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Miguel Sánchez-Romero

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DEPARTAMENTO DE ANÁLISIS ECONÓMICO:
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Demand for Private Annuities and Social Security: Consequences to Individual Wealth

Miguel Sánchez Romero[*][†]

Departamento de Análisis Económico: Teoría Económica e Historia Económica.

Universidad Autónoma de Madrid, E-28049, Cantoblanco, Madrid, Spain.

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Abstract

This paper focuses on comparing public and private individual wealth over the life-cycle, when individuals face an uncertain length of life. We also analyze how a fully funded and actuarially fair Social Security affects the desire to annuitize private wealth. Within this framework, we find that a social security system can contribute to reaching a higher national wealth, even when the economy is composed of selfish individuals. Thus, by means of some simulations we obtain the result that a payroll tax of 6 percent increases individual wealth up to 17 percent. This increment, however, is obtained under the assumption that insurance companies offer fair annuities. On the contrary, under an unfair private annuity market, individual wealth can decrease around 10 percent for the same payroll tax.

Key words: Actuarially Fair Funded Social Security, Crowding Out Effect, Public and Private Wealth Profiles

JEL classification: D01, D31, D81, D91, G11, H31

1 Introduction

It has been well known since Feldstein (1974)[1] that Social Security crowds out private saving. The intensity of this crowding out effect varies according to how Social Security is financed, the behavior of each individual in the economy, and the return yielded by public and private pensions.

First, we know that the negative impact of a funded social security system on steady-state capital stock is smaller, or zero, than that yielded by an unfunded system. Thus for example, Auerbach and Kotlikoff (1987), İmrohoroğlu et al. (1999), and Conesa and Krueger (1999) estimate under an unfunded Social Security that steady-state capital stock is reduced between 11 and 25 percent. On the contrary, under a funded Social Security and lifetime uncertainty, Eckstein et al. (1985), Abel (1985), and Hubbard (1987) demonstrate that the crowding out only occurs when selfish individuals have neither access to the annuity market, nor actuarially fair annuities.

Second, individual feelings can influence the intensity of the crowding out effect as well. In particular, the more altruistic an agent is, the greater her saving and, therefore, her wealth is. Hence, Fuster (1999) finds that an unfunded social security system with two-sided altruistic agents crowds out only 8 percent of the capital stock for a 44 percent replacement rate. Note that this value is much lower than those estimated for selfish individuals by Auerbach and Kotlikoff (1987), among others. Nevertheless, there does not exist a consensus among economists regarding the importance of altruistic feelings on individual's behavior. Thus, we shall assume that our individual is selfish hereinafter.

Third, it has also been quantified that Social Security does not reduce the stock of capital in the long-run, so long as public and private pensions yield the same return. Unfortunately, this result has been obtained assuming that the decision of purchasing annuities is exogenous. As a consequence, we cannot derive any relationship between the desire to purchase annuities and wealth over time.

In this paper, we analyze how the wealth accumulation process is affected when both the Social Security is funded and individuals endogenously purchase annuities. To do so, we develop an economy that incorporates financial companies, private insurances, and a funded Social Security. Consequently, individuals can invest their wealth in safe assets, risky assets, and annuities. Moreover, in order to make the decision of purchasing annuities endogenous, we have made the following five assumptions: i) our individual faces an uncertain lifespan, ii) the yield of annuities

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[1]This negative effect was found firstly by Feldstein (1974) in the case of an unfunded Social Security.

dominates that of bonds, iii) a negative asset position at the time of death is forbidden, iv) the consumer is selfish, and v) she has a bounded rationality (i.e., even though financial institutions do not allow individuals to die in debt, our agent does not make decisions considering this constraint). Under the first four assumptions, Yaari (1965) states that the consumer will fully annuitize her savings. However, Sanchez-Romero (2005) demonstrates, by adding the assumption number v), that the decision of purchasing annuities depends on the relationship between the present value of future non-capital earnings and the initial wealth. That is to say, he finds that private annuities are not purchased when public benefits are high. Therefore, this last finding suggests that the crowding out effect should be analyzed not only by studying what sort of social security system the economy has, but also whether individuals are willing to purchase annuities or not.

On the other hand, the implications of these five assumptions are consistent with the fact that the demand for annuities is small on average. Nonetheless, there are other factors that explain the lack of annuitization, although they are out of the scope of this paper. For example, bequest motive, annuity market imperfections such as the irreversibility of annuitization, or even risk sharing within families. The importance of any of these factors is, besides our assumption number v), that wealth accumulated at the age of retirement may change. This is in addition to the fact that wealth inequality, among descendants of people recently deceased, might increase over time.

Finally, it is worth noting that the utility function and the dynamic optimization method used throughout the paper to calculate the optimal portfolio differ from previous analysis. Thus, instead of using a CRRA utility function and the Hamilton-Jacobi-Bellman method, as Merton (1971) and Richard (1975) have done before, we use a mean-variance utility and the Lagrange method, in order to be consistent with the bounded rationality assumption. Under this setting, we find two important features. One, the optimal portfolio is affected by age. Second, the investment in risky assets is much lower than those obtained by Merton (1971). Therefore, the investment in safe assets is preferred according to this model than in previous analyses.

Throughout the paper we show that, when there is no Social Security or the payroll tax is equal to zero, our individual invests her wealth both in equities and in annuities. On the contrary, as the Social Security payroll tax increases, our agent is more willing to purchase bonds instead of annuities. According to this fact, we find that an actuarially fair funded social security system could increase the stock of capital in the long run if, and only if, our agent only purchases bonds at the beginning of her life-cycle. We simulate that the wealth increment, with a 6 percent payroll tax and private fair annuities, is close to 17 percent. However, under a private unfair annuity market

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this wealth increment is reduced, even to the point of decreasing wealth in the long run.

The remainder of this paper proceeds as follows. Section II explains the consumer's behavior when there is no Social Security. Here, we obtain the optimal portfolio choice with the intention of subsequently estimating how wealth evolves over time. In section III, we introduce a funded social security system. It will enable us to calculate how public and private wealth evolve according to different payroll taxes. Section IV describes the effects of a funded social security system on the demand for private annuities. Furthermore, we shall distinguish between actuarially fair annuities and unfair annuities. In Section V we make our final conclusions. An Appendix containing a detailed demonstration of optimal investment as well as consumption behavior finishes the paper.

2 Optimal Portfolio Choice under Uncertain Lifetime: Bonds, Equities, and Annuities

Individuals, who finance their future consumption using annuities, reduce the crowding out effect that is caused by an actuarially fair funded social security system, Abel (1986). Unfortunately, empirical research indicates that the value of the demand for annuities is small on average. Therefore, to substitute an unfunded Social Security by a funded one does not necessarily eliminate the crowding out effect.

