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José Luis Cendejas, Félix-Fernando Muñoz and Nadia Fernández-de-Pinedo

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DEPARTAMENTO DE ANÁLISIS ECONÓMICO: TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

A contribution to the analysis of historical economic fluctuations (1870-2010): filtering, spurious cycles and unobserved component modelling

José Luis Cendejas · Félix-Fernando Muñoz (✉) · Nadia Fernández-de-Pinedo

Abstract: Time series filtering methods such as the Hodrick-Prescott (HP) filter, with a consensual choice of the smoothing parameter, eliminate the possibility of identifying long swing cycles (e.g., Kondratieff type) or, alternatively, may distort periodicities that are in fact present in the data, giving rise, for example, to spurious Kuznets-type cycles. In this paper, we propose filtering Maddison's time series for the period 1870-2010 for a selection of developed countries using a less restrictive filtering technique that does not impose but rather estimates the cut-off frequency. In particular, we use unobserved component models that optimally estimate the smoothing parameter. Using this methodology, we identify cycles of periods mainly in the range of 4-7 years (Juglar type cycles), as well as a pattern of cyclical convergence that deepens with globalization processes. After 1950, a common business cycle factor grouping all economies is found.

Keywords: historical business cycles, spectral analysis, unobserved component models, Maddison's time series

JEL: C32, E32, N1

J.L. Cendejas
Instituto de Investigaciones Económicas y Sociales Francisco de Vitoria. Universidad Francisco de Vitoria. Pozuelo de Alarcón. 28223. Madrid. SPAIN
E-mail: joseluis.cendejas@iies-fv.es

F.-F. Muñoz (✉)
Departamento de Análisis Económico: Teoría Económica e Historia Económica. Universidad Autónoma de Madrid. 28049. Madrid. SPAIN
E-mail: felix.munoz@uam.es. Tel.: (+34) 91 497 43 95. Fax: (+34) 91 497 70 69

N. Fernández-de-Pinedo

Departamento de Análisis Económico: Teoría Económica e Historia Económica. Universidad Autónoma de Madrid. 28049. Madrid. SPAIN
E-mail: nadia.pinedo@uam.es.

1. Introduction

In a recent and suggestive paper, Diebolt (2014) claims to have identified a Kuznets-type cycle from a cliometric exercise based on the spectral analysis of Maddison's GDP series (Maddison, 2009; Bolt and Zanden, 2013). To this end, he previously proceeded to filter GDP series with the Hodrick-Prescott (1997) filter, and the spectra are estimated from the cycle component (deviation from the HP trend). In these spectra, a frequency corresponding to Kuznets-type cycles (approximately 20 years; Kuznets 1930, 1961) dominates. He identifies as well a common component for the economies in the sample.[1] Finally, Diebolt attributes the existence of Kuznets cycles to a demographic cycle that would manifest in housing and infrastructure demand and discards explanations of Kondratieff type.[2] An important problem with this filtering procedure is the possibility of inducing spurious cycles or other types of distortions in the filtered series when, for example, the smoothing parameter (in HP filter) is imposed *a priori* (e.g., Pedersen, 2001).

The main difficulty in the historical analysis of economic fluctuations, apart from the availability of reliable data, is the conceivable overlapping of waves of different periodicity (Schumpeter, 1939). In its origin, this was a fundamental question not satisfactorily resolved because of the insufficient statistical and computational tools (Nerlove et al., 1979). Although the beginnings of time series analysis took place in the thirties (Yule, 1927; Slutsky, 1937; Wold, 1938), the main econometric agenda was until the seventies centered on the linear regression model and its extension to simultaneous equation modelling (Epstein 1987, Morgan 1990, Hendry and Morgan 1995). The restatement of time series analysis in economics under the Box-Jenkins (1970) paradigm caused the return to the former interest on unobserved components.[3] The decomposition of economic time series in trend and cycle (in addition to the seasonal and the irregular components) is clearly related to notions of secular evolution (long swings), which is eventually linked to long-term growth, and business cycle dynamics. Fortunately, we dispose today of a panoply of techniques to efficiently address this problem (see Mills 2009); however, they are usually unknown and seldom applied in the analysis of historical time series.[4]

In this paper we propose estimating an unobserved component model to resolve this signal extraction problem, in which the smoothing parameter (a signal-to-noise ratio) is estimated optimally at the same time that the filtered components are obtained by means of the Kalman filter and the associated state space expression of the model (Harvey, 1989). This procedure does not introduce distortions by overweighting irrelevant frequencies or causing the appearance of inexistent cycles. Informally, we let the data "speak for

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[1] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Sweden, the UK, and the USA. In what follows, we add Spain and Switzerland.

[2] Discarding these types of results is trivial, as a sample of 140 observations would scarcely allow finding three complete cycles with a 50-year period.

[3] However, the Box-Jenkins methodology, based on differencing to achieve stationarity, eliminates long-term dynamics and obscures the cyclical one.

[4] An example is Cendejas & Font (2014), in which the price series of Hamilton have been modelled and analysed to obtain estimations of the common cyclical content of the Spanish historical inflation.

themselves". The rest of the paper is as follows. In Section 2, we expose, first, the univariate unobserved component modelling and its frequency domain implications; second, the usual static common factor methodology that we employ to explore the existence of common cycles from the components previously estimated; and third, a multivariate common factor model that embodies common cyclical variation. In Section 3, the cyclical components are estimated, the existence of common cyclical factors between economies is discussed and, according to this finding, the multivariate common factor model is estimated. In particular, an increasing cyclical coherence is found, especially after 1950. In accordance with the results, an historical interpretation of them is proposed. The paper ends with some concluding remarks.

2. Unobserved component modelling

The possibility of inducing periodicities not really present in the observed time series as a consequence of the filtering method has been known since the 30s. The so-called Yule-Slutzky effect (Yule, 1927; Slutzky 1937) consists in generating cyclical fluctuations only by summing and differencing a white noise process. Kuznets cycles of approximately twenty years have become a classical example of a "statistical artefact" (Adelman, 1965; Howrey, 1968). Kuznets transformed precisely the original series by averaging and differencing (Sargent 1979, pp. 248-251; Pedersen, 2001), causing the spectral gain of the implicit filter to show an important peak at the frequency of 20.25 years. In case of transforming a white noise process by this filter, a cycle of this period will be found. For time series distinct from white noise, this filter would favor the appearance of periods of approximately 20 years.

This distortion, in which the filter contains a cycle that passes into the filtered series, must be distinguished from the effects derived from imposing a cut-off frequency on economic series with the typical spectral shape, that is, series that concentrate variance in low frequencies (Granger, 1966). In this respect, Nelson and Kang (1981) show how trend removal of a random walk process induces pseudo-periodic behavior in the detrended series, and Nelson (1988) shows how a random walk could be incorrectly decomposed in a relatively smooth trend and in a cycle. Concerning mechanical detrending, when the smoothing parameter λ is imposed in the HP filter, Harvey and Jaeger (1993) show how this procedure gives rise to cyclical behavior, and propose structural models that simultaneously fit trend and cycle to avoid such pitfalls.[5] Cogley and Nason (1995) argue in a similar way. In all these cases, the periodicity found in the filtered series is not strictly spurious because the filter does not have a cycle, although some leakage and compression distortions have taken place.[6] In our view, in addition to the problem of correctly selecting the desired frequencies, it is important for these frequencies to be fundamental in business cycle dynamics. To

[5] A critical illustration of the consequences of employing detrending filters "mechanically" is Metz (2010).
[6] An ideal band-pass filter would prevent these distortions by excluding absolutely the undesired frequencies (Pedersen, 2001).

address these questions, it is proposed here to let the data locate the frequency in which the cyclical period is concentrated by optimally estimating the parameters in an unobserved component model.

2.1. The univariate model

The univariate model estimated here is the *Integrated Random Walk* (IRW) trend model (Young, 1984; Harvey, 1989, 2010; Kitawaga and Gersch, 1996). Its multivariate extension to a cyclical common factor model has been employed by Cendejas *et al.* (2014). The IRW model can be interpreted in terms of growth and acceleration of the variables involved, and it is consistent both with the classical business cycle (expansion and recession states depending on the sign of GDP growth) and endogenous growth theory, in which many models establish the stationary state as a constant growth state and, consequently, the transitional dynamics is a time path for which the second derivative is distinct from zero.

The univariate unobserved component model assumes that each of the observed series (which are expressed in logarithms) follows the equation

$$y_t = \mu_t + \varepsilon_t \tag{1a}$$

where μ_t is a non-stationary trend or level component and $\varepsilon_t \sim NID(0,\sigma_\varepsilon^2)$. In general terms, μ_t can be considered the signal and ε_t the noise, so we are facing a signal extraction problem. The trend μ_t is supposed to change with g_{t-1}

$$\mu_t = \mu_{t-1} + g_{t-1} \tag{1b}$$

where g_{t-1} can be interpreted as the underlying growth rate of y_t, and therefore, changes in g_t, Δg_t, represent the acceleration of y_t

$$g_t = g_{t-1} + a_{t-1} \tag{1c}$$

where a_{t-1} is the acceleration. The acceleration a_{t-1} is characterized as white noise

$$a_t = \eta_t \tag{1d}$$

with $\eta_t \sim NID(0,\sigma_\eta^2)$ and uncorrelated with ε_t. Model (1a) to (1d) is the so-called IRW trend model or "smooth trend" model because of the absence of a noise term in (1b). The sign of the growth g_t could indicate the phase of expansion or recession under a classical business cycle notion. By incorporating an acceleration component, we also consider declines and upturns in the growth rate. Additionally, as will be discussed later, the acceleration component is related in the frequency domain with the so-called growth cycle (upturns and downturns with respect to a trend) usually estimated by filtering with the HP filter. So, the IRW model serves as a unified and coherent framework for modelling both types of cyclical dynamics: classical and growth cycles.

The signal-to-noise ratio $q = \frac{\sigma_\eta^2}{\sigma_e^2}$ operates as a smoothing parameter and is the inverse of the smoothing parameter of the HP filter; that is, $\lambda = q^{-1}$. In particular, the lower is q (the higher is λ), the smoother is the trend component μ_t because the filter implied by the IRW model gives more weight to low frequencies when estimating the trend. Consequently, the detrended series, $y_t - \mu_t$, would incorporate a broader range of high frequencies. By imposing the value for $\lambda = 1600$ for quarterly series to obtain a cycle component, the range of frequencies is cutting out at a period of 9.9 years. For annual data, the usual value of $\lambda = 100$ divides the interval of frequencies at a period of 19.8 years. So, due to filtering by imposing λ, frequencies of a period longer than a certain duration will scarcely be present in the detrended series. In particular, if present, Kondratieff's long swings will not be found in annual series after detrending with $\lambda = 100$. On the contrary, the leakage distortion mentioned earlier will favor the appearance of cycles of approximately 20 years.

To avoid the risk of detecting cycles of distorted period, it is proposed here to estimate the signal-to-noise ratio optimally and let the data locate the frequency in which the cyclical period is concentrated. The state space form of the model (1a) to (1d) (see Appendix A) allows the variances in q to be estimated by maximum likelihood by using the Kalman filter (Harvey, 1989; Durbin and Koopman, 2001) and to obtain the predicted components (as conditioned by the information available up to $t-1$), the filtered components (as conditioned by the information available up to t) and the smoothed components (using the full sample). If we are interested in post-sample or historical analysis, the smoothed components are more appropriate. These components are the trend, the underlying growth rate, the acceleration and the deviation respect to the trend. In what follows, we analyze their properties in the frequency domain, that is, the spectral gain of the filters implied by these four components.

