

Flux avalanches triggered by microwave depinning of magnetic vortices in Pb superconducting filmsA. A. Awad,¹ F. G. Aliev,^{1,*} G. W. Ataklti,² A. Silhanek,^{2,3} V. V. Moshchalkov,² Y. M. Galperin,⁴ and V. Vinokur⁵¹*Departamento Fisica Materia Condensada, C-03, Universidad Autonoma de Madrid, E-28049, Madrid, Spain*²*INPAC, K.U. Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*³*Departement de Physique, Universite de Liege, B-4000 Liege, Belgium*⁴*Department of Physics, University of Oslo, P. O. Box 1048, Blindern, 0316 Oslo and Centre for Advanced Study, Drammensveien 78, NO-0271 Oslo, Norway*⁵*Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, Illinois 60439, USA*

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We observe abrupt changes in broadband microwave permeability of thin Pb superconducting films as functions of the microwave frequency and intensity, as well as of external magnetic field. These changes are attributed to vortex avalanches generated by microwave induced depinning of vortices close to the sample edges. We map the experimental results on the widely used theoretical model assuming reversible response of the vortex motion to ac drive. It is shown that our measurements provide an efficient method of extracting the main parameter of the model—*depinning frequencies*—for different pinning centers. The observed dependences of the extracted depinning frequencies on the microwave power, magnetic field, and temperature support the idea that the flux avalanches are generated by microwave induced thermomagnetic instabilities.

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I. INTRODUCTION

The electromagnetic response of type-II superconductors in a magnetic field is determined mostly by the dynamics of Abrikosov vortices.¹ Under applied current the vortices move and dissipate energy, unless they are pinned. The maximal current that superconductors can stand, a critical current J_c , is customarily defined by the average properties of the pinning.² Vortex depinning often leads to avalanches of magnetic flux, which stochastically emerge at weakest pinning points close to the edges of the sample.^{3–5} In thin films, irreproducible flux avalanches involving up to millions of vortices are usually observed.^{6–9} The avalanches are conventionally attributed to thermomagnetic instabilities induced by moving vortices.^{4,5,10,11} Indeed, the vortex motion locally heats the sample, resulting in a local decrease in the pinning strengths and thus developing a positive feedback loop triggering instability.^{12,13}

It is still not fully clear to what extent one can control the intermittent flux dynamics in type-II superconductors (see, e.g., Refs. 14 and 15). There exist several ways to trigger avalanches controllably, e.g., by laser⁷ or current¹⁶ pulses, due to microwave fields in superconducting resonators or coils,^{17–19} etc. The main feature of these methods is that the instability is triggered by a normal hot spot generated by the external perturbation. Therefore the detailed relationship between the characteristics of the excitation and those of flux pinning is not straightforward. Recent experiments, however, indicate that the microwave induced avalanches have very specific frequency dependence containing pronounced peaks at a few specific excitation frequencies²⁰ indicating a complex microwave vortex depinning.

In this paper, we report on unexpected abrupt changes in the microwave permeability of thin type-II superconducting Pb films as a function of frequency in the presence of an out-of-plane magnetic field. We interpret these changes as generated by flux avalanches triggered by microwave depinning of superconducting vortices at the sample edges.

The experimental results are described in terms of the linear-response model, where vortices are assumed to be trapped by a pinning potential $V(r)$ and oscillate under the applied ac field near its minimum $r = 0$. The pinning potential is quantified by the so-called Labush constant $\alpha_L = (\partial V / \partial r)_{r=0}$ characterizing its curvature at the minimum.^{21,22} If the applied frequency is high enough and exceeds the so-called depinning frequency $f_d = \alpha_L / 2\pi\eta$, the oscillating vortices cease to feel the detail of the pinning potential. This approach describes both single vortices, in a single vortex pinning regime, and vortex bundles, where collective effects become essential.²¹ In the latter case η is a friction constant for the vortex system.

Usually the information about the Labush constant was obtained from the data on the surface impedance on an assumption that only one kind of pinning is important;²³ see also references in Ref. 24. In this work, we show that broadband measurements of the permeability at different microwave powers allow determining the depinning frequencies in the cases when different defects are present. The method allows one to determine f_d for different sources of pinning in a more-or-less model-free way. This is the *central result* of the present work.

The knowledge of the vortex depinning frequencies opens the opportunity to control and manipulate superconducting vortices using relatively weak microwave excitations at specific frequencies. In the broader perspective, our paper introduces a different method to investigate depinning of the vortex system close to critical conditions.

