Type IIA flux vacua with mobile D6-branes and α'-corrections

Memoria de Tesis Doctoral presentada por

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Abstract

Type IIA flux compactifications have proven to be a rich framework to construct phenomenologically appealing string vacua. However, a better understanding of such a flux landscape in the presence of D-branes is required if one wants to find realistic vacuum solutions. In this thesis, we study perturbative Type IIA flux vacua with an underlying Calabi-Yau geometry, by mean of the flux-axion polynomial formalism.

In a first stage we consider type IIA Calabi-Yau orientifolds with background fluxes and rewrite the classical flux potential as a bilinear of flux-axion polynomials invariant under the discrete shift symmetries of the compactification. We perform a systematic search of purely closed string vacua, showing that one can easily rewrite the conditions for $\mathcal{N}=0$ Minkowski and $\mathcal{N}=1$ AdS in terms of simple algebraic equations on the axion polynomials. Then we turn to the search of vacua in compactifications with fluxes and mobile D6-branes. The presence of D6-brane moduli redefines the four-dimensional dilaton and complex structure moduli and simultaneously destroy the nice factorization between Kähler and complex structure moduli in the Kähler potential, complicating the search of vacua in terms of the effective Kähler potential and superpotential. Nevertheless, one may still express the F-term scalar potential as a bilinear of such polynomials, which allows us to find a new and more general class of $\mathcal{N}=0$ Minkowski vacua, which present a quite simple structure of contravariant F-terms. We compute the set of soft supersymmetry breaking terms for chiral models of intersecting D6-branes in such vacua, finding a quite universal pattern.

In a second stage we further study type IIA Calabi-Yau flux compactifications with perturbative α'-corrections. It is a well-known fact that the inclusion of such α'-corrections allows to construct the mirror duals of type IIB Calabi-Yau flux compactifications, in which the effect of flux backreaction is under control. We compute the α'-corrected scalar potential generated by RR and NS fluxes, and reformulate it as a bilinear of the flux-axion polynomials. The use of such invariants allows to express in a compact and simple way the conditions for $\mathcal{N}=0$ Minkowski and $\mathcal{N}=1$ AdS flux vacua, and to extract the effect of α'-corrections on them.

Resumen

Las compactificaciones de la teoría de cuerda de tipo IIA con flujos de fondo han demostrado ser un marco rico para construir vacíos de cuerdas fenomenológicamente atractivos. Sin embargo, si se quiere encontrar soluciones de vacío realistas, se requiere una mejor comprensión de este paisaje de flujos en presencia de D-branas. En esta tesis, se estudian vacíos perturbativos de tipo IIA con flujos y una geometría Calabi-Yau subyacente, por medio del formalismo de los polinomios de axiones.

En una primera etapa, se consideran Calabi-Yau *orientifolds* de tipo IIA en presencia de flujos y reescribimos el potencial clásico generado por los flujos como un bilineal de los polinomios de axiones, los cuales son invariantes bajo las simetrías discretas de la compactificación. Utilizando este formalismo se realiza una búsqueda sistemática de vacíos de cuerdas cerradas, demostrando que uno puede reescribir fácilmente las condiciones de vacíos de Minkowski no-supersimétricos y vacíos AdS supersimétricos en términos de simples ecuaciones algebraicas en los polinomios de axiones. Luego comenzamos la búsqueda de vacíos en compactificaciones que incluyen flujos y D6-branas móviles. La presencia de los moduli de D6-branas redefine los moduli de estructura compleja y el dilaton en cuatro dimensiones, y simultáneamente destruye la agradable factorización entre los moduli de Kähler y los moduli de estructura compleja en el potencial de Kähler, lo que complica la búsqueda de vacíos en términos del potencial de Kähler y el superpotencial. Sin embargo, el potencial escalar todavía puede ser expresado como un bilineal de los polinomios de axiones, lo que nos permite encontrar una clase nueva y más general de vacíos de Minkowski no-supersimétricos que presentan una estructura bastante simple de los F-terms contravariantes. Además, se calculan los términos de ruptura suave de supersimetría para los campos quirales que viven en las intersecciones de las D6-branas en tales vacíos, encontrando un patrón bastante universal.

En una segunda etapa, se estudian compactificaciones de tipo IIA en Calabi-Yau que incluyen flujos y correcciones perturbativas de α'. Es un hecho bien conocido que la inclusión de tales correcciones en el lado IIA, permite construir compactificaciones que son espejos duales de las compactificaciones de tipo IIB con flujos en las que la *backreaction* de los flujos está bajo control. Se calcula el potencial escalar generado por los flujos y que incluye las correciones de α', y lo reformulamos como un bilineal de los polinomios de axiones. El uso de tales invariantes permite expresar de manera compacta y sencilla las condiciones de vacíos de Minkowski no-supersimétricos y vacíos AdS supersimétricos y extraer los efectos de las correcciones de α' en estos vacíos.

Contents

Chapter 1

Motivation and overview

1.1 The prevailing paradigms of high energy physics

Our current understanding of the universe on large and small scales is based on two different theories describing together the four fundamental interactions observed in nature. On the one hand, we have Quantum Field Theory (QFT) which is the outcome of merging Quantum Mechanics and Special Relativity and it is the essential tool to understand microscopic physics. In particular, the Standard Model (SM) which is a chiral non-Abelian Yang-Mills theory based on the gauge group $SU(3) \times SU(2) \times U(1)$ provides a successful quantum description of electromagnetic, weak and strong interactions. Although the SM provides a remarkably successful description of almost all non-gravitational phenomena, there are some theoretical issues which are not answered satisfactorily by the SM, such as the existence of many free parameters which cannot be predicted by theory itself or the *bad* quantum behavior of its scalar sector, which leads to a destabilization of the hierarchy of mass scales, known as the hierarchy problem. Moreover, the experimental evidence of massive neutrinos is another signature of the incompleteness of the SM, since within the SM, neutrinos are strictly massless. Finally, further difficulties are found when trying to construct cosmological models based only on the SM particle content, since it does not contain a suitable candidate for the *Dark Matter* (DM) content of the universe, neither a viable candidate for *inflaton.*

On the other hand, General Relativity (GR) offers a classical field theory description of gravity, a force not yet included in the SM, yet the paradigm of GR forms the basis to our understanding of the large-scale structures in the universe. In this theory, gravity is intrinsically associated with the dynamics of spacetime itself in such a way that the geometry of spacetime is intrinsically determined by the distribution of the matter densities. Despite GR has been well tested by experiments, it faces some theoretical issues, such as the singularities that occasionally appear in the solutions of Einstein's field equation, which cannot be cured by the theory itself. Furthermore, this classical theory of gravity does not capture any quantum effects and does not allow to formulate a consistent quantum theory of gravity up to date. All these facts clearly indicate that both GR and SM are incomplete and cannot be considered as fundamental theories.

Looking for a theory of quantum gravity

One naively could try finding a quantum theory of gravity by using the same methods that effectively worked for the other three interactions. This can be achieved partially in the framework of QFTs on curved spacetimes, where gravity is mediated by a spin 2 particle called *graviton*, but unfortunately this approach fails irrevocably for Einstein's theory of GR. The reason why conventional methods of quantisation do not work for GR is the non-renormalisability of the theory. This means that the abundant divergences induced by quantum gravitational processes cannot be cured by the usual renormalization procedure, thus rendering the theory ill-defined at the Planck scale, which is energy scale at which quantum gravitational interactions become relevant, see [1]. This indicates that GR should be viewed as an effective low energy description of some more fundamental theory which regulates the theory in the ultraviolet (UV) regime, yet we do not know what this theory is.

Supersymmetry

A natural extension of the symmetry groups in QFTs is provided by supersymmetry, which is a fundamental symmetry relating bosons and fermions, which is mainly motivated by the hierarchy problem of the SM. In the simplest supersymmetric theory, each known particle has a superpartner which only differs in spin by 1/2 and are related by a supersymmetry transformation. Thus, supersymmetry basically doubles the particle content of the theory.

Supersymmetry also implies that fermions and bosons filling out a chiral supermultiplet are degenerated in mass. Since such a mass degeneracy between the ordinary particle and its supersymmetric partner has not yet been observed. This is clearly an indication that supersymmetry if exist, must be broken at low energies. Such that superpartners's masses are above the accessible energies in accelerator experiments. The simplest way to do this is by adding further terms to the Lagrangian that explicitly break supersymmetry and parametrize our ignorance of any underlying supersymmetry breaking mechanism. If only certain supersymmetry-breaking terms (gauge and Lorentz invariant) with dimensionful couplings are added to the Lagrangian, then the quadratic divergences still cancel but the mass degeneracy is removed, in that case supersymmetry is said to be *softly* broken. However, this set of soft supersymmetry breaking terms is not very appealing from a theoretical viewpoint because they are not only added by hand, they also violate the supersymmetry which with we started in the first place. Alternatively, if supersymmetry is an exact symmetry of nature and it is spontaneously broken, this means that the Lagrangian of the theory is invariant under supersymmetry transformations but the vacuum is not. On the other hand, the field that spontaneously breaks supersymmetry by acquiring a vacuum expectation value (VEV) has to be a SM singlet, otherwise the gauge symmetry would be simultaneously broken. Therefore it is necessary to introduce additional fields to the theory. Since these further fields do not interact (or have very tiny couplings) with the SM fields, they are said to be in the so-called *hidden sector*. This will naturally give rise to the question of how supersymmetry breaking is communicated from the hidden sector to the visible sector. Although there are various mechanism to explain such a transmission, a natural candidate is provided by gravitational interactions, which is known as *gravity mediation* and is reviewed in more details in section 6.3.1.

Even though a simple supersymmetric extension of the SM may provide an elegant resolution of the hierarchy problem and also potential candidates for Dark Matter, gravity is still missing within this framework. On the other hand, supersymmetric extensions of GR which is nothing but a local (gauge) generalization of supersymmetry also known as Supergravity, makes the UV divergences softer but the non-renormalizability still persists.

The need for a unifying theory

Despite the independent successes of GR and the SM in predicting physical processes in their respective domain of validity, they appear to be mutually incompatible, as can be intuitively checked from the field equations for the gravitational field:

$$G_{\mu\nu} \sim T_{\mu\nu} \tag{1.1}$$

The Einstein tensor on the left hand side encodes the classical geometry of space-time, while the energy momentum tensor on the right hand receives contributions from the quantised SM matter fields. This situation becomes unacceptable, in regimes where the interplay of quantum and gravitational effects become important, like early time cosmology or black hole physics. In these circumstances, one needs to combine both theories in order to describe the proper physics. However, finding such a unifying theory appears to be very challenging, the reason is concepts and observables that characterize both theories are strikingly different. The reason is clearly that the gravitational field is treated differently than the force and matter fields in the SM. A resolution of this clash of classical versus quantum mechanical physics is a prerequisite for the success of any unifying theory, from which the SM and GR must emerge as appropriate limits. Up to now, we do not know what this unifying theory is. Yet we have at least one possible candidate known as String Theory, which has been studied intensively from various directions in the last forty years. As we will see below, it has the potential to provide all the ingredients that shape our universe: gravity, gauge interactions, chiral fermions and scalar fields. For a more comprehensive introduction to the subject we recommend the textbook [2].

1.2 String Theory

String Theory postulates that the fundamental objects are one-dimensional objects (*strings* with a characteristic length $l_s = 2\pi\sqrt{\alpha'}$) rather than point-like particles, where α' is the Regge slope which is believed to be the only fundamental constant of the theory. As strings evolve in time, they sweep out a two-dimensional surface in spacetime known as the *worldsheet* of the string, instead of the worldline sweeped out by a point-like particle. Since strings can be either open or closed, one has various topologies for the *worldsheet* and string interactions are uniquely determined by the topology of the world-sheet. Moreover, the different vibrational massless modes of the fundamental strings are identified with different particles at low energies.

This simple idea leads to various interesting consequences. First, string interactions do not take place at a single point of spacetime but are smeared out into a region. Thus, the UV divergences encountered in the point-like description of QFT are removed. On the other hand, at large distances compared to the string scale, the spatial extension of strings effectively becomes a point, such that the resulting theory reproduces the particle behavior of QFT. Second, the consistency of the worldsheet theory imposes certain constraints on the dimensionality of space-time and this is something that no theory did before. Moreover, the theory dynamically determines its own coupling strength, with the latter being the expectation value of a scalar field known as *dilaton*.

The quantum mechanical consistency of the original theory also known as bosonic string theory, fixed the critical dimension of space-time to be $D = 26$. However, such a theory always contains a tachyon and lacks fermions in its spectrum, which renders the theory unrealistic. Tachyon-free string theories arise when introducing supersymmetry on the worldsheet. These string theory have a fixed spacetime dimension of $D = 10$ at weak coupling and exhibit also spacetime supersymmetry, such that they are known as superstring theories. Besides fundamental strings, the theory also contains extended p-dimensional objects called Dp-branes on which open strings can end, and carry basic charges of p-form gauge fields. A remarkable feature of these extended objects is that at arbitrarily large string coupling they become very light, even lighter than the fundamental strings and hence their behavior dominates the low energy dynamics.

If the fundamental scale is assumed to be close to the Planck scale, the massive string excitations whose masses are of order $k/\sqrt{\alpha'}$ are inaccessible at low energies. This implies that only the massless string excitations are reachable at low energies and therefore relevant for phenomenology. Therefore in order to obtain an effective description in terms of only massless string excitations, we must integrate out the massive string excitations. It was shown that the resulting effective theory is a ten-dimensional supergravity theory.

One interesting observation about string theory is that it was originally thought of a single, unifying theory but surprisingly there is not only one theory, instead, there are five consistent superstring theories, which are known as type I, type IIA, type IIB, heterotic $E_8 \times E_8$ and heterotic $SO(32)$. These superstring theories differ in their field content and the amount of supersymmetry preserved by the theory. The type II theories preserve $\mathcal{N} = 2$ supersymmetry and contain oriented closed strings, they differ by the kind of Dp-branes present in each theory. While heterotic theories preserve $\mathcal{N} = 1$ supersymmetry and only contain closed string yielding matter fields transforming either under $E_8 \times E_8$ and $SO(32)$. Whereas type I theory preserves $\mathcal{N} = 1$ supersymmetry and not only contain closed but also open strings.

The fact that there are five consistent superstring theories is somehow puzzling. However all these theories are not independent from each other, they are related through a web of dualities as displayed in figure 1.1[1]. In addition to those dualities, there is a generalization of T-duality known as *mirror symmetry*, which relates the physics of type IIA string theory compactified on a Calabi-Yau (CY) manifold $\mathcal{M}_6$ to that arising from type IIB string theory compactified on a mirror CY manifold $\tilde{\mathcal{M}}_6$. The existence of these dualities suggests that superstring theories might be unified into a single fundamental theory. Actually, they seem to be different limits of an eleven-dimensional theory known as M-theory, which is not fully understood up to date. However, there exist a unique supergravity theory in eleven dimensions, which can be interpreted as the low-energy limit of M-theory.

1.2.1 String compactifications

As mentioned in the previous section, superstring theories are consistently defined in ten-dimensions, but the observable world looks four-dimensional. Facing this problem, one has two options: Either we discard superstring theory completely and continue searching for a new fundamental theory; or requiring that six of the ten dimensions are compactified into a sufficiently small space, such that extra dimensions have avoided detection in high energy experiments so far. The latter

[1] A duality connects two apparently different theories which describe the same physics and hence they should not really be regarded as distinct theories.

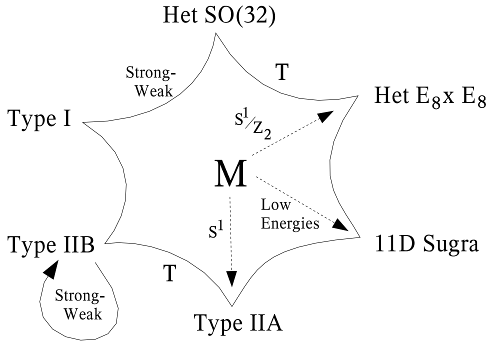

Figure 1.1: *The duality web of Superstring Theories.*

approach implies that the theory lives in a spacetime of the form

$$\mathcal{M}_{10} = \mathbb{M}^{3,1} \times \mathcal{M}_6 \tag{1.2}$$

where $\mathbb{M}^{3,1}$ represents a maximally symmetric four-dimensional space, i.e. de-Sitter (dS), Anti-deSitter (AdS) or Minkowski space, while $\mathcal{M}_6$ is some compact six-dimensional manifold. This strategy is known as compactification of string theory on $\mathcal{M}_6$ or string compactification for short. The size of compact space $\mathcal{M}_6$ could be of the same order of the string scale and highly curved. However, the known techniques to study such situations are very limited up to date. Therefore, the best-understood situation is provided by compactifications where the size of the space $\mathcal{M}_6$ is large compared to the fundamental scale and supersymmetry is broken at some scale below the compactification scale. In this regime, one can use the low energy supergravity description of superstring theory.

To obtain a four-dimensional effective theory one has to integrate the ten-dimensional theory over $\mathcal{M}_6$: this procedure known as dimensional reduction comes from an old idea dating back to Kaluza and Klein in the early 1920s. Their original idea was to unify gravity and electromagnetism through the addition of a tiny rolled-up fifth dimension. This unified theory takes an explicit form when considering five-dimensional GR on a space-time of the form $\mathbb{M}^{3,1} \times \mathbf{S}^1$, where the extra dimension is compactified on a circle of radius R. Although this theory had no phenomenological success, it showed us how the topology of the extra-dimensional space determines decisively the resulting four-dimensional physics, namely the matter content and forces. Although string compactifications involve a six-dimensional space (typically CY manifolds) which are much more complicated than a simple $\mathbf{S}^1$, the essence of this procedure is nevertheless the same, differing only in technical details. In the next chapter we will discuss some basic aspects of the geometry of CY manifolds and perform explicitly the dimensional reduction of type IIA string theory on them.

1.2.2 Moduli stabilisation

As will see in the next chapter, string compactifications typically introduce a huge amount of massless neutral fields which parametrize a continuous family of unequivalent *vacua*. These fields are often called *moduli* and their field space is known as the *moduli space*. It is useful to distinguish the different kinds of moduli present in generic string compactifications. The most obvious moduli are those parametrizing either the size or shape of subspaces on the internal space and are known as geometric moduli. Besides these fields, further moduli arise from expanding the p-form gauge potentials C_p in the appropriate bases of harmonic forms. Note that the transformation $C_p \to C_p + c_i\omega_p^i$ where ω_p^i are harmonic p-forms, leaves the field strength $G_{p+1} = dC_p$ unaltered and hence not affect the energy. These moduli enjoy a continuous shift symmetry $c_i \to c_i + a_i$ descending from the corresponding p-form gauge invariance of the ten-dimensional theory and therefore we refer to them as axions. The continuous shift symmetry holds to all orders in perturbation theory, but is broken nonperturbatively, by D-brane instantons. As a consequence, what remains is a discrete symmetry $c_i \to c_i + 2\pi$. Finally, there can be additional moduli associated with the positions of D-branes in the compactification manifold, which are called D-brane position moduli.

These moduli cannot remain massless for various reasons. First of all, we have not observed massless scalar fields in collider experiments, which implies that, if they exist, they should have masses above the accessible energies in current high energy experiments. Secondly, the presence of massless scalar fields in the low-energy effective theory would lead to deviations of the gravitational force law which has not been observed. For these reasons, moduli must get a mass sufficiently large in order to obtain a realistic four-dimensional vacua and the process by which moduli become massive is known as *moduli stabilization*. However, this is a non-trivial task and requires further refinements of the standard string compactifications. A lot of efforts have been made to find controllable mechanisms to stabilize moduli in string compactifications. A promising approach to deal with moduli stabilisation for all closed string moduli is provided by flux compactifications [3, 9, 10, 48], in which the inclusion of background fluxes generates scalar potentials for the moduli, allowing the uplifting of the undesired massless fields.

1.2.3 The string landscape

String compactifications immediately lead to other major problem, which is related to the fact that superstring theories allow for a huge number of consistent choices of the compact manifold $\mathcal{M}_6$. Since the resulting four-dimensional effective theory depends strongly on the chosen manifold, this naturally leads to an incredibly large set of effective theories. In addition to this ambiguity, masses and coupling constants appearing in the low energy effective theory are functions of the moduli. Although background fluxes generate potentials that stabilize moduli, fluxes obey a Dirac quantization condition and therefore take discrete values, which add to other discrete parameters of the compactification, such as D-brane charges. Consequently, the four-dimensional effective theory also depends on the choice of these discrete parameters and thus enlarging even more the ensemble of effective theories.

In general, each of these effective theories has a large discrete set of stable solutions known as four-dimensional string vacua, which interpreted as a set form the so-called *String Landscape*. A naive estimation of the number of type IIB flux vacua leads to $N_{\text{vacua}} \sim 10^{500}$ [4, 5] although in recent years this number has grown to $N_{\text{vacua}} \sim 10^{272000}$ in F-theory, the strong coupling limit of Type IIB string theory [6]. Depending on the chosen vacuum, physics may look one way or another, this does not mean that the fundamental laws that govern all of them are essentially different, they just may look different at low energies[2]. This large number of possibilities gives rise to the question whether among these vacua there is at least one that describes our universe, and if exist, is it picked out by some special mathematical property or is it just the result of some anthropic principle? Despite many efforts that have been devoted to understanding the string landscape, it is still poorly understood. Up to date, there is no known example of a stringy vacuum that accurately describes our universe and no successful vacuum selection principle has been proposed. In the absence of such a vacuum selection principle, one might tackle those questions by studying the statistical properties of the string landscape for extracting some insight, which can give us some indications on which region one should concentrate on to hopefully find realistic vacua [7].

Finally, even if our universe does not reside in this flux landscape, it would not necessarily imply that the theory is incorrect, since moduli stabilization by fluxes occurs within the effective supergravity approach, which is valid only at large volume (such that α'-corrections can be neglected) and small string coupling, implying that those vacua are found within some approximations and hence they constitute only a limited corner of the full landscape of string vacua. It could well be that our world lies outside this corner

1.3 Overview of the thesis

After this brief general introduction let us now turn to the concrete subjects of this thesis. The first part of the thesis contains some introductory chapters which provide the basic ideas as well as the mathematical tools that will be used

[2]This means that quantities as gauge group, the number of fermion generations, the value of the Cosmological Constant, etc, all depend on the particular vacuum selected.

throughout the thesis.

In chapter 2 we briefly review some basic aspects of type IIA flux compactifications. We start by reviewing compactifications of type IIA on Calabi-Yau three-folds in the absence of background fluxes and discuss the resulting $\mathcal{N} = 2$ effective action in terms of geometrical data of the CY manifold. Afterwards, we turn to type IIA compactifications on CY orientifolds, which are constructed as the quotient manifold of a Calabi-Yau three-fold modded out by the orientifold action. We discuss the orientifold projection which consists of perturbative symmetries of the type IIA string theory and whose presence projects out part of $\mathcal{N} = 2$ spectrum, allowing us to end up with a $\mathcal{N} = 1$ effective action. Then, we additionally allow for non-trivial background fluxes and discuss the superpotential that encodes type IIA flux potentials. Afterward we also add D6-branes hosting open string moduli to flux compactifications and discuss how the presence of such moduli modify the 4d effective action. Finally, we end up the chapter with a short review on the α'-corrections to the Kähler potential, which have to be taken into account in regions of the moduli space away from the large volume limit.

In chapter 3 we briefly present the axion-polynomial description of type IIA Calabi-Yau orientifolds. We start describing purely closed string compactifications in the axion polynomial language in section 3.2. These axion polynomials capture the axionic partners together with the flux quanta into shift-invariant combinations whose precise shapes are intimately connected to Freed-Witten anomaly cancelation. In section 3.4 we immediately turn to the axion polynomial formulation of type IIA flux vacua with D6-branes hosting open string moduli. In section 3.5 we present how perturbative α'-corrections also fit into the axion polynomial formalism. Finally, in section 3.6 we rewrite the potential generated by fluxes and D6-branes as bilinear of the flux-axion polynomials.

The aim of the second part of the thesis is to rephrase moduli stabilisation in the language of the axion polynomials invariant under the discrete shift symmetries of the four-dimensional effective theory, showing that such invariants provide an alternative and powerful method to search for vacua.

In chapter 4, we start rewriting two well-known examples of pure-closed string vacua in this language. More precisely, in section 4.1.1 we show that by solving the F- terms conditions for the dilaton and Kähler moduli in terms of the axion polynomials we are able to recover the conditions for $\mathcal{N} = 0$ Minkowski vacua, while the conditions for $\mathcal{N} = 1$ AdS vacua in terms of these polynomials are obtained in section 4.1.2. In section 4.3 we add mobile D6-branes to backgrounds with ISD fluxes and find a new and more general class of non-supersymmetric Minkowski vacua, with the open string moduli stabilised at non-trivial vevs. Finally, in section 4.4 we present a more direct approach to analyse the appearance of semi-definite positive scalar potentials and the corresponding Minkowski vacua.

In chapter 5, we turn to moduli stabilisation in the regime of moderately large volumes where α'-corrections must be taken into account. In section 5.1 we introduce the effect of perturbative α'-corrections in the Kähler sector and compute the resulting F-term scalar potential, again rewriting it in terms of α'-corrected axion polynomials. In sections 5.2 we use such invariants to express in a compact and simple way the conditions for $\mathcal{N}=0$ Minkowski and extract the effect of α'-corrections on them, reproducing the results of [8], while the condition for $\mathcal{N}=1$ AdS flux vacua are obtained in section 5.3, thus finding the α'-corrected version of the AdS vacua found in [9].

In the third part, we will present some phenomenological applications of our work. For this purpose, the low-energy effective actions for the matter fields living at the intersections of D6-branes are of special interest and in particular the terms arising from supersymmetry breaking. In chapter 6 we turn to study some physical observables arising from flux vacua in which supersymmetry is spontaneously broken in moduli sector. In section 6.2 we start rewriting the apparent and effective gravitino mass in term of the flux-axion polynomials and compute the resulting gravitino masses in the vacua studied in previous chapters. In section 6.3 we study the pattern of soft terms emerging from $\mathcal{N}=0$ Minkowski vacua with D6-branes. Finally, in section 6.4 we discuss the various energy scales present in the type IIA flux landscape to make sure the validity of our approach.

In chapter 7 we turn to the construction of a consistent supersymmetric DFSZ axion model in type IIA orientifolds with background fluxes and intersecting $D6$-branes. In section 7.1 we briefly review some basic aspects of the DFSZ axion model, while a consistent supersymmetric extension of this model is discussed in section 7.2. Section 7.3 contains a discussion of the various types of closed and open string axions present in type IIA compactifications and how one of them might be identified with the QCD axion. In section 7.5 we review some essential aspects of the $\mathbf{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifolds with intersecting D6-branes, and discuss their spectrum. Section 7.6 contains an explicit example of consistent supersymmetric DFSZ model on this toroidal setup. The structure of the flux-induced soft-terms in the Higgs-axion sector of the model, is discussed in section 7.6.1.

In the last part of the thesis, I present the conclusions and discuss the prospects for future work. Finally, several technical details have been relegated to the appendices. In appendix A we collect some properties and relations of the Kähler metrics on type IIA Calabi-Yau compactifications with mobile D6-branes. In Appendix B we discuss the type IIA superpotential in the presence of mobile D6-branes, and how this allows to deduce the redefinition of complex structure moduli by open string moduli. We relegate to Appendix C the technical details regarding the computations of the α'-corrected scalar potential. In Appendix D we discuss the Kähler metrics for open string fields at D6-branes intersections on toroidal backgrounds.

Part I

Type IIA flux compactifications with D6-branes and four-dimensional effective actions

Chapter 2

Type IIA flux compactifications: A Brief Review

The aim of this chapter is to briefly review type IIA flux compactifications with an underlying Calabi-Yau geometry and discuss the low energy effective action arising from this kind of setup, following mostly [10, 11]. For a nice review on string compactifications the reader may be also referred to [12–14].

2.1 Supersymmetry

As already mentioned, supersymmetric string theories live on a spacetime of the form 1.2. So far we have only required $\mathcal{M}_6$ to be compact and sufficiently small such that is not detectable in current experiments, however further constraints on $\mathcal{M}_6$ have to be imposed. Note that the structure of spacetime 1.2 implies that the 10-dimensional Lorentz group decomposes as $SO(9,1) \to SO(3,1) \times SO(6)$, therefore it forces $\mathcal{M}_6$ to have a $SO(6)$ group structure. Moreover, there are various reasons to consider string compactifications that preserve supersymmetry: such compactifications are under much better control and do not contain tachyons. On the other hand supersymmetric backgrounds allow to build phenomenologically interesting models of particle physics. Therefore an additional constraint on $\mathcal{M}_6$ comes from requiring to have some unbroken supersymmetry at the compactification scale.

The condition that compactification on $\mathcal{M}_6$ preserves some supersymmetry actually splits into two conditions, the first is related to the existence of globally well-defined spinors on $\mathcal{M}_6$ and the second that they are covariantly constant. Note that, globally well-defined spinors only exist on manifolds that have reduced structure, while the existence of covariantly constant on $\mathcal{M}_6$ is determined by its holonomy group Hol($\mathcal{M}_6$). This implies that the holonomy group of $\mathcal{M}_6$ must be a subgroup of $SO(6)$. In addition, Hol($\mathcal{M}_6$) must be one under which the decomposition of a spinor **4** of $SO(6)$ contains a singlet. When combining all these elements one finds that $\mathcal{M}_6$ has to be a manifolds of $SU(3)$ holonomy[1]. The spaces which admit a metric with this special holonomy are also known as Calabi-Yau manifolds. All these reasons make Calabi-Yau manifolds a suitable candidate for the compactification space.

2.2 Calabi-Yau manifolds

Before starting to review type IIA compactifications on Calabi-Yau manifolds, let first us give some basic definitions on this kind of manifolds. Calabi-Yau manifolds or CY N-folds for short are a particular class of Kähler manifolds. Strictly speaking, a CY N-fold of complex dimension N is a compact Kähler manifold with zero Ricci form, vanishing first Chern class and that has $SU(N)$ holonomy. It is not difficult to show that a compact Ricci-flat Kähler manifold automatically has vanishing first Chern class. The first Chern class of a manifold $\mathcal{M}_{2N}$ is defined as the first Chern class of the holomorphic tangent bundle

$$c_1(\mathcal{M}_{2N}) = \frac{1}{2\pi}\mathrm{Tr}R \tag{2.1}$$

where R is the Ricci two-form on $\mathcal{M}_{2N}$. This means that the Ricci form defines the first Chern class of the manifold and therefore it is easy to see that if a Kähler manifold admits a Ricci-flat metric then it has vanishing first Chern class. However, the converse, does a Kähler manifold with vanishing first Chern class admit a Ricci-flat metric, is much harder to prove. This was conjectured by Calabi and proved by Yau twenty years later. According to Yau's proof, given a complex manifold with a Kähler metric g, a Kähler form J and vanishing first Chern class, then there exists a unique Ricci-flat metric $\tilde{g}$, whose Kähler form $\tilde{J}$ is in the same Kähler class as J. The usefulness of Yau's theorem is that it is quite hard to directly determine whether or not $\mathcal{M}_{2N}$ admits a Ricci-flat metric. In fact, no explicit Ricci-flat metrics are known on any CY manifolds. However, it is quite simple to compute the first Chern class of $\mathcal{M}_{2N}$, and therefore to find examples with vanishing first Chern class.

Any $2N$-dimensional real manifold $\mathcal{M}_{2N}$ is an almost complex manifold whether it admits an almost complex structure I_m^n satisfying $I_m^n I_n^l = -\delta_m^l$ which is

[1]The spinor representation **4** of $SO(6)$ can be decomposed in representations of $SU(3)$ as $\mathbf{4} \to \mathbf{3} \oplus \mathbf{1}$. Since there is a $SU(3)$ singlet in the decomposition, then there exists a spinor that depends trivially on the tangent bundle of $\mathcal{M}_6$ and therefore it is well-defined and non-vanishing.

nothing but the map used to define local complex coordinates $dz^n = dx^n + i\, I^n_m\, dy^m$ that can be patched together globally in a consistent way[2]. In these complex coordinates, the Kähler form reads

$$J = g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}} \tag{2.2}$$

Moreover, any CY manifold admits a nowhere vanishing holomorphic $(N,0)$-form

$$\Omega = f(z^1, ..., z^N) dz^1 \wedge ....dz^N \tag{2.3}$$

obeying the algebraic conditions

$$J \wedge \Omega = 0, \qquad \frac{1}{N!} J^N = \left(\frac{i}{2}\right)^N \Omega \wedge \bar{\Omega} \tag{2.4}$$

In complex manifolds, p-forms and their cohomology classes are classified according to their number of holomorphic and antiholomorphic indices. More precisely, harmonic forms with p holomorphic and q antiholomorphic indices are representative of the cohomology class $H^{(p,q)}(\mathcal{M}_6)$. The dimensions of the cohomology groups are given by the Hodge numbers $h^{(p,q)} = \dim H^{(p,q)}(\mathcal{M}_6, \mathbb{Z})$ and these numbers are usually arranged in the so-called Hodge diamond.

From now on we only focus on CY three-folds, which is the case we are interested in, but most results can be extended straighforwardly to higher dimensional CY manifolds. For a CY three-fold, the Hodge diamond has the following structure

$$\begin{array}{ccccccc} & & & h^{(0,0)} & & & \\ & & h^{(1,0)} & & h^{(0,1)} & & \\ & h^{(2,0)} & & h^{(1,1)} & & h^{(0,2)} & \\ h^{(3,0)} & & h^{(2,1)} & & h^{(1,2)} & & h^{(0,3)} \\ & h^{(3,1)} & & h^{(2,2)} & & h^{(1,3)} & \\ & & h^{(3,2)} & & h^{(2,3)} & & \\ & & & h^{(3,3)} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{(1,1)} & & 0 & \\ 1 & & h^{(2,1)} & & h^{(2,1)} & & 1 \\ & 0 & & h^{(1,1)} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} . \tag{2.5}$$

All Hodge numbers on the left hand side of 2.5 are not independent each other, they are related by: complex conjugation $h^{(p,q)} = h^{(q,p)}$, Hodge duality $h^{(p,q)} = h^{(n-p,n-q)}$ and holomorpic duality $h^{(p,0)} = h^{(3-p,0)}$, $h^{(0,q)} = h^{(0,3-q)}$. Let us now introduce a basis for the various cohomology groups. The basis of 2-forms $l_s^{-2}\,\omega_A$ correspond to harmonic representatives of the classes in $H^{(1,1)}(\mathcal{M}_6, \mathbb{Z})$, which is dual to the basis $l_s^{-4}\tilde{\omega}^A$ of $H^{(2,2)}(\mathcal{M}_6, \mathbb{Z})$ and are dimensionless due to the

[2]Strictly speaking I^n_m is a generalization of the usual multiplication by $\pm i$ in complex analysis, it gives to each tangent space $T_p\,\mathcal{M}_{2N}$ to a point p in $\mathcal{M}_{2N}$ the structure of a complex vector space.

insertion of the string length l_s. While $(\alpha_\alpha, \beta^\kappa)$ forms a real symplectic basis of $H^{(3)}(\mathcal{M}_6, \mathbb{Z})$. The basis elements are chosen to satisfy

$$\begin{aligned} l_s^{-6} \int_{\mathcal{M}_6} \omega_K \wedge \tilde{\omega}^L &= \delta_K^L, \qquad K, L = 1, ...h^{(1,1)} \\ l_s^{-6} \int_{\mathcal{M}_6} \alpha_\alpha \wedge \beta^\kappa &= \delta_\alpha^\kappa, \qquad \alpha, \beta = 1, ...h^{(2,1)} \end{aligned} \tag{2.6}$$

with the remaining intersections being trivial. All non-trivial cohomology groups of CY three-folds and their basis elements are summarized in table 2.1. As we

cohomology group	dimension	basis
$H^{(1,1)}(\mathcal{M}_6, \mathbb{Z})$	$h^{(1,1)}$	ω_A
$H^{(2,2)}(\mathcal{M}_6, \mathbb{Z})$	$h^{(1,1)}$	$\tilde{\omega}^A$
$H^{(3)}(\mathcal{M}_6, \mathbb{Z})$	$2h^{(2,1)} + 2$	$(\alpha_\alpha, \beta^\kappa)$
$H^{(2,1)}(\mathcal{M}_6, \mathbb{Z})$	$h^{(2,1)}$	χ_K

Table 2.1: *Basis of harmonic forms in a CY three-fold and their basis elements.*

will discuss below, the four-dimensional massless modes of each ten-dimensional p-form gauge potential are in one-to-one correspondence with the harmonic forms on the internal space $\mathcal{M}_6$ and therefore their multiplicity is counted by the dimension of the non-trivial cohomolgy groups.

2.2.1 Moduli Space of CY three-folds

In the previous section, we briefly reviewed some basic aspects of the geometry and topology of CY manifolds. The aim of this section is to show that each of those CY three-folds is actually part of a continuous family of CY manifolds. As already mentioned, any CY manifold admits a metric g such that $R_{i\bar{j}}(g) = 0$. Given such a Ricci-flat metric, one can continuously perturb to a new metric $g+\delta g$ that also preserves the Ricci-flatness. There are two basic types of perturbations δg: those with pure and those with mixed type indices

$$\delta g = \delta g_{ij} dz^i dz^j + \delta g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} + c.c. \tag{2.7}$$

Since g is a Hermitian metric, perturbations with mixed type indices preserve the original index structure of g while those of pure type do not. The Ricci-flatness condition $R_{i\bar{j}}(g + \delta g) = 0$ implies that δg satisfies the Lichnerowicz equation

$$\nabla^q \nabla_q \delta g_{ij} + R_{ij}^{qr} \delta g_{qr} = 0 \tag{2.8}$$

which imposes strong restrictions on δg. In particular, it turns out that $\delta g_{i\bar{j}} dz^i \wedge \bar{z}^{\bar{j}}$ must be harmonic and hence it is uniquely associated to an element of $H^{(1,1)}(\mathcal{M}_6, \mathbb{Z})$. On the other hand, using the holomorphic three-form Ω_3, it can also be shown that $\Omega_{ijk} g^{k\bar{k}} \delta_{\bar{k}\bar{l}} dz^i \wedge dz^j \wedge dz^l$ is an element of $H^{(2,1)}(\mathcal{M}_6, \mathbb{Z})$. Hence, these two cohomology groups are associated with the space of deformations of an initial Ricci-flat metric on $\mathcal{M}_6$ to a nearby Ricci-flat metric. More precisely, deformations of the metric with mixed type indices correspond to deformations of the Kähler class J of $\mathcal{M}_6$, such deformations can be expanded in the basis of $H^{(1,1)}(\mathcal{M}_6, \mathbb{Z})$ as

$$J = t^A \omega_A, \qquad A = 1, ..., h^{(1,1)} \tag{2.9}$$

which gives rise to $h^{(1,1)}$ real scalars t^A that parametrize the sizes of 2-cycles of the internal manifold. These real deformations fit together with the real scalars b^A arising in the expansion of the B-field presents in Type IIA string theory, whose massless fluctuations are also expanded as $B_2 = b^A \omega_A$, to form $h^{(1,1)}$ complex scalar T^A defined through

$$J_c = B_2 + i e^{\frac{1}{2}\phi} J = T^A \omega_A \tag{2.10}$$

which are known as Kähler moduli. The additional insertion of the ten-dimensional dilaton ϕ indicates that the Kähler form J is expressed in the Einstein frame[3]. The complex fields T^A span the Kähler moduli space $\mathfrak{M}_K$ of the Calabi-Yau three-fold, which is a spacial Kähler manifold with Kähler potential

$$K_T = -\log\left(\frac{4}{3}\int_{\mathcal{M}_6} e^{\frac{3\phi}{2}} J \wedge J \wedge J\right) = -\log\left(\frac{i}{6}\mathcal{K}_{ABC}(T^A - \overline{T}^A)(T^B - \overline{T}^B)(T^C - \overline{T}^C)\right) \tag{2.11}$$

which is a cubic polynomial in $t^A = \mathrm{Im}(T^A)$, while B_2-axions do not enter in the Kähler potential. In the above Kähler potential we have used the dimensionless triple intersection numbers of the Calabi-Yau manifold

$$\mathcal{K}_{ABC} = \ell_s^{-6} \int_{\mathcal{M}_6} \omega_A \wedge \omega_B \wedge \omega_C, \tag{2.12}$$

The metric on $\mathfrak{M}_K$ can be straightforwardly computed from the Kähler potential (2.11) through [15]

$$G_{A\bar{B}} = \partial_{T^A} \partial_{\bar{T}^B} K_T = -\frac{3}{2}\left(\frac{\mathcal{K}_{AB}}{\mathcal{K}} - \frac{3}{2}\frac{\mathcal{K}_A \mathcal{K}_B}{\mathcal{K}^2}\right) = \frac{3e^{-\phi}}{2\mathcal{K}\ell_s^6}\int_{\mathcal{M}_6} \omega_A \wedge \star_6 \omega_B \tag{2.13}$$

[3]Throughout this thesis, we use the Einstein frame Kähler form J, which is related to Kähler form measured in the string frame J_s used in [10] by the relation $J = e^{-\frac{\phi}{2}} J_s$.

To simplify the above expressions we have used the following notation for the contractions of the intersection numbers with the Einstein-frame Kähler form

$$\mathcal{K} = \ell_s^{-6} \int_{\mathcal{M}_6} J \wedge J \wedge J = \mathcal{K}_{ABC} t^A t^B t^C, \quad \mathcal{K}_A = \ell_s^{-6} \int_{\mathcal{M}_6} \omega_A \wedge J \wedge J = \mathcal{K}_{ABC} t^B t^C$$
$$\mathcal{K}_{AB} = \ell_s^{-6} \int_{\mathcal{M}_6} \omega_A \wedge \omega_B \wedge J = \mathcal{K}_{ABC} t^C,$$

Furthermore, the function $\mathcal{G}_T = e^{-K_T}$ is a homogenous function of degree three in the geometric Kähler moduli t^A, which implies a no-scale condition for the Kähler potential K_T:

$$(K_T)_A (K_T)^{A\overline{B}} (K_T)_{\overline{B}} = 3. \tag{2.14}$$

The homogeneity of the function $\mathcal{G}_T$ also implies that $\mathcal{G}_T$ can be derived from a holomorphic pre-potential $\mathcal{F}$ by the relation:

$$\mathcal{G}_T = i \left(\overline{T}^A \mathcal{F}_{T^A} - T^A \overline{\mathcal{F}}_{\overline{T}^A} \right)_{T^0=1}, \tag{2.15}$$

In order to work with homogeneous (projective) coordinates on the Kähler moduli space we have included a complex coordinate T^0 which we set to unity after differentiation of the prepotential[4]. One can then easily check that the Kähler potential (2.11) results from the (tree-level) holomorphic pre-potential, valid at large internal volumes:

$$\mathcal{F}_{\text{tree}}(T) = -\frac{1}{3!} \frac{\mathcal{K}_{ABC} T^A T^B T^C}{T^0}. \tag{2.17}$$

In section 2.7, we will discuss perturbative α'-corrections to this pre-potential, which have to be taken into account in regions of the moduli space away from the large volume limit.

On the other hand, deformations of the metric with pure type indices correspond to deformations of the complex structure. To understand this, we first note that such deformations yield a metric which is no longer Hermitian, but it can be put back into Hermitian form by a suitable change of variables. However, this change of variables is necessarily not holomorphic as holomorphic coordinate changes cannot affect the index structure of a tensor. Hence the new metric is Hermitian with respect to a different complex structure. Those deformations of

[4]In case one prefers to work in the affine coordinate patch $(1, T^a)$, the relation between the homogenous function $\mathcal{G}_T$ and the pre-potential has to be properly adjusted:

$$-i\,\mathcal{G}_T = 2\mathcal{F} - 2\overline{\mathcal{F}} - (T^a - \overline{T}^a) \left(\frac{\partial \mathcal{F}}{\partial T^a} + \frac{\partial \overline{\mathcal{F}}}{\partial \overline{T}^a} \right). \tag{2.16}$$

the complex structure of $\mathcal{M}_6$ are parameterized by $h^{2,1}$ complex scalar fields z^κ, which are in one-to-one correspondence with harmonic $(2,1)$-forms through

$$\delta g_{\bar{i}\bar{j}} = -\frac{i}{||\Omega||^2}\, z^\kappa (\chi_\kappa)_{kl\bar{j}}\, \bar{\Omega}^{kl}_{\bar{i}} \ , \quad \kappa = 1, \dots, h^{(2,1)} \ , \tag{2.18}$$

where $||\Omega||^2 = \frac{1}{3!}\Omega_{ijk}\bar{\Omega}^{\bar{i}\bar{j}\bar{k}}$ and χ_κ form a basis of $H^{(2,1)}(\mathcal{M}_6, \mathbb{Z})$ which are related to the variation of Ω by mean of Kodaira's equation

$$\chi_\kappa = \partial_{z^\kappa}\Omega(z) + \Omega(z)\partial_{z^\kappa} K^{\rm cs} \tag{2.19}$$

As we argue in the next section, complex structure deformations z^κ fit together with C_3-axions to form the proper $\mathcal{N}=2$ chiral multiplets. The complex fields z^K span the complex structure moduli space $\mathfrak{M}_{\rm cs}$ of the Calabi-Yau three-fold, which also is a spacial Kähler manifold with Kähler potential

$$K_{cs} = -\log\left(\frac{i}{\ell_s^6}\int_{\mathcal{M}_6} \Omega_3 \wedge \overline{\Omega}_3\right) = -\log i\left(\overline{Z}^\kappa \mathcal{F}_\kappa - Z^\kappa \overline{\mathcal{F}}_\kappa\right). \tag{2.20}$$

with $(Z^\kappa, \mathcal{F}_\kappa)$ being the holomorphic periods depending only on the complex structure moduli

$$Z^\kappa(z) = \int_{\mathcal{M}_6} \Omega(z)\wedge\beta^\kappa \ , \qquad \mathcal{F}_\kappa(z) = \int_{\mathcal{M}_6}\Omega(z)\wedge\alpha_\kappa \ , \tag{2.21}$$

The periods $\mathcal{F}_\kappa$ play the role of first order derivatives of a pre-potential $\mathcal{F}^{cs}$ with respect to the periods Z^κ. More precisely, the periods $\mathcal{F}_K$ can be seen as homogeneous functions of degree one in the homogeneous projective coordinates Z^κ, such that the pre-potential $\mathcal{F}^{cs} = \frac{1}{2}Z^\kappa\mathcal{F}_\kappa$ is a homogeneous function of degree two. This implies that the holomorphic three-form Ω_3 admits an expansion of the form

$$\Omega_3(z) = Z^\kappa(z)\,\alpha_\kappa - \mathcal{F}_\kappa(z)\,\beta^\kappa \tag{2.22}$$

The metric on $\mathfrak{M}_{\rm cs}$ can be straightforwardly computed from the Kähler potential (2.20) and is given by [15]

$$G_{\alpha\bar{\kappa}} = \partial_{z^\alpha}\partial_{\bar{z}^\kappa} K_{\rm cs} = -\frac{\int_{\mathcal{M}_6}\chi_\alpha\wedge\bar{\chi}_\kappa}{\int_{\mathcal{M}_6}\Omega_3\wedge\bar{\Omega}_3} \tag{2.23}$$

As is well-known, the holomorphic three-form Ω_3 is only determined up to a complex rescaling $e^{-h(z)}$, which changes the Kähler potential (2.20) by a Kähler transformation

$$\Omega \to \Omega\, e^{-h(z)} \quad \Longrightarrow \quad \mathcal{K}_{\rm cs} \to \mathcal{K}_{\rm cs} + h + \bar{h} \tag{2.24}$$

By virtue of this rescaling symmetry, we can set one of the periods to one (usually denoted by Z^0) and work in an affine coordinate patch in which the complex structure deformations can be identified with the remaining $h^{(1,2)}$ periods as $z^\kappa = Z^\kappa/Z^0$. Consequently, the full moduli space of the compactification corresponds to the direct product of both special Kähler manifolds

$$\mathfrak{M}_K \times \mathfrak{M}_{\rm cs} \tag{2.25}$$

2.3 Type IIA compactifications on Calabi-Yau three-folds

In this section we review compactifications of the ten-dimensional type IIA supergravity on a Calabi-Yau three-fold in the absence of background fluxes. It is a $\mathcal{N}=2$ supersymmetric theory in ten dimensions, which is naturally obtained as the low energy limit of type IIA superstring theory[5]. In the following we will consider only the bosonic massless spectrum, which consists of two sectors. On the one hand, one has the dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{g}_{MN}$ and the two-form $\hat{B}_2$ in the NS-NS sector, while the gauge fields $\hat{C}_1$ and $\hat{C}_3$ arise in the R-R sector. Thus, our starting point is the ten-dimensional type IIA supergravity action in the Einstein frame

$$\begin{aligned} S^{(10)}_{\rm IIA} &= \int -\frac{1}{2}\hat{R}\star\mathbf{1} - \frac{1}{4}d\hat{\phi}\wedge\star d\hat{\phi} - \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3\wedge\star\hat{H}_3 - \frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\hat{F}_2\wedge\star\hat{F}_2 \\ &\quad -\frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\hat{F}_4\wedge\star\hat{F}_4 - \frac{1}{2}\hat{B}_2\wedge\hat{F}_4\wedge\hat{F}_4\ , \end{aligned} \tag{2.26}$$

where the NS and RR field strengths are defined as

$$\hat{H}_3 = d\hat{B}_2\ ,\quad \hat{F}_2 = d\hat{C}_1\ ,\quad \hat{F}_4 = d\hat{C}_3 - \hat{C}_1\wedge\hat{H}_3\ . \tag{2.27}$$

In order to obtain the four-dimensional effective theory, one has to perform a Kaluza-Klein reduction of the ten-dimensional fields. Upon dimensional reduction, the four-dimensionsional fields come from the zero modes of the ten-dimensional fields. It is well-known that there is one-to-one correspondence between zero modes of differential operators on a given space and the harmonic forms of that space and therefore their multiplicity is counted by the dimension of the cohomology groups of $\mathcal{M}_6$. Accordingly we expand the ten-dimensional fields in terms of harmonic forms on $\mathcal{M}_6$ as follows

$$\begin{aligned} \hat{C}_1 &= A^0(x)\ , \qquad \hat{B}_2 = B_2(x) + b^A(x)\,\omega_A\ ,\quad a = 1,\ldots,h^{(1,1)}\ , \\ \hat{C}_3 &= A^A(x)\wedge\omega_A + \xi^\kappa(x)\,\alpha_\kappa - \tilde{\xi}_\kappa(x)\,\beta^\kappa\ ,\quad \kappa = 0,\ldots,h^{(2,1)}\ . \end{aligned} \tag{2.28}$$

where the four-dimensional fields $b^A, \xi^\kappa, \tilde{\xi}_\kappa$ are scalars, A^0, A^A are four-dimensional one-forms, while $B_2(x)$ is a four-dimensional two-form. These massless fields fit together with the ones encoding deformations of the geometry to form $\mathcal{N}=2$ multiplets, which are summarized in table 2.2. Note that the two-form $B_2(x)$ can be dualised to a scalar a which results in one further hypermultiplet.

