$\Delta^{(\lambda)} \simeq \frac{\lambda_1^2}{4\lambda}\left[\frac{4c'^2\lambda_1^4}{\lambda_1^4+\lambda_2^4}+(c'-c'_0)^2+16(c-c_0)^2\frac{(g^2+g'^2)^2}{\lambda_1^4}\right]^{1/2} . \tag{6.66}$$

We see that there is a big difference between cases 1) and 2). In case 1), the coupling $\lambda_1$ varies between $\lambda_t$ at the lower limit of $m_h$ and $\lambda_t/\sqrt{2}$ at the upper limit, and it does not cost much to get $\lambda$ right. Therefore the associated tuning is always small. In case 2), $\lambda_1$ is of moderate size ($\sim \lambda_t$) near the lower limit on $m_h$ but grows significantly when $m_h$ increases (reaching $\lambda_1 \sim 4\pi$ near the upper limit). Then, getting $\lambda$ right requires small values of $c'$ and, being unnatural, this causes a sizeable tuning. Coming back to fig. 6.8, one can easily check that the dependence of the fine-tuning in that plot on $c$ and $m_h$ can be understood as a particular combination of the two effects shown in fig. 6.9.

[8]The small sensitivity to $c$ and the small difference between scenarios 1) and 2) which can be appreciated in fig. 6.9 is a subtle effect [not captured by the approximation (6.65)] due to the dependence of $\lambda$ on $c, c'$ and $\lambda_1$ (even though we are fixing $\lambda$). Such effects are discussed in Appendix C.

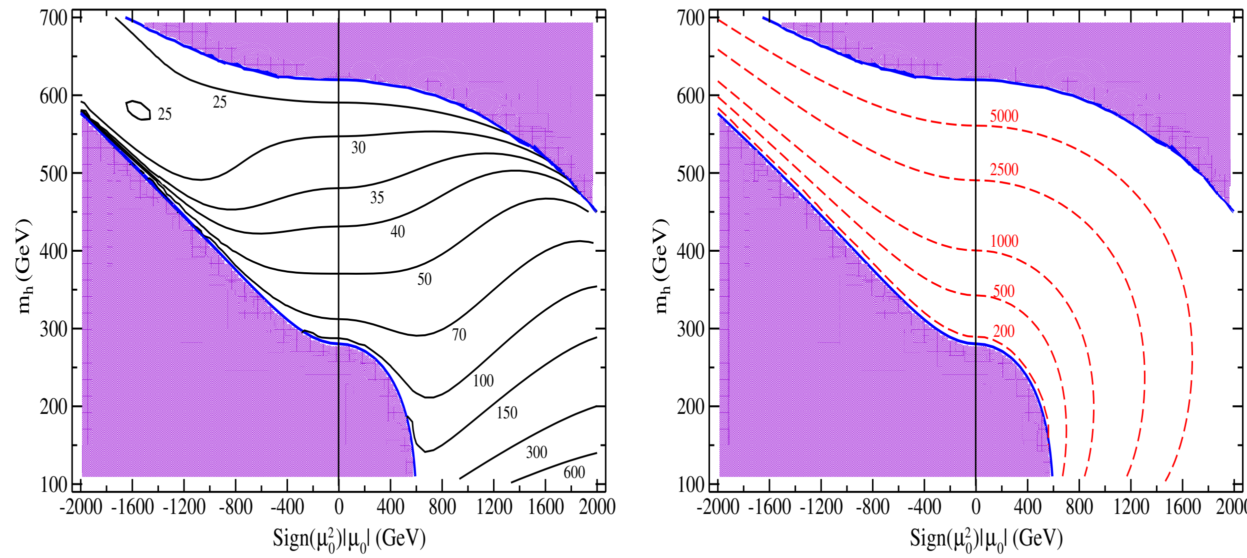



Figure 6.10: Fine-tuning contours in a Little Higgs model with $T$-parity, with a 'tree-level' $\mu_0^2$ mass parameter, using $c_0$ and $c_0'$ of eq. (6.24) as unknown parameters and setting $c = 0$. The left (right) plot corresponds to case 1 (2) of eq. (6.64).

Finally, let us consider the effect of two-loop (or 'tree-level') contributions to the Higgs mass parameter which, as mentioned, can allow Higgs masses below the (quite high) lower limit $m_h \geq 280$ GeV of fig. 6.8. We mimic this effect by adding a constant mass term $1/2\mu_0^2 h^2$ to the Higgs potential (allowing both signs of $\mu_0^2$). From the arguments given in previous sections, we do not expect big changes in the fine-tuning but it is interesting to consider this possibility as a way of accessing regions of lower Higgs mass, which are more attractive phenomenologically. Notice that eq. (6.63) is now enlarged by one more parameter, namely $\mu_0^2$. The resulting fine-tuning for cases 1) and 2) of eq. (6.64) is shown in fig. 6.10, (left and right plots, respectively), setting $c = 0$ (which nearly minimizes the fine-tuning). For Higgs masses accessible already with $\mu_0 = 0$, the fine-tuning does not change much, as expected, while for lower Higgs masses the fine-tuning increases [case 1)] or remains large [case 2)]. We see that case 1) continues to be the best option.

Figs. 6.8 and 6.10 summarize our results for the model analyzed in this section. As for the models of sections 6.1 and 6.2, the fine-tuning is always substantial ($\Delta > 10$) and usually comparable to (or higher than) that of the Little Hierarchy problem [$\Delta \gtrsim \mathcal{O}(100)$] and worse than in the MSSM. Notice also that the lowest fine-tuning, $\Delta \sim 25$, is obtained for large values of the Higgs mass, $m_h \gtrsim 500$ GeV, which is generically disfavoured from fits to precision electroweak observables [67]. In addition, such large values of $m_h$ are less satisfactory from the point of view of the Little Higgs philosophy: the Little Higgs mechanism is interesting because it might explain the lightness of the Higgs compared to the TeV scale.

## 6.4 The Simplest Little Higgs Model [64]

### 6.4.1 Structure of the Simplest Little Higgs model

We now depart from the group structure of the Littlest Higgs and consider a model, proposed in [64], that is based on a global $[SU(3)\times U(1)]^2/[SU(2)\times U(1)]^2$. The initial gauged subgroup is $[SU(3)\times U(1)_X]$ which gets broken to the electroweak subgroup. This spontaneous symmetry breaking produces 10 Goldstone bosons, 5 of which are eaten by the Higgs mechanism to make massive a complex $SU(2)$ doublet of extra $W'$s, $(W'^{\pm}, W'^{0})$, and an extra $Z'$. The remaining 5 degrees of freedom are: $H$ [an $SU(2)$ doublet to be identified with the SM Higgs] and $\eta$ (a singlet).

Explicitly, the spontaneous breaking is produced by the vevs of two scalar triplet fields, $\Phi_1$ and $\Phi_2$:

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} , \qquad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} . \tag{6.67}$$

These triplets transform under the global symmetry as

$$\Phi_1 \to e^{-i\alpha_1/3} U_1 \Phi_1 , \qquad \Phi_2 \to e^{-i\alpha_2/3} U_2 \Phi_2 , \tag{6.68}$$

where $U_i$ is an $SU(3)_i$ matrix and $e^{-i\alpha_i/3}$ are $U(1)_i$ rotations, with gauge transformations corresponding to the diagonal $U_1 = U_2$, $\alpha_1 = \alpha_2$. Using the broken generators, the Goldstone fluctuations around the vacuum (6.67) can be written as

$$\Phi_i = \exp\left[\frac{i}{f}\begin{pmatrix} 0 & 0 & h_i^+ \\ 0 & 0 & h_i^0 \\ h_i^- & h_i^{0*} & \eta_i/\sqrt{2} \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f_i \end{pmatrix} , \tag{6.69}$$

for $i = 1, 2$, with $f^2 = f_1^2 + f_2^2$. Identifying explicitly the linear combinations of $h_i$ and $\eta_i$ that correspond to the eaten Goldstones $(G^{\pm}, G^0, G_S)$ and the physical fields $(H, \eta)$ one gets

$$\Phi_1 \;=\; \exp\left[\frac{i}{f}\begin{pmatrix} 0 & 0 & G^+ \\ 0 & 0 & G^0 \\ G^- & G^{0*} & G_S/\sqrt{2} \end{pmatrix} + \frac{if_2}{ff_1}\begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & \eta/\sqrt{2} \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} ,$$

$$\Phi_2 = \exp\left[\frac{i}{f}\begin{pmatrix} 0 & 0 & G^+ \\ 0 & 0 & G^0 \\ G^- & G^{0*} & G_S/\sqrt{2}\end{pmatrix} - \frac{if_1}{ff_2}\begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & \eta/\sqrt{2}\end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f_2\end{pmatrix} . \tag{6.70}$$

The scalar kinetic part of the lagrangian is

$$\mathcal{L}_k = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 , \tag{6.71}$$

with

$$D_\mu \Phi_i = \partial_\mu \Phi_i - igW^a_\mu T^a \Phi_i + \frac{i}{3} g_x B^x_\mu \Phi_i , \tag{6.72}$$

corresponding to the $SU(3) \times U(1)_x$ gauged group. Obviously, $g$ corresponds to the $SU(2)$ gauge coupling while the relation between $g, g_x$ and the $U(1)_Y$ gauge coupling $g'$ is given by

$$\frac{1}{g'^2} = \frac{1}{3g^2} + \frac{1}{g_x^2} , \tag{6.73}$$

which simply fixes $g_x$ in terms of $g$ and $g'$. This initial tree-level lagrangian has a structure similar to eq. (6.9). In particular, $m^2$ and $\lambda$ are zero at this level.

As in previous models, in order to study the electroweak breaking, we need to consider the one-loop Higgs potential, for which we have to to compute the $h$-dependent masses of the model.

In the gauge sector one can write the masses of the gauge bosons in terms of $\Phi_{1,2}$. For this we find convenient to define the operator

$$\mathcal{O}_{12} \equiv \frac{1}{f^2}\left(f_1^2 f_2^2 - |\Phi_1^\dagger \Phi_2|^2\right) . \tag{6.74}$$

In a background of $\langle h^0 \rangle = h/\sqrt{2}$ and $\eta$, this operator can be expanded as

$$\mathcal{O}_{12} = \frac{1}{2}h^2 - \frac{1}{48}\frac{f^2}{f_1^2 f_2^2} h^2(4h^2 + \eta^2) + ... \tag{6.75}$$

Generically, one gets masses of the form

$$m^2_{H,L} = \frac{M^2}{2}\left[1 \pm \sqrt{1 - 4\kappa_{12}^2 \mathcal{O}_{12}/M^2}\right] , \tag{6.76}$$

where the subindices $H, L$ stand for heavy and light masses, $M$ is a generic mass of order $f$ and $\kappa_{12}$ is some combination of couplings. An expansion in powers of $\mathcal{O}_{12}$ gives

$$\begin{aligned} m_H^2 &= M^2 - \kappa_{12}^2 \mathcal{O}_{12} + \mathcal{O}(\mathcal{O}_{12}^2) , \\ m_L^2 &= \kappa_{12}^2 \mathcal{O}_{12} + \mathcal{O}(\mathcal{O}_{12}^2) . \end{aligned} \tag{6.77}$$

Besides the massless photon, the rest of gauge bosons have the following masses. For the charged $(W^\pm, W'^\pm)$, formula (6.76) holds with

$$M^2 = M_{W'}^2 \equiv \frac{1}{2}g^2 f^2 \ , \quad \kappa_{12}^2 = \frac{1}{2}g^2 \ . \tag{6.78}$$

