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FINE TUNING EXPLAINED? MULTIVERSES AND CELLULAR AUTOMATA

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Abstract

The objective of this paper is analyzing to which extent the multiverse hypothesis provides a real explanation of the peculiarities of the laws and constants in our universe. First we argue in favor of the thesis that all multiverses except Tegmark's «mathematical multiverse» are too small to explain the fine tuning, so that they merely shift the problem up one level. But the «mathematical multiverse» is surely too large. To prove this assessment, we have performed a number of experiments with cellular automata of complex behavior, which can be considered as universes in the mathematical multiverse. The analogy between what happens in some automata (in particular Conway's «Game of Life») and the real world is very strong. But if the results of our experiments can be extrapolated to our universe, we should expect to inhabit –in the context of the multiverse– a world in which at least some of the laws and constants of nature should show a certain time dependence. Actually, the probability of our existence in a world such as ours would be mathematically equal to zero. In consequence, the results presented in this paper can be considered as an inkling that the hypothesis of the multiverse, whatever its type, does not offer an adequate explanation for the peculiarities of the physical laws in our world.

Keywords: Astrophysics, Cosmology, Multiverse, Fine tuning, Cellular automata

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Introduction

The hypothesis that the universe where we live represents, not the whole physical reality, but a particular domain inside a much larger reality called “multiverse,” which includes many other universes, has received in recent years increasing attention in cosmology. This is because the multiverse appears on the horizon of some lines of research independent of one another. One of these lines is the study of the peculiarities of the structure of the laws and constants of nature. Given the current state of our knowledge,

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they seem very peculiar, in the sense that slight changes in those laws and constants would result in a universe unable to generate life, or indeed any form of complexity. Is the multiverse a good way to explain the peculiarities of the laws of nature? If so, such peculiarities would be simply an effect of the anthropic perspective: we can only observe the laws that are compatible with our existence.

In this article³ we suggest arguments for the thesis that this explanatory strategy concerning the peculiarities of the laws and constants of nature of our universe does not work. This happens because the different models of multiverse never have the right size: either they are too small, and the question about the peculiarities of the laws is simply shifted to the multiverse level, or to avoid this, one must postulate a multiverse so large that the simplicity of the laws of nature in our universe becomes problematic.

The first part of this argument -that almost all proposed multiverses are too small-, is well known in the specialized literature. All we have to do is to summarize the discussion here. However, to defend the second part -that a multiverse that avoids the previous problem becomes too big to understand the simplicity of the laws of nature in our world- we will offer a new argument.

We begin by pointing out that one of the most remarkable peculiarities of the laws of nature is that they do not show (at least so far) any temporal variation. This is a very special feature, because there are infinitely many possible laws and constants with a form similar to those in our world, except that they show some time dependence. Living in a very general multiverse, we should not expect to inhabit a world like ours, unless time dependence of the laws of nature is incompatible with the development of complex structures such as those we observe in our world. Is this the case?

In order to answer this question, we propose to study what happens to the structures generated by cellular automata. The reason for this suggestion is that there is a strong analogy between the structural properties of complex entities in our world (in particular, biological and chemical structures), and the structures that can be generated in certain cellular automata (those which are computationally complete, i.e. equivalent to a Turing machine or a digital computer). The analogy between both types of structures is so close, that cellular automata have been used for decades to support the study of issues such as the generation of complexity and the emergence of new properties in the evolution of life, the dynamics of ecosystems, neural interactions and so on.

We will ask, then, what would happen if the cellular automata rules (the equivalent to the laws of nature) that make it possible to develop structural configurations closely similar to the complex structures in our world, were time-dependent⁴.

³ A preliminary (and extended) version of this article is available at arXiv under the title «Is the Multiverse Hypothesis capable of explaining the Fine Tuning of Nature Laws and Constants? The Case of Cellular Automata». See: <http://arxiv.org/abs/1105.4278>

⁴ The suggestion of using cellular automata as simple models of possible universes in a multiverse has been already used by Paul Davies in his paper DAVIES (2007), but the goal of Davies is to make use of the automata to prove his conjecture regarding the origin of the bio-friendly laws of the universe. We think, however, that Davies conjecture (which includes the idea of physics and biology co-evolving in such a way that an apparently teleological behaviour in the universe emerges) is very speculative. And it seems not easy to get support for such ideas through automata models. Therefore our aim is simply to explore the use of cellular automata to test the (somewhat more conventional) multiverse explanation of the peculiar features of the nature laws in our universe.

On the basis of the simulations we are currently performing, we answer tentatively that a temporal independence of the rules is not necessary for the survival of complex structures in the automata-worlds. Therefore, if the analogy is valid, this feature would not be necessary for the existence of complexity in our world, and the laws of our universe are thus much simpler than we should expect, if we really live in the multiverse.

To develop this argument, the paper is divided into the following sections:

In the first section we briefly review the reasons which led to the proposal of the multiverse hypothesis as a possible (and acceptable in principle) scenario in cosmology.

In the second section, we consider some of the examples of fine tuning of the laws and constants of nature discussed in the latest years, since the use of the multiverse hypothesis to explain this type of data is what are questioning here.

In the third section we discuss the issue of which version of the multiverse should be assumed to eliminate fully the problem of the fine tuning of the universe. We shall show that only the «mathematical multiverse» proposed by Max Tegmark prevents the question of fine tuning to surface again in the multiverse context.

In the fourth section we suggest a way to test that hypothesis. We have selected cellular automata as examples of possible universes where we can test some of the «predictions» offered by Tegmark as regards the multiverse hypothesis.

In the fifth section we detail the experiments we have performed with cellular automata with respect to Tegmark's «predictions». They try to show how the behavior of cellular automata is affected by changes in their rules –the equivalent to the laws of nature in a universe– concerning their ability to develop complex structures.

In the sixth section we state the consequences of our study regarding the question of whether the multiverse hypothesis can explain the fine tuning of our universe. Our provisional answer is on the negative: if we accept the multiverse as the explanation of the fine tuning of universe, we should expect that the laws of this universe would be less simple than they are. In particular, it seems that we could expect that at least some of the laws and constants of nature should show a certain time dependence.

