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Modelling Functional Area and Commuting Flows

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Abstract

Traditionally large cities had also concentrated significant economic, social and demographic power due to economies of agglomeration. This article presents a hypothesis about modification of these economies by new technologies of information and communication, which change spatial coordinates and diminish the role of distance.

This paper presents an economic analysis of residential location of economic agents, with a special attention paid to urban dispersion. The paper derives the attractiveness of a city as the function of its population and studies the equilibrium split of population across cities. The main goal is to explain price formation and urban dispersion through production structure and wages determined by sizes and relative location of cities. The results show that different neighbouring municipalities become economically interrelated, and we need to shift our attention to a new spatial object of analysis —functional urban areas, which is a product of spatial aggregation of municipalities with respect to housing and labour markets. It implies also a policy

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change which should take into account this interdependence of neighbouring markets. For example, NUTS-III in the European Community should be replaced by functional areas.

Keywords: housing prices, cities, economies of agglomeration, location choice.

JEL Classification: H22, R12; R52, H54

1. Introduction

The goal of this article is to develop theoretical model that allow us to determine the spatial patterns of location of economic agents and to explain the process of residential dispersion in big cities caused by centrifugal forces.

The decentralization of productive system as the consequence of economic globalization leads to an emergence of a network of big cities, which concentrate economic power via the production of advanced services. Due to scale economies, these cities create additional demand for labour, which attracts significant migration flows. However, competition for space between productive and residential sectors inside these cities leads to high land rents and housing prices in central areas, which make them affordable only for business and services. Thus, CBD is created, while the workers have to commute with it from the periphery of the city. They locate in the so called functional area of the city and commute to the CBD as the place of their work. With an increase in technical possibilities to commute, the functional area is growing, and counter-urbanization is taking place. Berry (1976) was the first to describe this process of economic and demographic reverse of flows in big metropolitan areas in favour to periphery in the last decades of the 20th century.

In order to explain these processes we start from a theoretical model, which determines an optimal population for a city and equilibrium partition of population across two cities, or municipalities. We also study a comparative statics of this equilibrium with respect to external shocks.

1.1. *Observation and Hypothesis*

During the 19th and 20th centuries the process of urbanization was taking place; economic activity and population were concentrating in big cities. But later this central city became less concentrated, forming with its growing periphery a big city-region.

Why has this happened?

Due to the reduction of centripetal forces in favour of centrifugal. New transportation and communication technologies have reduced the benefits from agglomeration and thus have changed spatial coordinates. Mobility became less expensive and now takes less time. This has increased the potential distance of commuting for workers.

But transportation of goods also became less expensive, while the cost of information delivery depends very little on distance. This leads to globalization, but we will not focus on long-distance (inter-country) effects in this study.

2. Relation between Residential and Labour Markets

2.1. *Agglomeration and externalities*

Traditionally, the process of urbanization and agglomeration was explained from the point of view of urban economics, which have been derived in 1970ies (Henderson, 1974) on the basis of pioneering work of Alonso (1964).

The recent studies in the field of new economic geography were summarized in the book of Fujita, Krugman and Venables (1999): they explain the dynamics of urban hierarchy through an interplay of centrifugal and centripetal forces.

And there were practically no attempts to link together these approaches, in order to determine the relation, which exists between labour and housing markets. Tabuchi (1998) proposes a possibility of synthesis between Alonso and Krugman, or between urban agglomeration and dispersion. This paper attempts to follow this objective.

Let us consider the economies of agglomeration, which make large urban areas an attractor of population during the last decades. The global evolution of employment served as a positive shock for housing demand, especially in city centres, where real estate prices have been growing above the general level of inflation[1]. If we follow the classification of Glaeser et al (2001) of external and agglomeration economies, we can tell that externalities of Jacobs type (diversification as an engine of

[1] Of course, not all price rise was justified by this reason, bubbles also occurred. Annual appreciation of 15-30%, typical for Moscow, Barcelona and other cities in 2002,2003,2004 and 2005 cannot be explained by purely employment factor, but also contain an element of portfolio choice when financial markets are downward sloping.

innovation) are more relevant for large cities, while externalities of Marshall-Arrow-Romer type (related to specialization) represent a property of smaller cities.