A recent paper by Davidoff et al. (2005) suggests, among other reasons, that the lack of the demand for annuities may be caused by behavioral biases. Building on this idea, Sanchez-Romero (2005) proves that individuals with behavioral biases, such as bounded rationality, are more willing to purchase annuities the greater wealth is in relation to future non-capital earnings. Hence, *ceteris paribus*, we can expect that an actuarially fair funded Social Security causes a higher crowding out in economies with low private wealth.

The aim of this section, therefore, is to derive how individuals who live in an economy without Social Security accumulate assets to finance their future consumption at retirement. This result will be used as a benchmark to compare to the asset accumulation process derived by introducing a social security system. We develop an economy composed of financial companies which supply safe and risky assets (e.g. bonds and equities) and private insurances that offer annuities. The significance of the introduction of equities into the model is twofold. First, an economic model which studies private pensions needs to take into account how bonds and equities evolve. Second, if the agent has perfect foresight and short-selling is not constrained, then this model yields a

greater accumulation of wealth which may lead to an increase in the demand for annuities. This point will be analyzed at the end of this section.

The representative economic agent faces an uncertain lifetime. Her survival probability Ω is known in advance, but the age that she will die is unknown. T is the maximum age to which the agent can survive. In addition, our economic agent has three key features which affect her investment decision making. First, the consumer is selfish. She does not leave an intentional bequest at death. Second, following Sanchez-Romero (2005), the agent does not take into account that financial institutions do not allow individuals to die in debt. So, we can say that our agent has a bounded rationality. This assumption affects the demand for annuities. For example, in order to anticipate consumption, individuals purchase annuities when they are young, and reject using annuities when they are retired. Third, the individual temporarily modifies her consumption according to financial markets expectations. Concretely, she increases her consumption while she expects to gain money investing in financial markets. This last assumption makes the consumption decision stochastic. Thus, instead of using an expected utility function, we use a mean-variance utility $v(c, \sigma_c^2)$, which satisfies the conditions demonstrated in Tsiang (1972). Therefore, the consumer's utility at age x is depicted by the following function U:

$$U(x) = \int_x^T \frac{\Omega(s)}{\Omega(x)} \beta(s-x) v\left(c(s,x), \sigma_c^2(s,x)\right) ds, \text{ for all } x \in [0, T). \tag{1}$$

Where $c(s, x)$ is the mean consumption at age s, of an x year old consumer, $\sigma_c^2(s, x)$ is the consumption variance at age s, of an x year old consumer. The function v is at least twice differentiable, strictly increasing in $c(s, x)$, and decreasing in $\sigma_c^2(s, x)$. $\frac{\Omega(s)}{\Omega(x)}$ is the probability that an individual of age x will be alive at age s, and $\beta(s-x)$ is the time discount factor from age x to age s, or $e^{-\delta(s-x)}, \forall \delta \geq 0$.

Given a mean consumption level, the utility function (1) shows that the higher the consumption risk is, the lower the utility achieved by the consumer is. Hence, assuming that consumption variance is caused by risky asset investments, the consumer will maximize her consumption by investing in an efficient portfolio with the minimum variance and maximum expected return, as Sharpe (1964) and Markowitz (1952) suggest.

There are two alternative portfolios. The first one is composed by bonds and equities. The second one is composed by annuities and equities. Bonds and equities yield a safe interest rate r and a random interest rate α, respectively. Annuities, on the contrary, are lotteries contingent on the consumer mortality risk. Specifically, if the consumer survives at the end of the period, she

will receive the safe interest rate r plus a risk premium μ contingent on her mortality risk. But, if she does not survive at the end of the period, she will not receive anything.

Each period, our representative individual has an initial wealth k and a labor income y. The individual takes $y(s), \forall s \in [0, T)$ as given. These resources are allocated to both consumption and investment. Nonetheless, she must choose the portfolio in which she will compound her resources. Thus, the agent at age x faces two alternative budget constraints.

$$k(x) + \int_x^T \frac{R(s)}{R(x)} \left((\alpha(s) - r(s))e(s, x) + y(s) - c(s, x) \right) ds = 0, \tag{2}$$

and

$$k(x) + \int_x^T \frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)} \left((\alpha(s) - r(s))e(s, x) + y(s) - c(s, x) \right) ds = 0. \tag{3}$$

(2) and (3) are respectively the budget constraint when consumption is financed (besides by equities) by investing in conventional assets, and when consumption is financed by annuities. $e(s, x)$ is the amount of money invested in risky assets at age s, of an x year old consumer.[2] $\frac{R(s)}{R(x)}$ is the financial present value at age x, of a monetary unit received at age s, and $\frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)}$ is the actuarial present value; that is,

$$\frac{R(s)}{R(x)} = e^{-\int_x^s r(j)dj},$$

and

$$\frac{R(s)}{R(x)} \frac{\Omega(s)}{\Omega(x)} = e^{-\int_x^s (r(j)+\mu(j))dj}.$$

It is worth noting that neither (2) nor (3) constrain wealth to be nonnegative along the lifespan. Nevertheless, the economic agent never dies in debt under (3), but she could under (2). This is an important property that we shall use subsequently. Also, if the consumer decides to purchase annuities, she will not leave a bequest. But, in contrast, if she chooses to finance consumption by investing in bonds, she will unintentionally bequeath at death. Therefore, choosing either (2) or (3) has important consequences on income distribution inter and intra-generations. However, this fact is beyond the scope of this paper.

So far we have established the general framework from which an individual accumulates assets to finance her future consumption. Now, we shall proceed by explaining the solutions obtained by plugging a CRRA utility function ($u(\xi) = \frac{\xi^{1-\gamma}}{1-\gamma}, \gamma > 0$) into (1), and assuming that consumption variance at age s is proportional to risky investment variance at age s, of an x year old consumer.

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[2] Hereinafter, whenever the consumer will decide to purchase annuities, both mean consumption and money invested in risky assets will be denoted with a hat.

That is,

$$\sigma_c^2(s,x) = \eta^2 \sigma_\alpha^2(s) e^2(s,x), \text{ for all } s,x \in [0,T) \text{ with } s > x, \tag{4}$$

where $\eta > 0$ is the constant of proportionality and σ_α^2 is the equity variance.