1. Three roads to the multiverse

The hypothesis that our universe is only a particular domain within a much larger multiverse started to be considered a cosmological possibility in the last decade of the past century. Although a similar idea was offered in the fifties as a solution to the quantum measurement problem (Everett's «multiple worlds» interpretation), that problem is completely different from those tackled by the cosmologists upholding the multiverse hypothesis. Therefore, we will not consider here Everett's approach and its subsequent formulations.

The confluence of three different lines of research explains why a hypothesis as speculative and risky as the multiverse has been taken seriously in the latest years. These lines are the following:

- (1) Inflationary cosmology.
- (2) Diverse attempts to build a quantum gravity theory.

- (3) Research on the effect that a small modification in the structure of physical laws would have on the development of complex beings and life as we know it.

The multiverse question arises in the context of inflationary cosmology. The cosmic inflation hypothesis was proposed initially by Alan Guth in 1981. Guth was trying to explain two phenomena which the standard cosmological model does not explain: (1) the homogeneity of those regions in the universe which had never been able to interact [the so-called horizon problem] and (2) the fact that the universe seems to be approximately flat, which entails that, in the beginning of the expansion, the density parameter of the universe must have had a value extremely near to the critical density [the so-called flatness problem].

Guth proposed that the universe experienced a process of exponential expansion between 10^{-37} and 10^{-35} seconds after the Big Bang. At the end of this stretch of time, inflation was replaced by an expansion similar to that described by the standard model, which would still be valid, except for its application to the first stages of the universe.

The inflationary scenario offers an answer to the two mentioned questions. The horizon problem is solved because what today makes our observable universe proceeds from a very small region with mutual interactions before the exponential expansive phase. And the flatness problem is solved because the universe, as a consequence of inflation, has reached such dimensions that it appears to be practically flat, even though it may still possess some curvature.

Meanwhile, the initial Guth model, together with several other proposals made to explain the mechanism of inflation, have been proved unfeasible. Currently, the model which appears to present less problems –developed mainly by Andrei Linde, Alex Vilenkin and co-workers– suggests that the cosmic inflationary process never ends, that the universe expands exponentially forever, while here and there different domains are being formed, one of them our observable universe, which are regions of a much larger physical reality: regions where the potential pushing the inflation has reached a minimum value, where the exponential growth of the cosmos has stopped. As these regions are causally disconnected from one another, it seems that, if we accept the inflationary hypothesis, we should also accept the existence of the multiverse.

Another independent line of research which leads to the idea of the multiverse is the search for a quantum gravity theory. Briefly, it will suffice to say that we have at present two main different approaches towards this theory: the superstring hypothesis and the quantum loop theory, both of which end up in the idea of the multiverse. In the case of superstrings, the problem is that, instead of there being a single physical structure complying with the requirements considered fundamental for this frame, there are about 10^{500} (or as some say, 10^{1000}) possible structures. This is a regrettable situation, and the solution proposed by Susskind and others is that physical reality carries out all those possibilities. So we would live in a multiverse where all the possible universes within the frame of string theory are also real universes.

As to the quantum loop hypothesis, it so happens that the first tentative cosmological models being developed within this frame –Bojowald models– suggest that our universe suffered a process of collapse previous to the Big Bang described by the standard cosmology. This has reinforced Smolin's conjecture that this universe could have resulted from a gravitational collapse inside a larger physical reality. In other words, one

universe may give origin to another, inside one of its black holes. Once again, the multiverse scenario.

Finally, the multiverse hypothesis has been proposed as a solution to the problem of the fine tuning of physical constants and laws. This problem can be stated thus: since the eighties, we have a detailed standard model, both in cosmology and in particle physics, that makes it possible to analyze, theoretically and through computer simulations, questions such as the consequences for the cosmos of slight changes in some of the parameters exhibited by these models. The result of these researches has been the discovery that a certain number of these parameters, both in the standard cosmological model as in the standard particle physics model, give the impression of being finely tuned, in the sense that, if they had had values minimally different from those they actually have, life in the cosmos –and in many cases every complex structure– would have been physically impossible.

Some authors interpret this fact as an inkling that our universe is nothing but a single domain in a much larger reality, in such a way that we inhabit just that domain of reality where the appropriate conditions prevail for the existence of life as we know it.

The objective of this paper is analyzing to which extent the multiverse hypothesis provides a real explanation of the peculiarities of the laws and constants in our universe. But before discussing this, we should have an idea of what these peculiarities are. We will devote the next section to this issue.

2. The fact of the fine tuning of the universe

All along the last century, especially in its last decades, a surprising fact about the universe where we live has been discovered: the fact that its architecture possesses very peculiar properties, in the sense that very slight changes in the combination of physical laws and constants of nature would have the consequence that the cosmos would become a physical system hostile to the development of life and possibly hostile too to the development of any form of complex entities (at least those based on chemistry). The fact that the universe behaves following one of the (at least apparently) scarce hospitable combinations of laws and constants is known as the «fine tuning» of the laws of nature. In order not to leave this exposition in a too abstract plane, we shall mention a few concrete examples of this tuning. These examples have been taken from Robin Collins paper «The evidence of fine tuning», one of the clearest presentations of the matter.

(a) The cosmological constant:

«The smallness of the cosmological constant is widely regarded as the single greatest problem confronting current physics and cosmology. [...] Apart from some sort of extraordinary precise fine-tuning or new physical principle, today's theories of fundamental physics and cosmology lead one to expect [...] an extraordinary large effective cosmological constant, one so large that it would, if positive, cause space to expand at such an enormous rate that almost every object in the Universe would fly apart, and would, if negative, cause the Universe to collapse almost instantaneously back in on itself. This would clearly make the evolution of intelligent life impossible.

What makes it so difficult to avoid postulating some sort of highly precise fine-tuning of the cosmological constant is that almost every type of field in current

physics [...] contributes to the vacuum energy. [...] [When] physicists make estimates of the contribution to the vacuum energy from these fields, they get values of the energy density anywhere from higher 10^{53} to 10^{120} than its maximum life-permitting value»⁵.

