Scattering Forces from the Curl of the Spin Angular Momentum of a Light Field

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(Received 24 November 2008; published 20 March 2009)

Light forces on small (Rayleigh) particles are usually described as the sum of two terms: the dipolar or gradient force and the scattering or radiation pressure force. The scattering force is traditionally considered proportional to the Poynting vector, which gives the direction and magnitude of the momentum flow. However, as we will show, there is an additional nonconservative contribution to the scattering force arising in a light field with nonuniform helicity. This force is shown to be proportional to the curl of the spin angular momentum of the light field. The relevance of the spin force is illustrated in the simple case of a 2D field geometry arising in the intersection region of two standing waves.

DOI: 10.1103/PhysRevLett.102.113602 PACS numbers: 42.50.Wk, 37.10.Jk, 37.10.Vz, 87.80.Cc

Light carries energy and both linear and angular momenta that can be transferred to atoms, molecules, and particles. Demonstration of levitation and trapping of micronsized particles by radiation pressure dates back to 1970 and the experiments reported by Ashkin and coworkers [1]. Light forces on small particles are usually described as the sum of two terms: the dipole or gradient force and the radiation pressure or scattering force [2–4]. In analogy with electrostatics, small particles develop an electric dipole moment in response to the light's electric field. The induced dipole is then drawn by field intensity gradients which compete with radiation pressure due to momentum transferred from the photons in the beam. By fashioning proper optical field gradients, it is possible to trap and manipulate small particles with optical tweezers [2] or create atomic arrays in optical lattices [5,6]. Intense optical fields can also induce significant forces between particles [7].

In free space, radiation pressure is traditionally considered proportional to the Poynting vector, which gives the direction and magnitude of the momentum flow. However, there is an additional contribution to the scattering force that is usually neglected in the analysis of optical forces on small particles [4]. Although there was no intuitive picture about its physical origin, numerical calculations have recently shown that it can play an important role in determining the actual forces on nanometer sized particles [8]. The main purpose of this work is to show that this additional contribution is induced by the curl of the spin angular momentum of the light field.

While the transfer of linear momentum leads to a net force, the transfer of angular momentum can induce rotation of microscopic particles. The total angular momentum of a light field is given by the sum of *spin* and *orbital* contributions [9]. The *spin angular momentum* associated with circular polarization arises from the spin of individual photons, $\pm\hbar$. Spin-induced torques were already observed by Beth [10] in 1936. More recently, it has been shown [11,12] that for beams with helical phase fronts, the *orbital angular momentum* per photon in the propagation direction is an integer multiple of $\hbar$. This orbital angular momentum can be associated with the component of the Poynting vector that circulates about the beam axis (optical vortex). Transfer of both spin and orbital angular momentum has important applications in fields as diverse as optically driven micromachines and biosciences [13].

Although spin and orbital angular momentum are equivalent in many ways, they have, in general, different interaction properties [12]. For a small particle, the transfer of orbital angular momentum caused the particle to orbit around the beam axis while the spin transfer cause it to rotate on its own axis. As we will show, when the light field has a nonuniform spatial distribution of spin angular momentum (i.e., nonuniform helicity), an additional scattering force arises as a reaction of the particle against the rotation of the spin. We will illustrate the relevance of the spin force in the particularly simple case of a 2D field geometry arising in the intersection region of two standing waves [6,14,15].

Let us first discuss briefly the optical forces on a particle from the point of view of classical electrodynamics. The net force exerted on an arbitrary object is entirely determined by Maxwell's stress tensor $\boldsymbol{T}$ [16]. For simplicity, we consider the object in vacuum and in the presence of a harmonic electromagnetic field of frequency ω. The time average force can be written as [3,16]

$$\langle \mathbf{F} \rangle = \int d^3\mathbf{r} \nabla \langle \boldsymbol{T}(\mathbf{r}) \rangle = \int_A \langle \boldsymbol{T} \rangle \cdot \boldsymbol{n}\, dA, \qquad (1)$$

