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Effective Theories of Neutrino Masses

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Abstract

The importance of improving the bounds on those effective non-standard neutrino interactions (NSI) which are a signal of all fermionic-mediated Seesaws is stressed: they are revealed as non-unitarity of the leptonic mixing matrix, and at experimental reach for seesaw scales $\leq \mathcal{O}$(TeV). Some recent activity in the literature on other -theoretically not well motivated- ill-constrained NSI are also summarized. Furthermore, the status of the simplest Seesaw scenario with only two heavy neutrinos is reviewed. This model happens to be a explicit realization of the effective Minimal Flavour Violation approach. We derive the scalar potential for the fields whose background values are the Yukawa couplings of that model, and explore its minima. The Majorana character plays a distinctive role: the minimum of the potential allows for large mixing angles -in contrast to the simplest quark case- and predicts a maximal Majorana phase. This points in turn to a strong correlation between neutrino mass hierarchy and mixing pattern.

Keywords: seesaw, flavour

1. Introduction

The urge to build theories beyond the Standard Model of particle physics (BSM) is mainly fed by the experimental evidence for new **particle** physics: i) neutrino masses; ii) dark matter; iii) the matter-antimatter asymmetry of the universe.

Furthermore, within the Standard Model of particle physics (SM) itself, which is based on the gauge group $SU(3)_{QCD} \times SU(2) \times U(1)$, uncomfortable issues taunt us which take the form of severe fine-tunings of parameters. They illustrate the seemingly paradoxical situation that, while we understand quite reasonably the behaviour of the excitations over the SM vacuum, this is not the case for the vacuum itself, that is, the SM state of lowest energy: in QCD, the non-observation of the θ parameter characterizing its vacuum points to a numerical adjustment of at least ten orders of magnitude; for gravity, the contributions to the cosmological constant from our quantum field theories require an even larger readjustment of what would be its naively estimated value; and finally, the electroweak $SU(2) \times U(1)$sector also points to fine-tunings.

For this last sector, the main fine-tunings take two names: a) the electroweak hierarchy problem: a light (e.g. $\mathcal{O}$(100) GeV) Higgs particle is puzzling in case there is physics BSM to which the Higgs field may couple, with a scale larger than the electroweak one; b) the flavour puzzle. Many BSM theories have been built, with quite satisfactory answers to the electroweak hierarchy problem (supersymmetry at the electroweak scale, technicolor, and its modern variants, etc.), specially as long as fermions are not considered.

Fermions not only worsen quantitatively the electroweak hierarchy issue, but induce the most difficult problem that BSM theories usually face: the flavour puzzle and the smallness of flavour-changing effects in neutral currents. An acceptable BSM theory should improve both a) and b) or at least one of these issues without worsening the other; unfortunately, this is not the case by and large. Little if any progress has been made in the last thirty years or so in understanding the dynamics behind flavour. The important progress has rather been in the acquisition of precious data, essential to the formulation of the puzzle itself. Data as those provided by the B factories in the quark sector, for in-

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stance, and by the experimental searches of charged-lepton flavour-changing signals and by neutrino oscillation experiments. What light has the neutrino sector shed on the flavour puzzle? This talk will keep this question as guideline.

Fermion families mix under charged-current interactions with a startling different pattern for leptons and quarks: large mixings versus small ones, respectively. The scales of the mass spectra also differ by many orders of magnitude, with neutrinos in particular exhibiting tiny masses compared to charged leptons and quarks. Furthermore, the charged fermion spectrum is hierarchical, while it remains to be determined whether the neutrino spectrum is hierarchical (normal or inverted) or degenerate. It is plausible that the different mixing patterns may be related to the fact that the neutrino sector may include Majorana mass terms, but this cannot be determined in general.