Expanding in powers of $h$, one gets

$$\begin{aligned} m_{W'^\pm}^2(h) &= M_{W'}^2 - \frac{1}{4}g^2 h^2 + \mathcal{O}(h^4/f^2) \ , \\ m_{W^\pm}^2(h) &= \frac{1}{4}g^2 h^2 + \mathcal{O}(h^4/f^2) \ . \end{aligned} \tag{6.79}$$

For the $(Z'^0, Z^0)$ pair, again the masses are given by formula (6.76), now with

$$M^2 = M_{Z'}^2 \equiv \frac{2g^2}{3 - t_w^2} f^2 \ , \quad \kappa_{12}^2 = \frac{1}{2}(g^2 + g'^2) \ , \tag{6.80}$$

where $t_w \equiv g'/g$. An expansion in powers of $h$ gives

$$\begin{aligned} m_{Z'^0}^2(h) &= M_{Z'}^2 - \frac{1}{4}(g^2 + g'^2)h^2 + \mathcal{O}(h^4/f^2) \ , \\ m_{Z^0}^2(h) &= \frac{1}{4}(g^2 + g'^2)h^2 + \mathcal{O}(h^4/f^2) \ . \end{aligned} \tag{6.81}$$

Finally, for the complex $W'^0$

$$m_{W'^0}^2 = M_{W'}^2 \ , \quad \kappa_{12}^2 = 0 \ . \tag{6.82}$$

In the fermion sector, the Yukawa part of the lagrangian, reads

$$\mathcal{L}_Y = \lambda_1 u_1^c \Phi_1^\dagger \Psi_Q + \lambda_2 u_2^c \Phi_2^\dagger \Psi_Q + \text{h.c.} \ , \tag{6.83}$$

with generation indices suppressed (we only care about the third family). Here $\Psi_Q$ is an $SU(3)$ triplet (with $x$-charge 1/3) that contains the usual quark doublet while $u_{1,2}^c$ are $SU(3)$ singlets (with $x$-charge $-2/3$). A combination of $u_1^c$ and $u_2^c$ corresponds to the SM top quark field while the orthogonal combination gets a heavy mass with the third component of $\Psi_Q$. The explicit masses of these fields follow the pattern of (6.76) with

$$M^2 = M_T^2 \equiv \lambda_1^2 f_1^2 + \lambda_2^2 f_2^2 \ , \quad \kappa_{12}^2 = \lambda_t^2 \ , \tag{6.84}$$

where $\lambda_t$ is the SM top Yukawa coupling, given by

$$\frac{f^2}{\lambda_t^2} = \frac{f_1^2}{\lambda_2^2} + \frac{f_2^2}{\lambda_1^2} \ . \tag{6.85}$$

An expansion in powers of $h$ gives

$$\begin{aligned} m_T^2(h) &= M_T^2 - \frac{1}{2}\lambda_t^2 h^2 + \mathcal{O}(h^4/f^2) \ , \\ m_t^2(h) &= \frac{1}{2}\lambda_t^2 h^2 + \mathcal{O}(h^4/f^2) \ . \end{aligned} \tag{6.86}$$

From the generic formula for the masses in eq. (6.76), one sees that $\mathrm{Str}M^2$ is field independent and then the cancellation of $h^2$ terms holds to all order in $h$. Therefore, and in contrast with previous models, one-loop quadratically divergent corrections from gauge or fermion loops do not induce scalar operators to be added to the lagrangian. Then, no Higgs quartic coupling is present at this level.

Less divergent one-loop corrections do induce both a mass term and a quartic coupling for the Higgs. Using again the $\overline{\mathrm{MS}}$ scheme in Landau gauge[9] and setting the renormalization scale $Q = \Lambda$, it is straightforward to compute the one-loop potential including fermion and gauge boson loops once the masses are known as a function of $h$. Performing an expansion of this potential in powers of $h$, one gets [64]

$$V(h) = \frac{1}{2}\delta m^2 h^2 + \frac{1}{4}\left[\delta_1\lambda(h) - \frac{\delta m^2}{3}\frac{f^2}{f_1^2 f_2^2}\right] h^4 + ... \tag{6.87}$$

with

$$\begin{aligned}\delta m^2 &= \frac{3}{32\pi^2}\left[g^2 M_{W'}^2\left(\log\frac{\Lambda^2}{M_{W'}^2} + \frac{1}{3}\right) + \frac{1}{2}(g^2+g'^2)M_{Z'}^2\left(\log\frac{\Lambda^2}{M_{Z'}^2} + \frac{1}{3}\right)\right] \\ &- \frac{3}{8\pi^2}\lambda_t^2 M_T^2\left(\log\frac{\Lambda^2}{M_T^2} + 1\right) + ... \ ,\end{aligned} \tag{6.88}$$

and

$$\begin{aligned}\delta_1\lambda(h) &= -\frac{3}{128\pi^2}\left[g^4\left(\log\frac{M_{W'}^2}{m_W^2(h)} - \frac{1}{2}\right) + \frac{1}{2}(g^2+g'^2)^2\left(\log\frac{M_{Z'}^2}{m_Z^2(h)} - \frac{1}{2}\right)\right] \\ &+ \frac{3}{16\pi^2}\lambda_t^4\left(\log\frac{M_T^2}{m_t^2(h)} - \frac{1}{2}\right) + ... \ ,\end{aligned} \tag{6.89}$$

where the dots in eqs. (6.88) and (6.89) stand for subdominant contributions (in particular those from the $\eta$ and the Higgs field itself, which were also subdominant in previous models).

The radiatively induced Higgs mass, $\delta m^2$, is dominated as usual by the negative heavy Top contribution, which is again too large (being $M_T^2 \geq 4\lambda_t^2 f_1^2 f_2^2/f^2$) and now there is no bosonic contribution that can be used to compensate it. This problem is solved [64] by adding to the tree-level potential a mass $\mu^2$ for the triplets $\Phi_{1,2}$

$$\delta_0 V = \mu^2 \mathcal{O}_X \equiv \mu^2(2f_1 f_2 - \Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1) \ . \tag{6.90}$$

Such operator contributes to the Higgs potential the piece

$$\delta_0 V = \frac{1}{2}\mu_0^2 h^2 - \frac{1}{48}\frac{\mu_0^2 f^2}{f_1^2 f_2^2}h^4 + ... \tag{6.91}$$

[9] Our scheme differs from that used in [64], but the difference is numerically small.

where $\mu_0^2$ is given in terms of the fundamental mass parameter $\mu^2$ by

$$\mu_0^2 = \mu^2 \frac{f^2}{f_1 f_2} \ . \tag{6.92}$$

By choosing $\mu_0^2 > 0$ we get a positive contribution to the Higgs mass parameter that can compensate the heavy Top contribution in $\delta m^2$. The tree-level value of the Higgs quartic coupling from eq. (6.91) is then negative but the large (and positive) radiative corrections in eq. (6.89) can easily overcome that effect.

### 6.4.2 Fine-tuning analysis in the Simplest Little Higgs

In order to compute the fine-tuning in this model [27] we use the previous potential, (6.87) plus (6.91):

$$V(h) = \frac{1}{2}(\mu_0^2 + \delta m^2)h^2 + \frac{1}{4}\left[\delta_1\lambda(h) - \frac{f^2}{3f_1^2 f_2^2}\left(\delta m^2 + \frac{\mu_0^2}{4}\right)\right] h^4 + ... \tag{6.93}$$

As mentioned, it does not contain the subdominant contributions from $\eta$ and the Higgs field. The input parameters are now:

$$\{\lambda_1, \lambda_2, \mu^2, f_1, f_2\} \ . \tag{6.94}$$

Without loss of generality we can choose $f_1 \leq f_2$, in which case the UV cut-off is $\Lambda = 4\pi f_1$. Since we want $\Lambda = 10$ TeV (the scale of the Little Hierarchy problem) we also set $f_1 = 1$ TeV. As $f_1$ and $f_2$ are not the only mass scales in the problem (there is $\mu^2$ as well) it is important to include the fine-tuning associated to them, which might be large now.

The Higgs mass that results from the potential (6.93), after trading $\mu_0^2$ by $v$ using the minimization condition, can be computed as a function of $M_T^2$ for fixed $f_2/f_1$. For any pair $\{\lambda_1, \lambda_2\}$ that gives a particular value of $M_T^2$, there is another pair $\{\lambda_1, \lambda_2\} \to \{\lambda_2 f_2/f_1, \lambda_1 f_1/f_2\}$ that gives the same $M_T^2$. Therefore each choice of $M_T^2$ (to get a particular value of $m_h$) corresponds to two different solutions in terms of $\lambda_{1,2}$. We will refer to them as

$$1)\ \lambda_1 f_1 \leq \lambda_2 f_2 \ , \qquad 2)\ \lambda_1 f_1 \geq \lambda_2 f_2 \ . \tag{6.95}$$

As mentioned above, these two solutions are related by the interchange $\lambda_1 f_1 \leftrightarrow \lambda_2 f_2$. Fig. 6.11 gives the fine-tuning in the plane $\{m_h, f_2/f_1\}$ for these two cases. We see from these plots that the fine-tuning is sizeable and increases with $f_2/f_1$. From the bound $M_T \geq 2\lambda_t f_1 f_2/f$ and the fact that $\delta m^2$ and $\delta_1\lambda$ cannot be arbitrarily large, it follows that $m_h^2$ is limited to a certain range. This range depends on the value of $f_2/f_1$: for $f_2 = f_1$ one gets 163 GeV $\leq m_h \leq$ 606 GeV and a narrower range for larger $f_2/f_1$, as it can be seen in fig. 6.11.

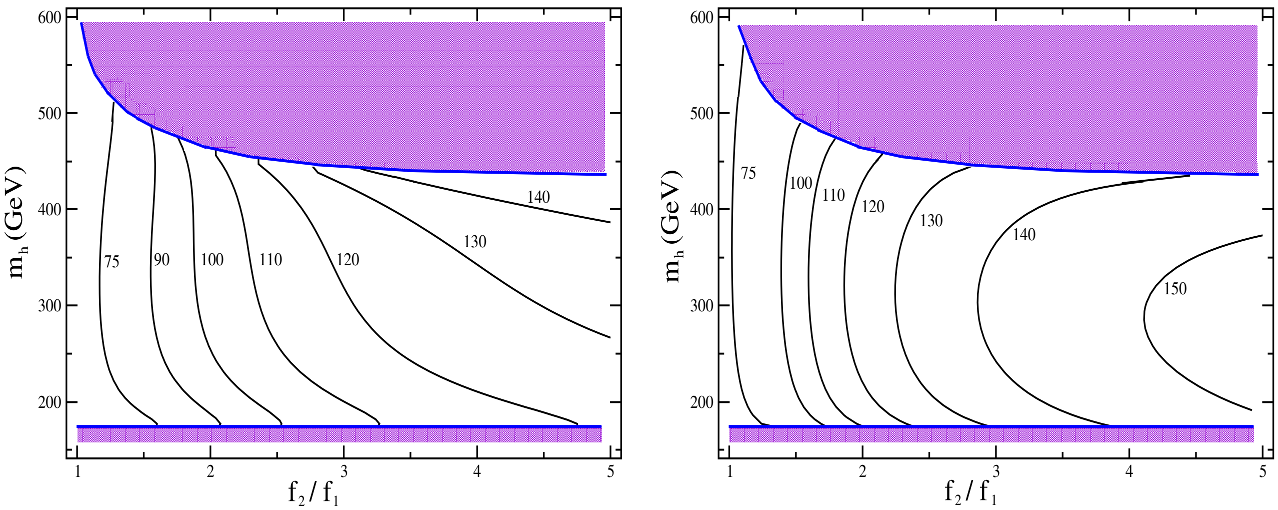



Figure 6.11: Fine-tuning contours for the Simplest Little Higgs model for cases 1) (left plot) and 2) (right plot) of eq. (6.95).