Inserting the field expansions 2.28 into the ten-dimensional Type IIA action 2.26 and integrating over the CY manifold, one obtains a standard four-

[5] For a nice review of $\mathcal{N}=2$ supergravity see [16]

Multiplet	Multiplicity	Bosonic components
gravity	1	$(g_{\mu\nu}, A^0)$
vector	$h^{(1,1)}$	(A^A, t^A, b^A)
hyper	$h^{(2,1)}$	$(z^\kappa, \xi^\kappa, \tilde{\xi}_\kappa)$
tensor	1	$(B_2(x), \phi, \xi^0, \tilde{\xi}_0)$

Table 2.2: *Bosonic components of the $\mathcal{N}=2$ multiplets for Type IIA supergravity compactified on CY three-folds.*

dimensional $\mathcal{N}=2$ ungauged supergravity action [17, 18]

$$\begin{aligned} S^{(4)}_{\text{IIA}} &= \int -\frac{1}{2}R * \mathbf{1} + \frac{1}{2}\text{Im}\,\mathcal{N}_{AB}\, F^A \wedge *F^B + \frac{1}{2}\text{Re}\,\mathcal{N}_{AB}\, F^A \wedge F^B \\ &\quad -G_{A\bar{B}}\, dT^A \wedge *d\bar{T}^B - h_{uv}\, d\tilde{q}^u \wedge *d\tilde{q}^v \ , \end{aligned} \tag{2.29}$$

In the gauge kinetic part the field strengths $F^A = dA^A$, while $G_{A\bar{B}}$ is the metric on the Kähler moduli space given in (2.13) and $\mathcal{N}_{AB}$ is the gauge-kinetic coupling matrix that encodes the couplings of the vector multiplets and can be directly computed from the holomorphic prepotential (2.17) by using equation (C.14) and its explicit form is given by

$$\text{Re}\,\mathcal{N} = \begin{pmatrix} -\frac{1}{3}\mathcal{K}_{ABC}b^A b^B b^C & \frac{1}{2}\mathcal{K}_{ABC}b^B b^C \\ \frac{1}{2}\mathcal{K}_{ABC}b^A b^C & -\mathcal{K}_{ABC}b^C \end{pmatrix} \tag{2.30}$$

$$\text{Im}\,\mathcal{N} = -\frac{\mathcal{K}}{6}\begin{pmatrix} 1 + 4G_{A\bar{B}}b^A b^B & -4G_{A\bar{B}}b^B \\ -4G_{A\bar{B}}b^B & 4G_{A\bar{B}} \end{pmatrix} \tag{2.31}$$

Afterwards, we turn to the couplings of the hypermultiplets in the action (2.29), which are encoded in the quaternionic metric h_{uv} and whose explicit form is

$$\begin{aligned} h_{uv}\, d\tilde{q}^u\, d\tilde{q}^v &= (dD)^2 + G_{\alpha\bar{\kappa}}\, dz^\alpha d\bar{z}^\kappa + \frac{1}{4}e^{4D}\left(da - (\tilde{\xi}_\kappa d\xi^\kappa - \xi^\kappa d\tilde{\xi}_\kappa)\right)^2 \\ &\quad -\frac{1}{2}e^{2D}(\text{Im}\,\mathcal{M}^{-1})^{\alpha\kappa}\left(d\tilde{\xi}_\alpha - \mathcal{M}_{\alpha\beta}d\xi^\beta\right)\left(d\tilde{\xi}_\kappa - \bar{\mathcal{M}}_{\kappa\lambda}d\xi^\lambda\right) \end{aligned} \tag{2.32}$$

where D is the four-dimensional dilaton defined as $e^D = e^\phi/\sqrt{\mathcal{V}}$, $G_{\alpha\bar{\kappa}}$ is the metric on the space of complex structure deformations defined in (2.23), while the complex coupling matrix $\mathcal{M}_{\alpha\kappa}$ depends on the complex structure deformations z^κ and can be directly computed from (2.21) by using equation (C.14).

Finally, the full moduli space for $\mathcal{N}=2$ compactifications has a local structure of the form

$$\mathfrak{M}_{\mathcal{N}=2} = \mathfrak{M}_K \times \mathfrak{M}_Q \tag{2.33}$$

where $\mathfrak{M}_K$ is again the special Kähler manifold spanned by the Kähler moduli (2.10), while $\mathfrak{M}_Q$ is a quaternionic manifold spanned by the scalars in the hypermultiplets and that has a special Kähler submanifold $\mathfrak{M}_{\mathrm{cs}}$ spanned by the complex structure deformations z^κ.

2.4 Type IIA compactifications on Calabi-Yau orientifolds

In this section we briefly review the four-dimensional low energy effective action arising from type IIA orientifold compactifications. Type IIA orientifolds can be constructed by starting from Type IIA string theory and modding out by the symmetry group $\mathcal{O} = \Omega_p \, (-1)^{F_L} \, \mathcal{R}$, which is assumed to be a symmetry of the original theory. This symmetry known as orientifold projection consists of worldsheet parity Ω_p exchanging left and right movers, a projection operator $(-1)^{F_L}$ counting the number of spacetime fermions in the left-moving sector and an internal involution $\mathcal{R}$ which acts non-trivially on $\mathcal{M}_6$, but leaves the four extended dimensions unchanged. Supersymmetry requires that the internal involution must act on the Kähler as follows

$$\mathcal{R}\,(J) = -J \tag{2.34}$$

On the other hand, the compatibility of the above condition with the Calabi-Yau condition $J \wedge J \wedge J \sim \Omega \wedge \bar{\Omega}$ implies that $\mathcal{R}$ acts non-trivially on the holomorphic 3-form Ω as

$$\mathcal{R}\,(\Omega) = e^{2i\theta}\,\bar{\Omega} \tag{2.35}$$

where θ is some phase which can be eliminated through a redefinition of Ω. The fix-point set of the anti-holomorphic involution $\mathcal{R}$ in $\mathcal{M}_6$ are three-cycles Π_{O6} supporting the internal part of the orientifold planes. These three-cycles have to be special Lagrangian submanifolds of $\mathcal{M}_6$, which is an immediate consequence of the constraints (2.34) and (2.35), namely

$$J\big|_{\Pi_{O6}} = 0, \qquad \mathrm{Im}\,\left(\mathrm{e}^{-\mathrm{i}\theta}\Omega_3\big|_{\Pi_{O6}}\right) = 0. \tag{2.36}$$

This means that those three-cycles are calibrated with respect to Re $\left(e^{-i\theta}\Omega_3\big|_{\Pi_{O6}}\right)$. As we will discuss in the next section, the above constraints will also affect the D6-branes present in the compactification, if they are demanded to preserve supersymmetry. After orientifold projection only the $\mathcal{O}$-invariant states are kept, thus by using the parity behavior of the massless bosonic states under Ω_p and $(-1)^{F_L}$ displayed in table 2.3 one can straightforwardly derive the parity of the

Field	Ω_p	$(-1)^{F_L}$
ϕ	even	even
g	even	even
B_2	odd	even
C_1	even	odd
C_3	odd	even

Table 2.3: *Parity behavior of the massless bosonic states under Ω_p and $(-1)^{F_L}$.*

$\mathcal{O}$-invariant states under the involution $\mathcal{R}$:

$$\mathcal{R}(\phi) = \phi, \qquad \mathcal{R}(g) = g, \qquad \mathcal{R}(B_2) = -B_2, \qquad \mathcal{R}(C_1) = -C_1, \qquad \mathcal{R}(C_3) = C_3 \tag{2.37}$$

As mentioned in the previous section the massless modes are in one-to-one correspondence with the harmonic forms on $\mathcal{M}_6$, therefore we now have to determine the splitting of the cohomology groups $H^{(p,q)}$ under the action of $\mathcal{R}$

$$H^{(p,q)}(\mathcal{M}_6, \mathbb{Z}) = H_+^{(p,q)}(\mathcal{M}_6, \mathbb{Z}) \oplus H_-^{(p,q)}(\mathcal{M}_6, \mathbb{Z}) \quad . \tag{2.38}$$

where $H_+^{(p,q)}(\mathcal{M}_6, \mathbb{Z})$ contains harmonics (p,q)-form even under $\mathcal{R}$ while $H_-^{(p,q)}(\mathcal{M}_6, \mathbb{Z})$ contains the odd ones. In table 2.4 we summarize all cohomology groups on $\mathcal{M}_6$ and their basis elements. Depending on the parity behavior given in (2.37) the $\mathcal{O}$-invariant states reside either in $H_+^{(p,q)}(\mathcal{M}_6, \mathbb{Z})$ or $H_-^{(p,q)}(\mathcal{M}_6, \mathbb{Z})$, consequently the four-dimensional spectrum will be reduced.

cohomology group	$H_+^{(1,1)}$	$H_-^{(1,1)}$	$H_+^{(2,2)}$	$H_-^{(2,2)}$	$H_+^{(3)}$	$H_-^{(3)}$
dimension	$h_+^{(1,1)}$	$h_-^{(1,1)}$	$h_-^{(1,1)}$	$h_+^{(1,1)}$	$h^{(2,1)}+1$	$h^{(2,1)}+1$
basis	ω_α	ω_a	$\tilde{\omega}^a$	$\tilde{\omega}^\alpha$	$(\alpha_K, \beta^\Lambda)$	$(\beta^K, \alpha_\Lambda)$

Table 2.4: *Splitting of the cohomology groups of $\mathcal{M}_6$ under the action of $\mathcal{R}$ and their basis elements.*

Due to the fact that the volume form $\mathrm{dvol}_{\mathcal{M}_6} \sim J \wedge J \wedge J$ is odd, one straightforwardly deduces that $h_-^{(3,3)} = 1$ and $h_+^{(3,3)} = 0$, then by using Hodge duality we further infer that $h_-^{(0,0)} = 0$ and $h_+^{(3,3)} = 1$. On the other hand, Hodge duality also requires that $h_+^{(1,1)} = h_-^{(2,2)}$ and $h_-^{(1,1)} = h_+^{(2,2)}$. It implies that the non-trivial intersection numbers are given by

$$l_s^{-6} \int_{\mathcal{M}_6} \omega_\alpha \wedge \tilde{\omega}^\beta = \delta_\alpha^\beta \ , \qquad \alpha, \beta = 1, \ldots, h_+^{(1,1)}, \tag{2.39}$$
$$l_s^{-6} \int_{\mathcal{M}_6} \omega_a \wedge \tilde{\omega}^b = \delta_a^b \ , \qquad a, b = 1, \ldots, h_-^{(1,1)}$$

Finally, under the action of $\mathcal{R}$ the symplectic basis $(\alpha_\kappa, \beta^\lambda) \in H^3(\mathcal{M}_6, \mathbb{Z})$ decomposes into the bases of $\mathcal{R}$-even 3-forms $(\alpha_K, \beta^\Lambda) \in H_+^3(\mathcal{M}_6, \mathbb{Z})$ and $\mathcal{R}$-odd 3-forms $(\beta^K, \alpha_\Lambda) \in H_-^3(\mathcal{M}_6, \mathbb{Z})$.[6] In this basis, the only non-trivial intersections are given by

$$l_s^{-6} \int_{\mathcal{M}_6} \alpha_K \wedge \beta^L = \delta_K^L, \qquad l_s^{-6} \int_{\mathcal{M}_6} \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda^\Sigma \tag{2.40}$$

Let us now determine the spectrum surviving the orientifold projection. From the equations (2.34) and (2.37) we immediately see that both J and B_2 are odd under $\mathcal{R}$ and therefore must be expanded in the basis of $\mathcal{R}$-odd forms $l_s^2 \omega_a \in H_-^{(1,1)}(\mathcal{M}_6, \mathbb{Z})$ as

$$J = t^a \omega_a, \qquad B_2 = b^a \omega_a, \qquad a = 1, ...h_-^{(1,1)} \tag{2.41}$$

In contrast with the unorientifolded theory, now the four-dimensional two-form $B_2(x)$ gets projected out due to that fact that $\mathcal{R}$ acts trivially on the flat dimensions. Just as in the $\mathcal{N} = 2$ theory, both J and B_2 can be combined to form complex scalars

$$J_c = T^a \omega_a, \qquad a = 1, ...h_-^{(1,1)} \tag{2.42}$$

This implies that orientifold projection reduces the number of Kähler moduli from $h^{(1,1)}$ to $h_-^{(1,1)}$. These fields also span a spacial Kähler manifold $\tilde{\mathfrak{M}}_K$, where Kähler potential and the metric on this manifold are trivial truncations of the Kähler potential (2.11) and the metric (2.13) respectively, namely replacing $T^A \to T^a$.

Let us now discuss how the orientifold projection affects to the complex structure deformations. We start expanding the holomorphic three-form Ω in the basis of $H_+^3 \oplus H_-^3$ as

$$\Omega = \mathcal{Z}^K \alpha_K - \mathcal{F}_\Lambda \beta^\Lambda + \mathcal{Z}^\Lambda \alpha_\Lambda - \mathcal{F}_K \beta^K \tag{2.43}$$

[6]Alternatively, one can chose a real symplectic basis such that all α-elements are chosen to be even $\alpha_K \in H_+^3$ while all β-elements are chosen to be odd $\beta^K \in H_-^3$ with intersection.

$$l_s^{-6} \int_{\mathcal{M}_6} \alpha_K \wedge \beta^I = \delta_K^I$$

Despite this particular choice of $H_+^3 \oplus H_-^3$ simplifies considerably the computations, here we use a more general basis which result to be more suitable to construct the mirror duals of the type IIB Calabi-Yau flux compactifications.

Then by inserting the above expansion back into the constraint (2.35) one obtains

$$\mathrm{Im}\left(e^{-i\theta}\mathcal{Z}^K\right) = \mathrm{Re}\left(e^{-i\theta}\mathcal{F}_K\right) = 0, \qquad \mathrm{Re}\left(e^{-i\theta}\mathcal{Z}^\Lambda\right) = \mathrm{Im}\left(e^{-i\theta}\mathcal{F}_\Lambda\right) = 0 \quad (2.44)$$

Note that the constraints $\mathrm{Im}\left(e^{-i\theta}\mathcal{Z}^K\right) = \mathrm{Re}\left(e^{-i\theta}\mathcal{Z}^\Lambda\right)$ yields a set of $h^{(2,1)}+1$ real conditions for $h^{(2,1)}$ complex structure deformations. On the other hand, the scale invariance of the holomorpic three-form Ω defined in 2.24 implies that one of these conditions is redundant. This means we can use the phase $e^{-h(z)}$ to trivially satisfy one of these constraints, while the remaining equations allow us to project out $h^{(2,1)}$ real scalars. Note that the remaining equations $\mathrm{Re}\left(e^{-i\theta}\mathcal{F}_K\right) = \mathrm{Im}\left(e^{-i\theta}\mathcal{F}_\Lambda\right) = 0$ should not be seen as equations determining the complex structure deformations, instead they yield a set of constraints on the periods. To preserve the scale invariance of Ω in the orientifolded theory will be useful to introduce a compesator field $\mathcal{C} = e^{-D-i\theta}e^{\frac{1}{2}K^{cs}(z)}$ with the transformation property $\mathcal{C} \to \mathcal{C}e^{\mathrm{Re}\,h}$, where D is the four-dimensional dilaton defined in the previous section. Accordingly, one can define the scale invariant function $C\Omega$ that depends on $h^{(2,1)}+1$ real parameters

$$\mathcal{C}\Omega = \mathrm{Re}\left(\mathcal{C}\mathcal{Z}^K\right)\alpha_K + i\mathrm{Im}\left(\mathcal{C}\mathcal{Z}^\Lambda\right)\alpha_\Lambda - \mathrm{Re}\left(\mathcal{C}\mathcal{F}_\Lambda\right)\beta^\Lambda - i\mathrm{Im}\left(\mathcal{C}\mathcal{F}_K\right)\beta^K \quad (2.45)$$

The next step is to expand the ten-dimensional fields C_1 and C_3 into harmonic forms of $\mathcal{M}_6$. Since C_1 is odd under $\mathcal{R}$, combined together with the fact that $\mathcal{M}_6$ posses no harmonic one-forms and $\mathcal{R}$ acts trivially on the flat dimensions, thus the entire C_1 is projected out. On the other hand, C_3 is even under $\mathcal{R}$ and therefore can be expanded as

$$C_3 = c_3(x) + A^\alpha(x)\wedge\omega_\alpha + \xi_\star^K\alpha_K - \xi_{\star\Lambda}\beta^\Lambda \quad (2.46)$$

where A^α are $h_+^{(1,1)}$ one-forms, while $\xi_\star^K, \xi_{\star\Lambda}$ are $h^{2,1}+1$ scalar fields and $c_3(x)$ is the four-dimensional part of the ten-dimensional field C_3, in four dimensions it can be dualised to a constant which will play the role of a further electric flux e_0. In order to find the proper complex fields will be useful to define the complex combination

$$\Omega_c = C_3 + i\mathrm{Re}\left(C\Omega\right) \quad (2.47)$$

As shown in [10], the C_3-axions fit together with the complex structure deformations of the CY metric to form complexified scalars of the $\mathcal{N}=1$ chiral multiplets:

$$N_\star^K = \ell_s^{-3}\int_{\mathcal{M}_6}\Omega_c\wedge\beta^K, \qquad U_{\star\Lambda} = \ell_s^{-3}\int_{\mathcal{M}_6}\Omega_c\wedge\alpha_\Lambda. \quad (2.48)$$

we refer to these fields as complex structure moduli. The fields (2.48) span a manifold $\mathfrak{M}_Q$ which maintains a Kähler structure with Kähler potential given by:

$$K_Q = -2\log\left(\frac{1}{4}\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)\mathrm{Re}\,(\mathcal{C}\mathcal{F}_\Lambda) - \frac{1}{4}\mathrm{Re}\,(\mathcal{C}\mathcal{Z}^\mathrm{K})\mathrm{Im}\,(\mathcal{C}\mathcal{F}_\mathrm{K})\right) = -\log(e^{-4D}), \quad (2.49)$$

The metric on this manifold is given by [10]

$$K_{K\bar{L}} = e^{2D} l_s^{-6} \int_{\mathcal{M}_6} \alpha_K \wedge \star \alpha_L, \qquad K_{\Lambda\bar{\Sigma}} = e^{2D} l_s^{-6} \int_{\mathcal{M}_6} \beta^\Lambda \wedge \star \beta^\Sigma, \qquad K_{\Lambda\bar{K}} = e^{2D} l_s^{-6} \int_{\mathcal{M}_6} \beta^\Lambda \wedge \star \alpha_K \tag{2.50}$$

Thus, after orientifold projection all the original $\mathcal{N} = 2$ multiplets break into $\mathcal{N} = 1$ multiplets, the resulting $\mathcal{N} = 1$ spectrum is summarized in table 2.4 To

multiplets	multiplicity	bosonic components
gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(1,1)}$	A^α
chiral multiplets	$h_-^{(1,1)}$	T^a
chiral multiplets	$h^{(2,1)} + 1$	$N_\star^K, U_{\star\Lambda}$

Table 2.5: *Bosonic components of the $\mathcal{N} = 1$ multiplets for Type IIA supergravity compactified on CY orientifolds.*

obtain the four-dimensional effective theory one has to insert the field expansions (2.42) and (2.46) into the ten-dimensional Type IIA action (2.26). However, this is equivalently to impose the orientifold projections on the $\mathcal{N} = 2$ action (2.29), such that one is left with

$$S_{\rm IIA}^{(4)} = \int -\frac{1}{2} R * \mathbf{1} - K_{I\bar{J}}\, dM^I \wedge * d\bar{M}^J + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\alpha\beta}\, F^\alpha \wedge * F^\beta + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\alpha\beta}\, F^\alpha \wedge F^\beta \tag{2.51}$$

Where M^I collectively denote the bosonic components of the chiral multiplets, namely $M^I = (T^a, N_\star^K, U_{\star\Lambda})$, the gauge-kinetic coupling matrix $\mathcal{N}_{\alpha\beta}$ is also a truncation of the $\mathcal{N} = 2$ gauge-kinetic coupling matrix and is given by $\mathrm{Re}\mathcal{N}_{\alpha\beta} = -\mathcal{K}_{\alpha\beta a} b^a$ and $\mathrm{Im}\mathcal{N}_{\alpha\beta} = \mathcal{K}_{\alpha\beta}$. Note that the above effective action does not include a potential term, therefore the tree-level superpotential has to vanish. Finally, the full moduli space for $\mathcal{N} = 1$ compactifications has the product structure

$$\tilde{\mathfrak{M}}_{\mathcal{N}=1} = \tilde{\mathfrak{M}}_K \times \tilde{\mathfrak{M}}_Q \tag{2.52}$$

where $\tilde{\mathfrak{M}}_K$ is a submanifold of the $\mathcal{N} = 2$ special Kähler manifold $\mathfrak{M}_K$ and $\tilde{\mathfrak{M}}_Q$ is a submanifold of the quaternionic manifold $\mathfrak{M}_Q$

2.5 Type IIA flux potential

As we have seen in the previous section, all scalar components of the $\mathcal{N} = 1$ multiplets have a flat potential. This is because the superpotential at tree level is trivial, but it is a well-known fact that non-trivial background generate scalar potentials for the closed string moduli [3, 10]. On the other hand, the presence of RR and NS fluxes will modify the internal background geometry, and therefore the Calabi-Yau geometry is no longer a solution to the equations of motion. However, one may address this obstacle by considering Calabi-Yau flux compactifications in the large volume limit, in which the fluxes are diluted. Moreover, if the typical energy scale of the fluxes is much lower than the compactification scale, one can argue that the spectrum discussed in the previous section is the same, except that some of the massless modes acquire a mass due to the fluxes.

From a ten-dimensional perspective, the democratic formulation of type IIA superstring theory offers the best starting point to capture the physics of string backgrounds with fluxes and D-branes. In this description, all RR gauge potentials C_{2p-1} with $p = 1, 2, 3, 4, 5$ are treated on equal footing and are grouped together in a polyform $\mathbf{C} = C_1 + C_3 + C_5 + C_7 + C_9$. Similarly to the NS 2-form B_2 they appear in the bosonic part of the type IIA supergravity action (6.42) through their associated field strengths $\mathbf{G} = G_0 + G_2 + G_4 + G_6 + G_8 + G_{10}$ and H_3. Apart from their equations of motion, these field strengths also have to satisfy the Bianchi identities, which in the absence of D-branes or other external sources read:

$$d(e^{-B_2} \wedge \mathbf{G}) = 0, \qquad dH_3 = 0 \tag{2.53}$$

On a compact manifold, the Bianchi identities imply that the polyforms $e^{-B_2} \wedge \mathbf{G}$ and NS 3-form H_3 are closed forms, such that these field strengths can be decomposed in terms of exact and harmonic forms:[7]

$$\mathbf{G} = e^{B_2} \wedge (d\mathbf{A} + \overline{\mathbf{G}}), \qquad H_3 = dB_2 + \overline{H}_3. \tag{2.54}$$

At the same time, the Bianchi identities written in this form allow to argue for the quantisation of the associated Page charge [19],

$$\frac{1}{\ell_s^{2p-1}} \int_{\pi_{2p}} dA_{2p-1} + \overline{G}_{2p} \in \mathbb{Z}, \qquad \frac{1}{\ell_s^2} \int_{\pi_3} dB_2 + \overline{H}_3 \in \mathbb{Z}, \tag{2.55}$$

arising through integration over the non-trivial homological cycles π_{2p} with $p = 1, 2, 3$ and π_3. The quantisation argument itself relies on the consistency of the field theory on a probe $(2p-2)$-brane wrapping a $(2p-1)$ cycle inside one of the

[7] The chosen form of the Bianchi identities allows to extract the solution for the RR field strengths in terms of the $\mathbf{A}$-basis instead of the $\mathbf{C}$-basis, which are related to each by a simple B_2-transformation, i.e. $\mathbf{A} = \mathbf{C} \wedge e^{-B_2}$.

non-trivial homological cycles π_{2p} or π_3. In the absence of localised sources such as D-branes, the gauge potentials $\mathbf{A}$ are well-defined everywhere and the non-trivial harmonic parts $\overline{G}_{2p}$ with $p = 0, 1, 2, 3$ and $\overline{H}_3$ with legs along the compactification manifold capture the quantised flux. For orientifold compactifications the internal p-cycles have to comply with the orientifold projection, such that the background flux can be characterised by virtue of flux quanta (m, m^a, e_a, e_0):[8]

$$\ell_s \overline{G}_0 = m, \qquad \frac{1}{\ell_s}\int_{\tilde{\pi}^a} \overline{G}_2 = m^a, \qquad \frac{1}{\ell_s^3}\int_{\pi_a} \overline{G}_4 = e_a, \qquad \frac{1}{\ell_s^5}\int_{\mathcal{M}_6} \overline{G}_6 = e_0, \quad (2.56)$$

with $\tilde{\pi}^a \in H_2^-(\mathcal{M}_6, \mathbb{Z})$ and $\pi_a \in H_4^+(\mathcal{M}_6, \mathbb{Z})$. The internal RR-fluxes $\overline{\mathbf{G}}$ are known to generate a perturbative superpotential for the Kähler moduli [20, 21]:

$$\ell_s W_T = \frac{1}{\ell_s^5}\int_{\mathcal{M}_6} \overline{\mathbf{G}} \wedge e^{J_c} = e_0 + e_a T^a + \frac{1}{2}\mathcal{K}_{abc} m^a T^b T^c + \frac{m}{6}\mathcal{K}_{abc} T^a T^b T^c\,. \quad (2.57)$$

The NS 3-form flux $\overline{H}_3$ on the other hand threads the $\mathcal{R}$-odd three-cycles $(B^K, A_\Lambda) \in H_3^-(\mathcal{M}_6, \mathbb{Z})$, which are the de Rahm duals to the $\mathcal{R}$-odd three-forms $(\beta^K, \alpha_\Lambda)$ introduced earlier. Similar as for the RR-fluxes, the quantised Page charge for the NS-flux background can be expressed in terms of the integer flux quanta (h_K, h^Λ):

$$\frac{1}{\ell_s^2}\int_{B^K} \overline{H}_3 = h_K, \qquad \frac{1}{\ell_s^2}\int_{A_\Lambda} \overline{H}_3 = -h^\Lambda. \quad (2.58)$$

The NS-fluxes generate in turn a linear superpotential for the complex structure moduli:

$$\ell_s W_Q = \frac{1}{\ell_s^5}\int_{\mathcal{M}_6} \Omega_c \wedge \overline{H}_3 = h_K N_\star^K + h^\Lambda U_{\star\Lambda}\,. \quad (2.59)$$

Unlike to the type IIB case, the richness of NS- and RR-fluxes in type IIA generates a superpotential for both Kähler moduli and complex structure moduli, which offers a controlled, perturbative method to deal with moduli stabilisation for all closed string moduli. Thus, the Type IIA flux potential is fully determined in terms of the Kähler potential and superpotential through

$$V_F = \frac{e^K}{\kappa_4^2}\left[K^{A\overline{B}} D_A W D_{\bar{B}} \bar{W} - 3\left|W\right|^2\right], \quad (2.60)$$

where we used the Kähler covariant derivatives $D_A W = \partial_A W + K_A W$ and summation over all closed string moduli is assumed.

2.6 Type II orientifolds with mobile D6-branes

In this section we turn to type IIA Calabi-Yau orientifolds with fluxes and D6-branes hosting open string moduli, dubbed mobile D6-branes in the following.

[8]We adhere to the conventions of [11] for the sign of the fluxes.

Fluxes and D-branes are the key ingredients that have allowed to build an abundance of phenomenologically interesting models from string compactifications [13, 22–24]. On the one hand, as discussed in the previous section fluxes generate potential for the closed string moduli and simultaneously provide sources for spontaneous supersymmetry breaking, which allows to build more general compactifications with fewer and fewer moduli. On the other hand, D-branes allow to construct realistic chiral gauge sectors, and to localise their degrees of freedom in a particular region of the compactification. Needless to say, when combining both ingredients in a given compactification one must do it consistently. In first instance this gives rise to constraints of topological nature, like avoiding Freed-Witten anomalies [25, 26]. At a finer level of detail, one must ensure to capture the dynamical effects that D-branes and fluxes exert on each other, as well as on the rest of the compactification. As we will see below, D-branes create potentials for certain closed string moduli and contribute to the four-dimensional light degrees of freedom with moduli of their own. Moreover, fluxes which are sourced by branes also create potentials for the open string moduli. Therefore, in order to properly describe the low energy effective dynamics all of these effects must be taken into account on equal footing.

Moreover, as we will discuss later on in chapter 6 whenever the vacuum breaks supersymmetry this generically results in a set of soft supersymmetry breaking terms for the charged matter fields living on the worldvolume of D6-branes. These terms can be computed from the effective low energy action as it has been carried out in section 6.3 following the standard supergravity analysis of [27–29].

2.6.1 D-brane action

Besides fundamental object, string theories also contain various extended p-dimensional objects called Dp-branes on which open strings can end. All Dp-branes become heavy in the limit $g_s \to 0$ and therefore they play an important role to gain insight into non-perturbative aspects of string theories. On the other hand Dp-branes have also provided for new ingredients in constructing phenomenological appealing string models as they give rise to Abelian gauge groups in type II superstring theories. The low-energy effective description of these objects is captured by the Dirac-Born-Infeld action plus the Chern-Simons action.

$$S_{\mathrm{DBI}} = -T_p \int_{\mathcal{M}_{p+1}} d^{p+1}\xi \, e^{-\phi} \sqrt{-det\left(P[E] - \frac{l_s^2}{2\pi}F\right)} \tag{2.61}$$

$$S_{CS} = \mu_p \int_{\mathcal{M}_{p+1}} d^{p+1}\xi P\left[\mathbf{C}\right] e^{\frac{l_s^2}{2\pi}F - P[B_2]} \tag{2.62}$$

The above actions describe the dynamics of the bosonic part of a single D-brane in a type II string theory backgrounds. Where T_p and μ_p are the tension and charge of the Dp-brane respectively, $E = e^{\frac{\phi}{2}}g + B_2$, $P[...]$ denotes the pullback into the worldvolume of a given Dp-brane and F corresponds to the field strength of the $U(1)$ gauge field living on its worldvolume. The polyform $\mathbf{C}$ contains all RR gauge potentials present in the theory, and only $(p+1)$-forms contribute to the integral in the Chern-Simons action. The fluctuations of a Dp-brane in the directions transverse to its worldvolume are encoded in the pullback and are parameterized by a set of uncharged scalar fields.

The Dp-brane may also carry gauge flux, in that case the field strength F has to be modified as

$$F = dA + f \tag{2.63}$$

where f is a harmonic two-form on the worldvolume of the Dp-brane. To preserve Lorentz invariance of the 4D theory these gauge fluxes must be forms on the internal part of the worldvolume. Note that Dp-branes also carry RR charges, because their couplings couple as extended objects to the RR gauge field C_{p+1}[9]. Besides this coupling: in the case of non-trivial F, Dp-branes also contain lower dimensional D-brane charges and therefore they interact also with lower degree RR p-form gauge fields, all these couplings to the bulk RR fields are also encoded in the Chern-Simons action.

When N Dp-branes coincide, the worldvolume gauge theory becomes non-Abelian[10]. As a consequence the effective actions in (2.61) have to be generalized, as shown in [31]. This nice feature of coincident Dp-branes is essential to build quantum field theories of the Yang-Mills type like the Standard Model, in type II string theories.

2.6.2 Supersymmetry and calibration conditions

In general Dp-branes lead to non-supersymmetric low energy theories, which are plagued by various instabilities due to runaway potentials for the moduli, unlike with supersymmetric setups which are under much better control. Nevertheless,

[9] Strictly speaking, the $p+1$-dimensional worldvolume couples naturally to the RR form C_{p+1}, which is nothing but a higher dimensional generalization of the electrodynamic coupling in which a point like particle couples to one-form gauge field.

[10] Open strings can have either both endpoints on the same stack of Dp-branes or on two different stacks. To distinguish the open strings that connect different Dp-branes one introduces the Chan-Paton (CP) labels and assigns a formal label λ^A to every open string. These CP labels can be represented by matrices that satisfy a Lie algebra as a symmetry group of open string interactions. In this way, the symmetries of open string scattering amplitudes turn out to be compatible with symmetry algebras $U(N)$, $SO(N)$ or $USp(N)$ [30].

the requirement of unbroken supersymmetry imposes strong conditions on the D-branes present in the compactification. Dp-branes preserving half of the original supersymmetries are known as BPS branes. The BPS condition requires that the brane tensions T_p and their charges μ_p are equal, which ensures stability since the net force between BPS branes vanishes [32, 33]. In addition, there are further constraints on the $(p-3)$-cycle Π_α wrapped by the Dp-brane. From now on we focus only on D6-branes which are the ones we are interested in. In [34] was shown that for a D6-brane preserves some supersymmetry, it has to wrap a special Lagrangian three-cycle Π_α on the internal space.

$$e^{\frac{\phi}{2}} J\big|_{\Pi_\alpha} = 0, \qquad \mathrm{Im}\left(\mathrm{e}^{-\mathrm{i}\theta_{\mathrm{D6}}}\mathrm{e}^{\frac{\phi}{4}}\,\mathcal{C}\Omega_3\big|_{\Pi_\alpha}\right) = 0 \tag{2.64}$$

Since the three-form Ω_3 is the natural calibration form for the ($SLag$) three-cycles on Calabi-Yau 3-folds, the three-cycle volume form for the supersymmetric D6-branes can be expressed as follows [34] for a given point in the Calabi-Yau moduli space

$$e^{-i\theta_{D6}}\, e^{\frac{\phi}{4}}\mathcal{C}\Omega\big|_{\Pi_\alpha} = d\mathrm{Vol}\big|_{\Pi_\alpha}. \tag{2.65}$$

Where the additional insertion of the ten-dimensional dilaton indicates that the above expressions are expressed in the Einstein frame. In presence of non-trivial gauge fluxes, the above conditions have to be complemented with a further condition

$$P[B_2] - \frac{l_s^2}{2\pi} F\big|_{\Pi_\alpha} = 0 \tag{2.66}$$

Finally, in order to ensure that D6-branes preserve the same supersymmetry as the O6-planes, they should be calibrated with the same phase θ, namely $\theta = \theta_{D6}$[11].

As mentioned above the D6-brane may carry gauge flux f which arises as the vacuum expectation value of the field strength. Since we are considering only massless modes of the gauge field one has that $dA\big|_{\Pi_\alpha} = 0$ which leads to $F\big|_{\Pi_\alpha} = f^\alpha$. Finally, the cancellation of the Freed-Witten anomaly [25] requires that the worldvolume gauge flux should be quantized $\frac{l_s^2}{2\pi} f^\alpha \in H^2(\Pi_\alpha, \mathbb{Z})$ and therefore we can expand it in terms of harmonic two-forms as

$$f^\alpha = n^\alpha_{Fi}\rho^i \tag{2.67}$$

where we have introduced the basis of harmonic two-forms $\ell_s^{-2}\rho^i \in \mathcal{H}^2(\Pi_\alpha, \mathbb{Z})$ and $n^\alpha_{Fi} \in \mathbb{Z}$.

As already discussed in the previous section, the fixed loci Π_{O6} under the involution $\mathcal{R}$ define the locations of O6-planes with negative charge and tension and wrapping one or more special Lagrangian ($SLag$) three-cycles. The O6-plane RR-charges have to be cancelled along the internal directions, which can

[11] From now on we will set all phases $\theta = \theta_{D6} = 0$.

be achieved by introducing D6-branes wrapping *SLag* three-cycles Π_α and filling out the four-dimensional spacetime. Note that once we add a D6-brane wrapping a three-cycle Π_α on the internal manifold we also have to add the orientifold image D6-brane wrapping the orientifold image three-cycle $\Pi_3' = \mathcal{R}(\Pi_3)$. Thus, in the absence of background fluxes, the RR tadpole cancellation conditions can be recast into constraints in homology

$$\sum_\alpha N_\alpha([\Pi_\alpha] + [\mathcal{R}\Pi_\alpha]) - 4[\Pi_{O6}] = 0, \tag{2.68}$$

where N_α indicates the number of D6-brane in each stack α.

Whenever a *SLag* three-cycle Π_α can be continuously deformed along a normal vector without violating the special Lagrangian condition, a D6-brane wrapped around it can change its embedding or position along its transverse internal directions. As a result it has a non-trivial moduli space, parametrised by one or more open string moduli. More precisely, if we pick a set $\{X_j\}$ of normal vectors to Π_α which preserve the *SLag* condition,[12] McLean's theorem states that the one-forms $\iota_{X_i} J\big|_{\Pi_\alpha}$ are proportional to harmonic one-forms in $\mathcal{H}^1(\Pi_a, \mathbb{Z})$. In this sense, a generic, infinitesimal deformation $X = \ell_s X_i \varphi^i$ is expected to yield $b_1(\Pi_\alpha)$ different position moduli φ^i. In order to properly define the chiral superfields for the open string moduli, we use the basis of harmonic two-forms $\ell_s^{-2}\rho^i \in \mathcal{H}^2(\Pi_\alpha, \mathbb{Z})$ and assign an open string modulus to each ρ^i as follows:

$$\Phi_\alpha^i = -\frac{1}{\ell_s^4}\int_{\Pi_\alpha}\left(\frac{\ell_s^2}{\pi}A - \iota_X J_c\right)\wedge\rho^i = T^b(\eta_{\alpha\, b})^i{}_j\varphi^j - \theta_\alpha^i = \hat{\theta}_\alpha^i + i\,\phi_\alpha^i. \tag{2.69}$$

In this expression A again represents the D6-brane gauge potential, which reduces along the internal directions to Wilson line degrees of freedom θ_α^i. By introducing the constant parameters $(\eta_{\alpha\, b})^i{}_j$,

$$(\eta_{\alpha\, b})^i{}_j = \frac{1}{\ell_s^3}\int_{\Pi_\alpha}\iota_{X_j}\omega_b\wedge\rho^i, \tag{2.70}$$

the implicit dependence of the open string moduli on the Kähler moduli has been extracted in the right hand side of (2.69). When extending the infinitesimal deformation to a finite deformation of the *SLag* three-cycle, the functional dependence of the open string moduli on the position moduli φ^i will no longer be linear and higher order powers in the position moduli have to be computed through a normal coordinate expansion. Roughly speaking, the term $(\eta_{\alpha\, b})^i{}_j\varphi^j$ in (2.69) then has to be replaced by a generic function $f_{\alpha\, b}^i(\varphi)$, which can further depend on the closed string geometric moduli t^a, n^K and u_Λ [11]. The open string modulus then reads

$$\Phi_\alpha^i = T^a f_{\alpha\, a}^i - \theta_\alpha^i = \hat{\theta}_\alpha^i + i\,\phi_\alpha^i. \tag{2.71}$$

[12]The preservation of the *SLag* condition along direction X_i can be expressed through the corresponding Lie-derivative, i.e. $\mathcal{L}_{X_i} J\big|_{\Pi_\alpha} = 0 = \mathcal{L}_{X_i}\Omega_3\big|_{\Pi_\alpha}$.

2.6.3 The $\mathcal{N} = 1$ four-dimensional effective theory

When introducing mobile D6-branes into the type IIA orientifold compactification, the full moduli space of the compactification does generically not correspond to a direct product of the closed string moduli space $\tilde{\mathfrak{M}}_K \times \tilde{\mathfrak{M}}_Q$ with the open string moduli space. For small field fluctuations around a chosen point in the moduli space, one can adopt the approach in which the calibration conditions for *SLag* three-cycles (2.64) and (2.65) are evaluated in a particular background with frozen closed string moduli. As such, only those small deformations of the D6-brane that respect the *SLag* conditions with respect to this background have to be considered. Even in this approach, the reduction of the ten-dimensional theory induces kinetic mixing between open string and bulk moduli, such that a redefinition of the complex structure moduli is necessary to identify the proper $\mathcal{N} = 1$ chiral superfields. Following the reasoning of appendix B.2, one deduces the following field redefinition for the complex structure moduli:

$$N^K = N_\star^K + \frac{1}{2}\sum_\alpha (g^K_{\alpha i}\theta^i_\alpha - T^a \mathbf{H}^K_{\alpha\, a}), \qquad U_\Lambda = U_{\star\Lambda} + \frac{1}{2}\sum_\alpha (g_{\alpha\,\Lambda\, i}\theta^i_\alpha - T^a\mathbf{H}_{\alpha\,\Lambda\, a}), \tag{2.72}$$

where the real functions $\mathbf{H}^K_{\alpha\, a}$ and $\mathbf{H}^K_{\alpha\,\Lambda\, a}$ are defined through the expressions:

$$\partial_{\phi^i_\beta}(t^a \mathbf{H}^K_{\alpha\, a}) = \delta_{\alpha\beta}\, g^K_{\alpha i}, \qquad \partial_{\varphi^j} g^K_{\alpha i} = \ell_s^{-3}\int_{\Pi_\alpha} \iota_{X_j}\beta^K \wedge \zeta_i, \tag{2.73}$$

and

$$\partial_{\phi^i_\beta}(t^a \mathbf{H}_{\alpha\,\Lambda\, a}) = \delta_{\alpha\beta}\, g_{\alpha\,\Lambda\, i}, \qquad \partial_{\varphi^j} g_{\alpha\,\Lambda\, i} = \ell_s^{-3}\int_{\Pi_\alpha} \iota_{X_j}\alpha_\Lambda \wedge \zeta_i. \tag{2.74}$$

with $\phi^i_\alpha = \mathrm{Im}\,(\Phi^{\mathrm{i}}_\alpha)$. The functions $g^K_{\alpha i}$ and $g_{\alpha\,\Lambda\, i}$ are chain integrals that allow to write the two-forms $\iota_X\beta^K$ and $\iota_X\alpha_\Lambda$ on the three-cycle Π_α in terms of the more appropriate basis of quantised harmonic two-forms ρ^i, related to the quantified one-forms ζ_i as $\ell_s^{-3}\int_{\Pi_\alpha}\zeta_i \wedge \rho^j = \delta_i{}^j$. As argued in appendix A of [11], the functions $g^K_{\alpha i}$ and $g_{\alpha\,\Lambda\, i}$ are homogeneous functions of degree zero in the moduli $\{t^a, n^K, u_\Lambda, \phi^i_\alpha\}$, which implies that also the functions $\mathbf{H}^K_{\alpha\, a}$ and $\mathbf{H}^K_{\alpha\,\Lambda\, a}$ are homogeneous functions of degree zero in the respective moduli. The field redefinition also has repercussions for the Kähler potential (2.49) depending on the complex structure moduli. More precisely, the function $\mathcal{G}_Q(n^k, u_\Lambda)$ hidden in the Kähler potential (2.49), as inherited from the $\mathcal{N} = 2$ Calabi-Yau compactifications, remains a homogeneous function of degree two in the geometric moduli, but has to be rewritten in terms of the redefined complex structure moduli and the open string moduli:

$$K_Q = -2\log\left[\mathcal{G}_Q\left(n^K + \frac{1}{2}t^a\sum_\alpha \mathbf{H}^K_{\alpha\, a}, u_\Lambda + \frac{1}{2}t^a\sum_\alpha \mathbf{H}_{\alpha\,\Lambda\, a}\right)\right]. \tag{2.75}$$

An immediate consequence of the moduli redefinition is the explicit dependence of the function $\mathcal{G}_Q$ on all geometric moduli $\{t^a, n^K, u_\Lambda, \phi^i_\alpha\}$, such that the moduli space obviously no longer factorises for type IIA orientifold compactification with D6-branes. Ignoring α'-corrections for K_T, the combined Kähler potentials $K_T + K_Q = -\log(\mathcal{G}_T\mathcal{G}_Q^2)$ still satisfy a no-scale condition:

$$K_A K^{A\overline{B}} K_{\overline{B}} = 7, \tag{2.76}$$

where the indices A and B sum over all closed and open string moduli, in line with the conventions used in appendix A to express some revelant properties of the full Kähler potential.