It may (or may not) be an important aspect of the puzzle the fact that neutrinos are optimal windows into the dark sectors of the universe. Indeed, neutrinos may mix with any exotic SM singlet fermion, heavy or light; and besides their interactions are not obscured by strong and electromagnetic ones. In fact, it is easy to establish the "dark portals" of the SM, by looking into the possible singlet combinations of SM fields with mass dimension (d) ≤ 4, so that they could be completed with singlet fields up to form a $d = 4$ term in a Lagrangian. It turns out that only two such combinations of SM fields can be written: $\phi^\dagger\phi$ and $\bar{\ell}_L\phi$, where ϕ and ℓ_L denote the SM Higgs doublet and lepton doublet, respectively. These are the SM portals into dark sectors such as dark matter. The combination $\phi^\dagger\phi$ is the *scalar portal*: it has mass dimension two and allow thus for completions with a singlet scalar field S of the general form $\phi^\dagger\phi S$ and $\phi^\dagger\phi|S|^2$, within a renormalizable $d = 4$ Lagrangian. The singlet combination $\bar{\ell}_L\phi$ is instead the *fermionic portal* into the dark sector: it allows a completion with a singlet fermion Ψ, of the form $\bar{\ell}_L\phi\Psi$, which is precisely reminiscent of the Yukawa couplings to singlet neutrinos in seesaw constructions. In addition, it may well happen that dark matter includes more fermionic components than the known neutrinos and even of different varieties (dark matter flavours); would this be the case, the deep understanding of the observed pattern of flavours may need the identification of those exotic components before it can be properly formulated and answered.

2. Exotic neutrino couplings

Neutrino oscillations have established the existence of lepton flavour violation in nature. Their interpretation as neutrino masses points most plausibly towards a new (Majorana) physics scale. As new physics has thus been signaled by neutrino masses, a natural question is whether new physics may also surface in exotic neutrino couplings. One popular variety are the so-called *Non-standard neutrino interactions* (NSI): four fermion effective couplings which do not conserve flavour and involve two neutrinos, e.g. $\bar{q}q\nu_\alpha\nu_\beta$ or $\bar{e}e\nu_\alpha\nu_\beta$ and analogous couplings.

A very interesting class of NSI are those predicted by fermionic-mediated seesaw models. These signals are revealed as a breakdown of unitarity of the leptonic U_{PMNS} matrix appearing in charged current W-neutrino-charged lepton couplings, a fact which is easily understood: if there are more than three neutral fermions in nature, all of them may mix and, while the total leptonic mixing matrix is certainly unitary, the 3×3 subset involving the fields observed up to now is not unitary. Integrating out the W field, this non-unitary mixing leads, at energies under the mass of the W boson, to 4-fermion couplings which do not preserve flavour in general, that is, to NSI couplings. Neutrino production, propagation and detection are all modified by the effect, and the bounds resulting from present "zero-distance" measurements (rare decays, modification on the Fermi constant G_F, universality tests, etc.) are very strong [1, 2] and superseed those from neutrino oscillations. Denoting by $\epsilon_{\alpha\beta}$ the strength of the modification of the weak coupling, $\epsilon_{\alpha\beta}\, G_F\, \bar{l}l\nu_\alpha\nu_\beta$, where l denotes charged leptons, ν neutrinos and greek indices indicate flavour, it turns out that present bounds are very stringent with $\epsilon_{\alpha\beta} < 10^{-2}$. It is very important to keep monitoring the unitarity of the leptonic mixing matrix, as an optimal way to detect the presence of fermions in nature which are singlets of the SM. The planned experiments on $\mu \to e\gamma$ and $\mu \to eee$ decays and specially those on $\mu - e$ conversion in nuclei, will improve the sensitivity by many orders of magnitudes in the $e - \mu$ sector. For the $\mu-\mu$ and $\mu-\tau$ sectors, future neutrino oscillation facilities such as the neutrino factory [3] could also reach $\epsilon_{\alpha\beta}$ sensitivies up to $O(10^{-3})$ in the $\nu_e - \nu_\tau$ and $\nu_\mu - \nu_\tau$ channels or even on the diagonal $\nu_\mu - \nu_\mu$ channel. Proposals based on beta-beams [4] for the $\nu_e - \nu_\tau$ channel or near detectors [24] in the $\nu_\mu - \nu_\tau$ channel can also be relevant.