The spectral gain of a filter measures the increase in amplitude of any specific frequency component of a time series. It is obtained by the Wiener-Kolmogorov (WK) formula (Whittle, 1983). To this end, we depart from (1a), in which the signal $\mu_t = \frac{\eta_{t-2}}{(1-L)^2}$. The WK filter (of a doubly infinite realization of a time series) that provides the minimum mean squared error of the signal is given by the ratio of the autocovariance generating functions of the signal μ_t and the series y_t. For the trend component, the filter is

$$\hat{\mu}_t = \frac{\frac{\sigma_\eta^2}{(1-L)^2(1-L^{-1})^2}}{\frac{\sigma_\eta^2}{(1-L)^2(1-L^{-1})^2} + \sigma_\varepsilon^2} y_t = \frac{q}{q+(1-L)^2(1-L^{-1})^2} y_t = \frac{q}{q+|1-L|^4} y_t \qquad (2)$$

where L^{-1} is the forward operator ($L^{-k}y_t = y_{t+k}$) and the convention $(1-L)(1-L^{-1}) = |1-L|^2$ is adopted. The spectral gain of the filter of μ_t is obtained by doing $L = e^{-i\omega}$ in (2), where $i = \sqrt{-1}$ is the imaginary number and ω the frequency, obtaining

$$G_\mu(\omega) = \frac{q}{q + 4(1-\cos\omega)^2} \,. \tag{3}$$

For the growth component g_t, (1a) is expressed as $y_{t+1} = \frac{g_t}{1-L} + \varepsilon_{t+1}$ with the signal $g_t = \frac{\eta_{t-1}}{1-L}$. The resulting WK filter is

$$\hat{g}_t = \frac{\dfrac{\sigma_\eta^2}{(1-L)(1-L^{-1})}}{\dfrac{\sigma_\eta^2}{(1-L)^2(1-L^{-1})^2} + \sigma_\varepsilon^2}\, y_{t+1} = \frac{(1-L)(1-L^{-1})q}{q + (1-L)^2(1-L^{-1})^2}\, y_{t+1} = \frac{|1-L|^2 q}{q + |1-L|^4}\, y_{t+1} \tag{4}$$

for which the spectral gain is

$$G_g(\omega) = \frac{2(1-\cos\omega)q}{q + 4(1-\cos\omega)^2} \tag{5}$$

This gain has a maximum at the frequency

$$\omega_{\max} = \arccos\left(1 - \left(\frac{q}{4}\right)^{1/2}\right) \tag{6}$$

For example, for $q = \lambda^{-1} = \{0.001,\ 0.01,\ 0.1,\ 1,\ 10\}$, the corresponding periods $p = \frac{2\pi}{\omega_{\max}}$ are $p = \{35.3,\ 19.8,\ 11.0,\ 6.0,\ 2.9\}$ units of time.

For the acceleration component, (1a) is expressed as $y_{t+2} = \frac{a_t}{(1-L)^2} + \varepsilon_{t+2}$ with the signal $a_t = \eta_t$; then, the WK filter is

$$\hat{a}_t = \frac{\sigma_\eta^2}{\dfrac{\sigma_\eta^2}{(1-L)^2(1-L^{-1})^2} + \sigma_\varepsilon^2}\, y_{t+2} = \frac{(1-L)^2(1-L^{-1})^2 q}{q + (1-L)^2(1-L^{-1})^2}\, y_{t+2} = \frac{|1-L|^4 q}{q + |1-L|^4}\, y_{t+2} \tag{7}$$

In addition, in the frequency domain,

$$G_a(\omega) = \frac{4(1-\cos\omega)^2 q}{q+4(1-\cos\omega)^2} \qquad (8)$$

The HP filter is the optimal filter when the trend follows an IRW (King and Rebelo, 1993). In the context of HP filtering, the cycle (growth cycle) is defined as the deviation with respect to the trend $\varepsilon_t = y_t - \mu_t$, and the corresponding WK filter is

$$\hat{C}_t^{HP} = y_t - \hat{\mu}_t = \left(1 - \frac{q}{q+|1-L|^4}\right) y_t = \frac{|1-L|^4}{q+|1-L|^4} y_t \qquad (9)$$

A comparison of (3), (5), (8) and (9) allows to verify that

$$G_{HP}(\omega) = \frac{4}{q}(1-\cos\omega)^2 G_\mu(\omega) = \frac{2}{q}(1-\cos\omega) G_g(\omega) = \frac{1}{q} G_a(\omega) \qquad (10)$$

with $G_{HP}(\omega)$ the spectral gain of the HP filter given q. These gains are represented in Figure 1. In this figure, the spectral gains of the filters for the trend, growth and acceleration components (the latter coinciding with that of $\hat{C}_t^{HP}$ when normalizing) have been represented as an example value $q = 0.01$ ($\lambda = 100$). The gains of the filters of $\hat{g}_t$ and $\hat{a}_t$ have been normalized in such a way that the gain is 1 in their maxima ($\omega_{\max}$ according to (6) and π , respectively, with the original gains being $\frac{\sqrt{q}}{2}$ and q). With regard to $\hat{\mu}_t$ and $\hat{C}_t^{HP}$, their maximum gains are reached at the frequencies 0 and π with a gain of 1 in both cases. In view of this figure, the estimated components select the range of frequencies present in the observed series with the weights corresponding to the gain (not normalized) of the corresponding filter. For example, in the trend, the frequencies will remain mainly below a period of 19.8 years; in the growth component; the frequencies around this period; and in the acceleration (or $\hat{C}_t^{HP}$) component above this period. When imposing the smoothing parameter λ in the HP filter, $\hat{C}_t^{HP}$ may exclude frequencies that are important in the observed series. On the contrary, the estimation of λ leaves the data to locate the maximum gain in $\hat{g}_t$, that is, following a classical business cycle concept and, according to this period, the implied growth cycle $\hat{C}_t^{HP}$. This method simultaneously estimates both types of cycles, allowing a more coherent analysis.

[Figure 1 about here]

2.2. Common factor analysis

Common factor models synthesize in few variables (unobserved factors) the common information present in a wider set of variables. Let $y_t = (y_{1t}, y_{2t}, ..., y_{nt})'$ be a vector of n time series.[7] The vector y_t can be reduced to a simpler structure of m unobserved variables called factors with $m < n$. In what follows, the standardized variables, $x_{it} = \frac{1}{\sigma_i}\left(y_{it} - \mu_i\right)$, are considered, where μ_i and σ_i are the mean and the standard deviation, respectively, of y_{it}. Each time series x_{it} can be written as

$$x_{it} = \lambda_{i1} f_{1t} + \lambda_{i2} f_{2t} + ... + \lambda_{im} f_{mt} + v_{it} = \lambda_i^{'} f_t + v_{it} \tag{11}$$

where λ_{ik} are the factor loadings, f_{kt} the factors, and v_{it} an idiosyncratic or specific error. In vector form, $x_t = \Lambda f_t + v_t$ with $x_t = (x_{1t}, x_{2t}, ..., x_{nt})'$, $\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1m} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nm} \end{bmatrix}$ the matrix of factor loadings, $f_t = (f_{1t}, f_{2t}, ..., f_{mt})'$ the vector of factors, and $v_t = (v_{1t}, v_{2t}, ..., v_{nt})'$ the vector of errors.

The orthogonal (or classical) factor model assumptions are the following:

(a) Factors are standardized and orthogonal, that is, $\mathrm{E}\left[f_t\right] = 0_m$ and $\mathrm{E}\left[f_t f_t'\right] = I_m$ with 0_m a column vector of dimension m and I_m the identity matrix.

(b) The vector of errors v_t verifies that $\mathrm{E}\left[v_t\right] = 0_n$ and its covariance matrix $\Sigma = \mathrm{E}\left[v_t v_t'\right] = diag(\sigma_{v_i}^2)$.

(c) Factors and specific errors are orthogonal $\mathrm{E}[f_t v_t'] = 0_{m \times n}$, with $0_{m \times n}$ a $m \times n$ matrix of zeros.

Under these assumptions, the matrix of correlations of x_t, $\Gamma = \mathrm{E}\left[x_t x_t'\right]$, can be written as

$$\Gamma = \Lambda\Lambda' + \Sigma \tag{12}$$

From Γ, a variance decomposition is obtained for every x_{it}. In the diagonal of Γ, the correlation of x_{it} (trivially equal to 1) is $\rho(x_{it}, x_{it}) = c_i^2 + \sigma_{v_i}^2 = 1$, with $c_i^2 = \sum_{k=1}^{m} \lambda_{ik}^2$ the communality of x_t, and $\sigma_{v_i}^2$ its uniqueness, both in percentage points. Outside the diagonal of Γ, the correlation between variables is

[7] As we are interested in a common business cycle factor, the vector of observations stacks the underlying growth components previously estimated for every economy; thus, $y_t = (\hat{g}_{1t}, \hat{g}_{2t}, ..., \hat{g}_{nt})'$.

expressed as the sum of the products of factor loadings, $\rho(x_{it}, x_{jt}) = \sum_{k=1}^{m} \lambda_{ik}\lambda_{jk}$. Finally, the factor loadings are the correlations between variables and factors

$$\rho(x_{it}, f_{jt}) = \mathrm{E}\left[\left(\sum_{k=1}^{m} \lambda_{ik} f_{kt} + v_{it}\right) f_{jt}\right] = \mathrm{E}\left[\sum_{k=1}^{m} \lambda_{ik} f_{kt} f_{jt} + v_{it} f_{jt}\right] = \lambda_{ij} \qquad (13)$$

given assumptions (a) and (c).

Maximum likelihood estimation of model (11) assumes that $x_t \sim N(0_n, \Lambda\Lambda' + \Sigma)$. Estimates of Λ and Σ are obtained under the constraint $\Lambda'\Sigma^{-1}\Lambda = \Delta$, with Δ a diagonal matrix. This condition assures the identification of the factor model.[8]

2.3. The multivariate common factor model

The multivariate common factor model generalizes the IRW model by assuming common accelerations (or common cyclical factors; remember the equivalence shown by equation (10)). So, equations (1a) and (1b) are merely sub-indexed accordingly

$$y_{i,t} = \mu_{i,t} + \varepsilon_{i,t} \qquad (14a)$$

$$\mu_{i,t} = \mu_{i,t-1} + g_{i,t-1} \qquad (14b)$$

with $\varepsilon_{i,t} \sim NID(0, \sigma^2_{\varepsilon_i})$ and uncorrelated across i 's in all leads and lags, while the underlying growth components are assumed to follow

$$g_{i,t} = g_{i,t-1} + \gamma_i a_{t-1} + a_{i,t-1} \qquad (14c)$$

Equation (14c) implies that changes in $g_{i,t}$, $\Delta g_{i,t}$, are the sum of a common acceleration component, a_t, shared with the other series in the model and an idiosyncratic or specific acceleration component, $a_{i,t}$. The parameter γ_i is the factor loading that acts as a scale factor that amplifies or reduces a_t (if positive; if

[8] The decomposition $x_t = \Lambda f_t + v_t$ is not unique. For any orthogonal $m \times m$ matrix P verifying that $PP' = P'P = I_m$, we have that $x_t = \Lambda f_t + v_t = \Lambda PP' f_t + v_t = \Lambda^* f_t^* + v_t$ is also an orthogonal factor model. The transformation $f_t^* = P' f_t$ with P orthogonal is called an orthogonal rotation in the m dimensional space. The selected rotation will depend on the interpretation to be given to the estimated factors for any particular application. Estimation by principal components imposes the constraint that $\Lambda\Lambda'$ is diagonal and by maximum likelihood, that it is $\Lambda'\Sigma^{-1}\Lambda$. Both methods resolve the non-identification problem.

negative, the variable would be countercyclical). Both acceleration components, a_t and $a_{i,t}$, are assumed to be white noise[9] processes

$$a_t = \eta_t \tag{14d}$$

$$a_{i,t} = \eta_{i,t} \tag{14e}$$

with $\eta_t \sim NID(0,1)$, $\eta_{i,t} \sim NID(0,\sigma^2_{\eta_i})$, mutually uncorrelated and with respect to $\varepsilon_{i,t}$ in all leads and lags. The variance of η_t is normalized to unity to allow the identification of the model. The specification of equation (14c) is based on the unobserved component model with a common cyclical factor proposed by Stock and Watson (1989, 1991) to obtain a coincidental economic indicator from first log-difference time series.

Given that the signal is $\mu_{i,t} = \dfrac{\gamma_i \eta_{t-2} + \eta_{i,t-2}}{(1-L)^2}$ and analogously to equation (2), the WK filter of the trend component is

$$\hat{\mu}_{i,t} = \frac{\dfrac{\gamma_i^2 + \sigma^2_{\eta_i}}{(1-L)^2(1-L^{-1})^2}}{\dfrac{\gamma_i^2 + \sigma^2_{\eta_i}}{(1-L)^2(1-L^{-1})^2} + \sigma^2_{\varepsilon_i}} y_{i,t} = \frac{q_i}{q_i + |1-L|^4} y_{i,t} \tag{15}$$

where the signal-to-noise ratio is now $q_i = \dfrac{\gamma_i^2 + \sigma^2_{\eta_i}}{\sigma^2_{\varepsilon_i}} = q_i^c + q_i^s$, with the ratios $q_i^c = \dfrac{\gamma_i^2}{\sigma^2_{\varepsilon_i}}$ and $q_i^s = \dfrac{\sigma^2_{\eta_i}}{\sigma^2_{\varepsilon_i}}$ expressing the signal-to-noise ratio decomposition due to the common and the specific components. In percentage terms

$$w_i^c + w_i^s = \frac{q_i^c}{q_i} + \frac{q_i^s}{q_i} = \frac{\gamma_i^2}{\gamma_i^2 + \sigma^2_{\eta_i}} + \frac{\sigma^2_{\eta_i}}{\gamma_i^2 + \sigma^2_{\eta_i}} = 1 \tag{16}$$

where the weightings, w_i^c and w_i^s, quantify the relative importance of the common and the specific variation on every unobserved component.