The agent maximizes (1) subject to either (2) or (3). Solving this economic problem yields two different consumption trajectories, which are quite similar to the uncertain lifetime case with just bonds and annuities.[3] Nevertheless, equities now modify the marginal utility of consumption and, consequently, the dynamic of consumption is also moved according to the expected evolution of asset returns. For example, consumption increases (resp. decreases) whenever the difference between asset returns also increases (resp. decreases). These consumption changes, nonetheless, are not high enough to produce consumption trajectories totally different from those obtained by Sanchez-Romero (2005). This circumstance is explained by the small investment in risky assets, depicted by any of the following equations:

$$e(x,x) = \frac{1}{\gamma} \left(\frac{\alpha(x) - r(x)}{\sigma_\alpha^2(x)} \right) \frac{\psi(x,x)}{\eta^2} c(x,x),$$

or

$$\hat{e}(x,x) = \frac{1}{\gamma} \left(\frac{\alpha(x) - r(x) - \mu(x)}{\sigma_\alpha^2(x)} \right) \frac{\hat{\psi}(x,x)}{\hat{\eta}^2} \hat{c}(x,x).$$

Where both ψ and $\hat{\psi}$ are functions whose range are restricted to the closed interval $[1,2]$. The first two components on the right side of the equality signal are similar to Merton (1971) and Richard (1975). However, the amount of money invested in risky assets depends on consumption, instead of depending on initial wealth and the present value of future non-capital earnings. As a consequence, this model yields portfolios which are mainly composed of either bonds or annuities.[4] In particular, the proportion of either bonds or annuities relative to equities raises as our individual ages. Thus, equities are the main investment when the economic agent is young, but as time goes by she prefers to hold safer investments.

On the other hand, so long as Social Security does not pay benefits, wealth is also held in annuities rather than in bonds. Both the bounded rationality and the liquidity constraint assump-

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[3]The reader will find the analytical solutions in the appendix.

[4]According to Tsiang (1972), the CRRA utility function $u(y)$ is convergent to $v(c,\sigma_c^2)$ if, and only if:

$$\eta \geq \left(\frac{\varepsilon(1+\gamma)}{2} + \frac{1}{\varepsilon\gamma} \right) \cdot \max \left\| \frac{\alpha(x) - r(x)}{\sigma_\alpha(x)} \right\|_{\forall x \in [0,T)},$$

where ε is a real number which satisfies that $\frac{\sigma_c}{c} \leq \varepsilon < 1$. In particular, Tsiang (1972) suggests a value of $\frac{1}{10}$ for ϵ, therefore we cannot expect high values of $\frac{\psi(x,x)}{\eta^2}$. Note that this condition corresponds to the non-annuitized wealth case. Thus, if we are interested in the value of $\hat{\eta}$, we should add the mortality risk premium to $r(x)$.

tions are key factors for this allocation process. Thus, unless individuals purchase annuities or they have sufficient capital, they are unable to anticipate consumption at the beginning of their life cycle. Therefore, we find that young individuals are more willing to purchase annuities in order to increase their consumption. However, the presence of annuities raises borrowed money and so, because individuals must repay their debts, the economic agents have a lower positive asset position upon retirement. This latter fact negatively affects the demand for annuities, Sanchez-Romero (2005). Nonetheless, they will buy insurances contingent on their death due to the lack of public benefits assumed so far.

In sum, in an economy without Social Security, we find that our agent allocates her wealth in a portfolio composed by equities and annuities. But, equities represent a small percentage of total wealth, and annuities decrease wealth held upon retirement among those individuals who have needed to borrow money at young ages.

3   Payroll Tax and Wealth-Age Profiles

Up to now, we have studied the asset accumulation process of an individual who lives in an economy without Social Security. Under this scenario, we have found that individuals mainly purchase annuities because it enables one to borrow money, and because it assures an income after retirement. In this section however we introduce an actuarially fair funded social security system that assures an income at retirement. Thus, Social Security levies a payroll tax τ on gross earnings, in exchange of a future benefit when people retire. According to this fact, we rewrite income as the following piecewise function:

$$y(s) = \begin{cases} (1 - \tau_e)w(s) & 0 \leq s < J \\ b(s) & s \geq J \end{cases}, \tag{5}$$

where $w(s)$ is the gross salary at age s, $b(s) = b$, for all s, is the flat public pension benefit received at retirement, and J is the age of retirement. We consider that the payroll tax is paid not only by the employee τ_e, but also by the employer τ_f. As a consequence, our representative individual receives an actuarially fair pension benefit equal to:

$$b = (\tau_e + \tau_f)\frac{\int_0^J R(s)\Omega(s)w(s)ds}{\int_J^T R(s)\Omega(s)ds}. \tag{6}$$

This assumption is introduced into the model because current social security regimes are jointly financed by employers and employees. In addition, the fact that Social Security is financed

by these two agents has important and interesting consequences on individual saving. For example, a funded social security system financed by employers and employees generates an increase in lifetime resources. The positive income effect caused by the system, however, differs according to the portfolio chosen by each individual. Thus, in a model without firms, Hubbard (1987) proves that an actuarially fair and funded system generates an increase in lifetime resources when individuals do not purchase actuarially fair annuities. Nevertheless, a system partially financed by employers generates an increase in lifetime resources as well, even when individuals purchase annuities. That is to say, substituting equation (5) and (6) into the budget constraint (3), and afterwards subtracting (3) with respect to the budget constraint without Social Security, we have that

$$\tau_f \int_0^J R(s)\Omega(s)w(s)ds. \tag{7}$$

This increment in resources correspond to those individuals who purchase actuarially fair annuities. (7) equals the pension financed by the employer; since we are assuming that $w(s)$ is the maximum gross salary, that the employer is willing to pay without Social Security. On the contrary, if our individual decides to finance her consumption with the portfolio composed by bonds, we will expect a greater increment in lifetime resources than if it is financed by annuities.[5] Repeating the previous process, but now with the budget constraint (2), we get that

$$\kappa \left(\tau_f \int_0^J R(s)\Omega(s)w(s)ds + \tau_e \int_0^J R(s)\left(\Omega(s) - \frac{1}{\kappa}\right)w(s)ds \right), \tag{8}$$

where κ is the difference in discount rates under certainty and uncertainty:

$$\kappa = \int_J^T R(s)ds \Big/ \int_J^T R(s)\Omega(s)ds > 1.$$

We have found according to (7) and (8) that an actuarially fair funded Social Security could raise lifetime resources. On the one hand, we know that the higher the income effect is, the greater the payroll tax is. On the other hand, the income effect also increases when our individual decides to invest in bonds, instead of doing so in annuities. Consequently, given a periodical earning such as (5), we can enumerate three causes that reduce private saving: i) a decrease in net salary, ii) an increase in consumption due to the positive income effect, and iii) a lower necessity of saving for retirement motive. Nevertheless, the decrease in private savings is offset by an increase in public savings. Therefore, it is not clear that the individual wealth[6] will be reduced in the long

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[5]Given that κ also depends on Ω, we expect that $\Omega(s) > \frac{1}{\kappa}$ for almost all ages (s) between x and J years old.