(b) The strong and the electromagnetic forces:

«A 50 percent decrease in the strength of the strong force, for instance, would undercut the stability of all elements essential for carbon-based life, with a slightly larger decrease eliminating all elements except hydrogen»⁶

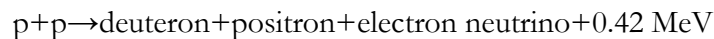
«[Around] a fourteen-fold increase in the electromagnetic force would have the same effect on the stability of elements as a 50 percent decrease in the strong force»⁷.

(c) Carbon production in stars:

«[A] change of more than 0.5% in the strength of the strong interaction or more than 4% in the strength of the Coulomb [electromagnetic] force would destroy either nearly all C or all O in every star. This implies that irrespective of stellar evolution the contribution of each star to the abundance of C or O in the ISM [interstellar medium] would be negligible. Therefore, for the above cases the creation of carbon-based life in our universe would be strongly disfavored»⁸

(d) The proton/neutron mass difference:

«The neutron is slightly heavier than the proton by about 1.293 MeV. If the mass of neutron were increased by another 1.4 MeV –that is, by one part in 700 of its actual mass of about 938 MeV– then one of the keys steps by which stars burn their hydrogen to helium could not occur [...]:



[...] On the other hand, a small decrease on the neutron mass of around 0.5 to 0.7 MeV would result in nearly equal numbers of protons and neutrons in the early stages of the Big Bang [...] resulting in an almost all-helium universe»⁹

Accepting thus that there is a delicate tuning of the laws and physical constants, without which the development of complex chemical structures would not have been possible, especially life (and most especially intelligent life, whose appearance requires doubtlessly that favorable conditions are maintained much longer than what is required by one-cellular life), the question is how to interpret this fact. What does this fine tuning tell us about the physical reality? Is it a meaningful datum, or mere chance? And if the former, what does it entail? What is it pointing at?

In the latest years, some authors suggest that fine tuning is an inkling that the cosmos is much wider than we assumed and is made of domains with different

⁵ COLLINS (2003) 180-181.

⁶ Ibidem 182-183. Taken from BARROW - TIPLER (1986) 326-327. This book from Barrow and Tipler is the very classical exposition on fine-tuning of the universe, and, as such, it is highly recommended for the interested reader.

⁷ Ibidem.

⁸ Ibidem 185. Cited from OBERHUMMER et al. (2000) 90.

⁹ Ibidem 186-187.

combinations of laws and constants, in which case we must inhabit one of those oases favorable to life in the middle of a mostly inhospitable physical whole. This is equivalent to propose the multiverse as the explanation of the fine tuning observed in the cosmos.

Now then, under which conditions can the multiverse really explain the fine tuning of our universe? We will tackle this question in the next section.

3. Types of multiverse and their adequacy to explain the fine tuning

To explain the fine tuning of the universe, the multiverse has to meet certain requirements. In the words of Bostrom:

«A multiverse theory can potentially explain cosmological fine-tuning, provided several conditions are met. To begin with, the theory must assert the existence of an ensemble of physically real universes. The universes in this ensemble would have to differ from one another with respect to the values of the fine-tuned parameters, according to a suitably broad distribution. If observers can exist only in those universes in which the relevant parameters take on the observed fine-tuned values (or if the theory at least implies that a large portion of all observers are likely to live in such universes), then an observation selection effect can be invoked to explain why we observe a fine-tuned universe. Moreover, in order for the explanation to be completely satisfactory, this postulated multiverse should not itself be significantly fine-tuned. Otherwise the explanatory problem would merely have been postponed; for we would then have to ask, how come the multiverse is fine-tuned? A multiverse theory meeting these conditions could give a relatively high conditional probability to our observing a fine-tuned universe»¹⁰

Let us now ask what multiverse could meet these requirements. In the specialized literature there are several types of entities called «multiverse». The main ones are the following:

1. On the first place, we find authors who give the name of multiverse to the infinite universe, following this reasoning: the observable universe, the environment which includes all the objects whose light has reached us since the Big Bang¹¹ to our time, has now a radius of about $4 \cdot 10^{26}$ meter; the volume of the corresponding sphere is called «the Hubble volume». It is thus a partial domain inside the infinite universe. Everything we could examine, whatever the power of our telescopes, is included inside our Hubble volume. If there is something beyond, we don't know, and it can never affect us causally. In fact, for us, its existence does not matter. Under a purely empiricist criterion (which we are far from enforcing) we could say that the assertion that there is something beyond the Hubble volume is not even scientific.

¹⁰ Bostrom (2007) 439-440.

¹¹ Actually we can only receive the light emitted after what is usually called the «surface of final dispersion», the instant when radiation uncoupled from matter. This happened about 100,000 years after the Big Bang. But these details are not important here. After all, 100,000 years are not much... at the cosmological scale.

Some authors suggest that we should consider every sphere of the same size as our observable universe as a full-fledged universe. Since an infinite universe would contain infinitely many spheres of this kind, they give it the name of multiverse.

Actually this terminology is a rather unfortunate choice, because it makes us take as a set of different entities what really makes a single physical system —the open universe— endowed with a high degree of unity, which cannot be decomposed in «sphere-universes» or any other cosmological sub-unities, except in an arbitrary way.

2. The second type of multiverse derives from physical hypotheses which have not reached an empiric support that would allow them to become standard theories, but a certain number of specialists trust that they are adequate to reality, and try to develop them and provide the empiric support they still lack. In this group we may rank the multiverse derived from the eternal inflation scenario proposed by Linde and Vilenkin; the multiverse containing the so-called «cosmic landscape», i.e. all the possible realizations of superstring theory; the scenario defended by Smolin of multiple universes generated in black holes; and similar conjectures. In this case, the term multiverse is used to represent something completely new —as against the infinite universe.— In all these scenarios, the various domains differ structurally from one another. This means that the laws of physics can be partially different in each domain, although all of them obey a common general physical structure. On the other hand, each of the cosmic domains is completely —or almost completely— causally disconnected from the others. Therefore each can be considered as an authentic «universe island» which would continue its evolution according to its own dynamics although the remainder of the universe disappeared in an immense cosmic cataclysm.
3. Finally, since a few years ago, the possible existence of a multiverse incomparably larger than the former is being discussed. This idea, proposed by the physicist Max Tegmark, consists in assuming that:

«[...] mathematical existence and physical existence are equivalent, so that *all* mathematical structures exist physically as well»¹².