That is why, advanced technologies and labour of high quality are attracted by dominant cities of the highest hierarchy. The reason is that research and development activity exhibits both higher fixed costs and associated economies of scale, which can break even only in urban agglomeration of a significant size. Big cities also possess the necessary infrastructure, qualified labour market and proximity to other activities. Thus, the process of growth of large urban areas becomes self-reinforcing.

On the other hand, if a firm operates under standard technology, it might also require a location in a city above some threshold size, due to scale economies, but the optimal city size for this productive activity will be less, since it may be unable to extract additional benefits in a large city that would compensate higher land rents.

In the city of the highest hierarchic level, we observe a vicious cycle: higher specialization, higher salary, expansion of employment, higher residential demand for land and higher prices for real estate. As a result, we observe both expansion and spatial dispersion of large cities.

2.2. *Alternative explanation*

Following Lopez Garcia (1992), the housing price reflects the characteristics of demand and supply through the supply of land, income level of population, interest rate, tax laws, etc. But these factors might not explain the differences of housing prices across municipalities and the differences in the growth rates of prices across them. In order to explain these differences, we need to introduce the spatial structure of an effective demand and to derive the potential of these municipalities from the following points of view:

- demand for space by productive capital,
- population mobility inside functional area due to the process of adjustment of residential preferences,
- net migration received by the considered functional area.

This effective demand is responsible for urban hierarchy, size of municipalities and spatial dynamics of the functional area. Reassignment of population and capital across locations in the process of this dynamics implies also an evolution of housing prices.

New technologies of communication and information change the spatial coordinates and allow populating areas more distant from the CBD. This reduces the benefits from agglomeration for a significant fraction of population and business. At the same time, the changes in organizational structure of firms reinforce the attractiveness of central city and diminish the attractiveness of the regions on a periphery.

All these factors and processes define the position of cities and municipalities in urban hierarchy and the level of attractiveness. The structure of residential market corresponds to monopolistic competition, where each location has its particular characteristics and thus represents a local partial monopoly.

3. Theoretical Model

We would like to have a theoretical basis for the competition between municipalities for labour force, which describes the flows of migration from the municipalities with lower benefits per capita to those with higher benefits. These flows exist since agents can participate in spatial arbitrage, voting by foot, like in Tiebout model. We also would like to see how external shock changes the equilibrium allocation of labour. While the empirical model for Barcelona functional area will be formulated for many municipalities, it makes sense to start with a simple theoretical model that can predict an equilibrium pattern of labour allocation across two cities.

Our theoretical model includes a highly stylized equilibrium story for an optimal city size, in the spirit of model of Henderson (1974). Then we suggest an economic explanation for commuting.

- *Assumptions:*

1. There exist two cities (1,2) and N identical agents, facing a binary choice: to live in city 1 or in city 2.
2. Every city i produces some benefits B(i) that attract agents, but to live in a city they have to pay city-specific costs C(i). For simplicity, we assume that benefits are derived from wages, while costs are related to housing prices. All other factors (ecology, climate, etc) can be counted as location-specific factors, that perturb initially symmetric model. Attractiveness of a city is the difference between benefits and costs: V(i)=B(i)-C(i). The flow of migrants is proportional to the differential in attractiveness between cities i and j: F(ij) = ? (V(j)-V(i)).

3. Every city has an industry with Leontief-type technology Q = min {K, L} (here K denotes capital and L-labour) and fixed costs F. The markets for capital and industrial product are perfectly competitive, with r being the price of capital, and p being the price of final good. The wage w depends on number of workers and thus depends on city size. We further assume that all population of the city is employed in this industry.
4. Every city has a radially symmetric structure with constant population density ?, CBD in its centre and internal transport costs linear in distance. Thus agents who live at its periphery are compensated for transport costs by lower location rent and housing price.

• *A Model of Optimal City*

We will first derive how wage depends on city size, and then will find the relationship between city size and housing price. Thus, we will obtain the attractiveness of a city as a function of its population.