II. SAMPLES AND EXPERIMENTAL SETUP

The 80-nm-thick Pb films were evaporated in a molecular-beam epitaxy (MBE) system at a working pressure of 7×10^{-8} Torr with a source material in the crucible of 99.9999% purity. In order to obtain a smooth film the substrate was cooled to 77 K, with growth rate of 5 Å/s, controlled by mass spectrometer. The average roughness estimated by AFM was 1.2 nm on an area of $1 \mu\text{m}^2$. Low angle x-ray diffraction (XRD) shows periodic oscillations of intensity as a function

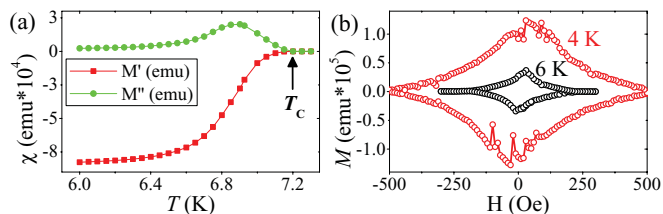


FIG. 1. (Color online) (a) Temperature dependence of real and imaginary parts of the susceptibility ($h = 1$ Oe, $f = 3837$ Hz). (b) Typical magnetization hysteresis curves indicating transition to the vortex avalanche regime at low temperatures.

of angle, a characteristic of highly homogeneous films. High angle XRD shows that Pb has a preferential growth along the (111) direction of the fcc lattice. The structure of the films is textured, meaning that actually not all grains are oriented along the (111) direction. Although the effects reported here were observed for Pb films with thickness between 50 and 90 nm and dimensions of 6×5 mm², we concentrate below on representative set data obtained on 80-nm-thick Pb films.

Shown in Fig. 1(a) are temperature dependences of the real and imaginary contributions to the susceptibility, which yield the critical temperature $T_c = 7.2$ K typical for Pb type-II superconducting thin films.^{3,25} Magnetization curves [Fig. 1(b)] indicate the presence of vortex avalanches at sufficiently low temperatures. The broadband measurements were done in the transmission configuration by means of a two-port (1,2) vector network analyzer (VNA) (see Ref. 26 for more details). The microwave drive field (\mathbf{h}_{rf}) up to 0.11 Oe is provided by a coplanar wave guide (CWG) in the TEM mode; Fig. 2(a). The sample was formed by cleaving after the film deposition; it was placed face down on the CWG to locate the sample edge above the central conductor so that \mathbf{h}_{rf} was parallel to the surface and nearly perpendicular to the film edge. During the measurements, the signal frequency was swept under constant bias magnetic field \mathbf{H} , applied perpendicular to the surface. The signal from the CWG to VNA (after calibration of the external cables and correction for small field-independent contributions due to geometric resonances) was analyzed with a model²⁷ based on the effective uncalibrated microwave permeability parameter,

$$U(f, H_n) = \pm i \left(\frac{\ln[S_{21}(f, H_n)]}{\ln[S_{21}(f, H_{ref})]} - 1 \right),$$

neglecting the effect of reflection. Here $S_{21}(f, H_n)$ and $S_{21}(f, H_{ref})$ are the frequency-dependent forward transmissions between ports 1 and 2 measured correspondingly at the applied field H_n , and the reference field H_{ref} (usually the maximum applied field). The imaginary part, $\text{Im} U$, describes the losses. In order to minimize the influence of field-independent “ripples” from the standing waves due to multiple reflection from the connectors, we carried out a differential analysis of $U(f, H)$ by using as H_{ref} the previous value of the magnetic field, H_{n-1} . Since $\ln S_{21}(f, H) \propto -\alpha(f, H)$, where $\alpha(f, H)$ is the signal attenuation between ports 1 and 2, the so-defined function $\text{Im}[U(f, H)]$ converts into $\text{Im}[U'(f, H)] \equiv \alpha^{-1}(\partial\alpha/\partial H)$. Along with the broadband capabilities, our method provides direct measurements of the vortex depinning frequencies through detection of avalanches triggered in a