Tegmark's suggestion results in the conception of a multiverse where every possible combination of laws and natural constants occurs in fact in one or another domain of reality. In the cosmic scenario proposed by Tegmark, there are no privileged mathematical structures, nor privileged initial or boundary conditions, nor physical constants of any type whose values are restricted to such and such concrete value. Every consistent mathematical structure is realized. (Or, more precisely, every consistent mathematical structure is a physical universe). The only reason behind the peculiarities of the universe we observe is anthropic: we just observe the world which is consistent with our own existence.

In fact, this third type of multiverse is the only one that provides us with an scenario which does not leave room for the question about the actual values of the physical constants, nor the question of why physical laws are what they are. That is, only a multiverse which realizes all the consistent mathematical structures seems a candidate with possibilities to solve the question of the fine tuning of the universe, for in the other multiverses this question appears again, this time in the multiverse frame.

To see that this is so, let us look at a concrete example: the cosmic landscape of string theory may contain about 10^{1000} structurally different universes, but they share

¹² TEGMARK (2004) 483.

common features, such as this: all of them possess physical laws of the quantum type. None of these universes may be ruled, for instance, by a Newtonian physics frame. This is an interesting detail, as the huge importance of quantum effects for the appearance of the chemical structures basic for life makes us suspect that, if the multiverse contained only worlds based on variations of classical physics, not one of them would be apt for the existence of life nor in general for the existence of complex chemical structures. Therefore, whatever the enormous size of the superstring cosmic landscape, it is still a biophile scenario which suggests design, in contrast with the unrealized possibilities of completely sterile multiverses.

Most of the multiverse models proposed up to now –those of Linde, Vilenkin, Susskind, Smolin, etc.– are subject to this problem: they only realize a too small number of all the possible physical structures. Therefore, the fact that one of them is apt for life and other complex systems is still surprising.

Let us underline this: to notice the limitation of these multiverses, at first sight so vast, what we must do is look at them from the –incomparably larger– perspective of all the mathematical structures which could be considered as the basis for the laws of a possible universe. In other words: in principle, if we start from the set of logically possible universes, it is possible to define in them a great variety of subsets (which would be the possible multiverses), as well as a great variety of mechanisms generating such subsets¹³. And each of these subsets of universes, together with each of their possible generating mechanisms, will possess certain features, more or less favorable for the development of complex structures, living beings, intelligent observers, or any other type of realities. What eventually takes us again to a situation of fine tuning which should be explained. In the words of Stoeger:

«If we do have good evidence, and an adequately specific model for, the multiverse to which our own universe belongs, thus providing some explanation for its bio-friendly characteristics, this would not be a complete –let alone an ultimate– explanation. We would still require an explanation for the existence and bio-friendly character of the multiverse itself (bearing in mind that there is no unique prescription for it) and for the process through which it emerged [...]»¹⁴.

For this reason, many authors have come to the conclusion that the postulate of this kind of multiverse just poses the problem of design in a new plane. In the words of Davies: «Multiverses merely shift the problem up one level»¹⁵.

How can this be solved? In principle, it seems that the only solution is the mathematical multiverse proposed by Tegmark. Evidently, the mathematical multiverse is quite different from the others. In this multiverse there are no mathematical structures (or no particular values of constants) privileged, thus there remains no room for design and choice (or chance). In other words, starting from the physical existence of all mathematical consistent structures, the objections placed (among others) by Davies and Stoeger do not apply.

¹³ Consult about this, for instance, the thoughts by Ellis, Kirchner and Stoeger about the set of physically possible universes and the different kinds of subsets (multiverses) definable in it. These thoughts can be found in the following papers: ELLIS – KIRCHNER – STOEGER (2003), and STOEGER – ELLIS – KIRCHNER (2004).

¹⁴ STOEGER (2007) 455.

¹⁵ DAVIES (2007) 497.

Let us thus look at the next question: Is the mathematical multiverse a viable explanation of the fine tuning of the laws and natural constants of our universe? Can we do something to test this scenario? Or is this just an unwarranted speculation? We will tackle this in the next section.

4. Predictions from the mathematical multiverse hypothesis. Cellular automata as a way to test them

It does not seem easy to derive, from the hypothesis of the mathematical multiverse (or actually from any other variant of the multiverse hypothesis) concrete predictions relative to our world. However, something can be done. If we start from the hypothesis that we live in a typical universe in the set of all the universes consistent with our existence –the so-called «mediocrity principle»¹⁶, which can be vindicated by means of statistical arguments– there are at least three assertions (proposed by Tegmark) which should take place. The first two can be formulated as follows:

«—**Prediction 1:** The mathematical structure describing our world is the most general among those consistent with our observations.

—**Prediction 2:** Our observations are the most general consistent with our existence.»¹⁷

The third statement is the following:

«Our future observations are the most general among those consistent with our past observations.»¹⁸

As in the case of the second prediction, what is being stated here is the fact that the behavior of the universe we observe cannot be more specific than what is strictly necessary to guarantee our existence. What will happen in the future should be determined only, and in the most general way, –consistent with the anthropic condition– by what has been observed in the past. Thus we should not expect that nature exhibits unnecessary

¹⁶ The «mediocrity principle» and the way in which this principle can be used to make predictions in the context of the multiverse hypothesis have been explained e.g. in VILENKIN (2006) chapter 14. Similar ideas (in a non cosmological context) had been proposed earlier by John Leslie and Richard Gott [See e.g. GOTT (1993)]. Gott called this principle «Copernican anthropic principle», but it is basically the same idea. Anthropic reasoning and predictions based in the «mediocrity principle» have been subject to some criticism. In the words of Vilenkin:

«The best we can hope for is to calculate the statistical bell curve. Even if we calculate it precisely, we will only be able to predict some range of values at a specified confidence level. Further improvements in the calculation will not lead to a dramatic increase in the accuracy of the prediction. If the observed value falls within the predicted range, there will still be a lingering doubt that this happened by sheer dumb luck, If it doesn't, there will be doubt that the theory might still be correct, but we just happened to be among a small percentage of observers at the tails of the bell curve.

