

Do interrelated financial markets help in forecasting stock returns?

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ABSTRACT

The interest in studying the interrelationships among financial markets is clear, specially for banks and financial institutions. Nevertheless there are not conclusive studies on this respect. In this paper we analyze the predictive power of the obvious random walk model for stock prices when compared with other univariate and multivariate alternatives that exploit the presence of common stochastic trends in the data. We address several issues: First, can we find one (or more) common growth factors that help us in improving the forecast accuracy of the stock price indexes? And second, within the family of unobserved components models, is there any one particularly specification for the trend well suited for explaining and forecasting financial stock market data?

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1. INTRODUCTION

In the last two decades we have witnessed a tremendous internationalisation of national economies at all levels and sectors. Increasing trade and cooperation and abolition of exchange controls among national governments have led to removal of barriers and to greater free flow of goods, services, as well as physical and human capital. This phenomenon is particularly striking in financial markets where banks and financial institutions have increased their cross-border investments once they have become aware of the potential benefits of international diversification. Theoretically, international portfolio diversification allows a reduction of the total risk by increasing the gains (particularly in the short-run) in foreign markets showing low correlations with the domestic stock market.

Consequently, the relationships among equity markets have been analyzed in many previous empirical studies showing mixed evidence. Using weekly and monthly data, Agmon (1972) finds no significant leads or lags among the common stocks of Japan, the USA and other European countries. Later, Dwyer and Haffer (1988) confirm these results (using daily data) for seven months before and after the October 1987 crash, for the same set of countries. Others, however, (e.g., Eun and Shim, 1989; Bertera and Mayer, 1990; and Kasa, 1992) show statistical evidence of a substantial amount of interdependence among international equity markets. Also, similar results are found when we look for links in major Asian or European countries (e.g., Chowdhury, 1994; Kwuan et al., 1995; Arshanapalli and Doukas, 1993 and Malliaris and Urrutia, 1992).

There are several reasons for this conflicting evidence in reporting international stock markets linkages. Most of them are related to the use of different econometric methodologies, different time periods, different data frequencies and the role played by the stock-market crash of October 1987. Additionally, the issue of nonlinearity has introduced certain confusion to the debate, given the absence of a testing methodology that formally attempts to discriminate between intrinsic nonlinearity and the one due to nonstationarity in the data (de Lima, 1998). If nonlinearity is a previous issue, then posterior causality tests should take into account this feature and, the traditional Granger (1969) test carried out within the linear framework should be substituted by the nonlinear Skalin and Terasvirta (1999) test based on the STAR model. A previous issue, however, is whether the STAR framework is the most appropriate model to capture nonlinearities in the stock market in the presence of structural breaks. Similar problems and puzzles are common when revising the empirical results

from cointegration. In particular, we have to be very careful when choosing the number of lags in the VAR model. Kasa (1992), for instance, finds that low-order VARs reveal little evidence of cointegration, while higher-order VARs provide much stronger evidence in favor of the cointegration hypothesis. Also, temporal aggregation seems to play an important role when testing for cointegration. He finds much stronger evidence of cointegration using quarterly data than using monthly observations.

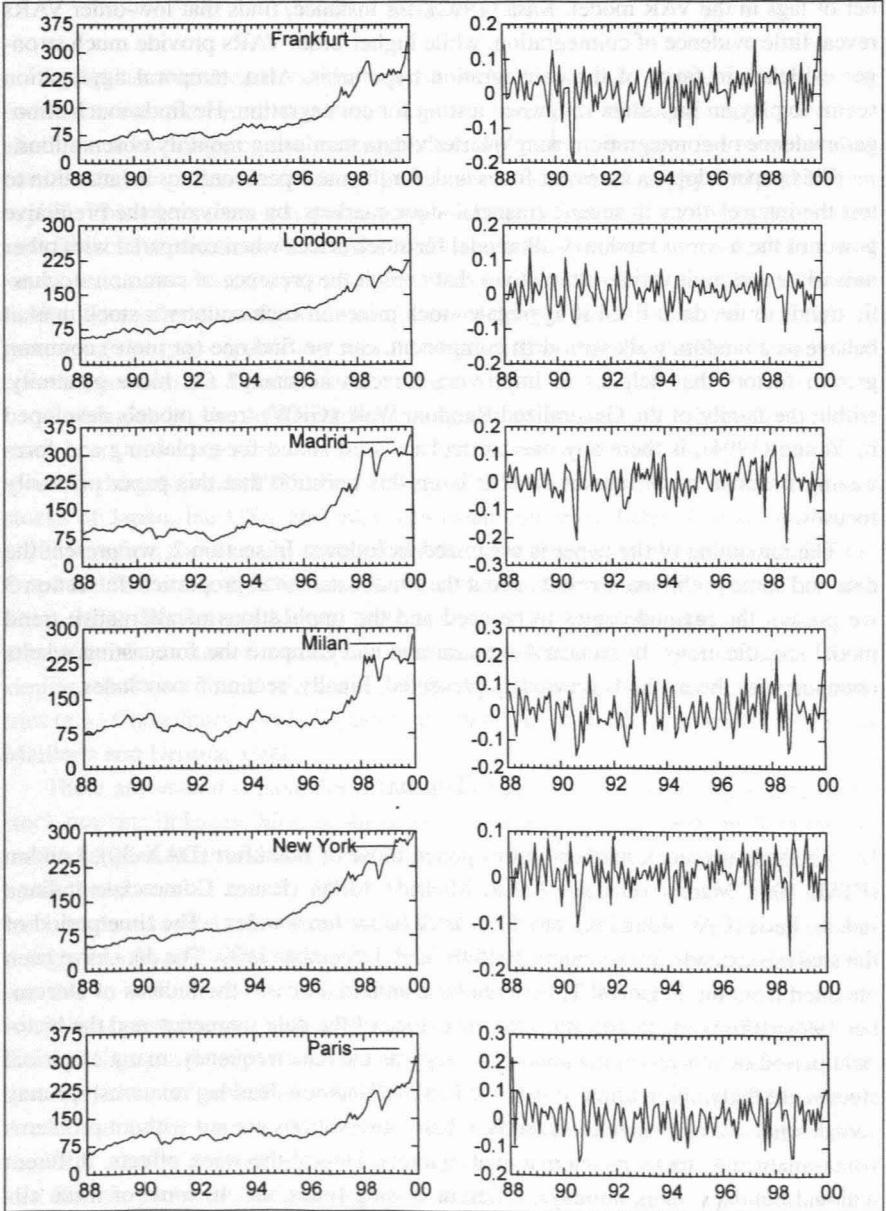
This paper adopts a narrower focus and, for the most part confines its attention to test the interrelations in several financial stock markets, by analyzing the predictive power of the obvious random walk model for stock prices when compared with other univariate and multivariate alternatives that exploit the presence of common stochastic trends in the data. Even if aggregate stock prices in each country's stock market behave as a random walk with drift component, can we find one (or more) common growth factors that help us in improving forecast accuracy? Or, more generally, within the family of the Generalized Random Walk (GRW) trend models developed by Young (1994), is there any one particularly well suited for explaining and forecasting financial stock market data? It is on this question that this paper primarily focuses.