• *How does the wage depend on city size?*

We assume that each city has a firm which operates with zero profit (monopolistic competitive environment). Thus, the benefits from sales should cover fixed costs and the rest should be split across labour and capital. For Leontieff production function, it is optimal to operate with equal size of inputs; hence, Q = K = L. Now we will rely on a stylized fact of perfect capital and imperfect labour mobility. Due to perfect competition in financial markets and perfect capital mobility, every unit of capital receives the same return r, while labour can get different returns in different cities. Hence, we get pL = (w+r) L + F. Since L=N(i), the expression for the wage is the following:

$$w(i) = p - r – F/N(i) \qquad (1)$$

In fact, the wage depends on city size only, w(N) = C - F/N. This function is concave in N: w'>0, w''<0. Very small cities simply cannot exist (wage should be negative), while for very large cities the wage approaches its upper limit: w(∞)=C.

• *Housing price and city size*

Consider a city as monocentric CBD, with r as the distance from the city centre (Alonso model). For simplicity, we assume linear transport cost and will abstract from distance effect on dwelling size as well as on the height of the buildings.

The housing price, P_h, is assumed to be a linear function of location rent R(r):

$$P_h(r) = R(r) + H$$

where H denotes construction cost, equal for all locations.

At the edge of the city, the location rent is equal to agricultural rent R_a, which is assumed to be a constant, independently on location. Then, the location rent in the centre $R(0) = R_a + t\, r^*$, where t is unit distance transport cost. Assume that a city has a radius r^*. Then its population equals to $N = ?\pi\, (r^*)^2$.

Hence, $r^* = \sqrt{N/(?\pi)}$. The housing price in city centre, $P_h(0) = R_a + H + t\, r^*$. It will be considered as housing price index, or cost of living, for the whole city: $P(i) = P_h(0)$. The reason is that only the residents in this point do not face transport cost, and this price exactly equals the sum of housing rental price plus internal transport cost for the whole city. This cost of living in a city contains a term proportional to a square root of its population:

$$P(i)= a + b \sqrt{N(i)} \qquad (2)$$

It is also concave in N, but the shape differs from w(N).

Now we can write a formula for city attractiveness:

$$V(i) = w(i) – P(i) = p - r - a – F/N(i) – b \sqrt{N(i)} \qquad (3)$$

The shape of attractiveness reminds one in the model of Henderson (1974): there exists a unique optimal city size. We can differentiate the formula (3) w.r.t. N(i) to find an optimal city size. The f.o.c. leads to the following expression:

$$2 F \sqrt{N} = b N^2, \text{ or } 4F^2 = b^2 N^3$$

The last equation has a unique positive root, $N^*=(2F/b)^{2/3}$. At this level of N*, the second derivative is negative, $V'' = - 3 b^2/(8F) < 0$, and we really have a maximum.

Proposition 1: Under assumptions 1-4, both wages and housing prices positively depend on city size. However, an interplay between these factors leads to a unique optimal city size, which creates the highest attractiveness for its citizens. Hence, the potential of city attractiveness has an inversed U-shape. First, the attractiveness increases with population growth, and then declines. It is the result of interplay between centripetal forces (due to economies of scale) and centrifugal forces (expulsion due to high residential prices caused by competition for scarce land in a city).

3.1. *Comparative statics of an equilibrium*

It is easy to show that $\partial N^*/\partial b<0$ and $\partial N^*/\partial F>0$. Thus, a decline in transportation cost b results in an increase of optimal city size. Perhaps, this is the reason of continuous growth of urban centres, especially in less developed countries (Mexico city, Bombay, San Paolo, etc). On the other hand, an increase in fixed costs F leads to an increase of city size. Another consequence of the uniqueness of optimal city size is the shape of attractiveness V(N): it starts as convex (IRS) function, and continues as concave (DRS) function.

However, if all population N has to be split across two cities, there is little chance that both cities can have optimal sizes. This question will be analyzed in the next subsection.