It's little wonder that, given a choice, physicists would not give up their old paradigm in favor of anthropic selection. But nature has already made her choice. We only have to find out what it is. If the constants of nature vary from one part of the universe to another, then, whether we like it or not, the best we can do is to make statistical prediction based on the principle of mediocrity» VILENKIN (2006) 151.

¹⁷ TEGMARK (1998) 4.

¹⁸ TEGMARK (2007) 120.

regularities along time. In this way, we can consider this third prediction a particular case of the second.

The problem, anyway, is that these assertions are so general that it is not easy to see how they could be refuted by means of concrete observations about the structure of our world.

In the ideal case, a scientist doing research in this area should be able to study several universes with laws similar to ours (to a certain extent), so as to test the result of altering in some way the laws of nature. Do we really observe the most general laws consistent with our existence? Are our future observations the most general among all those consistent with our past observations? For instance, up to now everything seems to point to the fact that neither the laws nor the constants of nature in our universe change with time. Does this mean that, if they exhibited a minimal variability, they would be unable to generate structures such as the vast variety of complex systems (chemical and biological) that we see in our world?

In the ideal case, the scientist would choose a sample of universes, some identical to ours, some with a certain variability of the laws of nature (or other variation providing those laws with a more general formulation than ours); and would find whether intelligent life, or, in general, complex chemical systems, becomes impossible in those universes. If this is not the case, we would be living in a universe with specially simple laws among those universes compatible with life as ours, and predictions 2 and 3 in Tegmark's proposal would be falsified.

Well, it is evident that we do not have a sample of universes to perform such a study. But perhaps we can reach the same goal in an indirect way, by studying a type of mathematical structures that can exist in multiple variations, and which generate worlds –at least in the context of the mathematical universe, where every consistent mathematical structure must be considered a world– where, depending on the rules and the initial or boundary conditions selected, complex entities may or may not appear and stay living.

Cellular automata¹⁹ (CA in short) make an interesting case of this type of mathematical structures. They consist of the following components: (1) a discrete space of dimension $n \in \mathbb{Z}$ divided in cells; (2) a finite set of possible states for each cell; (3) a certain number (the same for all cells) of neighboring cells; and (4) a transition rule that fixes the next state of each cell as a function of its current state and the states of its neighboring cells. Time is considered a set of discrete instants, i.e. $t \in \mathbb{Z}$. Alternatively, a CA can be seen as a set of finite deterministic automata (FDA) distributed in discrete cells along a regular grid. The inputs of the automata are the sets of states of their neighbors; the neighborhood is the same along the grid. CA can be one-dimensional (if the grid is a string of cells), bi-dimensional (when the grid is a surface), or higher-dimensional.

Cellular automata are very useful for the question we are trying to research, because they have two very interesting properties:

- (1) CA provide us with models of «universes» regulated by rules easy to describe. Thus, the development of an automaton-universe (provided with certain initial conditions) can easily be followed with the help of computers.
- (2) Some automaton-universes show close analogies to our cosmos.

¹⁹ See NEUMANN (1966).

The first analogy is simple and links the types of universes described by CA with those that arise from changes in the laws of our world. Just as most of the variations of our laws of nature lead to uninteresting universes without complex structures, while a few combinations (such as the laws of our universe) make the development of a wide variety of complex structures possible; in a similar way, most of the CA rules lead to «universes» which generate monotonous (steady or periodic) or chaotic configurations, while some rules make the development of a wide variety of complex structures possible.

Beyond this, it has also been shown that the set of complexity-generating CA produce dynamic systems with amazing parallels to physical systems of our world.

Reiner Hedrich summarizes this point as follows:

«It seems to give crucial parallels between these mathematical models and the real systems of our nature. Many of the concepts and characteristics that were already known by the systems of nature and older theoretical approaches are also found in the investigation of the macro-behavior of cellular automata. This includes, for example, symmetries, conservation laws, reversibility and irreversibility (time arrow), macroscopic order parameter, self-organization phenomena, periodicity and aperiodicity [...], light cone structures, non-separability of space areas as a result of interactions, deterministic chaos [...] and complex pattern formation»²⁰.

Against this background it seems justified to use CA as a base to explore the question of the typical characteristics of the mathematical structures that can produce a complexity-generating universe.

This conclusion may be further strengthened if we consider the research of Stephen Wolfram and his co-workers about the structures of all types of natural systems, not excluding those of living beings. These studies strongly support the view that the similarity between cellular automata generated systems and natural systems is not a mere analogy, but the dynamics of automata are precisely the mechanism that nature uses to generate the complex structures of our world²¹.

The most famous of the complex (sometimes called fractal) automata is the so-called «game-of-life», which we will tackle extensively in the next section. Such is the analogy between what happens in this automaton and the real world, that the «game-of-life» has been used by authors such as Daniel Dennett as an illustration of how a world ruled by a simple and strict physics may give rise to structures strongly analogous to living beings, in the sense that, like living beings, they should be described with a language of «intentions», «risk avoiding», «anticipation», «open opportunities», and so forth²².

All these facts make of cellular automata in general, and complex cellular automata of the «game-of-life» type in particular, a set of mathematical objects key for the study of the predictions of the mathematical multiverse. The CA provide us with possible worlds which sometimes contain classes of objects at least analogous in complexity to those in our own universe. This gives us the opportunity to investigate what happens to the complexity

²⁰ HEDRICH (1990) 201-202. For more information about this see HEDRICH (1990) 191-202 and HEDRICH (1994) 94-106.

²¹ See WOLFRAM (2002) cap. 7-8.

²² See DENNETT (2003) cap.2.

of those worlds when the rules of the interesting automata are made more general and complicated.

For instance, it is evident –at least to the degree of precision reached by our best instruments– that the laws and constants of nature do not experience any temporal variation in our world. Is this a necessary prerequisite for a universe to generate complex structures as ours? Or is this a case of a strange simplicity within the set of universes with rules that allow interesting structures to appear? That is, do we live, or not, in a typical universe in the set of those that generate dynamics similar to our world, as we should expect, according to the predictions of the mathematical universe?

We shall try to answer this question in the next section.