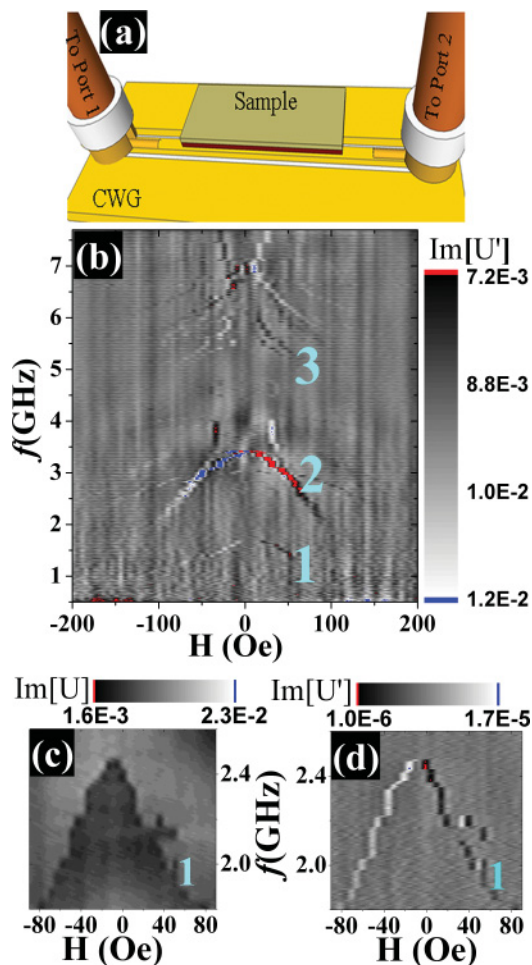


FIG. 2. (Color online) (a) Sketch of the CWG with the sample. (b) $\text{Im}[U'(f, H)]$ versus external magnetic field for $h_{rf} = 0.1$ Oe, $T = 6.1$ K. Only the branches $i = 1, 2, 3$ are further analyzed. Parts (c) and (d) compare $\text{Im}[U(f, H)]$ (H_{ref} is 50 mT) and $\text{Im}[U'(f, H)]$ measured for the mode $i = 1$ at $T = 6.1$ K, $h_{rf} = 0.06$ Oe.

region of excitation powers, magnetic fields, temperatures, and through continuous frequency sweeps, previously inaccessible to other techniques;^{23,28,29} see also references in Refs. 18 and 24. For example, a broadband technique based on the Corbino geometry^{23,29} mainly provides information on microwave vortex depinning *inside* the superconducting sample, namely, in its central part where the microwave field is largest. Our metallic coplanar waveguide technique, however, detects microwave vortex dynamics close to the film edge. In addition, metallic pads deposited on the sample in the Corbino disk geometry prevent avalanche creation. Therefore our main goal—investigation of the microwave-induced depinning via detection of the avalanches—becomes not accessible.

III. RESULTS AND DISCUSSION

Let us start with discussion of the broadband dynamic responses measured at a fixed radio-frequency (rf) drive power and temperature, for different fixed magnetic fields (increasing from some value <1000 Oe with a step of 5 Oe) by continuous rf frequency scan between 300 kHz and 8 GHz. Figure 2(b) shows our main finding: the anomalous variation

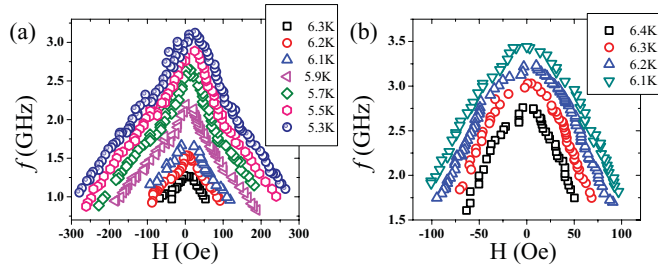


FIG. 3. (Color online) Magnetic-field dependences of SLAs $f_0^{(1)}$ (a) and $f_0^{(2)}$ (b) for different temperatures measured at the drive intensity 5 dBm.

of $\text{Im}[U'(f, H)]$ as a function of microwave frequency and magnetic field at fixed temperature. At certain drive frequencies $f_0^{(i)}$ and low enough H we observe spike-like anomalies (SLA) in $\text{Im}[U'(f, H)]$ (i.e., differential absorption as a function of frequency), which are nearly symmetric in magnetic field. At highest frequencies we observe even more complicated SLAs, which are also symmetric in H [see the branch $i = 3$ and other SLAs in Fig. 2(b)]. Figures 2(c) and 2(d) demonstrate comparison between $\text{Im}[U(f, H)]$ (H_{ref} is 500 Oe) and $\text{Im}[U'(f, H)]$. The similarity of these functions confirm the validity of our differential analysis.