To access lower values of $m_h$ one can add a piece $\lambda_0$ to the Higgs quartic coupling in the potential (6.93). This new term can result from the unknown heavy physics at the cut-off $\Lambda$. For $\lambda_0 < 0$ one can get values of $m_h$ below the lower bounds discussed before. In the presence of such term we should also worry about the quadratically divergent contributions of scalars to the Higgs mass parameter. From

$$\delta V_1^{\rm quad} = \frac{\Lambda^2}{32\pi^2}(m_h^2 + 3m_G^2 + m_\eta^2) \ , \tag{6.96}$$

where $m_h, m_G$ and $m_\eta$ are the tree-level masses of the Higgs, the electroweak Goldstones and $\eta$ respectively, one gets[10] (after substituting $\Lambda = 4\pi f_1$)

$$\delta_q m^2 = -\frac{5f^2}{8f_2^2}\mu_0^2 + 6\lambda_0 f_1^2 \ . \tag{6.97}$$

The piece proportional to $\mu_0^2$ is not particularly dangerous and can even be interpreted as a redefinition of the original $\mu_0^2$ parameter, while the second term, proportional to the new coupling $\lambda_0$, can be sizeable, thus having a significant impact on the fine-tuning. In the presence of these quadratically divergent corrections we expect to have a contribution to the

[10] Of course, this contribution is due to the fact that the Simplest model does not include additional fields to cancel the quadratic divergencies from loops of its scalar fields.

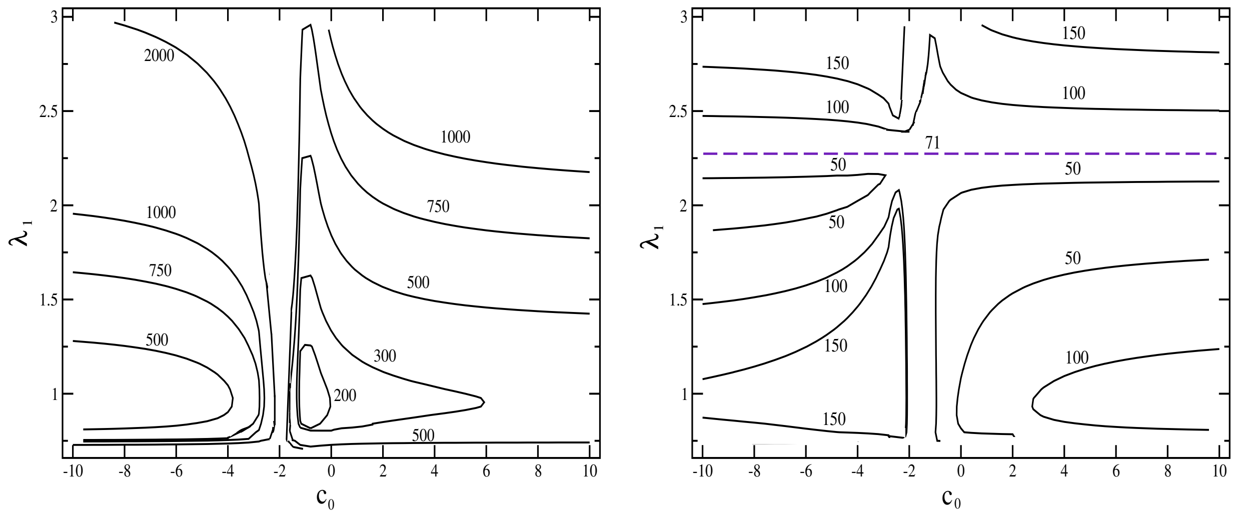



Figure 6.12: Fine-tuning contours for the Simplest Little Higgs model augmented by a 'tree-level' quartic coupling $\lambda_0$, with $m_h = 115$ GeV (left plot) and $m_h = 250$ GeV (right plot).

Higgs mass parameter of order $6\lambda_0 f_1^2$ already at the cut-off. Therefore we introduce such a mass term in the potential, multiplied by some unknown coefficient $c$, from the beginning. As we did in previous models, we then split $c$ into an unknown 'tree-level' contribution $c_0$ and a calculable radiative one-loop correction $c_1$, with $c = c_0 + c_1 = c_0 + 1$. Our potential is now

$$V(h) = \frac{1}{2}[\mu_0^2 + \delta m^2 + 6(c_0+1)\lambda_0 f_1^2]h^2 + \frac{1}{4}\left[\lambda_0 + \delta_1\lambda(h) - \frac{f^2}{3f_1^2 f_2^2}\left(\delta m^2 + \frac{\mu_0^2}{4}\right)\right]h^4 + ... \quad (6.98)$$

and the set of input parameters is enlarged to

$$\{\lambda_0, \lambda_1, \lambda_2, \mu^2, f_1, f_2, c_0\}\ . \quad (6.99)$$

Fig. 6.12 shows the fine-tuning associated to this modified potential in the plane $\{c_0, \lambda_1\}$ for $m_h = 115$ Gev (left plot) and $m_h = 250$ GeV (right plot) for $f_2 = f_1$. As expected, lower Higgs masses can now be reached, but there is a fine-tuning price to pay. As shown by the right plot, in the case of larger Higgs masses, already accessible for $\lambda_0 = 0$, the effect of the new parameters $c_0$ and $\lambda_0$ allows the fine-tuning to be reduced if such parameters are chosen appropriately, but the effect is never dramatic (for the sake of comparison, we show by a dashed line, the fine-tuning corresponding to $\lambda_0 = 0$). However, the fine-tuning gets worse in most of the parameter space.

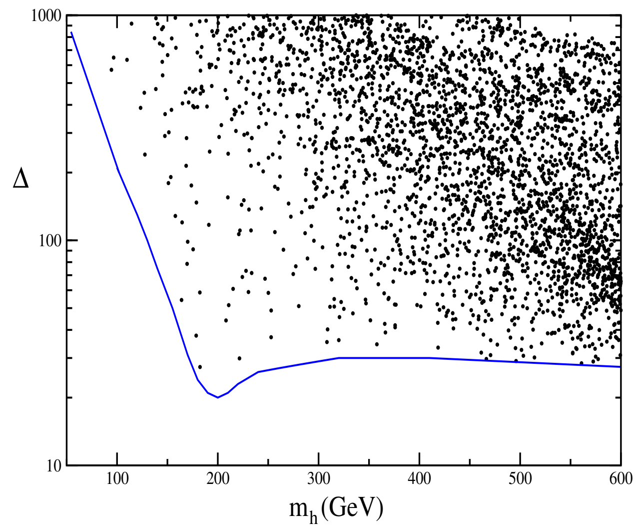



Figure 6.13: Scatter-plot of the fine-tuning in the Simplest Little Higgs model as a function of the Higgs mass.

From figs. 6.11 and 6.12, we can conclude that the fine-tuning in the Simplest LH model is similar to that of the models analyzed in previous sections: it is always significant and usually comparable to (or higher than) that of the Little Hierarchy problem [$\Delta \gtrsim \mathcal{O}(100)$]. Only for some small regions of parameter space is $\Delta$ comparable to the MSSM one ($\Delta \sim 20 - 40$ for $m_h \lesssim 125$ GeV); usually it is much worse. The last point is illustrated by the scatter-plot of fig. 6.13, which shows the value of $\Delta$ vs. $m_h$ for random values of the parameters given in (6.99) compatible with $v = 246$ GeV. More precisely, we have set $f_1 = f_2 = 1$ TeV and chosen at random $\lambda_0 \in [-2, 2]$, $\lambda_1 \in [\lambda_t/\sqrt{2}, 15]$ and $c_0 \in [-10, 10]$. The solid line gives the minimal value of $\Delta$ as a function of $m_h$ and has been computed independently (rather than deduced from the scatter plot). Clearly, the density of points gets sparser near this lower bound.

## 6.5 Conclusions

Little Higgs (LH) models try to solve the Little Hierarchy problem, making the Higgs a pseudo-Goldstone boson resulting from a spontaneously broken global symmetry. In these models quadratic divergences to the Higgs mass are absent at one-loop level, and no fine-

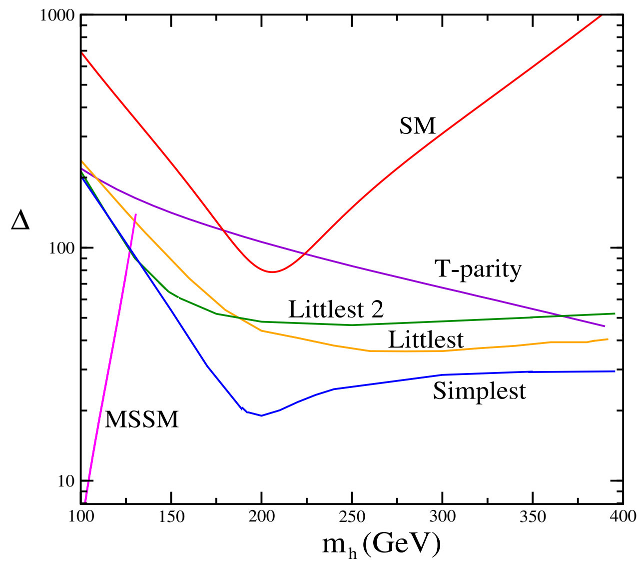



Figure 6.14: Comparative summary of the fine-tuning vs. $m_h$ for different scenarios. The curves for Little Higgs models (lines labeled "Littlest", "Littlest 2", "$T$-parity" and "Simplest") are lower bounds on the corresponding fine-tuning, see text for details.

tuning is required to keep the Higgs sufficiently light until a scale of the order 10 TeV : the "Little Hierarchy" is then stabilized.

We have rigorously analyzed the fine-tuning associated to the electroweak breaking process in Little Higgs (LH) scenarios, focussing on four popular and representative models, corresponding to refs. [61, 62, 63, 64].

Although LH models solve parametrically the Little Hierarchy problem [generating a Higgs mass parameter of order $f/(4\pi)$], our first conclusion is that these models generically have a substantial fine-tuning built-in, usually much higher than suggested by the rough considerations commonly made. This is due to the great amount of superstructure of these models giving implicit tunings between parameters that can be overlooked at first glance but show up in a more systematic analysis. This does not demonstrate, of course, that all LH models are necessarily fine-tuned, but it stresses the need of a rigorous analysis in order to claim that a particular model is not fine-tuned, especially if a quantitative statement is attempted (e.g. to compare its degree of fine-tuning with that of the MSSM). In this respect, the analysis presented here can also be helpful as a guide to the ingredients that typically increase the fine-tuning in LH models, in order to correct them in improved constructions.

We have quantified the degree of fine-tuning following the 'standard' criterion of Barbieri and Giudice [16], through a fine-tuning parameter $\Delta$, that can be computed in each model (e.g. $\Delta \simeq 100$ means a fine-tuning at the one percent level), finding that the four LH scenarios analyzed here present fine-tuning ($\Delta > 10$) in all cases and in most of their parameter space. The results are summarized in the plots of figs. 6.3 (for the Littlest Higgs), 6.7 (for the modified Littlest Higgs), 6.8 and 6.10 (for the Littlest Higgs with $T$-parity), and 6.11 and 6.12 (for the Simplest Little Higgs). Actually, the fine-tuning is comparable to or higher– sometimes much higher– than the one associated to the Little Hierarchy problem of the SM (given by the blue line of fig. 2.8 in chapter 2) in most of the parameter space of these models. Since LH models have been designed to solve the Little Hierarchy problem, we believe this is a serious drawback. Likewise, the fine-tuning is usually worse than that of supersymmetric models ($\Delta = 20 - 40$ for the MSSM and lower for other supersymmetric scenarios), which succeed at stabilizing a much larger hierarchy ($\Lambda \simeq M_{GUT}$ or $M_{Planck}$ rather than $\Lambda \simeq 10$ TeV).