5. Cellular automata considered as universes. Experiments on the influence on complex structures of changes in the laws

As already said, a cellular automaton can be seen as a set of finite deterministic automata distributed in discrete cells along a regular grid. When the grids are finite, boundary conditions become essential. They determine, for instance, which is the left neighbor of the leftmost cell.

A given CA can be tested (executed) with different initial conditions: the initial states of all the cells in the grid.

To explore what happens to the automata generating complex configurations when changes are made that turn their rules time-dependent, we have performed experiments with one and two-dimensional automata.

5.1 Experiments with one-dimensional cellular automata

The simplest one-dimensional CA is a linear string of cells, each containing an FDA. Each automaton in the string has the same set of n possible states; a set of neighbors, defined by the number (k) of neighboring cells to its left and to its right, the same for all the FDA; and a transition function (the rule of the CA, also the same for all), which defines the next state of each automaton in the string as a function of its own current state and the states of its neighbors.

In our experiments, we have worked with one-dimensional CA with the following properties:

- Number of states: $n=2$, represented by 0 and 1. State 0 will be called «dead» and state 1 will be called «alive».
- Maximum neighboring distance: $k=2$, which means that the number of neighbors for each cell is 4 (two to the left and two to the right).
- Therefore, the next state of each cell is a function of five binary variables (the state of the cell itself and its four neighbors). The number of different possible rules is thus 2^{32} . A given rule can be defined by a 32 bit binary string such as the following:

01001011010010110100101101001011, where each bit defines the next state of each cell for all the possible values of the five input variables in the natural order.

- An additional restriction is imposed on the rules to prevent spontaneous generation: when the current state of a cell is «dead» and the states of all its neighbours are «dead», the next state of the cell must be «dead». This means that the rule of the CA must always start by 0, and the number of possible rules becomes 2^{31} .
- For our tests, we have selected a CA grid with cyclic boundary conditions.

Stephen Wolfram classified²³ one-dimensional CA into four broad categories: (i) Class 1: ordered (static) behavior; (ii) Class 2: periodic behavior; (iii) Class 3: random or chaotic behavior; (iv) Class 4: complex behavior. The first two are totally predictable. Random CA are unpredictable. Somewhere in between, in the transition from periodic to chaotic, a complex, interesting behavior can occur.

Since the total number of rules is too large to allow for a systematic study, we have selected at random four automata with rules which generate a complex behavior, and explored what happens when one or two mutations are applied to these rules. Tables 1 and 2 show the results. The Complex/Chaotic corresponds to the case when the modified CA displays a complex behavior for some initial conditions, and a chaotic behavior for others. The following initial conditions have been used: a) A single one in the middle of a series of 600 zeros. b) 1100 repeated to make a 601 string. c) Three consecutive ones in the center of a 598 long string of zeros.

Table 1. What happens when all possible single mutations are applied to a CA with complex behavior.

Rule	Complex	Chaotic	Complex /Chaotic	Ordered/ Periodic	Total
0100101101001011 0100101101001011	21	1	0	9	31
0101011001101110 1110111010000000	9	20	2	0	31
0110011001100110 0110011001100110	9	14	3	5	31
0011110000111100 0011110000111100	10	12	2	7	31

Table 2. What happens when all possible consecutive double mutations are applied to a CA with complex behavior.

Rule	Complex	Chaotic	Complex /Chaotic	Ordered /Periodic	Total
0100101101001011	14	3	2	11	30

²³ See WOLFRAM (2002).

0100101101001011					
0101011001101110 1110111010000000	3	26	1	0	30
0110011001100110 0110011001100110	0	17	8	5	30
0011110000111100 0011110000111100	5	12	1	12	30

These tables show that the behavior of the automata changes from complex to ordered, periodic, or, most frequently, chaotic, when its rule (i.e. one of the laws of nature for a world regulated by its rule) is modified; but in a significant number of cases, the automaton maintains a complex behavior after the change.

The next tests were performed on the second CA in tables 1 and 2. This CA has been designed with a rule similar to the laws of our universe, according to the following considerations:

1. The rule is symmetric, i.e. the neighbors to the left have the same effect as the neighbors to the right.
2. Too few neighbors or too many neighbors tend to cause the «death» of the central cell (as in the Game of Life).
3. About one half situations cause cells to become «alive», the other half make them «dead». According to Langton, this makes complex behavior more probable.

As indicated in the tables, the rule for this automaton is the following: 01010110011011101110111010000000. If the rule of this CA is applied a single mutation, symmetry is lost and the complex behavior for the indicated initial conditions becomes chaotic. However, if we apply a double mutation which maintains symmetry, the behavior of the CA will still be complex. The same happens with other double mutations which also keep the symmetry.

We next complicated the «laws of nature» in this universe, applying the original rule for some time during the evolution of the CA, then applying a double mutation for some time, and letting the CA go back to its original rule. In general, we got a complex behavior, even though the evolution in each case is visibly different from the corresponding histories of the original CA. Finally, the double mutation was applied in every generation with a probability of 1 in 1000.

These experiments show that the CA-universes which exhibit an interesting behavior (i.e. a complex behavior) do not lose it automatically if they suffer certain changes in their rules. This suggests that the most general form of the laws of nature allows for a certain time variability in those laws.

Of course, the actual histories of the modified universes will be different. Also, from so general a perspective, we cannot say what will happen to a given structure in a complex world when its laws are complicated with changes of any type (for instance, by introducing temporal changes in the laws, as shown in the experiments). But we can certainly expect at least that many variations of such universes will allow the existence of entities of analogous complexity.

5.2 Experiments with bi-dimensional cellular automata

To study what happens to particular structures in a complex world when changes in the laws occur, we decided to analyze bi-dimensional automata of the Game-of-Life type²⁴, one of the best explored up-to-now.

The 2-D CA called the Game-of-Life was designed by John Conway. It consists of a matrix of cells, where each cell may take one of two states: alive and dead (respectively represented by one and zero). Each cell has eight neighbors. At every time step, also called a generation, each cell computes its new state by determining the states of the cells in its neighborhood and applying the transition rules to compute its new state. Every cell uses the same update rules and all the cells are updated simultaneously. The next state of a cell is determined by the rule B3/S23, which means that cells are born (go from the dead to the living state) if they have exactly three living neighbors, and survive (stay in the living state) if they have two or three living neighbors. In all other cases, a cell dies or remains dead.