[9] Alternatively, a_t and $a_{i,t}$ could follow autoregressive processes if some cyclical persistence is present. In this case, equations (14d) and (14e) would be $\phi(L)a_t = \eta_t$ and $\phi_i(L)a_{i,t} = \eta_{i,t}$ with $\phi(L)$ and $\phi_i(L)$ the respective autoregressive polynomials.

3. Empirical results and discussion

Hereafter, we present the estimation results of the univariate IRW model for the Maddison GDP series as well as the exploratory analysis of common factors together with an economic history interpretation. An important finding is the existence of a unique common factor grouping all the economies in the sample from 1950. This common factor allows the possibility to estimate the multivariate IRW model and to analyze the importance of the common business cycle on the national ones.

	Full sample estimations				Sub-sample estimations		
				Duration of the cycles (years)			
	σ_{ε}^2	σ_{η}^2	$q = \frac{\sigma_{\eta}^2}{\sigma_{e}^2}$	1870-2010	1870-1914	1915-1949	1950-2010
Australia	4.3883 (0.8703)	4.8872 (1.3080)	1.1137	5.8	8.5	(*)	7.1
Austria	24.8742 (5.4396)	26.6350 (8.1610)	1.0708	5.9	13.7	5.8	5.4
Belgium	4.5914 (1.0805)	5.6595 (1.7878)	1.2326	5.7	5.8	5.4	6.4
Canada	4.1845 (1.0981)	13.7684 (3.2089)	3.2903	4.3	6.7	3.0	5.3
Denmark	6.1112 (1.1534)	1.9107 (0.7616)	0.3127	8.2	(*)	8.2	7.5
Finland	4.3667 (1.2110)	10.0097 (2.8898)	2.2923	4.7	6.3	5.1	2.9
France	12.1750 (2.4517)	14.1541 (3.7978)	1.1626	5.8	(*)	5.4	4.9
Germany	10.6926 (4.3538)	47.2044 (14.3227)	4.4147	3.9	9.1	3.7	9.0
Italy	1.5217 (0.9678)	20.0013 (4.8295)	13.1441	2.5	11.2	(*)	6.5
Japan	19.6794 (3.6875)	11.8505 (3.6905)	0.6022	6.9	(*)	6.2	4.5
Netherlands	12.2539 (3.1796)	19.1652 (6.1052)	1.5640	5.3	11.2	5.5	4.4
Norway	4.3380 (0.8522)	2.9124 (0.9316)	0.6714	6.7	4.3	7.3	3.5
Spain	5.6174 (1.4779)	6.7669 (2.4505)	1.2046	5.7	15.5	4.8	2.9
Sweden	3.3951 (0.6457)	2.2577 (0.6929)	0.6650	6.7	15.9	6.1	5.5
Switzerland	13.1639 (2.4299)	1.6918 (0.8840)	0.1285	10.3	(*)	9.0	4.2
UK	0.7988 (0.3025)	6.3616 (1.3458)	7.9638	3.2	3.5	2.6	3.2
USA	4.3801 (1.2239)	17.5286 (3.9707)	4.0019	4.0	10.5	(*)	6.2
			Mean period	5.6	9.4	5.6	5.3
			Standard deviation	1.9	4.1	1.8	1.7

Table 1. Full sample estimated variances of the univariate IRW model (1) and duration of the cycles derived from the estimated parameter q according to equation (6) for full sample and sub-sample data. Note: In these cases, the important noisy content of the original data either produces a very low estimated q and a corresponding long period or passes into the signal, causing a very high q that prevents the period from being computed according to equation (6).

Table 1 shows the full sample estimated variances of the univariate IRW model (1) and the periods (duration of the cycles) derived from the estimated parameter q according to equation (6). Model (1) has also been estimated in sub-samples corresponding to pre-, inter- and post-war periods. The adequacy of the estimated trend and growth components $\hat{\mu}_t$ and $\hat{g}_t$ and the original series can be observed in the figures in Appendix B. Full sample estimations show a mean duration of 5.6 years with a standard deviation of 1.9 years. The range of durations is from the 2.5 years of Italy to the 10.3 years of Switzerland. The noisy content of the original time series might influence these results, but duration characteristics of long swings and Kuznets cycles are clearly excluded. A period in the range of 4 to 7 years is present in 12 of the 17 economies. For sub-samples, the mean duration increases to 9.4 years in the pre-war period with a high standard deviation of 4.1 years. These figures, in particular for some countries, are very influenced by the noisy content of the original data that cause the variance σ_η^2 to be very small and, consequently, also the noise-to-variance ratio. The mean durations in the inter- and post-war periods are 5.6 and 5.3 years, respectively, with standard deviations of 1.8 and 1.7 years. When the Maddison original series are supposed to be more reliable (post-WWII period), the durations are within the range from 2.9 years (Spain) to 9.0 years (Germany). For the USA and the post-war sub-sample, our estimated period (6.2 years) is not very different from that of the NBER (5.7 for the period 1945-2009).[10]

Table 2 shows the results of the estimation of the factor models according to subsection 2.2. The sample period has been split into the three major historical periods[11] previously considered. The main results are coherent with those observed in Table 1. A noisier pre-war period (see figures in Appendix B) goes hand-in-hand with the absence of a common factor grouping all economies. Two common factors have been found (Factor 1 and Factor 2). Factor 1 is significatively correlated with 10 of the 17 economies. The maximum correlations correspond to Germany (84%), Belgium (82%) and Austria (80%). The list of economies includes mainly continental Europe together with the USA and Canada, although, in these two cases, their communalities (the percentage of the variance explained by the factors) are small. Consequently, Factor 1 shows mainly a Central European business cycle. The second factor (Factor 2) significatively includes the Scandinavian economies (Norway, Sweden and perhaps, with a non-significant correlation, Finland), France, the UK and Spain. In general, the communalities show a moderate explanatory ability of the factor model coherent with the absence of global cyclical integration. The exceptions are Austria, Belgium, France and Germany, which exceed 50% mainly due to Factor 1. Factor 2 is mainly due to Norway and Sweden and could be interpreted as a peripheral factor grouping economies excluded from Factor 1 (negative loadings do not have any special interpretation in this context). When a third factor is added to the model, the communality increases minimally in some cases at the cost of some reduction in others; thus, no further integration, as a clear dependence on common factors, can be found

[10] http://www.nber.org/cycles.html. In relation to the average duration of business cycles for 13 developed countries, Bergman *et al.* (1998) obtain some different results. This is a consequence of both the different filtering methodology (they use a band-pass filter that imposes a range of duration of the business cycle between 2 and 8 years) and the databases employed.

[11] For a different periodization, see, among others, Foreman-Peck (2007) and Northrup (2005). The one employed here is fairly coincident with that of Maddison (2007).

(this also happened in the other periods). These factors and their relation with the estimated growth components, $\hat{g}_t$, can be seen in Figure 2.

	1870-1914			1915-1949			1950-2010		
	Factor 1 loadings	Factor 2 loadings	Commu-nality	Factor 3 loadings	Factor 4 loadings	Commu-nality	Factor 5 loadings	Factor 6 loadings	Commu-nality
Australia	0.23	0.05	0.05	-0.19	**0.56**	0.35	**0.49**	**0.62**	0.63
Austria	**0.80**	**-0.37**	0.78	0.20	**0.98**	1.00	**0.80**	**-0.31**	0.73
Belgium	**0.82**	-0.07	0.67	**0.85**	-0.01	0.73	**0.87**	0.15	0.78
Canada	**0.48**	0.08	0.24	-0.28	0.36	0.21	**0.61**	**0.58**	0.71
Denmark	**0.50**	0.11	0.26	**0.79**	-0.18	0.66	**0.75**	0.17	0.59
Finland	**0.46**	0.24	0.27	**0.47**	0.25	0.29	**0.69**	**0.33**	0.58
France	**0.65**	**0.54**	0.71	**1.00**	0.00	1.00	**0.99**	-0.07	0.98
Germany	**0.84**	-0.02	0.70	-0.27	**0.79**	0.70	**0.69**	**-0.32**	0.58
Italy	**0.35**	-0.28	0.20	**0.67**	0.35	0.56	**0.86**	-0.13	0.76
Japan	0.02	0.04	0.00	-0.12	**0.81**	0.67	**0.85**	-0.28	0.80
Netherlands	0.22	-0.03	0.05	**0.89**	0.14	0.80	**0.72**	0.23	0.57
Norway	0.14	**0.83**	0.71	**0.88**	-0.03	0.77	**0.50**	**0.31**	0.35
Spain	0.17	**0.36**	0.16	-0.16	0.06	0.03	**0.81**	0.22	0.70
Sweden	-0.05	**0.68**	0.46	**0.77**	-0.04	0.60	**0.71**	**0.38**	0.65
Switzerland	**0.33**	**-0.42**	0.28	**0.61**	0.10	0.38	**0.75**	-0.16	0.59
UK	0.19	**0.39**	0.19	-0.35	**0.48**	0.36	**0.40**	**0.63**	0.56
USA	**0.37**	-0.19	0.17	**-0.58**	0.34	0.45	**0.55**	**0.46**	0.52

Table 2. Estimated factor models. In bold: correlation with the common factor statistically significant at the 0.01 level.

[Figure 2 about here]

Factors 1 and 2 represent both the outcome of the first globalization and of the second industrialization process that took place within the period 1870 to 1914, also known as the *Belle Époque*. Despite the first modern crisis of overproduction – the Great Depression of 1873 –[12] it seems that "positive shocks" had a much deeper impact on the growth trend than "negative shocks". A considerable degree of economic openness, a solid international monetary system, and the consolidation of the central nation-states (Germany and Italy) favored stability and the integration of the international economy. It is necessary to

[12] Germany and the United States were affected by the crisis that started with the crack of Vienna on the 8th of May 1873 as a consequence of speculation, rising costs and declining corporate profitability. From Central Europe, the crisis moved to the Atlantic and reached the US in September 1873. The depression lasted until 1879. The industrial sector suffered markedly. Even the UK faced large bankruptcies (13,130) only in 1879. Prices and wages dropped (Flamant and Singer-Kerel 1971).

emphasize that the period of greater liberalization of world economy – migrations, capital and trade – occurred between 1870 and 1914. Industrial Enlightenment (Mokyr, 2010) in Great Britain and the spread of the Industrial Revolution provoked a convergent trend between European economies. The first-comers (Belgium, France, and Switzerland) and the latecomers (Germany, Italy and Austria-Hungary) enjoy the *European Pax* (Craig and García-Iglesias 2010: 124) under an opened trade area[13] after the Cobden-Chevalier treaty of 1860, with a parallel leading role of the UK and its *Pax Britannica* in the seas. In addition to trade liberalization, monetary stability was reached after the majority of European industrialized countries and the US (1879) joined the Gold standard (Germany in 1872, Belgium, France and Switzerland in 1878, and Canada in 1853).

Increased productivity lowered production costs in agriculture and manufacture industry, and the impact of technical improvements and transport revolution, together with the opening of the Suez Canal (1869), prompted the decline in international freight rates as well as in domestic transport costs with the spread of railroads in Europe and the transcontinental railroad in the US (1869). Intra-European migration first and massive international migration from 1875 had an enormous impact on wage convergence. International flows of labor and capital permitted the US and Canada to join the club. Nordic countries (Factor 2) also caught up (Bruland and Smith 2010) from the 1870s, taking advantage of natural resources, world capital and labor markets as well as the flow of new technological know-how[14]. Nordic economies sent forth work populations to the US and received large amounts of capital from France and Germany from the mid-nineteenth century. In fact, France and the UK became major exporters of skilled labor, machinery and capital in the nineteenth century thanks to its preeminent industrial leadership.

[Figure 3 about here]

The second period (1915-1949) shows a higher correlation of the growth components with the estimated factors (Factor 3 and Factor 4). In general (except for Canada and Spain)[15], the communalities are greater than in the pre-war period as a consequence of common devastating shocks such as the World Wars and the Great Depression of the 30s. The two factors include disjointed sets of economies. Factor 3 is identified with the growth component of France (correlation of 100%) and also includes Belgium, Denmark, Finland, Italy, the Netherlands, Norway, Sweden and Switzerland. Factor 4 is significatively correlated with Germany and also includes Australia, Austria, Japan and the UK. This grouping indicates the different cyclical patterns present, particularly from the 30s, as seen in Figure 3 when comparing both factors: They move in parallel until 1937; in this year, Factor 3 diminishes and recovers beginning in 1941, reaching a

[13] "International trade is perhaps the most important form of engagement with the world economy" (Nayyar 2009: 14).

[14] Nordic countries, in addition to their cultural proximity, had a late and quick industrialization based on institutional reforms that eliminated restrictions on business, innovation and credit (bank system). They combined rich natural resources such as forest, ore deposits, fishing, land and oil with a late integration in the globalization process, in addition to mergers and acquisitions between big firms and, from WWII, the expansion of the public sector and welfare system (Henning, Enflo, and Andersson 2011). The impressive progress made Nordic countries "an overachiever" (O'Rourke and Williamson, 1995: 8), although there were differences between countries, Sweden being the country that made the most rapid transition.

