12th International Workshop on Database and Expert Systems Applications. Proceedings. IEEE, 2001. 649-653

DOI: http://dx.doi.org/10.1109/DEXA.2001.953132

A realistic substrate for Small-world networks modeling

Carlos Aguirre[1 2] , Fernando Corbacho[1] and Ramón Huerta [1 3]
[1]Computer Engineering Department, Universidad Autonoma de Madrid, 28049 Madrid Spain
[2]Zentrum für Interdisziplinäre Forschung, Universitat Bielefeld, Bielefeld, Germany
[3]Institute for Nonlinear Science, University of California, San Diego, La Jolla, CA, 92093-0402

Abstract

Small-World networks are networks with high local clustering and small distance between the nodes. In order to study the properties of this kind of networks, Watts and Strogatz developed a method based on varying the probability of rewiring each edge on a regular graph. As initial substrate for the regular graph, some specific topologies are usually selected, as for example, ring-lattice or grids. These regular graphs are not suitable for modeling of certain hierarchical topologies as for example, holonic systems and Internet. We present a new regular substrate, that models more accurately topologies with certain hierarchical properties. We also investigate the dynamics of the diffusion of information packages over the network for different types of network substrates.

Key words: Small-world networks, networks modeling, hierarchical networks, Internet, biconnectivity.

1 Introduction

In the last few years, the growth of communication networks, multi-agent systems [2], holonic systems [3] and architectures [1], complex systems and specially the Internet have generated many studies focused on network topologies. In this framework, graphs represent the most adequate abstract representation of the network, where each node represents an agent and each edge represents a connection between two agents. There are many investigations that deal with graphs that model communication networks [10, 11]. In a previous work [9], the use of transit-stub networks is proposed as an improvement to model real networks, because they capture the most relevant topological characteristics (diameter, average degree, average path length, connectivity and biconnectivity). In [17] the use of scale-free networks is proposed since it models more accurately some real-life networks such as cell populations, social networks and Internet. A common aspect in all these studies is the limited set of graphs subjected to study, that is, star, ring, grid, regular, and random networks. Many interesting properties arise when the network topology itself is an intrinsic parameter that can vary with a given set of rules. This set of rules determines a dynamics over the set of edges in the graph.

In [6] a method to study the dynamic behavior of networks when the network is shifted from a regular, ordered network to a random one is proposed . The method is based on the random rewiring with a fixed probability p for each edge in the graph. When p is 0 we obtain the original regular graph, and when p is 1 we obtain a random graph. It was found that there is a range of values of p where there are short paths but the graph is highly clusterized. The topologies in this range are known as Small-World (SW) topologies. SW topologies present some very interesting features that make them very suitable for efficient transmission of commodities [13, 6]. And in fact they appear naturally in many real life networks.

As initial substrate for the generation of SW, the use of a ring-lattice or a grid is usually proposed [7, 13]. They are used because these graphs are connected, present a good transition from regular to random, and there is no special nodes on them. Multi-agent networks can be analyzed within the frame of Small-World topologies, yet the family of graphs generated by Watts and Strogatz method presents a very significant difference with many real multi-agent networks. In [9], the graphs generated present a high number of biconnected components [14], intuitively a biconnected component is a subnetwork connected by a set of outputs to the network. Furthermore, the number of biconnected components in the graph grows with the number of nodes in the graph. On the other hand, the substrate selected in [6] presents a single biconnected component, independently of the number of nodes in the graph. In many multi-agent networkts there are special nodes that connect backbones with subnetworks, and as discussed in [9], communication networks continue to grow in size and importance. For all these reasons realistic models of network topologies will be critical for the meaningful assessment of all kinds of algorithms and policies.

In this communication we present a theorem of existence, and a method to build regular graphs with a high number of biconnected components. We provide an analytic expression for the number of biconnected components. We also present by means of computer simulations the dynamical behavior of the agent state information broadcasting on a set of agents ranging from regular to random graphs with multiple biconnected components. These agent state broadcast of information is used, for example, in any link-state routing protocol as could be OSPF or IS-IS routing protocols [8].