Another class of NSI which continues to prompt active research is that of "why not" NSI, meaning by it exotic couplings which would not affect simultaneously neutrino production, propagation and detection, and for which no plausible theoretical motivation has been found. Among these, the couplings which would only affect neutrino propagation in matter are more loosely bound and attract attention. In practice the analysis reduces then to explore the point-like coupling $\bar{f} f \nu_\alpha \nu_\beta$, where f may be an electron, and up quark or a down quark. Being the couplings diagonal in f, they are not limited by the strong "zero-distance" bounds mentioned above. In my opinion, only those works which assume an $SU(2)\times U(1)$ gauge invariant formulation are worth considering, as the BSM theory **must** respect and enlarge the established symmetries of nature. When this is implemented, new constraints and severe limitations tend to appear unless extreme fine-tunings are accepted; in short, no reasonable BSM theory has been found up to now able to allow for ϵ values close to the (loose) present experimental bounds claimed for this matter effects. The actual values for the latter can be found in the analysis in Ref. [6], with allowed ϵ values $< 10^{-1}$ in the channels with $\alpha \neq \beta$. Stronger bounds can be argued for the $\mu-\tau$ sector from atmospheric data [7], implying $\epsilon_{\mu\tau} < 10^{-2}$.

A recent interesting development is that in Ref. [8], which explores the monojet and monophoton data obtained at LHC. Last year, the same data had allowed to deduce gorgeous bounds on DM couplings. The bounds obtained for neutrino NSI from the first fb^{-1} of LHC data are already comparable and in some channels better than the previous ones mentioned above [6] for matter-propagation NSI. Nevertheless, these type of LHC analyses are systematically limited and no important improvement in binding/detecting neutrino NSI is expected from LHC data in the future.

Considering on the other hand "why not" NSI affecting only neutrino production and/or detection, there has been some recent work on the bounds resulting from reactor data. The trading of NSI for θ_{13} in reactor data had been explored previously in Refs. [9, 10], while a recent work [11] concludes that, with three year data, Daya Bay will be sensitive to different NSI in production and detection.

3. Effective couplings with Minimal Flavour Violation

The Minimal Flavour Violation (MFV) ansatz [12, 13, 14, 15, 16, 17] is restrictive enough to allow to explore simultaneously the flavour puzzle for both quarks and leptons, including their widely different mixing pattern.

We had previously developed the simplest MFV See-Saw model in Ref. [18], that incorporates only two right-handed (RH) neutrinos (see also Ref. [19], where scenarios with three RH neutrinos where presented, that reduce in a certain limit to the two light neutrino case). This is a model which automatically falls in the MFV category, in which each Yukawa coupling can be associated to just one spurion, whose vacuum expectation value (vev) would correspond to the physical masses and mixings.

The Lagrangian for this variety of type I seesaw models with just two heavy singlet neutrinos N and N' reads

$$\begin{aligned} -\mathcal{L}_{mass} &= \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi}\,(YN + Y'N') \\ &+ \Lambda \overline{N'} N^c + h.c. \end{aligned} \tag{1}$$

where E_R denotes the RH charged lepton fields (flavour indices are implicit), $\tilde{\phi} = i\tau_2\phi^*$ and $\langle\phi\rangle \equiv \mathrm{v}/\sqrt{2}$ is the electroweak vev, with v= 246 GeV. With only two heavy neutrinos, one light neutrino remains massless - an open possibility to this date- and there is only one physical Majorana phase. Λ is a Majorana scale, while the Yukawa couplings for charged leptons Y_E, and for neutrinos Y and Y', are a matrix and vectors in flavour space, respectively. The Lepton Number (LN) symmetry is violated by the simultaneous presence of Y, Y' and Λ. The light neutrino matrix reflects these properties and takes the typical form in type I See-Saw models:

$$\bar{\nu}_L \, \frac{v^2}{2\Lambda} \left(Y\, Y'^T + Y'\, Y^T \right) \nu_L^c + h.c., \tag{2}$$

where $\Lambda \gg$ v has been assumed. Flavour changing effects may then occur even in the limit of LN conservation, for vanishing Y or Y' (but not both), and they may be observable for Λ values not much larger than the TeV scale [18, 24, 25, 26, 27]. We work in the basis in which the charged lepton Yukawa matrix is diagonal. Denoting by Y_ν the matrix constructed out of the two vectors Y and Y', $Y_\nu \equiv (Y, Y')$, the Yukawa couplings may be described by [18]

$$Y_E = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad Y_\nu = U\, f_{m_\nu} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}, \tag{3}$$