[15] The Spanish Civil War (1936-1939) may have affected this result.

maximum in 1946. Factor 4 diminishes from 1940 until 1945 and then recovers. Thus, the different cyclical effects of WWII are shown by these factors, both in the years of the war and the immediate post-war (e.g., the negative and significant correlation of the USA with Factor 3).

The inability to create a stable international system after WWI finally results in WWII. War was an exogenous factor with negative effect on economic output (Feinstein et al. 2008) and represents a downturn that involves multiple countries with different trajectories. The state has to intervene in the economy to solve restructuring economic problems. The instauration of a war economy means that investments in strategic sectors (heavy industry, railway network, and the arms industry) had to be prioritized at the expense of light industry and agriculture. In accordance with the disruption of commerce and agricultural production and the destruction of infrastructure, convertibility is suspended, affecting balance of payments adjustments. Finally, the post-war recovery and reparations from WWI, as well as changes in frontiers, with new countries emerging and others disappearing (Singleton 2007: 11)[16] – all of this allows discussion of de-globalization (Williamson, 1996). Wars and the depression of the 1930s stopped capital, migration and goods flows (Siegfreid, 1937: 90, in Bouvier 1999: 422), interrupting the tendencies that began in 1870. War is the opposite of peace, which "together with law and order, plays a key role in allowing the market to extend and creating increasing returns" (Foreman-Peck 2007:24).

Both wars had different final impacts on national economies. WWI interrupted the industrialization process of some countries due to the lack of foreign direct investment and technology imports. The United States and Japan, as well as peripheral countries, become the main beneficiaries of the European conflict after capturing markets abandoned by a Europe involved in war. The balance of economic power moved to the Pacific. WWI was a great business for certain economies. Between 1913 and 1929, the European neutrals (Nordic countries, the Netherlands, and Switzerland) experienced faster growth than the combatants. Except for the UK, which was particularly vulnerable to the dislocation of international markets, the European Allies outperformed the former Central powers. War imposed output losses on many countries and altered the long-term rate of growth of the UK.

When peace returned, the industrialization process accelerated, spurred by the recovery policies focusing on investments in modern technology (chemistry, electricity, cement). The recovery of the war was uneven. Financial costs and the economic consequences of the war prevented a return to the situation that prevailed in July 1914. Inflation and deficits were the main imbalances that affected in different ways the different countries in the early post-war years. The return to the gold standard at pre-war parities proved to be a difficult task for the European countries. Only neutrals and the UK achieved this objective. Finland, Belgium, France and Italy returned to gold with reduced parities and in fact enjoyed greater economic growth than the UK. Factor 3, by grouping these countries, would show the consequences of these

[16] As Singleton noted, geography changed. France received Alsace and Lorraine from Germany, but "Poland was created out of land formerly belonging to the German, Russian and Austro-Hungarian Empires. The heartland of the Austro-Hungarian Empire was divided into the independent nations of Austria, Hungary and Czechoslovakia. Russia was stripped of Lithuania, Latvia, Estonia and Finland …" (Singleton 2007).

adjustments on growth. Germany and Austria also returned to the gold standard after the large depreciations and the hyperinflation of 1920s.

When European production capacity returned to normal, both in agriculture and industry, the problem of overproduction emerged because the production capacity of non-European countries had greatly increased, and overproduction became chronic (Eichengreen 1992). The crisis lasted from three to four years, from 1929 to 1932-33. Protectionism since 1930 meant a return to quotas, import substitution policies and, in the case of some countries (Germany, Italy, Japan and Spain), tendencies toward economic autarky (Bouvier 1995: 381) and authoritarian regimes. As a result, economies tended to reduce imports and restrict capital flows and foreign trade was controlled by the state. The countries most affected were those that had been defeated in the war, while those only slightly affected were the Scandinavian countries: Denmark GDP did not decrease, and others only moderately decreased, such as Spain and Italy (Maddison, 2001). In general, small neutral countries such as Switzerland, the Netherlands, Denmark and Sweden managed to adapt to new models of competitiveness and discovered niche markets. Some of the great multinationals linked to these countries successfully managed to enter the world market and consolidate their positions in these years.

After the recovery from the Great Depression, both Germany and Japan began to prepare their economies for war. The unstable international market withdrew for the third time in less than thirty years. During WWII, the whole Continent, except four countries that remained neutral, was occupied. For the first time, their economies were unified under a single yoke. Hitler achieves a total economic and political reconstruction of Europe to make it self-sufficient. GDP did not increase throughout belligerent Europe, but the US doubled per capita income after recovering from the Great Depression.

[Figure 4 about here]

Finally, stage three (1950-2010) reflects the global convergence of Western Europe and Anglo-Saxon economies headed by the US and the return of a greater degree of economic openness as a consequence of the second globalization. In consequence, the post-war period is characterized by global cyclical integration as shown by the communalities in Table 2, in which values are more homogenous and, in most cases, higher than in the previous periods. Plots in Figure 4 show the increasing cyclical coherence during the period. Coherently, a common factor including all the economies is found (Factor 5), while the second factor of the model (Factor 6) captures some specific growth present in the Anglo-Saxon economies (Australia, Canada, UK and USA) mainly after the 80s (see Figure 4) as well as some Scandinavian specificity (Finland, Norway and Sweden). Factor 5 is identified, as in the previous period, with the growth component of France and could be viewed as a precursor to a European Union business cycle. It is important to note its decreasing profile. Precisely when comparing both factors, Factor 6 shows the higher growth present in the mentioned economies from the 80s, which can be related to the liberalizing policies present in these economies.

After the post-war (see Factor 5), a convergent trend started with the European recovery plan (Marshall Plan 1948-1957) and the establishment of the Bretton Woods System (1944-1971). The regulation of trade through GATT in 1947 and the Treaty of Rome, which established the European Community in 1957, and the European Free Trade Association two years later intensified economic integration. Japan and Western Europe did much better and greatly reduced the gap between their income and productivity levels with respect to those of the USA. In Western Europe, this catch-up process gave the opportunity to recuperate from the lost opportunities from the war. The Japanese catch-up process was spectacular. Japan, which had devoted a large part of its human and capital resources to military ends since the Meiji Period, had to complete de-militarization. This meant, as in the case of Germany, that its skills, organizational capacity and investment were devoted almost entirely to economic growth through capital intensive technology. During the 1980s, the end of the cold war and the integration of Eastern Europe caused convergence to accelerate, while the merger of the EU market and the adoption of the Maastricht Treaty in 1991 deregulated economic activities. Regional integration spread through multilateral free trade agreements, and customs unions or common markets spread over the five continents. In spite of the several frequent international periods of stagnation and stock market bubbles from the 1970s (oil price shocks and the great inflation of the 1970s and 1980s) until the 2007 crisis,[17] the convergence of the Western European, Japanese and Anglo-Saxon economies seems to be an incontestable fact.

In sum, two relevant factors seem to influence trends:[18] the relevance of international politics, understood as a process that enhances or boosts cooperation, and the significant role of the institutional framework, in particular as it is linked to international payment mechanisms and monetary arrangements, which in turn encourage trade.

Finally, to quantify the importance of this global business cycle factor in national economies, the multivariate common factor model presented in subsection 2.3 is estimated for the post-war period. In this way, we have a measure of the importance of the common factor grouping all the economies (i.e., a global factor) in business cycle dynamics. The estimation results are shown in Table 3. All the factor loadings are significatively distinct from zero, verifying the existence of a global common business cycle factor that displays the acceleration shared by all the economies in the sample. According to equation (16), the relative importance of this common behavior has been computed, with the result that it exceeds 50% in 15 cases (the exceptions being Norway and Spain) and 70% for 9 economies. The more important specific cyclical variation in decreasing order corresponds to Spain (78%), Norway (75%), Denmark (46%), Canada (43%), the UK (43%), the USA (42%) and Japan (41%). It must be noted that these estimates are averages for the period, and some progressive increase in the weighting of the common factor could have taken place in some cases. Across the specific components, $a_{i,t}$, some important positive correlations would support the possibility of estimating a minor common factor grouping Australia, Canada, Denmark, Norway, UK and the USA, which is consistent with the previous exploratory factor analysis. The periods obtained from q_i differ in some cases from those of Table 1. In general, the duration is lower than that of Table 1. In the

[17] From the mid-1980s until 2007, the gradual reduction of inflationary trends in the industrialized world was referred to as the "Great Moderation" thanks to the reduction in the volatility in GDP growth in Australia, Canada, the US, the UK, Germany, Japan, France and Italy (Summers 2005).

[18] For other approaches that examined business cycle fluctuations and monetary policy regimes, see Bergman et al., 1998; Piketty and Saez, 2006; and Milanovic, 2005.

range from 4 to 7 years, there are 6/14 economies; in the range 3 to 8, 11/14 economies. The mean duration is 4.0 years with a standard deviation of 1.2 years.

	$\sigma^2_{\varepsilon_i}$	$\sigma^2_{\eta_i}$	γ_i	Importance of the common cycle w_i^c	Importance of the specific cycle w_i^s	$q_i = \dfrac{\gamma_i^2 + \sigma^2_{\eta_i}}{\sigma^2_{\varepsilon_i}}$	Duration of the cycles (years)
Australia	1.4609 (0.5540)	0.2118 (0.2557)	0.8054 (0.3007)	0.75	0.25	0.5890	6.9
Austria	0.5080 (0.1760)	0.6766 (0.2926)	1.8492 (0.2708)	0.83	0.17	8.0630	3.1
Belgium	0.3192 (0.0999)	0.1418 (0.0845)	1.7890 (0.2205)	0.96	0.04	10.4709	2.8
Canada	1.2552 (0.4066)	1.1601 (0.5742)	1.2502 (0.3338)	0.57	0.43	2.1694	4.8
Denmark	1.4003 (0.4641)	0.9271 (0.5168)	1.0530 (0.3484)	0.54	0.46	1.4539	5.4
Finland	0.4982 (0.3115)	2.6375 (1.0529)	2.8090 (0.4150)	0.75	0.25	21.1317	(*)
France	0.2007 (0.0991)	0.0965 (0.1087)	1.4375 (0.1831)	0.96	0.04	10.7763	2.8
Germany	0.8839 (0.2585)	0.4017 (0.2104)	1.4443 (0.2718)	0.84	0.16	2.8146	4.5
Italy	0.5273 (0.2793)	0.7826 (0.4754)	1.5096 (0.2786)	0.74	0.26	5.8062	3.5
Japan	0.5646 (0.2406)	1.9343 (0.6883)	1.6792 (0.3271)	0.59	0.41	8.4199	3.1
Netherlands	0.8569 (0.9092)	0.6750 (1.2527)	1.6574 (0.3579)	0.80	0.20	3.9933	4.0
Norway	0.2250 (0.1383)	1.6356 (0.5698)	0.7331 (0.2316)	0.25	0.75	9.6578	2.9
Spain	0.7457 (0.3815)	3.6561 (1.3434)	1.0253 (0.4200)	0.22	0.78	6.3126	3.4
Sweden	0.0601 (0.2450)	2.1148 (1.1221)	1.8822 (0.3300)	0.63	0.37	94.1364	(*)
Switzerland	0.7739 (0.2236)	0.3556 (0.2005)	2.2352 (0.3018)	0.93	0.07	6.9151	3.3
UK	0.2301 (0.1842)	1.8047 (0.7453)	1.5356 (0.2839)	0.57	0.43	18.0904	(*)
USA	1.2732 (0.4393)	1.1731 (0.6079)	1.2774 (0.3297)	0.58	0.42	2.2030	4.8
						Mean period	**4.0**
						Standard deviation	**1.2**

Table 3. Estimations for the multivariate IRW model (14), importance of the common cycle and duration of the cycle derived from the estimated parameter q_i according to equation (6) for the post-war period 1950-2010 (Maddison data). (*) See note of Table 1.

These results have been compared with those obtained from the Penn World Tables (v8.0) for the same period. Except for the year 1986, both common cycle components are broadly similar (see Figure 5). The importance of the common cyclical behavior is close when compared with the Maddison data except for Spain (90%) and Denmark (70%), which is now greater, and Norway (56%), which is lower. With respect to the duration of the business cycle, the more important differences appear in Australia, Germany and Norway. The mean duration is 4.2 years with a standard deviation of 2.1 years.