2 Biconnected Small-Worlds

This section describes an algorithm to generate a SW graph with a high number of biconnected components in contrast to lattices. In the literature about Small-World, the ring-lattice is used as the initial graph substrate due to the following advantages: ring substrates are connected, have a good transition from "large" to "small", and have no special nodes. The ring-lattice substrate has some characteristics that makes it unsuitable for modeling some specific types of networks such as hierarquical multi-agent networks or Internet. These networks have special nodes, the ones that connect stubs with backbones, and seem to present a number of biconnected components higher than one [9]. On the other hand, as it can be seen in figure 4, the number of connected bicomponents in the ring lattice substrates is very close to 1 when p ranges from 0 to 1.

For a given graph G we use n as $|G|$ i.e., the number of nodes in the graph, and k_i as the number of neighbors the node i has. We denote k as $\sum_{i=1}^{n} k_i/n$ i. e. the average number of neighbors each node has.

As the initial substrate for our Small-World model of the hierarquical networks, we have developed a constructive method to build a finite regular graph with multiple biconnected components that we name stub-domain regular graphs. It is not clear a priori that such graphs exist and, if they exist, it needs to be proven that they can be built algorithmically for given values of n and k.

Here we present a theorem that establishes the existence of regular finite graphs with several biconnected components. The theorem is constructive i.e. it proves the existence by building such graphs. The algorithm is based on building a regular ring-lattice and then attach an almost-regular stub to each node in the central ring. This algorithm produces a k-regular graph with n nodes and a number of biconnected components higher than one. We show that, in this graph, for fixed k the number of biconnected components grows linearly with n.

Theorem 1: Provided that k is odd and $(k+3)$ divides n, there is a regular graph with n nodes, k neighbors per node, and $2n/(k+3)+1$ biconnected components.

Proof: The proof is constructive. First, we build a central ring-lattice with $n/k+3$ nodes, and $k-1$ neighbors per node. Then we build a graph for each node in the ring-lattice with $k+2$ nodes and each node connected to k neighbors, except one of them that should be connected only to $k-1$ neighbors (this is possible since k es odd, and therefore $k+2$ is also odd). This node must be connected to the corresponding node in the ring-lattice.

The previous algorithm generates a regular graph with n nodes and k neighbors per node. The number of biconnected components can be easily calculated which is $2n/(k+3)+1$. A similar theorem can be established when k is even.

3 Graphs Metrics Dynamic Behavior

In order to study the dynamic behavior of our graphs, we follow the standard procedure described in [6]. Other procedures to build small-world graphs, have been described by [15, 16], however they increase the number of edges and/or nodes in the graph, while the Watts-Strogatz method maintains constant the number of nodes and edges in the graph. Figure 2 displays the process of shifting a regular stub-domain graph to a random one. When $p=0$ no changes are made over the substrate and the graph remains unchanged. As p increases shortcuts appear in the graph. In the limit of $p=1$ a random graph is obtained.

The metrics used to characterize the graph are the characteristic path length L, the characteristic cluster C [7] and the number of biconnected components B [14] in the graph . These parameters are the most extensively studied in the existing literature. The diameter of the graph is tightly related to L (being in fact an upper bound)[12]. Therefore we consider L because it provides more specific information about how far the nodes are in the graph.