The game of life is particularly interesting to study the general form of the laws that give rise to complex structures, because in the last decades a large number of structures generated by this automaton have been studied, and several analogies between the dynamics of these structures and the dynamics of the chemical and biological systems in our world have been proposed.

Different variants of the game of life have been defined. HighLife, for instance, differs because its rule is B36/S23 (i.e. a cell is also born if it has 6 living neighbors). Life-3-4 has the rule B34/S34 (cells are born or survive if they have 3 or 4 living neighbors). Seeds is a CA with the rule B2/S (a cell is born if it has exactly two living neighbors, but it never survives). And so forth. For symmetric rules of the game-of-life type, the number of possible different rules is 2^{16} , most of which do not entail a complex behavior. If symmetry is not forced, the number of possible rules is much higher (2^{512}).

In our experiments, we have developed a genetic evolution program²⁵ which selects some of the most «interesting» initial conditions for Life and other rules and rule combinations: those which give rise to a good number of interesting small structures, specially gliders (which make it possible to design logical gates, and thus provide Life with the capability of universal computation), but also r-pentominos or exploders.

What is obtained thus (with such a program) is a measurement of the fecundity of a given rule, that is to say, i.e. its capacity to generate certain complex structures when suitable initial conditions occur. This allows us to compare the case of Life (that we already know generates a great amount of interesting objects) with those cases where we define time-dependent rules.

Table 3 shows some preliminary results obtained in our experiments, seven for each type of rule: Life, HighLife, a periodic mixed rule (Life/HighLife, with the Life rule applied for 25 generations, the HighLife rule for another 25, and so on, periodically), and another similar mixed rule (Life/B38S23), selecting for both gliders and r-pentominos. For comparison, row 4 in this table shows what happened when the evolutionary algorithm was

²⁴ See WOLFRAM (1986).

²⁵ See more technical details of this program and the related experiments in our paper: ALFONSECA - SOLER GIL (2012).

changed to select for a different type of object (two simple types of exploders). Just five experiments of the latter type have been performed.

Table 3. Summary of experiments as a function of rule type

Type of rules	Average life	Gliders	Perm. gliders	Glider life	R-Pent.	Exploders
Life	578	106	4	33.49	23	71
HighLife	386	41	0	53.63	5	24
L→HL→L→...	552	65	4	64.34	13	27
L→SL→L→...	930	170	1	31.22	44	137
Life (exploders selected)	938	79	1	24.04	26	88

A few conclusions derived from Table 3:

- Interesting behavior appears both with Life, Highlife and the two mixed rules. The highest score in the genetic algorithm corresponds to one experiment which used Life, and the highest number of gliders appeared in another experiment using the second mixed rule, but interesting behavior appeared sometimes for all the rules. It appears that the rules of Life and B38S23 are somewhat more prone to the appearance of «interesting» behavior than the rules of Highlife, while the mixed Life-Highlife rule occupies an intermediate position.
- The life length of an experiment is considered to end when the CA configuration goes into a static situation, where the states of all the cells remain the same forever (not necessarily dead), or a periodic configuration, where the states of the cells oscillate with a certain period.
- Gliders are generated much more frequently than r-pentominos. Three columns in Table 3 show the total number of different gliders generated by the CA with the evolved initial conditions for all the experiments associated to a given rule; the total number of permanent gliders generated (gliders that are never destroyed by colliding with other objects); and the average life of the gliders (permanent gliders display periodic behavior).
- When exploders were not selected for, they appear anyway, relatively frequently. On the other hand, the experiments where exploders are selected had a higher average life length.

The next set of experiments tried to find the effect of changing the rules during the execution of one of the automata generated in the previous examples. In this case, the initial conditions evolved for one type of laws (Life or HighLife) were applied to a CA which runs under those laws until generation 46, then changes to the opposite laws during the remainder of its «life». Thus, if the automaton was generated using the rules of Life, at generation 46 the rules would be changed to HighLife, and vice versa. The results can be seen at rows 3 and 4 in Table 4.

Table 4. Summary of experiments when initial conditions evolved for Life are used with the HighLife rules and vice versa

Rules evolved for A, used with B	Average life length	Gliders per exper.	R-Pent. per exper.	Exploders per exper.
Life	578	15.1	3.3	10.1
HighLife	386	5.9	0.7	3.4
Life / Life→HighLife	394	5.3	1.6	2.3
HighLife / HighLife→Life	1827	>16.7	2.9	many
Life / L→HL→L	791	14.4	3.9	5.9
Life / L→HL→L→...	478	9	2	4.1
L→HL→L→... / L→HL→L→...	552	8.3	1.9	3.9

From their observation we may get the following conclusions:

- The mixed rule of the form Life→HighLife (with initial conditions evolved for Life) generated a less complex behavior (shortest life, less gliders and other interesting objects) than the equivalent experiments where the rules of Life were allowed to apply always, but a slightly more complex behavior than those where the rules of HighLife applied always, with initial conditions evolved for HighLife.
- The mixed rule of the form HighLife→Life (with initial conditions evolved for HighLife) generated a behavior at least as complex (in fact slightly more complex) than those where initial conditions evolved for Life were applied to a CA running with the Life rules.

In the next set of experiments of this type, we started with CA with rules of the Life type and let it develop for 46 generations; then changed the rules to HighLife, executed them for 4 generations, and restored again the rules to Life for the remainder of their development. The initial conditions evolved for CA with Life rules where applied to these CA. Row 5 in table 4 show the results: a CA of this type performs comparably as that with the Life rule.

In the next set of experiments, we started with CA with rules of the Life type and let it develop for 46 generations; then changed the rules to HighLife, executed them for 4 generations, and restored again the rules to Life. This procedure was repeated periodically every 50 generations. The initial conditions evolved for CA with Life rules where applied to these CA. Row 6 in table 4 show the results: CA of this type performs slightly worse than those with the Life rule, but about the same as CA with periodic rules and initial conditions evolved for them (row 7 in the table).