We attribute the observed SLAs to *flux avalanches* triggered at the sample's edges by microwave induced vortex depinning (but not just depinning without considering avalanches). Indeed, (i) The SLAs appear only at low enough temperatures, typically, below $0.8T_c$. Our magnetization measurements, in accordance to previous reports,³⁰ indicate magnetization steps below $(0.6-0.7)T_c$. (ii) The SLAs diminish when maximum of the rf excitation is displaced from the edge to the film center or sample is covered by thermal grease. (iii) Last but not least, the SLAs are observed only above some threshold drive amplitude (see more details below).

Figure 3(a) shows the magnetic-field dependences of the SLA with $i = 1, 2$, $f_0^{(1,2)}(H)$, for different temperatures. The dependences are approximately quadratic. The SLA exists only at $T < 0.8T_c$; decreasing temperature expands the magnetic-field interval where it is detected and generally increases the characteristic frequency $f_0^{(i)}(0)$.

In order to elucidate the possible influence of rf heating on the vortex avalanches, we have investigated the influence of the microwave power on the SLAs. Shown in Fig. 4(a) are the frequency dependences of the differential absorption for different microwave powers in the range $(-20-5)$ dBm, the rf field amplitudes being $(0.0067-0.11)$ Oe. Figure 4(b) compares the dependence of the SLA amplitude ($i = 0$) on the microwave power at two different fixed temperatures. Within precision of our technique, SLAs are absent below some critical amplitude [see Fig. 4(b)]. The threshold value shows a tendency to increase when the temperature is reduced. These experimental facts further confirm that we are dealing with rf-triggered reproducible vortex avalanches.

Shown in Figs. 4(c) and 4(d) are temperature dependences of SLAs' positions, $f_0^{(i)}(T)$, $i = 1, 2, 3$. One notes that $f_0^{(1)}$ shows a clear tendency to decrease with temperature and

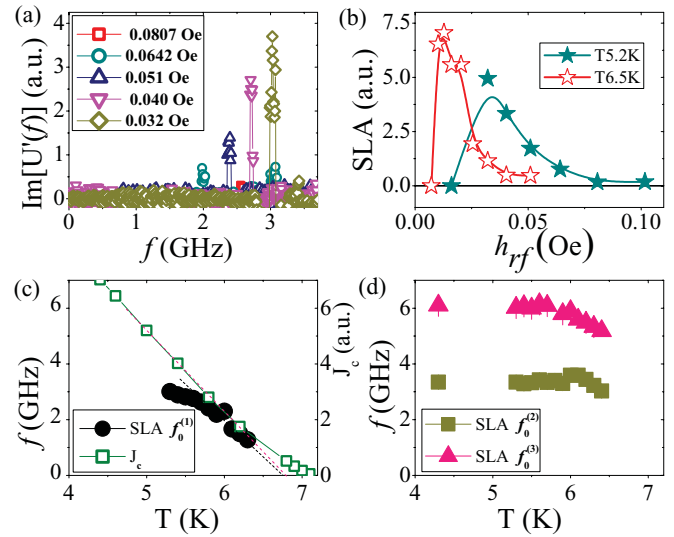


FIG. 4. (Color online) (a) Frequency dependence of microwave losses (SLA with $i = 0$) measured with different rf drive fields at temperature 5.2 K and bias field of 240 Oe. (b) Amplitude of SLA ($i = 1$) as a function of rf power and rf magnetic field measured at two different temperatures. (c) Temperature dependence of low field vortex depinning frequencies $f_0^{(1)}$ and $f_0^{(2,3)}(T)$ (d) for the same SLA shown in Fig. 2(b).

vanish at $T \rightarrow T_c$. At the same time, $f_0^{(3)}$ shows a much weaker temperature dependence. Interestingly, extrapolating $f_0^{(1)}(T)$ to $f = 0$ [see Fig. 4(c)] gives a critical temperature very close to that obtained from extrapolating to $H = 0$ the experimentally measured $J_c(T)$ from magnetization loops. The similarity of $f_0^{(1)}(T)$ and $J_c(T)$ supports the suggestion that the lowest frequency branch ($i = 1$) in SLAs is due to avalanches induced by vortex depinning from the weakest pinning sites. Indeed, these thermally activated vortex depinnings can limit the critical current. While lower depinning frequencies ($f_0^{(1)}$ branch) correspond to vortex depinning from weakest pinning sites with depinning barrier much reduced as temperature approaches T_c the pinning sites with stronger pinning barriers corresponding to the branches $f_0^{(2,3)}$ are less affected by temperature.