Figure 3 shows the behavior of L and C in a ring-lattice and in a regular stub-domain graph for both k even and odd when they are shifted from a regular to a random situation. Note that all of them present a clearly visible Small-World area (that is, low L and high C). However the decrease of L in the stub-domain models is smoother mainly due to the fact that characteristics paths are shorter in the case of a regular stub-domain substrate than in the case of a ring-lattice one. In the ring lattice substrate the value of the characteristic path L is very close to the maximum for any regular graph and has a very fast descent. In the case of the characteristic cluster C, no significant differences are found between both kinds of substrates. Figure 4 dispays the number of biconnected components in the graph as a function of p. Observe that the number of biconnected components is reduced to a very low number as p increases in an initially regular stub-graph substrate. However it remains constant

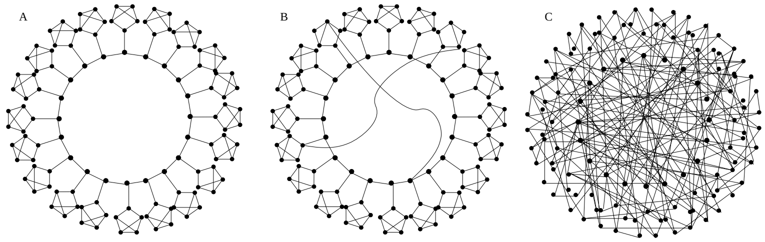

Figure 1. Stub-domain graphs for $n = 120$ and $k = 3$. (A) Regular stub-domain graph for $p = 0$. (B) Small-world stub-domain graph for $p = .02$. (C) Random graph for $p = 1$.

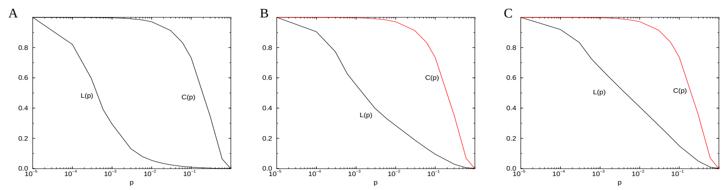

Figure 2. (A) L and C for ring-lattices with $n = 2982$ and $k = 8$. (B) Stub graphs (k odd) with $n = 2980$ and $k = 7$. (C) Stub graphs (k even) for $n = 2982$ and $k = 8$.

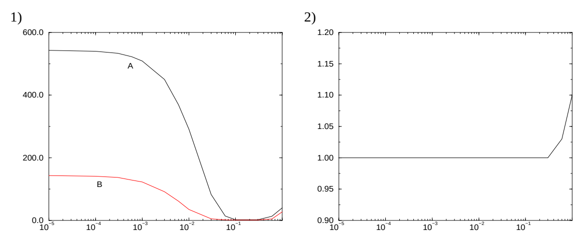

Figure 3. 1) (A) Number of biconnected components for stub graphs for $n = 2980$ and $k = 7$. 1) (B) Stub graphs for $n = 2982$ and $k = 8$. 2) Ring-lattice graphs for $n = 2982$ and $k = 8$.

when a ring-lattice graph is used.

4 Agent state information diffusion

Comparisons based on different topological metrics have provided differences between graph generation methods. However, as [9] pointed out, it does not mean much by itself unless a link to its function is established. Therefore we must study how different static topologies affect the performance in real networking problems, such as problems of diffusion of agents state information.

The overall goal is to model the paths (i.e. sequence of nodes) along which the information flows in an internetwork. Let us provide a brief sketch for an agent state information difussion algorithm. The algorithm is performed by each node in the following way: the node collects information relative to its current state, builds a package containing all the information, sends this package by selective diffusion to the rest of the agent, and, finally, the agent takes decisions based on its own information and on the information packages it receives from the other agents.

5 Dynamic Metrics

Two metrics are the most frequently used to measure the dynamic properties of a given network system, namely the error rate of information arriving at the nodes and the transfer time of information throughout the network. Both affect the QoS of a network. In our work, both measures are directly related to the Sustained Cell Rate (SCR), and the Cell Error Ratio (CER) parameters of the Quality of Service

(QoS) standard. SCR is usually a negotiable parameter, and the customer usually acquires a specific quantity of SCR. CER usually is not negotiable and is network inherent.