The remaining of the paper is organized as follows: In section 2, we present the data and some preliminary results about their main statistical properties. In section 3 we present the methodologies to be used and the implications of alternative trend model specifications. In section 4 we examine and compare the forecasting results obtained with the methods previously presented. Finally, section 5 concludes.

2. THE DATA

We study six stock markets in this paper, those of Frankfurt (DAX-30), London (FTSE 100), Madrid (General Index, Madrid), Milan (Banca Commerciale Italiana index), Paris (CAC 40 index), and New York (Dow Jones index). The time period of the analysis extends from January 1988 through December 1999. The data have been obtained from the Financial Times data base and, in all cases the indices of December 1994=100. Some justification for the choice of the data frequency and the historical period of analysis is mandatory. As regards the data frequency, many empirical studies use daily observations due to the fact that potential lead/lag relationships may vanish when we aggregate the data. But daily observations are not without problems when analyzing stocks in international markets. Day-of-the-week effects, different national holidays, bank holidays, different closing times, etc. In some of these cir-

Figure 1. Stock original indices and monthly returns for several markets: 1988(1)-1999(12)



cumstances, specially when national stock exchanges are closed, the index level is assumed to remain the same as that of the previous trading day. As regards the period of analysis, we have deliberately avoided the presence of the October 1987 stock-market crash given its strong influence in the study of the dynamics of stock-market returns.

Plots of the original indices I_t for each country and its monthly returns $r_t = \ln(I_t/I_{t-1}) \times 100$ are shown in Figure 1. One common characteristic of equity prices in the national stock markets is that over the sample period they follow an upward trend. Summary statistics for monthly percentage changes r_t are contained in Table 1. There are several noteworthy points. First, for Frankfurt, London, Madrid and New York the biggest drop occurred in August 1998. For Milan and Paris, however, the biggest drop occurred in August 1990. Second, note from the excess kurtosis and skewness tests that r_t for Frankfurt, Madrid and New York are highly nonnormal since there are too many large changes to be consistent with a normal distribution. Although not crucial

Table 1. Summary statistics and tests for monthly returns: 1988.1-1999.12

	FRKT	LON	MAD	MIL	NY	PAR
Mean	1.347	0.970	1.035	0.913	1.236	1.239
S.D.	5.794	4.148	5.887	6.745	3.916	5.993
Maximum	16.653	13.478	13.719	20.825	9.756	21.893
Minimum	-19.943	-10.610	-21.514	-15.364	-16.408	-15.095
Skewness ^(a)	-3.779*	-0.524	-3.363*	1.265	-4.000*	-0.951
Excess kurtosis ^(b)	5.019*	0.706	4.066*	0.081	5.679*	1.037
$\rho(1)$	-0.013	0.013	0.123	0.007	-0.127	-0.035
$\rho(2)$	-0.014	-0.166	-0.023	-0.003	0.012	-0.050
$\rho(12)$	0.030	-0.117	-0.019	0.078	-0.005	-0.030
Q(6) ^(c)	1.939	7.074	5.603	4.977	8.439	2.307
Q(12) ^(c)	6.475	13.522	11.150	14.532	14.621	11.096
VR(2) ^(d)	0.078	0.178	1.079	0.152	-1.550	-0.330
VR(4) ^(d)	-0.157	-1.277	0.830	0.067	-1.396	-0.147
VR(12) ^(d)	-0.522	-1.606	0.276	-0.114	-1.698	-0.087

(a) The sample skewness follows a $N(0,6/T)$, being T the sample size. H_0 : No skewness.