where $y \equiv \sqrt{(Y)^\dagger Y}$ and $y' \equiv \sqrt{(Y')^\dagger Y'}$. Here U denotes the PMNS mixing matrix: using the PDG notation, U is written as the product of three rotations and a matrix containing the Majorana phases, $U = R_{23}(\theta_{23})\, R_{13}(\theta_{13}, \delta)\, R_{12}(\theta_{12})\, \Omega$. Ω can be parametrised as $\Omega = \mathrm{diag}\{1, e^{i\alpha}, e^{-i\alpha}\}$ for NH and $\Omega = \mathrm{diag}\{e^{i\alpha}, e^{-i\alpha}, 1\}$ for IH. In the convention that will be used throughout the Letter in which $\Delta_{ij} \equiv m_j^2 - m_i^2 > 0$, all angles $\theta_{ij} \in [0, \pi/2]$, the Dirac CP phase $\delta \in [0, 2\pi)$ and the Majorana phase $\alpha \in [0, \pi]$. The term f_{m_ν} is a matrix function of neutrino masses: for the normal (NH) and inverted (IH) hierarchies it is defined as

$$\text{NH:} \qquad f_{m_\nu} = \frac{1}{\sqrt{m_{\nu_2}+m_{\nu_3}}}\begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix}, \quad (4)$$

with $(m_{\nu_2})^2 = \Delta m^2_{sol}$, $(m_{\nu_3})^2 = \Delta m^2_{atm} + \Delta m^2_{sol}$. and

$$\text{IH:} \qquad f_{m_\nu} = \frac{1}{\sqrt{m_{\nu_1}+m_{\nu_2}}}\begin{pmatrix} \sqrt{m_{\nu_1}} & 0 \\ 0 & \sqrt{m_{\nu_2}} \\ 0 & 0 \end{pmatrix}, \quad (5)$$

with $(m_{\nu_1})^2 = \Delta m^2_{atm} - \Delta m^2_{sol}$, $(m_{\nu_2})^2 = \Delta m^2_{atm}$. The Lagrangian in Eq. (1) can be rewritten in the basis in which the heavy singlet neutrino mass matrix is diagonal, leading to :

$$-\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T \quad (6)$$
$$+ \Lambda(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c., \quad (7)$$

where the neutrino Yukawa coupling $\tilde{Y}_\nu$ reads

$$\tilde{Y}_\nu = \frac{1}{\sqrt{2}} U f_{m_\nu} \begin{pmatrix} y+y' & -i(y-y') \\ i(y-y') & y+y' \end{pmatrix}. \quad (8)$$

3.0.1. Dynamical Yukawas

We will explore below the possible scalar potential for the lepton sector Yukawas, when they are promoted to dynamical fields in the model under discussion, and compare it with the analogous analysis for the quark sector. This work has been recently presented in Ref. [28].

In the limit of vanishing Yukawa couplings, $Y_E = \tilde{Y}_\nu = 0$, the Lagrangian in Eq. (7) presents an extended global non-Abelian symmetry (plus Abelian global symmetries which are not made explicit below),

$$\mathcal{G}_{fl} \sim SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N, \quad (9)$$

with $O(2)_N$ corresponding to orthogonal transformations between N_1 and N_2 (in the flavour basis, the Majorana mass term in Eq. (1) exhibits a $U(1)_N \times Z_2$ invariance in the (N, N') sector, isomorphic to $O(2)_N$). Notice that when $\mathcal{G}_{fl}$ is exact, LN is also a symmetry of the Lagrangian.

MFV suggests a dynamical origin for the Yukawa couplings of the theory. For instance, they may result from scalar fields, usually called flavons, taking a vev. In the simplest realisation of MFV, each Yukawa coupling is associated to a single scalar field, singlet under the SM gauge group and transforming under $\mathcal{G}_{fl}$ as:

$$\mathcal{Y}_E \sim (3, \bar{3}, 1), \qquad \mathcal{Y}_\nu \sim (3, 1, 2). \quad (10)$$

The charged-lepton flavon fields $\mathcal{Y}_E$ belong then to the bi-fundamental representation of $SU(3)_{\ell_L} \times SU(3)_{E_R} \subset \mathcal{G}_{fl}$, while the neutrino flavon $\mathcal{Y}_\nu$ belongs to the fundamental representation of $SU(3)_{\ell_L}$.