The transmission time is of critical importance in delay-guarantee systems such as industrial control [4] and mission-critical Internet Servers [5]. Network congestion is also of critical importance as it determines essentially the transfer time and the number of packets lost in their way.

6 Model Description

This section presents a generalization of the state information diffusion algorithm. One of the key common aspect is the diffusion of agents state information packages. This becomes increasingly important as the network becomes highly dynamic. In order to make the model network more realistic, a set of qualitative properties found in communication networks has been added to the model. In our simulations each node maintains a finite queue of packages. In the l first time instants l packages of a file containing routing information are sent by one fixed agent by selective diffusion to the whole set of nodes in the network. Each package has an unique identification number and a time to live (ttl) counter that decreases by one by each hop the package crosses. The package also maintains a list of visited nodes in order to trace the route the package has followed and to implement effectively the selective diffusion. When the ttl counter becomes 0, the package is considered obsolete and removed from the network. In addition, the package can become corrupted at any bit, including ttl, route or identification number. Packets are of fixed size.

At each time instant each node gets the first no-obsolete package (a package becomes obsolete if there is another copy of the package in the queue with a higher ttl or its ttl takes the value 0) from its queue and sends the package by selective diffusion. If the destination node is congested (i.e its queue is full) the package is removed from the network reflecting the congestion properties of the network. At every time instant all obsolete packages are removed from the queues. This allows queues to maintain the most recent version of the package. At each crossed hop, the package decrements its ttl and updates its route register. The package has a maximum number of hops that corresponds to its ttl, if the maximum is reached (i.e $ttl = 0$), the package is removed from the network. At each hop the package has a probability of becoming corrupted as it crosses links with certain probability of corrupting packages. The system stops when the whole set of nodes owns a copy of the complete file.

When the system stops the two metrics are computed. The time it takes to diffuse the file to the whole set of nodes between the total number of packages delivered.

The other metric we compute is the average error rate. We use the Hamming distance between the original packages and the packages at each node.

We have carried out our simulations for each of the different topologies obtained by applying the Watts-Strogatz method in a ring-lattice and stub-domain substrates. The next section presents the results obtained by varying the model parameters. All the metrics are averages over 100 different experiments.

7 Results

Figure 5 dislays the plots for the transfer time T and the error rate E. The left plot represents T both for ring-lattices (squares) and regular stub-domain graphs (circles). Note a very similar behavior between both dynamical metrics. The central plot shows the error rate for both substrates. Notice the lower error rate in ring-lattice substrates for high values of p. We can observe that, in both substrates, the average time T decreases and the error ratio E increases when p grows (i.e when the graph becomes more random). The reduction in the average time is due to shorter paths that are characteristic of random graphs (See fig. 2). The error ratio increases as clusters disappear and the wires become progressively longer. We can also see that in both models the error ratio E increases with p.

A clear difference between the two models is established when the average of the two measures is computed. If an average measure of the two parameters is used as in [13], an optimal point for building networks with optimal error-time delay ratio would be the networks corresponding to the small-world area for stub-domain graphs. However, for ring-lattice graphs a higher number of shortcuts are needed in order to achieve the minimum value.

8 Conclusions

We have developed a method to build initial regular substrates with different topological characteristics than the usual ones, as for example, the number of biconnected components. We studied a specific real-life dynamics over these new topologies, comparing it with the same dynamics over previously studied topologies.

We would like to conclude with several ideas. There exist regular graphs with k even and odd that present high number of biconnected components. These networks better resemble hierarquical networks than structures like stars, rings and grids. These new stub-domain networks present a similar small-world area when they are shifted from a regular to a random situation in a similar way as the ring-lattice substrates do. However stub-domain networks present a slower descent in its characteristic path. The selected substrate changes the values of the dynamical metrics of a well known internetworking problem over a network of agents.