Besides the mixing between open, Kähler and complex structure moduli at the level of the Kähler potential, mobile D6-branes also contribute to the superpotential through a bilinear coupling between some open string moduli and Kähler moduli:

$$W_{D6} = W^0_{D6} + \ell_s^{-1} \sum_\alpha \Phi^i_\alpha (n^\alpha_{F\,i} - n^\alpha_{a\,i} T^a). \tag{2.77}$$

with Φ^i_α defined in (2.71) in terms of a reference three-cycle Π^0_α. At this reference point in open string field space $\Phi^i_\alpha = 0$ and therefore the open string contribution to the superpotential is given by W^0_{D6}. The microscopic justification of this superpotential was derived in [35] and is reviewed in Appendix B, where we refer the reader for a more detailed analysis. Applications of the bilinear superpotential (2.77) to large field inflation were presented in [36, 37]. Thus, in the presence of mobile D6-brane the full superpotential is given by

$$W = W_T + W_Q + W_{D6} \tag{2.78}$$

where W_T is given by (2.57) and W_Q by (2.59) with the replacement $\{N^K_\star, U_{\star\Lambda}\} \to \{N^K, U_\Lambda\}$.

2.7 Perturbative α'-corrections

The previous sections provided a brief review of some important aspects about Type IIA string compactifications on Calabi-Yau orientifold with background fluxes, which were constructed in the large volume limit. If we go away from regions in the moduli space where the six-dimensional internal volume is huge, quantum corrections such as higher-derivative curvature corrections and world-sheet instanton corrections have to be taken into account. The aim of this section is to investigate how the perturbative α'-corrections modify the classical theory by considering how they affect the Kähler potential and superpotential in the four-dimensional $\mathcal{N} = 1$ supergravity description.

The $\mathcal{N}=1$ supergravity description of Type IIA orientifold compactifications with Kähler potential (2.11) is only reliable for sufficiently large internal volumes. Away from this limit, the Kähler potential is modified by the so-called α'-corrections, which break the no-scale structure of K_T for generic Calabi-Yau manifolds. In the regime of moderately large volumes in which the world-sheet instanton corrections can be neglected, the most relevant α'-corrections are those that descend from $(\alpha')^3 R^4$ curvature corrections in the ten-dimensional supergravity action. Following [8], such corrections can be incorporated into the pre-potential by virtue of the homogeneous coordinates $T^A = (T^0, T^a)$ on the Kähler moduli space. In homogeneous coordinates the most generic (perturbative) pre-potential is given by:

$$\mathcal{F}_{\rm per}(T) = -\frac{1}{6}\frac{\mathcal{K}_{abc}T^aT^bT^c}{T^0} + \frac{1}{2}K^{(1)}_{ab}T^aT^b + K^{(2)}_a T^aT^0 - \frac{i}{2}K^{(3)}(T^0)^2. \tag{2.79}$$

The first term is the usual tree-level Calabi-Yau volume from (2.11) and the remaining three terms encode different orders of curvature corrections in α'. The term proportional to $K^{(3)}$ corresponds to the $(\alpha')^3$-correction and is the only effective contribution to the Kähler potential. The parameter $K^{(3)} = -\frac{\zeta(3)}{(2\pi)^3}\,\chi_{\mathcal{M}_6} \in \mathbb{R}$ is proportional to the Euler characteristic $\chi_{\mathcal{M}_6}$ of the compactification manifold $\mathcal{M}_6$. The corrections $K^{(1)}_{ab}$ and $K^{(2)}_a$ correspond respectively to one-loop and two-loop corrections in α', yet do not have a ten-dimensional counterpart due to the lack of a ten-dimensional curvature polynomial with the appropriate features. Their presence can nevertheless be argued from mirror symmetry, which in fact allows to express them in terms of topological quantities of $\mathcal{M}_6$ like its triple intersection numbers and second Chern class, see e.g. [38, 39]. Their presence is however physically irrelevant at the level of the Kähler metrics, as confirmed by their absence in the Kähler potential that results from (2.79):

$$K_T = -\log\left(\frac{4}{3}\mathcal{K}_{abc}t^at^bt^c + 2K^{(3)}\right) = -\log\left(\frac{2}{3}\mathcal{K}(2+3\varepsilon)\right), \tag{2.80}$$

where we have defined $\varepsilon \equiv \frac{K^{(3)}}{\mathcal{K}}$ which captures the $(\alpha')^3$ curvature corrections to the compactification volume. As anticipated earlier, in the presence of these perturbative α'-corrections the classical no-scale condition for the Kähler potential (2.14) no longer holds and needs to be modified as well:

$$(K_T)_a (K_T)^{a\bar{b}} (K_T)_{\bar{b}} = \frac{3}{1-3\varepsilon}. \tag{2.81}$$

For generic Calabi-Yau compactifications with background fluxes, the (perturbative) α'-corrections to the Kähler moduli pre-potential (2.79) also induce curvature corrections to the (Kähler moduli) superpotential [8]. By rewriting the superpotential in terms of the homogeneous coordinates $T^A = (T^0, T^a)$, the α'-corrected superpotential can be obtained from the pre-potential (2.79):

$$l_s\,(W_T + W_Q) \;=\; \left(T^0, T^a, -\partial_{T^a}\mathcal{F}_{\rm per}, \partial_{T^0}\mathcal{F}_{\rm per}, N^K, U_\Lambda\right)_{T^0=1} \cdot \vec{q}\;, \tag{2.82}$$

with $\overrightarrow{q}$ being a vector of flux quanta $\overrightarrow{q} = \left(e_0, e_a, m^a, m, h_K, h^\Lambda\right)^T$. The superpotential W_Q for the complex structure moduli remains unchanged by the curvature corrections, while the part W_T with the Kähler moduli takes a similar form as (2.57),

$$W = \bar{e}_0 + \bar{e}_a T^a + \frac{1}{2}\mathcal{K}_{abc} m^a T^b T^c + \frac{m}{3!}\mathcal{K}_{abc} T^a T^b T^c - imK^{(3)} + h_K N^K + h^\Lambda U_\Lambda \quad (2.83)$$

upon taking into account the curvature correction $K^{(3)}$ and after introducing the curvature corrected flux quanta $\bar{e}_0 = e_0 - m^a K_a^{(2)}$ and $\bar{e}_a \equiv e_a - K_{ab}^{(1)} m^b + m K_a^{(2)}$. This clearly shows that the corrections $K_{ab}^{(1)}$ and $K_a^{(2)}$ become relevant in the presence of a superpotential for the Kähler moduli and cannot be ignored.

Chapter 3

Axion polynomials and Freed-Witten anomalies

In this chapter we briefly review the axion-polynomial description of type IIA Calabi-Yau orientifolds with fluxes and D6-branes presented in [40, 41]. I would like to point out that although most of this chapter is a review of the results obtained in those articles, here we also present some new results, in particular those regarding the axion-polynomials with α'-corrections discussed in section 3.5 which have been found in [42].

3.1 Type IIA flux potentials and axion polynomials

We have seen in section 2.5 that both Kähler and complex structure moduli develop an F-term scalar potential once NS and RR background fluxes are turned on. This scalar potential can be reproduced both by applying the usual 4d supergravity expression or by direct dimensional reduction. The latter approach involves integrating out the degrees of freedom associated to three-form fields in the four-dimensional Minkowski spacetime which give a non-vanishing contribution to the potential [3]. More precisely, we may reproduce the full F-term scalar potential (2.60) purely in terms of contributions coming from Minkowski three-forms by performing the dimensional reduction of type IIA supergravity in its democratic formulation discussed in section 2.5. In this chapter we will adopt this approach. As we will see below it allows to incorporate the open string moduli into the computation and derive a scalar potential for open and closed string modes simultaneously. To this end, we first define a set of Minkowski four-form field strengths coming from the dimensional reduction of the 10d-dimensional RR field strengths

$$G_4 = F_4^0 + ..., \quad G_6 = F_4^a \wedge \omega_a + ..., \quad G_8 = \tilde{F}_{4,a} \wedge \tilde{\omega}^a + ..., \quad G_{10} = \tilde{F}_4 \wedge \mathrm{dvol}_{\mathcal{M}_6} + ... \tag{3.1}$$

where the four-forms $(F_4^0, F_4^a, , \tilde{F}_{4,a}, \tilde{F}_4)$ have their legs along $\mathbb{R}^{1,3}$ and $\omega_a, \tilde{\omega}^a$ are the harmonic forms of $\mathcal{M}_6$ defined in section 2.4. Besides these Minkowski 4-forms, there are $h_+^{(2,1)} + 1$ further Minkowski 4-forms H_4^I arising from the NS sector through:

$$H_7 = H_4^I \wedge \alpha_I \tag{3.2}$$

where H_7 is the Hodge dual of H_3 and α_I are $\mathcal{R}$-odd three-forms $\alpha_I \in H^3_-(\mathcal{M}_6, \mathbb{Z})$.

As shown in [41], the kinetic terms for the ten-dimensional gauge fields can be dimensionally reduced to four dimensions and these Minkowski four-forms enter into the four-dimensional effective action as

$$S_{\text{eff}} = -\frac{1}{16\kappa_4^2} \int_{\mathbb{R}^{1,3}} Z_{AB} F_4^A \wedge *_4 F_4^B - \frac{1}{16\kappa_4^2} \int_{\mathbb{R}^{1,3}} Z^{AB} \rho_A \rho_B *_4 1 + \frac{1}{8\kappa_4^2} \int_{\mathbb{R}^{1,3}} F_4^A \rho_A \tag{3.3}$$

where ρ_A are polynomials of the closed string axions of the four-dimensional effective theory and we refer to them as axion-polynomials. They are related to the Minkowski four-forms by four-dimensional Hodge duality as

$$*_4 F_4^A = Z^{AB} \rho_B \tag{3.4}$$

As we will see in the next section, the polynomial coefficients in the different ρ_A are topological quantities of the compactification, like triple intersection numbers or flux quanta, such that the ρ_A are invariant under the discrete shift symmetries of the four-dimensional effective theory. On the other hand, the precise shape of each axion-polynomial is fully determined by the Freed-Witten anomalies of D-branes that appear as four-dimensional string defects.

By plugging the duality relation (3.4) back into the effective action (3.3) we can easily check that the first two terms in (3.3) cancel out, while the last term of (3.3) becomes a potential of the form

$$V = \frac{1}{8\kappa_4^2} Z^{AB} \rho_A \rho_B \tag{3.5}$$

Note that the above potential is nothing but the usual type IIA flux potential expressed in terms of the axion-polynomials. Coming up next we review and extend the reasoning of [40, 41] that led to the scalar potential (3.5). We extend this result in the sense that we consider both kinds of complex structure moduli (N^K, U_Λ) considered in the type IIA orientifold literature. Finally, in section 3.5 we show that perturbative α'-corrections also fit into the axion polynomial language.

3.2 Axion polynomials without open string moduli

Let us first consider the case without open string moduli. Following the philosophy of [41], but now taking into account the presence of both type of complex structure moduli defined in (2.48), the superpotential generated by RR and NS fluxes discussed in section 2.5 can be alternatively expressed as the product

$$W_T + W_Q = \vec{\Pi}^t \cdot \vec{\rho}, \qquad \ell_s \vec{\rho} = (R^{-1})^t \cdot \vec{q}, \tag{3.6}$$

of a saxion vector $\vec{\Pi}^t(t^a, n_\star^K, u_{\star\Lambda}) = (1, it^a, -\frac{1}{2}\mathcal{K}_{abc}t^bt^c, -\frac{i}{3!}\mathcal{K}_{abc}t^at^bt^c, in_\star^K, iu_{\star\Lambda})$ and an axion vector $\vec{\rho}$ of components ρ_A. The latter is given in terms of an $(2h_-^{11} + h^{21} + 3) \times (2h_-^{11} + h^{21} + 3)$ dimensional axion rotation matrix,

$$R(b^a, \xi^K, \xi_\Lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -b^a & \delta^a{}_b & 0 & 0 & 0 & 0 \\ \frac{1}{2}\mathcal{K}_{abc}b^bb^c & -\mathcal{K}_{abc}b^c & \delta^a{}_b & 0 & 0 & 0 \\ -\frac{1}{3!}\mathcal{K}_{abc}b^ab^bb^c & \frac{1}{2}\mathcal{K}_{abc}b^bb^c & -b^a & 1 & 0 & 0 \\ -\xi_\star^K & 0 & 0 & 0 & \delta^K{}_L & 0 \\ -\xi_{\star\Lambda} & 0 & 0 & 0 & 0 & \delta^\Sigma{}_\Lambda \end{pmatrix}, \tag{3.7}$$

and a charge vector $\vec{q}$ consisting of the quantised fluxes, i.e. $\vec{q} = (e_0, e_a, m^a, m, h_K, h^\Lambda)^t$. The factorised form of the superpotential enables to expose the multi-branched structure of the vacua for the closed string axions: the periodic shift symmetry of the axions leaves the action, potential and superpotential invariant provided that the flux quanta $\vec{q}$ are shifted simultaneously. Formally, the shift symmetries of the closed string axions are generated by the nilpotent matrices P_a, P_K and P^Λ,

$$P_a = \begin{pmatrix} 0 & -\vec{\delta}_a^t & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathcal{K}_{abc} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\vec{\delta}_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad P_K = \begin{pmatrix} 0 & 0 & 0 & 0 & -\vec{\delta}_K^t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad P^\Lambda = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -(\vec{\delta}^L)^t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3.8}$$

which mutually commute among each other. As such, the axion rotation matrix can be expressed in terms of these matrices through exponentiation:

$$R^t(b^a, \xi_\star^K, \xi_{\star\Lambda}) = e^{b^aP_a + \xi_\star^K P_K + \xi_{\star\Lambda}P^\Lambda}. \tag{3.9}$$

The matrix notation also allows to express elegantly the invariance of the theory under the axionic shift symmetries, which acts on the axion rotation matrix as:

$$(R^{-1})^t(b^a + r^a, \xi_\star^K + \varpi^K, \xi_{\star\Lambda} + \varpi_\Lambda) = (R^{-1})^t(b^a, \xi_\star^K, \xi_{\star\Lambda}) \cdot e^{-r^a P_a - \varpi^K P_K - \varpi_\Lambda P^\Lambda}, \tag{3.10}$$

with $r^a, \varpi^K, \varpi_\Lambda \in \mathbb{Z}$. The invariance of the superpotential is manifest provided the charge vector transforms as,

$$\vec{q} \quad \rightarrow \quad e^{r^a P_a + \varpi^K P_K + \varpi_\Lambda P^\Lambda} \vec{q}\,. \tag{3.11}$$

The shift symmetry implies the existence of a set of gauge-invariant axion polynomials $\ell_s \vec{\rho} \equiv (R^{-1})^t \cdot \vec{q}$, whose explicit component forms are given by,

$$\begin{array}{lcl} \ell_s \rho_0 & = & e_0 + e_a b^a + \frac{1}{2}\mathcal{K}_{abc} m^a b^b b^c + \frac{m}{6}\mathcal{K}_{abc} b^a b^b b^c + h_K \xi_\star^K + h^\Lambda \xi_{\star\Lambda}, \\ \ell_s \rho_a & = & e_a + \mathcal{K}_{abc} m^b b^c + \frac{m}{2}\mathcal{K}_{abc} b^b b^c, \\ \ell_s \tilde{\rho}^a & = & m^a + m b^a, \\ \ell_s \tilde{\rho} & = & m, \\ \ell_s \hat{\rho}_K & = & h_K, \\ \ell_s \hat{\rho}^\Lambda & = & h^\Lambda. \end{array} \tag{3.12}$$

3.3 Four-dimensional strings and Freed-Witten anomalies

The invariance under the axion shift symmetries is not coincidental, but relies microscopically on the cancellation of Freed-Witten anomalies for four-dimensional strings in the presence of background flux [41]. More concretely, each of the axions $(b^a, \xi_\star^K, \xi_{\star\Lambda})$ can be Hodge-dualised in four dimensions to its corresponding two-form coupling to four-dimensional strings. In type IIA backgrounds these axionic strings arise from NS5-branes wrapping the Poincaré-dual four-cycles $\mathrm{PD}(\omega_a)$ (b-type axionic strings) and D4-branes wrapping the Poincaré-dual three-cycles $\mathrm{PD}(\alpha_K)$ and $\mathrm{PD}(\beta_\Lambda)$ respectively (ξ-type axionic strings). In the presence of background RR-flux $\overline{G}_{2p}$ the b-type axionic strings develop a Freed-Witten anomaly in case $G_{2p}\big|_{\mathrm{PD}(\omega_a)}$ is non-trivial in cohomology, which can be mediated by emitting a $D(6-2p)$-brane wrapping the $(4-2p)$-cycle in the Poincaré dual class of $G_{2p}\big|_{\mathrm{PD}(\omega_a)}$. Similarly, the ξ-type axionic strings resolve the Freed-Witten anomaly in the presence of H_3-flux by emitting $D2$-branes, as summarised in table 3.1. The emitted D-branes form four-dimensional domain walls bounded by axionic strings that separate vacua in the axion moduli space with different RR- and/or NS-fluxes [43]. In this respect the domain walls are unstable under nucleation of holes bounded by axionic strings, which allows the axions to cross the domain wall by virtue of a monodromy generated by the matrices P_a, P_K and P^Λ. Under the axion monodromies the flux quanta will shift as prescribed in (3.25), such that both effects cancel each other out and all vacua for the axions are equivalent. It is also straightforward to verify that the field strengths in (2.54) remain invariant under such shift symmetries, which can be seen as a particular subset of gauge transformations.

String		**Flux**	**Domain Wall**	
Axion	Brane Set-up	type	Brane Set-up	Rank
$B_2 = b^a\omega_a$	NS5 on $[\pi_a] \in H_4^+(\mathcal{M}_6,\mathbb{Z})$	$\overline{G}_0 = m$	D6 on $[\pi_a]$	m
$B_2 = b^a\omega_a$	NS5 on $[\pi_a] \in H_4^+(\mathcal{M}_6,\mathbb{Z})$	$\overline{G}_2 = m^a\omega_a$	D4 on $[\mathrm{PD}(\overline{G}_2 \wedge \omega_a)]$	$\int_{\tilde{\pi}^a} \omega_c = \mathcal{K}_{abc}m^b$
$B_2 = b^a\omega_a$	NS5 on $[\pi_a] \in H_4^+(\mathcal{M}_6,\mathbb{Z})$	$\overline{G}_4 = e_a\tilde{\omega}^a$	D2 at point in $\mathcal{M}_6$	$\int_{\pi_a} \overline{G}_4 = e_a$
$C_3 = \xi_\star^K\alpha_K$	D4 on $[B^K] \in H_3^-(\mathcal{M}_6,\mathbb{Z})$	$H_3 = h_K\beta^K$	D2 at point in $\mathcal{M}_6$	$\int_{B^K} \overline{H}_3 = -h_K$
$C_3 = -\xi_{\star\Lambda}\beta^\Lambda$	D4 on $[A_\Lambda] \in H_3^-(\mathcal{M}_6,\mathbb{Z})$	$H_3 = h^\Lambda\alpha_\Lambda$	D2 at point in $\mathcal{M}_6$	$\int_{A_\Lambda} \overline{H}_3 = h^\Lambda$

Table 3.1: Summary of 4d axionic strings with their respective attached domain walls arising from Dp- and NS5-branes wrapping internal cycles on a Calabi-Yau manifold with internal flux.

3.4 Axion polynomials with open string states

As reviewed in section 2.6, backgrounds with localised sources such as D6-branes and O6-planes provide a much more intricate picture for type IIA compactifications with fluxes. On the one hand, their presence redefines the 4d fields that appear in the Kähler potential, modifying the Kähler metrics non-trivially. On the other hand, some open string moduli for mobile D6-branes will contribute to the superpotential through a bilinear coupling with the Kähler moduli. Fortunately, the particular (bi)linear structure of the last term in (2.78) allows for the factorisation of the superpotential (3.6) into geometric moduli, axions and flux quanta to go through in the presence of open string moduli as well:

$$\ell_s \left(W - W^0_{D6}\right) = \overrightarrow{\Pi}^t \cdot (R^{-1})^t \cdot \overrightarrow{q}\,, \tag{3.13}$$

where the saxion vector $\overrightarrow{\Pi}^t(t^a, n^K, u_\Lambda, \phi^i_\alpha) = (1, it^a, -\frac{1}{2}\mathcal{K}_{abc}t^bt^c, -\frac{i}{3!}\mathcal{K}_{abc}t^at^bt^c, in^K, iu_\Lambda, i\phi^i_\alpha, t^a\phi^i_\alpha)$ is now extended with the open string moduli ϕ^i_α, the charge vector $\overrightarrow{q} = (e_0, e_a, m^a,\ m, h_K, h^\Lambda, n^\alpha_{Fi}, n^\alpha_{ai})^t$ is extended with the open string quanta $(n^\alpha_{Fi}, n^\alpha_{ai})$ and the axion rotation matrix has to be enlarged with open string axions $\hat{\theta}^i_\alpha$:

$$R(b^a,\xi^K,\xi_\Lambda,\hat{\theta}^i_\alpha)=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -b^a & \delta^a{}_b & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2}\mathcal{K}_{abc}b^bb^c & -\mathcal{K}_{abc}b^c & \delta^a{}_b & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3!}\mathcal{K}_{abc}b^ab^bb^c & \frac{1}{2}\mathcal{K}_{abc}b^bb^c & -b^a & 1 & 0 & 0 & 0 & 0 \\ -\xi^K & 0 & 0 & 0 & \delta^K{}_L & 0 & 0 & 0 \\ -\xi_\Lambda & 0 & 0 & 0 & 0 & \delta^\Sigma{}_\Lambda & 0 & 0 \\ \hat{\theta}^i_\alpha & 0 & 0 & 0 & 0 & 0 & \delta^i{}_j & 0 \\ \hat{\theta}^i_\alpha b^a & \hat{\theta}^i_\alpha\delta^a{}_b & 0 & 0 & 0 & 0 & b^a\delta^i{}_j & \delta^i{}_j\delta^a{}_b \end{pmatrix}. \tag{3.14}$$

Also in the presence of open string axions, the rotation matrix can be generated by a set of nilpotent matrices through exponentiation:

$$R^t(b^a,\xi^K,\xi_\Lambda,\hat{\theta}^i_\alpha)=e^{b^a\mathbb{P}_a+\xi^K\mathbb{P}_K+\xi_\Lambda\mathbb{P}^\Lambda+\hat{\theta}^i_\alpha\mathbb{P}^\alpha_i}, \tag{3.15}$$

with the shift-generating matrices $(\mathbb{P}_a,\mathbb{P}_K,\mathbb{P}^\Lambda)$ forming the natural extension of their closed string counterparts (3.8):

$$P_a\to\mathbb{P}_a=\begin{pmatrix} P_a & \vec{0}^t & \vec{0}^t \\ \vec{0} & 0 & \vec{\delta}^t_j \\ \vec{0} & 0 & 0\end{pmatrix},\ P_K\to\mathbb{P}_K=\begin{pmatrix} P_K & \vec{0}^t & \vec{0}^t \\ \vec{0} & 0 & 0 \\ \vec{0} & 0 & 0\end{pmatrix}, \tag{3.16}$$

$$P^\Lambda\to\mathbb{P}^\Lambda=\begin{pmatrix} P^\Lambda & \vec{0}^t & \vec{0}^t \\ \vec{0} & 0 & 0 \\ \vec{0} & 0 & 0\end{pmatrix},$$

and the only new generator $\mathbb{P}^\alpha_i$ being associated to the shift symmetries of the open string axions:

$$\mathbb{P}^\alpha_i=\begin{pmatrix} 0&0&0&0&0&0&\vec{\delta}^t_j & 0\\ 0&0&0&0&0&0&0&\vec{\delta}^t_a\vec{\delta}^t_j\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}. \tag{3.17}$$

Under the shift symmetries of the closed string axions, the rotation matrix keeps its original transformation properties (3.10), and the addition of open string axions enforces the axion rotation matrix to transform under an additional set of shift symmetries associated to the open string axions, with $\lambda^i_\alpha\in\mathbb{Z}$:

$$(R^{-1})^t(b^a,\xi^K,\xi_\Lambda,\hat{\theta}^i_\alpha+\lambda^i_\alpha)=(R^{-1})^t(b^a,\xi^K,\xi_\Lambda,\hat{\theta}^i_\alpha)\cdot e^{-\lambda^i_\alpha\mathbb{P}^\alpha_i}. \tag{3.18}$$

Invariance of the superpotential under the combined axion shift symmmetries requires the charge vector to transform as well:

$$\overrightarrow{q} \to e^{r^a \mathbb{P}_a + \varpi^K \mathbb{P}_K \varpi_\Lambda \mathbb{P}^\Lambda + \lambda^i_\alpha \mathbb{P}^\alpha_i} \cdot \overrightarrow{q} \, . \tag{3.19}$$

These considerations thus naturally extend the observations reviewed in the previous section and allow to identify a set of shift-invariant axion polynomials $\ell_s \overrightarrow{\varrho} \equiv (R^{-1})^t \cdot \overrightarrow{q}$ including both closed and open string axions:

$$\begin{array}{rcl}
\ell_s \varrho_0 &=& e_0 + e_a b^a + \frac{1}{2}\mathcal{K}_{abc} m^a b^b b^c + \frac{m}{6}\mathcal{K}_{abc} b^a b^b b^c + h_K \xi^K + h^\Lambda \xi_\Lambda + n^\alpha_{Fi} \hat{\theta}^i_\alpha - n^\alpha_{ai} \hat{\theta}^i_\alpha b^a, \\
\ell_s \varrho_a &=& e_a + \mathcal{K}_{abc} m^b b^c + \frac{m}{2}\mathcal{K}_{abc} b^b b^c - n^\alpha_{ai} \hat{\theta}^i_\alpha, \\
\ell_s \tilde{\varrho}^a &=& m^a + m b^a, \\
\ell_s \tilde{\varrho} &=& m, \\
\ell_s \hat{\varrho}_K &=& h_K, \\
\ell_s \hat{\varrho}^\Lambda &=& h^\Lambda, \\
\ell_s \varrho^\alpha_i &=& n^\alpha_{Fi} - b^a n^\alpha_{ai}, \\
\ell_s \varrho^\alpha_{ai} &=& n^\alpha_{ai}.
\end{array} \tag{3.20}$$

The microscopic justification for the invariance under the axion shifts now runs [41] through the Hanany-Witten effect, which is in one-to-one correspondence with the Freed-Witten anomaly condition and allows to identify which combinations of flux quanta form invariant directions. Apart from assuring the consistency of four-dimensional axionic strings in flux backgrounds, the Freed-Witten anomaly conditions also serve to verify the microscopic compatibility between background fluxes and the D6-branes wrapping internal *SLag* three-cycles. In first instance, the NS-fluxes can induce Freed-Witten anomalies on the D6-brane worldvolume, unless the pullback of the NS 3-form field strength with respect to the wrapped three-cycle is an exact 3-form, see e.g [26]:

$$\int_{\Pi_\alpha} H_3 = 0. \tag{3.21}$$

On a formal footing, the requirement of vanishing Freed-Witten anomalies in a background B_2-field ensures the absence of global worldsheet anomalies in the fermionic sector of the open superstring attached to the D6-brane [25]. At the level of the 4d $\mathcal{N} = 1$ supergravity theory, a vanishing Freed-Witten anomaly implies that only the linear combination $h_K \xi^K + h^\Lambda \xi_\Lambda$ effectively enters in the superpotential, while all orthogonal combinations can be gauged under the open string $U(1)$ symmetries living on D6-branes [44, 45] without violating gauge invariance.

3.5 Axion polynomials and α'-corrections

In this section we study how the inclusion of the curvature corrections is compatible with the axionic shift symmetries of the superpotential. First, notice that its inclusion does not destroy the factorability of the superpotential in terms of geometric moduli and axions. Indeed, we can write the α'-corrected flux superpotential as

$$\ell_s\left(W_T + W_Q\right) = \overrightarrow{\Pi}^t \cdot Q^t \cdot (\overline{R}^{-1})^t \cdot \overrightarrow{q}, \tag{3.22}$$

provided that we modify the previous quantities. The saxion vector is now given by $\overrightarrow{\Pi}^t(t^a, n^K, u_\Lambda) = (1, it^a, -\frac{1}{2}\mathcal{K}_{abc}t^bt^c, -\frac{i}{3!}\mathcal{K}_{abc}t^at^bt^c - iK^{(3)}, in^K, iu_\Lambda)$, we have introduced a square matrix Q defined below, and the axion rotation matrix is given by

$$\overline{R}(b^a, \xi^K, \xi_\Lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -b^a & \delta^a{}_b & 0 & 0 & 0 & 0 \\ \frac{1}{2}\mathcal{K}_{abc}b^bb^c - K^{(1)}_{ab}b^b & -\mathcal{K}_{abc}b^c & \delta^a{}_b & 0 & 0 & 0 \\ -\frac{1}{3!}\mathcal{K}_{abc}b^ab^bb^c - 2K^{(2)}_a b^a & \frac{1}{2}\mathcal{K}_{abc}b^bb^c + K^{(1)}_{ab}b^b & -b^a & 1 & 0 & 0 \\ -\xi^K & 0 & 0 & 0 & \delta^K{}_L & 0 \\ -\xi_\Lambda & 0 & 0 & 0 & 0 & \delta^\Sigma{}_\Lambda \end{pmatrix}. \tag{3.23}$$

Second, the axion rotation matrix is still generated through exponentiation as in (3.9), but now by a modified set of nilpotent, commuting matrices $(\overline{P}_a, P_K, P^\Lambda)$. The shift-generator $\overline{P}_a$ for the Kähler axions is related to the previous version in (3.8) by conjugation with the charge matrix Q,

$$\overline{P}_a = Q^{-1}P_aQ, \qquad Q = \begin{pmatrix} 1 & 0 & -K^{(2)}_a & 0 & 0 & 0 \\ 0 & \delta^a_b & -K^{(1)}_{ab} & K^{(2)}_a & 0 & 0 \\ 0 & 0 & \delta^b_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta^K_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta^\Sigma_\Lambda \end{pmatrix}. \tag{3.24}$$

Given these simple extensions, the superpotential remains invariant under the shift symmetries of the closed string axions, provided that the flux quanta transform simultaneously as follows:

$$\overrightarrow{q} \quad \to \quad e^{r^a\overline{P}_a + \varpi^K P_K + \varpi_\Lambda P^\Lambda}\overrightarrow{q}\,. \tag{3.25}$$

The transformed flux vector has integer entries provided that $K^{(1)}_{ab}, K^{(2)}_a \in \mathbb{Z}$, which we will assume in the following. Finally, one may express the superpotential in terms of the previous rotation matrix as

$$\ell_s\left(W_T + W_Q\right) = \overrightarrow{\Pi}^t \cdot (R^{-1})^t \cdot \overrightarrow{\mathfrak{q}}, \qquad \overrightarrow{\mathfrak{q}} \equiv Q \cdot \overrightarrow{q} \tag{3.26}$$

Hence, also in the presence of α'-corrections one is encouraged to introduce gauge-invariant axion polynomials $\ell_s \overrightarrow{\rho} \equiv (R^{-1})^t \cdot \overrightarrow{q}$, which can be given explicitly in terms of the flux quanta,

$$\begin{aligned}
\overline{\rho}_0 &= \overline{e}_0 + \overline{e}_a b^a + \tfrac{1}{2}\mathcal{K}_{abc} m^a b^b b^c + \tfrac{m}{6}\mathcal{K}_{abc} b^a b^b b^c + h_K \xi_\star^K + h^\Lambda \xi_{\star\Lambda}, \\
\overline{\rho}_a &= \overline{e}_a + \mathcal{K}_{abc} m^b b^c + \tfrac{m}{2}\mathcal{K}_{abc} b^b b^c, \\
\tilde{\rho}^a &= m^a + m b^a, \\
\tilde{\rho} &= m, \\
\hat{\rho}_K &= h_K, \\
\hat{\rho}^\Lambda &= h^\Lambda.
\end{aligned} \tag{3.27}$$

where $\overline{e}_0$ and $\overline{e}_a$ are the curvature-corrected flux quanta as introduced before.

This flux redefinition can be interpreted microscopically by noticing that the curvature corrections $K_{ab}^{(1)}$ and $K_a^{(2)}$ induce lower-dimensional D-brane charge on D-branes wrapping internal cycles, see e.g. [46]. They will, in particular, induce lower-dimensional charge on D-brane domain walls with non-trivial internal worldvolumes. For instance, the $K_a^{(2)}$ curvature corrections induce D4-brane charges on D8-brane domain walls wrapping the full compactification space, and also create bound states of D6-D2 brane domain walls. The $K_{ab}^{(1)}$ curvature corrections on the other hand turn D6-brane domain walls into bound states of D6-D4 brane domain walls. The induced lower-dimensional D-brane charges due to the curvature corrections also imply that the Freed-Witten anomalies associated to the b-type axionic strings have to be cured by bound states of domain walls. To end up in a different vacuum separated by the bound states of domain walls, the b-axions have to undergo a monodromy transformation generated by the modified matrix $\overline{P}_a$ defined in (3.24). Alternatively, one may redefine the basis of domain walls (or, equivalently the flux basis) by the matrix Q such that the monodromy matrix is generated by P_a. The Freed-Witten anomaly cancelation for the ξ-type axionic strings on the other hand remains unaffected by the curvature corrections. These considerations offer a microscopic rationale behind the superpotential (3.26), which allows for a factorisation in which curvature corrections of order $\mathcal{O}(\alpha')$ and $\mathcal{O}(\alpha'^2)$ are incorporated into the set of shift-invariant axion polynomials (3.27). The $\mathcal{O}(\alpha'^3)$ curvature contributions represented by $K^{(3)}$ correct the overall volume of the internal space and therefore have to be included in the modified saxion vector $\overrightarrow{\Pi}^t$.

3.6 The bilinear form of the Type IIA flux potential

At the beginning of this chapter, we comment in advance that flux potentials can be expressed alternatively as a bilinear of the flux-axion polynomials, namely of the form $V = Z^{AB}\rho_A\rho_B$. The indices A, B run over all fluxes of the compactification, ρ_A are polynomials of the closed and open string axions of the 4d effective theory defined in previous sections and Z^{AB} is the inverse metric in the space of the axion polynomials [40, 41]. This description of flux potentials makes more transparent the discrete symmetries of the effective theory, where the polynomials ρ_A can be seen as the basic building blocks. Thus, any flux dependence of the scalar potential or any axion dependence which is not periodic must appear through a function of the polynomials ρ_A, or otherwise it will not respect the underlying discrete shift symmetry of the theory. Moreover, such a description of flux potentials may have interesting applications, for instance as we show in the next chapters it can facilitate the search of minima in flux potentials. In particular, we can easily rewrite the conditions for Minkowski and AdS vacua from the closed-string type IIA flux potential in this language, obtaining algebraic equations on the ρ_A that reproduce known results in the literature [8, 9],. A further advantage of rewriting the scalar potential as a bilinear is that one can easily incorparate the presence of D6-brane moduli. Indeed, in terms of the expression $V = Z^{AB}\rho_A\rho_B$ this only means that the indices A and B runs over more fluxes and that Z and the ρ's depend on more fields, but the structure of the potential remains the same. As we will argue in the next chapter, in this way one may easily add mobile D6-branes to, e.g., the class of flux compactifications analysed in [8], thus obtaining a more general class of non-supersymmetric Minkowski vacua in which the open string moduli are stabilised at non-trivial vevs. In the previous sections, we provided the explicit shape of the flux-axiom polynomials in purely closed-string setups, in the presence of open string moduli and finally in the presence of perturbative α'-corrections. Hence, the next step is to determine the explicit form of the metric in the flux-axion polynomial space.

In the absence of open string moduli

Let us first consider the case without open string moduli, in which the geometric moduli of the compactification reduce to $\left(t^a, n_\star^K, u_{\star\Lambda}\right)$ and their axionic partners are $\left(b^a, \xi_\star^K, \xi_{\star\Lambda}\right)$. Then, the flux potential can be written as [47]

$$V_F = \frac{1}{8\kappa_4^2}\rho_A(b,\xi_\star)\, Z^{AB}(t, n_\star, u_\star)\, \rho_B\,(b,\xi_\star), \tag{3.28}$$

Here ρ_A are the shift invariant axion polynomials defined in (3.12) and encode the dependence in the flux quanta and the axions, while the inverse metric on the axion polynomial space Z^{AB} is given by

$$Z^{AB} = 8e^K \begin{pmatrix} 4 & & & & & \\ & K^{a\bar{b}} & & & & \\ & & \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}} & & & \\ & & & \frac{1}{9}\mathcal{K}^2 & \frac{2}{3}\mathcal{K}n_\star^I & \frac{2}{3}\mathcal{K}u_{\star\Lambda} \\ & & & \frac{2}{3}\mathcal{K}n_\star^J & K^{IJ} & K^{I\Sigma} \\ & & & \frac{2}{3}\mathcal{K}u_{\star\Sigma} & K^{\Lambda J} & K^{\Lambda\Sigma} \end{pmatrix}. \tag{3.29}$$

Note that, the potential (3.29) is nothing but the flux potential (3.5), but now we allow for both type of complex structure moduli, while the above inverse metric is the natural generalization of the inverse metric eq. (3.2) of [41].

In the presence of open string moduli

Let us now consider the presence of open string moduli in the compactification. As argued in section 3.4, the presence of open string states implies the shift of certain closed string axion polynomials (3.12) and the appearance of a term contributing to the scalar potential [41]

$$V_{\mathrm{DBI}} = \sum_\alpha \frac{e^K}{\kappa_4^2 l_s^2} G_\alpha^{ij} \left(n^\alpha_{\mathcal{F}i} - n^\alpha_{ai} T^a\right)\left(n^\alpha_{\mathcal{F}j} - n^\alpha_{aj} \bar{T}^a\right) \tag{3.30}$$

arising from the dimensional reduction of the DBI action for D6-branes. Where the index α runs over pairs of D6-branes related by the orientifold action and G_α^{ij} is the inverse of the metric on the D6-brane position moduli space

$$G^\alpha_{ij} = \frac{e^{-\frac{\phi}{4}}}{8\mathcal{V} l_s^3} \int_{\Pi^0_\alpha} \zeta_i \wedge * \zeta_j \tag{3.31}$$

we defined $n^\alpha_{\mathcal{F}i} = n^\alpha_{Fi} - \frac{1}{2} g_i^K h_K - \frac{1}{2} g_{\Lambda i} h^\Lambda$. Since the terms (3.30) are also quadratic on closed- and open-string fluxes, one may easily see that the structure of the full scalar potential remains the same as in (3.28) if one keeps the axion polynomials as in (3.20) and takes the inverse metric on the axion polynomial space as

$$Z^{AB} = 8e^K \begin{pmatrix} 4 & & & & & & & \\ & K^{a\bar{b}} & & & & & & \\ & & \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}} & & & & & \\ & & & \frac{1}{9}\mathcal{K}^2 & \frac{2}{3}\mathcal{K} n^I_\star & \frac{2}{3}\mathcal{K} u_{\star\Lambda} & & \\ & & & \frac{2}{3}\mathcal{K} n^J_\star & K^{IJ} & K^{I\Sigma} & & \\ & & & \frac{2}{3}\mathcal{K} u_{\star\Sigma} & K^{\Lambda J} & K^{\Lambda\Sigma} & & \\ & & & & & & G^{ij}_{\alpha\beta} & \\ & & & & & & & G^{ij}_{\alpha\beta} t^a t^b \end{pmatrix}. \tag{3.32}$$

with $G^{ij}_{\alpha\beta} = G^{ij}_\alpha \delta_{\alpha\beta}$. Finally, in section 3.5 we showed that perturbative α'-corrections also fit into the axion-polynomial formalism. This fact is clearly an indication that the bilinear structure of the potential even persists in the presence of those corrections, which will be shown in more detail in chapter 5 and appendix C. For now, we just state that such a structure of the potential is kept.

Part II

Moduli stabilisation

Chapter 4

The Type IIA Flux Landscape

If type IIA orientifold compactifications ought to provide for vacuum solutions exhibiting the well-known features of our universe, the various open and closed geometric moduli have to be stabilised with sufficiently high masses. Fortunately, the richness of background NS- and RR-fluxes in type IIA offers a controlled, perturbative method to deal with moduli stabilisation for all closed string moduli [3, 9, 10, 44, 48].

In previous chapter we have argued that the classical flux potential can be rewritten as a bilinear $V = Z^{AB}\rho_A\rho_B$ in which the geometric moduli-dependent part is fully captured by the matrix Z^{AB}, while the polynomials ρ_A contain the dependence on the axions. Since such a factorisation is also observed in the perturbative superpotential induced by NS- and RR-fluxes, this gives rise to the question how moduli stabilisation respects this factorisation and can be formulated in terms of the axion polynomial language. This is precisely the aim of this chapter, to illustrate the general idea we choose two well-known examples from literature, i.e. non-supersymmetric Minkowski vacua and supersymmetric AdS vacua without D6-branes as toy examples. Finally, in section 4.3 we argue that the stabilisation of the open string moduli can be dealt with in a much more elegant way by using this formalism, and in particular it allows us to find new and more general classes of non-supersymmetric Minkowski vacua. This chapter is mainly based on the paper [47].

4.1 Flux vacua without D6-branes

The aim of this section is to reformulate the known flux stabilisation of closed string modul at $\mathcal{N} = 0$ Minkowski and $\mathcal{N} = 1$ AdS vacua but now using the axion polynomial formalism, obtaining algebraic equations on the ρ_A that reproduce known results in the literature.

4.1.1 $\mathcal{N} = 0$ Minkowski vacua

The imaginary self dual (ISD) flux vacua of type IIB can be T-dualised to type IIA flux vacua [8, 44] for which all RR-fluxes are switched on and the NS 3-form flux is turned on along only one $\Omega\mathcal{R}$-odd three-cycle. Following the symplectic basis choice of [8] in which the complex structure moduli $\{N_\star^K\}_{K\neq 0}$ are projected out, we can assume that the four-dimensional dilaton $N_\star^0 = S_\star = \xi_\star^0 + i\,\mathrm{Im}\,(\mathrm{S}_\star)$ factorises from the other complex structure moduli $U_{\star\Lambda}$ in the Kähler potential:

$$K_Q^{\mathrm{ISD}} = -\log\left[-i(S_\star - \overline{S}_\star)\right] - 2\log\left[\tilde{\mathcal{G}}_Q(u_{\star\Lambda})\right], \tag{4.1}$$

where $\tilde{\mathcal{G}}_Q(u_{\star\Lambda})$ is a homogeneous function of degree 3/2 with an implicit dependence on the geometric moduli $u_{\star\Lambda}$. More precisely, the functional dependence of $\tilde{\mathcal{G}}_Q$ can be expressed in terms of the rescaled periods $\mathrm{Im}\,(\mathrm{Z}^\Lambda) \equiv 2\mathrm{Re}\,(\mathcal{C}\mathrm{Z}^0)^{-1/2}\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)$ and upon inverting the relation $u_{\star\Lambda} = \partial_{\mathrm{Im}\,(\mathrm{Z}^\Lambda)}\tilde{\mathcal{G}}_Q$ the function $\tilde{\mathcal{G}}_Q$ can in principle be written in terms of the geometric moduli $u_{\star\Lambda}$. Finally, if we further assume that the only non-vanishing NS-flux is supported along the $\Omega\mathcal{R}$-odd three-form β^0, we obtain the generic superpotential for ISD fluxes,

$$\ell_s W_{\mathrm{ISD}} = h_0 S_\star + e_0 + e_a T^a + \frac{1}{2}\mathcal{K}_{abc}m^a T^b T^c + \frac{m}{6}\mathcal{K}_{abc}T^a T^b T^c, \tag{4.2}$$

which in terms of the axion polynomials reads

$$W_{\mathrm{ISD}} = is_\star\hat{\rho}_0 + \rho_0 + it^a\rho_a - \frac{1}{2}\mathcal{K}_a\tilde{\rho}^a - \frac{i}{6}\mathcal{K}\tilde{\rho}. \tag{4.3}$$

Given the specific form of the Kähler potential (4.1), the F-term scalar potential takes the form

$$\begin{aligned} V_F &= \frac{e^K}{\kappa_4^2}\left[K^{A\bar{B}}F_A F_{\bar{B}} - 3\left|W\right|^2\right] \qquad (4.4)\\ &= \frac{e^K}{\kappa_4^2}\left[K^{T^a\bar{T}^b}F_{T^a}F_{\bar{T}^b} + K^{S_\star\bar{S}_\star}F_{S_\star}F_{\bar{S}_\star} + K^{U_{\star\Lambda}\bar{U}_{\star\Lambda}}F_{U_{\star\Lambda}}F_{\bar{U}_{\star\Lambda}} - 3\left|W\right|^2\right]\\ &= \frac{e^K}{\kappa_4^2}\left[K^{T^a\bar{T}^b}F_{T^a}F_{\bar{T}^b} + K^{S_\star\bar{S}_\star}F_{S_\star}F_{\bar{S}_\star}\right] \end{aligned}$$

where in the last line we have used that by assumption $F_{U_{\star\Lambda}} = K_{U_{\star\Lambda}}W$ and the no-scale relation $K^{U_{\star\Lambda}\bar{U}_{\star\Lambda}}K_{U_{\star\Lambda}}K_{\bar{U}_{\star\Lambda}} = 3$ that arises from (4.1). Therefore, for these kind of vacua we recover a positive semidefinite flux potential whose absolute minima are reached whenever $F_{S_\star} = F_{T^a} = 0$. In general, the factorisable form (3.6) of the ISD flux superpotential enables us to simplify the F-terms for the dilaton $S_\star$ and Kähler moduli and express them entirely in terms of geometric moduli and the gauge-invariant axion polynomials (3.12). Focusing first on the F-term for the dilaton we obtain[1]

$$F_{S_\star} = -i\,\partial_{s_\star} W_{\rm ISD} + \frac{i}{2s_\star} W_{\rm ISD} = \frac{1}{2s_\star}\left(i\rho_0 - t^a\rho_a - \frac{i}{2}\mathcal{K}_a\tilde{\rho}^a + \frac{1}{6}\mathcal{K}\tilde{\rho} + s_\star\hat{\rho}_0\right), \quad (4.5)$$

where we have used the holomorphicity of the superpotential, i.e. $\partial_{\overline{S}_\star} W_{\rm ISD} = 0$, to obtain a first order derivative purely with respect to the four-dimensional dilaton $s_\star = \mathrm{Im}\,(\mathrm{S}_\star)$. Similar considerations can be made for the F-terms of the Kähler moduli,

$$\begin{aligned} F_{T^a} &= -i\,\partial_{t^a} W_{\rm ISD} + \tfrac{3i\mathcal{K}_a}{2\mathcal{K}} W_{\rm ISD} \\ &= \rho_a + i\mathcal{K}_{ab}\tilde{\rho}^b + \tfrac{3i\mathcal{K}_a}{2\mathcal{K}}\left(\rho_0 + it^b\rho_b - \tfrac{1}{2}\mathcal{K}_b\tilde{\rho}^b + \tfrac{i}{6}\mathcal{K}\tilde{\rho} + is_\star\hat{\rho}_0\right). \end{aligned} \quad (4.6)$$

Finally, a more elegant polynomial expression in terms of the geometric moduli and axion polynomials is found in the form of the linear combination $t^aF_{T^a}$,

$$t^aF_{T^a} = \frac{3i}{2}\rho_0 - \frac{1}{2}t^a\rho_a + \frac{i}{4}\mathcal{K}_a\tilde{\rho}^a - \frac{3}{2}\left(\frac{1}{6}\mathcal{K}\tilde{\rho} + s_\star\hat{\rho}_0\right). \quad (4.7)$$

When considering the expressions (4.5), (4.6) and (4.7) as polynomials in t^a simultaneously, the vanishing of the F-terms implies that their coefficients ought to vanish:

$$\tilde{\rho}^a = 0, \quad \rho_a = 0, \quad \frac{1}{6}\mathcal{K}\tilde{\rho} + s_\star\hat{\rho}_0 = 0, \quad \rho_0 = 0. \quad (4.8)$$

As we discuss in section 4.3, one can easily rederive these conditions from the bilinear form of the potential (3.28). The first set of equations $\tilde{\rho}^a = 0$ stabilise the Kähler axions in terms of the RR flux quanta:

$$b^a = -\frac{m^a}{m}, \quad (4.9)$$

while the second set of equations $\rho_a = 0$ represent a set of constraints on the flux quanta:

$$2me_a - \mathcal{K}_{abc}m^bm^c = 0. \quad (4.10)$$

Upon imposing these set of relations, the third and last equation stabilise the four-dimensional dilaton $\mathrm{Im}\,(\mathrm{S}_\star)$ and its axion $\xi^0_\star$ respectively in terms of flux quanta and the Kähler moduli:

$$h_0 s_\star = -\frac{m}{6}\mathcal{K}_{abc}t^at^bt^c, \qquad h_0\xi^0_\star = -\frac{1}{m^2}\left(e_0m^2 - \frac{1}{6}\mathcal{K}_{abc}m^am^bm^c\right). \quad (4.11)$$

Thus, the analysis of the F-terms for the dilaton and Kähler moduli in terms of the axion polynomials allows to easily extract the generic ISD vacua (4.8),

[1]To simplify the expressions, we use $\mathcal{K} = \mathcal{K}_{abc}t^at^bt^c$, $\mathcal{K}_a = \mathcal{K}_{abc}t^bt^c$, $\mathcal{K}_{ab} = \mathcal{K}_{abc}t^c$.

which reproduce the results of section 3.1 in [8] represented by the last four relations (4.9)-(4.11). In these vacua, the saxionic parts of the Kähler moduli and complex structure moduli remain unstabilised partly due to the no-scale symmetry for the complex structure moduli $U_{\star\Lambda}$. This no-scale symmetry combined with the vanishing F-terms for the dilaton and Kähler moduli imply a vanishing F-term scalar potential at the ISD vacuum, which corresponds to a non-supersymmetric Minkowski spacetime in four dimensions. Supersymmetry is then spontaneously broken by the non-vanishing F-terms of the complex structure moduli $U_{\star\Lambda}$, given that the on-shell superpotential for ISD flux vacua is non-vanishing for arbitrary Romans mass,

$$\langle W_{ISD}\rangle = -\frac{i}{3}\mathcal{K}\tilde{\rho}. \tag{4.12}$$

The structures of the F-terms in the complex structure moduli sector will be further analysed in chapter 7, in conjunction with the structures of flux-induced soft terms. As we will see in the next chapter, a more compelling moduli stabilisation scenario is achieved upon inclusion of the α'-corrections that deform the Kähler potential from (2.11) to (2.80). In that case one is also able to fix the geometric part of the Kähler moduli.