Gráfico 1. Optimal city size

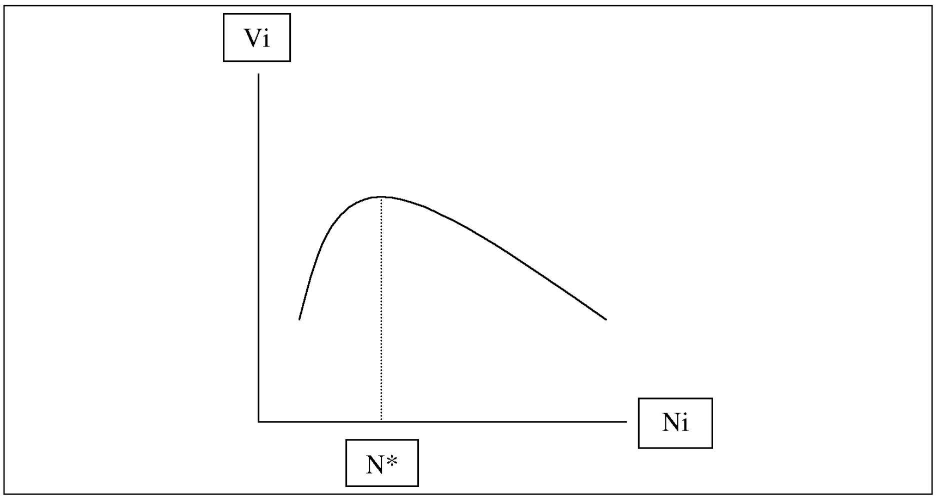
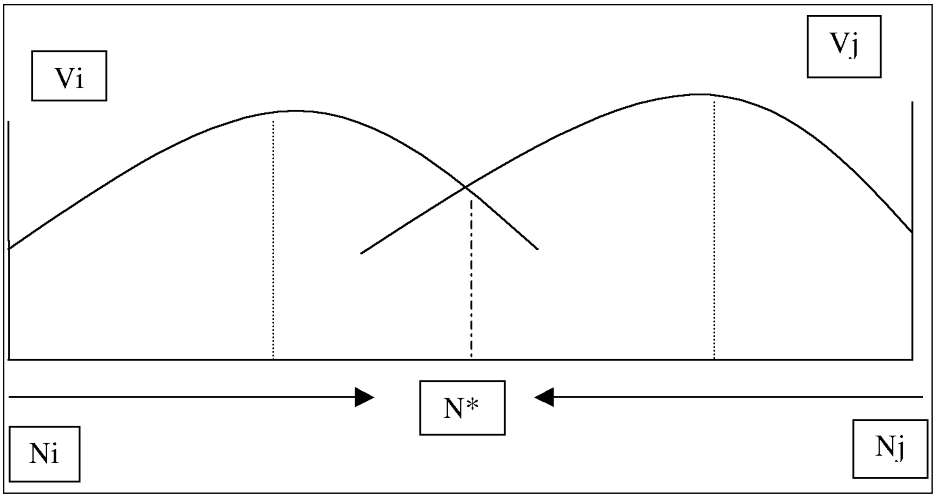