(b) The sample kurtosis follows a $N(3, 24/T)$. H_0 : No excess of kurtosis.

(c) The Portmanteau statistic $Q(k)$ follows a χ^2_k . H_0 : The first k autocorrelations are zero. Critical values for $\alpha=0:05$; are 12.6 for $k=6$ and 21.0 for $k=12$.

(d) Variance-ratio statistic for H_0 : the series follow a random walk with uncorrelated increments. The asymptotic distribution of the test-statistic is $N(0,1)$.

* Rejection of H_0 at the usual $\alpha=0:05$ significance level.

Table 2. Unit root tests for the levels and first differences of the logs of the indexes of Frankfurt, London, Madrid, Milan, New York and Paris 1988-99

	FRKT	LON	MAD	MIL	NY	PAR
Levels						
DF	0.285	-0.049	0.587	0.230	0.435	-0.109
ADF(1)	0.295	-0.069	0.337	0.201	0.573	-0.071
PP(1)	0.298	-0.058	0.469	0.224	0.533	-0.018
First differences						
DF	-11.725*	-11.735*	-10.459*	-11.602*	-13.544*	-12.110*
ADF(1)	-8.277*	-9.989*	-8.142*	-8.277*	-8.892*	-8.849*
PP(1)	-11.723*	-11.734*	-10.458*	-11.600*	-13.544*	-12.111*

DF: Dickey-Fuller test. ADF: Augmented Dickey-Fuller test. PP: Phillips-Perron test.

* Rejection of H_0 : There is a unit root, at the usual $\alpha = 0:05$ significance level.

for the main focus of this paper, this result is of some importance when testing for cointegration. Second, the first, second and twelve order autocorrelations and the Ljung-Box Q-statistics are also reported in Table 1. While the usual caveat concerning such a short sample needs to be kept in mind, all testing results do not allow us to reject a simple random walk model for the stock indices. Also, in none of the indexes there seems to be evidence of seasonality in the data since the 12-th order autocorrelation is not significant for any of them. Third, confirmation of the previous results is reinforced when we look at the variance-ratio (VR) asymptotically standard normal tests proposed by Lo and MacKinlay (1988). Similarly, the usual three unit root tests, the Dickey-Fuller, the Augmented Dickey-Fuller and the Philips-Perron tests (for the series in levels and first differences) reported in Table 2 are in agreement with the previous findings regarding the order of integration of the series. The Augmented Dickey-Fuller test was performed adding one lag of the differenced series to take into account the possibility of serial correlation. The Philips-Perron test used 1 lag for the variance correction. The critical values used were those supplied by MacKinnon (1991). In all cases, the monthly returns r_t seem to be $I(0)$ variables.

In addition to measures of volatility and nonnormality, another interesting feature of the stock index data is the contemporaneous correlation between monthly changes of the various national markets. These correlations are shown in Tables 3 and 4. Table 3 shows the sample correlations for the whole sample 1988-99 and Table 4 for the two subperiods 1988-95 and 1996-99 (in parenthesis the numbers for the 1996.1-1999.12 period). In general, the sample correlations tend to be higher to those reported in other studies related to the seventies and eighties (see, Tay-

Table 3. Monthly returns correlation matrix for the time period 1988:01-1999:12

	FRA	LON	MAD	MIL	NY	PAR
FRA	1.0					
LON	.58	1.0				
MAD	.61	.60	1.0			
MIL	.64	.50	.63	1.0		
NY	.62	.69	.57	.39	1.0	
PAR	.76	.60	.62	.60	.60	1.0

Table 4. Monthly returns correlation matrix for the time period 1988:01-1995:12 and in parenthesis for the time period 1996:01-1999:12

	FRA	LON	MAD	MIL	NY	PAR
FRA	1.0					
LON	.51 (.74)	1.0				
MAD	.52 (.71)	.53 (.75)	1.0			
MIL	.58 (.69)	.40 (.67)	.54 (.74)	1.0		
NY	.49 (.79)	.69 (.70)	.49 (.68)	.29 (.50)	1.0	
PAR	.70 (.86)	.57 (.69)	.53 (.76)	.46 (.79)	.56 (.67)	1.0

lor and Tonks, 1989 and Kasa, 1992). Also, in most cases, there appear to be a marked increase in the correlations of the European countries during the last 1996-99 part of the sample.

These last results are in agreement with those reported in Table 5 where the annual growth rates of r_t for the whole 1989-1999 period are shown. Here we can see that after 1996 the behavior of the international stock markets show generalized high growth rates in all countries as if the interrelations among stock markets has strengthened lately. The sample mean of the annual growth rates for the 1989-1995 period is 7.2 and its sample variance is 12.77, while for the 1996-1999 period are 26.2 and 24.9, respectively. A simple test for the equality of the means rejects this null hypothesis. The median values of the annual growth rates for these two periods are 8.0 for the first one and 26.3 for the second one. Before 1996 the cyclical evidence was mixed and some markets (with the exception of London and New York) showed similar negative growth rates during the 1990-1992 period.