We can make the previous statements more precise. Fig. 6.14 shows the fine-tuning $\Delta$ as a function of $m_h$ for different scenarios. The curve labelled "SM" represents the fine-tuning of the Little Hierarchy problem in the SM, as discussed in fig. 2.8 in chapter 2. The "MSSM" line shows the fine-tuning of the MSSM[11]. Then, for each LH model analyzed in sects. 1–4 of this chapter we have plotted (lines labeled "Littlest", "Littlest 2", "$T$-parity" and "Simplest") the minimum value of $\Delta$ accessible by varying the parameters of the model. Usually, only in a quite small area of parameter space of each model is the fine-tuning close to the lower bound shown, so the LH curves in fig. 6.14 are a very conservative estimate of the fine-tuning in the corresponding LH models. This point is illustrated by fig. 6.13 for the Simplest LH model (the best behaved): the lower line in that plot corresponds to the "Simplest" line in fig. 6.14. Now we see that the value of $\Delta$ for all these models is $\geq \mathcal{O}(100)$ in most of parameter space, and larger that $20 - 30$ in all cases. This fine-tuning is larger than the MSSM one, at least for the especially interesting range $m_h \lesssim 130$ GeV. Notice here that $m_h \gtrsim 135$ GeV is not available in the MSSM if the supersymmetric masses are not larger than $\sim 1$ TeV. This limitation does not hold for other supersymmetric models, e.g. those with low-scale SUSY breaking, as discussed in ref. [36], which are definitely in better shape than LH models concerning fine-tuning issues.

Regarding the specific ingredients that potentially increase the fine-tuning in LH models, we stress two of them. First, the LH lagrangian is generically enlarged with operators that have the same structure as those generated through the quadratically divergent radiative corrections to the potential (and are necessary for the viability of the models). Such operators

[11] This curve has been obtained for large $\tan\beta$ (which minimizes the fine-tuning) but disregarding stop-mixing effects (which can help in reducing the fine-tuning).

have two contributions: the radiative one (calculable) and the 'tree-level' one (arising from physics beyond the cut-off and unknown). Very often the required value of the coefficient in front of a given operator is much smaller than the calculable contribution, which implies a tuning (usually unnoticed) between the tree-level and the one-loop pieces (similar to the hierarchy problem in the SM). Second, the value of the Higgs quartic coupling, $\lambda$, receives several contributions which have a non-trivial dependence on the various parameters of the model. Therefore, to keep $\lambda$ small (as required to have $m_h$ in a phenomenologically acceptable region) an extra fine-tuning is required.

# Chapter 7

# Conclusions

## 7.1 Conclusions (English)

This thesis focusses on the Hierarchy problem of the Standard Model (SM) and in the naturalness of Electroweak Symmetry Breaking (EWSB) for specific examples of physics beyond the SM.

The Hierarchy problem of the SM gives an estimate of the scale of New Physics (NP), $\Lambda$, based on the sensitivity of the Higgs mass, $m_h$, to quadratic divergences. Imposing that the Higgs mass is not fine-tuned, i.e. that the quadratically divergent contributions are not much larger than $m_h$ itself, one obtains the estimate $\Lambda \leq$ 2-3 TeV for the SM cut-off (although in very specific situations this bound could be evaded). Then, according to this estimate, NP should appear at scales on the reach of the LHC at CERN. Although this argument is arguably naive [as it ignores other contributions to the Higgs mass (logarithmic and finite) which can be potentially large], the general analysis presented in this thesis, based on a model-independent study of the one-loop effective potential, shows that one can keep $\Lambda \leq$ 2-3 TeV as a conservative bound. In spite of being a very general argument, this kind of analysis still has limitations, and there could be some cases where the above result does not apply (e.g. all the dangerous contributions are absent due to some unknown reason). Therefore, to be on the safe side, one must perform the fine-tuning analysis on specific scenarios of NP in order to obtain more precise implications from naturalness arguments.

If one assumes that the SM is valid up to a very large cut-off scale (e.g. $M_{GUT} \sim 10^{15}$ GeV or $M_{Planck} \sim 10^{19}$ GeV), the Higgs mass suffers from an enormous fine-tuning, called the “Big Hierarchy” problem. On the other hand, if the considered cut-off is at 10 TeV (i.e.

the experimental lower bound on the effective scale of some higher order operators), there is still a fine-tuning, but of the order of 1%. The last is the so called "Little Hierarchy" problem. In the literature it has been proposed to alleviate this Little Hierarchy problem with simple modifications of the Higgs sector. In this work, specific proposals of two Higgs doublets models (2HDM) have been examined from naturalness arguments to confirme or not if they can raise the expected SM scale of new physics of 2-3 TeV. Then, this could lead to NP above the LHC reach. The conclusion for them is that, in general, they cannot improve the SM upper bound, keeping the expected NP inside LHC reach.

On the other hand, the evidence for neutrino masses is the first experimental result of physics beyond the ordinary SM. The simplest extension of the SM incorporates small neutrino masses by adding right-handed neutrinos with a large Majorana mass, $M_R$ (the "seesaw" mechanism). In this scenario there are two neutrino eigenstates for each generation, one very light ($m_\nu < 1$ eV) and the other very heavy (e.g. $M_R \sim 10^{13}$ GeV). The heaviest eigenstate contributes to the dangerous corrections to the Higgs mass and we have seen that fine-tuning arguments in the Higgs mass set an upper bound, $M_R \lesssim 10^7$ GeV, which spoils the natural small values for neutrino masses in the seesaw mechanism with natural neutrino Yukawa couplings of $\mathcal{O}(1)$. In conclusion, the case of right-handed seesaw neutrinos suffers a very important fine-tuning problem, which can not been ameliorated unless additional new physics is introduced, such as Supersymmetry.

In Supersymmetry (SUSY) quadratically divergent corrections to the Higgs mass are cancelled. Despite this feature, the new sparticles with masses of $\mathcal{O}(m_{\rm soft})$ give rise to logarithmic and finite contributions to the Higgs mass that lead to the usual fine-tuning upper bound $m_{soft} \equiv \Lambda \lesssim 2-3$ TeV. In the Minimal Supersymmetric Standard Model (MSSM) the upper bound is much more stringent, namely $m_{\rm soft} \lesssim$ few hundred GeV. In the MSSM, radiative corrections to the Higgs mass are required due to the smallness of the tree-level Higgs quartic coupling (not large enough to be consistent with LEP Higgs mass bound), this implies soft masses larger than $m_{\rm soft} > 300$ GeV. In consequence, the MSSM is already fine-tuned at the few percent level. The main reasons for this abnormally large fine-tuning are this smallness of the tree-level Higgs quartic coupling and the large coefficients of soft-terms contributions in the effective potential due to Renormalization Group running effects. The MSSM problems can be alleviated in alternative SUSY models. In this thesis, we have considered scenarios where the breaking of SUSY occurs at low scale (not far from the TeV). In such scenarios it is natural to have tree-level SUSY breaking contributions to the Higgs quartic coupling which can make it larger. This helps in evading the LEP Higgs mass bound without the need of large radiative corrections and, moreover, in such models RG effects do not play a significant role since the cut-off scale is much closer to the electroweak scale. All these improvements can cooperate to make EWSB much more natural than in the MSSM, and in this particular case the general bound $\Lambda \lesssim 2-3$ TeV can indeed be saturated.

Besides SUSY, there are other candidates for NP. Among them, Little Higgs (LH) scenarios are a recent proposal done to solve the Little Hierarchy problem of the SM, that is, to stabilize the Higgs mass sufficiently light until a scale at least of 10 TeV. In our fine-tuning analysis of four representative LH models, we have concluded that their fine-tuning is much larger than suggested by rough estimates. Because these models present a great amount of superstructure, there are "hidden" adjustments between the parameters of the model in order to have the correct EWSB, giving rise to a large amount of fine-tuning in most of the parameter space of the model. These implicit tunings can be overlooked at first glance but show up in a more systematic analysis. This unexpected high fine-tuning is mostly due to two reasons. First, the LH models have operators in their lagrangian with the same structure as the operators generated through the quadratic radiative corrections to the potential. These operators have two contributions: the radiative one (computable) and the 'tree-level' one (arising from physics beyond the cut-off and unknown). The required value of the coefficient in front of a given operator is often much smaller than the calculable contribution, which implies a tuning between the tree-level and the one-loop pieces. Second, the value of the Higgs quartic coupling, $\lambda$, receives several contributions which have a non-trivial dependence on the various parameters of the model. Keeping $\lambda$ in a phenomenologically acceptable region needs an extra fine-tuning. This does not demonstrate that all LH models are fine-tuned, but stresses the need of a careful analysis of this issue in model-building.

The last point of the above paragraph should be indeed applied to all the scenarios of NP. The fine-tuning analyses give us a naturalness criterion for the different kinds of scenarios, and together with electroweak precision tests they offer a guideline to NP properties and to the scales at which NP could be found. In this thesis, it has been found that the upper bound of 2-3 TeV for NP given by the Hierarchy problem of the SM is a conservative bound, and when specific scenarios are considered, the upper bounds from naturalness on the masses of new particles are even smaller than 2 TeV. This suggests that new physics is around the corner, at scales accessible to the next generation of high energy accelerator, such as LHC, starting next year. And from naturalness arguments and due to its successful properties, supersymmetry could be the favourite candidate to be observed.

## 7.2 Conclusiones (Castellano)

Esta tesis se centra en el problema de las jerarquías del Modelo Estándar y en la naturalidad de la ruptura de la simetría electrodébil en ejemplos específicos de física más allá del Modelo Estándar.

El problema de las jerarquías del Modelo Estándar da una estimación de la escala de nueva física, basada en la sensibilidad de la masa del Higgs a divergencias cuadráticas. Si imponemos que la masa del Higgs no esté sometida a un ajuste fino ó fine-tuning, es decir, que las divergencias cuadráticas no sean mucho mayores que la propia masa del Higgs, se obtiene una estimación en la escala de $\Lambda \leq$ 2-3 TeV, probablemente accesible para el LHC en el CERN. Aunque este argumento se podría considerar como simplista, ya que pueden exister situaciones en las que esta cota puede evadirse como hemos visto, el análisis general que se presenta en esta tesis basado en el estudio del potencial efectivo a un loop independiente de modelo, muestra que la cota estimada anteriormente funciona bien como una cota conservadora, ya que al estimarla se han ignorado otras contribuciones a la masa del Higgs que pueden ser potencialmente grandes. A pesar de ser un argumento general, este tipo de análisis tiene sus limitaciones, y podría aparecer algún caso donde la cota anterior fuera errónea (p. ej. alguna teoría donde todas las contribuciones peligrosas no aparecerieran debido a alguna razón desconocida). Por lo tanto, por precaución, se deben realizar los análisis de fine-tuning en escenarios específicos de nueva física para así obtener implicaciones más precisas de los argumentos de naturalidad.