[6]Hereinafter we call "individual wealth" as the sum of private and public wealth.

run. In fact, individual wealth may either increase or decrease depending on how the payroll tax modifies both public and private wealth over time. In particular, we expect that Social Security will raise (resp. reduce) individual wealth accumulated, so long as the elasticity of public savings with respect to the payroll tax is greater (resp. lower) than the absolute value of the elasticity of private savings with respect to the payroll tax.

In order to understand how individual wealth evolves over time, we simulate nine wealth profiles which differ according to the payroll tax and the proportion of the tax levied by each economic agent. To do so, we assume that bonds yield an annual constant interest rate r equal to 0.037. Equities yield an interest rate that follows an Ito process

$$\alpha(s)ds = r(s)ds + \sigma_\alpha dB(s),\ dB(s) \sim N(0, \sqrt{ds})$$

where σ_α equals 0.1. The individual satisfies every feature explained in section 2, with a γ-value of 2, and a time discount factor δ of 0.02. Moreover, we assume that the gross earning received by the individual, which is used to calculate these wealth profiles, is depicted by Figure 1 below.

Figure 1: Gross Earning Profile ($w(s)$)
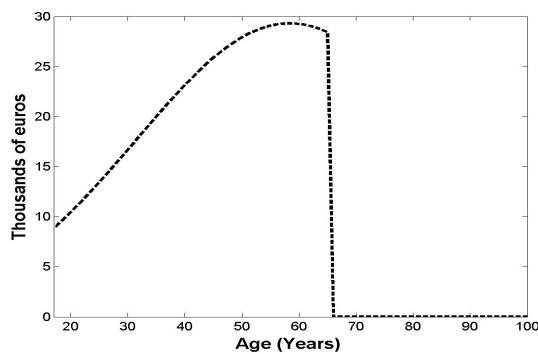
The actuarially fair funded Social Security offers an implied rate of return equal to the mortality hazard rate plus bonds return. Table 1 shows the annual pension benefits that our individual will receive for different payroll taxes (i.e. $\tau = \tau_e + \tau_f$). On the one hand, it shows that a total payroll tax of 3 percent roughly assure a benefit equal to the lowest income of her life. On the other hand, values of 6 and 8 percent points approximately guarantee 85 percent of her average earning and her highest earning, respectively. We have chosen these percentages because they are the most important three cases, which will be explained subsequently. In addition, a percentage greater than 8 percent makes no sense because it has perverse effects both on private saving and on the economy.

Table 1: BENEFITS (b)

Payroll Tax (τ)	Annual Pension Benefit
0,03	8.727,70
0,06	17.455,00
0,08	23.274,00

Note: The individual retires at the age of 65. The mortality hazard rate is assumed to follow the Gompertz's Law $\mu(s) = \alpha e^{\beta s}$, where α is equal to $9,221765 \cdot 10^{-5}$ and $\beta = 0,085277$.

Given this setup, Figure 2 shows that our individual borrows money at the beginning of her life-cycle in order to anticipate her consumption. However, the money borrowed decreases as the payroll tax increases (dotted square line). Note in Figure 2 that changing the total payroll tax τ from 3 to 6 raises individual wealth. By contrast, Figure 3 shows that a payroll tax of 8 percent leads our individual to not save for retirement (dotted line with an x mark); as a consequence total wealth is almost the same as an economy without Social Security (solid line).

Figure 2: WEALTH PROFILES: PAYROLL TAXES 3 AND 6 PERCENTS
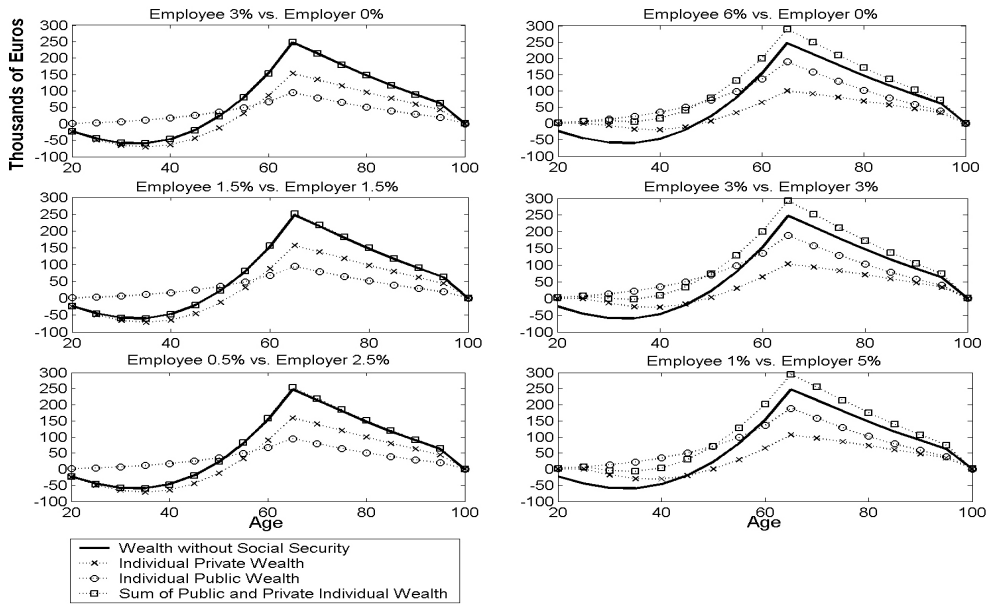
We have found that the increment of total wealth occurs because young individuals are not

Figure 3: WEALTH PROFILE: PAYROLL TAX 8 PERCENT
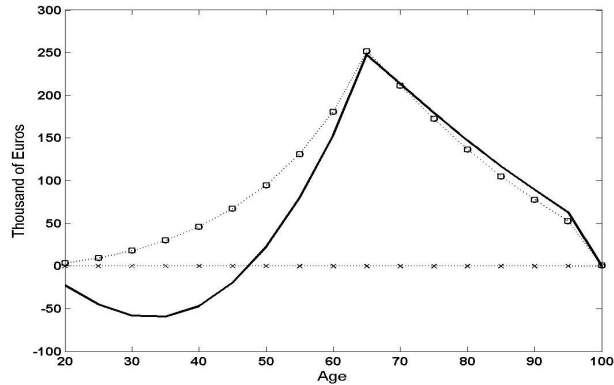
Note: In this case, there is no difference between the proportion

of the tax paid by each agent.

willing to purchase annuities. However, if our individual does not purchase annuities along her lifespan, as happens in Figure 3, there will not be such an increment. Hence, there must exist a payroll tax that maximizes individual wealth without strangling private savings. In this particular case the optimal payroll tax is equal to 6, as Table 2 shows.