The relation between the Yukawa couplings and the flavon vevs may be linear,

$$\frac{\langle \mathcal{Y}_E \rangle}{\Lambda_{fl}} \equiv Y_E, \quad \frac{\langle \mathcal{Y}_\nu \rangle}{\Lambda_{fl}} \equiv \tilde{Y}_\nu, \quad (11)$$

where Λ_{fl} represents the scale of the flavour dynamics, and we will stick here to this simplest assumption (in fact the results are more general, as only the transformation properties under the flavour group are relevant). With this assignment, the Yukawa terms in Eq. (7) have dimension 5; they are the leading non-trivial flavour invariant operators.

The scalar potential for the $\mathcal{Y}_E$ and $\mathcal{Y}_\nu$ fields must be invariant under the SM gauge symmetry and the flavour symmetry $\mathcal{G}_{fl}$. At the renormalisable level, the possible linearly-independent invariant terms reduce to:

$$\text{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right), \quad \text{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right), \quad \det(\mathcal{Y}_E), \quad (12)$$

$$\text{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2, \quad \text{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right), \quad (13)$$

$$\text{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2, \quad \text{Tr}\left(\mathcal{Y}_\nu \sigma_2 \mathcal{Y}_\nu^\dagger\right)^2, \quad (14)$$

from which the most general scalar potential V for the flavon fields can be constructed. Details can be found in Ref. [28]. The question now is whether the minimum of the potential can accommodate the physical values of masses and mixings and whether its coefficients need to be fine-tuned. For charged leptons, the choice of $\mathcal{Y}_E$ in the bifundamental mirrors that for the simplest quark case [29, 30, 31, 32]. It is worth noting that the determinant $\det(\mathcal{Y}_E)$, whose impact is to push towards degenerate charged lepton masses, is absent when the $\mathcal{G}_{fl}$ symmetry is enlarged to include the invariance under global phases, $\mathcal{G}_{fl} = U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$, present in the limit of vanishing Yukawa couplings. Furthermore, with this enlarged symmetry, the first non-renormalisable contributions to the scalar potential arise only at mass dimension 6, and would be suppressed by Λ^2_{fl}.

For the $\mathcal{Y}_\nu$ fields, an interesting issue is whether the potential minimum allows for the hierarchy $y \gg y'$ or $y' \gg y$, or in other words the approximate LN invariant case. The answer is positive, see Ref. [28].

In the spirit of an effective field theory approach, the analysis presented here consistently considers all possible operators up to mass dimension 5, invariant under the SM gauge symmetry and the flavour symmetry $\mathcal{G}_{fl}$. The first relevant contributions to the Yukawa lagrangian appear at dimension 5, while the scalar potential is non-trivial already at the renormalizable level, dim ≤ 4. The next to leading contributions appear in both sectors at mass dimension 6 and have a negligible impact on the determination of the mixing angles and the Majorana phase, that will be discussed in the following section. These contributions will thus not be further considered.

3.0.2. Mixing angles and Majorana Phase

Since mixing arises from the misalignment in flavour space of the charged lepton and the neutrino flavons, the only invariant relevant for mixing, at the renormalisable level, is

$$\mathcal{O}_{\rm mix} \equiv {\rm Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right) . \quad (15)$$

Substituting the expressions for the flavon vevs in Eq. (11), it follows that:

$$\mathcal{O}_{\rm mix} = \frac{2\Lambda_{fl}^4}{v^2 \sum m_{\nu_i}}[(y^2+y'^2)\sum_{l,i}|U^{li}|^2 m_l^2 m_{\nu_i} \quad (16)$$
$$+ \quad (y^2-y'^2)\left(i\, e^{2i\alpha}\sum_{l,i<j}(U^{li})^* U^{lj} m_l^2 \sqrt{m_{\nu_i} m_{\nu_j}} + c.c.\right)] .$$

This expression can be compared with the equivalent one for quarks in the bifundamental of the flavour group [32], given by

$${\rm Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) = \frac{4\Lambda_{fl}^4}{v^4}\sum_{i,j}|U_{CKM}^{ij}|^2 m_{u_i}^2 m_{d_j}^2 , \quad (17)$$

in an obvious notation. The first term in Eq. (16) for leptons corresponds to that for quarks in Eq. (17): the only difference is the linear -instead of quadratic- dependence on neutrino masses, as befits the See-Saw realisation. The second term in Eq. (16) has a strong impact on the localisation of the minimum of the potential and is responsible for the different results in the quark and lepton sectors: it contains the Majorana phase α and therefore connects the Majorana nature of neutrinos to their mixing.