Gráfico 2. Equilibrium split of population between two cities

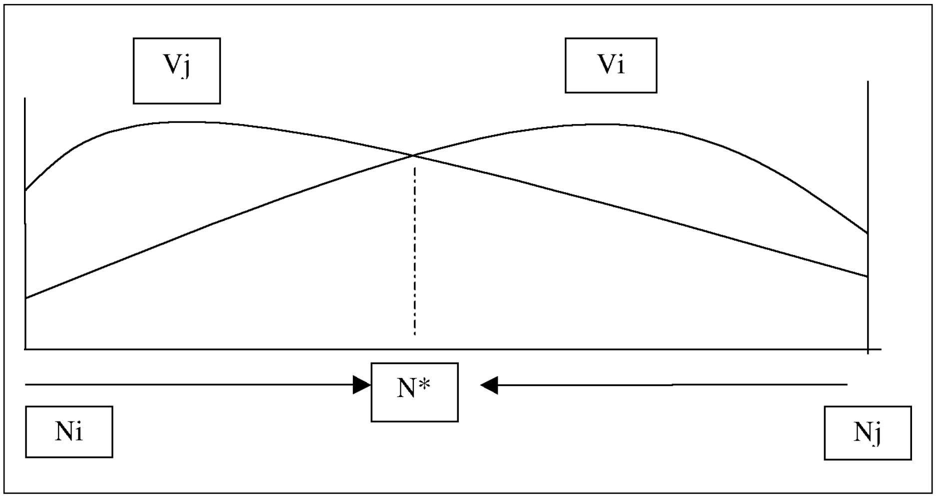


- *Competition between Two Cities for Labour: Two Types of Equilibrium*

3.2. *Stable equilibrium*

If we have to split all population N across two cities, the result depends on this N. Assume first, that cities are symmetric. If N>2N*, the curves intersect in the area of decreasing returns (every additional citizen negatively contributes to the utility of the rest). This equilibrium is stable (see Fig.2). Imagine that initial split of population is different from this equilibrium allocation. Then, according to assumption 2, the difference between potentials of attractiveness will create a flow of migrants from less attractive to more attractive city. This difference will continuously decline, until it fully disappears.

Gráfico 3. Unstable equilibrium



• U*nstable equilibrium*

On the other hand, if N<2N*, two curves intersect on IRS parts (Fig. 3). This equilibrium is unstable. Consider initial allocation different from equilibrium. If a marginal worker moves from less to more attractive city, the attractiveness of less attractive city drops even more, while for more attractive city it increases further. Thus, any deviation from equilibrium will be self-reinforcing. In fact, cities fight for additional workers in this case. While this is an interesting case, which may explain why historically not all cities were growing, but some of them ended in a decline, we will not focus on elaboration of these ideas in this paper.

While our formal analysis is provided for the system of two cities (municipalities), the results can be applied for a functional area of more complex structure. We can infer that there exists an optimal spatial distribution of population and capital[2].

[2] We cannot prove the theorem of uniqueness, there might be multiplicity of equilibria.

3.3. *Extension of the Model*

In reality, we have more than two cities. Also, not only direct benefits derived from wages and costs related to housing prices define the decisions of agents to locate in a particular city. There are also geographical factors (climate, beautiful views, sea, mountain, forest, lake, etc) that affect their decisions. Since we cannot model the influence of these factors in an explicit form, we demote them as Q. Since we have n different locations (cities, functional area or municipalities), we get n different utilities U(i) that an agent can obtain:

$$U(i)=U(V(i),Q(i)),\ i=1,2,\ldots,n,$$

where V(i) is defined by formula (3) and depends on salary and housing price, which both depend on city size N(i) (through capital, infrastructure and other economic factors and externalities). Note, that we can have U(i)=U(j), when $N(i)\neq N(j)$. Hence, the introduction of geographic factors allows explaining the difference in sizes of the cities (otherwise, all cities would have a unique optimal size).

Until now, we have been working with a static microeconomic model, and now we turn to its dynamic generalization. The static model formally corresponds to a short run horizon, where capital stock is fixed and geographic factors are static. We have seen that location economic net benefits V(i) are determined only by city population N(i). We can generalize the equilibrium theory, which was developed before for 2 cities without geographic externalities, for the case of n cities.

- *Equilibrium conditions.*

Let N(i) denotes the population of our city i, while N(j) represents the populations in the rest of locations. Then we have a stable equilibrium allocation of the population between all locations, if for every agent k from location i the following conditions hold:

1) $Uk(V(N(i)), Q(i)) \geq Uk(V(N(j)), Q(j))$, for all k from i,
2) $Uk(V(N(i)+1, Q(i)) \leq Uk(V(N(j)), Q(j))$, for all $j \neq i$,
3) $N(i)+ \sum N(j)=N$.

In other words, equilibrium allocation is stable if every agent locating in city i is not better off by moving to other location. Since agents in our model are assumed to be identical[3], there is no difference between utility of every citizen k in a particular city i, but only across cities, and this simplifies analysis. Besides, if additional agent will come to this location i, the utility of her (and the rest of citizens via externality effect) will decline. By this reason, we always have population above optimal level in equilibrium. But if it is below, nothing can stop a city from further growth before it reaches an optimal level. Alternatively, it can become depopulated. In this case, we have unstable equilibrium, which was considered before.