[Figure 5 about here]

	$\sigma^2_{\varepsilon_i}$	$\sigma^2_{\eta_i}$	γ_i	Importance of the common cycle w_i^c	Importance of the specific cycle w_i^s	$q_i = \frac{\gamma_i^2 + \sigma^2_{\eta_i}}{\sigma^2_{\varepsilon_i}}$	Duration of the cycles (years)
Australia	6.0408 (1.3807)	0.2227 (0.1771)	0.6334 (0.2978)	0.64	0.36	0.1033	10.9
Austria	0.4337 (0.1456)	0.6145 (0.2435)	1.6517 (0.2467)	0.82	0.18	7.7069	3.2
Belgium	0.6316 (0.2350)	0.5287 (0.3314)	2.6960 (0.3244)	0.93	0.07	12.3450	2.6
Canada	2.2468 (0.8770)	2.0016 (1.1399)	1.4628 (0.4157)	0.52	0.48	1.8432	5.1
Denmark	2.5383 (0.9334)	1.4902 (0.9966)	1.8462 (0.4369)	0.70	0.30	1.9299	5.0
Finland	0.0352 (0.2320)	14.6047 (3.1642)	4.0427 (0.6550)	0.53	0.47	(*)	(*)
France	0.5553 (0.1951)	0.1366 (0.1524)	2.3554 (0.2958)	0.98	0.02	10.2368	2.8
Germany	0.5522 (0.2736)	1.1374 (0.5792)	2.2821 (0.3274)	0.82	0.18	11.4908	2.7
Italy	1.0634 (0.3605)	0.9469 (0.4763)	2.3255 (0.3681)	0.85	0.15	5.9759	3.5
Japan	1.6097 (0.5981)	5.4433 (1.7105)	2.1183 (0.5034)	0.45	0.55	6.1691	3.5
Netherlands	1.2159 (0.4018)	0.7416 (0.4089)	2.1651 (0.3368)	0.86	0.14	4.4653	3.9
Norway	3.6579 (1.4370)	2.5125 (1.6567)	1.7777 (0.5421)	0.56	0.44	1.5509	5.3
Spain	3.1554 (0.8584)	1.0229 (0.5928)	3.0224 (0.4848)	0.90	0.10	3.2192	4.3
Sweden	(*)	6.7125 (1.3617)	2.8093 (0.4675)	0.54	0.46	(*)	(*)
Switzerland	0.5543 (0.2385)	1.4962 (0.5641)	2.6419 (0.3465)	0.82	0.18	15.2910	2.2
UK	0.5683 (0.2994)	1.9368 (0.8865)	2.0028 (0.3410)	0.67	0.33	10.4665	2.8
USA	1.7474 (0.7536)	1.4309 (0.9099)	1.1039 (0.3796)	0.46	0.54	1.5163	5.3
						Mean period	**4.2**
						Standard deviation	**2.1**

Table 4. Estimations for the multivariate IRW model (14), the importance of the common cycle and duration of the cycle derived from the estimated parameter q_i according to equation (6) for the post-war period 1950-2010 (Penn World Tables data, v 8.0). (*) See note of Table 1.

4. Concluding remarks

As noted previously, HP filtering with *a priori* smoothing parameter implies a selection of frequencies that may distort the analysis of business cycle duration and phases by exclusion and/or leakage of frequencies. The estimation method proposed here avoids this problem by estimating the signal-to-noise ratio, that is, allows the data to "speak for themselves". Additionally, the IRW model (both univariate and multivariate) incorporates, in a coherent framework, the possibility of estimating simultaneously both the classical business cycle and the growth cycle implied by the estimated signal-to-noise ratio. When applied to a sample of Maddison's GDP series, a classical business cycle of a duration in the range 4-7 years (Juglar-type cycles) is found, and there is no evidence of long swings or Kuznets-type cycles. Cyclical convergence is evident and very strong after 1950, when cyclical phases between economies are synchronized (one factor grouping all the economies has been found), and the standard deviation of the cyclical period is approximately one year.

In the pre-war and inter-war periods, a minor economic integration, the specific effects of the World Wars together with the more noisy content of the series, weaken the cyclical coherence. As O'Rourke and Williamson (O'Rourke & Williamson, 1995:7) noted, "global openness and convergence seem to be positively correlated; global autarky and convergence seem to be negatively correlated". Not all regions are synchronized with the national business cycles (Owyang *et al*. 2005), and not all economies are synchronized, but cyclical convergence seems to depend on capital and trade international flows if the effects of the industrial revolution (the first, second, and third) and the globalization process among national economies are considered.

Although European countries entered the nineteenth century with mercantilist policies that consolidated nation-states, a century later, all economies were linked by the need to validate international rules to regulate economic activity at a world level. Regional economic agreements in the interwar period (Oslo Group, Clearing Agreements, Rome or Ottawa Agreements or even Cartel Agreements) showed the crucial significance of cooperation over regional variations. The two world wars highlighted how easy it is to alter the domestic economic structure and international flows. Although the catch-up to modern economic growth followed diverse rhythms and timeframes, the negotiations before the end of WWII to gestate a world order and prevent another interwar period evidenced the importance of cooperation and multilateralism, both foundations of long globalization cycles.

Finally, although our estimations do not locate long swings, we cannot discard the existence of Kondratieff-type cycles.[19] Moreover, when investigating long-term processes affecting economic growth, some interesting extensions of this work would include the relationship between demographic stagnation (Gonzalo *et al.*, 2013) and the decreasing profile of (per capita) growth rates along the post-war period. Some tentative hypothesis concerns the existence of a demographic dividend (e.g., Roa and Cendejas, 2007). In any case, unobserved component modelling has proved to be a very useful tool for cliometric analysis due to the explicit consideration of long-term and medium-term (cycles) economic processes when the models are interpreted in the frequency domain.

[19] An interesting attempt in this sense is Metz (2011).

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FIGURES

Figure 1. Spectral gains of the filters of μ_t (continuous line in red), g_t (dashed line in blue) and a_t (dashed line in green) for $q = 0.01$. Horizontal axis: period in units of time.

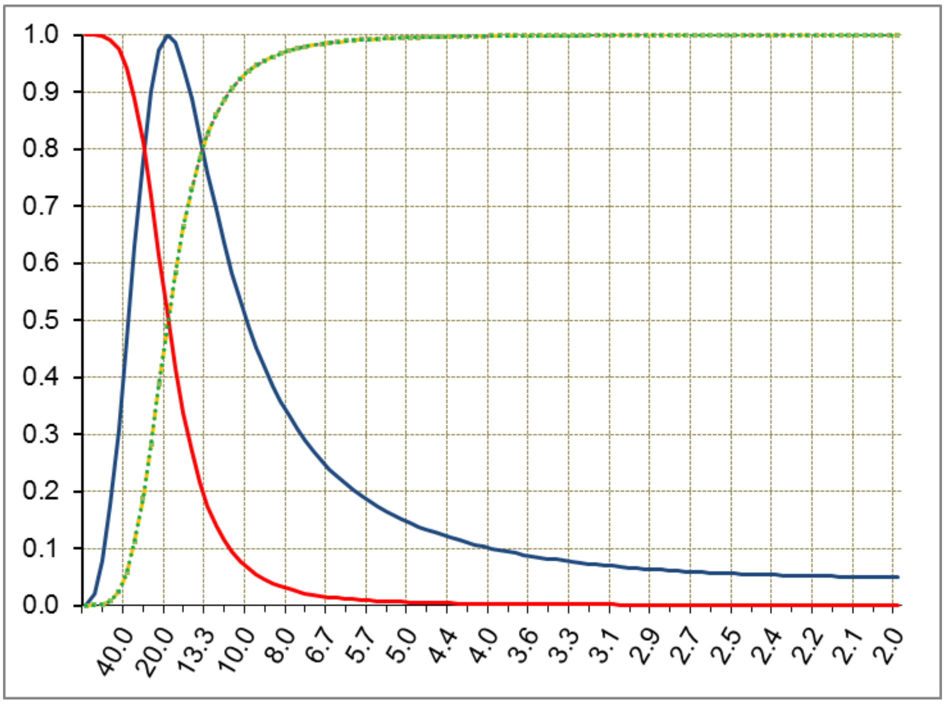
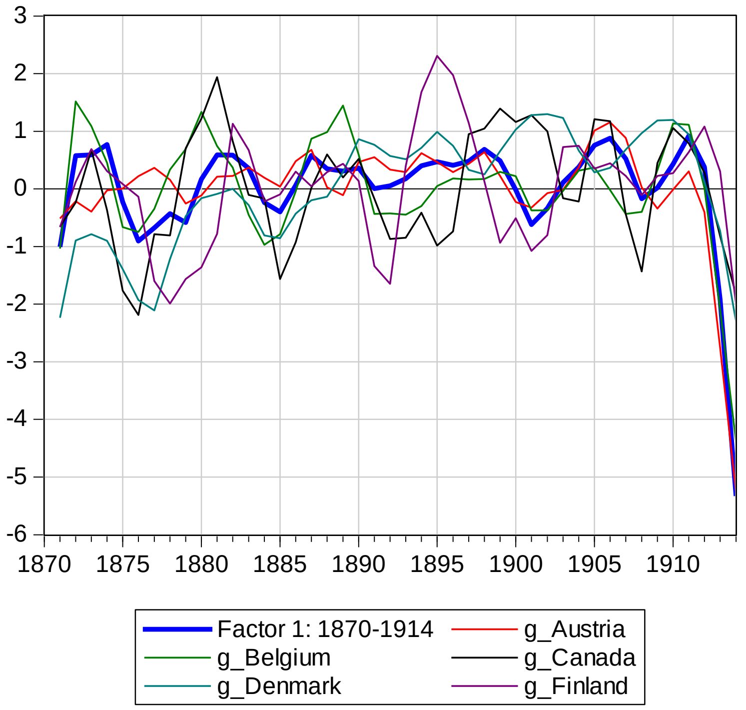


Figure 2. Common factors and underlying growth for the period 1870-1914. Normalized scale.



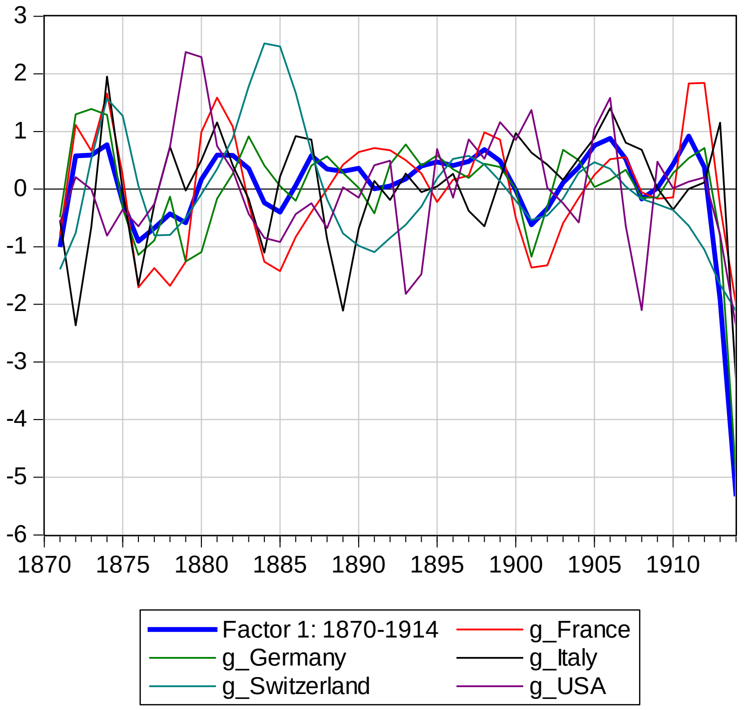
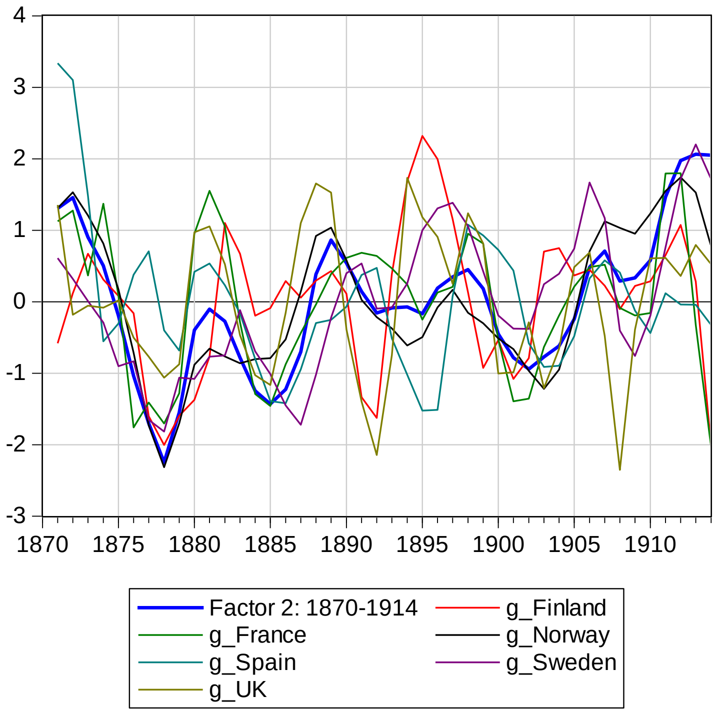


Figure 2 (cont.).