Table 2: INDIVIDUAL WEALTH AT THE AGE OF 65

τ_e ($\tau_f = 0$)	0	1	2	3	4	5	6	7	8
Percentage	100	100	100	100	100	105	117	89	102

Note: $100 = 247.519, 26$ euros.

This fact implies that even though a social security system leads young individuals to be worse off in terms of consumption, their wealth become greater as they age (if, and only if, the system has not excessively levied gross earnings). Thus, the system does not necessarily offset one public monetary unit by another private one. In fact, a different payroll tax can help to raise individual wealth.[7] Another alternative for raising individual wealth is to increase τ_f and decrease τ_e. However, we have not found significant changes to wealth, as Figure 2 shows, that could balance the negative effect on unemployment caused by the increment on the labor cost versus the capital cost.

In sum, an actuarially fair funded Social Security expels the demand for private annuities, but it may increase individual wealth as well. The former effect has been widely discussed since Feld-

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[7]Note that Table 2 is calculated under the assumption that τ_f equals 0.

stein (1974). By contrast, the latter effect results if, and only if, the following two circumstances take place: i) individuals voluntarily decide not to purchase annuities and ii) financial markets do not allow individuals to die in debt. Therefore, this result shows, contrary to previous research, that a social security system can contribute to reaching a higher national wealth, even when the economy is made up of selfish individuals.

4 Effects of a Fully Funded Social Security on the Demand for Private Annuities

It has been pointed out that an actuarially fair and fully funded social security system does not reduce the steady-state wealth whenever individuals are selfish and a private annuity market exists. In order to obtain this result, it is necessary to assume that individuals are rational. Otherwise, if individuals have a bounded rationality of the sort explained in this paper, the actuarially fair and fully funded Social Security can either increase or decrease steady-state wealth (see Table 2). Since the introduction of the system reduces the desire of purchasing annuities and, as a consequence, individuals can either have a greater individual wealth because they do not borrow money at young ages, or have a lower individual wealth because they do not save for retirement. Therefore, we analyze in this section the possible reasons for not investing in private annuities and how it affects individual wealth.

The introduction of this social security system yields two reasons for not investing in annuities. First, it causes a lower private wealth upon retirement[8] that reduces the desire of purchasing annuities. As it is explained by Sanchez-Romero (2005). Second, following Hubbard (1987), individuals may prefer bonds to annuities in order to achieve higher lifetime resources, see (8). Thereby, the higher the contribution to Social Security is, the greater the crowding out effect on the demand for private annuities is. However, the first reason is offset because we have assumed that financial institutions do not allow individuals to die in debt. Thus, Social Security may increase private wealth by inducing individuals to hold their wealth in the form of bonds; since once they purchase bonds instead of annuities, they are unable to borrow money and so they have a greater positive asset position earlier.[9] The intensity of these two opposite effects on private wealth is the key factor to determine whether or not the system produces a crowding out. In particular, we find

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[8]This is equivalent to say that Social Security reduces private saving for retirement.

[9]In order to realize this fact, compare in Figure 2 those charts on the left side with those on the right side.

that Social Security raises wealth while it does not cancel private saving for retirement.

This current section proceeds as follows. First, we explain the demand for private annuities when there is no Social Security. We use its annuity equivalent wealth values (AEW) as our baseline case. Subsequently, we divide this section in two subsections in order to give insight into how Social Security changes the demand for private annuities. One subsection shows an individual's behavior when private markets offer fair annuities, and the other subsection shows the individual's behavior when they offer unfair annuities. Both subsections contain tables and figures which depict the desire to purchase annuities for different payroll taxes and risk aversion coefficients.

We found in section 3 that young people prefer annuities to bonds in order to anticipate consumption. This is because financial institutions do not lend money unless people insure their wealth with life insurances. Later on, assuming an economy without public pensions, individuals prefer to purchase annuities in order to maintain their economic status. If they choose, by contrast, the alternative portfolio composed of bonds, then they have the risk of outliving their financial resources more quickly. Equivalently, in the case of holding their wealth in bonds, individuals may not have an income in the time just before death. Therefore, people always prefer to purchase annuities when there is no Social Security. Figure 4 below shows this statement for the representative agent introduced in the previous section. Note that AEW values[10] are higher than one, and thus annuities are preferred over bonds. AEW has a Λ-shape which means that this individual is more willing to purchase annuities as she approaches the date of retirement; while she is almost indifferent when choosing between bonds and annuities both at the beginning of her life-cycle and at the end.

The introduction of Social Security will move the AEW figure downwards. Thus, given that AEW has a Λ-shape, we have to expect that the system mainly affects our individual when young, conditioning her future decisions afterwards. In addition to age, Figure 4 also changes according to the behavior towards risk and the proportion of the load charged upon annuities.

4.1 *Perfect Life Insurance*

Private annuity markets, which offer actuarially fair life insurances, assure that individuals' life-time resources raise according to either (7) or (8). Consequently, every result already obtained is

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[10]The proportion of annuitized wealth that is necessary to achieve the utility level when the consumer has no access to the annuity market.

Figure 4: A.E.W. BY AGE WITHOUT SOCIAL SECURITY
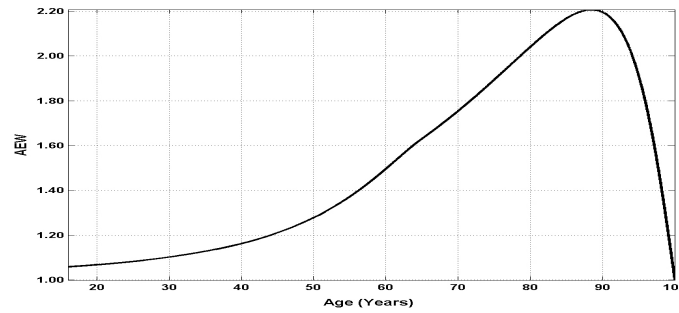
Note: An annuity equivalent wealth value lower (resp. greater)

than one means that the individual prefers (resp. does not prefer)

bonds to annuities.

applicable. Here, we focus on studying how private wealth is modified by different payroll taxes and risk aversion coefficients. This is because, following Sanchez-Romero (2005), the demand for private annuities mainly depends on private wealth and on future benefits. In order to analyze this fact, we will first pay attention to our agent at the age of 65, see Table 3 below. Second, we shall study with the help of Figure 5 the demand for private annuities in a dynamic perspective.