Si se asume que el Modelo Estándar es válido hasta una escala de cut-off alta (e.g. $M_{Planck}$), la masa del Higgs presenta un gran fine-tuning, llamado el "gran" problema de las jerarquías. Por otro lado, si se considera el cut-off en 10 TeV (el límite experimental inferior en la escala efectiva de algunos operadores de orden superior), sigue existiendo un fine-tuning, pero en este caso del 1%, llamado el "pequeño" problema de las jerarquías. Se ha propuesto en la literatura resolver el "pequeño" problema de las jerarquías modificando el sector de Higgs del MS. En esta tesis, se han analizado diferentes propuestas de modelos con dos dobletes de Higgs y se ha observado que, sin embargo, no son capaces de mejorar la cota de naturalidad del MS.

Por otro lado, el primer resultado experimental de física más allá del Modelo Estándar ordinario es la evidencia de masas para los neutrinos. La extensión más simple del MS para incorporar masas para los neutrinos es via el mecanismo de "seesaw", añadiendo neutrinos dextrógiros con un masa tipo Majorana muy grande, $M_R$. En este escenario hay dos autoestados de masa para cada generación, uno muy ligero ($m_\nu < 1$ eV) y otro muy pesado ($M_R \sim 10^{13}$ GeV). El autoestado pesado contribuye a la masa del Higgs a través de correciones radiativas. Hemos mostrado que para que esas correciones no supongan un gran fine-tuning, la masa $M_R$ debería ser $\lesssim 10^7$ GeV, por lo que para mantener masas my pequeñas para los neutrinos ligeros necesitaríamos acoplos de Yukawa no naturales, $\lesssim 10^{-7}$. Con esto concluimos que los neutrinos seesaw presentan un problema importante de fine-tuning que no puede mejorarse salvo que se introduzca nueva física adicional, como puede ser supersimetría.

En Supersimetría las correciones cuadráticamente divergentes a la masa del Higgs se cancelan. A pesar de esta propiedad, las nuevas partículas supersimétricas con masas $\mathcal{O}(m_{\rm soft})$ dan lugar a contribuciones logarítmicas y finitas a la masa del Higgs que conducen a la cota de naturalidad habitual, $m_{soft} \equiv \Lambda \lesssim 2-3$ TeV. En el modelo supersimétrico mínimo (MSSM) esta cota es mucho más severa, $m_{\rm soft} \lesssim 150$ GeV. Debido a que el acoplo cuártico del Higgs a nivel árbol es muy pequeño haciendo necesarias correciones radiativas a la masa del Higgs para que esta masa sea consistente con el límite inferior experimental, las masas de las partículas supersimétricas deben ser tales que $m_{\rm soft} > 300$ GeV, con lo que el MSSM resulta estar ajustado aproximadamente al 4 %. Otra razón de este gran ajuste es que el potencial efectivo tiene contribuciones de los términos soft con coeficientes grandes debido a efectos de running del grupo de renormalización, dando lugar a ajustes entre estos términos para que se dé una correcta ruptura electrodébil. Los problemas anteriores pueden ser atenuados en otros modelos supersimétricos. En esta tesis se ha considerado el escenario donde la ruptura de supersimetría ocurre a baja escala (no muy lejos del TeV). En estos escenarios se tiene de manera natural un acoplo cuártico del Higgs a nivel árbol mayor que el del MSSM. Esto hace que la masa del Higgs a nivel árbol sea consistente con las cotas experimentales y no se necesiten grandes correciones radiativas y, además, los efectos del grupo de renormalización no juegan un papel importante ya que la escala de cut-off está más próxima a la escala electrodébil. Estas mejoras hacen que la ruptura de la simetría electrodébil se realice de manera más natural que en el MSSM, y en este caso particular se ve también como el límite general de $\Lambda \lesssim 2-3$ TeV puede saturarse.

Además de la supersimetría, existen otros candidatos de nueva física. Entre ellos, los escenarios de Little Higgs son una propuesta reciente realizada para resolver el "pequeño" problema de las jerarquías del Modelo Estándar. En nuestro análisis del fine-tuning en cuatro modelos representativos de Little Higgs hemos concluido que su fine-tuning asociado a la ruptura de la simetría electrodébil es mucho mayor que lo sugieren las primeras estimaciones. Debido a que estos modelos presentan una gran cantidad de estructura interna, hay muchos ajustes "ocultos" entre los parámetros para que se de la correcta ruptura de la simetría electrodébil. Esto da lugar a una gran cantidad de fine-tuning en la mayoria del espacio de parámetros del modelo. Estos ajustes implícitos pueden pasar desapercibidos, pero salen a la luz cuando se lleva a cabo un análisis sistemático, como el que hemos realizado. Este inesperado fine-tuning es debido principalmente a dos razones. La primera es que estos modelos tienen operadores en su lagrangiano con la misma estructura que los operadores generados a través de correciones radiativas cuadráticas al potencial. Estos operadores tienen dos contribuciones: la radiativa (calculable) y la de nivel árbol (que proviene de física por encima del cut-off y desconocida). El valor requerido para el coeficiente en frente de un operador dado suele ser mucho menor que la contribución calculable, dando lugar a un ajuste entre la pieza calculable y la de nivel árbol. La segunda razón es que el valor del acoplo cuártico del Higgs recibe varias contribuciones que tienen una dependencia no trivial en los parámetros del modelo y para mantener este acoplo en la región aceptable fenomenológicamente se necesita un fine-tuning extra. Estas dos razones no demuestran que todos los modelos de Little Higgs estén ajustados de manera fina, pero hace hincapié en la necesidad de un análisis cuidadoso del fine-tuning en la construcción de modelos de este tipo.

En realidad, el análisis cuidadoso del fine-tuning debería ser realizado en todos los escenarios de física más allá del MS. Estos análisis nos dan un criterio de naturalidad para las diferentes clases de escenarios, y, junto con las medidas de precisión electrodébil, nos ofrecen una guía acerca de las propiedades de la nueva física y a que escalas podría ésta encontrarse, además de cuáles son los escenarios más favorables para ser descubiertos en próximos experimentos. En nuestro trabajo se ha observado que el límite de 2-3 TeV para nueva física dado por el problema de las jerarquías del Modelo Estándar es en realidad una cota conservadora y cuando se estudian escenarios específicos se encuentran límites de naturalidad incluso menores que esa escala para las nuevas partículas de cada modelo en concreto. Esto sugiere que la nueva física se encuentra a escalas accesibles para la nueva generación de experimentos de alta energía como es el LHC , cuyo funcionamiento empieza el año próximo. En estos momentos, de los argumentos de naturalidad y debido a sus propiedades exitosas, supersimetría parecería ser el candidato favorito de ser observado en el LHC.

# Appendix A

# General formulas in two Higgs doublet models

Here we consider a generic scenario where the Higgs sector consists of two $SU(2)_L$ doublets of opposite hypercharge, $H_1$ and $H_2$, as in many supersymmetric models [9]. The most general Higgs potential for such two Higgs doublet models (2HDM) is at tree-level:

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - \left[m_3^2 H_1 \cdot H_2 + \text{h.c.}\right] \\ &+ \frac{1}{2}\lambda_1|H_1|^4 + \frac{1}{2}\lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1 \cdot H_2|^2 \\ &+ \left[\frac{1}{2}\lambda_5(H_1 \cdot H_2)^2 + \lambda_6|H_1|^2 H_1 \cdot H_2 + \lambda_7|H_2|^2 H_1 \cdot H_2 + \text{h.c.}\right] . \end{aligned} \tag{A.1}$$

## A.1 Renormalization Group Equations

In this Appendix, we collect the one-loop Renormalization Group Equations (RGEs) [70] that are needed in the analysis presented in chapter 4 of this work. Schematically, the RGEs at one-loop take the form

$$\frac{dp_i}{dt} = \beta_i(p_1, p_2, ...), \qquad \text{with}\, t \equiv ln\mu^2, \tag{A.2}$$

where $\mu$ is the energy scale and the parameters $p_i$ stand for the Higgs boson self-couplings $\lambda_i$, the squared Yukawa coupling for the top $\lambda_t^2$ and the squared gauge couplings $g^2$, $g'^2$ and $g_3^2$.

We must distinguished between the non-supersymmetric 2HDM case and the supersymmetric case. The distinction is given depending on whether the scale $\mu$ where we are is above or below the scale of supersymmetry breaking, $M_{\mathrm{SUSY}}$.

- $\mu > M_{\mathrm{SUSY}}$

$$\begin{aligned} 16\pi^2\beta_{\lambda_t^2} &= \lambda_t^2\left(6\lambda_t^2 - \frac{16}{3}g_3^2 - 3g^2 - \frac{13}{9}g'^2\right) \\ 48\pi^2\beta_{g'^2} &= g'^4\left(10N_g + \frac{3}{2}N_H\right) \\ 48\pi^2\beta_{g^2} &= g^4\left(6N_g + \frac{3}{2}N_H - 18\right) \\ 48\pi^2\beta_{g_3^2} &= g_3^4\,(6N_g - 27)\,. \end{aligned} \tag{A.3}$$

Here $N_g = 3$ is the number of generations, $N_H = 2$ is the number of scalar doublets, and the top Yukawa coupling is $\lambda_t = \frac{g m_t}{\sqrt{2} M_W s_\beta}$.

- $\mu < M_{\mathrm{SUSY}}$

$$\begin{aligned} 16\pi^2\beta_{\lambda_t^2} &= \lambda_t^2\left(\frac{9}{2}\lambda_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right) \\ 48\pi^2\beta_{g'^2} &= g'^4\left(\frac{20}{3}N_g + \frac{1}{2}N_H\right) \\ 48\pi^2\beta_{g^2} &= g^4\left(4N_g + \frac{1}{2}N_H - 22\right) \\ 48\pi^2\beta_{g_3^2} &= g_3^4\,(4N_g - 33)\,. \end{aligned} \tag{A.4}$$

The anomalous dimensions of the two Higgs fields and the RGEs for the Higgs self-couplings and mass parameters (with the Higss-fermion couplings as specified in section 1 of chapter 4) are:

$$\begin{aligned} 64\pi^2\gamma_1 &= 9g^2 + 3g'^2 \\ 64\pi^2\gamma_2 &= 9g^2 + 3g'^2 - 12\lambda_t^2 \\ 16\pi^2\beta_{\lambda_1} &= 6\lambda_1^2 + \lambda_3^2 + (\lambda_3+\lambda_4)^2 + \lambda_5^2 + 12\lambda_6^2 + \frac{3}{8}[2g^4 + (g^2+g'^2)^2] - 32\pi^2\lambda_1\gamma_1 \\ 16\pi^2\beta_{\lambda_2} &= 6\lambda_2^2 + \lambda_3^2 + (\lambda_3+\lambda_4)^2 + \lambda_5^2 + 12\lambda_7^2 + \frac{3}{8}[2g^4 + (g^2+g'^2)^2] - 6\lambda_t^4 - 32\pi^2\lambda_2\gamma_2 \\ 16\pi^2\beta_{\lambda_3} &= (\lambda_1+\lambda_2)(3\lambda_3+\lambda_4) + 2\lambda_3^2 + \lambda_4^2 + \lambda_5^2 + 2\lambda_6^2 + 2\lambda_7^2 + 8\lambda_6\lambda_7 \end{aligned}$$

$$+\frac{3}{8}[2g^4+(g^2+g'^2)^2]-16\pi^2\lambda_3(\gamma_1+\gamma_2) \tag{A.5}$$

$$\begin{aligned}
16\pi^2\beta_{\lambda_4} &= \lambda_4(\lambda_1+\lambda_2+4\lambda_3+2\lambda_4)+4\lambda_5^2+5\lambda_6^2+5\lambda_7^2+2\lambda_6\lambda_7 \\
&\quad +\frac{3}{2}g^2g'^2-16\pi^2\lambda_4(\gamma_1+\gamma_2) \\
16\pi^2\beta_{\lambda_5} &= \lambda_5(\lambda_1+\lambda_2+4\lambda_3+6\lambda_4)+5\lambda_6^2+5\lambda_7^2+2\lambda_6\lambda_7-16\pi^2\lambda_5(\gamma_1+\gamma_2) \\
16\pi^2\beta_{\lambda_6} &= \lambda_6(6\lambda_1+3\lambda_3+4\lambda_4+5\lambda_5)+\lambda_7(3\lambda_3+2\lambda_4+\lambda_5)-8\pi^2\lambda_6(\gamma_1+\gamma_2) \\
16\pi^2\beta_{\lambda_7} &= \lambda_7(6\lambda_2+3\lambda_3+4\lambda_4+5\lambda_5)+\lambda_6(3\lambda_3+2\lambda_4+\lambda_5)-8\pi^2\lambda_7(\gamma_1+\gamma_2) \\
16\pi^2\beta_{m_1^2} &= 3m_1^2\lambda_1+m_2^2(2\lambda_3+\lambda_4)-6m_3^2\lambda_6-16\pi^2\gamma_1 m_1^2 \\
16\pi^2\beta_{m_2^2} &= 3m_2^2\lambda_2+m_1^2(2\lambda_3+\lambda_4)-6m_3^2\lambda_7-16\pi^2\gamma_2 m_2^2 \\
16\pi^2\beta_{m_3^2} &= -3m_1^2\lambda_6-3m_2^2\lambda_7+m_3^2(\lambda_3+2\lambda_4+3\lambda_5)-8\pi^2(\gamma_1+\gamma_2)m_3^2
\end{aligned} \tag{A.6}$$

