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Small-World Topology for Multi-Agent Collaboration

Carlos Aguirre
Carlos.Aguirre@ii.uam.es
Jaime Martínez-Muñoz
Jaime.Martinez@ii.uam.es
Fernando Corbacho
Fernando.Corbacho@ii.uam.es
Computer Engineering Department,
Universidad Autónoma de Madrid,
28049 Madrid (SPAIN).

Ramón Huerta
Ramon.Huerta@ii.uam.es
Computer Engineering Department,
Universidad Autónoma de Madrid,
28049 Madrid (SPAIN).
Institute for Nonlinear Science,
University of California,
San Diego, La Jolla, CA 92093-0402

Abstract

This paper studies a specific methodology for the design of different topologies in multi-agent networks with the central objective of maximizing agent collaboration. In order to obtain this feature we rely on the use of a recently discovered type of topology, namely "small-world" (SW) topology. This topology has been shown to present several advantages such as enhancement of signal-propagation speed, computational power, and synchronizability. We have extended the analysis to multi-agent networks searching for the topologies that maximize the flow of entities (data, energy, goods, etc...) with different complexities in the behaviour of each agent in the network.

1 Introduction

This paper presents the different collaboration dynamics that appear in networks of simple agents connected under different topologies. In particular we will show that a special kind of topologies called small-world (SW) defined by Watts and Strogatz (1998) [1] enhances agent collaboration by maximizing flow.

The cooperative behavior of large assemblies of dynamical elements has been the subject of many investigations both in dynamical systems [2, 3, 7] or in holonic systems [4, 5, 6]. In all these investigations the connectivity between the elements of the network were local, global all-to-all, or random sparse connectivity. However few research projects have investigated the influence that all the different connection topologies may play on the dynamics of the network [8]. Other research projects have attempted to design cooperation strategies in multi-agent networks. A comprehensive review is beyond the scope of this short article. We can only provide a few examples and list others. Cooperation among autonomous agents has been studied by the Distributed Artificial Intelligence (DAI) community for several years. Yet there has been a resurgence due to the growth of the autonomous agents scientific community. [9, 10, 11].

In this paper we present different topologies for multi-agent networks giving rise to the different functional properties. Small-world (SW) topologies maximize flow of entities while maintaining high reliability and network construction low cost.

On the other hand, random connection topologies also give rise to high flow yet they have high construction cost and low transmission reliability. Finally regular topologies show the lowest flow, although they have a low construction cost and high reliability. Hence, SW networks take advantage of the best features of regular and random networks.

To construct the multi-agent network we decompose the problem methodology in two main thrusts: single agent dynamics and network topologies. Then we study the dynamics of the multi-agent network under different configurations.

2 Single Agent Dynamics

Each agent in the multi-agent network handles information (or in general any other kind of commodity) and is capable to pass/communicate part of it to its neighbour agents in the network. In a general setting we define an agent by its perceptions, actions, believes, goals, and the corresponding mappings (Corbacho & Arbib, 1997). In this paper the internal structure of each individual agent is kept to a minimum to emphasize on the emergence of collective behavior. The network is composed of a set of autonomous agents communicated by connections which may have different properties. The dynamics of each agent in its most general form is described by

$$s_i(t+1) = f(s_i(t), g(w_{i,j}(t), s_j(t))) \qquad (1)$$

where N is the number of agents in the system and i, j verify $0 < i, j < N$, $s_i(t)$ represents the state of the agent i at time t and $w_{i,j}$ represents the effective connectivity matrix (neighborhood) of agent i. The input arriving from other agents is gathered by g. The manner in which the current state and the current input from the neighbor agents affects the agent's next state is computed by the transition mapping f. f, and $\mathbf{w}$ may be different in different multi-agent networks as it will be shown in Section 5 that describes specific systems. For instance f may have a saturation to reflect the fact that the agent capacity is limited. In all the different agent dynamics we will use

$$g(w_{i,j}(t), s_j(t)) = \sum_{j=1; j\neq i}^{N} w_{i,j}(t) s_j(t) \qquad (2)$$

but change the way in which $\mathbf{w}$ is computed for different multi-agent networks. $\mathbf{w}$ represents the effective connectivity in the sense that, for any two particular nodes i, j in the network that are connected by an edge $e_{i,j}$, this path will have an effective connectivity of $w_{i,j}$. In any topology where i and j are not connected by any path, $w_{i,j}$ will have no meaning (usually equivalent to $w_{i,j} = 0$).