Based on the empirical information of Tables 3,4 and 5, we initially propose the following groups for the multicountry dynamic factor models:

Table 5. Annual growth rates 1989-1999

	FRA	LON	MAD	MIL	NY	PAR
1989	28.1	21.6	8.6	23.0	21.8	40.9
1990	13.9	0.6	-14.5	-1.5	5.1	1.9
1991	-6.0	12.3	3.1	-13.4	11.5	-1.3
1992	3.7	4.2	-13.6	-17.4	11.3	4.1
1993	11.3	15.9	17.5	20.7	7.3	10.0
1994	16.4	4.5	16.7	24.6	7.2	-0.2
1995	0.5	7.3	-5.8	-10.3	19.2	-9.1
1996	21.7	14.6	23.8	2.3	27.7	13.6
1997	43.6	21.9	50.9	36.2	28.6	31.6
1998	35.5	20.8	47.3	60.4	15.8	34.6
1999	6.6	11.4	9.8	11.4	21.6	23.6
89-99	12.6	10.0	11.3	7.5	15.8	12.6

These multiperiod growth rates are calculated as $\left[\prod_{j=0}^{k-1} (1 + r_{t+j}) \right]^{1/k} - 1$, where r_t is the simple net return between dates $t-1$ and t . For the annual growth rates $k = 12$ and for the total period 1989-1999 $k = 120$.

Group 1 (G1): Madrid and Milan.

Group 2 (G2): Frankfurt and Paris.

Group 3 (G3): London and New York.

3. METHODOLOGIES

Several univariate and multivariate alternatives are used to explain monthly variation of stock market indices over time for seven countries. In all cases, the information set will be strictly restricted to the I_t series without attempting to expand it by using other inputs or leading indicator variables. In spite of the fact that the estimation and forecasting periods include at least two important outliers corresponding to August 1990 and 1998, our estimation and forecasting calculations will not omit any particular data points.

3.1. Univariate Models

The obvious benchmark in evaluating the forecasting performance of more complicated models is the random-walk with drift model, that we can write as (recall that $r_{it} = 100 \times [\ln(I_t) - \ln(I_t - 1)]$):

$$r_{it} = \alpha_{0i} + \varepsilon_{it} \quad (1)$$

Similarly, several variants of unobserved component (UC) models are also proposed. Being $y_t = \ln(I_t)$ we postulate the appropriate UC model to be:

$$y_t = T_t + P_t + \varepsilon_t \quad (2)$$

where T_t is the low frequency or *trend* component, P_t is a *perturbational* component around a long-run trend, which may be either a zero-mean stochastic component with fairly general statistical properties, or a sustained *periodic* or *seasonal* component; and finally, ε_t is a zero-mean, serially uncorrelated white noise component with variance σ_ε^2 .

The stochastic evolution of T_t is assumed to be described by a Generalized Random Walk (GRW) model defined by Young (1994) as:

$$\begin{bmatrix} T_t \\ D_t \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} T_{t-1} \\ D_{t-1} \end{bmatrix} + \begin{bmatrix} \delta & 0 \\ 0 & \theta \end{bmatrix} \begin{bmatrix} \eta_t \\ \xi_t \end{bmatrix} \quad (3)$$

where D_t denotes the local slope (trend derivative) of the trend, and η_t and ξ_t are normal white disturbances independent of each other and normally distributed such as $\eta_t \sim N(0; \sigma_\eta^2)$ and $\xi_t \sim N(0; \sigma_\xi^2)$. This general model comprises as special cases (see Young, 1994) the following alternatives:

- Scalar Random Walk (RW): $\alpha = \delta = 1; \quad \beta = \gamma = \theta = 0$
- Integrated Random Walk (IRW): $\alpha = \beta = \gamma = \theta = 1; \quad \delta = 0$
- Smooth Random Walk (SRW): $0 < \alpha < 1; \quad \beta = \gamma = \theta = 1; \quad \delta = 0$

In the three cases, their reduced form equation is given by:

$$T_t = \frac{1}{(1 - \alpha L)(1-L)} \xi_t \quad (4)$$

where L denotes the lag operator. The random walk plus drift model implies that $\alpha = \beta = \gamma = \theta = 1$; $\theta = 0$. Also GRW encompasses other well known models in the UC literature such as the *Local Linear Trend* (LLT: $\alpha = \beta = \gamma = \theta = \delta = 1$); and the *Damped Trend* (DT: $\alpha = \beta = \theta = \delta = 1$; $0 < \gamma < 1$) described in Harvey (1989). It is instructive to consider the nature of the prediction equations (within the Kalman filter) for the various GRW processes. In the RW case, it is obvious that the one step-ahead prediction \hat{T}_{hh-1} is simply the estimate obtained at the previous recursive time period \hat{T}_{h-1} . In the other two cases, however, an additional component D_t is introduced and estimated. By substituting in (3) from the D_t equation into the T_t equation we obtain:

$$\text{IRW: } \hat{T}_{hh-1} = \hat{T}_{h-1} + (\hat{T}_{h-1} - \hat{T}_{h-2}) = \hat{T}_{h-1} + \Delta\hat{T}_{h-1} \quad (5)$$