## A.2 Formulas for fine-tuning parameters in a supersymmetric 2HDM

The minimum of the 2HDM potential occurs in general at non-zero values of the neutral components of the Higgs doublets, $H_1^0$ and $H_2^0$ with $\tan\beta \equiv \langle H_2^0\rangle/\langle H_1^0\rangle$ and $\langle H_1^0\rangle = (v/\sqrt{2})\cos\beta$, $\langle H_2^0\rangle = (v/\sqrt{2})\sin\beta$. It is useful to write $V$ as a 'SM-like'potential for $v$:

$$V(v) = \frac{1}{2}m^2v^2 + \frac{1}{4}\lambda v^4 \ , \tag{A.7}$$

where $\lambda$ and $m^2$ are functions of $\tan\beta$ and the initial parameters of the theory, $p_\alpha$. Explicitly,

$$m^2 = \sum_{i=1}^{3} c_i(\beta)m_i^2(p_\alpha), \qquad \vec{c} = (c_\beta^2, s_\beta^2, -s_{2\beta}) \ , \tag{A.8}$$

and

$$\lambda = \sum_{i=1}^{7} d_i(\beta)\lambda_i(p_\alpha), \qquad \vec{d} = (\frac{1}{2}c_\beta^4, \frac{1}{2}s_\beta^4, s_\beta^2c_\beta^2, s_\beta^2c_\beta^2, s_\beta^2c_\beta^2, c_\beta^2 s_{2\beta}, s_\beta^2 s_{2\beta}) \ . \tag{A.9}$$

Minimization of $V$ with respect to $v$ and $\beta$ implies[1]

[1] With an abuse of notation we use the same symbols ($v$ and $\beta$) for the variables and their vacuum expectation values.

$$v^2 = \frac{-m^2}{\lambda} \ , \tag{A.10}$$

$$2\lambda \frac{\partial m^2}{\partial \beta} - m^2 \frac{\partial \lambda}{\partial \beta} = 0 \ . \tag{A.11}$$

In order to evaluate the fine-tuning in a generic theory of this kind with the absence of quadratic divergences (as it happens in supersymmetric models), we will use the fine-tuning parameters, $\Delta_{p_\alpha}$, introduced by Barbieri and Giudice [16]:

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta v^2}{v^2} = \Delta_{p_\alpha} \frac{\delta p_\alpha}{p_\alpha} \ . \tag{A.12}$$

Naturalness requires $\Delta_{p_\alpha} \lesssim \mathcal{O}(10)$. Applying eq. (A.12) to eq. (A.10) we get, after trading $\partial m^2/\partial\beta$ by $\partial\lambda/\partial\beta$ using eq. (A.11),

$$\Delta_p = \frac{p}{m^2} \left[ \frac{\partial m^2}{\partial p} + \frac{v^2}{2} \frac{\partial \lambda}{\partial \beta} \frac{d\beta}{dp} + v^2 \frac{\partial \lambda}{\partial p} \right] \ . \tag{A.13}$$

The dependence of $\beta$ on $p$, which is not explicit in the initial potential (A.1), can be extracted from eq. (A.11) by acting on it with $d/dp$, to obtain finally

$$\Delta_p = -\frac{p}{x} \left[ \left( 2\frac{\partial^2 m^2}{\partial \beta^2} + v^2 \frac{\partial^2 \lambda}{\partial \beta^2} \right) \left( \frac{\partial \lambda}{\partial p} + \frac{1}{v^2} \frac{\partial m^2}{\partial p} \right) - \frac{\partial \lambda}{\partial \beta} \frac{\partial^2 m^2}{\partial \beta \partial p} + \frac{\partial m^2}{\partial \beta} \frac{\partial^2 \lambda}{\partial \beta \partial p} \right] \ , \tag{A.14}$$

where

$$x \equiv \lambda \left( 2\frac{\partial^2 m^2}{\partial \beta^2} + v^2 \frac{\partial^2 \lambda}{\partial \beta^2} \right) - \frac{v^2}{2} \left( \frac{\partial \lambda}{\partial \beta} \right)^2 \ . \tag{A.15}$$

The dependence of $m^2$ and $\lambda$ on $\beta$ is determined by eqs. (A.8, A.9). In many cases, equations (A.13) and (A.14) admit expansions which are useful for fine-tuning estimates. If there exists a fine-tuning at all, there must be some cancellation between the various contributions to $m^2$, say $\mathtt{m}_\mathtt{i}^2$, which generically implies $\partial m^2/\partial p = \mathcal{O}(\mathtt{m}_\mathtt{i}^2/p) \gg \mathcal{O}(m^2/p)$. Then, the last two terms within the brackets in eq. (A.13) are suppressed by a factor $\mathcal{O}(m^2/\mathtt{m}_\mathtt{i}^2)$, and

$$\Delta_p \simeq \frac{p}{m^2} \frac{\partial m^2}{\partial p} = -\frac{p}{\lambda v^2} \frac{\partial m^2}{\partial p} \ . \tag{A.16}$$

The same result can be obtained from eq. (A.14).

Let us now consider how the previous results are modified by radiative corrections. As is well-known, the one-loop correction to the effective Higgs potential in a supersymmetric theory (using the $\overline{\text{DR}}$ renormalization scheme) is given by

$$\delta_1 V = \frac{1}{64\pi^2}\sum_a N_a M_a^4(H)\left[\log\frac{M_a^2(H)}{Q^2} - \frac{3}{2}\right] , \tag{A.17}$$

where $Q$ is the renormalization scale, $M_a^2(H)$ is the $H$-dependent mass eigenvalue of the particle $a$ and $N_a$ its multiplicity (taken negative for fermions). $\delta_1 V$ modifies the minimization conditions as well as $m_h$. However, it is possible to reproduce these results by using appropriately one-loop corrected $m_i^2$, $\lambda_i$ parameters in the tree-level expressions, e.g. the minimization equations (A.10, A.11) [71]. In this way, one can still use all the previous (tree-level-like) equations (A.13–A.16) for fine-tuning estimates. In particular, the dominant contribution to the fine-tuning is still given by eq. (A.16) but expressed in terms of the one-loop corrected parameters.

Now, one expects $\delta_1 m_i^2 = \mathcal{O}(Nh^2\tilde{m}^2/(32\pi^2))$, $\delta_1\lambda_i = \mathcal{O}(Nh^4/(32\pi^2))$, where $h$ is the coupling constant of a field with multiplicity $N$ to the Higgses and $\tilde{m}^2$ is a typical soft mass. Moreover, there can be a logarithmic factor $\sim \log(\tilde{m}^2/m_t^2)$. Clearly $\delta_1 m_i^2$ are smaller than the typical $\mathcal{O}(\tilde{m}^2)$ tree-level contributions, so they do not affect the degree of fine-tuning. On the other hand, $\delta_1\lambda_i$ can be relevant if the tree-level values are small, as it happens for instance in the MSSM (but not in models with sizeable $\lambda_{\text{tree}}$). These corrections are normally dominated by the top-stop sector with coupling $h_t = \sqrt{2}m_t/(v\sin\beta)$, which besides being $\mathcal{O}(1)$ has large multiplicity, $N_L + N_R = 12$. If some of the Higgs self-couplings, $\lambda_i$, are initially large, say $\mathcal{O}(1)$, they can also contribute substantially to $\delta_1\lambda_i$, though the multiplicity is smaller than for the stops. However, when $\delta_1\lambda_i \ll \lambda_{\text{tree}}$ such corrections can be ignored for the fine-tuning issue.

Consequently, for fine-tuning estimates, we approximate the radiative corrections by the logarithmic stop contribution (more sophisticated expressions for $\delta_1\lambda_i$ can be found in [72]):

$$\delta_1\lambda_2 = \frac{3h_t^4}{8\pi^2}\log\frac{M_{\text{SUSY}}^2}{m_t^2} . \tag{A.18}$$

In particular the approximate formula given in eq. (A.16) simply gets corrected by a factor $\lambda_{\text{tree}}/\lambda_{1-\text{loop}}$.

# Appendix B

# LEP Higgs bounds

The main Higgs production mechanism of the physical CP-even scalars $\mathcal{H}_\alpha^0 = h^0, H^0$ at LEP is $e^+e^- \to Z^0\mathcal{H}_\alpha^0$. The Higgs production cross-section is

$$\sigma_{Z\mathcal{H}_\alpha} = \xi_{\mathcal{H}_\alpha}^2 \sigma_{Zh}^{SM}(m_{\mathcal{H}_\alpha}^2) \ , \tag{B.1}$$

where $\sigma_{Zh}^{SM}(m^2)$ is the SM production cross-section for a Higgs with mass $m$ [73] and the prefactor $\xi_{\mathcal{H}_\alpha}$ measures the coupling $Z^0Z^0\mathcal{H}_\alpha^0$ relative to the SM value. In a generic 2HDM the linear combination along the breaking direction[1] $h_\parallel \equiv h_1^{0r}\cos\beta + h_2^{0r}\sin\beta$ has a coupling to $ZZ$ of SM strength while the orthogonal combination $h_\perp \equiv h_1^{0r}\sin\beta - h_2^{0r}\cos\beta$ does not couple to $ZZ$. In the basis $\{h_\parallel, h_\perp\}$ the mass eigenstates $h^0, H^0$ read

$$h^0 = \xi_h h_\parallel + \xi_H h_\perp \ , \quad H^0 = \xi_H h_\parallel - \xi_h h_\perp \ , \tag{B.2}$$

with $\xi_h^2 + \xi_H^2 = 1$. That is, the coupling $\mathcal{H}_\alpha^0 Z^0 Z^0$ is proportional to the amount of $h_\parallel$ that enters in the composition of $\mathcal{H}_\alpha^0$. From the definition of $\tan\beta$ and that of the mixing angle of the two CP-even Higgs bosons $h^0, H^0$:

$$\begin{aligned} h^0 &= h_2^{0r}\cos\alpha - h_1^{0r}\sin\alpha \ , \\ H^0 &= h_1^{0r}\cos\alpha + h_2^{0r}\sin\alpha \ , \end{aligned} \tag{B.3}$$

we obtain the familiar expressions

$$\xi_h^2 = \sin^2(\alpha - \beta) \ , \quad \xi_H^2 = \cos^2(\alpha - \beta) \ . \tag{B.4}$$

[1] We write $H_1^0 = (v_1 + h_1^{0r} + ih_1^{0i})/\sqrt{2}$ and a similar formula for $H_2^0$.