4.1.2 $\mathcal{N} = 1$ AdS vacua

As soon as the no-scale structure for the complex structure moduli $U_{\star\Lambda}$ is broken by the presence of additional NS-fluxes, both the complex structure moduli and Kähler moduli can be stabilised to non-trivial values simultaneously. Considering all RR- and NS-fluxes turned on in a type IIA flux compactification, the geometric moduli, Kähler axions and one linear combination of complex structure axions can be stabilised supersymmetrically or non-supersymmetrically, yielding a four-dimensional Anti-de Sitter vacuum [9, 44]. Once more, the axion polynomials provide a very elegant way to find supersymmetric vacua by analysing the F-terms:

$$\begin{aligned} F_{N_\star^K} &= \hat{\rho}_K - i\tfrac{\mathrm{Im}\,(\mathcal{C}\mathcal{F}_K)}{2\mathcal{G}_Q}\left(W_T + W_Q\right), \\ F_{U_{\star\Lambda}} &= \hat{\rho}^\Lambda + i\tfrac{\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)}{2\mathcal{G}_Q}\left(W_T + W_Q\right), \\ F_{T^a} &= \rho_a + i\mathcal{K}_{ab}\tilde{\rho}^b - \tfrac{1}{2}\mathcal{K}_a\tilde{\rho} + \tfrac{3i}{2}\tfrac{\mathcal{K}_a}{\mathcal{K}}\left(W_T + W_Q\right). \end{aligned} \tag{4.13}$$

In order to solve for the full set of vanishing F-terms, let us first sum up strategically the complex structure F-terms

$$\sum_{K=0}^{h} n_\star^K F_{N_\star^K} + \sum_{\Lambda=0}^{h} u_{\star\Lambda} F_{U_{\star\Lambda}} = \sum_{K=0}^{h} \hat{\rho}_K n_\star^K + \sum_{\Lambda=0}^{h} \hat{\rho}^\Lambda u_{\star\Lambda} + 2i\left(W_T + W_Q\right) = 0, \tag{4.14}$$

such that the real part and complex part lead to two separate conditions:

$$\rho_0 - \frac{1}{2}\mathcal{K}_a\tilde{\rho}^a = 0, \qquad n_\star^K\hat{\rho}_K + u_{\star\Lambda}\hat{\rho}^\Lambda = \frac{1}{3}\mathcal{K}\tilde{\rho} - 2t^a\rho_a. \tag{4.15}$$

Also the F-terms of the Kähler moduli can be summed up as

$$t^a F_{T^a} = \frac{5}{2} t^a \rho_a - \frac{3}{4}\mathcal{K}\tilde{\rho} + \frac{3i}{2}\rho_0 + \frac{i}{4}\mathcal{K}_a \tilde{\rho}^a, \tag{4.16}$$

leading to two more conditions for vanishing F-terms:

$$\frac{3}{2}\rho_0 + \frac{1}{4}\mathcal{K}_a \tilde{\rho}^a = 0, \qquad \frac{5}{2} t^a \rho_a - \frac{3}{4}\mathcal{K}\tilde{\rho} = 0. \tag{4.17}$$

Combining all four relations allows us to express the stabilisation conditions for the moduli in terms of the axion polynomials:

$$\rho_0 = 0, \quad \tilde{\rho}^a = 0, \quad \rho_a = \frac{3}{10}\tilde{\rho}\,\mathcal{K}_a. \tag{4.18}$$

The first condition expresses the fact that a linear combination of complex structure axions is stabilised, while the second condition stabilises the Kähler axions:

$$h_K \xi_\star^K + h^\Lambda \xi_{\star\Lambda} = -\frac{e_0 m^2 - m e_a m^a + \frac{1}{3}\mathcal{K}_{abc} m^a m^b m^c}{m^2}, \qquad b^a = -\frac{m^a}{m}. \tag{4.19}$$

The third condition stabilises the geometric part of the Kähler moduli in terms of the fluxes. Inserting the identified solutions back into the F-terms for the complex structure moduli enables to write down the stabilisation conditions for the complex structure moduli in terms of their "dual" periods and the overall volume $\mathcal{K}$:

$$\mathcal{G}_Q \frac{\hat{\rho}_K}{\mathrm{Im}\,(\mathcal{C}\mathcal{F}_\mathrm{K})} = -\mathcal{G}_Q \frac{\hat{\rho}^\Lambda}{\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)} = \frac{1}{15}\tilde{\rho}\,\mathcal{K}. \tag{4.20}$$

To arrive at these relations, we imposed the vacuum expectation value for the superpotential in supersymmetric AdS vacua, which can be obtained by imposing the vacuum constraints on the axion polynomials:

$$\langle W_\mathrm{AdS} \rangle = -\frac{2i}{15}\mathcal{K}\tilde{\rho}. \tag{4.21}$$

One can check that the conditions (4.18) and (5.33) are equivalent to the vanishing F-term conditions (5.26). Hence, the vacuum relations found in [9] for supersymmetric AdS vacua can be derived very elegantly by virtue of the axion polynomial language.

Similarly to the ISD flux vacua, the supersymmetric AdS vacua are only realised in the presence of a non-vanishing Romans' mass $m \neq 0$, and are modified when taking into account the effect of α'-corrections. As we will argue in the next chapter, in this case the modification is less dramatic, because the classical scenario already stabilises all moduli, but their value will be nevertheless shifted from their previous value.

4.2 The Cosmological Constant in flux vacua

Both classes of vacuum solutions above have been obtained by solving for vanishing F-terms in the four-dimensional $\mathcal{N} = 1$ supergravity description. For non-vanishing F-terms, the vacuum solutions have to be determined by minimising the F-term scalar potential (2.60). However, Instead of solving for vanishing F-terms, vacuum configurations can be determined more generically by requiring that the first order derivatives of the scalar potential with respect to the moduli vanish. Due to the properties of the rotation matrix (3.9) the constraint equations for the axionic directions can be rephrased as orthogonality conditions between the vector $\vec{\rho}$ and its descendants $P_a\vec{\rho}$, $P_K\vec{\rho}$ or $P^\Lambda\vec{\rho}$:

$$\begin{aligned} \vec{\rho}^{\,T} Z^{-1} P_a \vec{\rho} &= 4\rho_0\rho_a + K^{c\bar{d}}\mathcal{K}_{dab}\rho_c\tilde{\rho}^b - \tfrac{4}{9}K_{\bar{b}a}\mathcal{K}^2\tilde{\rho}^b\tilde{\rho} = 0, \\ \vec{\rho}^{\,T} Z^{-1} P_K \vec{\rho} &= 4\rho_0\hat{\rho}_K = 0, \\ \vec{\rho}^{\,T} Z^{-1} P^\Lambda \vec{\rho} &= 4\rho_0\hat{\rho}^\Lambda = 0. \end{aligned} \tag{4.22}$$

These three constraint equations are solved simultaneously for $\rho_0 = 0$ and $\tilde{\rho}^a = 0$: two constraints on the axion polynomials that are common among the ISD flux vacua and supersymmetric AdS flux vacua, and are responsible for stabilising a linear combination of complex structure axions and all Kähler axions in terms of the flux quanta. The three constraint equations have to be supplemented by the vacuum conditions arising along the geometric moduli directions. In the case of ISD flux vacua, the vacuum conditions for the geometric moduli correspond to setting the following equations to zero,

$$\begin{aligned} \vec{\rho}^{\,T}\partial_{t^a}(Z^{-1})\vec{\rho} &= \vec{\rho}^{\,T}(Z^{-1})\vec{\rho}\;\partial_{t^a}K + 8e^K\left[\rho_c\partial_{t^a}K^{c\bar{d}}\rho_d + \mathcal{K}_a\tilde{\rho}\left(\tfrac{2}{3}\mathcal{K}\tilde{\rho} + 4s_\star\hat{\rho}_0\right)\right], \\ \vec{\rho}^{\,T}\partial_{s_\star}(Z^{-1})\vec{\rho} &= \vec{\rho}^{\,T}(Z^{-1})\vec{\rho}\;\partial_{s_\star}K + 8e^K\hat{\rho}_0\left[\tfrac{4}{3}\mathcal{K}\tilde{\rho} + 8s_\star\hat{\rho}_0\right], \\ \vec{\rho}^{\,T}\partial_{u_{\star\Lambda}}(Z^{-1})\vec{\rho} &= \vec{\rho}^{\,T}(Z^{-1})\vec{\rho}\;\partial_{u_{\star\Lambda}}K, \end{aligned} \tag{4.23}$$

where the solutions $\rho_0 = 0$ and $\tilde{\rho}^a = 0$ to the axion constraint equations have already been taken into account on the right-hand side. One can see that the derivative $\partial_{u_{\star\Lambda}}K$ is proportional to the quotient $\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)/\mathcal{G}_{\mathrm{Q}}$, and therefore a homogeneous function of $u_{\star\Lambda}$ of degree -1. As a result, the third relation in (4.23) vanishes in regions of the moduli space where the supergravity approximation is no longer valid, i.e. vanishing three-cycle volumes ($\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda) = 0, \forall\Lambda$) or three-cycles with infinite volumes, unless the four-dimensional vacuum energy proportional to $\vec{\rho}^{\,T}(Z^{-1})\vec{\rho}$ vanishes for the compactification. The vacuum conditions for the Kähler moduli sector and 4d dilaton in Minkowski vacua further lead to the constraints $\rho_a = 0$ and $\frac{1}{6}\mathcal{K}\tilde{\rho} + s_\star\hat{\rho}_0 = 0$, which complete the set of constraint equations (4.8) for the ISD flux vacua. Clearly, the axion polynomials jargon allows for a more systematic search of perturbative flux vacua, but it also reveals that many such flux vacua are related to each other through the shift symmetries (3.25) and should therefore not be counted as independent vacua.

Identifying the constraints on the axion polynomials for a particular vacuum configuration also allows to determine the perturbative value of the cosmological constant. To extract information about the cosmological constant from the axion polynomials, it is insightful to rewrite the inverse metric Z^{AB} in (3.29) into a block-diagonal form,

$$Z^{AB} = 8e^K \text{diag}\left(4, K^{a\bar{b}}, \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}}, -\frac{\mathcal{K}^2}{3}, \begin{pmatrix} K^{IJ} & K^{I\Sigma} \\ K^{\Lambda J} & K^{\Lambda\Sigma} \end{pmatrix}\right), \tag{4.24}$$

by rotating the axion polynomials to a new basis of axion polynomials:

$$\overrightarrow{\rho}_{\text{new}} = \left(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho}, \hat{\rho}_K - \frac{i\mathcal{K}}{3} K_{N^K_\star}\tilde{\rho}, \hat{\rho}^\Lambda - \frac{i\mathcal{K}}{3} K_{U_{\star\Lambda}}\tilde{\rho}\right), \tag{4.25}$$

where we have used the homogeneity of the complex structure Kähler potential (2.49). Taking into account the expression for the F-terms of the complex structure moduli (5.26), the vector (4.25) can be reinterpreted in a slightly more suggestive way:

$$\overrightarrow{\rho}_{\text{new}} = \left(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho}, F_{N^K_\star} - K_{N^K_\star}\left(W_T + W_Q + \frac{i}{3}\mathcal{K}\tilde{\rho}\right), F_{U_{\star\Lambda}} - K_{U_{\star\Lambda}}\left(W_T + W_Q + \frac{i}{3}\mathcal{K}\tilde{\rho}\right)\right). \tag{4.26}$$

The virtue of this new basis of axion polynomials lies in the possibility to understand each vacuum as a positive, null-like or negative norm with respect to the diagonalised inverse metric. The ISD flux vacua (with vanishing dilaton and Kähler moduli F-terms) for instance are characterised by the constraint equations (4.8) on the axion polynomials and are represented by the vector $\overrightarrow{\rho}_{\text{new}} = \tilde{\rho}\left(0, 0, 0, 1, 0, -\frac{i\mathcal{K}}{3}K_{U_{\star\Lambda}}\tilde{\rho}\right) = (0, 0, 0, \tilde{\rho}, 0, F_{U_{\star\Lambda}})$. This vector corresponds to a null-like vector with respect to the metric Z^{AB}, in line with the vanishing vacuum energy for non-supersymmetric Minkowski vacua.[2] SUSY AdS vacua, on the other hand, have vanishing F-terms in all sectors. From the relations (4.18) we obtain the vector $\overrightarrow{\rho}_{\text{new}} = \left(0, \rho_a, 0, \tilde{\rho}, -K_{N^K_\star}\left(W_T + W_Q + \frac{i}{3}\mathcal{K}\tilde{\rho}\right), -K_{U_{\star\Lambda}}\left(W_T + W_Q + \frac{i}{3}\mathcal{K}\tilde{\rho}\right)\right)$ $= \tilde{\rho}\left(0, \frac{3}{10}\mathcal{K}_a, 0, 1, -\frac{i}{5}\mathcal{K}K_{N^K_\star}, -\frac{i}{5}\mathcal{K}K_{U_{\star\Lambda}}\right)$, which forms a negative norm vector whose length corresponds to the negative cosmological constant for the AdS minimum:

$$\langle V_F\rangle_{\text{AdS}} = -3\frac{e^K}{\kappa_4^2}\left(\frac{2}{15}\tilde{\rho}\mathcal{K}\right)^2. \tag{4.27}$$

4.3 Flux vacua with mobile D6-branes

As already discussed in section 2.6, backgrounds with localised sources such as D6-branes and O6-planes provide a much more intricate picture for type IIA

[2] In fact, as we will see in the next section, the choice of Kähler potential (4.1) together with $\hat{\rho}_\Lambda = 0$ implies a positive semi-definite scalar potential minimised by this $\overrightarrow{\rho}_{\text{new}}$.

compactifications with fluxes. When searching for four-dimensional flux vacua in the presence of D-brane moduli, these must be considered simultaneously with the closed string moduli when minimising the potential, as opposed to adding them at a later stage of the analysis. This is manifest when using the standard $\mathcal{N} = 1$ prescription for computing the F-term scalar potential in terms of a Kähler potential and superpotential. For instance, in the case of Calabi-Yau compactifications the presence of open string moduli redefines the complex structure moduli and dilaton that appear in the Kähler potential, modifying the Kähler metrics non-trivially [11, 49–51]. In particular, the factorised metric structure between Kähler and complex structure moduli, inherited from the unorientifolded $\mathcal{N} = 2$ parent theory, is lost whenever open string moduli are considered [11]. This in turn implies that the no-scale properties of closed-string moduli potentials, a key ingredient to find certain classes of flux vacua as the one discussed in section 4.1.1, may be modified or even lost when open string moduli are taken into account. This gives rise to the question which kind of stable type IIA vacua exist in the presence of mobile D6-branes, and in particular whether one can construct Minkowski and AdS vacua analogous to the ones considered in sections 4.1.1 and 4.1.2. On the one hand, in the case of $\mathcal{N} = 1$ AdS vacua the strategy to find such vacua is rather straightforward, as one must look for points in field space where all the F-terms vanish. On the other hand, the search for $\mathcal{N} = 0$ Minkowski vacua is less obvious. Indeed, just as in [52] the pattern of F-terms that corresponds to stable $\mathcal{N} = 0$ Minkowski vacua relies on having a semi-definite scalar potential. In turn, the latter relies on the absence of certain fluxes in the superpotential and in the factorisation of the dilaton, Kähler and complex structure moduli in the Kähler potential. However, such a factorisation is lost as soon as mobile D6-branes appear in the construction, due to the four-dimensional field redefinition (2.72). Therefore, it is not clear that the no-scale properties of certain type IIA flux vacua can still be maintained in the presence of mobile D6-branes[3].

4.3.1 $\mathcal{N} = 0$ Minkowski vacua

The aim of this section is to investigate whether one can achieve stable $\mathcal{N} = 0$ four-dimensional Minkowski vacua in the presence of mobile D6-branes, where the stability is guaranteed by the semi-definiteness of the (classical) scalar potential. Rather than taking the ten-dimensional approach of [53], we will address this question in terms of the four-dimensional effective theory discussed above. We will first show how to obtain a semi-definite F-term scalar potential by means of its standard four-dimensional supergravity expression and a simple set of assumptions. In the next section we will show that one can recover the same result

[3]Notice that the same observation could be made for type IIB compactifications with D3 and D7-branes.

by using the formalism that rewrites the scalar potential as a bilinear of axion polynomials. In chapter 7 we will analyse the structure of the soft terms that arises for this kind of vacua.

The standard 4d supergravity perspective

As already mentioned, the presence of mobile D6-branes creates a non-trivial mixing in the metric between Kähler, complex structure and open string moduli. Nevertheless, as pointed out in [11] and [41], under certain assumptions the inverse metric $K^{A\bar{B}}$ displays a simplified structure.[4] First, even if $\partial_a\partial_{\bar{b}}K$ changes in the presence of mobile D6-branes, we have that $K^{\bar{b}a}$ remains the inverse of the previous Kähler moduli metric $\partial_a\partial_{\bar{b}}K_K$ (without open string moduli). Second, the rest of the components read:

$$K^{\bar{a}i} = f_b^i K^{b\bar{a}}, \tag{4.28a}$$

$$K^{\bar{j}i} = G_{\rm D6}^{ij} + K^{a\bar{b}} f_a^i f_b^j, \tag{4.28b}$$

$$K^{\bar{I}a} = -\frac{1}{2} K^{\bar{b}a}\mathbf{H}_b^I, \tag{4.28c}$$

$$K^{\bar{I}i} = -\frac{1}{2}\left[G_{\rm D6}^{ij}\, g_j^I + K^{\bar{b}a} f_a^i\, \mathbf{H}_b^I\right], \tag{4.28d}$$

$$K^{\bar{J}I} = \mathbf{N}^{IJ} + \frac{1}{4}\left[K^{\bar{b}a}\,\mathbf{H}_b^J\mathbf{H}_a^I + G_{\rm D6}^{ij}\, g_i^I g_j^J\right], \tag{4.28e}$$

where as before the indices a, b label Kähler moduli, I, J label dilaton and complex structure moduli and i, j label open string moduli, absorbing the index α for simplicity. Here the functions $\mathbf{H}_a^I$, f_a^i and g_i^I are defined as in section 2.6. Finally, $G_{\rm D6}^{ij}$ is the inverse of the open string metric defined in (3.31) and $\mathbf{N}^{IJ}$ is the inverse of the complex structure metric without mobile D6-branes

$$\mathbf{N}_{K\Lambda} = \frac{1}{4}\partial_{n_\star^K}\partial_{u_{\star\Lambda}} K_Q, \tag{4.29}$$

with K_Q taken as a function of $n_\star^K$, $u_{\star\Lambda}$ as in (2.49).

Using the relations (4.28) one can write the part of the F-term scalar potential that contains only derivatives of the superpotential with respect to the moduli as

$$\begin{aligned} K^{A\bar{B}}D_AWD_{\bar{B}}\overline{W} &= K^{a\bar{b}}\left[D_a + f_a^iD_i - \frac{1}{2}\mathbf{H}_a^K D_K\right]W\left[D_{\bar{b}} + f_b^iD_{\bar{i}} - \frac{1}{2}\mathbf{H}_a^K D_{\bar{K}}\right]\overline{W} \\ &+ G_{\rm D6}^{ij}\left[D_iW - \frac{1}{2}g_i^K D_KW\right]\left[D_{\bar{j}}\overline{W} - \frac{1}{2}g_j^L D_{\bar{L}}\overline{W}\right] \\ &+ \mathbf{N}^{IJ}D_IWD_{\bar{J}}\overline{W} \end{aligned} \tag{4.30}$$

[4]One can derive eqs.(4.28) by assuming that the zero degree functions $\mathbf{H}$ in (2.72) only depend on the D6-brane position variables φ^i, as it happens for instance in the case of toroidal orbifolds.

which is a sum of positive definite terms. This rewriting is crucial in order to match the scalar potential derived from dimensional reduction with the one obtained from the standard supergravity formula [11,41]. If in addition we consider a Kähler potential of the form (4.1), namely

$$K_Q = -\log(2s_\star) - K_{\tilde{Q}}(u_{\star\Lambda}), \tag{4.31}$$

then the entries of $\mathbf{N}_{K\Lambda}$ mixing the dilaton and the complex structure moduli $u_{\star\Lambda}$ will vanish, and the same will hold for its inverse. As a result, the contribution coming from the last line of (4.30) will split as

$$\mathbf{N}^{IJ} D_I W D_{\bar{J}} \overline{W} = \mathbf{N}^{SS} D_S W D_{\bar{S}} \overline{W} + \mathbf{N}^{\Lambda\Sigma} D_\Lambda W D_{\bar{\Sigma}} \overline{W} \tag{4.32}$$

Finally, if we assume that the fields U_Λ do not enter into the superpotential and use the corresponding no-scale relation we obtain

$$\mathbf{N}^{\Lambda\Sigma} D_\Lambda W D_{\bar{\Sigma}} \overline{W} = 3|W|^2, \tag{4.33}$$

that cancels the gravitational term in the F-term scalar potential. Therefore, with similar assumptions as for the ISD closed string vacua and the Kähler metric relations (4.28), we obtain a semi-definite positive scalar potential and the corresponding 4d Minkowski vacua.

The conditions for such vacua amount to imposing the following relations,

$$\begin{aligned} D_S W &= 0, & (4.34)\\ D_i W &= \frac{1}{2} g_i^\Lambda D_\Lambda W, & (4.35)\\ D_a W &= \frac{1}{2}\left(\mathbf{H}_a^\Lambda - f_a^i g_i^\Lambda\right) D_\Lambda W, & (4.36) \end{aligned}$$

which is slightly weaker than imposing the cancellation of the F-terms for S, T^a and Φ^i. To rewrite these conditions in a simple form, let us note that by eq.(2.73) $\partial_{\phi^i} u_{\star\Lambda} = \frac{1}{2} g_i^\Lambda$ and that the same assumptions that led to (4.28) imply $\partial_{t^a} u_{\star\Lambda} = \frac{1}{2}(\mathbf{H}_a^\Lambda - f_a^i g_i^\Lambda)$. We then have that they amount to

$$\begin{aligned} D_S W &= 0, & (4.37)\\ D_i W &= (\partial_i K_{\tilde{Q}}) W, & (4.38)\\ D_a W &= (\partial_a K_{\tilde{Q}}) W. & (4.39) \end{aligned}$$

Alternatively, one may consider the contra-variant expressions of the F-terms

$$F^A \equiv K^{A\overline{B}} \overline{D}_B \overline{W}, \tag{4.40}$$

which allow to designate in which moduli sector supersymmetry is broken spontaneously. Indeed, by imposing the vacuum conditions (4.34)-(4.36) and using the expressions (4.28) for the inverse metric on the moduli space, the only non-vanishing on-shell component is the F-term for the complex structure moduli U_Λ:

$$F^\Lambda = N^{\Lambda\overline{\Sigma}} K_{\overline{\Sigma}} \overline{W}_0 = -2iu_{\star\Lambda}\overline{W}_0. \tag{4.41}$$

Note that this relation forms the natural extension of the on-shell F-terms in type IIA closed string ISD flux vacua. Also in the presence of open string moduli (associated to mobile D6-branes) supersymmetry is spontaneously broken by the non-vanishing F-terms in the complex structure moduli sector, prompting us to label the class of such non-supersymmetric Minkowski vacua as *complex structure dominated* (CSD) vacua. In chapter 6 we will analyse different phenomenological aspects of these $\mathcal{N} = 0$ flux vacua with non-vanishing on-shell F-terms in the complex structure moduli sector, dubbed CSD vacua for short. In particular we will study the pattern of soft term resulting from this spontaneous breaking of supersymmetry in the complex structure sector

Finally, to determine the vacuum expectation value of the superpotential $\overline{W}_0$, the axion polynomial formalism turns out to be extremely useful once the vacuum conditions (4.34)-(4.36) are rewritten in terms of vacuum constraints on the axion polynomials, as we discuss in the next section.

4.4 The axion polynomial perspective

While the reasoning used above to obtain $\mathcal{N} = 0$ Minkowski vacua fits better with the existing literature on string compactifications, there is a more direct approach to analyse the appearance of semi-definite positive scalar potentials and the corresponding Minkowski vacua. Indeed, instead of describing the scalar potential in terms of a Kähler and superpotential one may consider its expression as a bilinear of axion polynomials, as directly obtained from dimensional reduction. As we will see, one can reproduce similar conditions as above for the semi-positive definiteness of the scalar potential, except that now no assumption on the Kähler metrics must be made.

As a warm up, let first us consider the well-know ISD case without mobile D6-branes, for which the potential can be expressed as in (3.28). In this language, the assumption (4.31) translates into the vanishing of the off-diagonal components $K^{I\Lambda}$ in (3.29). When switching to the new basis of axion polynomials $\overrightarrow{\rho}_{\rm new}$ in (4.25), this metric becomes

$$Z^{AB} = 8e^K \text{diag}\left(4, K^{a\bar{b}}, \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}}, -\frac{\mathcal{K}^2}{3}, K^{S\bar{S}}, K^{\Lambda\bar{\Sigma}}\right), \tag{4.42}$$

while

$$\overrightarrow{\rho}_{\rm new} = \left(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho}, \hat{\rho}_0 - \tilde{\rho}\mathcal{K}\frac{i}{3}K_S, \hat{\rho}^\Lambda - \tilde{\rho}\mathcal{K}\frac{i}{3}K_\Lambda\right). \tag{4.43}$$

Imposing that the complex structure moduli $U_{\star\Lambda}$ do not enter the superpotential is equivalent to require that $\hat{\rho}^\Lambda = 0$. Then, using the no-scale relation $K^{\Lambda\bar{\Sigma}}K_\Lambda K_{\bar{\Sigma}} = -K^{\Lambda\bar{\Sigma}}K_\Lambda K_\Sigma = 3$ one find an exact cancellation between the contribution of the Romans mass component $\tilde{\rho}$ of (4.43) and the last one. As a result the scalar potential (3.28) can be explicitly written as

$$V_F = \frac{e^K}{\kappa_4^2}\left(4\rho_0^2 + K^{a\bar{b}}\rho_a\rho_b + \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}}\tilde{\rho}^a\tilde{\rho}^b + K^{S\bar{S}}\left(\hat{\rho}_0 - \tilde{\rho}\mathcal{K}\frac{i}{3}K_S\right)^2\right), \tag{4.44}$$

which is clearly semi-definite positive and vanishes if and only if the conditions 4.8 are met. In this way, we directly recover the relations for the axions polynomials obtained in section 4.1 without having to consider any particular pattern for the F-terms.

Similarly, we may apply this strategy to the case of CSD vacua (with mobile D6-branes), where now the vector of axion polynomials has the components 3.20. From the results of section 3 of [41] adapted to our conventions for quantised fluxes, one obtains that inverse metric takes the diagonalised form

$$Z^{AB} = 8e^K \text{diag}\left(4, K_K^{a\bar{b}}, \frac{4}{9}\mathcal{K}^2(K_K)_{a\bar{b}}, -\frac{\mathcal{K}^2}{3}, \mathbf{N}^{S\bar{S}}, \mathbf{N}^{\Lambda\bar{\Sigma}}, G^{ij}_{\text{D6}}, G^{ij}_{\text{D6}}\right), \tag{4.45}$$

in the following basis of axion polynomials

$$\vec{\varrho}_{\text{ new}} = \left(\varrho_0, \varrho_a', \tilde{\varrho}^{a\prime}, \tilde{\varrho}, \hat{\varrho}_0 - \tilde{\varrho}\mathcal{K}\frac{i}{3}K_S, -\tilde{\varrho}\mathcal{K}\frac{i}{3}K_{U_\Lambda}, \varrho_i', t^a\varrho_{ai}\right). \tag{4.46}$$

Here we have defined

$$\varrho_a' = \varrho_a + f_a^i\varrho_i - \frac{1}{2}\mathbf{H}_a^0\hat{\varrho}_0, \tag{4.47}$$

$$\tilde{\varrho}^{a\prime} = \tilde{\varrho}^a - (\mathcal{K}^{ab}t^c f_c^i + \mathcal{K}^{ac}t^b f_c^i)\varrho_{bi}, \tag{4.48}$$

$$\varrho_i' = \varrho_i - \frac{1}{2}g_i^0\hat{\varrho}_0 \tag{4.49}$$

and we have already imposed that $\mathbf{N}^{S\Lambda} = 0$ and that $\hat{\varrho}^\Lambda = 0$. Again, we find a cancellation between the quadratic terms in the 4^{th} and 6^{th} entry of 4.46. This results into a semi-definite positive, bilinear scalar potential of the form

$$V_F = \frac{e^K}{\kappa_4^2}\Bigg\{4\varrho_0^2 + K_K^{a\bar{b}}\varrho_a'\varrho_b' + \frac{4}{9}\mathcal{K}^2(K_K)_{a\bar{b}}\tilde{\varrho}^{a\prime}\tilde{\varrho}^{b\prime} + \mathbf{N}^{S\bar{S}}\left(\hat{\varrho}_0 - \tilde{\varrho}\mathcal{K}\frac{i}{3}K_S\right)^2 + G^{ij}_{\text{D6}}\left[\varrho_i'\varrho_j' + t^a t^b\varrho_{ai}\varrho_{bj}\right]\Bigg\}, \tag{4.50}$$

We then find that the conditions for a Minkowski vacuum are

$$\varrho_0 = 0, \tag{4.51a}$$

$$\varrho_a = \frac{1}{2}\left(\mathbf{H}_a^0 - f_a^i g_i^0\right)\hat{\varrho}_0, \tag{4.51b}$$

$$\tilde{\varrho}^a = \mathcal{K}^{ab}\phi^i \varrho_{bi}, \tag{4.51c}$$

$$\hat{\varrho}_0 = -\frac{\mathcal{K}}{6s_\star}\tilde{\varrho}, \tag{4.51d}$$

$$\varrho_i = \frac{1}{2}g_i^0\hat{\varrho}_0, \tag{4.51e}$$

$$t^a\varrho_{ai} = 0, \tag{4.51f}$$

and that whenever they are satisfied the superpotential takes the value

$$W_0 = 2is_\star\hat{\varrho}_0 = -\frac{i\mathcal{K}}{3}\tilde{\varrho}\,. \tag{4.52}$$

Equivalently, at these vacua we have $\vec{\varrho}_{\text{new}} = \Big(0,0,0,\tilde{\varrho},0,F_{U_\Lambda},0,0\Big)$. One can easily check that these relations are equivalent to eqs.(4.37)-(4.39) if one uses eq. 2.73 and assumes that $\partial_{t^a}(t^a\mathbf{H}_a^0) = \mathbf{H}_a^0 - f_a^i g_i^0$.

Chapter 5

Type IIA flux vacua with α'-corrections

In chapter 3 we discussed how the axion polynomial language allows to incorporate perturbative α'-corrections in type IIA Calabi-Yau orientifold compactifications with background fluxes. This insight will allow us to extract the bilinear structure of the scalar potential in terms of the modified axion polynomials, but the intricacies of the curvature corrections make the search for vacua of the full perturbative scalar potential quite demanding. This chapter is therefore devoted to exploiting well-known methods for vacua searches in this context. More precisely, we will extend the analysis carried out in the previous chapter for non-supersymmetry Minkowski vacua and supersymmetric Anti-de Sitter vacua in terms of the axion polynomials, to include the effect of curvature corrections. For simplicity, here we will not consider vacua with mobile D6-branes. This chapter is mainly based on the paper [42].

5.1 The α'-corrected scalar potential

Since the factorability of the superpotential into saxions and shift-invariant axion polynomials persists in the presence of perturbative α'-corrections, one is naturally driven to the question how the modified form of the scalar potential looks like. The most straightforward path to obtain the four-dimensional scalar potential in the presence of background fluxes and perturbative α'-corrections consists in computing it directly from the F-term scalar potential (2.60) by inserting the Kähler potential (2.80) and superpotential (2.83) discussed in chapter 2.

In practice, the explicit computation of this scalar potential is drastically simplified by deconstructing the expression into three components and applying the elegant formulation of the axion polynomials to the fullest for each component. The first term consists purely of the derivatives of the superpotential with respect to the closed string moduli and requires us to use the modified expressions for the Kähler metric as discussed in appendix C.1:

$$\begin{aligned}\partial_\alpha W K^{\alpha\overline{\beta}}\partial_{\overline{\beta}}\overline{W} = K^{a\bar{b}}\overline{\rho}_a\overline{\rho}_b + \tfrac{4}{9}\mathcal{K}^2(1+\tfrac{3}{2}\varepsilon)^2 K_{a\bar{b}}\tilde{\rho}^a\tilde{\rho}^b + \tfrac{1+6\varepsilon}{1-3\varepsilon}(\mathcal{K}_a\tilde{\rho}^a)^2 + \tfrac{1}{3}\mathcal{K}^2\tilde{\rho}^2\tfrac{(1+\frac{3}{2}\varepsilon)^2}{1-3\varepsilon}\\ -\tfrac{4}{3}\tilde{\rho}\mathcal{K}\,\overline{\rho}_a t^a\,\tfrac{(1+\frac{3}{2}\varepsilon)^2}{1-3\varepsilon} + K^{N\overline{L}}\hat{\rho}_K\hat{\rho}_L + K^{N\overline{\Lambda}}\hat{\rho}_K\hat{\rho}_\Lambda + K^{\Sigma\overline{L}}\hat{\rho}_\Sigma\hat{\rho}_L + K^{\Sigma\overline{\Lambda}}\hat{\rho}_\Sigma\hat{\rho}_\Lambda.\end{aligned} \tag{5.1}$$

The second component consists of terms without derivatives of the superpotential:

$$\begin{aligned}K_\alpha K^{\alpha\overline{\beta}}K_{\overline{\beta}}|W|^2 - 3|W|^2 &= \left(\tfrac{3}{1-3\varepsilon}+4\right)|W|^2 - 3|W|^2\\ &= \tfrac{4-3\varepsilon}{1-3\varepsilon}\left[(\mathrm{Re}W)^2 + (\mathrm{Im}W)^2\right]\end{aligned} \tag{5.2}$$

where the real and imaginary part of the superpotential can be read off as a function of the axion polynomials directly from the modified superpotential (2.83). The third and last component consists of the remaining terms containing derivatives of the superpotential, which can be simplified by virtue of relation (C.4) and the holomorphicity of the superpotential:

$$\begin{aligned}K_\alpha W K^{\alpha\overline{\beta}}\partial_{\overline{\beta}}\overline{W} + \partial_\alpha W K^{\alpha\overline{\beta}}K_{\overline{\beta}}\overline{W} = -4\tfrac{1+\frac{3}{2}\varepsilon}{1-3\varepsilon}\Big(\mathrm{Re}(W)t^a\partial_{t^a}\mathrm{Re}(W) + \mathrm{Im}(W)t^a\partial_{t^a}\mathrm{Im}(W)\Big)\\ -4\mathrm{Im}(W)\left(n^K\hat{\rho}_K + u_\Lambda\hat{\rho}^\Lambda\right).\end{aligned} \tag{5.3}$$

In order to arrive at the simplest expression for the F-term scalar potential further simplifications and manipulations have to be made, which will be discussed at length in appendix C.2. For now, we state the end result of the computation, expressed in terms of the (modified) axion polynomials ($\overline{\rho}_0, \overline{\rho}_a, \tilde{\rho}^a, \tilde{\rho}, \hat{\rho}_K, \hat{\rho}^\Lambda$):

$$\begin{aligned}V_F \;=\; \tfrac{e^{K_T+K_Q}}{\kappa_4^2}\Big\{&4\overline{\rho}_0^2 + K^{a\bar{b}}\overline{\rho}_a\overline{\rho}_b + \tfrac{4}{9}\mathcal{K}^2\left(1+\tfrac{3}{2}\varepsilon\right)^2 K_{cd}\tilde{\rho}^c\tilde{\rho}^d + \tfrac{1}{9}\tilde{\rho}^2\mathcal{K}^2\left(1+\tfrac{3}{2}\varepsilon\right)^2\\ &+\tfrac{4}{3}\tilde{\rho}\mathcal{K}\left(1+\tfrac{3}{2}\varepsilon\right)\left(\hat{\rho}_K n^K + \hat{\rho}^\Lambda u_\Lambda\right) + K^{K\overline{L}}\hat{\rho}_K\hat{\rho}_L\\ &+K^{K\overline{\Sigma}}\hat{\rho}_K\hat{\rho}_\Sigma + K^{\Lambda\overline{L}}\hat{\rho}_\Lambda\hat{\rho}_L + K^{\Lambda\overline{\Sigma}}\hat{\rho}_\Lambda\hat{\rho}_\Sigma\\ &+\tfrac{\varepsilon}{1-3\varepsilon}\Big[9\left(\overline{\rho}_0 + \tfrac{1}{2}\mathcal{K}_b\tilde{\rho}^b\right)^2 + 9(n^K\hat{\rho}_K + \hat{\rho}^\Lambda u_\Lambda)^2\\ &\qquad -9\left(t^a\overline{\rho}_a + \tfrac{1}{6}\tilde{\rho}\mathcal{K}(1-3\varepsilon)\right)^2\Big]\Big\},\end{aligned} \tag{5.4}$$

where now

$$e^{K_T+K_Q} = \frac{e^{4D}}{\frac{4}{3}\mathcal{K}(1+\frac{3}{2}\varepsilon)} \equiv \frac{e^{4\phi}}{8\mathcal{V}^3(1+\frac{3}{2}\varepsilon)^3}. \tag{5.5}$$

One notices immediately that the bilinear structure of the F-term scalar potential prevails in the presence of curvature corrections, such that the scalar potential can still be written as,

$$V_F = \frac{1}{8\kappa_4^2}\,\vec{\overline{\rho}}^{\,t}\cdot\mathcal{Z}^{-1}\cdot\vec{\overline{\rho}}, \tag{5.6}$$

where the inverse metric $\mathcal{Z}^{-1}$ is now modified by the $K^{(3)}$ curvature corrections expressed in terms of the parameter ε and reads

$$\begin{aligned}\mathcal{Z}^{-1} &= 8e^{K_T+K_Q}\left[\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & K^{a\bar{b}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{9}\mathcal{K}^2 K_{a\bar{b}}(1+\frac{3}{2}\varepsilon)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mathcal{K}^2}{9}(1+\frac{3}{2}\varepsilon)^2 & \frac{2}{3}\mathcal{K}n^L(1+\frac{3}{2}\varepsilon) & \frac{2}{3}\mathcal{K}u_\Sigma(1+\frac{3}{2}\varepsilon) \\ 0 & 0 & 0 & \frac{2}{3}\mathcal{K}n^K(1+\frac{3}{2}\varepsilon) & K^{K\bar{L}} & K^{K\bar{\Sigma}} \\ 0 & 0 & 0 & \frac{2}{3}\mathcal{K}u_\Lambda(1+\frac{3}{2}\varepsilon) & K^{\Lambda\bar{L}} & K^{\Lambda\bar{\Sigma}} \end{pmatrix}\right. \\ &\left. + \frac{\varepsilon}{1-3\varepsilon}\begin{pmatrix} 9 & 0 & \frac{9}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & -9t^a t^b & 0 & -\frac{3}{2}\mathcal{K}t^a(1-3\varepsilon) & 0 & 0 \\ \frac{9}{2}\mathcal{K}_b & 0 & \frac{9}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -\frac{3}{2}\mathcal{K}t^b(1-3\varepsilon) & 0 & -\frac{\mathcal{K}^2}{4}(1-3\varepsilon)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9n^K n^L & 9n^K u_\Sigma \\ 0 & 0 & 0 & 0 & 9n^K u_\Lambda & 9u_\Lambda u_\Sigma \end{pmatrix}\right].\end{aligned} \tag{5.7}$$

Due to the curvature corrections, i.e. $\varepsilon \neq 0$, off-diagonal terms enter in the symmetric matrix. As we will see in the next section, this complicates the search for extrema of the scalar potential at a technical level, but conceptually one may apply the same principles as in [47] to explore the set of vacua in the presence of α'-corrections.

5.2 Non-supersymmetric Minkowski vacua

In this section, we investigate how the ISD flux vacua discussed in section 4.1.1 are modified in the regions of moduli space where the perturbative α'-corrections cannot be neglected. As argued in section 2.7, the Kähler potential for the Kähler moduli is modified by the $(\alpha')^3$-correction to expression (2.80), while the ISD superpotential also requires modifications due to lower order α'-corrections. In particular we have that for this case the expression (3.26) reduces to

$$W_{\text{ISD}} = \overline{\rho}_0 + i\overline{\rho}_a t^a - \frac{1}{2}\mathcal{K}_a\tilde{\rho}^a - \frac{i}{3!}\mathcal{K}\tilde{\rho} - iK^{(3)}\tilde{\rho} + is\hat{\rho}_0. \tag{5.8}$$

Since the α'-corrections do not violate the no-scale symmetry in the U_Λ-complex structure moduli sector, the last equality of (4.4) still holds, and the same reasoning as above applies to arrive at the vacuum configuration for the ISD flux background. That is, we may derive the Minkowski vacuum conditions by imposing the vanishing of the F-terms for the dilaton and Kähler moduli. The dilaton modulus comes with the following F-term in the presence of perturbative

α'-corrections

$$F_S = \frac{1}{2s}\left(i\overline{\rho}_0 - t^a\overline{\rho}_a - \frac{i}{2}\mathcal{K}_a\tilde{\rho}^a + \frac{1}{6}\mathcal{K}\tilde{\rho} + K^{(3)}\tilde{\rho} + s\hat{\rho}_0\right), \tag{5.9}$$

while the corrected F-term for the Kähler moduli reads

$$F_{T^a} = \overline{\rho}_a + i\mathcal{K}_{ab}\tilde{\rho}^a - \frac{1}{2}\mathcal{K}_a\tilde{\rho} + \frac{2i\mathcal{K}_a}{\frac{4}{3}\mathcal{K} + 2K^{(3)}}W_{\text{ISD}}. \tag{5.10}$$

We may now set both quantities to zero and solve the resulting algebraic equations explicitly. As in [47], we may simplify such computations by first considering the following linear combination

$$\begin{aligned} t^aF_{T^a}\left(\tfrac{4}{3}\mathcal{K} + 2K^{(3)}\right) \;&=\; 2i\mathcal{K}\overline{\rho}_0 + t^a\overline{\rho}_a(-\tfrac{2}{3}\mathcal{K} + 2K^{(3)}) + i\mathcal{K}_a\tilde{\rho}^a\left(\tfrac{1}{3}\mathcal{K} + 2K^{(3)}\right) \\ &\quad -\mathcal{K}\tilde{\rho}\left(\tfrac{1}{3}\mathcal{K} - K^{(3)}\right) - 2\hat{\rho}_0 s\mathcal{K}. \end{aligned} \tag{5.11}$$

The combined set of the algebraic equations that describe the vacuum constraints for ISD flux vacua can be simply expressed in terms of the redefined axion polynomials (3.27). At a first stage one can see that the vanishing of (5.9) and (5.11) is equivalent to

$$\begin{array}{ll} \overline{\rho}_0 = 0, & -t^a\overline{\rho}_a + \frac{1}{6}\mathcal{K}\tilde{\rho} + K^{(3)}\tilde{\rho} + s\hat{\rho}_0 = 0, \\ \tilde{\rho}^a = 0, & t^a\overline{\rho}_a(-\frac{2}{3}\mathcal{K} + 2K^{(3)}) + \mathcal{K}\tilde{\rho}\left(-\frac{1}{3}\mathcal{K} + K^{(3)}\right) - 2\hat{\rho}_0 s\mathcal{K} = 0. \end{array} \tag{5.12}$$

Notice that the conditions $\overline{\rho}_0 = 0$ and $\tilde{\rho}^a = 0$ are essentially similar to the uncorrected case (4.8), while now we no longer have that $\overline{\rho}_a = 0$. The set of equations $\tilde{\rho}^a = 0$ stabilises the Kähler axions through the same flux quanta as in absence of α'-corrections, and the axionic partner of the dilaton ξ^0 is stabilised by virtue of the condition $\overline{\rho}_0 = 0$, such that its vacuum expectation value can be expressed purely in terms of the curvature corrected flux quanta $\overline{e}_0$ and $\overline{e}_a$:

$$h_0\xi^0 = -\frac{1}{3m^2}\left(\mathcal{K}_{abc}m^am^bm^c - 3\overline{e}_am^am\right) - \overline{e}_0. \tag{5.13}$$

Notice as well that the condition $\tilde{\rho}^a = 0$ and the vanishing eq (5.10) imply that $\overline{\rho}_A \propto \mathcal{K}_a$, and so solving (5.12) is equivalent to the vanishing of (5.9) and (5.10). The remaining two set of equations are solved simultaneously by the relations

$$\frac{1}{6}\mathcal{K}\tilde{\rho} + s\hat{\rho}_0 = \tilde{\rho}K^{(3)}\frac{\frac{1}{6}\mathcal{K} + K^{(3)}}{\frac{4}{3}\mathcal{K} - K^{(3)}}, \qquad \overline{\rho}_a = \overline{e}_a - \mathcal{K}_{abc}\frac{m^bm^c}{2m} = \tilde{\rho}K^{(3)}\frac{\frac{3}{2}\mathcal{K}_a}{\frac{4}{3}\mathcal{K} - K^{(3)}}, \tag{5.14}$$

which clearly reduce to the previous conditions in the limit $K^{(3)} \to 0$. They also provide explicit vacuum relations for the dilaton in terms of the flux quanta and curvature corrections:

$$h_0s = -\frac{1}{6}m\mathcal{K} + mK^{(3)}\frac{\frac{1}{6}\mathcal{K} + K^{(3)}}{\frac{4}{3}\mathcal{K} - K^{(3)}} = -\frac{1}{6}m\left(\mathcal{K} + 6K^{(3)}\right) + \frac{t^a}{2m}\left(2m\overline{e}_a - \mathcal{K}_{abc}m^bm^c\right), \tag{5.15}$$

as well as for the Kähler moduli:

$$\mathcal{K}_a = \frac{(8\mathcal{K} - 6K^{(3)})}{9mK^{(3)}} \ell_s \overline{\rho}_a = \frac{(4\mathcal{K} - 3K^{(3)})}{9m^2 K^{(3)}} \left(2\overline{e}_a m - \mathcal{K}_{abc} m^b m^c\right) . \tag{5.16}$$

in agreement with the results of section 4.2 in [8].[1] Finally, one may insert the value of the stabilised moduli into the expression (5.8) to obtain the on-shell value of the superpotential for this set of vacua:

$$\langle W_{ISD} \rangle = -\frac{i}{3}\tilde{\rho}\left(\mathcal{K} + \frac{3}{2}K^{(3)}\right) \frac{\mathcal{K} - 3K^{(3)}}{\mathcal{K} - \frac{3}{4}K^{(3)}}. \tag{5.17}$$

As discussed in section 5 of [47] this quantity controls the effective gravitino mass for this set of vacua and, to some extent, the whole spectrum of flux-induced soft-terms in models of intersecting D6-branes. It would be interesting to extract the phenomenological consequences of the α'-corrected spectrum of soft-terms in semi-realistic intersecting D6-brane models, a task that we leave for the future.