Two lepton families

In the illustrative two generation case, in which the flavour symmetry is $\mathcal{G}_{fl} = SU(2)_\ell \times SU(2)_{E_R} \times O(2)$ (plus Abelian symmetries), the mixing term in the potential Eq. (16) can be written as follows:

$$g\mathcal{O}_{\rm mix} \propto g\Big\{(m_e^2+m_\mu^2)(y^2+y'^2)(m_{\nu_2}+m_{\nu_1}) +$$
$$+ \quad (m_\mu^2 - m_e^2)\Big[(m_{\nu_2}-m_{\nu_1})(y^2+y'^2)\cos 2\theta +$$
$$+ \quad (y^2-y'^2)2\sqrt{m_{\nu_2}m_{\nu_1}}\sin 2\alpha \sin 2\theta\Big]\Big\} , \quad (18)$$

where θ is the mixing angle and α the Majorana phase. This formula shows explicitly the relations expected on physical grounds, between the mass spectrum and non-trivial mixing: i) the dependence on the mixing angle disappears in the limit of degenerate charged lepton masses; ii) it also vanishes for degenerate neutrino masses if and only if $\sin 2\alpha = 0$; iii) on the contrary, for $\sin 2\alpha \neq 0$ the dependence on the mixing angle remains, as it is physical even for degenerate neutrino masses; iv) the α dependence vanishes when one of the two neutrino masses vanishes or in the absence of mixing, as α becomes then unphysical.

The minimisation with respect to the Majorana phase and the mixing angle leads to the constraints:

$$(y^2-y'^2)\sqrt{m_{\nu_2}m_{\nu_1}}\sin 2\theta \cos 2\alpha = 0 ,$$
$${\rm tg}2\theta = \sin 2\alpha \frac{y^2-y'^2}{y^2+y'^2}\frac{2\sqrt{m_{\nu_2}m_{\nu_1}}}{m_{\nu_2}-m_{\nu_1}} . \quad (19)$$

The convention used implies that $\sin\theta_{ij} > 0$ and $\cos\theta_{ij} > 0$. The first condition predicts then that the *Majorana phase is maximal,* $\alpha = \{\pi/4, 3\pi/4\}$, *for non-trivial mixing angle*. The relative Majorana phase between the two neutrinos is therefore $2\alpha = \pm\pi/2$ which implies no CP violation due to Majorana phases. On the other hand, Eq. (22) establishes a link between the mixing strength and the type of spectrum, which indicates *a maximal angle for degenerate neutrino masses, and a small angle for strong mass hierarchy.* The sign of $\cos 2\theta$ is selected by the absolute minimum of the potential, which depends on the sign of g, see Eq. (18): $\cos 2\theta$ negative (positive) for $g > 0$ ($g < 0$). The sign of the last term in Eq. (18) in turn determines the sign of $\sin 2\alpha$ for given y, y'.

As an illustrative exercise within the two-family scenario, let us consider the “solar” (1, 2) sector: can the observed value of θ_{12} be accounted for in this framework? The answer is affirmative: $\cos 2\theta_{12}^{exp} > 0$ and $\sin 2\theta_{12}^{exp} > 0$ and the scalar potential can accommodate these facts at its minimum for $g < 0$; if $y > y'$ ($y < y'$) then $\tan 2\theta_{12} > 0$ for $\alpha = \pi/4$ ($\alpha = 3\pi/4$), see Eq. (19). This simple line of reasoning has the advantage that it extends straightforwardly to the three lepton generation case, which in this model adds a massless neutrino and thus a vanishing third $\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger$ eigenvalue.

Three lepton families

For three generations, the model under discussion deals with two light massive neutrinos, a massless one and only one Majorana phase. For the angle and Majorana phase spanned by the two light massive eigenstates, Eqs. (19) apply. Nevertheless, the other two mixing angles are forced to vanish by construction, and this minimal version of the model is not viable. This failure is intimately related to the fact that in the model discussed above one of the light neutrinos is massless. For more precisions, please see Ref. [28].