Table 3: PRIVATE WEALTH AT THE AGE OF 65

| Payroll Tax	Risk Aversion Coefficient		
τ_e vs. τ_f	$\gamma = 0.75$	$\gamma = 2$	$\gamma = 5$
3-0	292.550,10[a]	153.032,06[a]	114.137,23[a]
1,5-1,5	298.355,56[a]	156.744,85[a]	117.266,63[a]
0,5-2,5	302.225,87[a]	159.220,04[a]	119.352,89[a]
6-0	213.502,81[a]	100.001,84[a]	55.353,57[a]
3-3	222.280,32[a]	103.725,10[a]	58.056,57[a]
1-5	227.218,18[a]	106.177,29[a]	59.878,10[a]
8-0	0,00[b]	0,00[b]	0,00[b]
4-4	0,00[b]	0,00[b]	45.476,75[a]
1,33-6,67	0,00[b]	0,00[b]	49.068,69[a]

[a] The individual decides to annuitize her private wealth.

[b] The individual prefers to hold her private wealth in bonds.

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Table 3 above shows whether our individual purchases annuities (superscript a) or not (superscript b) according to her private wealth and her risk aversion coefficient. Thus, the table contains three important features: i) given the gross earning profile of Figure 1 and using Table 1, we find that our individual purchases annuities so long as the payroll tax is lower than 8 percent. This is an important result not only because she achieves, according to Table 2, a greater wealth, but also because it assures a periodical income up to her death. ii) it is worth noting that in this model the risk aversion coefficient causes two opposite effects upon the demand for private annuities. On the one hand, it is well known that the higher the risk aversion coefficient is, the greater the desire of an agent to purchase annuities is. However, on the other hand, we see in Table 3 that the lower the γ value is, the greater the private wealth at the age of retirement is. Thus, the agent is more willing to purchase annuities. In sum, once again the risk aversion coefficient does not explain the demand for annuities. Finally, iii) private wealth increases as the proportion of the payroll tax paid by the employer increases. This wealth increment nonetheless is not high enough to balance the resources paid by the employer[11] except for the case in which both the individual is quite risk adverse and the payroll tax is greater or equal than 8 percent.

Figure 5: A.E.W. BY AGE WITH SOCIAL SECURITY AND FAIR ANNUITIES
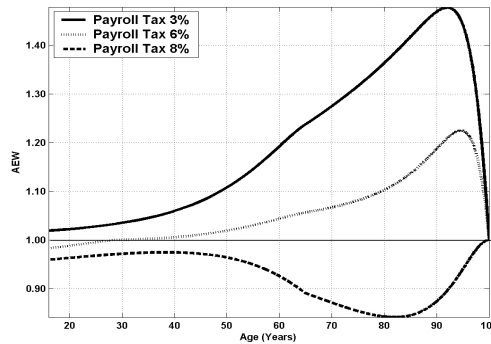
In addition to the static analysis presented in Table 3, Figure 5 above shows the AEW values by age associated with the following payroll taxes: (3-0), (6-0) and (8-0). The solid line plots how our individual always prefers annuities to bonds with a payroll tax of 3 percent. A payroll tax of 6 percent (dotted line) causes our individual to decide to purchase bonds instead of annuities at the beginning of her life-cycle. The dashed line plots how she always purchases bonds with a payroll tax of 8 percent. Therefore, AEW values by age are pushed downwards as the payroll tax increases (in order to see how AEW by age evolves, compare Figure 5 with Figure 4). However,

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[11] According to equation (6) the amount of the benefit received only depends on the total payroll tax, i.e. $\tau_e + \tau_f$.

once the individual has decided to purchase bonds along the rest of her life, the AEW by age has an inverted Λ-shape. As a consequence, we do not expect that she will reject her investment once she has decided the asset in which she allocates her wealth. In fact, as seen in Figure 5, financial institutions will have to offer greater returns in order to be able to make people change from one asset to another.[12]

4.2 *Imperfect Life Insurance*

In the real world we do not find actuarially fair annuities. In general, annuities are loaded by insurers with the intention of financing reserves, administrative costs, commissions, and profits. Therefore, it is more realistic to analyze previous results when life insurances do not offer fair annuities. The first consequence of this fact is that an actuarially fair funded Social Security offers a higher rate of return than private annuities, and hence individuals achieve a greater wealth by investing in public pensions than in private annuities. Second, an imperfect annuity market cannot offset those annuities offered by the social security system. This situation causes both an income effect and a substitution effect that change the demand for private annuities when fair life insurances were offered. Specifically, a lower annuity return increases present consumption and diminishes future consumption due to the substitution effect. Thus, our individual either consumes all her income if she invests in bonds, or borrows more money at the beginning of her life-cycle, and subsequently increases her saving, in the case of investing in annuities. On the other hand, given that public benefits are actuarially fair, an imperfect private annuity market reduces the income effect produced by investing in bonds. In order to show this fact, we assume for the sake of simplicity that annuities yield the following rate of return at age s:[13]

$$r(s) + (1 - \varpi)\mu(s), \text{ for all } s \in [x, T),$$

where $\varpi \in (0, 1)$ is the percentage of load over the mortality hazard rate. Note that we use this formula in order to satisfy that the yield of annuities still dominates that of bonds. Thereby, (7)

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[12]Thus, if policy makers aim to annuitize private pension plans, then it is convenient to undertake policies when people are between 30 and 50 years old.

[13]Now, Ω has been transformed to $\hat{\Omega}$ which has the following formula:

$$\hat{\Omega}(x) = e^{-(1-\varpi)\int_0^x \mu(j)dj} > \Omega(x), \text{ for all } x \in [0, T).$$

converges to (8) as we give to ϖ a value close to 1. Thus (7) is now rewritten as

$$\hat{\kappa}\left(\tau_f \int_0^J R(s)\Omega(s)w(s)ds + \tau_e \int_0^J R(s)\left(\Omega(s) - \frac{\hat{\Omega}(s)}{\hat{\kappa}}\right)w(s)ds\right), \qquad (9)$$

where $\hat{\kappa}$ is the difference in discount rates under unfair annuities and fair ones:

$$\hat{\kappa} = \int_J^T R(s)\hat{\Omega}(s)ds \Big/ \int_J^T R(s)\Omega(s)ds > 1.$$

From (9) we derive, whenever insurers offer unfair annuities, that the positive income effect caused by switching from annuities to bonds is diminished. According to this effect, annuities are now more preferred than bonds. However, the latter cannot balance the substitution effect. Indeed, we can see comparing Tables 3 and 4 below, that bonds are now more preferred than annuities at the age of 65.