To summarize, the experimental data suggest that SLAs in the GHz range are due to vortex avalanches triggered by the edge vortex depinning occurring at well defined frequencies of the rf drive. The edge vortices are usually the most strongly pinned by the edge defects and therefore can act as “nails” for the vortex system keeping it in the critical state. The edges recover the superconducting state after the avalanches expand inside the film.¹⁷ This scenario is also supported by the experimental fact that the observed effects are stronger and symmetric vs applied external magnetic field with rf drive applied across one of the film edges. The observed nonmonotonous dependence of the amplitude of SLAs on the rf power could be understood as follows. The external rf drive induces microwave currents and related alternating Lorentz force periodically “shaking” edge vortices back and forth. Below some particular (for each branch i) threshold power the rf drive does not depin vortices (see, e.g., mode $i = 1$ in Fig. 4).

Microwave power exceeding a particular threshold releases vortices at the sample edge that, in turn, facilitates the thermomagnetic instability resulting in flux avalanches (observed as SLAs). Further increase of the rf power slightly heats the sample (estimated temperature increase at the maximum rf drive does not exceed 0.1 K) decreasing in this way the magnitude of SLAs. We stress here that the dependence of the SLAs' amplitudes on the rf power [Fig. 4(b)] reflects abrupt changes in the losses due to avalanches triggered by the edge vortex depinning. According to the linear-response model,^{21,22} the absorbed power $Q_{vm}(f)$ can be represented as

$$Q_{vm}(f) = Q_c \frac{f^2}{f^2 + f_d^2}, \quad f_d = \frac{\alpha_L}{2\pi\eta}, \quad Q_c \propto \frac{J_0^2}{\eta}. \quad (1)$$

Here J_0 is the microwave current amplitude while f_d is the so-called depinning frequency. Hence the absorption is low at $f \ll f_d$ and it saturates at $f \gtrsim f_d$. Therefore it is natural to expect that for a given drive intensity the conditions for nucleating of the thermomagnetic instability are frequency dependent, the characteristic frequency being f_d . The depinning frequency (similarly to critical current) is determined by the details of vortex pinning.²⁴ In particular, it is very different in bulk and film superconductors;³¹ in the case of films, it strongly depends on the film thickness approaching from the MHz (bulk superconductors) to GHz range for thin films.²³ Each type of defects has its own f_d , therefore one can expect successive depinning of different defects.

Along the simplified model^{13,32} of the thermomagnetic instability (which ignores thermal conductance along the film) its onset is usually related to the so-called "stability parameter" $\beta = Q_{vm}/(\gamma C + C_h)\Delta T$. The absorbed microwave power Q_{vm} is balanced by heating of the Pb film, $\gamma C \Delta T$ (γ is the density of Pb), during the avalanche process in combination with heat removal into the substrate ($C_h \Delta T$) where C_h is the heat-transfer coefficient of the metal-substrate interface. Our experimental observations could be qualitatively understood from the analysis of the dependence of the thermal stability factor β on the external microwave power and magnetic field. Since $Q_{vm}(f)$ rapidly increases at $f_0 \sim f_d$ one can expect that at this frequency the thermomagnetic instability starts. Assuming that $\beta = 1$ at some value Q_0 of the absorbed power and using Eq. (1) we get that the instability threshold corresponds to the frequency,

$$f_0 = f_d \sqrt{\frac{Q_c}{Q_0 - Q_c}}. \quad (2)$$

Therefore increasing the microwave power (and the rf amplitude J_0) leads to a decrease of the signal frequency required for reaching the instability criterion, $\beta = 1$. This is also in agreement with the experiment; see Fig. 4(a). The observed dependence of the critical frequency on the magnetic field can be explained as follows. On the one hand, $Q_{vm} \propto H$ through the total magnetic flux. On the other hand, the heat capacity of the vortex system is field independent. It can also include proportional to magnetic-field quasiparticle contribution.^{13,33} Therefore the stability parameter could be expected to depend on the external field as $\beta \sim H/(H + \text{const})$, which linearly increases at small fields and should consequently reduce the

critical frequency with increasing magnetic field, as observed; see Figs. 3(a) and 3(b).