$$\text{SRW: } \hat{T}_{hh-1} = \hat{T}_{h-1} + \alpha(\hat{T}_{h-1} - \hat{T}_{h-2}) = \hat{T}_{h-1} + \alpha\Delta\hat{T}_{h-1} \quad (6)$$

where $\Delta\hat{T}_{h-j} = \hat{T}_{h-j} - \hat{T}_{h-j-1}$ is the rate of change of the estimate between the $(h - j - 1)$ and the $(h - j)$ recursions. In other words, while the RW prediction is constant at the level of the prediction origin, the IRW predicts linearly with a constant slope equal to its last rate of change at the prediction origin, and the SRW allows a range of intermediate possibilities between the RW and the IRW models as a function of α . These parametric variations seem to provide reasonable one step-ahead forecasts when we have in mind that these predictions are automatically corrected by the smoothing equations of the Kalman filter as soon as the new data point is available. In the case of the IRW type models we assume that $\sigma_\eta^2 = 0$. Then the variance of ξ_t is the only unknown in (4) and it can be defined by the noise variance ratio (NVR):

$$\text{NVR} = \frac{\sigma_\xi^2}{\sigma_\varepsilon^2} \quad (7)$$

This NVR uniquely defines the IRW models for the trend and the difficulties associated with the «choice» or estimation of the NVR value, are discussed later.

Clearly, the presence of α introduces an additional parameter that has to be identified from the data and optimized, thus introducing potential practical difficulties.

However, we can alleviate this problem by noting from (4) that α could be approximated by the «second» positive real root of the autoregressive AR(p) representation of the original series. If we allow for a large value of p to be consistent with the data frequency used in this paper, and estimate AR(12) models for the $\ln I_t$ (using the whole sample) we find the following results¹:

1. In all cases there is a positive real unit root in agreement with the unit root test results depicted in Table 2.
2. The range of values of the second real positive root lies between .70 for the case of Frankfurt, to .84 for the case of Milan. Only in the case of London there is no evidence of a second near-unity root.
3. The remaining pairs of conjugate complex roots do not indicate evidence of well defined cyclical or seasonal patterns in the data.

Experience with the GRW class of models has shown that the IRW model is particularly useful for the estimation of smooth trends in economic data (see e.g. Young *et al* 1999, García-Ferrer and Bujosa, 2000, García-Ferrer and Poncela, 2002). Other authors (see, Harvey and Koopmans, 1997) claim that many financial time series follow random walks and attempting to impose an underlying smooth trend on them would be totally misleading. This is why an “intermediate” option like the SRW could be seen as a compromise between both extremes.

There is also a different nonlinear trend alternative that we would like to explore. If we examine the smoothed estimate of the stochastic input ξ_t to the IRW model, which is obtained by doubly differencing the trend estimate, and find significant, heavily correlated variations describable by an AR(p)model such that

$$\phi(L)\xi_t = \zeta_t$$

where ζ_t is a zero mean white-noise process with variance σ_ζ^2 . If this is the case, we can extend the IRW model to the more complex double-integrated autoregressive (DIAR) model analyzed in Young (1994) and García-Ferrer *et al* (1996). The DIAR model is defined by its associated $NVR = \sigma_\zeta^2/\sigma_\epsilon^2$ and the estimated AR coefficients of the $\phi(L)$ polynomial.

¹ The reader should be aware that these results are extremely sensitive to the specification of the AR order and should be taken with care. Simple as it is, however, it provides an initial estimate of α that we can optimized later on.

3.2. Multivariate Models

We will use the nonstationary factor model (FM) described in Peña and Poncela (2002) for the multivariate analysis of the data. Let $\mathbf{y}_t = (y_{1,t}, \dots, y_{m,t})'$, $y_{i,t} = \ln(I_{i,t})$, $i = 1, \dots, m$, $t = 1, \dots, T$, be an m -dimensional vector of observed log indexes. It is assumed that the vector of the observed series, \mathbf{y}_t , can be written as a linear combination of a smaller number (r) of unobserved variables, called common factors, $r < m$, and m specific components,

$$\mathbf{y}_t = \mathbf{P} \mathbf{f}_t + \mathbf{n}_t; \quad (8)$$

where \mathbf{f}_t is the r -dimensional vector of **common factors**, \mathbf{P} is the factor loading matrix, and \mathbf{n}_t is the vector of **specific or idiosyncratic components**.

In our case, the common factors can be common trends and common stationary factors, so $\mathbf{f}'_t = [\mathbf{T}'_t; \mathbf{f}'_{0,t}]$.