where A is any arbitrary closed surface enclosing the object and

$$\nabla T = \epsilon_0 \mathcal{E}(\nabla \cdot \mathcal{E}) + \epsilon_0 (\nabla \times \mathcal{E}) \times \mathcal{E} + \mu_0 \mathcal{H}(\nabla \cdot \mathcal{H}) + \mu_0 (\nabla \times \mathcal{H}) \times \mathcal{H} \quad (2)$$

with

$$T_{ij} = \epsilon_0 \mathcal{E}_i \mathcal{E}_j - \mu_0 \mathcal{H}_i \mathcal{H}_j - \delta_{ij} \tfrac{1}{2} (\epsilon_0 |\mathcal{E}|^2 + \mu_0 |\mathcal{H}|^2). \quad (3)$$

Notice that the magnetic and electric field vectors, $\mathcal{E}$ and $\mathcal{H}$, correspond to the total electromagnetic field including both the external and scattered fields [4]. For a small particle, the forces can also be expressed in terms of the external fields. The total field in the vacuum outside the particle, $\mathcal{E}$, can be written as the sum of external (polarizing or incoming) **E** and scattered fields. For a spherical particle (with a size a much smaller than the wavelength λ) with relative permittivity $\epsilon(\omega)$ and located at $\mathbf{r} = \mathbf{r}_1$,

$$\mathcal{E}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) + \mathbf{G}(\mathbf{r} - \mathbf{r}_1) \cdot \mathbf{p} = \mathbf{E}(\mathbf{r}) + \alpha \mathbf{G}(\mathbf{r} - \mathbf{r}_1) \mathbf{E}(\mathbf{r}_1), \quad (4)$$

where **G** is the free space (dyadic) Green function,

$$G_{ij}(\mathbf{r}) = (k_0^2 \delta_{ij} + \partial_i \partial_j) g(r); \qquad g(r) = \frac{e^{ik_0 r}}{4\pi\epsilon_0 r}, \quad (5)$$

$k_0 = \omega/c$ is the wave number, $\mathbf{p} = \alpha \mathbf{E}(\mathbf{r})$ is the induced dipole, and the polarizability α is given by [17]

$$\alpha = \frac{\alpha_0}{1 - i\alpha_0 k_0^3/(6\pi\epsilon_0)}, \qquad \alpha_0 = 4\pi\epsilon_0 a^3 \frac{\epsilon - 1}{\epsilon + 2}. \quad (6)$$

The time averaged force [Eq. (1)] can be rewritten in terms of the dipole moment and the external polarizing field as [18]

$$\langle F(\mathbf{r}_1) \rangle = \frac{1}{2} \mathrm{Re}\left\{ \sum_i p_i \nabla E_i^*(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_1} \right\} = \frac{1}{2} \mathrm{Re}\left\{ \sum_i \alpha E_i(\mathbf{r}_1) \nabla E_i^*(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_1} \right\}. \quad (7)$$

Taking into account that for harmonic fields $\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}$ and the identity

$$\sum_i E_i \nabla E_i^* = (\mathbf{E} \cdot \nabla) \mathbf{E}^* + \mathbf{E} \times (\nabla \times \mathbf{E}^*), \quad (8)$$

the dipolar force [Eq. (7)] can be rewritten as the sum of three terms [4,8]:

$$\langle \boldsymbol{F} \rangle = \frac{1}{4} \mathrm{Re}\{\alpha\} \nabla |\mathbf{E}|^2 + \sigma \frac{1}{2} \mathrm{Re}\left\{ \frac{1}{c} \mathbf{E} \times \mathbf{H}^* \right\} \quad (9)$$

$$+ \sigma \frac{1}{2} \mathrm{Re}\left\{ i \frac{\epsilon_0}{k_0} (\mathbf{E} \cdot \nabla) \mathbf{E}^* \right\}, \quad (10)$$

where we have made use of the definition of the total cross section of the particle $\sigma \equiv k_0 \mathrm{Im}\{\alpha\}/\epsilon_0$. The first term is the familiar gradient force. The second term is easily identified as the radiation pressure: $\sigma \langle \boldsymbol{S} \rangle / c$ ($\langle \boldsymbol{S} \rangle$ being the time averaged Poynting vector). The third term [Eq. (10)] is usually neglected in the discussions on optical forces on small particles [3] since it is zero when the field has a single plane wave component [4]. Surprisingly, it has not received special attention until recently [8]. In the discussion below, we provide a simple explanation of its physical origin.