3 Local vs. Global Control

The global principle from where we depart is the maximization of the flow for a large number of agents. There are in general two extreme approaches, on one hand global rules (totalitarian systems) that will respond to a measure of the total information in the system, and on the other hand local rules (democratic systems) inspired by biological systems (for example: hebbian rules and virus replication). We understand that global rules imply expensive computational procedures whereas local adaptation means cheap procedures that do not need a supervisor. We investigate under which conditions local interactions can lead to a good performance as compared to global maximization. Moreover, we will pursue the measure of the structural properties of the different network topologies in order to prove the advantages of using small-world topologies to increase the flow in the multi-agent network.

In what follows we will introduce a global function to be maximized $G(p)$ and we shall show that local interactions can give rise to the global maximization when the topology (dependent on p) lies within the small-world region.

4 Network Topologies for Agent Collaboration

We have tested three different kinds of connectivity patterns: regular, random and small-world. In all cases we included $N = 1000$ agents. To interpolate between regular and random networks we follow the procedure described by Watts and Strogatz [1] which we summarize here for convenience. Starting from a ring lattice with N vertices and k edges per vertex, we rewire each edge at random with probability p. This procedure allows us to "tune" the graph between regularity $(p = 0)$ and randomness $(p = 1)$, and probe the intermediate region $0 < p < 1$ where the small-world topology lies. We also quantify the structural properties of these graphs, following Watts and Strogatz (1998), by their characteristic path length $L(p)$ and clustering coefficient $C(p)$. Where $L(p)$ is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices. On the other hand, the clustering coefficient $C(p)$ is defined as follows. suppose that a vertex v has k_v neighbors; then at most $k_v(k_v - 1)/2$ edges can exist between them. Let C_v denote the fraction of these allowable edges that actually exist. Define C as the average of C_v over all v. Fig. 1a replicates that of Watts and Strogatz [1] for ease of reference and to verify our computations.

5 Results: Specific Test Cases

We are working towards a framework for multi-agent collaboration. Yet we must verify the framework on different specific examples. We have designed several multi-agent systems simulations where the main goal of the collaboration will be the maximization of flow through the network. Next we introduce a set of experiments under different conditions (i.e. different agent dynamics and topologies) to explore the space of configurations. For every multi-agent system we will define a global function that we would like to be maximized by the overall network. Then we check whether local interactions can lead to the maximization of this function. In general the features that we would like to maximize are *efficiency*, and *reliability* while minimizing the multi-agent network *construction cost*. Let us now introduce several examples that show the global behavior of a network of multiple agents under different dynamics.

Experiment 1: A unit of reproducible commodity (data, believes, information, etc...) is generated in one (randomly chosen) of the agents (source) $s_{i=source}(t_0) = 1$ and is distributed (copied) across the network of agents. This could represent, for example, the behavior of a set of Internet routers with a routing algorithm based on inundation routing (used in many military networks). Each agent can interact with all the agents it is directly connected to in the