We assume that there are r_1 common trends and r_2 common stationary factors. As in the univariate case, we suppose that each common trend can also be described by a Generalized Random Walk model defined as in (3). Let $\mathbf{T}'_{i,t} = [\mathbf{T}'_{i,t}; D_{i,t}]$; where as in the univariate case $D_{i,t}$ is the local slope of the i -th trend, $\mathbf{a}'_{i,t} = [\eta'_{i,t}; \eta'_{i,t}]$, $F_i = \begin{bmatrix} \alpha_i & \beta_i \\ 0 & \gamma_i \end{bmatrix}$ and $Q_i = \begin{bmatrix} \delta_i & 0 \\ 0 & \theta_i \end{bmatrix}$, $i = 1, \dots, r_1$, then the common trends is assumed to have the following Markovian representation

$$\mathbf{T}_t = \mathbf{F} \mathbf{T}_{t-1} + \mathbf{Q} \mathbf{a}_t; \quad (9)$$

where $\mathbf{T}'_t = [\mathbf{T}'_{1,t} \dots \mathbf{T}'_{r_1,t}]$, $\mathbf{a}'_t = [\mathbf{a}'_{1,t} \dots \mathbf{a}'_{r_1,t}]$ and \mathbf{F} and \mathbf{Q} are the block diagonal matrices with blocks equal to F_i and Q_i , respectively. As in the univariate case each of the common trends can be a RW, IRW or SRW, imposing the same kind of restrictions as for the scalar case. We also assume that each of the common stationary factors follows an $\text{AR}(p)$, $i = 1, \dots, m$ process

$$\mathbf{A}(L) \mathbf{f}_{0,t} = \mathbf{a}_{0,t}, \quad (10)$$

$$r_2 \times r_2 \quad r_2 \times 1 \quad r_2 \times 1$$

where $\mathbf{A}(L) = \mathbf{I} - \mathbf{A}_1L - \dots - \mathbf{A}_pL^p$ is a diagonal matrix of polynomials in L , being $p = \max(p_i)$, $i = 1, 2, \dots, m$, such that the determinantal equation $|\mathbf{I} - \mathbf{A}_1z - \dots - \mathbf{A}_pz^p| = 0$ has all its roots outside the unit circle, and $\tilde{\mathbf{a}}_t = (\mathbf{a}'_t \quad \mathbf{a}'_{0,t}) \sim N_t(\mathbf{0}, \Sigma_\alpha)$, is serially uncorrelated,

$$\mathbf{E}(\tilde{\mathbf{a}}_t \tilde{\mathbf{a}}'_{t-h}) = \mathbf{0}, \quad h \neq 0 \quad (11)$$

After extracting the common dynamic structure, we will assume that each one of the specific components, $n_{i,t}$, where $\mathbf{n}_t = (n_{1,t}, \dots, n_{m,t})'$ associated to each of the series follows an univariate AR(s) model,

$$n_{it} = \alpha_{0i} + \alpha_{1i}n_{it-1} + \alpha_{2i}n_{it-2} + \dots + \alpha_{si}n_{it-s} + e_{it} \quad (12)$$

for $i = 1, \dots, m$. The sequence of vectors $\mathbf{e}_t = (e_{1,t}, \dots, e_{m,t})'$ are normally distributed, have zero mean and diagonal covariance matrix Σ_e . We assume that the noises from the common factors and specific components are also uncorrelated for all lags,

$$\mathbf{E}(\tilde{\mathbf{a}}_t \mathbf{e}'_{t-h}) = \mathbf{0}, \quad \forall h \quad (13)$$

To write the model in state space form, rewrite (9) and (10) together to get the transition equation

$$\begin{bmatrix} \mathbf{T}_t \\ \mathbf{f}_{2,t} \\ \mathbf{f}_{2,t-1} \\ \mathbf{f}_{2,t-2} \\ \vdots \\ \mathbf{f}_{2,t-p+1} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \dots & \mathbf{A}_{p-1} & \mathbf{A}_p \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{t-1} \\ \mathbf{T}_{2,t-1} \\ \mathbf{f}_{2,t-2} \\ \mathbf{f}_{2,t-3} \\ \vdots \\ \mathbf{f}_{2,t-p} \end{bmatrix} + \begin{bmatrix} \mathbf{Qa}_t \\ \mathbf{a}_{0,t} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \quad (14)$$

The measurement equation is given by

$$y_t = [P_1 \quad P_2 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0] \begin{bmatrix} T_t \\ f_{2,t} \\ f_{2,t-1} \\ f_{2,t-3} \\ \vdots \\ f_{2,t-p+1} \end{bmatrix} + n_t, \quad (15)$$

being $P = [P_1 \quad P_2]$, P_1 the $m \times 2r_1$ submatrix of the factor loading matrix associated to the common trends, whose every other column is just a column of zeros (that can be suppressed if any of the common trends is just a scalar RW), and P_2 the $m \times 2r_2$ submatrix of the factor loading matrix associated to the common stationary factors. Model (14) and (15) can be written in a compact way as

$$y_t = \hat{P}z_t + n_t, \quad (16)$$

$$z_t = Gz_{t-1} + Ru_t, \quad (17)$$

where $P = [P_1 \quad P_2 \quad 0 \quad \dots \quad 0]$, $z_t = [T_t \quad f_{2,t} \quad f_{2,t-1} \quad \dots \quad f_{2,t-p+1}]'$, $R = [I_r \quad 0_{(p-1) \times r}]$, $u_t = [Qa_t \quad a_{0,t} \quad 0 \quad \dots \quad 0]$ and

$$G = \begin{bmatrix} I & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & A_1 & A_2 & A_3 & \dots & A_{p-1} & A_p \\ 0 & I & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & I & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & I & 0 \end{bmatrix} \quad (18)$$