In order to study the dynamics in the middle run, we introduce the effect of shocks on the structure of equilibrium.

- *Effects of Shocks*

Consider the effect of a shock on equilibrium allocation, in the case of stable equilibrium. Assume that a positive shock results in a multiplication of attractiveness of city i by some factor $\beta>1$. Thus, new attractiveness for city i is given by $V^*(N(i))=\beta V(N(i))$, while for cities $j\neq i$ it stays at the old level. Now the curves will intersect in a new point, and equilibrium will shift. The dynamics of process is also clear: after the shock there exists a difference between cities i and j in attractiveness, and it drives migration, until new equilibrium is achieved. Similarly, an effect of negative shock can be considered.

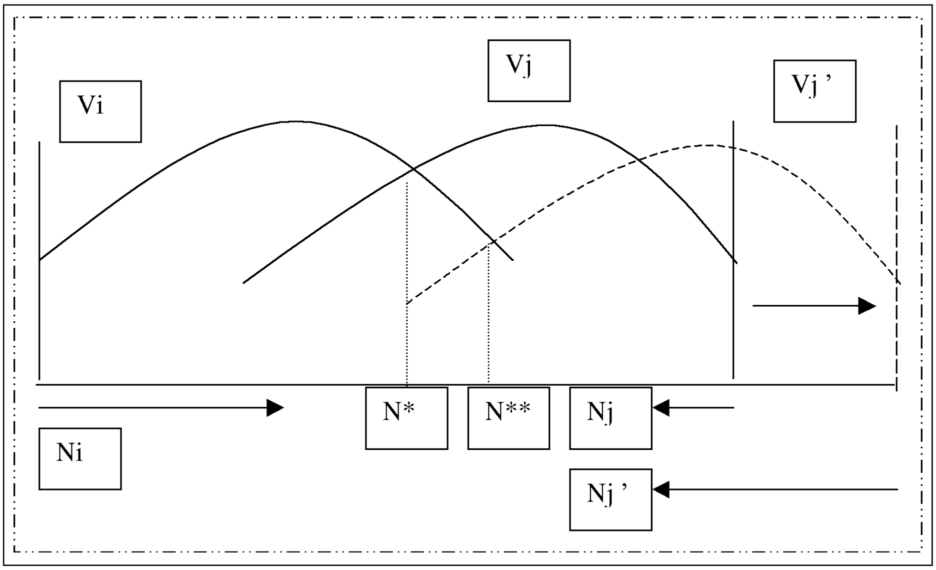
While traditionally economists think in the terms of demand or supply shocks, we find it convenient to introduce also internal and external shocks, from the point of view of location i. Further on, we assume that internal shocks are those which shift the schedule of the curve U(i), either affecting economic costs or benefits in this location, or perturbing geographic attractivity. Equilibrium can also be affected if nothing happens in municipality i, but occurs to municipality j.

How the change of spatial coordinates (caused by technological progress in transportation) will affect the equilibrium split of population? What will be the impact of particular projects and policies on this split?

[3] In fact, they may differ not only in income, but also in geographic preferences. Then we are no longer in the environment of the assumptions of our model.

What is the effect of additional stock of migrants arriving to some municipality? This shifts the total population N and the current position of population in this municipality along the curve. On the other hand, the shapes of all curves U(i) are not affected. New equilibrium population split will thus differ from the previous. It might involve additional re-migration. Generally, migration shock causes a decline in utility. This result can be easily shown analytically: if we have a stable equilibrium (Fig.2), the positive shift of N causes two curves to intersect at a level with lower value of utility. On the other hand, in the environment of unstable equilibrium (the phase of city growth), migration inflow can be mutually beneficial.