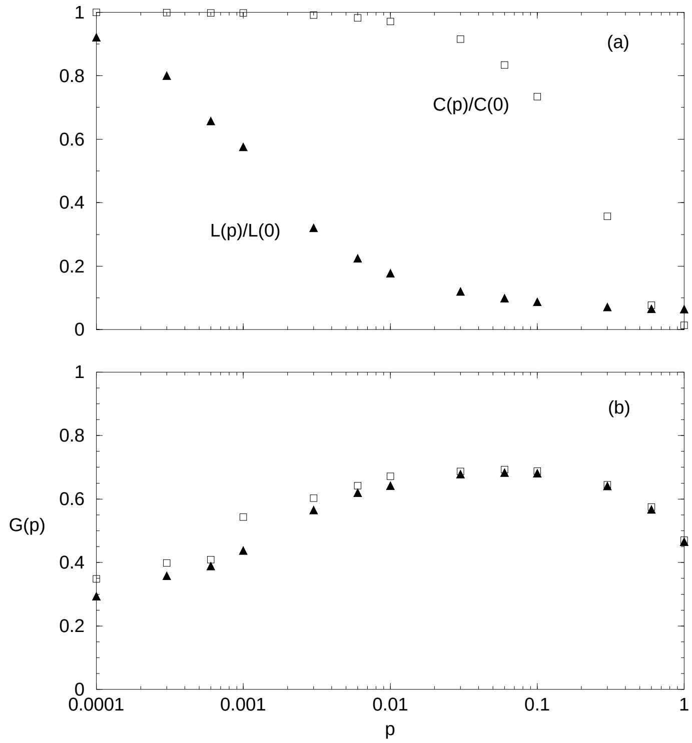

Figure 1. A. Characteristic path length $L(p)$ and clustering coefficient $C(p)$ for the family of randomly rewired graphs ($N = 1000$ and $k = 10$), normalized to the values $L(0)$ and $C(0)$ of the regular case. B. Global function $G(p)$ for the whole range of networks for Experiment 1. The triangles correspond to a single random target and the squares to the case when all the nodes in the network have received the information. Both curves are averages over 100 realizations of the simulation.

network. Each connection has a transmission *reliability* between pairs of agents. The transmission reliability reflects the fact that longer connections are more prone to loss (message lost in Internet, LANs, etc.). This is implemented by assigning a transmission probability $p_{i,j}$ to each connection in the network, assigning lower probabilities to longer connections and calculating $w_{i,j}(t)$ accordingly,

$$w_{i,j}(t) = \begin{cases} 1 & \text{if } p_{i,j} > \gamma \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where γ is a random variable in the interval $[0, 1]$ under an uniform distribution. The transition function corresponds to

$$f(s_i(t), in) = \begin{cases} 0 & \text{if } s_i(t) = 0 \text{ and } in = 0 \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

Initially we compute the construction cost for the different topologies $O(p)$. The construction cost is computed by considering all the links of a regular graph with the same cost and considering the links created during the re-wiring procedure described in section 4 with a higher cost (twice the value of a short link). Then we allow the agents to interact and we measure the transmission efficiency as the time it takes for the commodity (in the source) to reach all the nodes in the network $T(p)$ (as well as half of them). We then construct a function to be maximized that takes into account both the cost and the transmission reliability, namely

$$G(p) = \frac{1}{2}((1 - \frac{T(p)}{T_{max}}) + (1 - \frac{O(p)}{O_{max}})) \quad (5)$$

where T_{max} corresponds to the maximum time it takes for the unit of commodity reach all the nodes in the network for all networks, and O_{max} corresponds to the maximum cost for all the networks. In fig. 1B we plot $G(p)$ for each of the different networks characterized by its probability p. Notice that the maximum value occurs in the region in which a high $C(p)$ and a low $L(p)$ occur simultaneously. This is precisely the S-W region. The triangles correspond to a single target node and the squares to the case when the information has arrived to all the nodes, in both cases averaged over 100 trials.

Experiment 2: A packet of non-reproducible commodity (energy, chemical substance, etc...) is sent to one and only one of the neighbors due to the nature of this kind of commodity. To avoid any *a priori* bias the neighbor is selected randomly. The agent dynamics is described by (1), (2), (4), and

$$w_{i,j}(t) = \begin{cases} 1 & \text{if } j = \alpha \text{ and } p_{i,j} > \gamma \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where α is a discrete random variable in the interval $[1, k]$ under an uniform distribution. The function $G(p)$ for each of the different networks is represented in Fig. 2A. Again, the maximum value occurs in the region in which a high $C(p)$ and a low $L(p)$ occur simultaneously.

Experiment 3: Each agent may have any number of units of commodity p_l with $l = 1 \ldots s$. Yet for ease of analysis we still assume it can only send one unit at a time. Each agent maintains a queue of commodities q_i, and sends the first unit of commodity in its queue (removing the unit from its queue) to one of its neighbors (except for the target agent s_{target} that does not send any commodity). The receiving agent inserts the unit of commodity at the end of its queue. Again the neighbor that receives the unit of commodity is selected randomly. In this experiment we consider $T(p)$ as the time it takes for all the units to reach a randomly selected target agent (averaged over 100 simulations) using (5) as function to be maximized. In this case let $q_i(t)$ represent the state of the queue of agent i at time t, and the operators $R(q)$ and $A(q, s)$ the standard 'remove from head' and 'add packet s to tail' queue operators and $N(q)$ represent no operation over the queue. According to this, the agent dynamics are described by

$$q_i(t{+}1) = \begin{cases} A(q_i(t), R(q_j(t))) & \text{if } w_{i,j} = 1 \text{ and } j \neq target \\ N(q_i(t)) & \text{otherwise} \end{cases} \quad (7)$$

where $w_{i,j}$ is defined as in (8). Fig. 2B shows $G(p)$ for the different topologies generated by probability varying p. Surprisingly, in spite of the very different system dynamics, $G(p)$ has a very similar shape as in the other experiments pointing out the robustness of the framework. Again the maximum values of $G(p)$ are obtained in the S-W region.