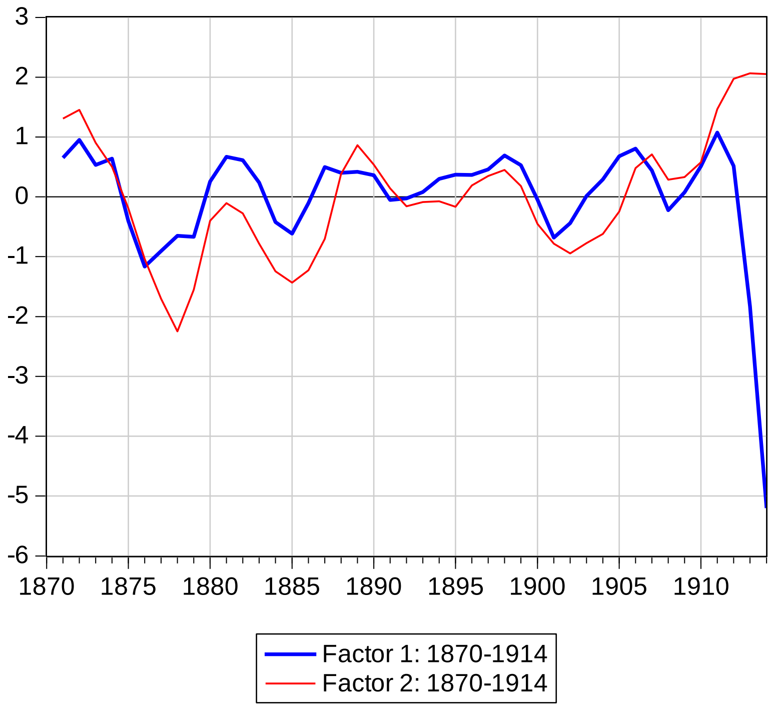
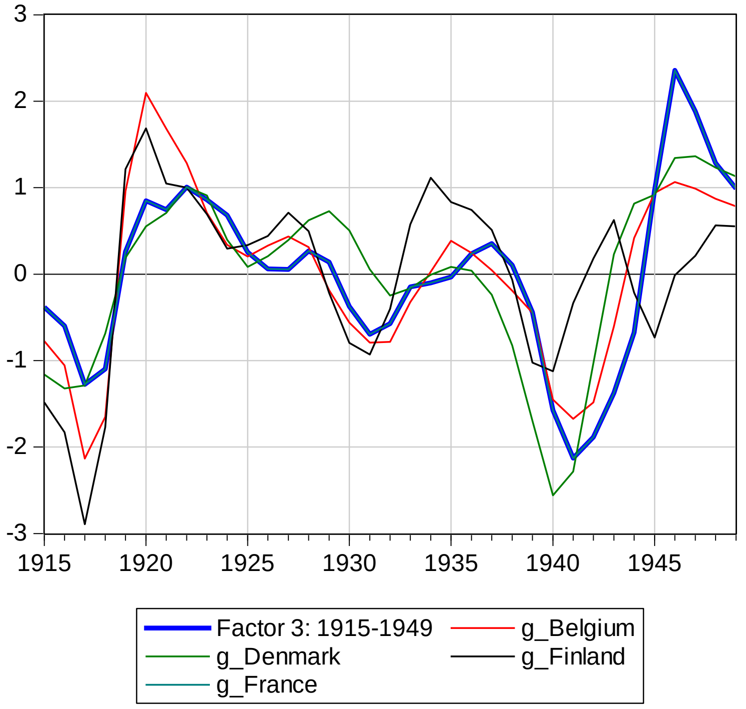
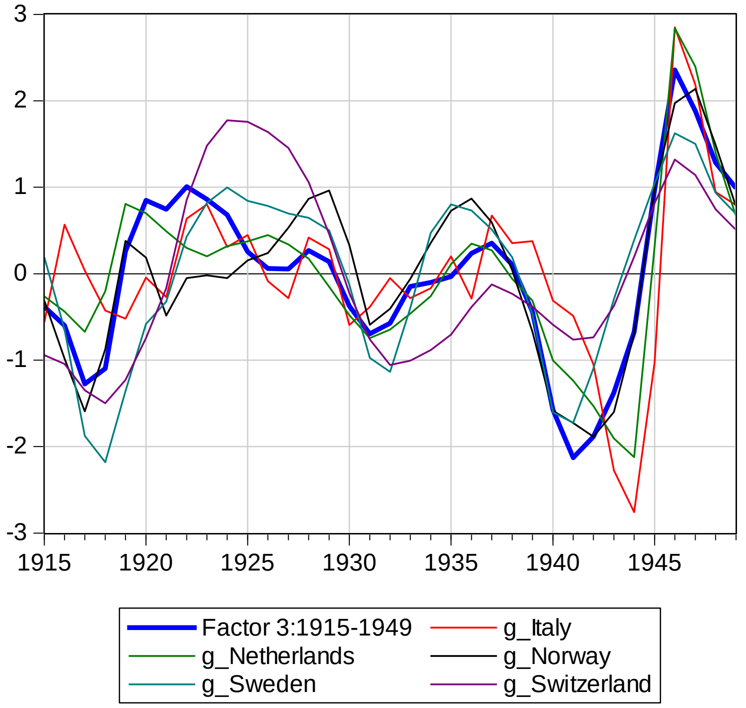


Figure 3. Common factors and underlying growth for the period 1915-1949. Nomalized scale.





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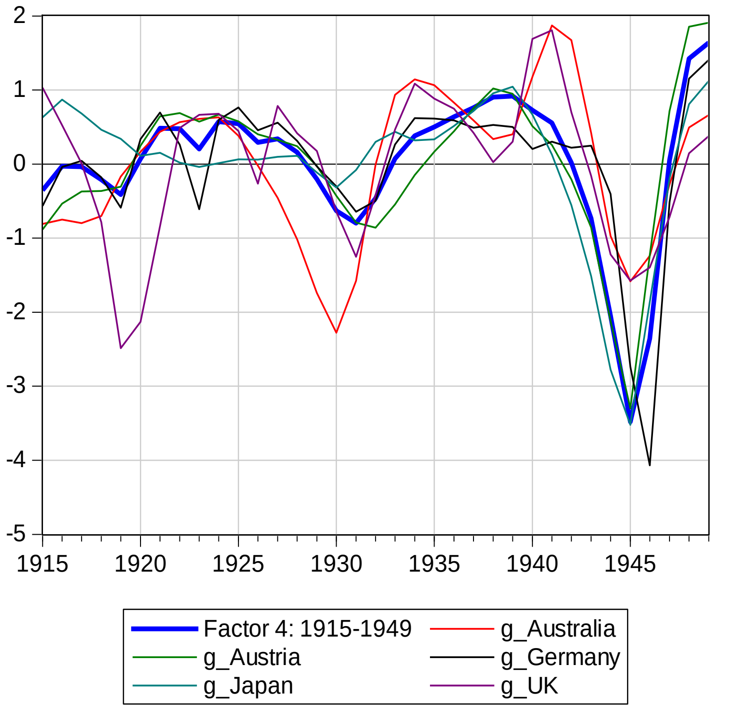


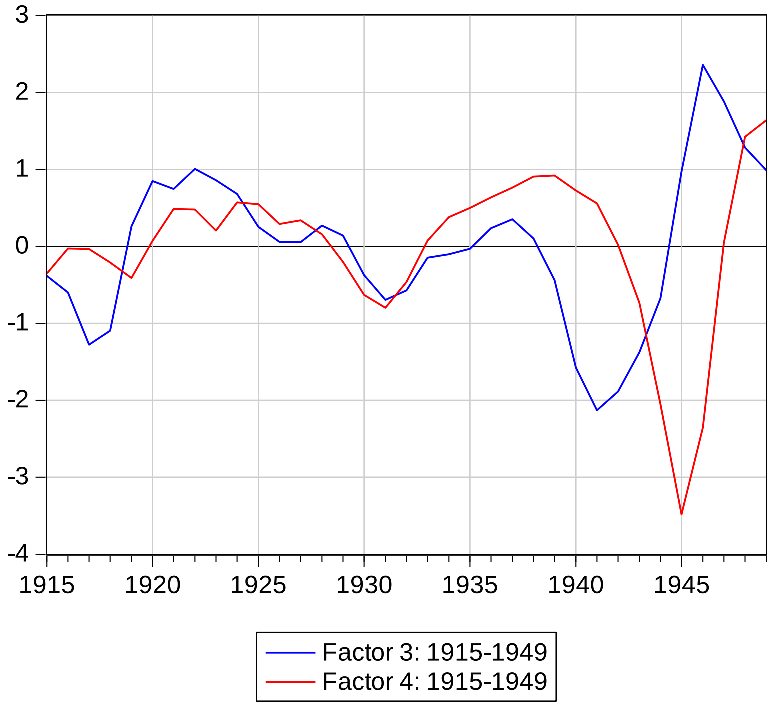
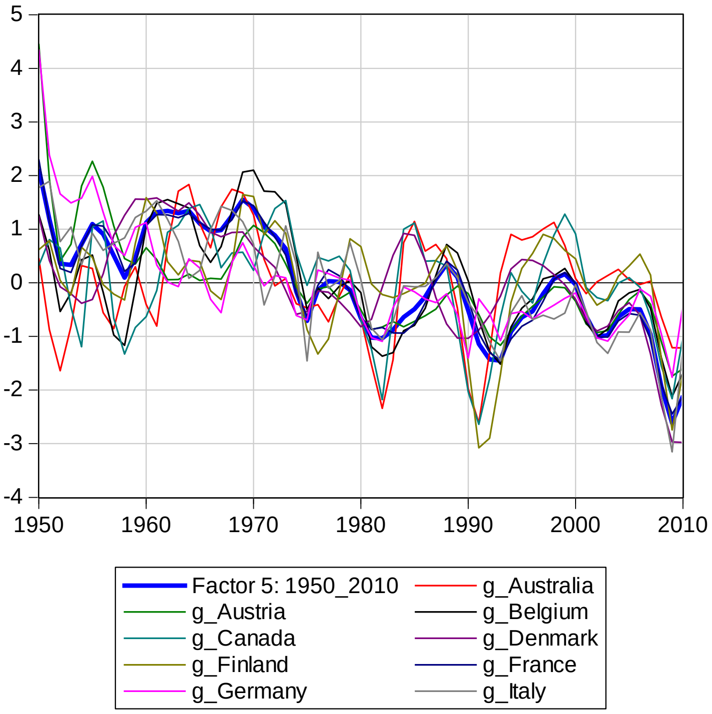
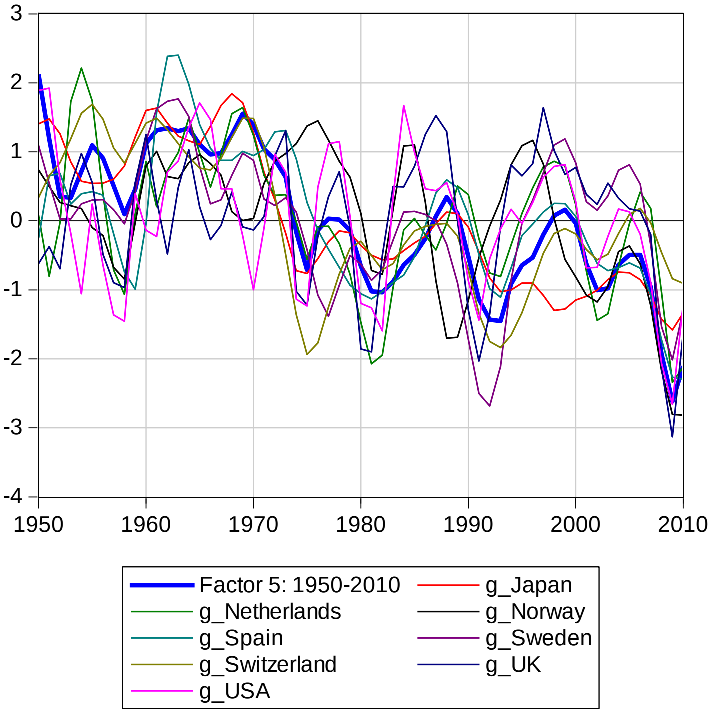


Figure 4. Common factors and underlying growth for the period 1950-2010. Normalized scale.