From the first equality in (4.4), that only relies on the choice of Kähler metrics (4.1) and of NS-fluxes $h^\Lambda = 0$, it is clear that the scalar potential is positive semi-definite, as one would expect from the mirror construction in [52]. As discussed in the previous chapter, one should be able to see this same feature directly from the bilinear formulation (5.6) of V. Because of the more complicated expression for $\mathcal{Z}^{-1}$ when α'-corrections have been taken into account, showing the positive semi-definiteness of V in this case is more involved. Nevertheless, as we discuss in Appendix C.3 under the above assumptions one can rewrite (5.6) as

$$V_F = \frac{1}{8\kappa_4^2} \vec{\rho}^{\,t}_{\rm ISD} \cdot \mathcal{G}^{-1} \cdot \vec{\rho}_{\rm ISD}, \tag{5.18}$$

where $\vec{\rho}_{\rm ISD}$ is a shorter vector than $\vec{\rho}$, containing as many entries as RR fluxes, but whose entries are no longer only axion dependent but instead

$$\vec{\rho}_{\rm ISD} = \begin{pmatrix} \overline{\rho}_0 \\ \overline{\rho}_a + \frac{27\varepsilon}{4(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)} \frac{\mathcal{K}_a}{\mathcal{K}} s\hat{\rho}_0 \\ \tilde{\rho}^a \\ \tilde{\rho} + \frac{6(1-\frac{3}{4}\varepsilon)}{(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)} \frac{s}{\mathcal{K}}\hat{\rho}_0 \end{pmatrix} \tag{5.19}$$

[1] To compare with the results in [8], one needs to take into account the difference in conventions for the definition of the fluxes, like a flip in the sign in the Romans' parameter m

and the symmetric matrix $\mathcal{G}^{-1}$ is given by

$$\begin{aligned}\mathcal{G}^{-1} \;=\; 8e^{K_T+K_Q}&\left[\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & K^{a\bar b} & 0 & 0 \\ 0 & 0 & \frac{4}{9}\mathcal{K}^2\left(1+\frac{3}{2}\varepsilon\right)^2 K_{a\bar b} & 0 \\ 0 & 0 & 0 & \frac{\mathcal{K}^2}{9}\left(1+\frac{3}{2}\varepsilon\right)^2 \end{pmatrix} + \right.\\ &\left. +\frac{\varepsilon}{1-3\varepsilon}\begin{pmatrix} 9 & 0 & \frac{9}{2}\mathcal{K}_a & 0 \\ 0 & -9t^a t^b & 0 & -\frac{3}{2}\mathcal{K}t^a(1-3\varepsilon) \\ \frac{9}{2}\mathcal{K}_b & 0 & \frac{9}{4}\mathcal{K}_a\mathcal{K}_b & 0 \\ 0 & -\frac{3}{2}\mathcal{K}t^b(1-3\varepsilon) & 0 & -\frac{\mathcal{K}^2}{4}(1-3\varepsilon)^2 \end{pmatrix}\right]. \end{aligned} \tag{5.20}$$

One can easily check that this matrix is positive definite and, in fact, corresponds to the Kähler moduli metric derived from the Kähler potential (2.80), as a quick comparison with (5.7) shows. As such, the minima of the potential will only be attained when each of the entries of the vector (5.19) vanish, or in other words upon imposing:

$$\bar\rho_0 = 0, \quad \tilde\rho^a = 0, \quad \bar\rho_a = -\frac{27\varepsilon}{4(1-3\varepsilon)\left(1+\frac{3}{2}\varepsilon\right)}\frac{\mathcal{K}_a}{\mathcal{K}}s\hat\rho_0, \quad \tilde\rho = -\frac{6\left(1-\frac{3}{4}\varepsilon\right)}{(1-3\varepsilon)\left(1+\frac{3}{2}\varepsilon\right)}\frac{s}{\mathcal{K}}\hat\rho_0. \tag{5.21}$$

It is easy to convince oneself that these conditions are equivalent to the relations satisfied in non-supersymmetric Minkowski vacua. Indeed, inserting the last relation in the third one we find that the latter is equivalent to

$$\bar\rho_a = \frac{9\varepsilon\mathcal{K}_a}{8\left(1-\frac{3}{4}\varepsilon\right)}\tilde\rho, \tag{5.22}$$

which is nothing but the second equation in (5.14). Similarly, the last relation in (5.21) can be rewritten as

$$\frac{\mathcal{K}}{6}\tilde\rho + s\hat\rho_0 = \tilde\rho\varepsilon\frac{\mathcal{K}}{8}\frac{1+6\varepsilon}{1-\frac{3}{4}\varepsilon}. \tag{5.23}$$

which is equivalent to the first equation in (5.14).

These relations reproduce precisely the proposal of [8] to stabilise the Kähler moduli by virtue of α'-corrections. Whether or not this stabilisation mechanism for the Kähler moduli is consistent relies on the possibility of finding a solution to the polynomial equation (5.16) for large values of the Kähler moduli. In order for the α'-correction on the left-hand side to counter-balance the tree-level overall volume, the RR-flux quanta and in particular Roman's mass m have to be chosen appropriately without over-shooting the RR tadpole cancellation conditions. Notice that at the end of this procedure, the complex structure moduli still remain unstabilised in these non-supersymmetric Minkowski vacua.

5.3 Supersymmetric AdS vacua

Just as for $\mathcal{N}=0$ Minkowski vacua, α'-corrections will also affect the conditions that describe supersymmetric AdS vacua in type IIA compactifications. In this case we expect that the effect of α'-corrections is a priori less dramatic, in the sense that Kähler moduli and complex structure moduli are already stabilised in their absence. Nevertheless, taking into account such corrections may be crucial in setups where moduli are stabilised at moderately large volumes. As we will see in the following, the axion polynomial formalism allows to treat such vacua in a somewhat equal footing as the previous case, and to easily extend the results discussed in the previous chapter, where perturbative α'-corrections were neglected.

To analyse α'-corrected $\mathcal{N}=1$ AdS vacua we consider a general Kähler potential

$$K = -\log\left(\frac{4}{3}\mathcal{K} + 2K^{(3)}\right) - 2\log\left[\frac{1}{4}\mathrm{Im}\,(\mathcal{C}\mathrm{Z}^{\Lambda})\mathrm{u}_{\Lambda} - \frac{1}{4}\mathrm{n}^{\mathrm{K}}\mathrm{Im}\,(\mathcal{C}\mathcal{F}_{\mathrm{K}})\right] \tag{5.24}$$

and a superpotential given by:

$$W = \overline{\rho}_0 + i\overline{\rho}_a t^a - \frac{1}{2}\mathcal{K}_a\tilde{\rho}^a - \frac{i}{6}\mathcal{K}\tilde{\rho} - iK^{(3)}\tilde{\rho} + in^K\hat{\rho}_K + iu_{\Lambda}\hat{\rho}^{\Lambda}. \tag{5.25}$$

Following the strategy of [47], we write the different F-terms in terms of axion polynomials and set them to zero:

$$\begin{array}{rcl} F_{N^K} & = & \hat{\rho}_K - i\frac{\mathrm{Im}\,(\mathcal{C}\mathcal{F}_{\mathrm{K}})}{2\mathcal{G}_Q}\left(W_T + W_Q\right) = 0, \\ F_{U_{\Lambda}} & = & \hat{\rho}^{\Lambda} + i\frac{\mathrm{Im}\,(\mathcal{C}\mathcal{Z}^{\Lambda})}{2\mathcal{G}_Q}\left(W_T + W_Q\right) = 0, \\ F_{T^a} & = & \overline{\rho}_a + i\mathcal{K}_{ab}\tilde{\rho}^b - \frac{1}{2}\mathcal{K}_a\tilde{\rho} + \frac{2i\mathcal{K}_a}{\frac{4}{3}\mathcal{K}+2K^{(3)}}\left(W_T + W_Q\right) = 0. \end{array} \tag{5.26}$$

Analogously to our previous discussion, simpler equations are obtained when we consider certain linear combinations of complex structure F-terms

$$\sum_{K=0}^{h} n_{\star}^K F_{N_{\star}^K} + \sum_{\Lambda=0}^{h} u_{\star\Lambda} F_{U_{\star\Lambda}} = \sum_{K=0}^{h} \hat{\rho}_K n_{\star}^K + \sum_{\Lambda=0}^{h} \hat{\rho}^{\Lambda} u_{\star\Lambda} + 2i\left(W_T + W_Q\right) = 0, \tag{5.27}$$

from where we find the following relations:

$$\overline{\rho}_0 - \frac{1}{2}\mathcal{K}_a\tilde{\rho}^a = 0, \qquad n_{\star}^K\hat{\rho}_K + u_{\star\Lambda}\hat{\rho}^{\Lambda} = \frac{1}{3}\mathcal{K}\tilde{\rho} - 2t^a\rho_a + 2K^{(3)}\tilde{\rho}. \tag{5.28}$$

The same can be done with the Kähler moduli F-terms, obtaining:

$$\left(\frac{4}{3}\mathcal{K} + K^{(3)}\right)t^a F_{T^a} = t^a\overline{\rho}_a\left(\frac{10}{3}\mathcal{K} + 2K^{(3)}\right) + i\mathcal{K}_a\tilde{\rho}^a\left(\frac{4}{3}\mathcal{K} + 2K^{(3)}\right) - \mathcal{K}\tilde{\rho}\left(\mathcal{K} + 3K^{(3)}\right), \tag{5.29}$$

were we have used (5.28) to simplify the rhs. It is easy to see that this last one can vanish if $\tilde{\rho}^a = 0$, which in turn implies that $\overline{\rho}_a \propto \mathcal{K}_a$ and the vanishing of (5.29) is the only non-trivial F-term condition in the Kähler sector. Combining such a condition with the first equation in (5.28) one obtains the following vacuum relations

$$\overline{\rho}_0 = 0, \qquad \tilde{\rho}^a = 0, \qquad \overline{\rho}_a = \frac{3}{10}\tilde{\rho}\,\mathcal{K}_a \left[\frac{\mathcal{K} + 3K^{(3)}}{\mathcal{K} + \frac{3}{5}K^{(3)}}\right], \tag{5.30}$$

which generalise the conditions obtained in [47]. Comparing to eq. (3.36) therein, only the third condition is essentially different from the uncorrected case. On the one hand, since the first two conditions are the ones that implement the stabilisation of Kähler axions and one linear combination of complex structure moduli, their vacuum expectation values in terms of the fluxes will have a similar form as in [47]

$$h_K \xi^K_\star + h^\Lambda \xi_{\star\Lambda} = -\frac{\overline{e}_0 m^2 - m\overline{e}_a m^a + \frac{1}{3}\mathcal{K}_{abc}m^a m^b m^c}{m^2}, \qquad b^a = -\frac{m^a}{m}. \tag{5.31}$$

On the other hand, the geometric part of the Kähler moduli, which are stabilised in terms of the background fluxes by the third condition in (5.30), will be affected nontrivially by the $(\alpha')^3$-correction term $K^{(3)}$.

To proceed, we may insert these conditions and the second equation in (5.28) to obtain the vacuum expectation value for the superpotential in these AdS vacua, finding that

$$\langle W_{\text{AdS}}\rangle = -\frac{2i}{15}\tilde{\rho}\frac{\left(\mathcal{K} - 3K^{(3)}\right)\left(\mathcal{K} + \frac{3}{2}K^{(3)}\right)}{\mathcal{K} + \frac{3}{5}K^{(3)}}. \tag{5.32}$$

Combined with the vanishing conditions for the F-terms in the complex structure sector, this allows to write down the stabilisation conditions for the complex structure moduli in terms of their "dual" periods:

$$\mathcal{G}_Q \frac{\hat{\rho}_K}{\text{Im}\,(\mathcal{C}\mathcal{F}_{\text{K}})} = -\mathcal{G}_Q \frac{\hat{\rho}^\Lambda}{\text{Im}\,(\mathcal{C}\mathcal{Z}^\Lambda)} = \frac{1}{15}\tilde{\rho}\,\frac{\left(\mathcal{K} - 3K^{(3)}\right)\left(\mathcal{K} + \frac{3}{2}K^{(3)}\right)}{\mathcal{K} + \frac{3}{5}K^{(3)}}. \tag{5.33}$$

Again, these geometric moduli are directly affected by the cubic correction term $K^{(3)}$, in sharp contrast with the axionic moduli.

Part III

Applications in String Phenomenology

Chapter 6

Fluxed supersymmetry-breaking and soft terms

In the previous chapters we discussed various classes of non-supersymmetric string vacua, and in particular, in section 4.3 we obtained a new class of non-supersymmetric Minkowski vacua with open string moduli stabilised at non-trivial vevs. It provides a nice examples of string vacua in which supersymmetry is spontaneously broken due to background fluxes. A first manifestation of broken supersymmetry in this class of vacua are the non-vanishing F-terms in the complex structure moduli sector, however the genuinely physical observables resulting from spontaneous supersymmetry breaking correspond to the gravitino mass and soft terms for the visible sector (chiral matter charged under gauge symmetries). The aim of this chapter is to establish a connection between those physical observables and flux-axion polynomials. This chapter is based on part of the paper [47].

6.1 Fluxed supersymmetry-breaking

The perturbative toolbox in $\mathcal{N} = 1$ supergravity to obtain a supersymmetry-breaking vacuum consists in coupling gravity to chiral multiplets subject to a non-trivial superpotential. The vacuum configuration of the resulting F-term scalar potential then determines the sign and value of the vacuum-energy, indicating whether the vacuum of the four-dimensional theory corresponds to an Anti-de Sitter, Minkowski or de Sitter spacetime. To discriminate supersymmetric from non-supersymmetric vacua it suffices to analyse the F-terms and identify at least one chiral superfield with a non-vanishing F-term in case of non-supersymmetric vacua. In that case, the fermionic partner inside the chiral superfield serves as the massless Goldstino, which is absorbed by the gravitino through the super-Brout-Englert-Higgs mechanism [54, 55]. The would-be mass of the gravitino in the Lagrangian, also dubbed *apparent* gravitino mass in [56], is proportional to the vacuum expectation value of the superpotential,

$$m_{3/2}^2 = \frac{1}{\kappa_4^4} e^K |W|^2. \tag{6.1}$$

Note, however, that a non-vanishing apparent gravitino mass does not imply supersymmetry is spontaneously broken, as is the case for the supersymmetric AdS vacua introduced in section 4.1.2. To evaluate whether supersymmetry is spontaneously broken, it is more appropriate to consider an *effective* gravitino mass [56],

$$\overline{m}_{3/2}^2 = m_{3/2}^2 + \frac{1}{3} V_F = \frac{1}{3} e^K F_A K^{A\overline{B}} F_{\overline{B}}, \tag{6.2}$$

whose scale is set by the (non-vanishing) F-terms of the chiral multiplets. This relation between the effective gravitino mass and the F-terms of the chiral multiplets has been obtained by virtue of the expression for the F-term scalar potential (2.60). When evaluating the value of the effective gravitino mass in the vacuum of the theory, its value corresponds to the on-shell apparent gravitino mass corrected by the vacuum energy for curved spacetimes. The evaluation of these formulae for ISD flux vacua and supersymmetric AdS vacua will follow shortly. For now, we summarise the various background vacua that can potentially emerge from an $\mathcal{N} = 1$ supergravity theory coupled to chiral supermultiplets in table 6.1. The

background	$m_{3/2}^2$	$\langle V \rangle$	$\overline{m}_{3/2}^2$
SUSY Minkowski	0	0	0
non-SUSY Minkowski	> 0	0	> 0
SUSY AdS	> 0	< 0	0
non-SUSY AdS	> 0	< 0	> 0
non-SUSY dS	> 0	> 0	> 0

Table 6.1: Overview of four-dimensional vacuum configurations in $\mathcal{N} = 1$ supergravity coupled to chiral supermultiplets with the corresponding apparent gravitino mass, vacuum energy and effective gravitino mass.

4d low-energy effective field theory for type IIA orientifold compactifications is (partly) captured by an $\mathcal{N} = 1$ supergravity theory coupled to chiral supermultiplets, with scalar components played by closed and open string moduli. Hence, by studying the vacuum structure of the F-term scalar potential we can both determine the consistency of the compactification as well as the physics of the four-dimensional spacetime.

6.2 Gravitino masses in the ρ-picture

In the previous chapters we argued that perturbative flux vacua are easily identified in terms of constraints on the shift-invariant axion polynomials (3.12) or (3.20). The next step is to rewrite the physical observables resulting from spontaneous supersymmetry breaking such as gravitino masses in terms of the flux-axion polynomials

6.2.1 Vacua without open string moduli

Let us first restrict ourselves to the case without D6-brane. By exploiting the factorability of the perturbative flux superpotential the apparent gravitino mass (6.1) can be expressed in terms of the axion polynomials (3.12) as follows,

$$m_{3/2}^2 = \frac{1}{\kappa_4^4} e^K \rho_A (\Pi^\dagger \ltimes \Pi)^{AB} \rho_B, \tag{6.3}$$

where the purely saxion-dependent matrix $\Pi^\dagger \ltimes \Pi$ reads more explicitly,

$$\Pi^\dagger \ltimes \Pi = \begin{pmatrix} 1 & 0 & -\frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & t^a t^b & 0 & -t^a \frac{\mathcal{K}}{6} & t^a n_\star^K & t^a u_{\star\Lambda} \\ -\frac{1}{2}\mathcal{K}_b & 0 & \frac{1}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -t^b\frac{\mathcal{K}}{6} & 0 & \left(\frac{\mathcal{K}}{6}\right)^2 & -\frac{\mathcal{K}}{6} n_\star^K & -\frac{\mathcal{K}}{6} u_{\star\Lambda} \\ 0 & t^b n_\star^I & 0 & -n_\star^I \frac{\mathcal{K}}{6} & n_\star^I n_\star^K & n_\star^I u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & -u_{\star\Sigma}\frac{\mathcal{K}}{6} & u_{\star\Sigma} n_\star^K & u_{\star\Sigma} u_{\star\Lambda} \end{pmatrix}, \tag{6.4}$$

when expressed in the basis of axion polynomials $\overrightarrow{\rho} = \left(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho}, \hat{\rho}_K, \hat{\rho}^\Lambda\right)$.

Also the effective gravitino mass (6.5) can be expressed in terms of the axion polynomials by working out the F-terms for the Kähler and complex structure moduli explicitly. When neglecting open string moduli or considering compactifications without D6-branes, the factorability of the closed string moduli space translates into a factorisation of the F-terms per sector:

$$\overline{m}_{3/2}^2 = \frac{1}{3} e^K \overrightarrow{\rho}^{\,T} \left(\mathbb{F}_T + \mathbb{F}_{UN\star}\right) \overrightarrow{\rho}, \tag{6.5}$$

where the matrix $\mathbb{F}_{UN\star}$ for the complex structure moduli is given by,

$$\mathbb{F}_{UN\star} = \begin{pmatrix} 4 & 0 & -2\mathcal{K}_a & 0 & 0 & 0 \\ 0 & 4t^a t^b & 0 & -2t^a\frac{\mathcal{K}}{3} & 2t^a n_\star^I & 2t^a u_{\star\Lambda} \\ -2\mathcal{K}_b & 0 & \mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -2t^b\frac{\mathcal{K}}{3} & 0 & 4\left(\frac{\mathcal{K}}{6}\right)^2 & -\frac{\mathcal{K}}{3} n_\star^K & -\frac{\mathcal{K}}{3} u_{\star\Lambda} \\ 0 & 2t^b n_\star^I & 0 & -\frac{\mathcal{K}}{3} n_\star^I & K^{N^I \overline{N}^K} & K^{N^I \overline{U}_\Lambda} \\ 0 & 2t^b u_{\star\Sigma} & 0 & -\frac{\mathcal{K}}{3} u_{\star\Sigma} & K^{U_\Sigma \overline{N}^K} & K^{U_\Sigma \overline{U}_\Lambda} \end{pmatrix}, \tag{6.6}$$

and the matrix $\mathbb{F}_T$ for the Kähler moduli sector reads,

$$\mathbb{F}_T = \begin{pmatrix} 3 & 0 & \frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & t^a t^b - \frac{2}{3}\mathcal{K}\mathcal{K}^{ab} & 0 & t^a \frac{\mathcal{K}}{6} & t^a n_\star^I & t^a u_{\star\Lambda} \\ \frac{1}{2}\mathcal{K}_b & 0 & \frac{3}{4}\mathcal{K}_a\mathcal{K}_b - \frac{2}{3}\mathcal{K}\mathcal{K}_{ab} & 0 & 0 & 0 \\ 0 & t^b \frac{\mathcal{K}}{6} & 0 & 3\left(\frac{\mathcal{K}}{6}\right)^2 & \frac{1}{2}\mathcal{K} n_\star^K & \frac{1}{2}\mathcal{K} u_{\star\Lambda} \\ 0 & t^b n_\star^I & 0 & \frac{1}{2} n_\star^I \mathcal{K} & 3 n_\star^I n_\star^K & 3 n_\star^I u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & \frac{1}{2} u_{\star\Sigma}\mathcal{K} & 3 u_{\star\Sigma} n_\star^K & 3 u_{\star\Sigma} u_{\star\Lambda} \end{pmatrix}, \tag{6.7}$$

both expressed in the basis of axion polynomials $\overrightarrow{\rho} = \left(\rho_0, \rho_a, \tilde{\rho}^a, \tilde{\rho}, \hat{\rho}_K, \hat{\rho}^\Lambda\right)$. The expressions for the apparent and effective gravitino mass have only taken into account the chiral multiplets from the closed string sector. As long as the superpotential remains factorisable in the sense of section 3.4 when including open string chiral multiplets, the expressions for the gravitino masses can be straightforwardly generalised, which will be the focus of the next section.

Supersymmetric AdS flux vacua

Let us now apply the above considerations to the supersymmetric AdS vacua discussed in section 4.1.2. As already known this class of vacua is represented by the vector $\overrightarrow{\rho}_{\text{AdS}} = \tilde{\rho}\left(0, \frac{3}{10}\mathcal{K}_a, 0, 1, -\frac{i}{5}\mathcal{K}K_{N_\star^I}, -\frac{i}{5}\mathcal{K}K_{U_{\star\Lambda}}\right)$. In this vacuum configuration, the apparent gravitino mass happens to have a non-vanishing value proportional to Romans mass $\tilde{\rho}$:

$$m_{3/2}^2 = \frac{1}{\kappa_4^4} e^K \left(\frac{2\mathcal{K}}{15}\tilde{\rho}\right)^2. \tag{6.8}$$

The effective gravitino mass in the supersymmetric AdS vacua vanishes, as can be checked explicitly by evaluating expression (6.5) for the axion vector $\overrightarrow{\rho}_{\text{AdS}}$. The vanishing effective gravitino mass should not surprise us at all, as it is fully in line with the vanishing F-terms and the (negative) vacuum energy for the supersymmetric AdS vacua, which equates in absolute value to three times the value of the apparent gravitino mass.

Non-supersymmetric Minkowski flux vacua

A well-known example of non-supersymmetric vacua is provided by backgrounds with ISD fluxes, as discussed in section 4.1.1. Considering the factorisation of the dilaton as in (4.1) for the ISD flux set-up, the purely saxion-dependent matrix $\Pi^\dagger \ltimes \Pi$ in the apparent gravitino mass takes the form,

$$\Pi^\dagger \ltimes \Pi = \begin{pmatrix} 1 & 0 & -\frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & t^a t^b & 0 & -t^a\frac{\mathcal{K}}{6} & t^a s_\star & t^a u_{\star\Lambda} \\ -\frac{1}{2}\mathcal{K}_b & 0 & \frac{1}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -t^b\frac{\mathcal{K}}{6} & 0 & \left(\frac{\mathcal{K}}{6}\right)^2 & -\frac{\mathcal{K}}{6}s_\star & -\frac{\mathcal{K}}{6}u_{\star\Lambda} \\ 0 & t^b s_\star & 0 & -s_\star\frac{\mathcal{K}}{6} & s_\star^2 & s_\star u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & -u_{\star\Sigma}\frac{\mathcal{K}}{6} & s_\star u_\Sigma & u_{\star\Sigma}u_{\star\Lambda} \end{pmatrix}. \tag{6.9}$$

The apparent gravitino mass for the ISD flux vacua, represented by the axion vector $\overrightarrow{\rho}_{\rm ISD} = \tilde{\rho}\left(0,0,0,1,0,-\frac{i}{3}\mathcal{K}K_{U_\Lambda}\right)$ also scales with Romans' mass $\tilde{\rho}$:

$$m_{3/2}^2 = \frac{1}{\kappa_4^4} e^K \left(\frac{\mathcal{K}}{3}\tilde{\rho}\right)^2 . \tag{6.10}$$

In this class of vacua, the effective gravitino mass does not vanish, which can be verified explicitly when writing out the F-terms by virtue of the axion polynomials:

$$\overline{m}_{3/2}^2 = \frac{1}{3} e^K \overrightarrow{\rho}^{\,T} \left(\mathbb{F}_T + \mathbb{F}_{S_\star} + \mathbb{F}_{U_\star}\right) \overrightarrow{\rho} = \frac{1}{3} e^K \left(\frac{\mathcal{K}}{3}\tilde{\rho}\right)^2 , \tag{6.11}$$

where the matrix $\mathbb{F}_{S_\star}$ for the dilaton sector is given by,

$$\mathbb{F}_{S_\star} = \begin{pmatrix} 1 & 0 & -\frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & t^a t^b & 0 & -t^a \frac{\mathcal{K}}{6} & -t^a s_\star & t^a u_{\star\Lambda} \\ -\frac{1}{2}\mathcal{K}_b & 0 & \frac{1}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -t^b\frac{\mathcal{K}}{6} & 0 & \left(\frac{\mathcal{K}}{6}\right)^2 & \frac{\mathcal{K}}{6}s_\star & -\frac{\mathcal{K}}{6}u_{\star\Lambda} \\ 0 & -t^b s_\star & 0 & s_\star\frac{\mathcal{K}}{6} & s_\star^2 & -s_\star\, u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & -u_{\star\Sigma}\frac{\mathcal{K}}{6} & -s_\star\, u_{\star\Sigma} & u_{\star\Sigma}u_{\star\Lambda} \end{pmatrix}, \tag{6.12}$$

the matrix $\mathbb{F}_{U_\star}$ for the complex structure moduli sector reads,

$$\mathbb{F}_{U_\star} = \begin{pmatrix} 3 & 0 & -\frac{3}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & 3t^a t^b & 0 & -t^a \frac{\mathcal{K}}{2} & 3t^a s_\star & t^a u_{\star\Lambda} \\ -\frac{3}{2}\mathcal{K}_b & 0 & \frac{3}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 \\ 0 & -t^b\frac{\mathcal{K}}{2} & 0 & 3\left(\frac{\mathcal{K}}{6}\right)^2 & -\frac{\mathcal{K}}{2}s_\star & -\frac{\mathcal{K}}{6}u_{\star\Lambda} \\ 0 & 3t^b s_\star & 0 & -s_\star\frac{\mathcal{K}}{2} & 3s_\star^2 & s_\star\, u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & -u_{\star\Sigma}\frac{\mathcal{K}}{6} & s_\star\, u_{\star\Sigma} & K^{\Lambda\Sigma} - u_{\star\Sigma}u_{\star\Lambda} \end{pmatrix} \tag{6.13}$$

and the matrix $\mathbb{F}_T$ for the Kähler moduli takes the form,

$$\mathbb{F}_T = \begin{pmatrix} 3 & 0 & \frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 \\ 0 & t^a t^b - \frac{2}{3}\mathcal{K}\mathcal{K}^{ab} & 0 & \frac{1}{6}\mathcal{K}t^a & t^a s_\star & t^a u_{\star\Lambda} \\ \frac{1}{2}\mathcal{K}_b & 0 & \frac{3}{4}\mathcal{K}_a\mathcal{K}_b - \frac{2}{3}\mathcal{K}\mathcal{K}_{ab} & 0 & 0 & 0 \\ 0 & \frac{1}{6}\mathcal{K}t^b & 0 & 3\left(\frac{\mathcal{K}}{6}\right)^2 & \frac{1}{2}\mathcal{K}s_\star & \frac{1}{2}\mathcal{K}u_{\star\Lambda} \\ 0 & t^b s_\star & 0 & \frac{1}{2}\mathcal{K}s_\star & 3s_\star^2 & 3s_\star u_{\star\Lambda} \\ 0 & t^b u_{\star\Sigma} & 0 & \frac{1}{2}\mathcal{K}u_{\star\Sigma} & 3s_\star u_{\star\Sigma} & 3u_{\star\Sigma}u_{\star\Lambda} \end{pmatrix}. \tag{6.14}$$

The non-vanishing value for the effective gravitino mass is due to the non-vanishing F-terms for the complex structure moduli in the ISD flux vacua, which can be verified explicitly in the axion polynomial language. The factorability of the moduli sectors allows in this case to clearly extract the U-dominated character of the supersymmetry-breaking in type IIA ISD flux vacua.

6.2.2 Vacua with open string moduli

As discussed in section 4.3, mobile D6-branes alter the vacuum structure of the four-dimensional effective theory. Subsequently, the pattern of supersymmetry-breaking in the presence of mobile D6-branes needs further exploration to assess how it defers from the pure closed string case. To this end, we first consider the apparent gravitino mass, which can still be factorised in a bilinear form in terms of the purely saxion-dependent matrix $\Pi^\dagger \ltimes \Pi$,

$$\Pi^\dagger \ltimes \Pi = \begin{pmatrix} 1 & 0 & -\frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 & 0 & t^a\phi^i \\ 0 & t^a t^b & 0 & -t^a\frac{\mathcal{K}}{6} & t^a n^K & t^a u_\Lambda & t^a\phi^i & 0 \\ -\frac{1}{2}\mathcal{K}_b & 0 & \frac{1}{4}\mathcal{K}_a\mathcal{K}_b & 0 & 0 & 0 & 0 & -\frac{1}{2}\mathcal{K}_b t^a\phi^i \\ 0 & -t^b\frac{\mathcal{K}}{6} & 0 & \left(\frac{\mathcal{K}}{6}\right)^2 & -\frac{\mathcal{K}}{6}n^K & -\frac{\mathcal{K}}{6}u_\Lambda & -\frac{\mathcal{K}}{6}\phi^i & 0 \\ 0 & t^b n^I & 0 & -n^I\frac{\mathcal{K}}{6} & n^I n^K & n^I u_\Lambda & n^I\phi^i & 0 \\ 0 & t^b u_\Sigma & 0 & -u_\Sigma\frac{\mathcal{K}}{6} & u_\Sigma n^K & u_\Sigma u_\Lambda & u_\Sigma\phi^i & 0 \\ 0 & t^b\phi^j & 0 & -\frac{\mathcal{K}}{6}\phi^j & n^K\phi^j & u_\Lambda\phi^j & \phi^i\phi^j & 0 \\ t^b\phi^j & 0 & -\frac{1}{2}\mathcal{K}_b t^a\phi^j & 0 & 0 & 0 & 0 & t^a t^b\phi^i\phi^j \end{pmatrix}, \tag{6.15}$$

expressed in terms of the axion polynomial basis $\vec{\varrho}^{\,T} = \left(\varrho_0, \varrho_a, \tilde{\varrho}^a, \tilde{\varrho}, \hat{\varrho}_K, \hat{\varrho}^\Lambda, \varrho_i, \varrho_{ai}\right)$.

Nevertheless, the relevant quantity to consider for vacua with (spontaneously) broken supersymmetry is the effective gravitino mass (6.5), whose explicit bilinear expression in terms of the axion polynomials becomes extremely involved upon inclusion of D6-brane moduli. More precisely, it is the mixing between closed and open string moduli sectors that prevents us from writing down the F-terms as axion polynomial bilinears by virtue of the simple matrices $\mathbb{F}_S$, $\mathbb{F}_U$ and $\mathbb{F}_T$, as in the closed string ISD flux case. Instead we look at the effective gravitino mass as the scalar product between the co-variant and contra-variant F-term vectors,

$$\overline{m}_{3/2}^2 = \frac{1}{3}e^K\left(F_aF^a + F_SF^S + F_\Lambda F^\Lambda + F_iF^i\right) \tag{6.16}$$

and express both vectors explicitly in terms of the axion polynomials. The co-variant F-term vectors contain two contributions both linear in the axion polynomials,

$$\begin{pmatrix} F_a \\ F_S \\ F_\Lambda \\ F_i \end{pmatrix} = \begin{pmatrix} 0 & \delta_a^b & i\mathcal{K}_{ab} & -\frac{1}{2}\mathcal{K}_a & 0 & 0 & 0 & -i\phi^j \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_\Lambda^\Sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_i^j & -it^a \end{pmatrix} \cdot \vec{\varrho} + \begin{pmatrix} K_{T^a} \\ K_S \\ K_\Lambda \\ K_{\Phi^i} \end{pmatrix} \vec{\Pi}^t \cdot \vec{\varrho}, \tag{6.17a}$$

and similarly the contra-variant F-term vector can be written as the sum of two linear terms in the axion polynomials,

$$\begin{pmatrix} F^a \\ F^S \\ F^\Lambda \\ F^i \end{pmatrix} = \begin{pmatrix} 0 & K^{a\bar{b}} & -iK^{a\bar{c}}\mathcal{K}_{cb} & -\frac{1}{2}K^{a\bar{c}}\mathcal{K}_c & K^{a\overline{S}} & K^{a\overline{\Sigma}} & K^{a\bar{j}} & iK^{a\bar{j}}t^b \\ 0 & K^{S\bar{b}} & -iK^{S\bar{c}}\mathcal{K}_{cb} & -\frac{1}{2}K^{S\bar{c}}\mathcal{K}_c & K^{S\overline{S}} & K^{a\overline{\Sigma}} & K^{S\bar{j}} & iK^{S\bar{j}}t^b \\ 0 & K^{\Lambda\bar{b}} & -iK^{\Lambda\bar{c}}\mathcal{K}_{cb} & -\frac{1}{2}K^{\Lambda\bar{c}}\mathcal{K}_c & K^{\Lambda\overline{S}} & K^{\Lambda\overline{\Sigma}} & K^{\Lambda\bar{j}} & iK^{\Lambda\bar{j}}t^b \\ 0 & K^{i\bar{b}} & -iK^{i\bar{c}}\mathcal{K}_{cb} & -\frac{1}{2}K^{i\bar{c}}\mathcal{K}_c & K^{i\overline{S}} & K^{i\overline{\Sigma}} & K^{i\bar{j}} & iK^{i\bar{j}}t^b \end{pmatrix} \cdot \overrightarrow{\varrho}$$
$$+ \begin{pmatrix} -2it^a \\ -2is \\ -2iu_\Lambda \\ -2i\phi^i \end{pmatrix} \overrightarrow{\Pi}^\dagger \cdot \overrightarrow{\varrho} \,, \tag{6.17b}$$

where we used the expressions (4.28) for the inverse metrics on the moduli space and the first order derivatives (A.11) of the Kähler potential to simplify the second term. An alternative (and more explicit) representation of the contravariant F-terms can be found in [41].

CSD vacua

Let us now apply the above results to the $\mathcal{N} = 0$ Minkowski vacua discussed in section 4.3, which rely on weaker vacuum constraints than the ISD flux vacua. Upon evaluating the apparent gravitino mass for the CSD vacuum conditions in (4.51), one easily retrieves the same functional dependence as for the ISD flux vacua:

$$m_{3/2}^2 = \frac{1}{\kappa_4^4} e^K \left(\frac{\mathcal{K}}{3} \tilde{\varrho} \right)^2 . \tag{6.18}$$

Upon evaluating the F-term vectors in the CSD vacua (4.51),

$$(\overrightarrow{F_A})^t = \left(\frac{1}{2}(\mathbf{H}_{a\Lambda} - f_a^i g_{i\Lambda}) F_\Lambda, 0, F_\Lambda, \frac{1}{2} g_i^\Lambda F_\Lambda \right), \qquad (\overrightarrow{F}^A)^t = \left(0, 0, -2iu_{\star\Lambda}\overline{W}_0, 0\right), \tag{6.19}$$

one can immediately deduce that only the complex structure moduli sector provides a non-vanishing contribution to the effective gravitino mass:

$$\overline{m}_{3/2}^2 = \frac{1}{3} e^K F_\Lambda F^\Lambda = \frac{1}{3} e^K \left(\frac{\mathcal{K}}{3} \tilde{\varrho} \right)^2 . \tag{6.20}$$

Note that the functional dependence of the effective gravitino mass for these CSD or $\mathcal{N} = 0$ Minkowski vacua is precisely the same as for the pure ISD flux vacua.

6.3 Flux-induced soft terms on D6-branes

For phenomenological purposes, we are interested in the Lagrangian for the charged matter fields living on the D6-brane worldvolume and in particular the terms arising from supersymmetry breaking. Upon including D6-branes into a type IIA flux vacuum in which supersymmetry is spontaneously broken in the moduli sector, this spontaneous supersymmetry-breaking is transmitted to the matter sector through gravitational interactions (gravity mediation mechanism) which results in a set of soft terms for the open string excitations living on the D6-branes.

6.3.1 Gravity-mediated supersymmetry breaking

In this section we review some basics aspects of the gravity mediation mechanism by recalling the results of [27–29]. In order to extract the structure of the soft supersymmetry breaking terms one usually distinguishes between the visible sector composed of the massless open string excitations (with vanishing vacuum expectation values) on the one hand and the hidden sector of closed string moduli on the other hand. Given that the D6-brane displacement moduli provide for more generic vacua in the presence of background fluxes, we choose a more suitable factorisation of the $\mathcal{N}=1$ chiral multiplets: on the one hand open string excitations transforming in bifundamental representations of the D6-brane gauge theories denoted collectively by $\mathcal{O}^\alpha$ (and its hermitian conjugate $\overline{\mathcal{O}}^{\overline{\alpha}}$), and on the other hand the "hidden" sector of closed string moduli and D6-brane displacement moduli denoted by $\mathcal{H}\in\{T^a,N^K,U_\Lambda,\Phi^i\}$. Subsequently, the Kähler potential and superpotential can then be expanded around the background values of the hidden sector moduli:

$$\begin{aligned} K(\mathcal{H},\overline{\mathcal{H}},\mathcal{O},\overline{\mathcal{O}}) &= K^0(\mathcal{H},\overline{\mathcal{H}})+K_{\alpha\overline{\beta}}(\mathcal{H},\overline{\mathcal{H}})\mathcal{O}^\alpha\overline{\mathcal{O}}^{\overline{\beta}}+\left[\frac{1}{2}Z_{\alpha\beta}(\mathcal{H},\overline{\mathcal{H}})\mathcal{O}^\alpha\mathcal{O}^\beta+h.c.\right]+\ldots, \\ W(\mathcal{H},\mathcal{O}) &= W_0(\mathcal{H})+\frac{1}{2}\mu_{\alpha\beta}(\mathcal{H})\mathcal{O}^\alpha\mathcal{O}^\beta+\frac{1}{6}Y_{\alpha\beta\gamma}(\mathcal{H})\mathcal{O}^\alpha\mathcal{O}^\beta\mathcal{O}^\gamma+\ldots. \end{aligned} \tag{6.21}$$

In this expansion, the Kähler potential $K^0=K_T+K_Q$ contains the Kähler potentials for the dilaton, Kähler moduli, complex structure moduli and open string displacement moduli, while the functions $K_{\alpha\overline{\beta}}(\mathcal{H},\overline{\mathcal{H}})$ represent the Kähler metrics for the open string excitations with vanishing vacuum expectation value (at the level of the supergravity analysis). The superpotential $W_0(\mathcal{H})$ encompasses the perturbative RR- and NS-flux superpotential as well as the bilinear superpotential as in (2.78), while the quadratic and Yukawa couplings between the open string modes arise from non-perturbative effects such as worldsheet instantons and potentially D-brane instantons. The soft terms for the open string modes follow by inserting the expansion for the Kähler potential and superpotential into the F-term scalar potential (2.60), and taking the limit $\kappa_4\to\infty$ while keeping the apparent gravitino mass $m_{3/2}$ fixed:

$$V_{\text{soft}}=m^2_{\alpha\overline{\beta}}\mathcal{O}^\alpha\overline{\mathcal{O}}^{\overline{\beta}}+\left[\frac{1}{6}A_{\alpha\beta\gamma}\mathcal{O}^\alpha\mathcal{O}^\beta\mathcal{O}^\gamma+\frac{1}{2}B_{\alpha\beta}\mathcal{O}^\alpha\mathcal{O}^\beta+h.c\right], \tag{6.22}$$

where the various soft term parameters depend on the closed string and D6-brane displacement moduli (evaluated at their vacuum expectation value):[1]

$$\begin{aligned}
m^2_{\alpha\overline{\beta}} &= (m^2_{3/2} + \frac{V_0}{M^2_{Pl}})K_{\alpha\overline{\beta}} - e^{K^0/M^2_{Pl}}\overline{F}^{\overline{m}}\left(\partial_{\overline{m}}\partial_n K_{\alpha\overline{\beta}} - \partial_{\overline{m}}K_{\alpha\overline{\gamma}}K^{\overline{\gamma}\delta}\partial_n K_{\delta\overline{\beta}}\right)F^n, \\
A_{\alpha\beta\gamma} &= \frac{\overline{\mathcal{W}}_0}{|\mathcal{W}_0|}e^{K^0/M^2_{Pl}}F^m\left[\partial_m K^0\, Y_{\alpha\beta\gamma} + D_m Y_{\alpha\beta\gamma}\right], \qquad (6.24) \\
B_{\alpha\beta} &= \frac{\overline{\mathcal{W}}_0}{|\mathcal{W}_0|}e^{K^0/2M^2_{Pl}}\Big\{e^{K^0/2M^2_{Pl}}F^m\left[\partial_m K^0\,\mu_{\alpha\beta} + D_m\mu_{\alpha\beta}\right] - m_{3/2}\mu_{\alpha\beta} \\
&\quad +(2m^2_{3/2} + \frac{V_0}{M^2_{Pl}})Z_{\alpha\beta} - m_{3/2}e^{K^0/2M^2_{Pl}}\overline{F}^{\overline{m}}\partial_{\overline{m}}Z_{\alpha\beta} + m_{3/2}e^{K^0/2M^2_{Pl}}F^m D_m Z_{\alpha\beta} \\
&\quad -e^{K^0/M^2_{Pl}}\overline{F}^{\overline{m}}F^n D_n\partial_{\overline{m}}Z_{\alpha\beta}\Big\}.
\end{aligned}$$

The soft terms depend both on universal data, such as the F-terms[2] and the Kähler-potential K^0, and on model-dependent input data captured through the moduli-dependent Kähler metrics $K_{\alpha\overline{\beta}}$ and coupling parameters $Z_{\alpha\beta}$, $\mu_{\alpha\beta}$, and $Y_{\alpha\beta\gamma}$. Nonetheless, these soft terms do not correspond to the physical parameters as long as the kinetic terms for the open string states are not written in their canonical form. To eliminate the closed string moduli dependence from the open string kinetic terms, an appropriate field redefinition of the open string excitations is required. In case the kinetic terms are all diagonal, i.e. $K_{\alpha\overline{\beta}} = K_\alpha\delta_{\alpha\beta}$, such a field redefinition is rather straighforward,

$$\mathcal{O}^\alpha \to \hat{\mathcal{O}}^\alpha = K_\alpha^{1/2}\mathcal{O}^\alpha. \qquad (6.25)$$

By virtue of this field redefinition, the physical soft terms for the physical open string excitations $\hat{\mathcal{O}}^\alpha$ reduce to a much simpler form:

$$\begin{aligned}
m^2_\alpha &= (m^2_{3/2} + V_0) - e^{K^0}F^{\overline{m}}F^n\partial_{\overline{m}}\partial_n\log K_\alpha, \qquad (6.26) \\
\hat{A}_{\alpha\beta\gamma} &= \hat{Y}_{\alpha\beta\gamma}F^m\left(\partial_m K^0 + \partial_m\log Y_{\alpha\beta\gamma} - \partial_m\log(K_\alpha K_\beta K_\gamma)\right), \\
\hat{B}_{\alpha\beta} &= \hat{\mu}_{\alpha\beta}\left[e^{K^0/2}F^m\left(\partial_m K^0 + \partial_m\log\mu_{\alpha\beta} - \partial_m\log(K_\alpha K_\beta)\right) - m_{3/2}\right], \\
M_i &= \frac{1}{2}(\mathrm{Im}\, f^{-1})e^{K^0/2}F^m\partial_m f,
\end{aligned}$$

[1]To simplify the formulae for the soft terms, we introduced the notations:

$$\begin{aligned}
D_m Y_{\alpha\beta\gamma} &= \partial_m Y_{\alpha\beta\gamma} - \left(K^{\delta\overline{\rho}}\partial_m K_{\overline{\rho}\alpha}Y_{\delta\beta\gamma} + (\alpha\leftrightarrow\beta) + (\alpha\leftrightarrow\gamma)\right), \\
D_n\mu_{\alpha\beta} &= \partial_m\mu_{\alpha\beta} - \left(K^{\delta\overline{\rho}}\partial_m K_{\overline{\rho}\alpha}\mu_{\delta\beta} + (\alpha\leftrightarrow\beta)\right), \qquad (6.23) \\
D_n Z_{\alpha\beta} &= \partial_m Z_{\alpha\beta} - \left(K^{\delta\overline{\rho}}\partial_m K_{\overline{\rho}\alpha}Z_{\delta\beta} + (\alpha\leftrightarrow\beta)\right).
\end{aligned}$$

[2]Note that the expression for the F-terms in this thesis differs by a factor $e^{-K^0/2M^2_{Pl}}$ from the expressions usually encountered in the literature. This deliberate choice allows to extract an overall exponential factor $e^{K^0/M^2_{Pl}}$ from the non-universal contribution to soft terms, in line with the factorisation of the scalar potential (2.60) and the gravitino mass (6.1).

where we now also included the soft gaugino masses and introduced the physical Yukawa couplings and μ-terms:

$$\hat{Y}_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|\hat{W}|}\, e^{K^0}\, \left(K_\alpha K_\beta K_\gamma\right)^{-1/2} Y_{\alpha\beta\gamma}, \qquad \hat{\mu}_{\alpha\beta} = \frac{\hat{W}^*}{|\hat{W}|}\, e^{K^0}\, \left(K_\alpha K_\beta\right)^{-1/2} \mu_{\alpha\beta}, \tag{6.27}$$

apart from setting $Z_{\alpha\beta} = 0$.