Gráfico 4. The change of population in city j from Nj to Nj' leads to a new equilibrium N** after re-migration. The utility level is lower



Speaking about the case of Catalonia, we can focus on two effects. In 1960ies, its particular municipalities have received a significant inflow of migrants from other Spanish regions. Currently, the municipalities along Mediterranean coast have a potential to attract retired population from European Union.

- *Internal and external shocks*

Internal shocks change utility of citizens in location i. For shocks of different nature, either the economic attractiveness can be shifted upwards (positive economic shock) or downwards (negative shock), or geographical attractiveness can change, due to some location externality (ecological disaster, effects of global warming, etc). If U(i) shifts upwards, this city i becomes more attractive, and receives additional migrants from other areas. The overall effect is likely to be positive (higher utility at new equilibrium), although city may loose some of its short-run gains from the positive shock in the middle run.

In an integrated world, one might be affected when something happens with your neighbour. Suppose, the curve U(j) got a downward shift due to geographic factors (ecological disaster). Then, migrants from that area to other regions, including i, would increase the population in i further away from its optimal level, and new utility will be lower.

3.4. *Modelling of Commuting Flows*

These commuting flows can never be modelled in a fully rational framework, since then any small difference of utilities results in a shift of all the population across locations. In fact, we deal with unobservable individual-specific variables, like the cost of shifting, attachment to particular location, etc. As a result, the difference in economic attractiveness across locations results only in flows of finite size, which increase with the difference.

What are these variables, that are responsible for commuting decisions? We have mentioned already that attractiveness is proportional to the difference between wages, w(i), and housing prices, P(i). Hence, these two variables, should be included. Since agents can also choose the size of their house (and we observe a difference in average housing size across municipalities), this variable can also be explanatory.

But how to measure geographic factors? It is possible to find a proxy variable: net migration inflow over a certain period to municipality i, MF(i). This variable captures average general attractiveness of location i with respect to all other locations in its historical perspective. This means that attractiveness could depend on time, and we need to aggregate. While economic factors could change over time in relative terms (for example, Barcelona city was a centre of attraction before 1980, and then began to loose its population, partly because of high housing prices), geographic factors are less likely to change so rapidly. Hence, aggregation over time will filter out some economic variable factors, while keep geographic.

Some other variables can also be responsible for commuting decision. For example, we can observe a variable: share of labour force in industry of intermediate goods. While all locations are able to create similar per capita employment levels in services oriented on local population (tourism is a separate case), location of an industrial factory creates significant amount of additional jobs (for example, SEAT in Martorell). Thus, even if equilibrium salaries would not exhibit a premium with respect to other locations, local unemployment rates would drop and labour force from other, less favourable areas, could exploit spatial arbitrage with respect to a higher probability of employment.

- *Why do agents commute and not migrate?*

If migration would be the only way to utilize spatial arbitrage for labour, it could bring the system to the equilibrium in the long run. Since regional shocks occur from time to time, most of the time the system is out of equilibrium. Thus, we observe some migration and commuting flows, which will be studied later empirically. On the other hand, positive commuting flows are also consistent with asymmetric equilibrium. Assume that:

$$wi - H\,Pi = wj - H\,Pj$$

(here H is the size of housing), but wj >wi. Then agents have more incentive to commute from i to j, then to migrate to j. This is a typical case of Barcelona city, which was studied in Mascarilla and Yegorov (2002).

- *Assumptions of revealed preferences in the basis of empiricals studies*

Assumption: if the mobility data show that a significant per cent age of agents live in municipality different from their work, *ceteris paribus*, if an individual does not migrate closer to a place of his work, it is because she reveals her preferences towards residential location. Thus, observing the commuting flows, we can make conclusions about preferences for residential locations.

Variable endogenous:

M ij-share (takes value between 0 and 1) of workers who reside in municipality i and commute to the place of their work in community j (measures the potential of attraction of location j for labour residing in i),

$$M_{ij} = \frac{\text{Number of commuting } i \rightarrow j}{\text{Workers residing in } i}$$

4. Conclusions

1. The reduction of relative weight of transport costs in economic decisions is one of manifestations of globalization. It changes spatial coordinates and makes the efficient borders across regions endogenous. Different neighbouring municipalities become economically interrelated, and we need to shift our attention to a new spatial object of analysis-functional urban areas, which is a product of spatial aggregation of municipalities with respect to housing and labour markets. It implies also a policy change which should take into account this interdependence of neighbouring markets. For example, NUTS-III in the European Community should be replaced by functional areas.