In the alternative scenario considered in section 4.2, at tree-level, the mass matrix for CP-even Higgses (in the basis $\{h_1^{0r}, h_2^{0r}\}$) is

$$\begin{aligned} M^2_{\mathcal{H}^0_\alpha} &= \begin{bmatrix} m_\parallel^2 c_\beta^2 + m_\perp^2 s_\beta^2 & (m_\parallel^2 - m_\perp^2) c_\beta s_\beta \\ (m_\parallel^2 - m_\perp^2) c_\beta s_\beta & m_\parallel^2 s_\beta^2 + m_\perp^2 c_\beta^2 \end{bmatrix} \\ &= \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} \begin{bmatrix} m_\parallel^2 & 0 \\ 0 & m_\perp^2 \end{bmatrix} \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} . \end{aligned} \tag{B.5}$$

This implies that $h_\parallel$ and $h_\perp$ are in fact mass eigenstates and means in particular that only $h_\parallel$ could be produced at LEP. For some choice of parameters (like in set A, used in section 4.2), $h_\parallel$ turns out to be the heavy state, and its mass makes it kinematically inaccessible at LEP. The light state turns out to be $h_\perp$ and even if it is light, it does not couple to $Z^0$ and therefore it is not produced.

At one-loop the previous situation changes. There are corrections to the mass matrix (B.5), the main one being

$$\delta\langle h_2^{0r}| M^2_{H_i^0} | h_2^{0r}\rangle = \frac{3m_t^4}{\pi^2 v^2 s_\beta^2} \log\frac{M_{\rm SUSY}}{m_t} , \tag{B.6}$$

that induce deviations of the mass eigenstates from $h_\parallel$ and $h_\perp$, making $\xi_h$ and $\xi_H$ different from 1 and 0. Working out the expression for the one-loop corrected $\alpha$, we arrive at the simple result

$$\xi_h^2 = \frac{(m_H^2 - m_{h_\parallel}^2)c_\beta^2 + (m_{h_\perp}^2 - m_h^2)s_\beta^2}{m_H^2 - m_h^2} , \tag{B.7}$$

and

$$\xi_H^2 = \frac{(m_H^2 - m_{h_\perp}^2)s_\beta^2 + (m_{h_\parallel}^2 - m_h^2)c_\beta^2}{m_H^2 - m_h^2} . \tag{B.8}$$

As discussed before, $\{h_\parallel, h_\perp\}$ are the tree-level mass eigenstates.

In order to implement the LEP bound in this alternative scenario, we conservatively impose that $\sigma_{Z\mathcal{H}_\alpha}$ should be smaller than $\sigma^{SM}_{Zh}(m_h^2)$ evaluated at $\sqrt{s} = 209$ GeV and $m_H = 115$ GeV (the ultimate LEP bound on the SM Higgs mass). This requirement can be represented as an upper limit on $\xi^2_{\mathcal{H}_\alpha}$ as a function of $m_h$. A more refined bound can be found on the experimental papers [74, 75].

Another possible Higgs production mechanism is associated production $e^+e^- \to A^0\mathcal{H}^0_\alpha$, with cross section given by [73]

$$\sigma_{A\mathcal{H}_\alpha} = (1-\xi^2_{\mathcal{H}_\alpha})\bar{\lambda}\sigma^{SM}_{Zh}(m^2_{\mathcal{H}_\alpha}) \ , \tag{B.9}$$

where $\bar{\lambda}$ is a kinematical factor. The non observation of this process sets a limit on our model. We implement this limit by using the experimental bound on the coefficient $(1-\xi^2_{\mathcal{H}_\alpha})$ derived *e.g* in [76] as a function of $m_{\mathcal{H}_\alpha}+m_A$. (We are conservative in using that experimental curve, which applies strictly to the case $m_{\mathcal{H}_\alpha} \simeq m_A$, and in assuming $\sim$ 100% branching ratios $A \to b\bar{b}$ and $\mathcal{H}_\alpha \to b\bar{b}$.) When $(1-\xi^2_{\mathcal{H}_\alpha}) \simeq 1$, this limit reads $m_{\mathcal{H}_\alpha}+m_A \lesssim 195$ GeV.

Finally, charged Higgs production ($e^+e^- \to H^+H^-$) does not give constraints in this scenario because $m_{H^\pm} \simeq 95$ GeV while the experimental limit is around $m_{H^\pm} \gtrsim 80$ GeV [77].

# Appendix C

# Fine-tuning estimates with constraints

Let $F(x_i)$ be a quantity that depends on some input parameters $x_i$ $(i = 1, ..., N)$, considered as independent. The fine-tuning in $F$ associated to $x_i$ is $\Delta_i$, defined by

$$\frac{\delta F}{F} = \Delta_i \frac{\delta x_i}{x_i} \ . \tag{C.1}$$

It is convenient for the following discussion to switch to vectorial notation and define

$$\vec{\Delta} F \equiv \left\{ \frac{\partial \log F}{\partial \log x_i} \right\} \ , \tag{C.2}$$

which is a vector of dimension $N$ with components $\Delta_i$, and is simply the gradient of $\log F$ in the $\{\log x_i\}$ space. Based on the statistical meaning of $\Delta_i$, we define the total fine-tuning associated to the quantity $F$ as

$$\Delta F \equiv \left[ \sum_i \Delta_i^2 \right]^{1/2} = ||\vec{\Delta} F|| \ . \tag{C.3}$$

This definition can be interpreted in the following statistical sense: if the input parameters $x_i$ are allowed to vary (randomly and independently) around their values, with $\delta x_i / x_i$ following gaussian distributions of width $\sigma_i$, then $\delta F/F$ in eq.( C.1) follows a gaussian distribution of width (squared)

$$\sigma_F^2 = \sum_i \Delta_i^2 \sigma_i^2, \tag{C.4}$$

which can be taken as a measure of the fine-tuning in $F$: notice that the quantity $(\delta F)^2$ is expected to take a value of order $F^2\sigma_F^2$. Now, it is sensible to assume that an unknown parameter $x_i$ may vary within a range of order $x_i$ (this is a common assumption in fine-tuning discussions) and hence to take $\sigma_i \sim 1$. Then, the definition in eq.( C.3) follows. In the case of an input parameter which is measured with some non-negligible experimental uncertainty $\epsilon_i^{exp}$ one should use $\sigma_i = \epsilon_i^{exp}/x_i$ in eq.( C.4). This is equivalent to rescaling the corresponding $\Delta_i$ as $\Delta_i \longrightarrow \Delta_i\sigma_i$ in eq. (C.3) [19].

Next suppose that the $x_i$ are not independent but are instead related by a number of (experimental or theoretical) constraints $G_\alpha^{(0)}(x_i) = 0$ ($\alpha = 1, ..., m$ with $m < N$) so that, when one computes the fine-tuning in $F$, one is only free to vary the input $x_i$'s in such a way that the constraints are respected. In order to compute the "constrained fine-tuning" in $F$ we first define, for each constraint, the vector $\vec{\Delta}G_\alpha^{(0)} = \{\partial G_\alpha^{(0)}/\partial \log x_i\}$ which is normal to the $G_\alpha^{(0)} = 0$ hypersurface in the $\{\log x_i\}$ space. We then use the Gramm-Schmidt procedure to get from the vectors $\vec{\Delta}G_\alpha^{(0)}$ an orthonormal set, $\vec{\Delta}G_\alpha$, that satisfies

$$\vec{\Delta}G_\alpha \cdot \vec{\Delta}G_\beta = \delta_{\alpha\beta} \ . \tag{C.5}$$

Then we can find the constrained fine-tuning simply projecting the unconstrained $\vec{\Delta}F$ on the $G_\alpha = 0$ manifold [which coincides with the $G_\alpha^{(0)} = 0$ manifold]:

$$\vec{\Delta}F\Big|_G = \vec{\Delta}F - \sum_\alpha (\vec{\Delta}F \cdot \vec{\Delta}G_\alpha)\vec{\Delta}G_\alpha \ . \tag{C.6}$$

Finally,

$$\Delta F|_G = \left|\left|\vec{\Delta}F\Big|_G\right|\right| = \left[(\Delta F)^2 - \sum_\alpha (\vec{\Delta}F \cdot \vec{\Delta}G_\alpha)^2\right]^{1/2} \ . \tag{C.7}$$

As it was to be expected, the constrained fine-tuning, $\Delta F|_G$, is always smaller that the unconstrained fine-tuning $\Delta F$.

The previous procedure can also be seen as a change of coordinates in the "euclidean" $\{\log x_i\}$ space [which leaves eq. (C.3) invariant], such that the first $m$ new coordinates $\{\log y_\alpha\}$ span the same subspace as the $\vec{\Delta}G_\alpha^{(0)}$ vectors. These $m$ coordinates have to be simply eliminated from eq. (C.3), as they are fixed by the constraints, while the remaining ones are totally unconstrained. In this way the final expression (C.7) is recovered.

Note that if $F$ does not depend on some of the parameters, say $\{x_a\}$, but some of the constraints do, the constrained fine-tuning will generically depend on the value of $\{x_a\}$, even if the other parameters remain the same. This is in fact a perfectly logical result. Notice that the fine-tuning quantity, $\Delta F$, measures the relative change of $F$ against the relative changes in the $x_i$ parameters. Imagine a function $F = F(x_1)$ and a constraint $G^{(0)} = x_1+x_2+x_3-C = 0$. If $x_2, x_3 \ll x_1$, the value of $x_1$ is essentially fixed and thus $\Delta F|_G$ should be small (if $x_2, x_3$ are allowed to change a 100%, $x_1$ is only allowed to change in a very small relative range). In the opposite case, if $x_2, x_3 \gg x_1$ (for the same value of $x_1$) the $x_1$ parameter can be freely varied and thus $\Delta F|_G \simeq \partial \log F/\partial \log x_1$. Therefore, $\Delta F|_G$ does depend on $x_2$ and $x_3$ even if $F = F(x_1)$. We have found this effect in some of the scenarios studied (although it always had a mild impact on the final fine-tuning); see the section dedicated to the Littles Higgs with $T$-parity.

# Appendix D

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# Bibliography


[1] S. Glashow, Nucl. Phys. **22** (1961) 579; S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264; A. Salam, in "Elementary Particle Theory", ed. N. Svartholm, Almqvist and Wiksells, Stockholm (1969) p. 367.

[2] M. Gell-Mann, Phys. Lett. 8 (1964) 214; G. Zweig, CERN-Report 8182/TH401 (1964); H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47 (1973) 365; D. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346; G. 't Hooft, Marseille Conference on Yang–Mills fields (1972).

[3] P. W. Higgs, Phys. Lett. **12** (1964) 132; Phys. Rev. Lett. **13** (1964) 508; Phys. Rev. **145** (1968) 1156.

[4] LEP Collaborations and the LEP Electroweak Working Group, hep-ex/0511027.

[5] S. Weinberg, Phys. Rev. D **13** (1976) 974 and Phys. Rev. D **19** (1979) 1277; L. Susskind, Phys. Rev. D **20** (1979) 2619.