To end this analysis, we decided to perform a few experiments using completely different types of cellular automata:

- The first we tried was Life-3-4, defined by the rule B34/S34. It resulted not to be amenable to this kind of experiments: there are no small long-lived structures similar to gliders, and therefore evolutionary algorithms do not seem to work; they fail to improve the best score, which is typically reached randomly in the first

generation at a very low value, and remains there for the remainder of the evolutionary process.

- Then we tried Seeds, a CA defined by the rule B2/S. With this apparently radical rule, however, it is possible to generate structures similar to gliders which move one step in a certain direction from one generation to the next. We selected for this structure in our evolutionary process. This CA has the problem that the number of cells alive increases quickly (in our experiments this happened always before the 100th «generation»), finally covering about 20% of the available space, and their distribution is more or less chaotic, which produces the effect that any glider that may appear from this point on will be quickly smothered by the neighboring cells and will stay there for just one or two generations. This chaotic behavior seems to stay forever, which means that the CA never reaches a static or periodic situation. To reduce this effect, we restricted the «life» of the CA to the first 60 generations and counted the number of gliders which appeared and their duration. In the two experiments performed, 6 and 7 gliders were produced (respectively) with an average duration of 11.4 generations.
- Finally we tried the following mixed case: starting with CA of the Life or HighLife type, we let them develop for 46 generations; then we changed the rules to 3-4-Life, executed them for 4 generations, and restored the rules to Life or HighLife. Table 5 shows the results. In some cases, the automata could not recover their former complexity after the change: not a single glider was produced after the original rule was restored. In other cases, however, new gliders were generated. We can conclude that even this drastic change in the rules decreases only moderately the average complexity of the development, i.e. sometimes the complexity of a given experiment is destroyed, but in other cases the CA is able to recover and proceeds to generate new complex behavior for a reasonable number of generations.

Table 5. Experiments with Life/3-4-Life mixed rules

Rules evolved for A, used with B	Life length	Gliders	R-Pent.	Exploders
Life / Life→Life-3-4→Life	341	9.6	3.9	3.4
HighLife / HighLife→Life-3-4→HighLife	260	5.6	2.3	4
Life / Life	578	15.1	3.3	10.1
HighLife / HighLife	386	5.9	0.7	3.4

Our conclusion: CA with mixed (time-dependent) rules are fully as capable to generate interesting behavior as those with «pure» rules, in some cases even more so.

We are currently working on the confirmation of the previous results by performing a larger number of experiments. We are also looking with more care at CA periodic rules that keep the resulting automata computationally complete, as it seems to happen with every time-dependent combination of Life and B38S23.

6. Discussion of the results

In this paper, we have checked on the fact that the only version of the multiverse that could be a suitable candidate for explaining the fine tuning of the laws of our universe to make the existence of complex entities in general and of intelligent beings in particular possible is Tegmark's mathematical multiverse. Then we have focused on the question whether the peculiar laws of our universe can be explained from the hypothesis that ours is a typical «complex-universe» or «life-enabling universe» among the set of all the worlds which includes all the consistent mathematical structures.

In order for this to be the case, the laws of our universe should be the most general among those consistent with our existence. To test this, in the previous section we have analyzed and proved that the most general form of those complex universes (which must exist, according to Tegmark's hypothesis) whose structure corresponds to that of cellular automata, are those whose rules exhibit some kind of temporal variability.

If this result can be extrapolated –and we should not forget that there are numerous authors from Martin Gardner to Daniel Dennett who have suggested the existence of a very close relation between the «game of life» and our universe²⁶–, it would imply that our universe is not typical at all, since it attains a high degree of complexity with laws and physical constants specially simple, as they are not a function of time. On the other hand, it is obvious that the number of possible universes compatible with life which would exhibit some kind of temporal dependence in their laws and physical constants, while keeping within the allowable margins of values, must be infinitely more probable than those of our type, which means that the probability of our existence in a world such as ours would be mathematically equal to zero²⁷.

²⁶ A recent and very interesting study of cellular automata (including the game of life) and its relationship to the real universe is Mainzer - Chua (2012). Of course, the use of classical cellular automata has its limitations, since most likely the real universe is a quantum universe. In the words of Mainzer and Chua:

«[...] classic deterministic cellular automata are only approximate models of physical reality, which is governed by the principles of quantum physics [...]. Quantum cellular automata (QCA) would be more adequate but, of course, not as easy to understand as the toy world of classical cellular automata. [...] In principle, it is possible to transform the concept of quantum systems into QCA» Mainzer - Chua (2012) 105.

Some authors as Seth Lloyd have investigated «toy» models of a quantum universe (considered as a quantum computer). Lloyd has found that such toy universes evolve complexity and structure naturally and with high probability [See e.g. Lloyd (1999)]. This encourages us to think that a transposition of the experiments performed here to the context of quantum cellular automata would show that the generation of complexity by means of time-dependent rules is perfectly possible in the quantum scenario. But this conjecture should be obviously tested in further research.

²⁷ If the universal *constants* in our universe are really constant (as most studies seem to imply), then our universe can be represented as a point in the configuration space of all the possible values of the constants. A universe where the *constants* were actually variable would be represented by a curve. If those universes are to be compatible with life, the point and the curve must lay within the subset of the configuration space that makes that compatibility possible. However, the number of points in a subset of space is a continuum-like infinite, while the number of curves in the same space is a different infinite, infinitely much larger than the continuum. Therefore, the probability of our having been born by chance in a *constant* universe (the quotient of both infinities) would be zero.

We are aware that current physics allows for some of the constants in our universe not to be so, as shown by the present debate on the constancy of the fine structure constant. But it cannot be denied that, in the worst case, all our constants are *almost constant*, which means that, although the actual probability of being in a universe like ours may not be exactly zero, it would still be very (perhaps vanishingly) small.

In consequence, the results presented in this paper can be considered as an inkling that the hypothesis of the multiverse, whatever its type, does not offer an adequate explanation to the peculiarities of the physical laws in our world. Multiverses are either too small or too large to explain fine-tuning. All the multiverses which have been proposed, except the «mathematical multiverse», are too small, so that they merely shift the problem of fine tuning up one level. But the «mathematical multiverse» is too large, in the sense that, in its context, the simplicity of our world becomes inexplicable.

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