Let us compare the experimentally measured depinning frequencies with those one could expect based on average material characteristics. The typical value of $f_d = \alpha_L/2\pi\eta$ can be estimated using $k_p = \pi B_c^2/\mu_0$ (where B_c is the thermodynamic critical field) and $\eta(T) = \phi_0\mu_0 B_{c2}(T)/\rho_n$.³⁴ Here ϕ_0 is the magnetic-flux quantum, B_{c2} is the upper critical field, ρ_n is the normal-state resistivity. For the temperature range of interest ($T = 5\text{--}6$ K) with $B_c = 50$ Oe, $B_{c2} = 500$ Oe, and $\rho_n = 4 \mu\Omega \text{ cm}$ we get $f_d \approx 400$ MHz, which is a factor of 6–10 smaller than the SLA frequencies observed experimentally. This is, of course, a very crude estimate since the parameters used (particularly the resistivity) could be different at the film edge where the rf-induced vortex depinning takes place.

It is worth noting that a particular set of observed depinning frequencies is a "fingerprint" determined by details of the edge pinning in a given sample. Since the depinning frequencies for different defects are controlled by disorder, f_d is expected to be sensitive to thermal annealing. Indeed, we found that while increasing the temperature to 20 K does not influence the observed effects, heating to 150 K keeps unchanged only zero-field SLA frequencies, while "annealing" of the defects with thermal excursion to 300 K during 5 h changes both the critical frequencies $f_0^{(i)}$ and their field dependences.

Previous usage of superconducting striplines and coplanar waveguides was aimed mainly at the improvement of microwave transmission.³⁵ Our results, however, show that in principle superconducting stripline-based techniques^{17,35} might also be employed for broadband detection of vortex depinning frequencies due to generated avalanches, once impedance mismatch between normal rf launchers and superconducting transmission striplines is minimized. The presence of multiple depinning frequencies in the experiments with superconducting strips was probably observed in Ref. 35. The authors, however, did not consider reproducible spikes in the microwave losses as evidence of the vortex depinning frequencies.

IV. CONCLUSIONS

In conclusion, using normal-metal coplanar waveguide we have studied broadband microwave permeability of thin superconducting films close to its edge. We have found pronounced spikelike anomalies of the absorption versus rf frequency at constant perpendicular dc magnetic field. Several observed features, such as significant dependences of the anomalies on temperature and on the position of the maximum of the rf radiation relative to the sample's edge, pronounced threshold in the rf intensity, lead us to the conclusion that the spikes is a manifestation of flux avalanches. The avalanches are *triggered at the edge* by local increase in the temperature owing to rf-induced vortex depinning.

A straightforward analysis based on conventional models shows that the spikes can be related to a set of discrete depinning frequencies corresponding to edge defects of different kind. Therefore we provide a method of extracting

depinning frequencies corresponding to edge defects. Precise knowledge of these frequencies of superconducting vortices could be of importance for understanding vortex pinning, control over the vortex avalanches in high-frequency applications of superconductors, and an effective way for noise reduction from trapped vortices in sensitive superconducting quantum interference device detectors through microwave assisted vortex depinning. Future structural studies of the sample, as well as magneto-optic experiments in the presence of broadband microwave sweeps with fine power control, could

unveil details of avalanche processes triggered by rf induced vortex depinning from sample edges.