Table 4: PRIVATE WEALTH UNDER UNFAIR ANNUITIES AT THE
AGE OF 65

Payroll Tax	Load	Risk Aversion Coefficient		
τ_e	ϖ	$\gamma = 0.75$	$\gamma = 2$	$\gamma = 5$
	0,25	245.527,90[a]	148.750,39[a]	119.05753[a]
3	0,50	199.331,52[a]	142.438,29[a]	124.514,48[a]
	0,75	141.062,91[b]	131.942,28[a]	130.045,47[a]
	0,25	0,00[b]	100.890,77[a]	57.383,95[a]
6	0,50	0,00[b]	33.473,84[b]	60.787,67[a]
	0,75	0,00[b]	33.473,84[b]	66.450,95[a]
	0,25	0,00[b]	0,00[b]	0,00[b]
8	0,50	0,00[b]	0,00[b]	0,00[b]
	0,75	0,00[b]	0,00[b]	0,00[b]

[a] The individual decides to annuitize her private wealth.

[b] The individual prefers to hold her private wealth in bonds.

We realize in Table 4 that private wealth decreases more markedly as the risk aversion lowers (see columns with $\gamma = 2$ and $\gamma = 0,75$) and the load increases. Instead, a γ value equal to 5 yields a higher private wealth under unfair annuities than under fair ones. This is so because she

prefers bonds to annuities at the beginning of her life-cycle and, as a consequence, she cannot borrow money because she simply consumes her income during this period. Moreover, we have used three different loads $\{0,25; 0,5; 0,75\}$ with the aim of showing how the demand for private annuities mainly depends on the relationship between private wealth and the present value of future earnings. Thus, it is worth noting that any of these loads yield, by definition, an annuity internal rate of return greater than that of bonds ($r = 0,037$); in particular, at the age of 65 they are equal to $\{0,049; 0,046; 0,042\}$ respectively. Therefore, the more unfair annuities are, the greater the present value of future benefits with respect to current private wealth is. Thus, the individual is less willing to purchase annuities.[14] In addition to the relationship between private wealth and future earnings, the risk aversion coefficient γ has to be considered as well, given that it determines the threshold private wealth from which our individual switches her investments from annuities to bonds. For example, Table 4 shows that, when annuities are not fair, an individual with both a γ equal to $0,75$ and an annual pension benefit of 17.455 euros[15] decides not to invest either in bonds, nor in annuities, for retirement. By contrast, in the subsection 4.1, Table 3 shows that under the same features our individual accumulates $213.502,81$ euros by investing in fair annuities. Thus, we can note that the threshold private wealth is easily reached, so long as the risk aversion decreases and the load increases.

Table 5: INDIVIDUAL WEALTH AT THE AGE OF 65 UNDER UNFAIR ANNUITIES ($\gamma = 2$)

τ_e ($\tau_f = 0$)	0	1	2	3	4	5	6	7	8
$\varpi = 0,25$	102	101	99	98	97	106	117	89	102
$\varpi = 0,50$	104	101	99	96	95	107	90	89	102
$\varpi = 0,75$	107	102	97	91	97	94	90	89	102

We have used as benchmark $100 = 247.519,26$ euros, which corresponds to the individual wealth achieved under fair annuities and no Social Security (see Table 2). Note that individual wealth is greater than our benchmark case for payroll taxes 1, 5 and 6. Nonetheless, the difference is reduced, and is even negative, as the load approaches to one.

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[14]Read proposition 1 in Sanchez-Romero (2005).

[15]See Table 1.

In sum, assuming an actuarially fair funded Social Security, individual wealth at the age of retirement is negatively affected by an unfair annuity market. This is so, unless policy makers decide to either reduce the payroll tax below 3 percent, or increase it up to 8 percent (see Table 5). However, if we take the first decision, we expect that people will outlive their financial resources faster and, consequently, their consumption will decrease as time goes by. Therefore, looking at consumption trajectories depicted in Figure 6 below, we recommend levying a payroll tax of 8 percent, not only because it assures an income after retirement, but also because individual wealth is not depleted before death.

Figure 6: CONSUMPTION, INDIVIDUAL WEALTH AND A.E.W. BY AGE, WITH SOCIAL SECURITY AND UNFAIR PRIVATE ANNUITIES (ϖ=0,50; $\gamma = 2$)
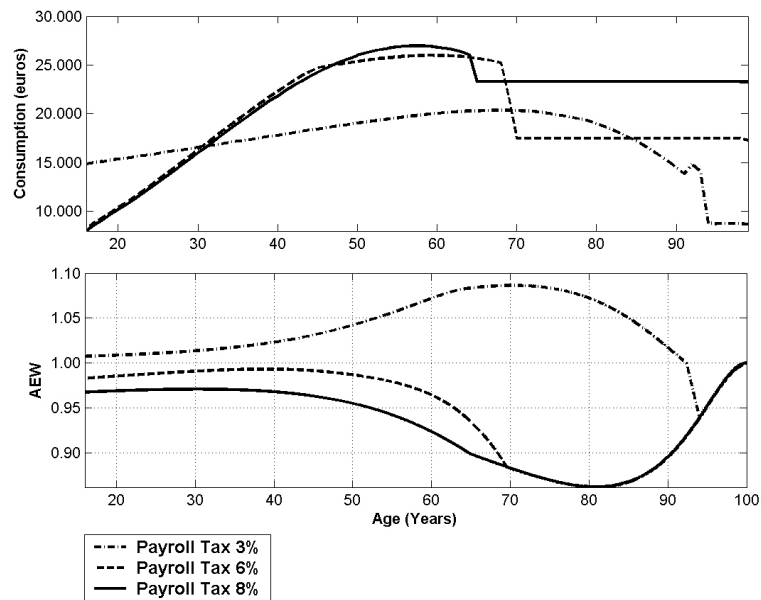
Note: annuities are only purchased when the payroll tax
is lower than 3 percent.

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5 Concluding Remarks

This paper presents new results about the crowding out effect produced by an actuarially fair funded Social Security on the stock of capital. We find that our consumer is more willing to purchase bonds, instead of annuities, as the payroll tax levied increases. On the one side, Social Security diminishes private wealth upon retirement which reduces the desire of purchasing annuities. On the other side, our individual may prefer bonds to annuities in order to achieve higher lifetime resources. We also find that, although this social security system expels the demand for private annuities, it may increase individual wealth. This latter fact nonetheless only happens so long as our individual voluntarily decides not to purchase annuities at the beginning of her life-cycle and, furthermore, that financial markets do not allow individuals to die in debt.

These findings show, contrary to previous research, that a social security system can contribute to reach a higher national wealth, even when the economy is composed by selfish individuals. For example, some simulation exercises presented here point out that a payroll tax of 6 percent increases individual wealth up to 17 percent points. This increment however is obtained under the assumption that private insurers offer fair annuities. Thus, on the contrary, under an unfair private annuity market, individual wealth can decrease around a 10 percent for the same payroll tax.