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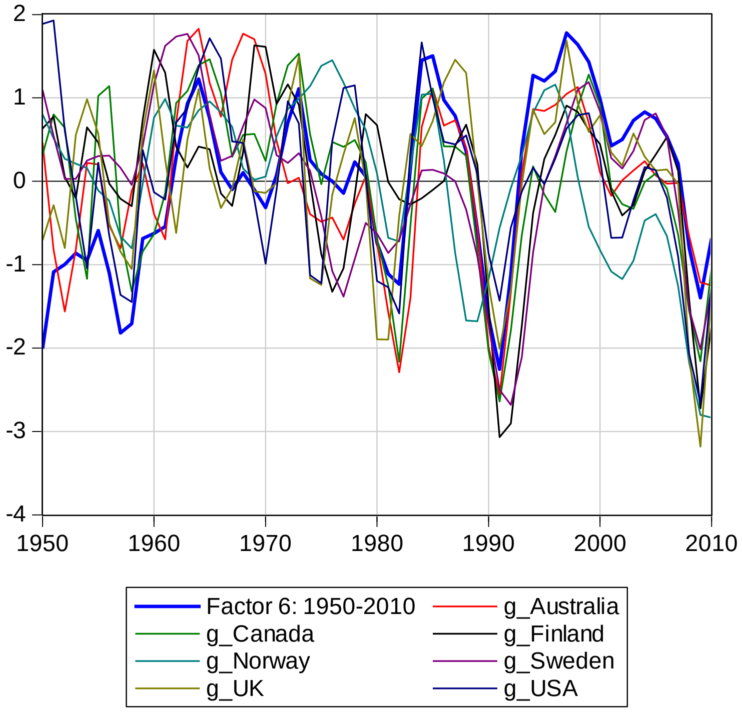


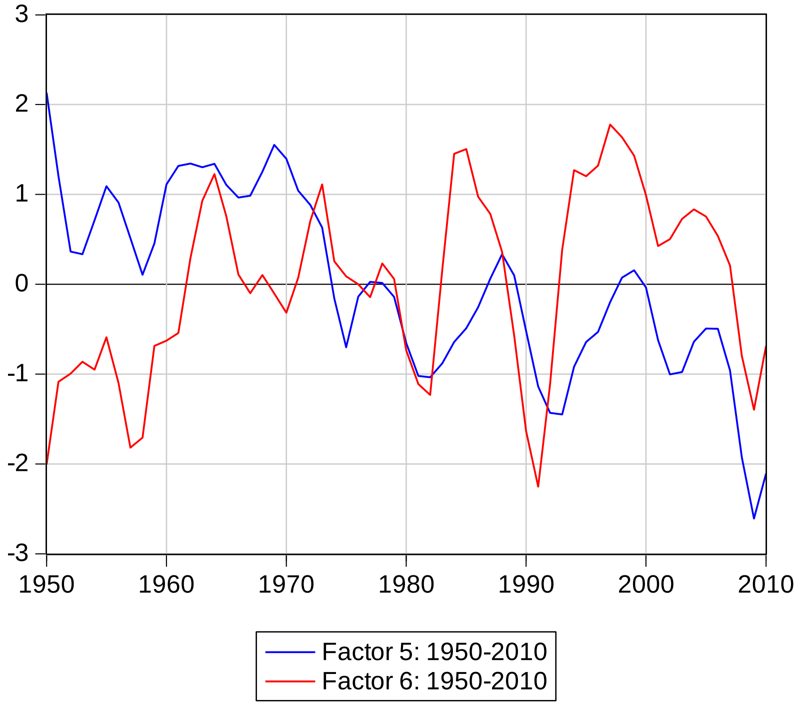
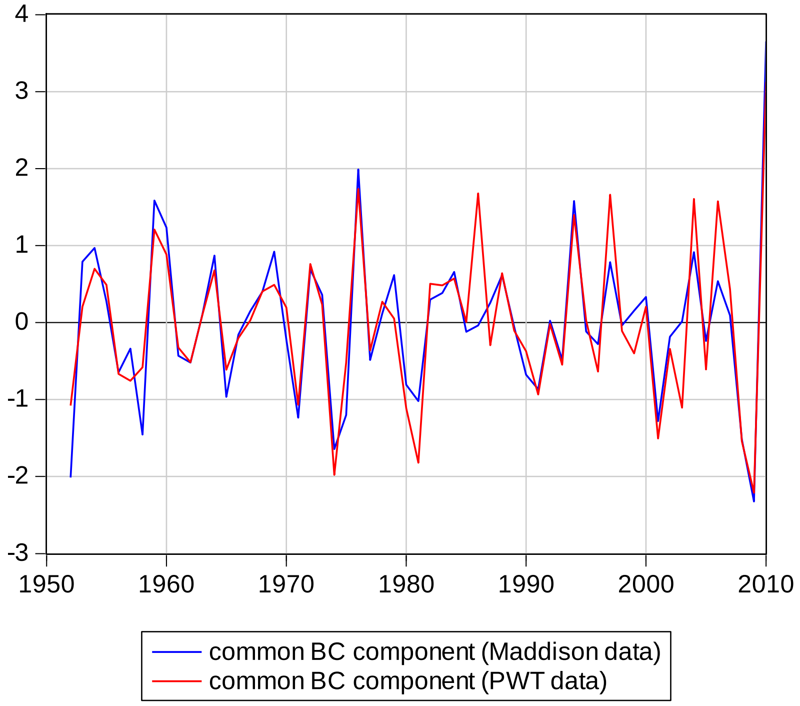


Figure 5. Common business cycle factors ($\hat{a}_t$) from Maddison and PWT data for the period 1950-2010.



Appendix A. State space representation of the univariate and the multivariate models

State space representation consists of two equations. The measurement equation relates the observed variable with the unobserved components and the observation noise. For IRW model (1), basically coincides with equation (1a)

$$\left[y_t\right]=\left[1 \quad 0 \quad 0\right]\begin{bmatrix}\mu_t \\ g_t \\ a_t\end{bmatrix}+\left[\varepsilon_t\right] \tag{A1}$$

The state transition equation represents the dynamics of the unobserved components. According to equations (1b) to (1d), this is

$$\begin{bmatrix}\mu_t \\ g_t \\ a_t\end{bmatrix}=\begin{bmatrix}1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mu_{t-1} \\ g_{t-1} \\ a_{t-1}\end{bmatrix}+\begin{bmatrix}0 \\ 0 \\ \eta_t\end{bmatrix} \tag{A2}$$

Gaussianity and orthogonality assumptions of the error terms imply that $\varepsilon_t \sim N\left(0,\sigma_\varepsilon^2\right)$ and $\xi_t \sim N\left(0_{3\times 1},Q\right)$, where $\xi_t=\begin{bmatrix}0 \\ 0 \\ \eta_t\end{bmatrix}$ and $Q=\begin{bmatrix}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_\eta^2\end{bmatrix}$. By doing $H=\left[1 \quad 0 \quad 0\right]$, $\beta_t=\begin{bmatrix}\mu_t \\ g_t \\ a_t\end{bmatrix}$ and $F=\begin{bmatrix}1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0\end{bmatrix}$, the state space representation of the system (A1)-(A2) in compact form is $\left.\begin{matrix}y_t=H\beta_t+\varepsilon_t \\ \beta_t=F\beta_{t-1}+\xi_t\end{matrix}\right\}$. Estimation of the vector of variances $\left\{\sigma_\varepsilon^2,\sigma_\eta^2\right\}$ is obtained by maximizing the likelihood function of the one-step ahead prediction errors (Harvey, 1989; Durbin and Koopman, 2001).

For the multivariate IRW model (14) and for illustrative purposes, let us assume two time series, $i=1,\ 2$, with specific acceleration components $a_{1,t}$ and $a_{2,t}$, and that the common acceleration component, a_t, follows an autoregressive model such as $a_t=\phi a_{t-1}+\eta_t$. From equation (14a), we have that the measurement equation relating observed variables with unobserved components is

$$\begin{bmatrix}y_{1,t} \\ y_{2,t}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mu_{1,t} \\ g_{1,t} \\ \mu_{2,t} \\ g_{2,t} \\ a_t \\ a_{1,t} \\ a_{2,t}\end{bmatrix}+\begin{bmatrix}\varepsilon_{1,t} \\ \varepsilon_{2,t}\end{bmatrix} \tag{A3}$$

According to equations (14b) to (14e), the state transition equation, is

$$\begin{bmatrix} \mu_{1,t} \\ g_{1,t} \\ \mu_{2,t} \\ g_{2,t} \\ a_t \\ a_{1,t} \\ a_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \gamma_1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \gamma_2 & 0 & 1 \\ 0 & 0 & 0 & 0 & \phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mu_{1,t-1} \\ g_{1,t-1} \\ \mu_{2,t-1} \\ g_{2,t-1} \\ a_{t-1} \\ a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \eta_t \\ \eta_{1,t} \\ \eta_{2,t} \end{bmatrix} \tag{A4}$$

Gaussianity and orthogonality assumptions of the error terms imply that $\varepsilon_t \sim N\left(0_{2x1}, R\right)$ and $\xi_t \sim N\left(0_{7x1}, Q\right)$, where $\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$, $\xi_t = \begin{bmatrix} 0_{4x1} \\ \eta_t \\ \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}$, $R = \begin{bmatrix} \sigma^2_{\varepsilon_1} & 0 \\ 0 & \sigma^2_{\varepsilon_2} \end{bmatrix}$, $Q = \begin{bmatrix} 0_{4x4} & 0_{4x1} & 0_{4x1} & 0_{4x1} \\ 0_{1x4} & 1 & 0 & 0 \\ 0_{1x4} & 0 & \sigma^2_{\eta_1} & 0 \\ 0_{1x4} & 0 & 0 & \sigma^2_{\eta_2} \end{bmatrix}$ and 0_{mxn} is a mxn matrix of ceros. By doing $y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\beta_t = \begin{bmatrix} \mu_{1,t} \\ g_{1,t} \\ \mu_{2,t} \\ g_{2,t} \\ a_t \\ a_{1,t} \\ a_{2,t} \end{bmatrix}$ and

$F = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \gamma_1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \gamma_2 & 0 & 1 \\ 0 & 0 & 0 & 0 & \phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, the state space representation of the system (A3)-(A4) in compact form is

$\left.\begin{aligned} y_t &= H\beta_t + \varepsilon_t \\ \beta_t &= F\beta_{t-1} + \xi_t \end{aligned}\right\}$. Estimation of the vector of parameters $\left\{\sigma^2_{\varepsilon_i}, \sigma^2_{\eta_i}, \gamma_i, \phi\right\}$ is obtained as previously outlined for the univariate model.

Appendix B. Comparison of the estimated components with the original series

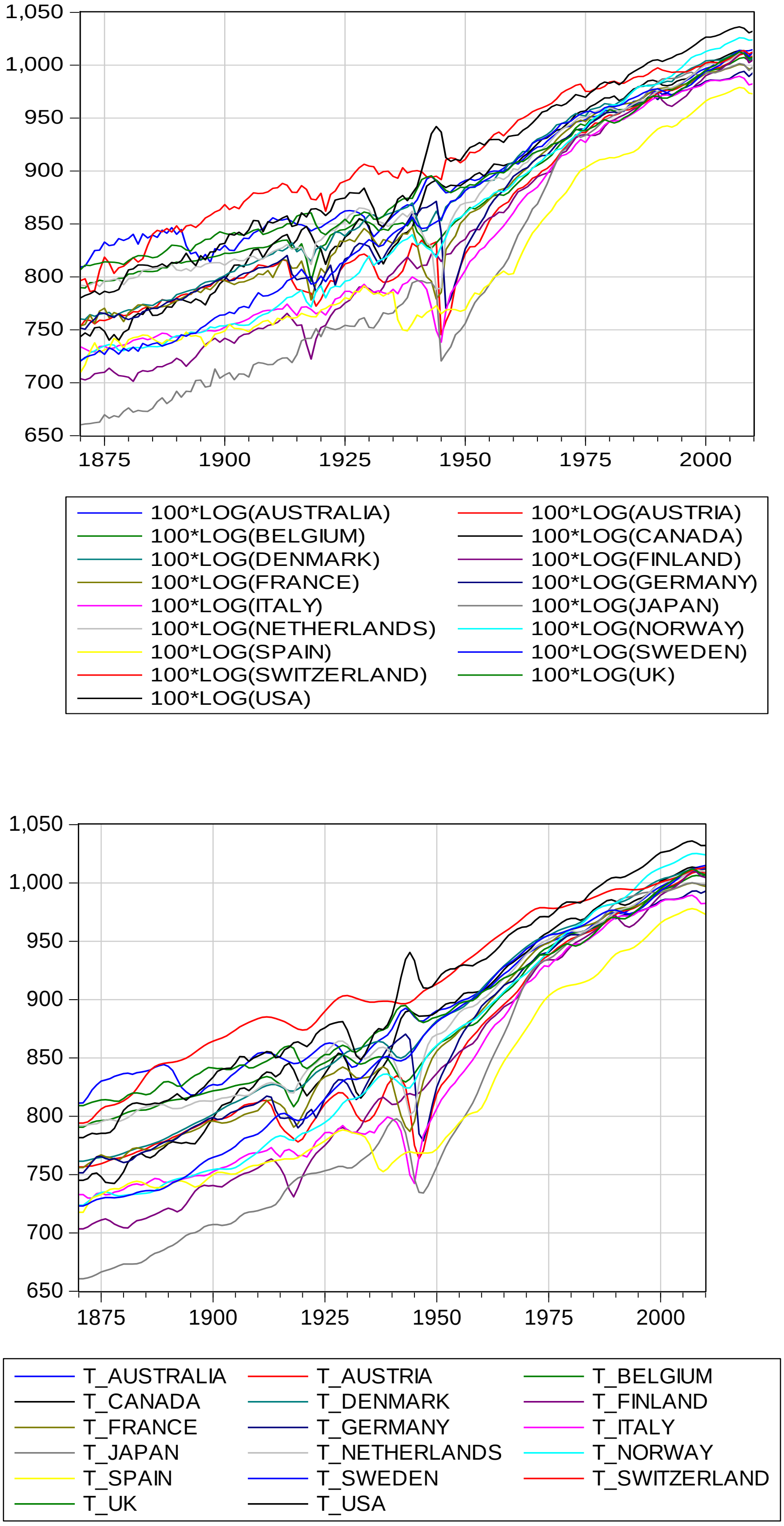


Figure B1. Original series and trends $\hat{\mu}_t$ (*T_country*)