Let us consider the following identity,

$$-2i\mathrm{Im}\{\{\mathbf{E}^* \cdot \nabla)\mathbf{E}\} = (\mathbf{E} \cdot \nabla)\mathbf{E}^* - (\mathbf{E}^* \cdot \nabla)\mathbf{E} = \nabla \times (\mathbf{E} \times \mathbf{E}^*),$$

valid for the external field with $\nabla \cdot \mathbf{E} = 0$. We can then rewrite Eq. (10) as

$$\sigma \frac{1}{2} \mathrm{Re}\left\{ i \frac{\epsilon_0}{k_0} (\mathbf{E} \cdot \nabla) \mathbf{E}^* \right\} = \sigma c \nabla \times \left(\frac{\epsilon_0}{4\omega i} (\mathbf{E} \times \mathbf{E}^*) \right). \quad (11)$$

In the right-hand side of Eq. (11), we can readily identify

$$\langle \boldsymbol{L}_S \rangle = \frac{\epsilon_0}{4\omega i} \{\mathbf{E} \times \mathbf{E}^*\} \quad (12)$$

as the time averaged spin density of a transverse electromagnetic field [9]. We can then finally write the total force on a small particle as

$$\langle \boldsymbol{F} \rangle = \mathrm{Re}\{\alpha\} \left\{ \nabla \frac{1}{4} |\mathbf{E}|^2 \right\} + \sigma \left\{ \frac{1}{c} \langle \boldsymbol{S} \rangle \right\} + \sigma \{ c \nabla \times \langle \boldsymbol{L}_S \rangle \}. \quad (13)$$

This is the main result of this Letter. The scattering force, proportional to the total cross section σ, can be written as the sum of two contributions: the traditional radiation pressure term, proportional to the Poynting vector, and a curl force associated to the nonuniform distribution of the spin density of the light field. By definition, the later

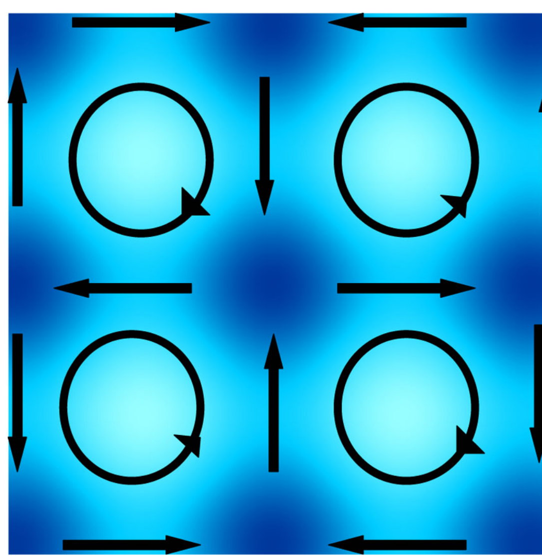

FIG. 1 (color online). s polarization: Energy flow vortices and field intensity distribution map in the intersection region of two standing waves ($\phi = 90°$). The bright areas correspond with the minimum of the electric field intensity. The arrows indicate the sense of the rotation of the vortices.

corresponds to a nonconservative force. When the light is linearly polarized, the curl term is identically zero.

Let us illustrate the relevance of the different contributions to the optical force in the particularly simple case of a 2D field geometry arising in the intersection region of two standing waves oriented along the x and y axes. The force field in this geometry depends critically on the light polarization. For p polarization (when the magnetic field is along the z axis or the electric field is polarized on the x-y plane) this geometry has been extensively used to trap and manipulate atoms in an optical lattice [6]. In contrast, for s polarization (when the electric field is along the z axis) the interference between the two standing waves leads to a lattice of radiation pressure vortices [14], which inhibits atom trapping. Interestingly, as we will see, while the spin curl force depends critically on polarization, the Poynting vector (and consequently, the radiation pressure) is exactly the same for both polarizations.