Directionality constraints:

In many MAS, the connections between agents cannot be considered as bidirectional, in the following experiment we followed the networks generation procedure described in section 4 but considering the links as directed edges in the graph.

The results obtained resembled the ones obtained when the connections where considered as bidirectional. The small-world area in directed graphs is also present at the same range of probabilies than for undirected graphs. The maximum value of the function $G(p)$ was obtained again in the small-world area. Figures are ommited as they are very similar to the images obtained when considering undirected graphs.

6 Discussion

In this paper we have presented a set of multi-agent simulations that can be applied to specific real-life examples in a better way than other approaches that don't take in account factors such as cost, loss of information, dissipation of energy, etc. A specific example could be the electric power grid; this has been already investigated by [1] (for the electric power grid of the western United States), our approach allows to introduce another important feature: the cost. The introduction of the cost in the analisys of the different possible topologies allows a realistic study of the more suitable topology for this particular system. Another example could be a network of robots trying to interchange information within an environment containing several obstacles.

For instance, two robots are connected if they are able to interchange information (one or both can move in order to get a distance short enough to establish communication). Long connections could mean that there are some robots that can move through some obstacles that others don't. These long distance robots could have a very high cost compared with normal robots and will also spend much more energy in their movements.

Networks of Internet routers could be another interesting real-life example of system that can use these results in order to maximize the information flow in the network with the lowest cost. In [11] a Collective Intelligence approach is proposed in order to control Internet traffic routing. We strongly believe that this approach can be complemented with a convenient selection of the network topology. There are many other systems and dynamics that can improve their performance through the use of these SW topologies.

7 Conclusion

In this paper we have investigated a variety of possible network topologies. Each one gives rise to different properties. We have reviewed both regular and random topologies and have introduced new results on small-world networks. The main conclusion of this research project is that small-world topologies seem to give rise to cheaper, more efficient, and more reliable cooperation channels for multi-agent systems.

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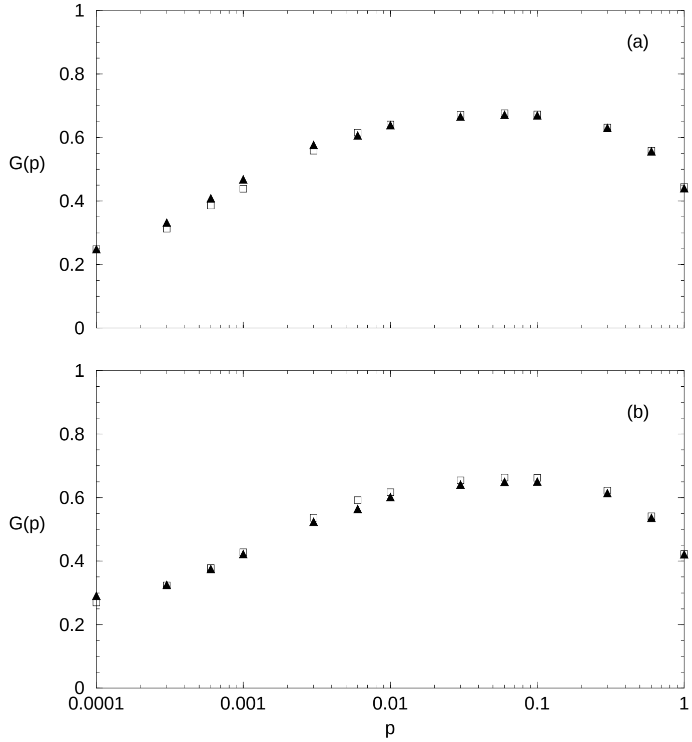

Figure 2. Global function $G(p)$ for the whole range of networks. Symbols and parameters are like those in fig.1

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