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- ¹A. A. Abrikosov, *Sov. Phys. JETP* **5**, 1174 (1957).
- ²G. Blatter *et al.*, *Rev. Mod. Phys.* **66**, 1125 (1994).
- ³M. Menghini, R. J. Wijngaarden, A. V. Silhanek, S. Raedts, and V. V. Moshchalkov, *Phys. Rev. B* **71**, 104506 (2005).
- ⁴I. S. Aranson, A. Gurevich, M. S. Welling, R. J. Wijngaarden, V. K. Vlasko-Vlasov, V. M. Vinokur, and U. Welp, *Phys. Rev. Lett.* **94**, 037002 (2005).
- ⁵V. V. Yurchenko *et al.*, *Low Temp. Phys.* **35**, 619 (2009).
- ⁶M. R. Werthheimer *et al.*, *J. Phys. Chem. Solids* **28**, 2509 (1967).
- ⁷P. Leiderer, J. Boneberg, P. Brull, V. Bujok, and S. Herminghaus, *Phys. Rev. Lett.* **71**, 2646 (1993).
- ⁸U. Bolz *et al.*, *Europhys. Lett.* **64**, 517 (2003).
- ⁹C. A. Duran, P. L. Gammel, R. E. Miller, D. J. Bishop, *Phys. Rev. B* **52**, 75 (1995).
- ¹⁰I. Aranson, A. Gurevich, and V. Vinokur, *Phys. Rev. Lett.* **87**, 067003 (2001).
- ¹¹A. L. Rakhmanov, D. V. Shantsev, Y. M. Galperin, and T. H. Johansen, *Phys. Rev. B* **70**, 224502 (2004); D. V. Denisov *et al.*, *Phys. Rev. Lett.* **97**, 077002 (2006); *Phys. Rev. B* **73**, 014512 (2006).
- ¹²R. Mints *et al.*, *Rev. Mod. Phys.* **53**, 551 (1981).
- ¹³E. R. Nowak, O. W. Taylor, L. Liu, H. M. Jaeger, T. I. Selinder, *Phys. Rev. B* **55**, 11702 (1997).
- ¹⁴A. V. Silhanek, S. Raedts, and V. V. Moshchalkov, *Phys. Rev. B* **70**, 144504 (2004).
- ¹⁵E. Altshuler and T. H. Johansen, *Rev. Mod. Phys.* **76**, 471 (2004); R. J. Zieve, T. F. Rosenbaum, H. M. Jaeger, G. T. Seidler, G. W. Crabtree, and U. Welp, *Phys. Rev. B* **53**, 11849 (1996).
- ¹⁶A. V. Bobyl *et al.*, *Appl. Phys. Lett.* **80**, 4588 (2002).
- ¹⁷G. Ghigo *et al.*, *J. Appl. Phys.* **102**, 113901 (2007).
- ¹⁸G. Ghigo *et al.*, *Appl. Phys. Lett.* **94**, 052505 (2009).
- ¹⁹P. Cuadra-Solís *et al.*, *J. Supercond. Nov. Mag.* **24**, 395 (2010).
- ²⁰P. Cuadra-Solís *et al.*, *Physica C* **468**, 805 (2008).
- ²¹A. Koshelev *et al.*, *Physica C* **173**, 465 (1991).
- ²²A. M. Campbell, *J. Phys. C* **2**, 1492 (1969); **4**, 3186 (1951); E. H. Brandt, *Phys. Rev. Lett.* **67**, 2219 (1991); E. Brandt, *Rep. Prog. Phys.* **58**, 1465 (1995); A. M. Campbell *et al.*, *Adv. Phys.* **50**, 465 (2001).
- ²³M. Golosovsky, M. Tsindlekht, H. Chayet, and D. Davidov, *Phys. Rev. B* **50**, 470 (1994); *Supercond. Sci. Technol.* **9**, 1 (1996); E. Silva *et al.*, *ibid.* **24**, 024018 (2011); N. Pompeo and E. Silva, *Phys. Rev. B* **78**, 94503 (2008).
- ²⁴M. Bonura *et al.*, *Physica C* **468**, 2372 (2008).
- ²⁵G. Dolan *et al.*, *Phys. Rev. Lett.* **30**, 603 (1973).
- ²⁶J. F. Sierra *et al.*, *Appl. Phys. Lett.* **93**, 172510 (2008).
- ²⁷S. S. Kalarickal *et al.*, *J. Appl. Phys.* **99**, 093909 (2006).
- ²⁸J. Gittleman *et al.*, *Phys. Rev. Lett.* **16**, 734 (1966).
- ²⁹J. C. Booth *et al.*, *Rev. Sci. Instrum.* **65**, 2082 (1994).
- ³⁰H. Radovan *et al.*, *Phys. Rev. B* **68**, 224509 (2003); S. Hébert *et al.*, *ibid.* **67**, 224510 (2003).
- ³¹D. Janjušević *et al.*, *Phys. Rev. B* **74**, 104501 (2006).
- ³²M. N. Wilson, *Superconducting Magnets* (Oxford University Press, New York, 1983).
- ³³A. Carr *et al.*, *Phys. Rev. B* **68**, 174519 (2003).
- ³⁴J. Bardeen *et al.*, *Phys. Rev.* **140**, A1197 (1965).
- ³⁵Y. Wang *et al.*, *IEEE Trans. Microwave Theor. Tech.* **53**, 2348 (2005).