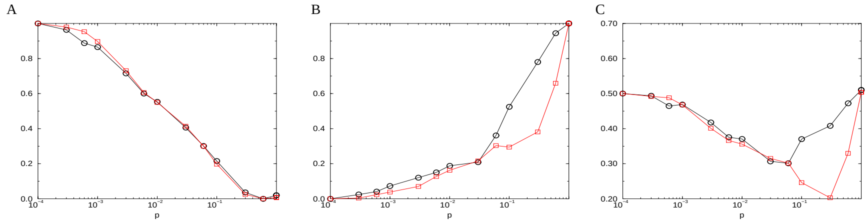

Figure 4. (A) T .(B) E.(C) (T+E)/2 for stubs-domain graphs with values $n = 2980$ and $k = 7$ (circles) and ring-lattice with values $n = 2980$ and $k = 8$ (squares).

The small-world area is an optimal point for error/time ratio average for stub-domain graphs.

We thank the Ministerio de Ciencia y Tecnología (BFI 2000-015). This work has been partially supported by the project ”The Science of Complexity” (ZiF, Bielefeld Universität, Germany).

References

[1] Fletcher, M.; Garcia-Herreros, E.; Chirstensen J.H.;Deen S.M and Mittman R. An Open Architecture for Holonic Cooperation and Autonomy Workshop on Database and Expert Systems Applications, Londres, IEEE Comput. Soc, p.224-230, 2000

[2] Corbacho F. and Arbib M. A. Schema-based Learning: Towards a Theory of Organization for Adaptive Autonomous Agents. Proceedings of the First International Conference on Autonomous Agents. Santa Monica, CA, 1997, 520.

[3] Pěchouček, M.; Mařík, V. and Štepánková O. Coalition information in Manufacturing Multi-agent systems Workshop on Database and Expert Systems Applications, Londres, IEEE Comput. Soc, p.241-246, 2000

[4] McGhee, J. Grimble, M.J. and Mowforth, P. Knowledge-based systems for industrial control IEE Books, 1990

[5] Chowdhury, A. Burger, E. and Grossman D. DRS: A Fault tolerant Network Routing system for mission Critical Distributed applications, IEEE-IC3N Sixth International Conference On Computer Communication and Networks, 1997.

[6] Watts D. J. and Strogatz S. H. Collective dynamics of small-world networks. Nature 393, 440. 1998,

[7] Watts D. J. Small Worlds: The dynamic of Networks between Order and Randomness Princeton University Press, 1999

[8] Tanenbaum A. S. Computer Networks. London Prentice Hall 1996

[9] Zegura, E. W.; Calvert, K. L. and Donahoo M. J. A Quantitative Comparison of Graph-Bases Models for Internet Topology. IEEE/ACM Transactions on Networking, Vol.5 No. 6 1997.

[10] Doar M. and Leslie I. How bad is naive multicast Routing ? Proc IEEE INFOCOM’93, 82-89, 1993

[11] Wei L. and Estrin D. The trade-offs of multicast trees and algorithms, Int. Conf, Computer Communications and Networks Aug 1994.

[12] Bollobas B. Random Graphs. Orlando, FL: Harcourt Brace Jovanovich 1985.

[13] Aguirre, C.; Martinez-Munoz, J.; Corbacho, F.; Huerta,R. Small-world topology for multi-agent collaboration. Proceedings 11th International Workshop on Database and Expert Systems Applications, Londres, IEEE Comput. Soc, p.231-5, 2000

[14] Sedgewick R. Algorithms in C++ Addison-Wesley Publishing Company, 1990

[15] Newman M.E.J. and Watts D.J. Renormalization group analysis of the Small-World network model. Physical Review E 60, 7332-7342,2000

[16] Dorogovtsev, S.N. and Mendes J.F.F. Exactly solvable analogy of small-world networks. Avaiable as cond-mat/9903426 1999

[17] Albert, R.; Jeong, H and Barabási A. Error and Attack tolerance of complex networks Nature, Vol 406, 27 July 2000