The model, as stated, is not identified since for any $(r_1 + pr_2) \times (r_1 + pr_2)$ non singular matrix H , the observed series y_t can be expressed in terms of a new set of factors,

$$y_t = \hat{P}^*z_t^* + n_t, \quad (19)$$

$$z_t^* = G^*z_{t-1}^* + R^*u_t^* \quad (20)$$

where $\mathbf{P}^* = \mathbf{P}\mathbf{H}^{-1}$, $\mathbf{z}_t^* = \mathbf{H}\mathbf{z}_t$, $\mathbf{u}_t^* = \mathbf{H}\mathbf{u}_t$, $\mathbf{G}^* = \mathbf{H}\mathbf{G}\mathbf{H}^{-1}$, $\mathbf{R}^* = \mathbf{H}\mathbf{R}\mathbf{H}^{-1}$ and $\Sigma_a^* = \mathbf{H}\Sigma_a\mathbf{H}'$. Models (16), (17) and (19), (20) are identical from the point of view of the available data.

To solve the identification problem, some restrictions are needed. We will follow the ones in Harvey (1989) and set $\Sigma_a = \mathbf{I}$, and $p_{ij} = 0$, if $i > j$, being $\mathbf{P} = [p_{ij}]$.

With respect to the forecasting functions of the different trend models considered, they are as in the univariate case (see, equations 5 and 6). Nevertheless, they enter into each of the series forecasting functions through the factor loadings. So, the same common trend can affect the various components of a vector of time series in different ways.

4. EMPIRICAL RESULTS

After we have used the 1988(1)-1998(12) sample for estimation, we report in this section the forecasting performance of alternative models for the 1999(1)-1999(12) period. We focus on one-step-ahead predictions, so that each model is re-estimated twelve times, starting with the 1988(1)-1998(12) sample, and including each time one additional month in the sample, to obtain the prediction for the next month. Our benchmark model is the random walk with drift for the individual monthly log indexes of the different stock markets. This model is not only related to the market efficiency hypothesis, but also is an obvious parametrization given the empirical information provided in Table 1. Before presenting the forecasting results, some comments regarding the estimation of the univariate and dynamic factor models are mandatory.

As regards the univariate unobserved components models, estimation of the NVRs for the IRW and DIAR models plus the estimation of the additional parameter α in the case of the SRW model is not without problems. Given the absence of well defined cyclical or seasonal peaks in the data, the optimization approach proposed by Young *et al* (1999) provides results that are extremely sensitive to the specification of the AR order model for the $\ln I_t$. In terms of statistical fitting criteria, best results can be summarized as follows:

1. For the case of the IRW models, the range of estimated NVR values for the trend component oscillates between 0.5 and 1.5, providing strong evidence against the hypothesis of smooth trends on this data set. As regards the perturbational component, most estimation results tend to favor an AR(4) structure in all countries.

Given these uncertainties in the estimation process, we have performed a forecasting sensitivity analysis for several NVRs values and several AR structures. For the range of values reported above, forecasting results are relatively robust. Therefore, all the forecasting results for this model presented in Tables 6 through 8, are based on a $NVR = 1$ for the trend and an $AR(4)$ for the perturbations, for all countries. The same type of results and comments are applicable to the case of the DIAR models.

2. In the case of the SRW models, the nonlinear least squares estimates of α range from .78 to .92, very much in agreement with the initial estimates stated in section 3.1. These results tend to favor the IRW alternative against the RW one. Therefore, forecasting results for this model should not differ considerably from the ones using the IRW alternative.

The univariate random walk plus drift unobserved component model is estimated by maximum likelihood. As regards the multivariate factor models, we fit a bivariate model for each of the three groups depicted in section 2. Statistical, geographical and historical reasons allow us to form the three groups. We look at the correlation matrix (see tables 3 and 4) and found the highest correlation .76 between the Frankfurt and Paris stock returns. This correlation grows to .86 for the last part of the sample (1996:01 to 1999:12). These two indexes constitute our first group. The second highest correlation is found between the London and New York returns with a value of .69. This constitutes our second group. The remaining two markets (Madrid and Milan) with a correlation as high as .63 (that goes up to .74 for the sample period of 1996:01-1999:12) constitutes our third and final model. An alternative approach could be to build a large factor model for the six indexes. This is not done here for several reasons. First, some returns do not show a correlation high enough. For instance, the correlation between the Milan and New York returns is .39 (and it goes down to .29 for the first part of the sample, 1988:12- 1995:12). Second, multiperiod 1995 annual returns are positive for the London and New York indexes, close to zero for Frankfurt and negative for the remaining markets. This disparity of behavior in the stock markets is also shown for other periods. And third, a large multivariate factor model for six indexes could imply some "ad-hoc" zero restrictions (see, Harvey, 1989, pp. 450-51) due to identification requirements.