The importance of these findings raise some questions for future research. The most important is to determine the optimal payroll tax under an unfunded Social Security. Since, given the increasingly concern in developed countries about the feasibility of the social security system, a similar finding, as the one presented here, could contribute not only to decrease the payroll tax for future generations of workers, but also to give new reasons for maintaining the current social security system.

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Appendix

Our agent decides each time whether to annuitize her wealth or not. This circumstance lies on the assumption (v) (bounded rationality) introduced in this model. As a consequence, our individual compares the utility reported by annuitizing her wealth with not doing so. Thus, we maximize her expected utility twice regarding either equation (2) or equation (3). But, because the algebra in both processes are similar, we shall only derive the optimal consumption and investment at age x, when our individual decides not to annuitize her wealth.

Optimal Consumption and Investment at age x under Annuitized Wealth.

Assuming that our agent at age x maximizes equation (1), subject to (2) and (4) then, we can compute the optimal allocation process as an isoperimetric problem, whose equation is

$$\Im \equiv \Im(\mathbf{c}, \mathbf{e}, \lambda(x)) = \int_x^T \frac{\Omega(s)}{\Omega(x)} \beta(s-x) \left(\frac{c(s,x)^{1-\gamma}}{1-\gamma} - \frac{\gamma}{2} \frac{\sigma_\alpha^2(s)\eta^2 \mathrm{e}^2(s,x)}{c(s,x)^{1+\gamma}} \right) ds$$
$$+ \lambda(x) \left(k(x) + \int_x^T \frac{R(s)}{R(x)} \left(\theta(s)\sigma_\alpha(s)\mathrm{e}(s,x) + y(s) - c(s,x) \right) ds \right)$$

where $\theta(s) = \frac{\alpha(s)-r(s)}{\sigma_\alpha(s)}$.

The first-order conditions at age x for $\mathbf{c}$, $\mathbf{e}$ and $\lambda(x)$, respectively, are

$$c(x,x)^{-\gamma} + \frac{\gamma(1+\gamma)}{2}\sigma_\alpha^2(x)\eta^2\mathrm{e}^2(x,x)c(x,x)^{-2-\gamma} - \lambda(x) = 0, \tag{10}$$

$$-\gamma\sigma_\alpha^2(x)\eta^2\mathrm{e}(x,x)c(x,x)^{-1-\gamma} + \lambda(x)\theta(x)\sigma_\alpha(x) = 0, \tag{11}$$

$$k(x) + \int_x^T \frac{R(s)}{R(x)} \left(\theta(s)\sigma_\alpha(s)\mathrm{e}(s,x) + y(s) - c(s,x) \right) ds = 0. \tag{12}$$

Now, we should follow the next six steps in order to derive $c(x,x)$ and $e(x,x)$. Firstly, we derive the function $e(x,x)$ from (11). Second, we plug $e(x,x)$ into (10) and multiply both sides of the equation by $c(x,x)^\gamma$. Third, let define the function

$$\varphi(s,x) = \frac{\lambda(x)}{\beta(s-x)} \frac{R(s)}{R(x)} \frac{\Omega(x)}{\Omega(s)} c(s,x)^\gamma, \forall s \in [x,T] \tag{13}$$

and introduce it into the last equation. Thus, by solving the second-order equation in the variable $\varphi(s,x)$, it is easy to prove that $\Im$ is maximized if, and only if:

$$\varphi(s,x) = \frac{1 - \sqrt{1 - 2\frac{1+\gamma}{\gamma}\left(\frac{\theta(s)}{\eta}\right)^2}}{\frac{1+\gamma}{\gamma}\left(\frac{\theta(s)}{\eta}\right)^2} \quad \text{for all } x \in [0,T).$$

Fourth, using (13) and $\varphi(s,x)$, we obtain that $c(s,x)$ and $e(s,x)$ are

$$c(s,x) = \left(\frac{1}{\lambda(x)}\right)^{\frac{1}{\gamma}} \psi_x(s), \tag{14}$$

$$e(s,x) = \frac{1}{\gamma}\left(\frac{\alpha(s)-r(s)}{\sigma_\alpha^2(s)}\right)\frac{\varphi(s,x)c(s,x)}{\eta^2}, \tag{15}$$

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where $\psi_x(s) = \hat{\varphi}^{\frac{1}{\gamma}}(s) \left(\frac{\Omega(s)}{\Omega(x)} \frac{R(x)}{R(s)} \beta(s-x) \right)^{\frac{1}{\gamma}}$ for all $s \in [x,T)$. Fifth, by plugging equations (14) and (15) into (12), the lagrangian multiplier satisfies:

$$\left(\frac{1}{\lambda(x)} \right)^{\frac{1}{\gamma}} = \frac{k(x) + \int_x^T \frac{R(s)}{R(x)} y(s) ds}{\int_x^T \frac{R(s)}{R(x)} \psi_x(s) \left(1 - \frac{1}{\gamma} \varphi(s,x) \frac{\theta^2(s)}{\eta^2} \right) ds}. \tag{16}$$

Sixth and last, we introduce (16) into (14). So, the rate of expenditure on consumption and the amount of money invested in risky assets at age x are equal to

$$c(x,x) = \psi_x(x) \frac{k(x) + \int_x^T \frac{R(s)}{R(x)} y(s) ds}{\int_x^T \frac{R(s)}{R(x)} \psi_x(s) \left(1 - \frac{1}{\gamma} \varphi(s,x) \frac{\theta^2(s)}{\eta^2} \right) ds}, \tag{17}$$

and

$$e(x,x) = \frac{1}{\gamma} \left(\frac{\alpha(x) - r(x)}{\sigma_\alpha^2(x)} \right) \frac{\varphi(x,x) c(x,x)}{\eta^2}. \tag{18}$$

Nonetheless, we still need to prove that (17) and (18) are maximums as well as (1) converges to a mean-variance utility function. Thus, $\Im$ satisfies the set of sufficient conditions for a regular interior maximum,

$$\begin{vmatrix} \Im_{\mathbf{cc}} & \Im_{\mathbf{ce}} \\ \Im_{\mathbf{ec}} & \Im_{\mathbf{ee}} \end{vmatrix} > 0,$$

where if $e(s,x) > 0$ (resp. < 0) then $\Im_{\mathbf{ce}} = \Im_{\mathbf{ec}} > 0$ (resp. < 0). And finally, following Tsiang (1972), we apply the following two constraints in order that a CRRA utility function converges to our mean-variance utility function:

1. $\frac{\sigma_c(s,x)}{c(s,x)} < \varepsilon, \forall s \in [x,T)$, where ε is an infinitesimal.

2. $1 - 2\frac{1+\gamma}{\gamma} \left(\frac{\theta(s)}{\eta} \right)^2 \geq 0, \forall s \in [x,T)$.

∎