[6] R. J. Davis, D. S. Harmer and K. C. Hoffman, Phys. Rev. Lett. **20**, (1968) 1205; B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); K. Lande *et al.*, Nucl. Phys. Proc. Suppl. **77**, 13 (1999); A. I. Abazov *et al.* [SAGE Collaboration], Phys. Rev. Lett. **67**, (1991) 3332; W. Hampel *et al.* [GALLEX Collaboration], Phys. Lett. B **447**, (1999) 127; T. A. Kirsten [GNO Collaboration], Nucl. Phys. Proc. Suppl. **118** (2003) 33; S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. B **539** (2002) 179; R. Becker-Szendy *et al.*, Nucl. Phys. Proc. Suppl. **38**, (1995) 331; S. Fukuda *et al.* [Kamiokande Collaboration], Phys. Lett. **B335**, (1994) 237; Y. Ashie *et al.* [Super-Kamiokande Collaboration], arXiv:hep-ex/0404034.

[7] Y. A. Golfand and E. P. Likhtman, JETP Lett. **13** (1971) 323; D. V. Volkov and V. P. Akulov, Phys. Lett. B **46** (1973) 109; J. Wess and B. Zumino, Nucl. Phys. B **70** (1974) 39.

[8] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513** (2001) 232; N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. **86** (2001) 4757; N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP **0208** (2002) 021.

[9] H. E. Haber and Y. Nir, Nucl. Phys. B **335** (1990) 363.

[10] G. F. Giudice, Int. J. Mod. Phys. **A19** (2004) 835.

[11] R. Barbieri and A. Strumia, "The 'LEP paradox'," [hep-ph/0007265];
R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, "Electroweak symmetry breaking after LEP1 and LEP2," Nucl. Phys. B **703**, 127 (2004) [hep-ph/0405040].

[12] M. J. G. Veltman, Acta Phys. Polon. B **12** (1981) 437.

[13] M. B. Einhorn and D. R. T. Jones, Phys. Rev. D **46** (1992) 5206.

[14] C. F. Kolda and H. Murayama, JHEP **0007** (2000) 035 [hep-ph/0003170].

[15] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP **0411**, 057 (2004) [hep-ph/0410298].

[16] R. Barbieri and G. F. Giudice, Nucl. Phys. B **306** (1988) 63.

[17] B. de Carlos and J. A. Casas, Phys. Lett. B **309** (1993) 320 [hep-ph/9303291].

[18] G. W. Anderson and D. J. Castaño, Phys. Lett. B **347** (1995) 300 [hep-ph/9409419]; Phys. Rev. D **52** (1995) 1693 [hep-ph/9412322]; Phys. Rev. D **53** (1996) 2403 [hep-ph/9509212].

[19] P. Ciafaloni and A. Strumia, Nucl. Phys. B **494** (1997) 41 [hep-ph/9611204].

[20] D. Whiteson [CDF Collaboration], arXiv:hep-ex/0605106.

[21] R. Barbieri, T. Gregoire and L. J. Hall, [hep-ph/0509242].

[22] R. Barbieri and L. J. Hall, [hep-ph/0510243].

[23] R. Barbieri, L. J. Hall and V. S. Rychkov, [hep-ph/0603188].

[24] J. A. Casas, J. R. Espinosa and I. Hidalgo, to appear.

[25] C. P. Burgess and D. London, Phys. Rev. D **48** (1993) 4337 [hep-ph/9203216].

[26] J.F. Gunion, H. E. Haber, G.L. Kane and S. Dawson, "The Higgs Hunter's Guide", Addison-Wesley, CA,1990

[27] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP **0503**, 038 (2005) [hep-ph/0502066].

[28] K. Hagiwara, S. Matsumoto, D. Haidt and C. S. Kim, Z. Phys. C **64**, 559 (1994) [Erratum-ibid. C **68**, 352 (1995)] [arXiv:hep-ph/9409380].

[29] Z. Chacko, H. S. Goh and R. Harnik, [hep-ph/0506256].

[30] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.
H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, (1974) 451.

[31] M. Gell-Mann, P. Ramond and R. Slansky, edited by P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315; T. Yanagida, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[32] F.Vissani, Phys. Rev. D **57** (1998) 7027 [hep-ph/9709409]; J. A. Casas, V. Di Clemente, A. Ibarra and M. Quirós, Phys. Rev. D **62** (2000) 053005 [hep-ph/9904295]; J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [hep-ph/0103065].

[33] L Girardello and M.T.Grisaru, Nucl. Phys. B **194**, 65 (1982).

[34] H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75. J. F. Gunion and H. E. Haber, Nucl. Phys. B **272** (1986) 1. J. F. Gunion and H. E. Haber, Nucl. Phys. B **278** (1986) 449.

[35] See *e.g.* the appendix of J. A. Casas, J. R. Espinosa and H. E. Haber, Nucl. Phys. B **526** (1998) 3 [hep-ph/9801365].

[36] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP **0401**, 008 (2004) [hep-ph/0310137].

[37] S. Dimopoulos and G. F. Giudice, Phys. Lett. B **357** (1995) 573 [hep-ph/9507282].

[38] K. Agashe and M. Graesser, Nucl. Phys. B **507** (1997) 3 [hep-ph/9704206].

[39] P. H. Chankowski, J. R. Ellis and S. Pokorski, Phys. Lett. B **423** (1998) 327 [hep-ph/9712234].

[40] R. Barbieri and A. Strumia, Phys. Lett. B **433** (1998) 63 [hep-ph/9801353].

[41] P. H. Chankowski, J. R. Ellis, M. Olechowski and S. Pokorski, Nucl. Phys. B **544** (1999) 39 [hep-ph/9808275].

[42] G. L. Kane and S. F. King, Phys. Lett. B **451** (1999) 113 [hep-ph/9810374].

[43] L. Giusti, A. Romanino and A. Strumia, Nucl. Phys. B **550** (1999) 3 [hep-ph/9811386].

[44] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84** (2000) 2322 [hep-ph/9908309].

[45] K. Agashe, Phys. Rev. D **61** (2000) 115006 [hep-ph/9910497].

[46] M. Bastero-Gil, G. L. Kane and S. F. King, Phys. Lett. B **474** (2000) 103 [hep-ph/9910506].

[47] A. Romanino and A. Strumia, Phys. Lett. B **487** (2000) 165 [hep-ph/9912301].

[48] G. L. Kane, J. Lykken, B. D. Nelson and L. T. Wang, Phys. Lett. B **551** (2003) 146 [hep-ph/0207168].

[49] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **257** (1991) 83; Phys. Lett. B **262** (1991) 477; Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85** (1991) 1; Phys. Lett. B **262** (1991) 54; H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66** (1991) 1815; R. Barbieri, M. Frigeni and M. Caravaglios, Phys. Lett. B **258** (1991) 167.

[50] M. Olechowski and S. Pokorski, Nucl. Phys. B **404** (1993) 590 [hep-ph/9303274].

[51] G. L. Kane, B. D. Nelson, T. T. Wang and L. T. Wang, [hep-ph/0304134].

[52] M. Bastero-Gil, C. Hugonie, S. F. King, D. P. Roy and S. Vempati, Phys. Lett. B **489** (2000) 359 [hep-ph/0006198].

[53] K. Harada and N. Sakai, Prog. Theor. Phys. **67** (1982) 1877; D. R. Jones, L. Mezincescu and Y. P. Yao, Phys. Lett. B **148** (1984) 317; I. Jack and D. R. Jones, Phys. Lett. B **457** (1999) 101 [hep-ph/9903365]; L. J. Hall and L. Randall, Phys. Rev. Lett. **65** (1990) 2939; F. Borzumati, G. R. Farrar, N. Polonsky and S. Thomas, Nucl. Phys. B **555** (1999) 53 [hep-ph/9902443]; S. P. Martin, Phys. Rev. D **61** (2000) 035004 [hep-ph/9907550].

[54] A. Brignole, F. Feruglio and F. Zwirner, Nucl. Phys. B **501** (1997) 332 [hep-ph/9703286].

[55] N. Polonsky and S. Su, Phys. Lett. B **508** (2001) 103 [hep-ph/0010113]; Phys. Rev. D **63** (2001) 035007 [hep-ph/0006174].

[56] A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, Nucl. Phys. B **666** (2003) 105 [hep-ph/0301121].

[57] N. Arkani-Hamed and S. Dimopoulos, [hep-th/0405159]; G. F. Giudice and A. Romanino, [hep-ph/0406088]; N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, [hep-ph/0409232].

[58] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.

[59] H. Georgi and A. Pais, Phys. Rev. D **10** (1974) 539. H. Georgi and A. Pais, Phys. Rev. D **12** (1975) 508.

[60] D. E. Kaplan and M. Schmaltz, JHEP **0310** (2003) 039 [hep-ph/0302049].

[61] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207** (2002) 034 [hep-ph/0206021].

[62] M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D **69** (2004) 075002 [hep-ph/0310039].

[63] H. C. Cheng and I. Low, JHEP **0408** (2004) 061 [hep-ph/0405243].

[64] M. Schmaltz, JHEP **0408**, 056 (2004) [hep-ph/0407143].

[65] I. Low, W. Skiba and D. Smith, Phys. Rev. D **66** (2002) 072001; S. Chang and J. G. Wacker, arXiv: hep-ph/0303001; C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D **68** (2003) 035009; T. Gregoire, D. R. Smith and J. G. Wacker, arXiv: hep-ph/0305275; W. Skiba and J. Terning, Phys. Rev. D **68** (2003) 075001.

[66] A. Manohar and H. Georgi, Nucl. Phys. B **234** (1984) 189; H. Georgi, Benjamin/Cummings, (Menlo Park, 1984); H. Georgi and L. Randall, Nucl. Phys. B **276** (1986) 241.

[67] J. Erler and P. Langacker, [hep-ph/0407097];
in S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592** (2004) 1.

[68] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D **67**, 115002 (2003) [hep-ph/0211124];

Phys. Rev. D **68**, 035009 (2003) [hep-ph/0303236];
J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP **0310** (2003) 062 [hep-ph/0211218];
T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Rev. D **67**, 095004 (2003) [hep-ph/0301040];
M. C. Chen and S. Dawson, Phys. Rev. D **70** (2004) 015003 [hep-ph/0311032].

[69] M. Soldate and R. Sundrum, Nucl. Phys. B **340** (1990) 1.

[70] H. E. Haber and R. Hempfling, Phys. Rev. D **48** (1993) 4280 [arXiv:hep-ph/9307201].

[71] See *e.g.* sect. 3 in B. de Carlos and J. R. Espinosa, Nucl. Phys. B **503** (1997) 24 [hep-ph/9703212].

[72] M. Carena, J. R. Espinosa, M. Quirós and C. E. Wagner, Phys. Lett. B **355** (1995) 209 [hep-ph/9504316]; H. E. Haber, R. Hempfling and A. H. Hoang, Z. Phys. C **75** (1997) 539 [hep-ph/9609331].

[73] See *e.g.* M. Spira and P. M. Zerwas, [hep-ph/9803257].

[74] A. Heister *et al.* [ALEPH Collaboration], Phys. Lett. B **526** (2002) 191 [hep-ex/0201014]; J. Fernandez [DELPHI Collaboration], [hep-ex/0307002]; G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **27** (2003) 311 [hep-ex/0206022]; P. Achard *et al.* [L3 Collaboration], Phys. Lett. B **545** (2002) 30 [hep-ex/0208042].

[75] G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **18** (2001) 425 [hep-ex/0007040].

[76] [LEP Higgs Working Group Collaboration], [hep-ex/0107030].

[77] [LEP Higgs Working Group for Higgs boson searches Collaboration], [hep-ex/0107031].