2. The different level of land rents and housing prices in different cities can be explained through the process of agglomeration, a self-reinforcing process driven by externalities, which leads to changes in wages and population sizes in cities. Our theoretical models predicts a particular functional form for wages and housing prices defined through the population of a city. When several cities compete for population in this environment of externalities and scale economies, different types of equilibrium can emerge. A stable equilibrium represents such a population split, when there is no incentive for re-migration. Due to differences in geographic factors, not all economic factors are equalized. An unstable equilibrium represents a potential for city growth, at the expense of other cities.

5. References

Alonso, O. (2001): «Metropolitan areas and public infrastructure». Investigaciones Económicas, vol.XXV (1), págs. 139-169.

Alonso, W (1964): Location and land use. Cambridge. Harward University Press.

BEESON, P. E.; DEJONG, D. N. y TROESKEN, W. (2001): «Population growth in U.S. counties, 1840-1990». Regional Science and Urban Economics, 31, págs. 669-699.

BERRY, B. J. (1976): Urbanization and counterurbanization, Arnold, New York

CAMAGNI, R. (2002): «Razones, principios y cuestiones para la política de desarrollo especial en una era de globalización, localización y trabajo en red», en Subirats et al, Redes, territorios y gobierno, Diputación de Barcelona, Barcelona, 321-350.

FUJITA, M.; KRUGMAN, P.; VENABLES, A. (1999): The Spatial economy. Cities, Regions and international trade. The MIT press, Cambridge, Massachussetts, London, England.

GLAESER, E. y SAPHIRO, J. (2001): «Is there a new urbanism? The growth of US cities in the 1990s». Harvard Institute of Economic Research. Discussion Paper 1925. Harvard University. Cambridge, Massachusetts.

HARRIS, C. D. (1954): The Market as a Factor in the Localization of Industry in the U.S. Annals of the Association of American Geographers, 44. (pàg 315-348).

HENDERSON, Vernon J. (1974): The sizes and types of cities. American Economic Review, 64 (pàg. 640-656).

LÓPEZ GARCÍA, M. A. (1992): Algunos aspectos de la economía y la política de la vivienda. - Investigaciones Economicas, vol. 16.

MASCARILLA, O. y YEGOROV, Y. (2002): El modelo de compensación monocéntrico, niveles de agregación territorial y efectos del AVE: el caso de Barcelona, Working paper, University of Barcelona.

MILLS, E. S. y HAMILTON, B. W. (1984): Urban Economics. 3ºEd. Glenview, Illinois. Scott, Foresman & Co.

MILLS, Edwin S. (1975): Economia Urbana. Editorial Diana. Mexico.

MUTH, R. F. (1969): Cities and Housing. Chicago. Chicago university press.

NEL·LO, O. (1999): «Área metropolitana de Barcelona. Dinàmiques metropolitanes a la regió de Barcelona», en Papers. Regió Metropolitana de Barcelona.

PUGA, D. (1998). Urbanisation Patterns: European vs. less developed countries. Journal of Regional Science, 38 (pàg. 231-252).

RICHARDSON, H. W. (1986). Economía Regional y Urbana. Alianza Universidad Textos.

ROBACK, J. (1982): «Wages, rents and quality of life». Journal of Political Economy, 90, págs. 1257-1278.

ROSEN, S. (1979): «Wage-based indexes of urban quality of life», en P.Mieszkowski & M. Straszheim (eds.), Current Issues in Urban Economics. Baltimore. Johns Hopkins University Press.

SMITH, Christine (Ed.): Location Theory and General Theory. Selected Papers of Walter Isard, volume 1, McMillan. 1990.

TAKATOSHI TABUCHI (1998): Urban agglomeration and Dispersion: a Synthesis of Alonso and Krugman, Journal of Urban Economics 44, 333-351.