6.3.2 The axion polynomial picture

In section 6.1 it was shown that the factorability of the closed string and D6-brane displacement moduli in terms of shift-invariant axion polynomials and geometric moduli can be extended to the expressions for the gravitino masses, which serve as order parameters for flux-induced supersymmetry-breaking. Given the structure of the soft terms it is very tempting to expose their factorable character by rewriting them in terms of the shift-invariant axion polynomials and geometric moduli as well. To this end, we consider the orientifold projection suited for the ISD flux vacua with closed string moduli (T^a, S, U_Λ) and turn to their respective (contra-variant) F-terms depending linearly on the axion polynomials as denoted in (6.17b). At this point it suffices to insert the F-term expressions back into the soft terms (6.26) in order to relate the soft terms to the axion polynomials. Let us now be more explicit and provide the detailed dependence of the soft terms on the axion polynomials.

Soft Masses

Focusing first on the soft masses m_α^2, we employ the results of the previous section to rewrite them in a matrix notation:

$$m_\alpha^2 = \frac{1}{\kappa_4^2} e^{K^0} \varrho_A \left(\left(\Pi^\dagger \ltimes \Pi\right)^{AB} + \frac{1}{8} Z^{AB} - \left(\mathbb{M}^\dagger\, \text{ק}\, \mathbb{M}\right)^{AB} \right) \varrho_B \tag{6.28}$$

where the Kähler metric matrix ק,

$$\text{ק} = \begin{pmatrix} \partial_{\overline{T}^a}\partial_{T^b} \log K_\alpha & \partial_{\overline{T}^a}\partial_S \log K_\alpha & \partial_{\overline{T}^a}\partial_{U_\Sigma} \log K_\alpha & \partial_{\overline{T}^a}\partial_{\Phi_\alpha^j} \log K_\alpha \\ \partial_{\overline{S}}\partial_{T^b} \log K_\alpha & \partial_{\overline{S}}\partial_S \log K_\alpha & \partial_{\overline{S}}\partial_{U_\Sigma} \log K_\alpha & \partial_{\overline{S}}\partial_{\Phi_\alpha^j} \log K_\alpha \\ \partial_{\overline{U}_\Lambda}\partial_{T^b} \log K_\alpha & \partial_{\overline{U}_\Lambda}\partial_S \log K_\alpha & \partial_{\overline{U}_\Lambda}\partial_{U_\Sigma} \log K_\alpha & \partial_{\overline{U}_\Lambda}\partial_{\Phi_\alpha^j} \log K_\alpha \\ \partial_{\overline{\Phi}_\alpha^i}\partial_{T^b} \log K_\alpha & \partial_{\overline{\Phi}_\alpha^i}\partial_S \log K_\alpha & \partial_{\overline{\Phi}_\alpha^i}\partial_{U_\Sigma} \log K_\alpha & \partial_{\overline{\Phi}_\alpha^i}\partial_{\Phi_\alpha^j} \log K_\alpha \end{pmatrix}, \tag{6.29}$$

is introduced to capture the model-dependent[3] contributions to the soft masses and the matrix $\mathbb{M}$ collects all saxion-dependent terms appearing in the contra-variant F-term vector (6.17b). For generic Calabi-Yau manifolds the explicit

[3]The epithet "model-dependent" refers to the freedom of choice regarding the D6-brane configuration once a Calabi-Yau orientifold background is chosen.

expressions for the Kähler metrics is beyond the scope of present-day computational technology, such that the model-dependent contributions seem to remain unknown at first sight. Nevertheless, closer inspection of the F-term expressions and the Kähler metric matrix ק suggest that it is sufficient to know the scaling behaviour of the Kähler metrics K_α to fully determine the model-dependent part of the soft masses. Let us clarify this bold statement by evaluating the soft masses in the CSD vacua represented by the constraints (4.51). In these CSD vacua, supersymmetry is broken by the F-terms of the complex structure moduli, i.e. $(\overrightarrow{F}^A)^t = \big(0, 0, F^{U_\Lambda}, 0\big)$, such that the model-dependent part of the soft terms reduces to:

$$\overrightarrow{\varrho}^t \cdot \mathbb{M}^T \,\text{ק}\, \overline{\mathbb{M}} \cdot \overrightarrow{\varrho} = e^{K_0} |W_0|^2 u_{\star\Lambda} u_{\star\Sigma} \partial_{u_{\star\Lambda}} \partial_{u_{\star\Sigma}} \log K_\alpha. \tag{6.30}$$

Under the assumption that the Kähler metrics on generic Calabi-Yau manifolds can be locally approximated by their counterparts on toroidal orbifolds discussed in appendix D, we consider the Kähler metrics K_α to be homogeneous functions of degree n_α in the complex structure moduli $u_{\star\Lambda}$. Hence, it follows straightforwardly that $u_{\star\Lambda} u_{\star\Sigma} \partial_{u_{\star\Lambda}} \partial_{u_{\star\Sigma}} \log K_\alpha = -n_\alpha$, which leads to a simple expression for the soft masses (6.28) in terms of the gravitino mass:

$$m_\alpha^2 = m_{3/2}^2 (1 + n_\alpha). \tag{6.31}$$

To find the scaling dimension (or modular weight) n_α for an open string state $\mathcal{O}^\alpha$ we further exploit the knowledge of Kähler metrics for intersecting D6-branes on toroidal orbifold compactifications. Similarly to the toroidal orbifold set-up, we distinguish two different sectors based on the origin of the charged open string state:

(i) Vector-like/Non-chiral matter:
Whenever two supersymmetric D6-branes intersect on a continuous subspace along the internal Calabi-Yau orientifold, their intersection number follows by computing the Euler characteristic of the intersection space.[4] Thus, in case of a codimension 5 intersection with topology $S^1 \simeq \mathbb{RP}^1$, their intersection number is zero due to the vanishing Euler characteristic. Yet the intersection of two such D6-branes can provide for vector-like pairs of $\mathcal{N} = 1$ chiral multiplets. To our knowledge a systematic study of

[4]When calculating the intersection number for two overlapping surfaces, one of the surfaces has to be deformed along normal directions [57,58]. Due to the special Lagrangian property of the cycles considered in this paper, the normal deformations can be mapped to vector fields in the tangent bundle of the intersection space by McLean's theorem. The intersection number is then computed as the number of simple zeros for non-vanishing sections of the tangent bundle, which is equal to the Euler characteristic of the intersection space by the Poincaré-Hopf index theorem.

vector-like matter at intersecting D6-branes has not yet been undertaken for generic Calabi-manifolds and the Kähler metrics for such states are therefore unknown. Though, we expect that the Kähler metrics for vector-like matter can be modelled locally around the intersection locus by homogeneous functions of the closed string moduli and that they exhibit the same scaling behaviour as their counter-parts on toroidal orbifolds. Under this assumption, we can exploit the structure of the Kähler metric (D.12) for vector-like matter on toroidal orbifolds and distinguish between two cases: the Kähler metrics are homogeneous functions of degree -1 in the complex structure moduli (thus with modular weight $n_\alpha = -1$), in which case the vector-like matter states do not acquire soft masses. The other option occurs for Kähler metrics that are homogeneous of degree $-1/2$ in the complex structure moduli and $-1/2$ in the dilaton (with modular weight $n_\alpha = -\frac{1}{2}$), for which the vector-like matter does acquire a soft mass $m_\alpha^2 = \frac{m_{3/2}^2}{2}$.

(ii) Chiral Matter:
Two supersymmetric D6-branes can intersect in a single point of the internal space, in which case a chiral $\mathcal{N} = 1$ supermultiplet in the bifundamental representation is supported at the codimension 6 intersection. Also for these chiral matter states the Kähler metrics on generic Calabi-Yau manifolds are unknown, but a modelisation in terms of homogeneous functions depending on the closed string moduli is undoubtedly possible around the intersection locus. As such, we expect the Kähler metrics for chiral matter states to exhibit to same scaling behaviour as their counterparts (D.13) computed for toroidal orbifolds with modular weight $n_\alpha = -\frac{3}{4}$. This implies that the chiral matter states always acquire soft masses in CSD flux vacua of the order $m_\alpha^2 = \frac{m_{3/2}^2}{4}$.

Soft Terms in Type IIA non-SUSY Minkowski vacua with D6-branes	
Soft masses	$m_\alpha^2 = m_{3/2}^2(1 + n_\alpha)$
A-terms	$\hat{A}_{\alpha\beta\gamma} = \hat{Y}_{\alpha\beta\gamma} m_{3/2} \,(3 + n_\alpha + n_\beta + n_\gamma)$
B-terms	$\hat{B}_{\alpha\beta} = \hat{\mu}_{\alpha\beta} m_{3/2} \,(2 + n_\alpha + n_\beta)$
Gaugino masses	$M_i = m_{3/2}$

Table 6.2: Summary of the soft terms in CSD vacua represented by the constraints (4.51). A coefficient n_α represents the modular weight (degree of the complex structure moduli in the Kähler metrics) for the open string excitation $\mathcal{O}^\alpha$.

A-terms, B-terms and Gaugino Masses

In type IIA compactifications, Yukawa or cubic interactions involving chiral matter states arise from worldsheet instantons α'-corrections, which correspond to two-dimensional surfaces with boundaries along the intersecting three-cycles [59, 60]. The holomorphic character of the two-dimensional surfaces, with the topology of a disc, ensures that the cubic couplings contribute to the superpotential. The amplitude $Y_{\alpha\beta\gamma}$ of the three-point coupling in (6.21) is an exponential function depending on the surface area, which can be expressed in terms of Kähler moduli. The amplitude $Y_{\alpha\beta\gamma}$ can also include holomorphic couplings to the open string moduli encoding the D6-brane position and Wilson line, such that $\mathcal{H} \in \{T^a, \Phi_\alpha^i\}$ for cubic interactions. The fact that the complex structure moduli do not enter in the holomorphic piece of the Yukawa interactions has immediate consequences for the flux-induced A-terms in (6.26), which can be similarly written in matrix notation by virtue of the matrix $\mathbb{M}$:

$$\hat{A}_{\alpha\beta\gamma} = -i\hat{Y}_{\alpha\beta\gamma}\ \left(\partial_{\overrightarrow{\mathcal{H}}} K^{0\,t} + \overrightarrow{\gamma}^{t}\right) \cdot \mathbb{M} \cdot \overrightarrow{\rho}, \tag{6.32}$$

allowing to expose the dependence on the axion polynomials. In this expression we distinguish between a model-independent contribution presented by the vector $\partial_{\overrightarrow{\mathcal{H}}} K^{0\,t} \equiv \left(\partial_{T^a} K^0, \partial_S K^0, \partial_{U_\Lambda} K^0, \partial_{\Phi^i_\alpha} K^0\right)$ and a model-dependent contribution in terms of the vector $\overrightarrow{\gamma}$:

$$\overrightarrow{\gamma} = \left(\begin{array}{c} \partial_{T^a} \log Y_{\alpha\beta\gamma} - \partial_{T^a} \log(K_\alpha\, K_\beta\, K_\gamma) \\ \partial_S \log Y_{\alpha\beta\gamma} - \partial_S \log(K_\alpha\, K_\beta\, K_\gamma) \\ \partial_{U_\Lambda} \log Y_{\alpha\beta\gamma} - \partial_{U_\Lambda} \log(K_\alpha\, K_\beta\, K_\gamma) \\ \partial_{\Phi^i_\alpha} \log Y_{\alpha\beta\gamma} - \partial_{\Phi^i_\alpha} \log(K_\alpha\, K_\beta\, K_\gamma) \end{array} \right). \tag{6.33}$$

The structure of the vector $\overrightarrow{\gamma}$ implies that it is sufficient to know the functional dependence of the Yukawa-coupling $Y_{\alpha\beta\gamma}$ on the hidden sector moduli $\mathcal{H}$ and the modular weights of the Kähler metrics to determine the model-dependent contribution to the A-terms. Once again such a strong statement can be best clarified with the CSD vacua (4.51) as an example. In these $\mathcal{N}=0$ vacua with F-term vector $(\overrightarrow{F}^A)^t = \left(0, 0, F^{U_\Lambda}, 0\right)$, there are only contributions from the complex structure moduli sector to the A-terms:

$$\begin{array}{rcl} \hat{A}_{\alpha\beta\gamma} & = & \hat{Y}_{\alpha\beta\gamma} \left(\frac{\partial_{u_{\star\Lambda}} \tilde{\mathcal{G}}_Q}{\tilde{\mathcal{G}}_Q} - \frac{1}{2}\partial_{u_{\star\Lambda}} \log Y_{\alpha\beta\gamma} + \frac{1}{2}\partial_{u_{\star\Lambda}} \log(K_\alpha\, K_\beta\, K_\gamma)\right) e^{K_0/2} \frac{2}{3}\mathcal{K}\tilde{\varrho} u_{\star\Lambda} \\ & = & \hat{Y}_{\alpha\beta\gamma} m_{3/2}\, (3 + n_\alpha + n_\beta + n_\gamma)\,. \end{array} \tag{6.34}$$

To arrive at the last step, we used that $\tilde{\mathcal{G}}_Q$ is a homogeneous function of degree $3/2$ in the complex structure moduli, that the Kähler metrics K_α are also homogeneous functions of degree n_α in the complex structure moduli, and that holomorphic Yukawa couplings generated by worldsheet instantons do not depend on the complex structure moduli.

In a similar fashion quadratic couplings in the superpotential (6.21) might result from worldsheet instantons [23], and these will again be independent from the complex structure moduli. In non-supersymmetric vacua the quadratic couplings give rise to physical B-terms, which can be decomposed in model-independent and model-dependent pieces:

$$\hat{B}_{\alpha\beta} = \hat{\mu}_{\alpha\beta} \left[-i \left(\partial_{\vec{\mathcal{H}}} K^{0\,t} + \vec{\gimel}^{\,t} \right) \cdot \mathbb{M} \cdot \vec{\rho} - m_{3/2} \right], \tag{6.35}$$

where the only model-dependent contribution is encoded in the vector $\vec{\gimel}$:

$$\vec{\gimel} = \begin{pmatrix} \partial_{T^a} \log\, \mu_{\alpha\beta} - \partial_{T^a}\, \log(K_\alpha K_\beta) \\ \partial_S \log\, \mu_{\alpha\beta} - \partial_S\, \log(K_\alpha K_\beta) \\ \partial_{U_\Lambda} \log\, \mu_{\alpha\beta} - \partial_{U_\Lambda}\, \log(K_\alpha K_\beta) \\ \partial_{\Phi^i_\alpha} \log\, \mu_{\alpha\beta} - \partial_{\Phi^i_\alpha}\, \log(K_\alpha K_\beta) \end{pmatrix}. \tag{6.36}$$

Also in this case, the knowledge about the modular weights of the Kähler metrics and the functional dependence of the coupling $\mu_{\alpha\beta}$ on the closed string moduli, i.e. $\log \mu_{\alpha\beta}$ is a homogeneous function of degree 0, are sufficient to determine the physical B-terms. Using the CSD vacua (4.51) as an explicit example, we obtain the following expressions:

$$\begin{aligned} \hat{B}_{\alpha\beta} &= \hat{\mu}_{\alpha\beta} \left(\tfrac{\partial_{u_{\star\Lambda}} \tilde{\mathcal{G}}_Q}{\tilde{\mathcal{G}}_Q} - \tfrac{1}{2} \partial_{u_{\star\Lambda}} \log\, \mu_{\alpha\beta} + \tfrac{1}{2} \partial_{u_{\star\Lambda}} \log(K_\alpha\, K_\beta) \right) e^{K_0/2} \tfrac{2}{3} \mathcal{K} \tilde{\varrho} u_{\star\Lambda} - \hat{\mu}_{\alpha\beta} m_{3/2} \\ &= \hat{\mu}_{\alpha\beta} m_{3/2} \left(2 + n_\alpha + n_\beta \right). \end{aligned} \tag{6.37}$$

In order for worldsheet instantons to contribute to the superpotential, the associated quadratic and cubic couplings of open string states in the superpotential (6.21) have to form singlets under the full gauge group supported by the D6-branes. In case this field theory selection rule is violated for massive Abelian gauge groups by a coupling, it will not result from a worldsheet instanton, but there exist a completely different set of non-perturbative corrections that can generate such couplings, namely D-brane instantons [61–64]. These Euclidean D2-branes wrap completely along internal special Lagrangian three-cycles and are non-perturbative in the string coupling. Furthermore, the amplitude of a D-brane instanton correction depends holomorphically on complex structure moduli. In that case, the functional dependence of the D-brane instanton will provide for an additional model-dependent contribution to the A-terms and B-terms.[5]

Last but not least, also gaugino masses are expected to arise from spontaneous supersym-metry-breaking in the moduli sector with non-vanishing F-terms.

[5]In principle both quadratic and cubic couplings in the superpotential can arise from D-brane instantons and subsequently give rise to B-terms and A-terms that differ from (6.37) and (6.34) respectively. More precisely, due to the exponential structure of such instanton amplitudes one can immediately deduce that $\log \mu_{\alpha\beta}$ and $\log Y_{\alpha\beta\gamma}$ are homogeneous functions of degree 1 in the complex structure moduli (when poly-instanton corrections are neglected), such that

In order to compute these gaugino mass, the functional dependence of the holomorphic gauge kinetic function is indispensable. The gauge kinetic functions f_α for gauge theories on D6-branes follow directly from the dimensional reduction of the D-brane Chern-Simons and Dirac-Born-Infeld action [50,51]. For a D6-brane wrapping a three-cycle Π_α, the (tree-level) gauge kinetic function f_α is a linear, holomorphic function of the dilaton and/or the complex structure moduli,[6]

$$f_\alpha = c_\alpha S_\star + \sum_\Lambda d_\alpha^\Lambda U_{\star\Lambda}, \tag{6.39}$$

where the integers c_α and d_α^Λ encode information about the three-cycle geometry. To arrive at the gaugino masses, we first rewrite their expression in matrix form by virtue of the F-term factorisation (6.17b):

$$M_\alpha = \frac{1}{2} e^{K^0/2} \mathrm{Im}\,(\mathrm{f}_\alpha{}^{-1})(\partial_{\vec{\mathcal{H}}} \mathrm{f}_\alpha)^{\mathrm{t}} \cdot \mathbb{M} \cdot \vec{\varrho}, \tag{6.40}$$

where we introduced the vector $(\partial_{\vec{\mathcal{H}}} f_\alpha)^t = \left(\partial_{T^a} f_\alpha, \partial_S f_\alpha, \partial_{U_\Lambda} f_\alpha, \partial_{\Phi_\alpha^i} f_\alpha\right)$ as a shorthand notation. The linear dependence on the complex structure moduli in (6.39) is sufficient knowledge to determine the gaugino masses in a supersymmetry-breaking vacua. Evaluating the gaugino masses for D6-branes with $c_\alpha = 0$ in the CSD vacua (4.51), for instance, leads to the familiar expression:

$$M_\alpha = \frac{1}{2\,\mathrm{Im}\,(\mathrm{f}_\alpha)} \sum_\Lambda d_\alpha^\Lambda u_{\star\Lambda} \frac{2}{3} \mathcal{K} \tilde{\varrho} e^{K^0/2} = m_{3/2}, \tag{6.41}$$

that equates the gaugino mass and the gravitino mass.

A summary of the soft terms in CSD vacua is offered by table 6.2. Our results generalise previous results in the literature, in the sense that they also apply to vacua with open string moduli. Indeed, typical soft-term scenarios in type IIB ISD flux vacua correspond to spontaneously broken supersymmetry with non-vanishing F-terms in the Kähler moduli sector [66–68], which corresponds via

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the respective B-terms and A-terms take the form:

$$\begin{aligned} \hat{B}_{\alpha\beta} &= \hat{\mu}_{\alpha\beta} m_{3/2} \left(2 + n_\alpha + n_\beta - \log \mu_{\alpha\beta}\right), \\ \hat{A}_{\alpha\beta\gamma} &= \hat{Y}_{\alpha\beta\gamma} m_{3/2} \left(3 + n_\alpha + n_\beta + n_\gamma - \log Y_{\alpha\beta\gamma}\right), \end{aligned} \tag{6.38}$$

and acquire a moduli-dependent contribution.

[6] The tree-level expression for the gauge coupling follows directly from the dimensional reduction of the DBI-action. However, such a KK reduction does not offer a fully holomorphic expression for the gauge kinetic function in the presence of open string D-brane moduli. Only one-loop corrections to the gauge kinetic functions [65] allow for a proper holomorphic gauge kinetic function, depending on the redefined complex structure moduli. Such a computation goes beyond the scope of this paper.

mirror symmetry to non-vanishing F-terms in the complex structure moduli sector for Type IIA ISD flux vacua. We find that CSD vacua have the same structure of contravariant F-terms as ISD flux vacua. Therefore, upon assuming that the chiral fields Kähler metrics are homogeneous polynomials, we obtain a similar soft term structure. Modelling the Kähler metrics for the chiral open string states as homogeneous polynomials in the geometric moduli is mostly inspired by the known results for toroidal models as summarised in appendix D, yet it has been adopted as a standard practice in the literature [69–71] to parameterise the Kähler metrics for generic Calabi-Yau manifolds. Here, we fully exploit the scaling behaviour of the Kähler metrics to simplify the model-dependent contributions to the soft terms as much as possible.

6.4 The validity of the Type IIA flux landscape

Our previous efforts have been devoted to deriving the vacuum structure, spontaneous supersymmetry-breaking and soft terms for perturbative flux vacua in terms of the shift-invariant axion polynomials. A hidden premise behind this approach is the consideration that the low-energy effective description for flux compactifications (with D6-branes) is captured by a four-dimensional $\mathcal{N}=1$ supergravity theory. To asses the validity of the premise and guarantee the overall consistency of a flux compactification (with D6-branes), one has to determine the geometric scales at which distinct particle states acquire their mass and argue for an adequate separation of scales.

The first geometric scale to determine in terms of the compactification data is the string mass scale, which follows upon comparison between the Einstein-Hilbert action and the four-dimensional effective field theory arising from the dimensional reduction of the ten-dimensional type IIA supergravity action. More precisely, we start from the kinetic terms for the massless bosonic type IIA string states in the string frame:

$$\mathcal{S} = -\frac{1}{2\kappa_{10}^2}\int e^{-2\phi}\left[\mathfrak{R}\star_{10}\mathbf{1} - 4d\phi\wedge\star_{10}d\phi + \frac{1}{2}H_3\wedge\star H_3\right] - \frac{1}{8\kappa_{10}^2}\int\sum_{p=0}^{5} G_{2p}\wedge\star_{10}G_{2p}, \tag{6.42}$$

where $\mathfrak{R}$ corresponds to the ten-dimensional Ricci scalar, H_3 to the NS 3-form field strength and G_{2p} to the RR-form field strengths as introduced in section 2.5. The conversion to the Einstein frame requires a rescaling of the ten-dimensional metric, i.e. $G^{(10)} \to G_E^{(10)} = e^{(\phi-\phi_0)/2}G^{(10)}$, while an overall rescaling of the four-dimensional metric in the form $g_E^{(4)} \to \frac{\mathcal{V}_E^0}{\mathcal{V}_E}g_E^{(4)}$ sneaks into the six-dimensional volume-dependence of the string mass scale:

$$M_s^2 = \frac{g_s^2}{4\pi}\frac{M_{Pl}^2}{\mathcal{V}_E^0}. \tag{6.43}$$

In this expression the string coupling constant $g_s = e^{\phi_0}$ is related to the *vev* of the ten-dimensional dilaton and $\mathcal{V}_E^0$ corresponds to the (dimensionless) volume of the Calabi-Yau orientifold evaluated at the vacuum for the geometric moduli in the Einstein frame.

In the presence of background fluxes along the internal dimension a perturbative potential (3.28) for the geometric moduli and axions arises upon the dimensional reduction of the ten-dimensional supergravity action (6.42) to four dimensions. This scalar potential matches precisely the F-term scalar potential from the $\mathcal{N} = 1$ supergravity analysis with the Kähler potentials given by (2.11) and (2.49) and the superpotential by (3.6) for the pure closed string sector. As we reviewed in previous sections, the inclusion of (mobile) D6-branes into the compactification can be easily mediated through a redefinition of the complex structure moduli whose Kähler potential is subsequently given by (2.75), while the superpotential is extended by the bilinear term (2.78). This supergravity analysis is valid for small string coupling and large internal volume, for which the string mass scale obviously lies below the Planck mass scale. As a second criterion for the validity of the supergravity analysis one has to ensure that the tower of massive Kaluza-Klein states decouples from the massless KK-modes, such that the effective field theory below the KK-scale consists purely of the (massless) $\mathcal{N} = 1$ chiral multiplets containing the Kähler moduli, complex structure moduli and open string moduli (as well as other massless open string excitations). Strictly speaking, it is unknown how to determine the KK mass scale for compactifications on generic Calabi-Yau manifolds, yet an adequate approximation follows [72] from toroidal compactifications with characteristic radius size $R = R_s \ell_s$. If we use the dimensionless radius R_s as a proxy for the internal volume $\mathcal{V}_s^0$, i.e. $\mathcal{V}_s^0 = (2\pi R_s)^6$ expressed in the string frame, we find a Kaluza-Klein mass scale of the order

$$M_{KK} \sim 2\pi \frac{M_{\rm s}}{(\mathcal{V}_s^0)^{1/6}} \sim \frac{g_s \sqrt{\pi} M_{Pl}}{(\mathcal{V}_E^0)^{2/3}}. \tag{6.44}$$

Thus, the $\mathcal{N} = 1$ supergravity analysis represents the effective field theory description of four-dimensional type IIA compactifications for energy scales below the KK-mass scale, and other mass generating effects should yield masses below this scale. For instance, the moduli masses induced by perturbative NS-fluxes take the following form,

$$M_{\rm mod} \sim \frac{N_{\rm flux}}{4\pi} \frac{M_{\rm s}}{\sqrt{\mathcal{V}_E^0}} \sim \frac{N_{\rm flux}}{4\pi} \frac{g_s M_{Pl}}{\mathcal{V}_E^0}, \tag{6.45}$$

and lie below the KK-scale for large internal volumes $\mathcal{V}_s^0 > 1$. This scaling of the moduli masses in perturbative type IIA flux vacua can be obtained following the same reasoning as in [72]: the rescaling of the ten-dimensional metric considered above allows to express all relevant quantities, such as the Kähler potential and superpotential, in the Einstein frame, after which the scaling with the internal volume can be deduced for the physical moduli masses in the vacuum configuration.

For closed string ISD flux vacua and the CSD vacua in (4.51), supersymmetry is spontaneously broken in the complex structure moduli sector and a non-vanishing gravitino mass is induced:

$$m_{3/2} = \overline{m}_{3/2} \sim g_s |\mathcal{W}_0| \frac{M_s}{\mathcal{V}_E^0} \sim g_s^2 |\mathcal{W}_0| \frac{M_{Pl}}{(\mathcal{V}_E^0)^{3/2}}, \tag{6.46}$$

where $\mathcal{W}_0 = \ell_s W_0$ is dimensionless. This gravitino mass clearly lies below the KK mass scale for large internal volumes. Moreover, as we have shown in the previous section and summarised in table 6.2, all soft terms in such vacua are proportional to the gravitino mass, such that also the soft masses for the chiral open string excitations lie below the KK-scale. Hence, $\mathcal{N} = 0$ Minkowski vacua with (partly) stabilised moduli through perturbative background fluxes easily satisfy the naïve mass hierarchy that is required to justify a Wilsonian effective field theory approach. Furthermore, in the supergravity limit one can also argue from the ten-dimensional equations of motion that the ten-dimensional dilaton is bounded from above, such that the perturbative type IIA flux vacua with non-vanishing Romans mass are inherently weakly coupled in the string coupling [73].

A more profound worry about the validity of type IIA flux vacua with Romans mass $m \neq 0$ concerns [74] their proper existence as solutions of ten-dimensional supergravity. In first instance, it is not a priori clear whether a Calabi-Yau manifold can be considered a proper compactification background in the presence of internal fluxes. In the case of type IIA ISD flux vacua this worry seems unfounded, as we expect the fluxes to be diluted at large volume such that warping or other back-reaction effects on the Calabi-Yau metric can be neglected to first order, similarly to the mirror dual ISD flux vacua in type IIB. The supersymmetric AdS vacua on the other hand require a more careful treatment to ensure that they are genuine $\mathcal{N} = 1$ supersymmetric backgrounds with an $SU(3)$ structure. To solve the ten-dimensional equations of motion for Minkowski or Anti-de Sitter compactifications it suffices [75] to solve for the supersymmetry variations of the dilatini and gravitini, alongside the Bianchi identities for the RR- and NS-field strengths. By virtue of the pure spinor formulation of generalized complex geometry one can then show that supersymmetric AdS vacua solve the supersymmetry variations with a constant dilaton and form a special subclass of the Lüst-Tsimpis AdS vacua [75, 76].

Secondly, to obtain a full-fledged 10d supergravity solution also the Bianchi identities have to be satisfied in the presence of sources. In the case of the RR two-form flux G_2 solving the Bianchi identity might be more involved due to the presence of sources: the NS-three-form acts as a magnetic source for G_2 in the presence of a non-vanishing Romans mass. Apart from background fluxes the Bianchi identity for G_2 one also has to take into account the RR-charges of the

D6-branes and O6-planes, as presented in equation (B.1). As the smooth H-flux distribution cannot be cancelled against the localized charges of the O6-planes, it is impossible to solve this Bianchi identity for a two-form flux G_2 consisting only of a harmonic and exact component.[7] Adding D6-branes to the mix can help to alleviate the RR tadpoles along the internal directions, but do not help to mediate the non-closedness of the G_2-flux. In order to see how the addition of mobile D6-branes alters the type IIA compactifications with ISD fluxes, we included them in section 4.3 and observed that they give rise to $\mathcal{N} = 0$ CSD vacua, with physically observable features such as a gravitino mass and soft masses. The similarities between the pure ISD flux vacua and the CSD vacua invite to add mobile D6-branes to the known supersymmetric AdS vacua and search for full-fledged 10d supergravity solutions on Calabi-Yau orientifold or more generic $SU(3) \times SU(3)$ structure backgrounds, such that the supersymmetry variations for the dilatini and gravitini still vanish in the modified vacuum structure with D6-branes.

[7] In the literature smeared O6-planes were proposed [77] as a solution to solve the Bianchi identities for the RR two-form flux. However, it is not a priori clear [78] that smearing O-planes offers consistent approximate solutions to the string theory equations with localised O-planes. Fortunately, solutions with localised O6-planes do exist in massive type IIA supergravity [79], such that the search for consistent, global type IIA vacua with fluxes, O-planes and D-branes is a well-defined scientific problem.

Chapter 7

Building a supersymmetric DFSZ Axion Model

The aim of this chapter is to build a consistent supersymmetric DFSZ axion model in type IIA orientifolds with background fluxes and intersecting $D6$-branes supporting charged chiral fermions and non-abelian gauge symmetries. Along the line of previous chapters, background fluxes used for the stabilization of the closed string moduli simultaneously provide the source for spontaneous supersymmetry breaking which induces soft terms for the matter fields living at the D6-brane intersections. Using the gravity mediation mechanism reviewed in chapter 6, we compute the flux-induced soft terms for a simple DFSZ axion model on toroidal backgrounds.

The model proposed here does not pretend to be a realistic DFSZ model, but rather to illustrate how such a model could be realized in the context of type IIA flux compactifications. Of course, a lot of work still have to be done in order to obtain a fully realistic model. In particular, will be needed a lot of efforts to obtain stringy vacua with exactly the DFSZ spectrum and a positive cosmological constant.

7.1 The strong CP problem and the DFSZ Axion Model

Strong interaction phenomena has revealed that discrete symmetries as charge conjugation C, parity P and time reversal T are independently good symmetries of nature. Therefore, the quantum chromodynamics (QCD) based on the gauge group $SU(3)$ must respect any combinations of these discrete symmetries to be accepted as the theory of strong interactions. However, among these discrete symmetries, the CP symmetry is not necessarily respected in QCD due to the nonzero QCD vacuum angle θ, this outstanding problem is known as the strong CP problem. Seem to be that the most attractive solution to the strong CP problem is provided by axion [80, 81].

Peccei and Quinn (PQ) proposed long ago, the existence of a further global symmetry $U(1)_{PQ}$ which is spontaneously broken and the axion corresponds to the Goldstone boson of the spontaneously broken symmetry [82,83]. In the model proposed by Peccei-Quinn-Weinberg-Wilczek (PQWW), the breaking scales of the electroweak and global $U(1)_{PQ}$ symmetry coincide, and consequently non-negligible contributions of axions to hadronic decay products involving heavy quarks in the initial state are expected. Since such effects have not been observed in hadronic processes, the most simple PQWW axion model was ruled out quickly.

The breaking scale of the global PQ symmetry can be decoupled from the electroweak scale by introducing a further scalar field σ which is neutral under the SM gauge group, but it is charged under $U(1)_{PQ}$. This mechanism can be implemented in two different ways. The first, was proposed by Kim, Schifman, Vainshtein and Zakharov (KSVZ), in which the axion couples to a vector-like heavy quark pair charged under the gauge group $SU(3)$ and the global symmetry $U(1)_{PQ}$ [84]. Models of this kind are referred to as KSVZ axion models. The second implementation was proposed by Dine, Fischler, Sredenicki and Zhitnitsky (DFSZ), in which the axion couples to two Higgs doublets which are charged under the $U(1)_{PQ}$, consequently the SM fermions also carry PQ charge [85, 86]. From now on we will focus on this kind of models, referred to as DFSZ axion models.

Let us start reviewing the main features of the DFSZ axion model, such as spectrum, the scalar potential and axion mass. The spectrum of this model consists of three generations of quarks and leptons charged under $SU(3)_{\mathrm{QCD}} \times SU(2)_L \times U(1)_Y$ plus two Higgs doublets charged under $SU(2)_L \times U(1)_Y$ introduced to break the electroweak gauge group, besides a complex scalar field σ introduced to break the global $U(1)_{PQ}$ symmetry spontaneously. Given this spectrum, the two Higgs doublets (H_u, H_d) couple to quarks and leptons through the usual Yukawa couplings. While the most general scalar potential invariant under these symmetries is given by the DFSZ potential

$$\begin{aligned} V_{DFSZ} &= \lambda_u \left(H_u^\dagger H_u - \frac{v_u^2}{2} \right)^2 + \lambda_d \left(H_d^\dagger H_d - \frac{v_d^2}{2} \right)^2 + \lambda_\sigma \left(|\sigma|^2 - \frac{v_\sigma^2}{2} \right)^2 \\ &+ \left(c_1 H_u^\dagger H_u + c_2 H_d^\dagger H_d \right) \sigma^\dagger \sigma + c_3 \left(H_u \cdot H_d \sigma^2 + h.c. \right) + c_4 |H_u \cdot H_d|^2 + c_5 |H_u^\dagger \cdot H_d|^2 \end{aligned} \tag{7.1}$$

To simplify the above expression we used $H_u \cdot H_d = H_u^i \epsilon_{ij} H_d^j$ as well as the standard decomposition of the Higgs doublets into charged and uncharged components, and their VEVs

$$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}, \quad \langle h_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad \langle h_d^0 \rangle = \frac{v_d}{\sqrt{2}} \tag{7.2}$$

The local minimum of the scalar potential (7.1) corresponds to electroweak symmetry breaking vacua

$$H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix} \tag{7.3}$$

in which the W and Z bosons as well as quarks and leptons acquire masses through the Higgs mechanism.

Note that, the presence of the term $c_3\left(H_u \cdot H_d \sigma^2 + h.c.\right)$ in the scalar potential (7.1) implies that the Higgs doublets have to be charged under the global $U(1)_{PQ}$ symmetry because of the complex scalar σ transforms non-trivially.

$$\sigma \to e^{iq_\sigma \theta}\sigma, \quad \Longrightarrow \quad H_u \to e^{iq_u\theta} H_u, \qquad H_d \to e^{iq_d\theta} H_d \tag{7.4}$$

such that $q_u + q_d = -2q_\sigma$. As a consequence, the quarks carry a charge under the global $U(1)_{PQ}$ symmetry as well. The transformations of the quarks and leptons and Higgs bosons under $U(1)_{PQ}$ can be easily deduced from the Yukawa couplings (7.9) and read

$$\begin{aligned} Q_L \to e^{iq_Q\theta} Q_L, \qquad & u_R \to e^{i\tilde{q}_u\theta} u_R, , \qquad & d_R \to e^{i\tilde{q}_d\theta} u_d \\ L \to e^{iq_L\theta} L, \qquad & e_R \to e^{iq_e\theta} e_R, , \qquad & \nu_R \to e^{i\nu_R\theta} \nu_R \end{aligned} \tag{7.5}$$

with $q_Q + \tilde{q}_u = q_L + q_\nu = -q_u$ and $q_Q + \tilde{d}_u = q_L + q_e = -q_d$. This leads to two inequivalent consistent choices of charge assignments with either left-handed quarks Q_L uncharged and right-handed quarks (u_R, d_R) charged or vice versa. In any case, there is unique choice for the $U(1)_{PQ}$ charge of the Higgses, given by $q_u = q_d = 1$. The discussion for leptons is completely analogous.

In order for the axion to remain invisible at low energies, the $U(1)_{PQ}$ symmetry must be broken at energies much higher than the scale of electro-weak symmetry breaking, which implies a hierarchy of VEVs

$$v_\sigma \gg \sqrt{v_u^2 + v_d^2} \tag{7.6}$$

Thus, the $U(1)_{PQ}$ is an anomalous global symmetry and the corresponding Goldstone-boson (axion) for this broken symmetry acquires a mass due to instanton effects.The physical axion corresponds to the mixing of the neutral CP-odd Higgs-bosons and the argument of the complex scalar field σ. However, by imposing the hierarchy of vevs (7.6) one has that the physical axion comes from mainly the argument of σ and whose mass is determined using Bardeen-Tye methods and read [87]

$$m_\theta^2 = \frac{f_\pi^2}{f_\theta^2} m_\pi^2 N^2 \frac{m_u m_d}{(m_u + m_d)^2} \sim \left(74\,\mathrm{keV}\frac{250\,\mathrm{GeV}}{f_\theta}\right)^2 \tag{7.7}$$

where N is the number of quark doublets, m_u, m_d, m_π are the masses of up- and down-type quarks and pions respectively, while f_π, f_θ are the decay constants of pions and axions respectively. On the other hand, the coupling of the axion to matter is determined by the constant f_θ, which also sets the strength of the axion coupling to gluons through

$$\mathcal{L}_\theta \supset -\frac{1}{32\pi^2}\frac{\theta(x)}{f_\theta}\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right) \tag{7.8}$$

Note that, compared to the original PQ axion model, the axion-gluon coupling as well as the couplings to ordinary matter are suppressed by a factor $r = \sqrt{v_u^2 + v_d^2}/f_\theta$, which implies that the production of axions is reduced by a factor r^2.

7.2 A consistent supersymmetric DFSZ axion model

The existence of two Higgses in the DFSZ model suggests that such a model can be easily generalized to a supersymmetric one. It is a well-known fact that a supersymmetric extension of the Standard Model requires at least two Higgs-doublets to satisfy anomaly cancellation conditions as well as mass-generation for quarks and leptons through the Yukawa-couplings in (7.9). Moreover, implementing the axion solution in a supersymmetric framework also requires promoting the axion to a superfield Σ, which introduces two further fields: the saxion and the axino. At first sight seems that the minimal extension of the supersymmetric DFSZ axion model consists of three generations of quarks and leptons charged under $SU(3)_{\mathrm{QCD}} \times SU(2)_L \times U(1)_Y$, one up-type Higgs H_u, one down-type Higgs H_d, besides the SM singlet Σ charged under the additional $U(1)_{PQ}$ symmetry. In this minimal supersymmetric version of the DFSZ model, the two Higgs doublets (H_u, H_d) couple to quarks and leptons through the usual Yukawa couplings

$$W = Y_u\, Q_L \cdot H_u U_R + Y_d\, Q_L \cdot H_u D_R + Y_e\, L \cdot H_d E_R \tag{7.9}$$

where Y_i are 3×3 Yukawa matrices and we have suppressed generation indices. In addition to the renormalizable superpotential of eqn. (7.9), further gauge-invariant terms

$$W = \lambda L \cdot L\bar{E}_R + \tilde{\lambda} L \cdot Q_L \bar{D}_R + \hat{\lambda} L \cdot H_u + \epsilon \bar{U}_R \bar{D}_R \bar{D}_R \tag{7.10}$$

Consistent supersymmetric DFSZ Axion Model		
Field	$SU(3) \times SU(2)_{\left(U(1)_Y \times U(1)_{PQ}\right)}$	Components
Q_L	$3(\mathbf{3},\mathbf{2})_{(1/6,1)}$	$(\tilde{q}_L, Q_L)$
U_R	$3(\bar{\mathbf{3}},\mathbf{1})_{(-2/3,0)}$	$(\tilde{u}_R, u_R)$
D_R	$3(\bar{\mathbf{3}},\mathbf{1})_{(1/3,0)}$	$(\tilde{d}_R, d_R)$
L	$3(\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	$(\tilde{l}, L)$
E_R	$3(\mathbf{1},\mathbf{1})_{(1,0)}$	$(\tilde{e}_R, e_R)$
H_u	$(\mathbf{1},\mathbf{2})_{(1/2,1)}$	$(\tilde{h}_u, H_u)$
H_d	$(\mathbf{1},\mathbf{2})_{(-1/2,-1)}$	$(\tilde{h}_d, H_d)$
Σ	$(\mathbf{1},\mathbf{1})_{(0,2)}$	(σ, χ)
$\tilde{\Sigma}$	$(\mathbf{1},\mathbf{1})_{(0,-2)}$	$(\tilde{\sigma}, \tilde{\chi})$
Φ	$(\mathbf{1},\mathbf{1})_{(0,0)}$	$(\tilde{\phi}, \tilde{\Phi})$

Table 7.1: Summary of the superfields for a consistent supersymmetric DFSZ axion model and their quantum numbers.

are allowed in the MSSM superpotential, which violate both lepton and baryon numbers. However, looking at table 7.1 we notice that only the $\hat{\lambda}$ term is not necessarily perturbatively forbidden by the global $U(1)_{PQ}$ symmetry.

On the other hand, embedding the Higgs-axion potential (7.1) into a supersymmetric model is more involved. As in any supersymmetric theory, this potential should be expressed in terms of F-terms, D-terms and soft terms contributions:

$$V_{\text{DFSZ}} = V_F + V_D + V_{\text{soft}} \tag{7.11}$$

The F-term contribution can be computed form a superpotential of the form [88]

$$W = \kappa \Sigma H_u \cdot H_d \tag{7.12}$$

and read explicitly as

$$V_F = \sum_i \left| \frac{\partial W_{\text{DFSZ}}}{\partial \phi^i} \right|^2 = |\kappa|^2 |H_u \cdot H_d|^2 + |\kappa|^2 \left(|H_u|^2 + |H_d|^2 \right) |\Sigma|^2 \tag{7.13}$$

Whereas the gauge structure of the model yields the following D-term contribution

$$V_D = \frac{1}{8}(g_Y^2 + g_2^2) \left(|H_u|^2 - |H_d|^2 \right)^2 + g_2^2 |H_u^\dagger H_d|^2 + \frac{g_2^2}{8} \left(|H_u|^2 - |H_d|^2 + 2|\Sigma|^2 \right) \tag{7.14}$$

where g_i are the corresponding gauge couplings. Finally, the set of soft terms respecting the gauge symmetries of the model is given by

$$V_{\text{soft}} = m_u^2|H_u|^2 + m_d^2|H_d|^2 + m_\sigma|\Sigma|^2 + c\Sigma H_d \cdot H_u \tag{7.15}$$

The above soft masses $m \sim \mathcal{O}(m_{3/2})$, these terms are all determined by the breaking of the underlying supergravity model, as argued in chapter 6. Note that, once the scalar component of the superfield Σ gets a non-vanishing VEV $\langle\sigma\rangle = f_\sigma$, the superpotential term (7.15) behaves effectively as a μ-term given by $\mu_{\text{eff}} = \kappa f_\sigma$. This implies that the coupling κ has to be tuned to $10^{-9} - 10^{-10}$, in order to get the proper breaking of the electroweak symmetry.

As shown in [89], such a supersymmetric extension of the DFSZ axion model is inconsistent, because of when minimizing the Higgs-axion potential, the saxion acquires a negative squared mass. As also pointed out in [89], this model can be done consistent by adding further fields and a superpotential of the form

$$W = \lambda\Phi\left(\tilde{\Sigma}\Sigma - \frac{1}{4}f^2\right) \tag{7.16}$$

The additional field $\tilde{\Sigma}$ carries PQ charge opposite to Σ, while Φ is PQ neutral. The spectrum for this minimal consistent DFSZ model is summarized in table 7.1. Thus, besides the soft terms in (7.15) one also must take into account terms of the form

$$V_{soft} = m_\phi^2|\Phi|^2 + m_{\tilde{\sigma}}^2|\tilde{\Sigma}|^2 + c_\Phi\Phi\tilde{\Sigma}\Sigma \tag{7.17}$$

7.3 Stringy axions and the QCD axion

As already mentioned, type IIA superstring theory contains a plethora of axions as well as Abelian gauge bosons arising from different sectors of the theory. Possibly, the best known example of stringy axions is provided by axions arising from the Kaluza-Klein reduction of massless p-forms gauge fields appearing in the closed string spectrum. The shift symmetry of these CP-odd real scalars descends from the gauge invariance of the ten-dimensional p-forms gauge field, while the axion decay constant is set by the non-canonical prefactor in the kinetic term of the axion appearing in the low energy effective action upon dimensional reduction. In D-brane constructions, the axial coupling to the QCD field strength (7.8) comes from the dimensional reduction of the D6-brane Chern-Simons action in (2.61), more precisely from the term

$$S_{CS} \supset \frac{M_s^3}{4\pi}\int_{\mathbb{R}^{1,3}\times\Pi_\alpha} C_3 \wedge \text{Tr}\left(F_\alpha \wedge F_\alpha\right) \tag{7.18}$$

where Π_α is the three-cycle wrapped by the D6-brane and F_α is the field strength of the $U(1)$ gauge field living on its worldvolume. In order to obtain the low-energy effective action for the closed string axions we first expand $C = \xi_\star^K \alpha_K + \xi_{\star\Lambda}\beta^\Lambda$ as in section 2.4, and then perform the dimensional reduction of the kinetic term $F_4 \wedge \star F_4$ together with the Chern-Simons term (7.18), which yields

$$S_\xi = \int_{\mathbb{R}^{1,3}} \frac{\pi}{8} M_s^8 \mathcal{V} \partial_\mu \xi^i \partial^\mu \xi^i - \frac{M_s^3 \mathcal{V}_{\Pi_\alpha}}{4\pi} \xi^i \mathrm{Tr}\left(F_\alpha \wedge F_\alpha\right) \tag{7.19}$$

where ξ^i collectively denotes the C_3-axions ξ^K and ξ_Λ and $\mathcal{V}_{\Pi_\alpha}$ is the volume of the three-cycle Π_α. The above effective action can be brought into its canonical form by simple rescaling of the closed string axions ξ^i. After bringing the effective actio into its standard form we can read off the axion decay constant

$$f_\xi = \frac{\mathcal{V}^{1/2}}{8\sqrt{2\pi}\mathcal{V}_{\Pi_\alpha}} M_\mathrm{s} \tag{7.20}$$

From the above expression we immediately see that the axion decay constant for closed string axions is proportional to the string scale M_s, which makes problematic to identify closed string axions with the QCD axion.