For s-polarized fields (see Fig. 1), the field of the two crossed standing waves in the interference region can be written as [14]

$$E_z(x, y; \omega) = \frac{E_0}{\sqrt{2}}(\{e^{ik_0x} - e^{-ik_0x}\} + e^{i\phi}\{e^{ik_0y} - e^{-ik_0y}\}) = \frac{2iE_0}{\sqrt{2}}(\sin k_0x + e^{i\phi}\sin k_0y), \quad (14)$$

where ϕ is the phase shift between the two beams. In this case, the electric field polarization is constant and there is no contribution from the spin angular momentum, i.e.,

$$\mathbf{F}^{(s)} = \mathrm{Re}\{\alpha\}\frac{1}{4}\nabla|E_z|^2 + \sigma\left\{\frac{1}{c}\langle \boldsymbol{S}\rangle^{(s)}\right\},$$

where

$$\frac{|E_z|^2}{2|E_0|^2} = \sin^2 k_0x + \sin^2 k_0y + 2\cos\phi\sin k_0x\sin k_0y. \quad (15)$$

It is easy to show that in this optical field the radiation pressure term leads to a nonconservative curl force for any finite phase shift. As a matter of fact, it can be explicitly written as

$$\frac{1}{c}\langle \boldsymbol{S}\rangle^{(s)} = c\nabla\times\langle \boldsymbol{L}_O\rangle, \quad (16)$$

$$\langle \boldsymbol{L}_O\rangle \equiv \frac{\epsilon_0}{\omega}|E_0|^2\sin\phi\cos k_0x\cos k_0y\,\mathbf{u}_z. \quad (17)$$

The curl force can be associated to the orbital angular momentum, $\boldsymbol{L}_O$, arising as a consequence of the rotation of the Poynting vector around the field nodes.

Instead of a small spherical particle, we could consider the forces acting on a long nanorod oriented along the z axis. In this case, the whole problem is two-dimensional (2D). However, all the results for s polarization remain unchanged [19] provided we take the appropriate polarizability, $\alpha \leftrightarrows \alpha^{(2D)}$. For a cylindrical nanorod of radius a and relative permittivity ϵ, we have [20]

$$\alpha^{(2D)} = \pi\epsilon_0 a^2(\epsilon - 1)\left[1 - i\frac{\pi}{4}(k_0a)^2(\epsilon - 1)\right]^{-1}. \quad (18)$$

It is worth mentioning that it has been suggested that for 2D problems where the polarization of the electric field is constant (s polarization) the optical force derives from a scalar potential [21]. This would seem to question the classical separation between gradient and scattering forces and, in particular, the nonconservative curl force term [Eq. (16)]. The apparent contradiction is related to the difference between total and external or polarizing fields: In the simpler case of a 2D problem, where both the object and the incident field have translational invariance along the z axis, the expression of the force from the Maxwell stress tensor [Eq. (1)] simplifies considerably. In that case, all the components of the fields can be expressed from the z components of the electric and magnetic fields [21]. If we further assume s polarization, i.e., $\mathcal{E} = \mathcal{E}_z(x, y)\mathbf{u}_z$, $\mathcal{H}_z$ vanishes and we have

$$\nabla \boldsymbol{T} = -2\epsilon_0\mathcal{E}_z\nabla\mathcal{E}_z; \qquad \nabla\langle \boldsymbol{T}\rangle = -\frac{\epsilon_0}{2}\nabla|\mathcal{E}_z|^2, \quad (19)$$

$$\langle \mathbf{F}\rangle = -\int d^3\mathbf{r}'\nabla'\left(\frac{\epsilon_0}{2}|\mathcal{E}_z(\mathbf{r}')|^2\right). \quad (20)$$

The force is then related to the (integral of the) gradient of the energy density $(\epsilon_0/2)|\mathcal{E}_z|^2$ of the total field $\mathcal{E}$ [21]. However, it is worth noticing that the force itself cannot, in general, be written as a gradient of the object's potential energy (i.e., the integral of the gradient is not the gradient with respect to the particle's coordinates). As a matter of