4. Conclusions

Would the LHC data continue not to show new particles other than the Higgs-like resonance at a mass ~ 125 GeV, in the present and new future, a profound impact would follow on our ideas about the electroweak hierarchy problem. Either it was an inapproppiate BSM guideline to worry about the lightness of the Higgs mass, or simply the new physics should lie around the TeV (as for instance happens in models with a strong dynamics for the Higgs even if it is light). Seesaw models of neutrino masses introduce a large Majorana mass, which will contribute to the Higgs mass through loop corrections, and thus feed the electroweak hierarchy problem unless the Majorana scale is not much larger than the TeV scale. Viable seesaw models with such low scales are those in which LN is separated from the effective scale of lepton flavour physics; these include a large class of models in the literature, such as "inverse seesaw" models and other constructions.

For fermionic mediated seesaw scenarios (such as type I, type II and their generalizations),the constructions with seesaw scales near the TeV may induce sizeable departures of unitarity in the leptonic mixing matrix and thus be at discovery reach in the present and planned ensemble of experiments. This underlines the relevance of pushing further those limits and, in general, of all experiments exploring lepton flavour changing process, with a singularly promising panorama for $\mu - e$ conversion in nuclei experiments, besides neutrino oscillation ones.

With respect to the outstanding theoretical question of whether the strong mixing pattern found in the leptonic sector - in contrast to the small mixing and hierarchical spectrum of the quark sector - is related to the possible Majorana character of the neutrino fields, we have explored the possibility of a dynamical origin of Yukawa couplings, promoting them to scalar fields that take a vev. The implementation of the Majorana character needs to refer to an explicit model of Majorana neutrino masses. The potential can then be determined in general, under the only requirement of SM gauge invariance and invariance under the underlying flavour symmetry. Much as it was the case for the analogous analysis for quarks [29, 30, 32], the latter turns out to be strongly restrictive. Our woek contains the first analysis of this type when Majorana neutrinos are present.

We concentrated here in one of the simplest possible MFV models of Majorana neutrino masses: a See-Saw model with only two extra heavy singlet neutrinos with approximate lepton number symmetry[18]. We determined the corresponding scalar potential and explored its minima. While for quarks the minimal assumption of associating each Yukawa coupling to just one flavon led to the conclusion of no mixing for a renormalisable potential [29, 30, 31, 32], the presence of the Majorana character introduces radically new ingredients. Generically, the flavour group is enlarged and different invariants are allowed in the potential. The toy model with only two families demonstrates that non-trivial Majorana phases and mixing angles may be selected by the potential minima and indicates a novel connection with the pattern of neutrino masses: i) large mixing angles are possible; ii) there is a strong correlation between mixing strength and mass spectrum; iii) the relative Majorana phase among the two massive neutrinos is predicted to be maximal, $2\alpha = \pi/2$, for non-trivial mixing angle; moreover, although the Majorana phase is maximal, it does not lead to CP violation, as it exists a basis in which all terms in the Lagrangian are real. These novel results are intimately related with the well-known fact that, in the presence of non-trivial Majorana phases, there may be physical mixing even for degenerate neutrino masses. In consequence, the results are expected to hold as well for general fermionic See-Saw scenarios.

Nevertheless, for that same model but with the three lepton generations considered, the minimum of the potential cannot correspond to the measured values of masses and mixings. This is probably linked to the necessary masslessness of the third left-handed neutrino in the minimal model, which imposes a fixed hierarchy with respect to the massive modes. More freedom is expected in models with three massive neutrinos, a work which is on progress.

The possibility of accommodating large mixing angles and Majorana phases is reminiscent of the typical anarchy pattern [21, 22, 23], in which the probability peaks at maximum values of the phases. Nevertheless, the present approach goes beyond anarchy not only in that it has dynamical content, but in that it sheds light on the difference in mixing strength in the quark and lepton sectors, while anarchy cannot deal with the quark sector. Furthermore, the model discussed here has an advantage with respect to models based on discrete symmetry, because usually in the latter the mass spectrum and the

Majorana phases are not correlated to the mixing angles. The strong correlation found between the neutrino mass spectrum, the mixing strength and the Majorana phase is also expected to hold for general fermionic See-Saw models, being linked only to the kind of invariants entering the scalar potential. We will investigate this possibility in a separate paper.

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