An additional obstacle for closed string axions to solve the strong CP-problem appears when addressing moduli stabilisation of their saxionic partners [90]. For a saxion stabilised supersymmetrically by non-perturbative corrections, its associated axion is also stabilised with the same mass. The no-go theorem in [90] further indicates that the presence of massless axions implies tachyonic directions in the scalar potential. However, if some of the saxions are stabilised by perturbative effects, like for instance α'-corrections, the no-go theorem can be circumvented, as was explicitly shown in the context of the Large Volume Scenario in [91]. Whereas for unfixed closed string axions, their axion decay constant are still proportional to the string scale, such that their appropriateness to serve as the QCD axion is strongly correlated with an intermediate string scale ($M_s \sim 10^{12}$GeV).

In D-brane setups, $U(1)$ symmetries appear as the centers of unitary gauge groups supported by the corresponding D-brane and gauge anomalies are canceled by the generalized Green-Schwarz mechanism [92] reviewed in the next section. Moreover, an anomalous $U(1)$ symmetry acquires a Stückelberg mass proportional to string scale by eating a closed string axion. In that case, the $U(1)$ survives perturbatively as a global anomalous symmetry that is only broken by non-perturbative effects. In addition to the closed-string axions discussed above, further axions may arise from the open string sector in Type IIA string theory More explicitly, these open string axion correspond to the phases of complex scalar fields charged under the anomalous $U(1)$ symmetry. At the string scale, the bosonic part of the effective Lagangian for an open string axion takes the following form

$$\mathcal{L}_{axion-U(1)} = |(\partial_\mu + iqB_\mu)\Sigma|^2 + \frac{1}{2}\left(\partial_\mu \xi + M_s B_\mu\right)^2 \tag{7.21}$$

where Σ denotes the complex scalar field with charged under the anomalous $U(1)$ symmetry with charge q, B_μ is the $U(1)$ gauge potential, while ξ represents the closed string axion eaten by B_μ in the Stückelberg mechanism. Thus, the open string axion σ arises as the phase of the complex field Σ

$$\Sigma = \frac{v + \tilde{\phi}(x)}{\sqrt{2}} e^{i\frac{\sigma}{v}} \tag{7.22}$$

where $\tilde{\varepsilon}(x)$ denotes the fluctuations of the open string saxion around its vacuum expectation value v. After inserting the expression for back into the Lagrangian, one obtains the following action by keeping only track of the CP-odd scalars

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2 + \left(qv\partial^\mu \sigma + M_s \partial^\mu \xi\right) B_\mu + \frac{1}{2}(q^2v^2 + M_s^2)B_\mu B^\mu \tag{7.23}$$

The above effective lagrangian can be brought into its standard form by a $SO(2)$ transformation of the fields ξ and σ

$$\chi = \frac{M_s \xi + qv\sigma}{\sqrt{M_s^2 + q^2v^2}}, \qquad \psi = \frac{M_s \sigma - qv\xi}{\sqrt{M_s^2 + q^2v^2}}, \tag{7.24}$$

Notice that the linear combination χ is the one eaten by the $U(1)$ gauge boson to get massive

$$m_B^2 = M_s^2 + q^2v^2 \tag{7.25}$$

while the orthogonal combination ψ remains massless. Similarly to the closed string case, open string axions also provide for an axion coupling to the topological QCD charge density (7.8), where the open string axion decay constant $f_\sigma = qv$. Performing the $SO(2)$ rotation also in this part of the Lagrangian yields the axion-gluon couplings in the (χ, ϕ)-basis one obtains

$$f_\chi = \frac{1}{2}\sqrt{M_s^2 + q^2v^2}, \qquad f_\psi = \frac{M_s qv\sqrt{M_s^2 + q^2v^2}}{M_s^2 + q^2v^2} \tag{7.26}$$

For models where the Stückelberg mass is much heavier than the scale at which Σ acquires a VEV, i.e. $M_s \gg v$, the axion χ eaten by the gauge boson consists primarily of the closed string axion χ. The orthogonal massless state ψ on the other hand will then be mostly composed of the open string axion σ. Notice that this will also be reflected in the decay constants of the respective axions: $f_\chi \sim f_\xi = M_s$ and $f_\psi \sim f_\sigma = qv$. Hence, in this configuration the string scale M_s can be much higher than 10^{12}GeV, as the presence of an open string axion provides another candidate for the QCD axion.

7.4 Generalized Green-Schwarz Mechanism

Before constructing a explicit model, we would like to discuss about the consistency of the four-dimensional effective theories emerging from D-brane constructions. An unquestionably consistency requirement is the cancellation of anomalies. On the one hand, the absence of non-Abelian gauge anomalies is guaranteed upon cancellation of the RR tadpoles in the underlying string theory. On the other hand, Abelian, mixed Abelian-non-Abelian and mixed Abelian-gravitational anomalies are not necesarly canceled upon cancellation of the RR tadpoles. However, string theory provides an additional mechanism to cancel such anomalies. This is the so-called Green-Schwarz (GS) mechanism [93] which can be generalized to intersecting D-brane models [92]. Let us start discussing the mixed Abelian-non-Abelian anomalies in more detail. The mechanism is based on the fact that the RR-charges of the D6-branes allow couplings between the RR-forms and the gauge fields living on the worlvolume of the D6-branes, captured by the Chern-Simons action. More precisely, for any stack of $D6_\alpha$-branes there exist Chern-Simons couplings of the form

$$\int_{\mathbb{R}^{1,3}\times\Pi_\alpha} C_3\wedge \mathrm{Tr}\left(F_\alpha\wedge F_\alpha\right), \qquad \int_{\mathbb{R}^{1,3}\times\Pi_\alpha} C_5\wedge \mathrm{Tr}\left(F_\alpha\right), \tag{7.27}$$

Following the philosophy of [92] we introduce a basis of even $\left([\alpha_K],[\beta^\Lambda]\right)$ and odd $\left([\beta^K],[\alpha_\Lambda]\right)$ homological three-cycles on the internal space such that

$$[\alpha_I]\cdot[\beta^K]=\delta_I^K, \qquad [\alpha_\Lambda]\cdot[\beta^\Sigma]=\delta_\Lambda^\Sigma \tag{7.28}$$

Now we expand every three-cycle Π_α in this basis as follows

$$\Pi_\alpha=p_\alpha^K[\alpha_K]+r_{\alpha,\Lambda}[\beta^\Lambda], \qquad \Pi_\alpha=p_\alpha^\Lambda[\alpha_\Lambda]+r_{\alpha,K}[\beta^K] \tag{7.29}$$

This basis allows us to define the four-dimensional fields Φ and their Hodge duals two-forms B

$$\begin{aligned}\tilde{\Phi}_K &= \int_{[\alpha_K]} C_3, \qquad B_2^K=\int_{[\beta^K]} C_5 \\ \tilde{\Phi}^\Lambda &= \int_{[\beta^\Lambda]} C_3, \qquad B_{2,\Lambda}=\int_{[\alpha_\Lambda]} C_5\end{aligned} \tag{7.30}$$

Thus, the generic couplings (7.27) can be dimensionally reduced to four dimensions and yield couplings of the form

$$\int_{\mathbb{R}^{1,3}}\left(p_\alpha^K\tilde{\Phi}_K+r_{\alpha\Lambda}\tilde{\Phi}^\Lambda\right)\mathrm{Tr}\left(F_\alpha\wedge F_\alpha\right), \qquad N_\alpha\int_{\mathbb{R}^{1,3}}\left(p_\alpha^\Lambda B_{2,\Lambda}+r_{\alpha K}B_2^K\right)\wedge\mathrm{Tr}\left(F_\alpha\right) \tag{7.31}$$

where the prefactor N_α comes from the normalization of the $U(1)$ generator and summation over the indices K and Λ is understood. Then, we can combine both couplings into a Green-Schwarz diagram yielding the proper contribution to cancel the mixed anomaly coming from triangular diagrams. In this way, one can show that the coefficient of this amplitude is given by

$$\mathcal{A}_{\alpha\beta\beta} = \frac{N_\alpha}{2}\left(I_{\alpha\beta} + I_{\alpha\beta'}\right) \tag{7.32}$$

which has the precisely form that cancels the field theoretic anomaly. As mentioned in the previous section, the couplings of the form $B_2 \wedge F_\alpha$ also induce a mass proportional to the string scale for the anomalous $U(1)_\alpha$ gauge field. In that case, the $U(1)$ disappears as gauge symmetry of the effective theory, but survive perturbatively as a global anomalous symmetry which is only broken by non-perturbative effects, like for instance D-brane instantons couplings to the RR axions. Therefore, such perturbative global symmetries are very suitable to serve as Peccei-Quinn symmetries, as we will show through an explicit example in the next section. A relevant observation at this point is that not only anomalous $U(1)$s but also some non-anomalous may have $B_2 \wedge F_\alpha$ couplings and therefore get massive through the Stückelberg mechanism. While any massless combination $U(1)_X = \sum_\alpha c_\alpha U(1)$ must be orthogonal to those that acquire a GS mass, indicating that they are non-anomalous and is determined by the condition

$$\sum_\alpha N_\alpha (Q_{\alpha K} - Q_{\alpha' K} + Q_\alpha^\Lambda - Q_{\alpha'}^\Lambda) c_\alpha = 0, \qquad \text{for all } K \text{ and } \Lambda \tag{7.33}$$

where $Q_{\alpha K} = [\Pi_\alpha] \cdot [\alpha_K]$ and $Q_\alpha^\Lambda = [\Pi_\alpha] \cdot \left[\beta^\Lambda\right]$. Such $U(1)$ factors remain as gauge symmetries of the low energy theory.

7.5 The $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O}$ orientifold

As already mentioned at the beginning of the chapter, our aim is to implement a consistent supersymmetric DFSZ model in type IIA orientifolds with background fluxes and intersecting D6-branes supporting chiral and gauge sectors. The easiest way to build it, is considering toroidal orientifolds, of which the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O})$ is the simplest.

In this section we briefly review the basics of type IIA compactified on the orientifold $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O}$ with intersecting D6-branes preserving $\mathcal{N} = 1$ supersymmetry in four dimensions. We will skip details related to the closed string moduli sector resulting from this class of backgrounds, which can be computed using standard techniques, for more details see appendix D. Focusing us mostly on the (charged) open string sector. To fully appreciate the phenomenological aspects of intersecting D6-brane models on this toroidal orientifold, a proper understanding of the background geometry is required. Therefore, our starting point is a brief summary of the essential geometric aspects of the orbifold $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, which is the singular limit of a CY three-fold. Considering a factorisable six-torus $\mathbf{T}^6 = \mathbf{T}^2_{(1)} \times \mathbf{T}^2_{(2)} \times \mathbf{T}^2_{(3)}$, the action of the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is described by a rotation of the complex coordinate $z^i = x^i + i\tilde{u}^i y^i$ parameterizing each two-torus $\mathbf{T}^2_{(i)}$ with $\tilde{u}^i = \frac{R^i_y}{R^i_x} e^{i\vartheta}$ being the corresponding complex structure parameter[1]

$$\begin{aligned} \Theta_1 : z^k &\to e^{2\pi i v_k} z^k, \qquad & \vec{v} &= (\frac{1}{2}, -\frac{1}{2}, 0) \\ \Theta_2 : z^k &\to e^{2\pi i w_k} z^k, \qquad & \vec{w} &= (0, \frac{1}{2}, -\frac{1}{2}) \end{aligned} \tag{7.34}$$

where Θ_1 generates the $\mathbb{Z}_2$ part of the orbifold group acting on $\mathbf{T}^2_{(1)} \times \mathbf{T}^2_{(2)}$, while Θ_2 generates the $\mathbb{Z}_2$ part acting on $\mathbf{T}^2_{(2)} \times \mathbf{T}^2_{(3)}$. The action of the orbifold group constraints the shape of the $\mathbb{Z}_2$ invariant two-tori to be either rectangular or tilted, as shown in figure 7.1, which are defined by the periodicities $z^i \sim z^i + R^i_x$, $z^i \sim z^i + iR^i_y$ and $z^i \sim z^i + R^i_x + \frac{i}{2}R^i_y$, $z^i \sim z^i + iR^i_y$ respectively [94, 95]. For simplicity, here we will consider only rectangular two-tori, but most results can be generalized straightforwardly to tilted two-tori. Besides the orbifold action (7.34) one must specify the choice of discrete torsion which relates both $\mathbb{Z}_2$ group generators. As argued in [96], there are two inequivalent choices, whose twisted cohomologies are either $(h^{(1,1)}_{\text{tw}}, h^{(2,1)}_{\text{tw}}) = (48, 0)$ or $(h^{(1,1)}_{\text{tw}}, h^{(2,1)}_{\text{tw}}) = (0, 48)$ [2]. For concreteness, we will consider the second case, dubbed $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion. Such backgrounds have Hodge numbers $(h^{(1,1)}, h^{(2,1)}) = (3, 51)$, which means that the resulting orbifold is the singular limit of a CY three-fold with Euler characteristic $\chi = 2(h^{(1,1)} - h^{(2,1)}) = -96$.

On the other hand, the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ has to be extended by the orientifold projection $\mathcal{O}$ defined in section 2.4, for this particular background the anti-holomorphic involution $\mathcal{R}$ acts on the complex coordinates as

$$\mathcal{R}(z^k) = \bar{z}^k \tag{7.35}$$

The gauging of $\mathcal{O}$ creates fixed O6-planes and the location of those planes is given by sets of points fixed under the elements of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O}$. More precisely, there are four kind of O6-planes associated to the actions of $\mathcal{O}$, $\mathcal{O}\Theta_1$,$\mathcal{O}\Theta_2$ and $\mathcal{O}\Theta_1\Theta_2$. Since all these O6-planes have compact transverse directions and carry RR-charges, for consistency one needs to include a certain amount of D6-branes canceling exactly those charges. Specifically, we introduce stacks of N_α D6-branes wrapping a factorisable three-cycle $[\Pi_\alpha]$ parametrized by the wrapping numbers

[1] In the rectangular case ($\vartheta = 0$), the complex coordinates reduce to $z^i = x^i + i\tau_i y^i$ with $\tau_i = \frac{R^i_y}{R^i_x}$.

[2] Cohomology groups can be split into untwisted and twisted contributions $H^{(p,q)} = H^{(p,q)}_{\text{untw}} + H^{(p,q)}_{\text{tw}}$. The untwisted Hodge number $h^{p,q}_{\text{untw}}$ is given by the number of (p,q)-forms invariant under the orbifold group, while the twisted Hodge number $h^{(p,q)}_{\text{tw}}$ recives contributions from all isolated singularities in the orbifold, see [96] for more details.

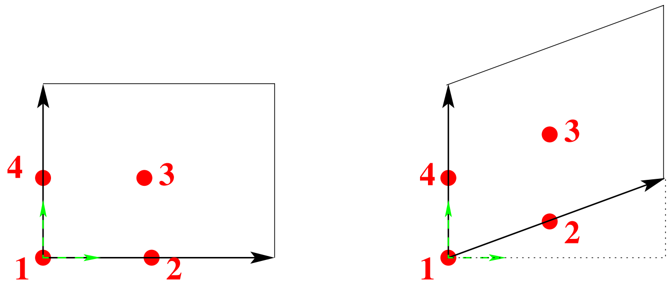

Figure 7.1: *The $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{R}$ invariant two-tori: (left) rectangular two torus, (right) tilted two-torus, which are parameterised by $b = 0, \frac{1}{2}$, respectively.*

(n^i_α, m^i_α), which encodes the one-cycle geometry on each two-torus $\mathbf{T}^2_{(i)}$.

$$[\Pi_\alpha] = \bigotimes_{i=1}^{3} \left(n^i_\alpha[\pi^i] + m^i_\alpha[\pi^{i+1}]\right) \tag{7.36}$$

Where $[\pi^i]$, $[\pi^{i+1}]$ are the homology classes of the fundamental one-cycles of $\mathbf{T}^2_{(i)}$, defined in appendix D. Since the orientifold projection acts non-trivially on the wrapping numbers as $(n^i_\alpha, m^i_\alpha) \to (n^i_\alpha, -m^i_\alpha)$. Then the RR tadpole cancellation condition (2.68) explicitly reads as in (D.9).

Moreover, we are interested in supersymmetric models, the three-cycle wrapped by the D6-brane must be a SLag three-cycle (2.65). In this background, Ω and J are given by (D.2) and (D.4) respectively, then one can easily check that any factorisable D6-brane satisfies the constraint $J|_{\Pi_\alpha} = 0$, while the second condition in (2.65) is satisfied if

$$\sum_{i=1}^{3} \vartheta^i_\alpha = 0 \mod 2\pi \tag{7.37}$$

where $\vartheta^i_\alpha = Arctan\left(\frac{m^i_\alpha}{n^i_\alpha}\tau_i\right)$ is the angle between the D6-brane and the x-axis on $\mathbf{T}^2_{(i)}$.

The open string spectrum

Let us now proceed to analyse the open string spectrum resulting from this kind of constructions. To do so, it will be useful to distinguish different matter sectors based on the origin of the charged open string fields. On the one hand, fields arising from strings with endpoints in the same stack of D6-branes N_α, are said to be in the $\alpha\alpha$-sector. This sector is invariant under Θ_1 and

Θ_2, and is exchanged with the $\alpha'\alpha'$-sector by the action of $\Omega_p\mathcal{R}$. On the other hand, the Θ_1 projection the gauge group breaks $U(N_\alpha)$ to $U\left(N_\alpha/2\right) \times U\left(N_\alpha/2\right)$, while Θ_2 identifies both factors, leaving $U\left(N_\alpha/2\right)$. This sector contains $U\left(N_\alpha/2\right)$ gauge bosons, plus three $\mathcal{N}=1$ chiral supermultiplets Φ^i_α transforming in the adjoint representation of the gauge group. In the case of D6-branes parallel to some O6-plane, the resulting gauge group is $USp(N_\alpha)$ with three $\mathcal{N}=1$ chiral supermultiplets transforming in the two-index antisymmetric representation [97].

On the other hand, matter fields arising from strings streched between two distinct stacks of D6-branes N_α and N_β, are said to be in the $\alpha\beta$-sector, which is invariant under the orbifold action, and is mapped to the $\alpha'\beta'$-sector by the action of $\Omega_p\mathcal{R}$. The matter content in this sector is given by $I_{\alpha\beta}$ chiral fermions in the bifundamental representation $\left(N_\alpha/2, \bar{N}_\beta/2\right)$, where the intersection number of the wrapped three-cycles $I_{\alpha\beta}$ is defined as

$$I_{\alpha\beta} = [\Pi_\alpha] \cdot [\Pi_\beta] = \prod_{i=1}^{3} \left(n^i_\alpha m^i_\beta - m^i_\alpha n^i_\beta\right) \tag{7.38}$$

The sign of $I_{\alpha\beta}$ determines the chirality of the corresponding fermion[3]. For supersymmetric intersections, further massless scalar fields fill out the corresponding supermultiplet. Moreover, we obtain a similar results in the $\alpha\beta'$ sector, which contains $I_{\alpha\beta'}$ fermions in the representation $\left(N_\alpha/2, N_\beta/2\right)$, plus additional massless scalar fields filling out the corresponding supermultiplet.

Finally, matter fields arising from strings streched between the stacks of D6-branes related by the orientifold action, are said to be in the $\alpha\alpha'$ sector. This sector contains $n_{\mathrm{sym,a}}$ chiral fermions in the two-index symmetric representation and $n_{\mathrm{asym,a}}$ chiral fermions in the two-index antisymmetric representation, with

$$n_{\mathrm{sym,a}} = \frac{1}{2}\left(I_{\alpha\alpha'} - I_{\alpha,O6}\right), \qquad n_{\mathrm{asym,a}} = \frac{1}{2}\left(I_{\alpha\alpha'} + I_{\alpha,O6}\right) \tag{7.39}$$

where $I_{\alpha,O6} = [\Pi_\alpha]\cdot[\Pi_{O6}]$ and $[\Pi_{O6}]$ denotes the complete set of O6-planes $[\Pi_{O6}] = [\Pi_{\mathcal{O}}] + [\Pi_{\mathcal{O}\Theta_1}] + [\Pi_{\mathcal{O}\Theta_2}] + [\Pi_{\mathcal{O}\Theta_1\Theta_2}]$.

As expected, the cancellation of the RR tadpoles (D.9) guarantees the absence of the non-Abelian anomalies for the chiral spectrum discussed above. However, mixed Abelian-non-Abelian anomalies do not cancel automatically when RR tadpoles conditions are satisfied. These anomalies are canceled by the GS mechanism discussed in section 7.4. Using the basis of homology three-cycles in (D.7), we can write the axionic couplings (7.31) explicitly as

$$p^0_\alpha \int_{\mathbb{R}^{1,3}} \tilde{\Phi}_0 \operatorname{Tr}\left(F_\alpha \wedge F_\alpha\right), \qquad r_{\alpha,i} \int_{\mathbb{R}^{1,3}} \tilde{\Phi}^i \operatorname{Tr}\left(F_\alpha \wedge F_\alpha\right) \tag{7.40}$$

[3]In our conventions, D6-branes with positive intersection number will give rise to left-handed fermions, while negative intersection numbers give rise to right-handed fermions.

where $p^0_\alpha = n^1_\alpha n^2_\alpha n^3_\alpha$ and $r_{\alpha,i} = n^i_\alpha m^j_\alpha m^k_\alpha$ with $i \neq j \neq k$. While the couplings of the untwisted RR two-forms to the $U(1)$ field strength F_α of each stack α are given by

$$N_\alpha r_{\alpha,0} \int_{\mathbb{R}^{1,3}} B^0_2 \wedge \mathrm{Tr} F_\alpha, \qquad N_\alpha p^i_\alpha \int_{\mathbb{R}^{1,3}} B_{2,i} \wedge \mathrm{Tr} F_\alpha \tag{7.41}$$

where $r_{\alpha,0} = -m^1_\alpha m^2_\alpha m^3_\alpha$ and $p^i_\alpha = m^i_\alpha n^j_\alpha n^k_\alpha$ with $i \neq j \neq k$. Using the couplings (7.40) we can determine the closed string axions that couple anomalously to the gauge-invariant field strength, while the couplings (7.41) allow to determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the GS mechanism. Finally, the condition for a combination of $U(1)$'s to remain massless can be written explicitly as

$$\sum_\alpha c_\alpha N_\alpha m^1_\alpha m^2_\alpha m^3_\alpha = \sum_\alpha c_\alpha N_\alpha m^1_\alpha n^2_\alpha n^3_\alpha = \sum_\alpha c_\alpha N_\alpha n^1_\alpha m^2_\alpha n^3_\alpha = \sum_\alpha c_\alpha N_\alpha n^1_\alpha n^2_\alpha m^3_\alpha = 0 \tag{7.42}$$

7.6 An explicit D6-brane model

In this section, we turn to the construction of a consistent supersymmetric DFSZ axion model, by using the machinery of intersecting D6-branes on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O}$ orientifold, discussed in the previous section. Before proceeding to the explicit construction of the model, it is important to emphasize some key ingredients. We have seen that the $SU(2)_L \times U(1)_Y$ singlet fields $\Sigma, \tilde{\Sigma}$ have PQ charges that are twice the charges of the Higgs-doublets H_u and H_d respectively. In intersecting D6-brane models, such states can be realized as antisymmetric representations of $U(2)$ or symmetric representations of $U(1)$ [98]. Therefore, we need to introduce a stack of D6-branes giving rise to such states. In particular, we will consider a model, in which those fields are realized as antisymmetric states under $U(2)_b$. On the other hand, to obtain a consistent supersymmetric model, a superpotential of the form (7.16) has to be included. As we argue below, the cubic coupling in this superpotential can be implemented as the Yukawa coupling among two $\mathcal{N} = 1$ chiral multiplets (Higgs doublets) and one $\mathcal{N} = 2$ multiplet (SM singlet Σ), whereas the realization of the last term in (7.16) is more involved, as we argue below, it might be generated by D6-branes wrapped on a non-factorisable three-cycle.

With all these details in mind, let us start the search for a set of D6-branes realizing the spectrum shown in table 7.1 as well as the superpotentials (7.12) and (7.16) . As shown in [99], one needs to choose some two-tori to be tilted in order to get MSSM-like models with an odd number of generations of quarks and leptons. However, if we relax the condition of three generations of quarks and leptons, one can easily find a configuration of intersecting D6-branes yielding a massless open string spectrum close to the one displayed in table 7.2. Concretaly, we will consider a set of D6-branes consisting on six stacks, the relevant geometric data of such a configuration is summarized in table 7.2. For simplicity, we have used stacks of D6-branes passing through some $\mathbb{Z}_2 \times \mathbb{Z}_2$ fixed point, such that they coincide with their orbifold images, but not with their orientifold images. The first four stacks of $D6_\alpha$-branes are characterized by being factorisable and reproduce the chiral part of the spectrum. This part of the spectrum is determined by computing the intersection numbers between distinct stacks of D6-branes and applying the results of the previous section. In addition to this set of factorisable D6-branes, we also add a stack of N_e D6-branes wrapping a non-factorisable three-cycle

$$\Pi_e = \pi_1 \otimes \left(n_e^2 \pi_3 + m_e^3 \pi_6\right) \otimes \left(n_e^3 \pi_5 + m_e^2 \pi_4\right) \tag{7.43}$$

which provides the source for a bilinear superpotential of the form (2.77).

This set of D6-branes by itself does not satisfy the RR tadpole cancellation conditions (D.9). Hence, additional hidden D6-branes and fluxes are required to satisfy the cancellation of the RR tadpoles. Since here we are only interested in the construction of a DFSZ model, to study the pattern of the soft supersymmetry breaking terms in this model, we will not deal here with these global issues of the compactification, leaving it for future work.

The set of factorisable D6-branes preserves supersymmetry if and only if $\tau_1 = \tau_2 = \tau_3 = 1$, this condition implies on the other hand that the complex structure moduli $u_{\star 1} = u_{\star 2} = u_{\star 3} = u_\star$. Notice that the additional D6-brane required for tadpole cancellation should be compatible with this condition, otherwise supersymmetry would be broken in this sector. Moreover, one can easily check that the non-factorisable three-cycle Π_e is Lagrangian if and only if $n_e^2 m_e^2 T^2 - n_e^3 m_e^3 T^3 = 0$, while the condition $\operatorname{Im}\Omega|_{\Pi_e} = 0$ is automatically satisfied. Hence, for the non-factorisable D6-branes we expect a F-term potential encoded in the superpotential (2.77), whenever the lagrangian condition is not satisfied. In order to compute this superpotential, one has to determine the geometric quantities (B.12), to do so we first identify the harmonic one-form $l_s^{-1}\zeta_1 = dx^1$ compatible with the D-brane normal deformation $X = \frac{1}{2} l_s \partial_{y^1}$ parallel to π_2. Then we use the basis of $\Omega\mathcal{R}$-odd harmonic two-forms defined in appendix D into the equation (B.12), to obtain

$$n_{21}^e = \frac{1}{l_s^3}\int_{\Pi_e} \omega_2 \wedge \zeta_1 = n_e^2 m_e^2 = 1, \qquad n_{31}^e = \frac{1}{l_s^3}\int_{\Pi_e} \omega_3 \wedge \zeta_1 = -n_e^3 m_e^3 = -1 \tag{7.44}$$

Plugging this result back into (2.77) and assuming vanishing worldvolume fluxes, one has

$$W_{\mathrm{D6}} - W_{\mathrm{D6}}^0 = -\Phi_e^1 \left(n_{21}^e T^2 + n_{31}^e T^3\right) = -l_s^{-1} \Phi_e^1 \left(T^2 - T^3\right) \tag{7.45}$$

Thus, the relevant part of the spectrum consists of two generations of quarks and leptons as well as one generation of right-handed neutrinos. In addition, it

D6-brane configuration on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O})$		
Stack	$(n^1_\alpha, m^1_\alpha) \times (n^2_\alpha, m^2_\alpha) \times (n^3_\alpha, m^3_\alpha)$	Angles
$N_a = 6$	$(1,1) \times (1,1) \times (0,-1)$	$\left(\frac{\pi}{4}, \frac{\pi}{4}, -\frac{\pi}{2}\right)$
$N_b = 4$	$(1,0) \times (1,1) \times (1,-1)$	$\left(0, \frac{\pi}{4}, -\frac{\pi}{4}\right)$
$N_c = 2$	$(0,1) \times (0,-1) \times (1,0)$	$\left(\frac{\pi}{2}, -\frac{\pi}{2}, 0\right)$
$N_d = 2$	$(1,-1) \times (1,-1) \times (0,1)$	$\left(-\frac{\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{2}\right)$
$N_e = 2$	$(1,0) \times (1,1)_{36} \times (1,1)_{54}$	

Table 7.2: Set of D6-branes realizing a consistent supersymmetric DFSZ model on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O})$ orientifold.

also contains two Higgs multiplets, a non-chiral pair $\Sigma + \tilde{\Sigma}$ in the antisymmetric representation of $U(2)_b$ and the open string modulus Φ^1_e which can be identified with the PQ neutral field in (7.16). This spectrum is summarized in table (7.3), additional adjoint fields are also present but they are not listed, since they do not play an important role in the discussion.

This set of D6-branes gives rise to the gauge group $U(3)_a \times U(2)_b \times USp(2)_c \times U(1)_d$, where the factor $USp(2)$ arises from the stack N_c sitting on top of the $\mathcal{O}\Theta_1\Theta_2$-invariant plane. In this particular model, there are four $U(1)$ symmetries, with charges Q_a, Q_b, Q_c and Q_d and some of them can be interpreted in terms of common SM global symmetries. More concretely, $Q_a = 3B$ and $Q_d = -L$ where B and L are the baryon and lepton numbers, while Q_c is twice the third component of the right handed weak isospin familiar from left-right symmetric models. Finally, Q_b has the features of a Peccei-Quinn symmetry, as we argue below. Let us now analyse the structure of the $U(1)$ anomally cancellation in this model. It is easy to check that there are two anomaly free $U(1)$s given by $\frac{1}{3}U(1)_a + U(1)_d$ and $U(1)_c$ and two anomalous ones, which are given by $3U(1)_a - U(1)_d$ and $U(1)_b$. Using the equation (7.41) one can see that the untwisted RR two-forms couple to the $U(1)$s in the model as follows:

$$\begin{aligned} & B_2^0 \wedge (3\mathrm{Tr}\, F_a - \mathrm{Tr}\, F_d) \\ & B_{2,2} \wedge \mathrm{Tr}\, 2F_b \\ & B_{2,3} \wedge (-3\mathrm{Tr}\, F_a - 2\mathrm{Tr}\, F_b + \mathrm{Tr}\, F_d) \end{aligned} \tag{7.46}$$

while $B_{2,1}$ has no coupling to any $U(1)$ field strength. The dual RR axions have couplings

$$\begin{aligned}
&\tilde{\Phi}_0 \mathrm{Tr}\,(F_b \wedge F_b) \\
&\tilde{\Phi}^1 \left(-\mathrm{Tr}\,(F_a \wedge F_a) - \mathrm{Tr}\,(F_b \wedge F_b) - \mathrm{Tr}\,(F_d \wedge F_d)\right) \\
&\tilde{\Phi}^2 \left(-\mathrm{Tr}\,(F_a \wedge F_a) - \mathrm{Tr}\,(F_d \wedge F_d)\right) \\
&\tilde{\Phi}^3 \left(-\mathrm{Tr}\,(F_a \wedge F_a) - \mathrm{Tr}\,(F_d \wedge F_d)\right)
\end{aligned} \tag{7.47}$$

Following the discussion in section 7.4, one immediately sees that the combination of the couplings for the untwisted RR two-forms B_2^0, $B_{2,2}$ and their duals leads to the cancellation of the mixed anomalies for $3U(1)_a - U(1)_d$ and $U(1)_b$ respectively. Whereas the RR two-form $B_{2,3}$ couples to a non-anomalous linear combination of $U(1)$s, rendering the corresponding gauge boson massive. Thus, the original gauge group breaks down to $SU(3)_a \times SU(2)_b \times USp(2)_c \times U_{B-L} \times U(1)^3_{\mathrm{massive}}$. Giving a VEV to some of the complex scalars encoding the D6-brane position and Wilson line yields a breaking $USp(2)_c \to U(1)_c$. In this way one is left with $SU(3)_a \times SU(2)_b \times U(1)_Y \times U_{B-L} \times U(1)^3_{\mathrm{massive}}$, where the hypercharge is defined as the massless combination of $U(1)$s:

$$Q_Y = \frac{1}{6}Q_a + \frac{1}{2}Q_c + \frac{1}{2}Q_d \tag{7.48}$$

Finally, since the $U(1)_b \subset U(2)_b$ has a mixed $SU(3)_a$-anomaly, hence it is identified with a Peccei-Quinn symmetry $U(1)_{\mathrm{PQ}}$. Thus, the SM singlets Σ and $\tilde{\Sigma}$ realized as antisymmetric states under $U(2)$ carry PQ charges 2 and -2 respectively.

The Yukawa couplings $Y_{\alpha\beta\gamma}$ among Higgs fields and two fermions are exponentially suppressed $Y_{\alpha\beta\gamma} \sim e^{A_{\alpha\beta\gamma}}$, with $A_{\alpha\beta\gamma}$ being the area of the string worldsheet spanning the triangle with vertices at the intersections and sides on the D6-branes, measured in string units[4]. Therefore, these couplings depend both on the Kähler and open string moduli, as argued in the previous chapter. These cubic coupling can be computed explicitly in this toroidal setup, by using the techniques developed in [59, 60]. For the massless spectrum displayed in table 7.3, the couplings allowed in the superpotential are

$$\mathcal{W} = \mathcal{W}_{\mathrm{Yuk}} + \mathcal{W}_{\mathrm{DFSZ}} \tag{7.49}$$

with

$$\begin{aligned}
\mathcal{W}_{\mathrm{Yuk}} &= Y_u\, Q_L \cdot H_u U_R + Y_d\, Q_L \cdot H_d D_R + Y_e\, L \cdot H_d E_R + Y_\nu\, L \cdot H_u N_R \\
\mathcal{W}_{\mathrm{DFSZ}} &= \lambda \Phi_e^1 \Sigma \tilde{\Sigma} - l_s^{-1} \Phi_e^1 \left(T^2 - T^3\right) + \kappa\, \Sigma H_u \cdot H_d
\end{aligned} \tag{7.50}$$

[4]Since different families are located at different intersections, the corresponding triangles have different areas that increase linearly with the family index, yielding thus an exponential hierarchy of the Yukawa couplings.

Massless open string spectrum.				
Sector	Field	$SU(3)_a \times SU(2)_b$	$U(1)_Y \times U(1)_{B-L}$	$U(1)_{PQ}$
ab'	Q_L	$2\times\ (\mathbf{3},\mathbf{2})_{(1,1,0,0)}$	$(\frac{1}{6},\frac{1}{3})$	1
ac'	U_R	$(\overline{\mathbf{3}},\mathbf{1})_{(-1,0,-1,0)}$	$(-\frac{2}{3},-\frac{1}{3})$	0
ac	D_R	$(\overline{\mathbf{3}},\mathbf{1})_{(-1,0,1,0)}$	$(\frac{1}{3},-\frac{1}{3})$	0
bd	L	$2\times\ (\mathbf{1},\mathbf{2})_{(0,1,0,-1)}$	$(-\frac{1}{2},-1)$	1
cd'	E_R	$(\mathbf{1},\mathbf{1})_{(0,0,1,1)}$	$(1,1)$	0
cd	N_R	$(\mathbf{1},\mathbf{1})_{(0,0,-1,1)}$	$(0,1)$	0
bc	H_u	$(\mathbf{1},\overline{\mathbf{2}})_{(0,-1,1,0)}$	$(\frac{1}{2},0)$	-1
bc'	H_d	$(\mathbf{1},\overline{\mathbf{2}})_{(0,-1-1,0)}$	$(-\frac{1}{2},0)$	-1
bb'	$\Sigma+\tilde{\Sigma}$	$(\mathbf{1},\mathbf{1_{Anti}})_{(0,\pm 2,0,0)}$	$(0,0)$	± 2
ee	Φ_e^1	$(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	(0,0)	0

Table 7.3: Summary of the massless open string states for the D6-brane configuration in table 7.2 and their charges under the extra symmetries $U(1)_{a,b,c,d}$.

Finally, lepton-number violating interactions like $Q_L \cdot LD_R$ and $L \cdot LE_R$ are forbidden perturbatively by $U(1)_{PQ}$ charge conservation, while the baryon number-violating interaction $U_R D_R D_R$ and the lepton-violating interaction $H_u \cdot L$ are perturbatively allowed. As a final remark, we would like to point out that once the linear combination $T^2 - T^3$ gets a non-vanishing VEV, the bilinear coupling $l_s^{-1}\Phi_e^1 (T^2 - T^3)$ in (7.50) behaves effectively as a term $\frac{1}{4}\lambda f^2 \Phi_e^1$ with $\frac{1}{4}\lambda f^2 = l_s^{-1}\langle T^2 - T^3\rangle$, yielding thus the last term of 7.16.

7.6.1 Soft-terms in the Higgs-axion sector

As a last topic, we study the structure of the soft supersymmetry breaking terms for the D6-brane model presented above. Here, we are mostly interested in the soft terms arising in the Higgs-axion sector of the model. Following the philosophy of chapter 6, a spontaneous breaking of supersymmetry due to NS and RR fluxes induce soft-terms for the matter fields living on D6-branes. For concreteness, we are going to embed the above set of D6-branes into the class of non-supersymmetric CSD vacua discussed in section 4.3, since it provides a suitable string vacuum to perform the explicit computation of the flux-induced soft terms. In this class of vacua the contra-variant F-terms of the dilaton an Kähler moduli vanish and supersymmetry is spontaneously broken in the complex structure sector.

As argued before, any prediction of the soft supersymmetry breaking terms requires a knowledge of the Kähler metric for the matter fields as well as the couplings appearing in the superpotential. With this purpose, let us first work out the Kähler metrics for matter fields relevant in our discussion. The Kähler metrics for matter fields on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{O}$ orientifold were obtained by computing string scattering amplitudes among open string matter fields on the D6-branes and bulk moduli fields [100–103] and collected in appendix D. The Higgs multiplets in table 7.3 are chiral $\mathcal{N} = 1$ supermultiplets supported at the codimension 6 intersection of the stacks N_b and N_c. In that case the matter field metric takes the generic form (D.13) and explicitly read

$$K_{H_u} = K_{H_d} = \frac{C_{bc}}{4\sqrt{2t^1t^2t^3}\,\sqrt[4]{s_\star u_\star^3}} \tag{7.51}$$

where the constant $C_{bc} = \Gamma(1/4)^2/\Gamma(3/4)^2$. Whereas the Standard Model singlets Σ and $\tilde{\Sigma}$ arise from stacks of D6-branes that coincides in the first two-torus (Codimension 5 intersection) and intersect at a point along the remaining two-tori. Therefore, they form a non-chiral pair of $\mathcal{N} = 1$ chiral supermultiplets whose Kähler metric takes the generic form (D.12) and read

$$K_\Sigma = K_{\tilde{\Sigma}} = \frac{1}{4u_\star\sqrt{t^2t^3}} \tag{7.52}$$

Since the open string modulus Φ resides in the non-factorisable stack N_e, its Kähler metric should be derived from the Kähler potential mixing open and closed string moduli (2.75), instead of using the usual expression of the Kähler metric for the $\mathcal{N} = 1$ supermultiplets supported at the codimension 3 intersections given in (D.10). In the toroidal case, the field redefinition (2.72) becomes

$$\begin{aligned} n^K &= n_\star^K - \frac{1}{4}\sum_\alpha \left(Q_\alpha^K\right)_{ij} \left[(t^a\eta_{\alpha a})^{-1}\right]^j_k \phi_\alpha^i \phi_\alpha^k \\ u_\Lambda &= n_{\star\Lambda} - \frac{1}{4}\sum_\alpha \left(Q_{\alpha\Lambda}\right)_{ij} \left[(t^a\eta_{\alpha a})^{-1}\right]^j_k \phi_\alpha^i \phi_\alpha^k \end{aligned}$$

where we have defined the quantities

$$\left(Q_\alpha^K\right)_{ij} = \frac{\partial g_{\alpha i}^K}{\partial \varphi^j} = l_s^{-3}\int_{\Pi_\alpha} \iota_{X^j}\beta^K \wedge \zeta^i, \qquad \left(Q_{\alpha\Lambda}\right)_{ij} = \frac{\partial g_{\alpha\Lambda i}}{\partial \varphi^j} = l_s^{-3}\int_{\Pi_\alpha} \iota_{X^j}\alpha_\Lambda \wedge \zeta^i \tag{7.53}$$

$$\left(\eta_{\alpha a}\right)^i_j = l_s^{-3}\int_{\Pi_\alpha} \iota_{X^j}\omega_a \wedge \rho^i \tag{7.54}$$

where ρ^i are defined as Poincare duals to the harmonic one-forms ζ^i on Π_α. To compute these quantities we use the D-brane normal deformation $X = \frac{1}{2}l_s\partial_{y^1}$

together with the basis of $\Omega\mathcal{R}$-odd harmonic three-forms given in (D.1) and we obtain

$$\left(Q_e^0\right)_{11} = -m_e^2 m_e^3 = -1, \qquad \left(Q_{e1}\right)_{11} = n_e^2 n_e^3 = 1 \tag{7.55}$$

while $\left(Q_{e\Lambda=2,3}\right)_{11} = 0$ vanish as the interior products of $\alpha_{\Lambda=2,3}$ with respect to such a deformation vanish. This implies that only the dilaton S and the complex structure modulus U_1 will be redefined by the open string moduli Φ_e^1. The next step is to compute the quantities $\left(\eta_{\alpha a}\right)_j^i$. A straightforward computation leads to

$$\left(\eta_{e1}\right)_1^1 = l_s^{-3} \int_{\Pi_\alpha} (-dx^1) \wedge \rho^1 = -1 \tag{7.56}$$

Using all these results, one can easily see that the open string modulus Φ_e^1 enters in the Kähler potential as follows

$$K^Q = -\mathrm{Log}\left(S - \bar{S} + \frac{1}{4}\frac{\left(\Phi_e^1 - \bar{\Phi}_e^1\right)}{T^1 - \bar{T}^1} + ...\right) - \mathrm{Log}\left(U_1 - \bar{U}_1 - \frac{1}{4}\frac{\left(\Phi_e^1 - \bar{\Phi}_e^1\right)}{T^1 - \bar{T}^1} + ...\right) + ... \tag{7.57}$$

where dots denote the dependence on the remaining open and closed string moduli. A power series expansion of the above Kähler potential in terms of the open string modulus Φ_e^1 leads to the following Kähler metric

$$K_{\Phi_e^1 \bar{\Phi}_e^1} = \frac{u_\star - s_\star}{4 s_\star u_\star t^1} \tag{7.58}$$

Taking a look at the expressions (7.51) and (7.52) we notice that they are homogeneous function of degree $-3/4$ and -1 in the complex structure moduli respectively. Plugging those values into the equation (6.31) we find that the Higgs multiplets have soft masses

$$m_{H_u}^2 = m_{H_d}^2 = \frac{1}{4} m_{3/2}^2 \tag{7.59}$$

with gravitino mass given by (6.20), while the SM singlets Σ and $\tilde{\Sigma}$ on the other hand do not acquire soft masses.

Since the Kähler metric for the QP neutral Φ is not a homogenous function of the complex structure moduli, its soft mass has to be computed obligatorily from the equation (6.28). In this way, after plugging the Kähler metric (7.58) into the equation (6.28) and evaluate properly the vacuum constrainsts, we obtain

$$m_{\Phi_e^1}^2 = m_{3/2}^2 \left(\frac{u_\star}{(s_\star - u_\star)^2}\right) \tag{7.60}$$

which is non-tachyonic but contains a non-universal contribution. In addition to the above soft masses, the cubic couplings $\Sigma H_u H_d$ in (7.50) will induce A-term of the form

$$A_{\Sigma H_u H_d} = \frac{1}{2} \hat{\kappa} m_{3/2}^2 \tag{7.61}$$

To arrive at this expression, we have used the modular weights $n_{H_u} = n_{H_d} = -3/4$ and $n_\Sigma = -1$ into the equation (6.34), while the physical coupling $\hat{\lambda} = i\kappa/\sqrt{K_\Sigma K_{H_u} K_{H_d}}$. Finally, the cubic coupling $\tilde{\Phi}\Sigma\tilde{\Sigma}$ yields a A-term of the form

$$A_{\Phi\Sigma\tilde{\Sigma}} = \hat{\lambda} m_{3/2}^2 \left(7 + \frac{s_\star}{s_\star - u_{\star 1}}\right) \tag{7.62}$$

Part IV

Conclusions and Appendices

Chapter 8

Conclusions

Let us conclude by summarising the main results of this thesis and by outlining the prospects for future work. This thesis has been devoted to offer a novel perspective on perturbative type IIA flux vacua with an underlying Calabi-Yau geometry. These four-dimensional vacua correspond to local minima of the four-dimensional scalar potential arising from the dimensional reduction of the ten-dimensional IIA supergravity action on Calabi-Yau orientifolds with background fluxes and D6-branes. Earlier studies of this scalar potential revealed its very simple structure consisting of a symmetric matrix depending solely on the geometric moduli and acting as a metric on the space of axion polynomials. These axion polynomials capture the axionic partners together with the flux quanta into shift-invariant combinations whose precise shapes are intimately connected to Freed-Witten anomaly cancelation. This bilinear structure of the scalar potential in terms of the axion polynomials even persists in the presence of D6-branes accompanied with displacements moduli, referred to as mobile D6-branes in this thesis, albeit with the proper addition of open string moduli and axions. Similarly, the shape of the open string axion polynomials can be related to the Hanany-Witten effect.

Moreover, in chapter 3 we shown that both the bilinear structure and the separate dependence into axions and saxions is maintained even in the presence of perturbative α'-corrections. More precisely, we have seen that certain α'-corrections modify the definition of the flux-axion polynomials ρ_A, in the sense that they redefine the basis of quantised fluxes. Others, namely the cubic correction $K^{(3)}$ that enters in the Kähler potential as in (2.80), only affect the expression for Z^{AB}. This constitutes a proof of concept that the bilinear form of the scalar potential is valid for a large set of flux vacua. It also supports the idea that the factorised dependence into axions and saxions should occur as long as it is a good approximation to assume that fluxes do not affect the 4d Kähler metrics of the light fields, or in other words that Z^{AB} is independent of the ρ_A.

In the second part of this thesis we apply this novel approch on perturbative type IIA flux vacua with (partly) stabilised moduli and their physical properties at the level of four-dimensional $\mathcal{N} = 1$ supergravity. At large volume the four-dimensional scalar potential can equally be obtained from the F-term scalar potential of an $\mathcal{N} = 1$ supergravity coupled to chiral multiplets consisting of Kähler moduli, complex structure moduli and open string moduli. The background fluxes yield a perturbative superpotential for the closed and open string moduli, such that its form can be expressed as a linear function of the axion polynomials with saxion-dependent coefficients. It is precisely the complete factorisation of the superpotential in terms of geometric moduli and axion polynomials that lies at the heart of our search for vacuum configurations of the four-dimensional $\mathcal{N} = 1$ supergravity. By solving the F-terms in terms of the axion polynomials we are able to recover the $\mathcal{N} = 1$ supersymmetric AdS vacua and the $\mathcal{N} = 0$ Minkowski vacua with ISD fluxes for purely closed string compactifications.