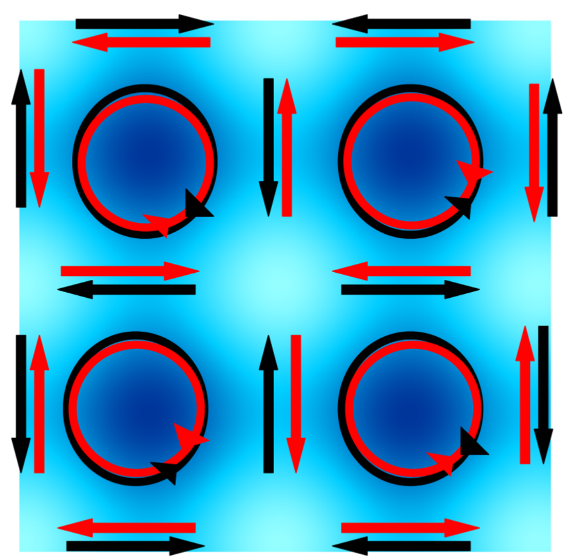

FIG. 2 (color online). p polarization: Energy flow vortices and field intensity distribution map in the intersection region of two standing waves ($\phi = 90°$). The bright areas correspond with the minimum of the electric field intensity. The dark arrows indicate the sense of the energy flow (Poynting vector) and the clear ones are associated to the curl of the spin density L_s.

fact, the actual force can be nonconservative as discussed above.

Let us now turn to the standing wave geometry in p polarization (see Fig. 2). In the interference region, the field of the two crossed standing waves is now given by

$$H_z(x, y; \omega) = -\epsilon_0 c \frac{2iE_0}{\sqrt{2}} (\sin k_0 x + e^{i\phi} \sin k_0 y) \tag{21}$$

and

$$E_x = \frac{2E_0}{\sqrt{2}} e^{i\phi} \cos k_0 y \tag{22}$$

$$E_y = -\frac{2E_0}{\sqrt{2}} \cos k_0 x. \tag{23}$$

The total force is then given by the gradient force and it is independent of the phase shift

$$\boldsymbol{F}^{(p)} = \mathrm{Re}\{\alpha\}\tfrac{1}{2}|E_0|^2 \boldsymbol{\nabla}(\cos^2 k_0 x + \cos^2 k_0 y). \tag{24}$$

It is known [14] that in this configuration, the light field acquires two different types of polarization gradients for the two characteristic values, $\phi = 0°$ and $\phi = 90°$, of the time phase difference between the two standing waves. When $\phi = 0°$, the spin density is zero and the polarization is linear everywhere, only its direction varies. In the $\phi = 90°$ case, there exists a 2D array of straight lines parallel to the z axis where the light exhibits circular polarization with an alternating sign. There is a continuous change to linear polarization when one moves away from those locations [14]. It is interesting to note that the radiation pressure term is exactly the same as in s polarization [Eqs. (16) and (17)], $\langle \boldsymbol{S} \rangle^{(s)} = \langle \boldsymbol{S} \rangle^{(p)}$. It is easy to see that the spin density $\boldsymbol{L}_S$ in this case is exactly given by $-\boldsymbol{L}_O$ defined in Eq. (17). Then, the two scattering force contributions cancel each other leading to a pure conservative force.

In summary, light forces on small (Rayleigh) particles can be written as the sum of three terms: the dipolar or gradient force, the radiation pressure force, and the spin curl force. The last (nonconservative) term was shown to be relevant when the particle is in a light field with nonuniform helicity spatial distribution. Spin curl forces were shown to play a key role to understanding light forces even in the apparently simple interference light field generated by two crossing standing waves. They could then be the most relevant for the optical control of particles close to scattering objects (e.g., structured surfaces) since, even with incoming plane wave illumination, scattering may lead to interference fields where the helicity is not uniform.

We acknowledge helpful discussions with F. Scheffold. This work was supported by the Spanish MEC through the Consolider *NanoLight* (CSD2007-00046), FIS2005-05137 and FIS2006-11170-C02-02 projects, Microseres-CM and by the Spanish-French PICASSO program (HF2007-0068).

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