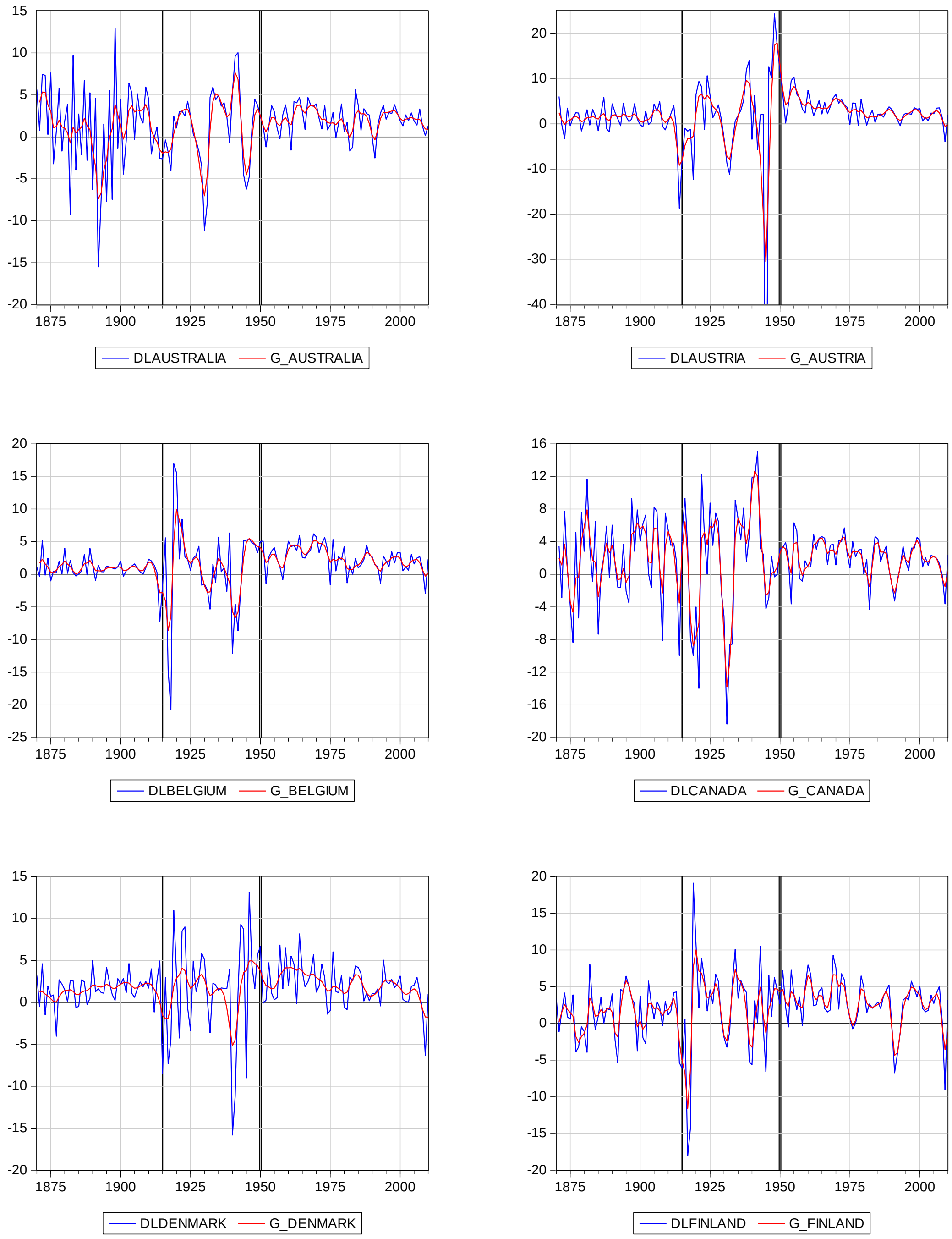


Figure B2. Rates of growth (*DLcountry*) and growth components $\hat{g}_t$ (*G_country*)

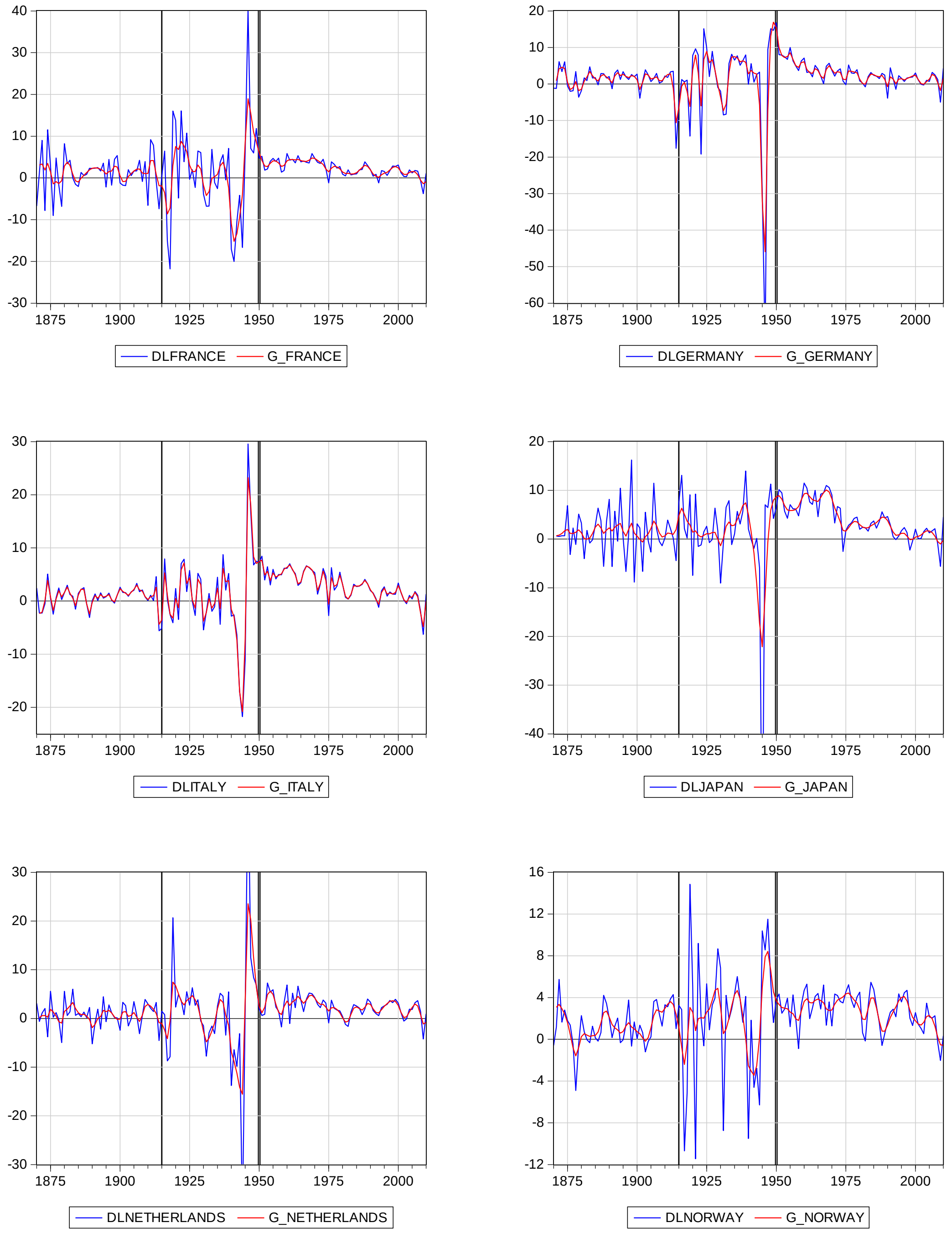


Figure B2 (cont.). Rates of growth (*DLcountry*) and growth components $\hat{g}_t$ (*G_country*)

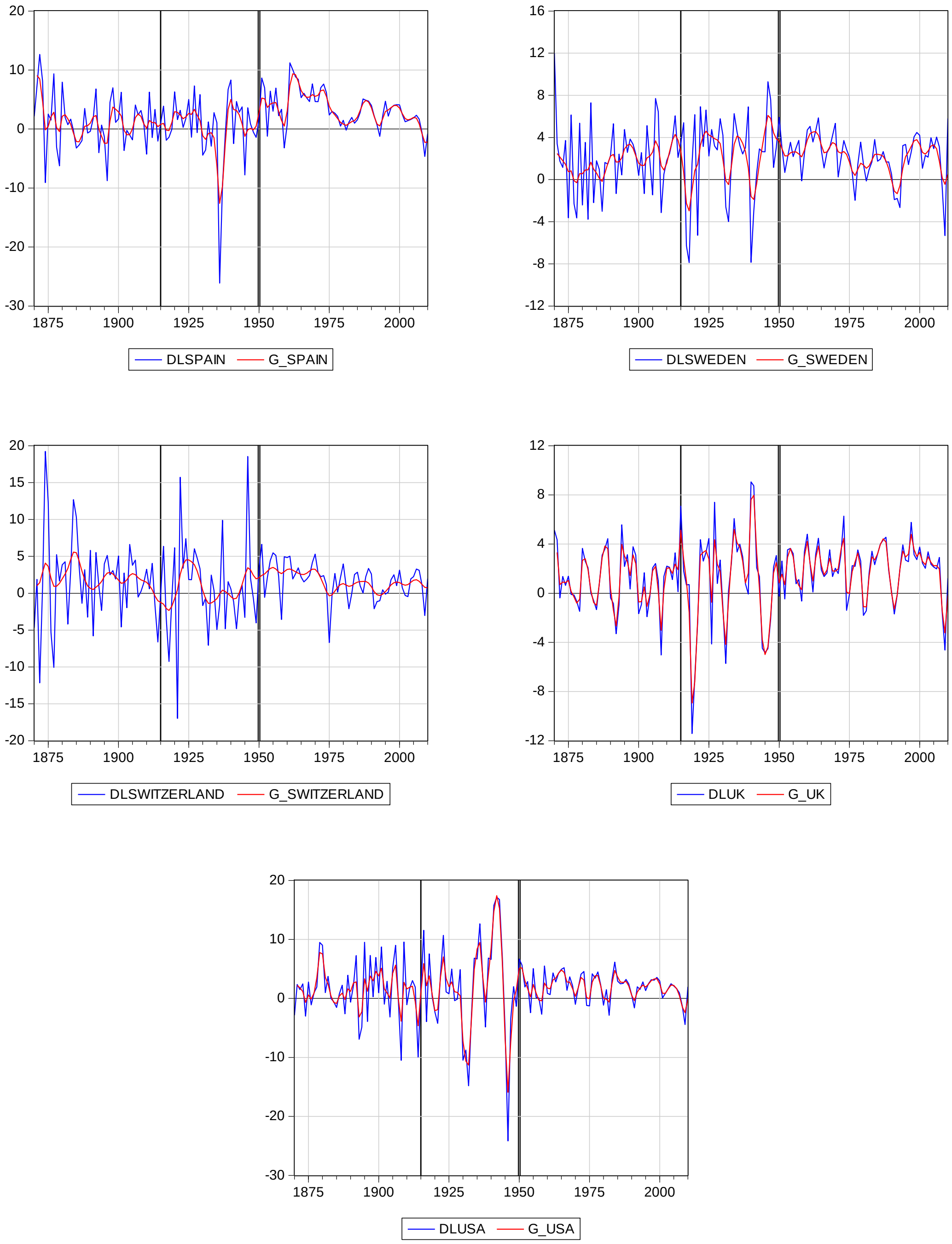


Figure B2 (cont.). Rates of growth (*DLcountry*) and growth components $\hat{g}_t$ (*G_country*)