The estimation of the three bivariate factor models is made by maximum likelihood through the EM algorithm of Dempster *et al* (1977) using the Kalman filter and fixed interval smoother. We fit a common factor model for each of the three groups. It is assumed that each pair of indexes within a group is generated by a common trend plus some specific dynamics. We tried several specifications for the common trends

exposed in the previous section and found that the best forecasting results were obtained when the common trends were modeled as a RW plus drift. These bivariate models are given by

$$\begin{bmatrix} y_{1,t}^i \\ y_{2,t}^i \end{bmatrix} = \begin{bmatrix} p_1 & 0 \\ p_2 & 0 \end{bmatrix} \begin{bmatrix} T_t^i \\ D_t^i \end{bmatrix} + \begin{bmatrix} n_{1,t}^i \\ n_{2,t}^i \end{bmatrix}$$

$$\begin{bmatrix} T_t^i \\ D_t^i \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{t-1}^i \\ D_{t-1}^i \end{bmatrix} + \begin{bmatrix} a_{i,t} \\ 0 \end{bmatrix}$$

$$n_{1,t}^i \sim AR(p^i_1)$$

$$n_{2,t}^i \sim AR(p^i_2)$$

where the superindex $i = 1, 2, 3$ stays for the three groups and the subindexes 1,2 for each stock market within a group. For brief of exposition, we only show these later results on tables 6 through 8, but the results from the remaining specifications of the common trend in the multivariate models are available from the authors upon request.

Table 6. RMSE of prediction for the Group1 of indexes: Madrid and Milan

Models	MAD	MIL	Mean
Naive	4.06	5.56	4.81
IRW	4.59	5.76	5.17
SRW	4.21	5.71	4.96
DIAR	4.44	5.76	5.10
RW+D	4.05	5.36	4.70
FM	4.22	5.07	4.64

Table 7. RMSE of prediction for the Group 2 of indexes: Frankfurt and Paris

Models	MAD	MIL	Mean
Naive	6.75	5.06	5.91
IRW	6.69	4.69	5.69
SRW	6.84	5.07	5.95
DIAR	6.52	4.50	5.51
RW+D	6.73	5.06	5.89
FM	6.26	4.92	5.59

Table 8. RMSE of prediction for the Group 2 of indexes: London and New York

Models	MAD	MIL	Mean
Naive	3.30	3.93	3.62
IRW	3.43	4.42	3.92
SRW	3.39	4.15	3.77
DIAR	3.42	4.30	3.86
RW+D	3.29	3.79	3.41
FM	4.14	3.49	3.82

Forecasting results based one-step-ahead RMSE for the six markets are presented in Tables 6 to 8². We will discuss them separately since the conclusions differ slightly among the different groups. For the Madrid-Milan group, results are presented in Table 6. The univariate random walk plus drift unobserved components model has the lowest RMSE for Madrid and the bivariate factor model shows the lowest value in the case of the Milan market. When looking at the aggregate mean, we find that the factor model has the lowest RMSE value among the different alternatives. For this particular group, the remaining three univariate unobserved component models do not show any improvement over the benchmark random walk with drift model. For the Frankfurt-Paris group, the bivariate factor model shows the best results for the case of Frankfurt, and the DIAR model is best in the case of Paris. When looking at the aggregate mean, again the DIAR model has the lowest RMSE value overall. These results are shown in Table 7. Finally, for the London and New York group, the univariate random walk with drift model has the lowest value for the London market, and the bivariate factor model in the case of New York. For this group, the aggregate mean of the benchmark random walk model is the lowest among the different alternatives. These results are shown in Table 8. In summary, the factor model presents the lowest RMSE for 3 out of the 6 indexes analyzed, the "naive" RW plus drift model in 2 out of 6 cases and the DIAR model in one case. Before concluding this section, a further comment regarding the leading influence of the New York market on the remaining markets is mandatory. This result has been stressed in the literature by several authors using several periodicities (see, Gerrits and Yüce, 1999). For this particular historical period and set of countries, however, our results indicate that New York does not Granger cause any of the remaining stock markets. In none of the cases, the coefficients of the estimated regression equations are statistically significant. Also the

² We have chosen as a criterion for our comparisons the RMSE since it is the most widely used criterion, in spite of its limitations.

RMSEs obtained for the forecast period show values considerably larger than those obtained by any of the alternatives presented in Tables 6 to 8. (Empirical results are available from the authors upon request).

5. CONCLUSIONS

In this paper we have analyzed monthly stock indexes of six important financial markets from January 1988 through December 1999. One common characteristic of the equity prices over the sample period is their upward trend. Other summary statistics also indicate the presence of nonnormality, lack of seasonal behavior, high volatility and confirmed evidence that monthly returns seem to be higher to those reported in other studies. In most cases, there appear to be a marked increase in the correlations. In particular, after 1996 the behavior of the international markets show generalized higher growth rates in all countries as if their interrelationship has strengthened lately. We have gone one step further and confined our attention in analyzing the predictive power of the obvious random walk model when compared with other univariate (linear and nonlinear) and multivariate alternatives that exploit the presence of common stochastic trends in the data. We have focused on one-step-ahead predictions for the 1999(1)-1999(12) period, having used the 1988(1)-1998(12) sample for estimation. As regards the forecasting results, the unobserved components model with a random walk plus drift for the trend gives slighter, although systematic (for all countries) better results than the benchmark model (the usual random walk plus drift model) fitted to the monthly returns. The remaining unobserved components models (with different specifications for the trend) do not seem to outperform the benchmark model. The shrinkage effect that it is produced through the factor model further improves the overall mean of the prediction RMSE for all the monthly returns.

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