After adding mobile D6-branes to the compactification, the search for local minima of the scalar potential appears at first sight to be much more energy-consuming, as the mixing between closed and open string moduli provides for an extra level of complexity. However, the language of axion polynomials allows to treat these cases in the same way as the pure closed string vacua. More precisely, when generalising the ISD flux set-up by adding D6-branes one can still take advantage of the no-scale symmetry in the complex structure moduli sector to rewrite the scalar potential as a positive semidefinite function, under mild assumptions about the functional dependence of the Kähler potential on closed and open string moduli. This positive semidefinite scalar potential has a local $\mathcal{N} = 0$ Minkowski minimum, in which the F-terms for the dilaton, Kähler moduli and open string moduli satisfy relations that are weaker than the ISD case. Yet, to expose which sectors break supersymmetry spontaneously, it suffices to look at the contra-variant F-terms in the complex structure moduli sector, which are the only non-vanishing ones for these vacuum configurations and thereby earned them the name complex structure dominated (CSD) vacua. Alternatively, these CSD vacua can also be derived by exploiting the bilinear structure of the open-closed string scalar potential, in which case the vacuum conditions are formulated in terms of the axion polynomials. Once again, the elegant language of the axion polynomials allows to expose the equivalence between the F-term conditions and the axion polynomial vacuum conditions.

In chapter 5, we started the search for vacua in regions of the moduli space where the internal volume is only moderately large. Armed with the explicit expressions for the α'-corrected flux-axion polynomials and $\mathcal{Z}^{AB}$, we have written down the full scalar potential and analysed several of its vacua. We have first considered the class of Minkowski vacua studied in [8], and shown that in this case the potential can be written as a bilinear positive definite form (5.18), as expected from mirror symmetry. The vanishing of each of the entries of the vector (5.19) gives the vacuum conditions for this class of compactifications, and reproduces the results in [8]. Second, we have considered how α'-corrections modify the vacuum conditions of supersymmetric AdS flux vacua, following the same strategy as in [47] and rewriting the vanishing F-term conditions in terms of axion polynomials and solving for them. As in the case of Minkowski vacua, we have found that the cubic correction $K^{(3)}$ only affects the stabilisation of geometric of saxionic moduli, while the other two corrections also affect (implicitly) the stabilisation of axions.

Determining the on-shell F-terms is a necessary step to understand whether a four-dimensional vacuum preserves supersymmetry or not. To assess physically whether supersymmetry is spontaneously broken in the vacuum, it suffices to evaluate the (effective) gravitino mass on-shell. A simple method to do precisely that takes advantage of the off-shell expression for the gravitino mass, which exhibits a bilinear form in the axion polynomials, similarly to the scalar potential. In chapter 6 we show that this factorisation in terms of geometric moduli and axion polynomials can also be extended to the soft terms for massless open string excitations located at the intersections of two distinguishable D6-branes. These soft-terms, resulting from the background fluxes through gravity mediation, also take on a (bi)linear expression in terms of the axion polynomials. Hence, this implies that gravitino masses and soft terms are universal for flux vacua that are related through each other by the axion shift symmetries, which is displayed explicitly in terms of the axion polynomials. Here, we have extended the analysis for the soft terms to the CSD vacua, yet their on-shell values exhibit similar scalings with the gravitino mass as the well-studied ISD flux vacua. This similarity suggests a universal pattern for the soft terms in vacua with complex structure dominated supersymmetry breaking.

A proper look at the ISD flux vacua and the CSD vacua shows that only part of the moduli is stabilised. Upon inclusion of α'-corrections in the ISD flux vacua, flat directions associated with the geometric parts of the Kähler moduli are lifted. However, the complex structure moduli still remain flat directions in this class of vacua, due to the no-scale property in the complex structure moduli sector. Hence, additional stabilising effects have to be added to the compactification to obtain a stable vacuum configuration. One could take into account various non-perturbative contributions to the superpotential (and Kähler potential), such as worldsheet instantons and D-brane instantons, which would however manifestly break the bilinear description in terms of the axion polynomials. It would be illuminating to develop a formalism that combines the perturbative and non-perturbative contributions to the superpotential and allows for elegant methods to determine the vacua of the compactification, in a similar fashion as we explained here for the axion polynomial language.

Moreover, it would also be interesting to extend the results obtained in chapter 4 to include more general classes of type IIA flux vacua. On the one hand, one could consider flux compactifications on non-Calabi-Yau geometries [44,53,104–113]. On the other hand, one may consider compactification with more general open string sectors, like models containing coisotropic D8-branes [114–117]. In particular, it would be interesting to see if one can generalise the CSD vacua of section 4.3 to any of these cases, and then compute the corresponding spectrum of soft terms. Since we have addressed the latter from a 4d effective theory approach, it would be important to develop a microscopic picture of the generation of such soft terms, equivalent to the microscopic computations made in the context of type IIB/F-theory flux backgrounds [53, 67, 118–126]. One may then compare such soft terms with the results of table 6.2, and use this to either confirm or correct our Ansatz for the Kähler metrics of the chiral open string modes. It would also be interesting to explore the implications of these results for the phenomenological applications of type IIA flux vacua like, e.g., revisit the cosmological scenarios in [36, 37].

Finally, it would be interesting to generalise the CSD vacua to include perturbative α'-corrections and see how such corrections modify the scalar potential in [41] and the corresponding vacua analysed in 4.3. On the one hand, it would also be interesting to see how the effect of α'-corrections modifies the spectrum of soft masses in non-supersymmetric flux vacua, extending the analysis carried out in section 6.3. On the other hand, it would also be interesting to compute the effect of perturbative α'-corrections for non-Calabi-Yau geometries. In general, we expect that our results help to achieve a wider understanding of type IIA compactifications with fluxes, D-branes and perturbative α'-corrections and, eventually, a better overview of the landscape of flux vacua.

Appendix A

Kähler metrics in Type IIA CY orientifolds with mobile D6-branes

Aim of this appendix is to collect together the expressions of the Kähler metrics in Type IIA CY orientifolds with mobile D6-branes, whose derivation would interrupt the flow of the main text.

A.1 Kähler Potentials in Type IIA CY orientifolds

Type IIA compactifications on Calabi-Yau orientifolds naturally come with moduli spaces parameterised by Kähler moduli and complex structure moduli. The moduli spaces inherit a Kähler geometry from the $\mathcal{N}=2$ compactifications on the Calabi-Yau manifolds before orientifolding, with the Kähler metric given by the second order derivative of the Kähler potential:

$$K = K_T + K_Q = -\log(\mathcal{G}_T \mathcal{G}_Q^2). \tag{A.1}$$

The product $\mathcal{G} = \mathcal{G}_T \mathcal{G}_Q^2$ is a homogeneous function of degree seven in the geometric moduli $\psi^A \in \{t^a, n_\star^K, u_{\star\Lambda}\}$ of the closed string sector:

$$\psi^A \partial_A \mathcal{G} = \left(t^a \partial_{t^a} + n_\star^K \partial_{n_\star^K} + u_{\star\Lambda} \partial_{u_{\star\Lambda}} \right) \mathcal{G} = 7\mathcal{G}, \tag{A.2}$$

indicating that the moduli form homogeneous coordinates on the moduli space subject to the scaling transformations,

$$t^a \to \lambda\, t^a, \qquad n_\star^K \to \tilde{\lambda}\, n_\star^K, \quad u_{\star\Lambda} \to \tilde{\lambda} u_{\star\Lambda}. \tag{A.3}$$

From these homogeneous functions the Kähler metric can be determined straightforwardly,

$$K_A = -\frac{1}{2i}\frac{\partial_A \mathcal{G}}{\mathcal{G}}, \tag{A.4}$$

$$K_{A\overline{B}} = -\frac{1}{4}\left(\frac{\partial_A\partial_B\mathcal{G}}{\mathcal{G}} - \frac{\partial_A\mathcal{G}\partial_B\mathcal{G}}{\mathcal{G}^2}\right). \tag{A.5}$$

The homogeneous property of the function $\mathcal{G}$ (A.2) implies some additional relations, such as

$$K^{A\overline{B}}K_{\overline{B}} = -2i\psi^A, \tag{A.6}$$

and the no-scale relation,

$$K^{A\overline{B}}K_A K_{\overline{B}} = 7, \tag{A.7}$$

and also allows to extract a simple relation for the inverse metric,

$$K^{A\overline{B}} = \frac{2}{3}\psi^A\psi^B - 4\mathcal{G}\mathcal{G}^{AB}, \tag{A.8}$$

with $\mathcal{G}^{AB}$ the inverse of $\partial_A\partial_B\mathcal{G}$.

A.2 Kähler metrics with mobile D6-branes

As discussed in section 2.4, in the absence of D6-branes the moduli space corresponds to the direct product of the Kähler and complex structure moduli space, which allows for an independent scaling transformation on both sectors with $\lambda \neq \tilde{\lambda} \in \mathbb{C}$. In the presence of D6-branes wrapping *SLag* three-cycles Π_α with $b^1(\Pi_\alpha) \neq 0$, a redefinition of the complex structure moduli induces a mixing between all closed and open string moduli, as discussed in section 2.6, such that the scaling symmetries acting on the Kähler and complex structure moduli are identified $\lambda = \tilde{\lambda}$. Nonetheless, $\mathcal{G}$ is still a homogeneous function of degree seven in terms of the geometric moduli $\psi^A \in \{t^a, n^K, u_\Lambda, \phi^i_\alpha\}$.

Let us now specify these relations in the presence of n D6-branes wrapping *SLag* three-cycles $\Pi_{\alpha\in\{1,\ldots,n\}}$ and the symplectic basis choice with $\{N^K\}_{K\neq 0} = 0$, as considered in section 4.3, such that the Kähler potential for the type IIA orientifold compactification reads:

$$K_T = -\log\left(\frac{4}{3}\mathcal{K}_{abc}t^a t^b t^c\right), \tag{A.9}$$

$$K_Q = -\log\left[s + \frac{1}{2}t^a\mathbf{H}^0_{\alpha\, a}\right] - 2\log\left[\tilde{\mathcal{G}}_Q\left(u_\Lambda + \frac{1}{2}t^a\mathbf{H}_{\alpha\Lambda\, a}\right)\right]. \tag{A.10}$$

To obtain analytic relations for the metric, we will further assume that the functions $\mathbf{H}^K_{\alpha\,a}$ and $\mathbf{H}^K_{\alpha\,\Lambda\,a}$ depend only on the geometric moduli $\{t^a, \phi^i_b\}$. Such a functional dependence is characteristic for toroidal backgrounds, but is also expected to be a good approximation in the large volume and large complex structure regions of the moduli space for more generic Calabi-Yau manifolds. Under this assumption the first order derivatives of the Kähler potential are given by

$$\begin{aligned}
&K_S = \tfrac{i}{2s+t^a\mathbf{H}^0_{\alpha\,a}}, \qquad\qquad K_{U_\Lambda} = i\tfrac{1}{\tilde{\mathcal{G}}_Q}\partial_{u_\Lambda}\tilde{\mathcal{G}}_Q,\\
&K_{T^a} = \tfrac{3i\mathcal{K}_{abc}t^bt^c}{2\mathcal{K}} + \tfrac{i}{4s+2t^b\mathbf{H}^0_{\alpha\,b}}\partial_{t^a}(t^c\mathbf{H}^0_{\alpha\,c}) + \tfrac{i}{2\tilde{\mathcal{G}}_Q}\partial_{u_\Lambda}(\tilde{\mathcal{G}}_Q)\partial_{t^a}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}),\\
&K_{\Phi^i_\alpha} = \tfrac{i}{4s+2t^b\mathbf{H}^0_{\alpha\,b}}\partial_{\phi^i_\alpha}(t^a\mathbf{H}^0_{\alpha\,a}) + \tfrac{i}{2\tilde{\mathcal{G}}_Q}\partial_{u_\Lambda}(\tilde{\mathcal{G}}_Q)\partial_{\phi^i_\alpha}(t^a\mathbf{H}_{\alpha\,\Lambda\,a}).
\end{aligned} \tag{A.11}$$

Upon introducing the row vectors

$$\mathbf{H}^0_T = \frac{1}{2}\partial_{t^a}(t^c\mathbf{H}^0_{\alpha\,c}), \quad \mathbf{H}_{\Lambda\,T} = \frac{1}{2}\partial_{t^a}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}), \tag{A.12}$$

$$\mathbf{H}^0_\Phi = \frac{1}{2}\partial_{\phi^i_\alpha}(t^c\mathbf{H}^0_{\alpha\,c}), \quad \mathbf{H}_{\Lambda\,\Phi} = \frac{1}{2}\partial_{\phi^i_\alpha}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}), \tag{A.13}$$

and the matrices

$$K_{\hat{S}\overline{\hat{S}}} = \frac{1}{(2\hat{s}+t^a\mathbf{H}^0_{\alpha\,a})^2}, \tag{A.14}$$

$$K_{\hat{U}_\Lambda\overline{\hat{U}}_M} = \frac{1}{2}\left(\frac{\partial_{\hat{u}_\Lambda}\tilde{\mathcal{G}}_Q\partial_{\hat{u}_M}\tilde{\mathcal{G}}_Q}{\tilde{\mathcal{G}}_Q^2} - \frac{\partial_{\hat{u}_\Lambda}\partial_{\hat{u}_M}\tilde{\mathcal{G}}_Q}{\tilde{\mathcal{G}}_Q}\right), \tag{A.15}$$

$$\begin{aligned}
\Xi_{T^a\overline{T}^b} = {} & -\frac{3}{2}\left(\frac{\mathcal{K}_{ab}}{\mathcal{K}} - \frac{3}{2}\frac{\mathcal{K}_a\mathcal{K}_b}{\mathcal{K}^2}\right) + \frac{i}{4}K_{\hat{S}}\,\partial_{t^a}\partial_{t^b}(t^c\mathbf{H}^0_{\alpha\,c})\\
& + \frac{i}{4}K_{\hat{U}_\Lambda}\,\partial_{t^a}\partial_{t^b}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}),
\end{aligned} \tag{A.16}$$

$$\Xi_{T^a\overline{\Phi}^j_\beta} = \frac{i}{4}K_{\hat{S}}\,\partial_{t^a}\partial_{\phi^j_\beta}(t^c\mathbf{H}^0_{\alpha\,c}) + \frac{i}{4}K_{\hat{U}_\Lambda}\,\partial_{t^a}\partial_{\phi^j_\beta}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}), \tag{A.17}$$

$$\Xi_{\Phi^j_\alpha\overline{\Phi}^j_\beta} = \frac{i}{4}K_{\hat{S}}\,\partial_{\phi^i_\alpha}\partial_{\phi^j_\beta}(t^c\mathbf{H}^0_{\alpha\,c}) + \frac{i}{4}K_{\hat{U}_\Lambda}\,\partial_{\phi^i_\alpha}\partial_{\phi^j_\beta}(t^c\mathbf{H}_{\alpha\,\Lambda\,c}), \tag{A.18}$$

the Kähler metric $\mathbf{K}_{AB}$ on the full moduli space can be written in an elegant way:

$$\mathbf{K}_{AB} = \begin{pmatrix} \mathbf{1} & 0 & 0 & 0\\ 0 & \mathbf{1} & 0 & 0\\ (\mathbf{H}^0_T)^t & (\mathbf{H}_{\Lambda\,T})^t & \mathbf{1} & 0\\ (\mathbf{H}^0_\Phi)^t & (\mathbf{H}_{\Lambda\,\Phi})^t & 0 & \mathbf{1} \end{pmatrix} \begin{pmatrix} K_{\hat{S}\overline{\hat{S}}} & 0 & 0 & 0\\ 0 & K_{\hat{U}_\Lambda\overline{\hat{U}}_M} & 0 & 0\\ 0 & 0 & \Xi_{T^a\overline{T}^b} & \Xi_{T^a\overline{\Phi}^j_\beta}\\ 0 & 0 & \Xi_{\Phi^i_\alpha\overline{T}^b} & \Xi_{\Phi^j_\alpha\overline{\Phi}^j_\beta} \end{pmatrix} \begin{pmatrix} \mathbf{1} & 0 & \mathbf{H}^0_T & \mathbf{H}^0_\Phi\\ 0 & \mathbf{1} & \mathbf{H}_{\Lambda\,T} & \mathbf{H}_{\Lambda\,\Phi}\\ 0 & 0 & \mathbf{1} & 0\\ 0 & 0 & & \mathbf{1} \end{pmatrix}. \tag{A.19}$$

From this expression we can straightforwardly determine the inverse Kähler metric $\mathbf{K}^{AB}$:

$$\mathbf{K}^{AB} = \begin{pmatrix} \mathbf{1} & 0 & -\mathbf{H}^0_T & -\mathbf{H}^0_\Phi \\ 0 & \mathbf{1} & -\mathbf{H}_{\Lambda\,T} & -\mathbf{H}_{\Lambda\,\Phi} \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1} \end{pmatrix} \begin{pmatrix} K^{-1}_{\hat{S}\overline{\hat{S}}} & 0 & 0 & 0 \\ 0 & K^{-1}_{\hat{U}_\Lambda \overline{\hat{U}}_M} & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{Ξ^{-1}}} \\ 0 & 0 & & \end{pmatrix} \begin{pmatrix} \mathbf{1} & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ -(\mathbf{H}^0_T)^t & -(\mathbf{H}_{\Lambda\,T})^t & \mathbf{1} & 0 \\ -(\mathbf{H}^0_\Phi)^t & -(\mathbf{H}_{\Lambda\,\Phi})^t & 0 & \mathbf{1} \end{pmatrix}, \tag{A.20}$$

where Ξ^{-1} denotes the inverse of the matrix with entries $\Xi_{T^a\overline{T}^b}$, $\Xi_{T^a\overline{\Phi}^j_\beta}$, $\Xi_{\Phi^i_\alpha\overline{T}^b}$ and $\Xi_{\Phi^i_\alpha\overline{\Phi}^j_\beta}$.

Appendix B

Superpotentials with mobile D6-branes

In this appendix we briefly review the microscopic origin of the D6-brane superpotential and how the redefined complex structure moduli enter in the superpotential. When considering orientifold compactifications with D6-branes and their orientifold images, one has to be aware that their RR-charges act as magnetic sources for the field strength G_2, such that the Bianchi identities (2.53) have to be modified accordingly:

$$\ell_s^2 \left(dG_2 - mH_3\right) = -\sum_\alpha N_\alpha \Big[\delta^3(\Pi_\alpha^0) + \delta^3(\mathcal{R}\Pi_\alpha^0)\Big] + 4\delta^3(\Pi_{O6}), \tag{B.1}$$

where the right-hand side considers the bump-like delta-function currents sourced by the D6-branes wrapping reference three-cycles Π_α^0 their respective orientifold images $\mathcal{R}\Pi_\alpha^0$, and the O6-planes. The field strength G_2 is globally well-defined upon imposing the modified RR tadpole cancellation conditions in the presence of NS 3-form flux and Romans mass m:

$$\sum_\alpha N_\alpha([\Pi_\alpha^0] + [\mathcal{R}\Pi_\alpha^0]) - 4[\Pi_{O6}] - m[\Pi_{H_3}] = 0, \tag{B.2}$$

where $[\Pi_{H_3}]$ corresponds to the Poincaré-dual three-cycle of the NS-flux H_3. Note that in the absence of H_3-flux, the RR tadpole condition implies the existence of a four-chain $\mathcal{C}_4^0$ connecting the D6-branes and their orientifold images to the O6-planes, i.e. $\partial\mathcal{C}_4^0 = \sum_\alpha N_\alpha\,(\Pi_\alpha^0 + \mathcal{R}\Pi_\alpha^0) - 4\Pi_{O6}$.

The Lagrangian condition (2.64) also has to be modified in the presence of worldvolume fluxes including the $U(1)$ field strength $F = dA$:

$$J_c\big|_{\Pi_\alpha} - \frac{\ell_s^2}{2\pi}F = 0. \tag{B.3}$$

In regions of the closed string moduli space where this condition is violated, a non-vanishing contribution to the superpotential arises that is capable of breaking the $\mathcal{N} = 1$ supersymmetry in four dimensions,

$$\Delta W = \frac{1}{\ell_s^5} \int_{\mathcal{C}_4^\alpha} \left(J_c - \frac{\ell_s^2}{2\pi} \tilde{F}_\alpha \right) \wedge \left(J_c - \frac{\ell_s^2}{2\pi} \tilde{F}_\alpha \right), \tag{B.4}$$

where the four-chain $\mathcal{C}_4^\alpha$ connects a three-cycle Π_α that is a homotopic deformation of the reference three-cycle Π_α^0, in line with the philosophy of section 2.6. The field strength $\tilde{F}_\alpha$ is the extension of the D6-brane worldvolume field strength to the four-chain. Microscopically, there exist two separate effects that yield a non-vanishing superpotential ΔW as a function of the open string moduli associated to the three-cycle deformations. The first effects comes from turning on a worldvolume flux:

$$\frac{\ell_s^2}{2\pi} F_\alpha = \frac{\ell_s^2}{2\pi} dA_\alpha + n_{Fi}^\alpha \rho^i, \qquad n_{Fi}^\alpha \in \mathbb{Z}, \tag{B.5}$$

such that the evaluation of (B.4) leads to a superpotential containing a linear term in the open string moduli:

$$\ell_s \Delta W^{(1)} = n_{Fi}^\alpha \Phi_\alpha^i. \tag{B.6}$$

A second contribution is due to the backreaction on the closed string fluxes following the homotopic deformation of a *SLag* three-cycle $\Pi_\alpha^0 \to \Pi_\alpha$. More precisely, after the deformation the backreacted RR-fluxes $\mathbf{G} = \mathbf{G}^0 + q_\alpha \Delta_\alpha \mathbf{G}$ can be decomposed into a component $\mathbf{G}^0$ that satisfies the Bianchi identities in the reference configuration (with vanishing worldvolume flux)

$$\ell_s^2 d_H \mathbf{G}^0 = - \left(\sum_\alpha N_\alpha \left(\delta^3(\Pi_\alpha^0) + \delta^3(\mathcal{R}\Pi_\alpha^0) \right) - 4\delta^3(\Pi_{O6}) \right) \wedge e^B, \tag{B.7}$$

and a component $\Delta_\alpha \mathbf{G}$ capturing the change in fluxes under the deformation:

$$\ell_s^2 d_H \Delta_\alpha \mathbf{G}^0 = N_\alpha \delta^3(\Pi_\alpha^0) \wedge e^B - N_\alpha \delta^3(\Pi_\alpha) \wedge e^{B - \frac{\ell_s^2}{2\pi} F}. \tag{B.8}$$

B.1 Open-Closed Superpotentials

In the absence of H_3-flux, both of these equations can be solved [41] in terms of bump delta-functions associated with the appropriate four-chains:

$$\mathbf{G}^0 = -\frac{1}{\ell_s} \delta^2(\mathcal{C}_4^0) \wedge e^B, \qquad \Delta_\alpha \mathbf{G} = -\frac{1}{\ell_s} \delta^2(\mathcal{C}_4^\alpha) \wedge e^{B - \frac{\ell_s^2}{2\pi} F}. \tag{B.9}$$

The four-chain $\mathcal{C}_4^0$ has been introduced above for the reference configuration, while the second four-chain $\mathcal{C}_4^\alpha$ connects the deformed three-cycle and reference three-cycle such that the delta-function satisfies $\ell_s d\,\delta^2(\mathcal{C}_4^\alpha) = N_\alpha \delta^3(\Pi_\alpha) - N_\alpha \delta^3(\Pi_\alpha^0)$. In the reference configuration the polyforms $e^{-B} \wedge \mathbf{G}_0$ still allow to define quantised Page charges, but the harmonic pieces of $\mathbf{G}_0$ are tied to their co-exact components resulting from the presence of localised sources. Similarly, the back-reacted polyforms $e^{-B} \wedge \mathbf{G}$ ought to allow for the definition of conserved Page charges upon deformation, which implies that the harmonic parts of $\Delta_\alpha \mathbf{G}$ are completely determined by their co-exact piece. The presence of a harmonic component for $\Delta_\alpha G_2$ can give rise to a superpotential contribution ΔW involving open string moduli. To see how this precisely happens, we follow the same logic as in [35,127] and consider the integral of $\Delta_\alpha G_2$ wedged with the closed four-form $J \wedge \omega_2$:

$$\int_{\mathcal{M}_6} \Delta_\alpha G_2 \wedge J \wedge \omega_2 = \int_{\mathcal{C}_4^\alpha} J \wedge \omega_2, \tag{B.10}$$

which is non-vanishing for a harmonic two-form ω_2. For an infinitesimal deformation X of the *SLag* three-cycle as in section 2.6, the chain integral reduces to an integral over the three-cycle,

$$\int_{\mathcal{C}_4} J \wedge \omega_2 = \int_{\Pi_\alpha} \iota_X J \wedge \omega_2, \tag{B.11}$$

which implies the existence of a non-trivial two-cycle in $H_2(\Pi_\alpha, \mathbb{Z})$, Poincaré dual to the one-form $\iota_X J$, for non-vanishing values. By using the more appropriate basis of one-forms ζ^i from section 2.6, the condition can be written out more explicitly through the D6-brane displacement parameters n_{ai}^α,

$$n_{ai}^\alpha = \frac{1}{\ell_s^3} \int_{\Pi_\alpha} \omega_a \wedge \zeta_i \quad \in \mathbb{Z}. \tag{B.12}$$

If at least one of the parameters $n_{ai}^\alpha \neq 0$, the evaluation of (B.4) gives rise to a superpotential consisting of a bilinear term mixing open string moduli and Kähler moduli:

$$\ell_s \Delta W^{(2)} = -n_{ai}^\alpha \Phi_\alpha^i T^a. \tag{B.13}$$

Consequently, the most generic four-dimensional effective superpotential for type IIA flux compactifications with (non-rigid) D6-branes includes an additional supersymmetry-breaking term mixing open string moduli and Kähler moduli as in equation (2.78). In this expression, W_{D6}^0 denotes the constant contribution to the D6-brane superpotential evaluated for the reference three-cycles Π_α^0:

$$W_{D6}^0 = \frac{1}{2\ell_s^5} \int_{\mathcal{C}_4^0} \left(J_c - \frac{\ell_s^2}{2\pi} \tilde{F}_\alpha \right) \wedge \left(J_c - \frac{\ell_s^2}{2\pi} \tilde{F}_\alpha \right), \tag{B.14}$$

in the absence of H-flux.

B.2 Superpotentials and Redefined Complex Structure Moduli

For flux compactifications with non-vanishing H_3-flux, the Bianchi identities (B.7) and RR tadpole conditions (B.2) no longer imply the existence of a four-chain $\mathcal{C}_4^0$ connecting the full set of D6-branes and O6-planes for the reference configuration. Instead the solutions (B.9) of the Bianchi identities have to be adjusted appropriately, as derived for the first time in Appendix B.1 of [41]. Here, we review and extend the reasoning that led to eq.(B.11) there, which allowed to deduce the expression for the redefined complex structure moduli N^K in term of the open string moduli. More precisely, we extend this result in the sense that we consider both kinds of complex structure moduli (N^K, U_Λ) considered in the type IIA orientifold literature.

Following [41] we first consider the type IIA flux superpotential

$$-\,iW \,=\, \frac{1}{\ell_s^6}\int_{\mathcal{M}_6} e^{-\phi}\mathrm{Re}\ \Omega_3 \wedge \mathrm{H} - \mathrm{i}\mathbf{G}\wedge \mathrm{e}^{\mathrm{iJ}} \tag{B.15}$$

which is manifestly gauge invariant and globally well-defined. Then one can split the RR flux background $\mathbf{G}$ into two pieces

$$\mathbf{G} \,=\, \mathbf{G}^0 + \sum_\alpha \Delta_\alpha \mathbf{G} \tag{B.16}$$

with $\mathbf{G}^0$ satisfying the Bianchi identities and quantisations conditions for the reference configuration, and $\Delta_\alpha\mathbf{G}$ representing the change in $\mathbf{G}$ as we replace the D6-brane at Π_α^0 with the one at Π_α. We find that

$$\mathbf{G}^0 = -j_0 - H\wedge C_3 + e^B \wedge \bar{\mathbf{G}} + \ldots \tag{B.17}$$

and

$$\Delta_\alpha \mathbf{G} \,\simeq\, \frac{1}{\ell_s^2}\delta(\Pi_\alpha)\wedge\left(\sigma A - \frac{1}{2}\sigma^2 A\wedge F\right)\wedge e^B - \frac{1}{\ell_s}\delta(\mathcal{C}_4^\alpha)\wedge\left(e^B - \varpi_4\right) \tag{B.18}$$

where $\mathcal{C}_4^\alpha$ is a four-chain such that $\partial\mathcal{C}_4^\alpha = \Pi_\alpha - \Pi_\alpha^0$, and ϖ_4 is the co-exact form such that $d\varpi_4 = H\wedge B$. Replacing this into (B.15) one obtains

$$W \,=\, \frac{1}{\ell_s^6}\int_{\mathcal{M}_6}\Omega_c\wedge H + \bar{\mathbf{G}}\wedge e^{J_c} + \frac{2}{\ell_s^4}\int_{\Pi_\alpha}\sigma A\wedge(J_c - \sigma F) - \frac{1}{\ell_s^5}\int_{\mathcal{C}_4^\alpha} J_c^2 - \varpi_4 + W_0\,. \tag{B.19}$$

From this last expression one can extract the closed and open-string moduli dependence of the superpotential. We are mainly interested in the terms proportional to the H-flux quanta, which are defined by

$$H \,=\, h_K\beta^K + h^\Lambda\alpha_\Lambda\,. \tag{B.20}$$

Then we have that the first piece of (B.19) contributes as

$$\frac{1}{\ell_s^6}\int_{\mathcal{M}_6}\Omega_c\wedge H \;=\; h_K N_\star^K + h^\Lambda U_{\star\Lambda}\,. \tag{B.21}$$

To evaluate the remaining dependence on the H-flux quanta we split the B-field on the four-chain $\mathcal{C}_4^\alpha$ as

$$B|_{\mathcal{C}_4^\alpha} \;=\; b^a\omega_a + \tilde{B} \tag{B.22}$$

with $\tilde{B}$ the co-exact piece of the B-field satisfying $d\tilde{B} = H|_{\mathcal{C}_4^\alpha}$. Given this split one can see that $\varpi_4|_{\mathcal{C}_4^\alpha} = \frac{1}{2}\tilde{B}\wedge\tilde{B}|_{\mathcal{C}_4^\alpha}$. We then find that the third and fourth terms in (B.19) contain the terms

$$-\frac{1}{\ell_s^4}\int_{\mathcal{C}_4^\alpha} J_c\wedge\tilde{B} + \frac{2}{\ell_s^4}\int_{\Pi_\alpha}\sigma A\wedge\tilde{B} \;=\; -\frac{1}{2}\aleph_{a\,\alpha}T^a + \frac{1}{2}\left(h_K g^K_{i\alpha\,i} + h^\Lambda g_{\alpha\,\Lambda\,i}\right)\theta^i_\alpha$$

where

$$g^K_{\alpha\,i} \;=\; \frac{2}{\ell_s^4}\int_{\mathcal{C}_4^\alpha}\beta^K\wedge\tilde{\zeta}_i \qquad \text{and} \qquad g_{\alpha\,\Lambda\,i} \;=\; \frac{2}{\ell_s^4}\int_{\mathcal{C}_4^\alpha}\alpha_\Lambda\wedge\tilde{\zeta}_i. \tag{B.23}$$

with $\tilde{\zeta}_i$ the extension of the one-form ζ_i of Π_α to $\mathcal{C}_4^\alpha$, and

$$\aleph_{a\,\alpha} = \frac{2}{\ell_s^4}\int_{\mathcal{C}_4^\alpha}\tilde{B}\wedge\omega_a\,. \tag{B.24}$$

Finally, generalising the computation below eq.(A.31) of [41] to a background flux of the form (B.20) one easily deduces that

$$\aleph_{a\,\alpha} \;=\; \frac{1}{2}\left(h_K\mathbf{H}^K_{\alpha\,a} + h^\Lambda\mathbf{H}_{\alpha\,\Lambda\,a}\right) \tag{B.25}$$

with the definitions of $\mathbf{H}^K_{\alpha\,a}$ and $\mathbf{H}_{\alpha\,\Lambda\,a}$ given in the main text.

Therefore, putting all these results together one finds that the superpotential depends on the H-flux quanta as

$$\begin{aligned} W \;&=\; h^K\left[N_\star^K + \frac{1}{2}\sum_\alpha(g^K_{\alpha i}\theta^i_\alpha - T^a\mathbf{H}^K_{\alpha\,a})\right] \\ &+\; h_\Lambda\left[U_{\star\Lambda} + \frac{1}{2}\sum_\alpha(g_{\alpha\,\Lambda\,i}\theta^i_\alpha - T^a\mathbf{H}_{\alpha\,\Lambda\,a})\right] + \ldots \end{aligned} \tag{B.26}$$

obtaining the following redefinition for the complex structure moduli of the compactification

$$N^K = N_\star^K + \frac{1}{2}\sum_\alpha(g^K_{\alpha i}\theta^i_\alpha - T^a\mathbf{H}^K_{\alpha\,a}), \qquad U_\Lambda = U_{\star\Lambda} + \frac{1}{2}\sum_\alpha(g_{\alpha\,\Lambda\,i}\theta^i_\alpha - T^a\mathbf{H}_{\alpha\,\Lambda\,a})\,. \tag{B.27}$$

Appendix C

Full Computation of the α'-Corrected Potentials

In this appendix we present the details regarding the computations of the α'-corrected potentials.

C.1 α'-Corrected Kähler Potentials

As pointed out in section 2.7, the no-scale symmetries (A.6) and (A.7) rely on the hidden assumption that we consider large volume regions in the Kähler moduli space. Away from this large volume limit, perturbative curvature corrections in α' have to be taken into account, which alter the Kähler potential for the Kähler moduli space but maintain the factorability of the closed string moduli space. As such, the modified Kähler potential derived in (2.80) allows to compute the Kähler metric in the same manner as equation (A.4) using the modified function $\mathcal{G}_T$:

$$K_a = \frac{3i}{2}\frac{\mathcal{K}_a}{\mathcal{K}\left(1+\frac{3}{2}\varepsilon\right)} \tag{C.1}$$

$$K_{a\bar{b}} = -\frac{3}{2}\frac{1}{\mathcal{K}^2\left(1+\frac{3}{2}\varepsilon\right)^2}\left(\mathcal{K}(1+\frac{3}{2}\varepsilon)\mathcal{K}_{ab}-\frac{3}{2}\mathcal{K}_a\mathcal{K}_b\right), \tag{C.2}$$

while the inverse metric in the presence of perturbative α'-corrections is given by:

$$K^{a\bar{b}} = -\frac{2}{3}\mathcal{K}\left(1+\frac{3}{2}\varepsilon\right)\left(\mathcal{K}^{ab}-3\frac{t^a t^b}{\mathcal{K}(1-3\varepsilon)}\right). \tag{C.3}$$

Subsequently, relation (A.6) is modified as well in the Kähler moduli sector due to the curvature corrections:

$$K^{a\bar{b}}K_{\bar{b}} = -2it^a \frac{1+\frac{3}{2}\varepsilon}{1-3\varepsilon}, \tag{C.4}$$

which immediately implies the violation of the no-scale symmetry:

$$K^{a\bar{b}}K_a K_{\bar{b}} = \frac{3}{1-3\varepsilon}. \tag{C.5}$$

This set of relations for the Kähler moduli sector turned out to be crucial for the computation of the F-term scalar potential computed in the next section.

C.2 α'-Corrected Scalar Potential

Next, we discuss the computation of the F-term scalar potential in full detail and highlight some manipulations that help us to arrive at the more elegant bilinear form of the scalar potential in equation (5.6). The philosophy used in section 5.1 consists in decomposing the F-term scalar potential in three separate terms and write each term as a function of the (α'-corrected) axion polynomials in the simplest form possible. Given that the Kähler potentials still factorise between the Kähler moduli and complex structure moduli sector, the term containing the derivatives of the superpotential can be written as,

$$\begin{aligned}\partial_\alpha W K^{\alpha\overline{\beta}}\partial_{\overline{\beta}}\overline{W} &= \partial_{T^a} W K^{T^a\overline{T}^b}\partial_{\overline{T}^b}\overline{W} + \partial_{N^K} W K^{K\overline{L}}\partial_{N^L} W + \partial_{N^K} W K^{K\overline{\Lambda}}\partial_{\overline{U}_\Lambda} W \\ &\quad + \partial_{U_\Sigma} W K^{\Sigma\overline{L}}\partial_{N^L} W + \partial_{U_\Sigma} W K^{\Sigma\overline{\Lambda}}\partial_{\overline{U}_\Lambda} W \\ &= K^{a\bar{b}}(\overline{\rho}_a - \tfrac{1}{2}\tilde{\rho}\mathcal{K}_a)(\overline{\rho}_b - \tfrac{1}{2}\tilde{\rho}\mathcal{K}_b) + K^{a\bar{b}}\mathcal{K}_{ac}\tilde{\rho}^c\mathcal{K}_{bd}\tilde{\rho}^d \\ &\quad + K^{N\overline{L}}\hat{\rho}_K\hat{\rho}_L + K^{N\overline{\Lambda}}\hat{\rho}_K\hat{\rho}_\Lambda + K^{\Sigma\overline{L}}\hat{\rho}_\Sigma\hat{\rho}_L + K^{\Sigma\overline{\Lambda}}\hat{\rho}_\Sigma\hat{\rho}_\Lambda.\end{aligned} \tag{C.6}$$

Inserting the expression for the inverse Kähler metric (C.3) on the Kähler moduli space allows to simplify this relation to the expression in (5.1). Moreover, this expression can be further rewritten as,

$$\begin{aligned}\partial_\alpha W K^{\alpha\overline{\beta}}\partial_{\overline{\beta}}\overline{W} &= K^{a\bar{b}}\overline{\rho}_a\overline{\rho}_b + \tfrac{4}{9}\mathcal{K}^2(1+\tfrac{3}{2}\varepsilon)^2 K_{a\bar{b}}\tilde{\rho}^a\tilde{\rho}^b + \tfrac{1+6\varepsilon}{1-3\varepsilon}(\mathcal{K}_a\tilde{\rho}^a)^2 + \tfrac{1}{3}\mathcal{K}^2\tilde{\rho}^2\tfrac{(1+\frac{3}{2}\varepsilon)^2}{1-3\varepsilon} \\ &\quad -\tfrac{4}{3}\tilde{\rho}\mathcal{K}\left(\mathrm{Im}W + \tfrac{1}{6}\tilde{\rho}\mathcal{K} + \tilde{\rho}\mathcal{K}\varepsilon - \hat{\rho}_K n^K - \hat{\rho}^\Lambda u_\Lambda\right)\tfrac{(1+\frac{3}{2}\varepsilon)^2}{1-3\varepsilon} \\ &\quad + K^{N\overline{L}}\hat{\rho}_K\hat{\rho}_L + K^{N\overline{\Lambda}}\hat{\rho}_K\hat{\rho}_\Lambda + K^{\Sigma\overline{L}}\hat{\rho}_\Sigma\hat{\rho}_L + K^{\Sigma\overline{\Lambda}}\hat{\rho}_\Sigma\hat{\rho}_\Lambda,\end{aligned} \tag{C.7}$$

by eliminating $\overline{\rho}_a t^a$ through the expression for ImW,

$$\mathrm{Im}W = \overline{\rho}_a t^a - \frac{1}{6}\tilde{\rho}\mathcal{K} - \tilde{\rho}\mathcal{K}\varepsilon + \hat{\rho}_K n^K + \hat{\rho}^\Lambda u_\Lambda. \tag{C.8}$$

The second component (5.2) is a consequence of imposing the no-scale symmetry in the complex structure moduli sector and the modified relation (C.5) in the presence of perturbative α'-corrections. The third component (5.3) of the F-term scalar potential results from the factorisation of the Kähler potentials (A.1) for Kähler moduli and complex structure moduli, after which one uses the relation (A.6) for the complex structure moduli sector and the modified relation (C.4) for the Kähler moduli sector. Then, we combine the second component (5.2) and third component (5.3), upon multiplication by $(1-3\varepsilon)$, to obtain a simplified expression written entirely in terms of the axion polynomials,

$$\begin{aligned}
&(4-3\varepsilon)|W|^2 - 4\left(1+\tfrac{3}{2}\varepsilon\right)\left[\mathrm{Re}W t^a\partial_{t^a}\mathrm{Re}W + \mathrm{Im}W t^a\partial_{t^a}\mathrm{Im}W\right] - 4(1-3\varepsilon)\mathrm{Im}W(n^L\hat{\rho}_L + u_\Lambda\hat{\rho}^\Lambda)\\
&= (4-3\varepsilon)\overline{\rho}_0^2 + 9\mathcal{K}_a\tilde{\rho}^a\overline{\rho}_0\varepsilon - (\mathcal{K}_a\tilde{\rho}^a)^2(1+\tfrac{15}{4}\varepsilon) - 9\varepsilon\,\mathrm{Im}W\left(t^a\overline{\rho}_a - \hat{\rho}_K n^K - \hat{\rho}^\Lambda u_\Lambda\right)\\
&\quad +\tilde{\rho}\mathcal{K}\mathrm{Im}W\left(\tfrac{4}{3} - \tfrac{1}{2}\varepsilon(1-6\varepsilon)\right).
\end{aligned} \tag{C.9}$$

Adding up the three components correctly, one can then deduce the final expression for the scalar potential including the perturbative α'-corrections, namely equation (5.4), by manipulating the end result further and separating zeroth order terms $\mathcal{O}(\varepsilon^0)$, similar to the ones that appear in the inverse metric (3.29), from higher order ε-corrections.

C.3 Alternative Computation of the ISD Scalar Potential

In section 5.2 a positive semi-definite form of the scalar potential in the presence of ISD flux was used to extract the non-supersymmetric Minkowski vacuum configuration in term of the axion polynomials. The precise form of this scalar potential can be derived by a series of computations that start from the T-dual Type IIB picture. In Type IIB the scalar potential associated to ISD flux vacua is explicitly positive definite when expressed in terms of the background ISD G_3 fluxes, see e.g. appendix A of [52],

$$\begin{aligned}
V_{GKP} &= \frac{1}{24V_{\mathcal{M}_6}}\int_{\mathcal{M}_6}\frac{|G_3+i\star_6 G_3|^2}{\mathrm{Im}\,(\tau)}\\
&= \frac{1}{2V_{\mathcal{M}_6}\mathrm{Im}\,(\tau)}\int_{\mathcal{M}_6}(\mathrm{Re}\,\mathrm{G}_3 - \star_6\mathrm{Im}\,\mathrm{G}_3)\wedge\star_6(\mathrm{Re}\,\mathrm{G}_3 - \star_6\mathrm{Im}\,\mathrm{G}_3),
\end{aligned} \tag{C.10}$$

with the three-form flux $G_3 = F_3 - \tau H_3$ and $\tau = C_0 + i\,e^{-\phi}$ the (four-dimensional) axio-dilaton. According the appendix A of [8] the Type IIB three-form flux can be T-dualised to the closed string fluxes of Type IIA compactifications, when one considers the following decomposition of the G_3-flux in terms of harmonic three-forms,

$$G_3 = -\left(e_0 + Sh_0, e_a, -m, m^a\right) \cdot \begin{pmatrix} \beta^0 \\ \beta^a \\ -\alpha_0 \\ -\alpha_a \end{pmatrix}. \tag{C.11}$$

In order to evaluate the scalar potential for the ISD-flux background, one needs to determine the Hodge duals of the harmonic three-forms [8, 10],

$$\star_6 \begin{pmatrix} \beta^I \\ -\alpha_I \end{pmatrix} = \mathcal{M}^{-1} \begin{pmatrix} \alpha_I \\ \beta^I \end{pmatrix}. \tag{C.12}$$

The transformation matrix $\mathcal{M}^{-1}$ can be further decomposed in terms of the matrices $\mathcal{R} = \mathrm{Re}\,(\mathcal{N}_{\mathrm{IJ}})$ and $\mathcal{I} = \mathrm{Im}\,(\mathcal{N}_{\mathrm{IJ}})$,

$$\mathcal{M}^{-1} = \begin{pmatrix} \mathcal{I}^{-1} & -\mathcal{I}^{-1}\mathcal{R} \\ -\mathcal{R}\mathcal{I}^{-1} & \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} \end{pmatrix} = \begin{pmatrix} \mathbb{I} & \\ -\mathcal{R} & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathcal{I}^{-1} & \\ & \mathcal{I} \end{pmatrix} \begin{pmatrix} \mathbb{I} & -\mathcal{R} \\ & \mathbb{I} \end{pmatrix}, \tag{C.13}$$

which follow form the moduli-dependent matrix $\mathcal{N}_{IJ}$ computed directly [10] from a pre-potential $\mathcal{F}$,

$$\mathcal{N}_{IJ} = \overline{\mathcal{F}}_{IJ} + 2i\,\frac{(\mathrm{Im}\,\mathcal{F})_{\mathrm{IK}}\mathrm{X}^{\mathrm{K}}(\mathrm{Im}\,\mathcal{F})_{\mathrm{JL}}\mathrm{X}^{\mathrm{L}}}{X^K(\mathrm{Im}\,\mathcal{F})_{\mathrm{KL}}\mathrm{X}^{\mathrm{L}}}, \tag{C.14}$$

where X^K represent the homogeneous coordinates used to parameterise the corresponding moduli space. In the absence of perturbative α'-corrections one can insert the tree-level pre-potential (2.17) for the Kähler moduli sector to obtain the respective matrices, while the inclusion of the perturbative curvature corrections requires us to use the modified pre-potential (2.79). In the latter case, the resulting transformation matrix $\mathcal{M}^{-1}$ can be decomposed as

$$\mathcal{M}^{-1} = -\frac{3}{2}\mathbb{Q}^t \cdot \mathbb{R}^{-1} \cdot \mathbb{G}^{-1} \cdot \mathbb{R}^{-1t} \cdot \mathbb{Q}, \tag{C.15}$$

with $\mathbb{R}$ the axion rotation matrix,

$$\mathbb{R} = \begin{pmatrix} 1 & & & \\ -b^i & \delta^i_j & & \\ \frac{1}{6}\mathcal{K}_{ijk}b^ib^jb^k & -\frac{1}{2}\mathcal{K}_{ijk}b^jb^k & 1 & b^i \\ \frac{1}{2}\mathcal{K}_{ijk}b^jb^k & -\mathcal{K}_{ijk}b^k & & \delta^i_j \end{pmatrix} \tag{C.16}$$

and the lower order curvature corrections $K^{(1)}_{ab}$ and $K^{(2)}_a$ encoded in the matrix $\mathbb{Q}$,

$$\mathbb{Q} = \begin{pmatrix} 1 & 0 & 0 & -K^{(2)}_a \\ 0 & \delta^a_b & -K^{(2)}_b & -K^{(1)}_{ab} \\ 0 & 0 & \delta^a_b & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{C.17}$$

Furthermore, the symmetric matrix $\mathbb{G}^{-1}$ incorporates the curvature corrections proportional to $K^{(3)}$ in the form of the parameter ε,

$$\mathbb{G}^{-1} = \frac{1}{\mathcal{K}(1+\frac{3}{2}\varepsilon)}\left[\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & K^{a\bar{b}} & 0 & 0 \\ 0 & 0 & \frac{\mathcal{K}^2}{9}\left(1+\frac{3}{2}\varepsilon\right)^2 & 0 \\ 0 & 0 & 0 & \frac{4}{9}\mathcal{K}^2\left(1+\frac{3}{2}\varepsilon\right)^2 K_{a\bar{b}} \end{pmatrix} + \frac{\varepsilon}{1-3\varepsilon}\begin{pmatrix} 9 & 0 & 0 & \frac{9}{2}\mathcal{K}_a \\ 0 & -9t^a t^b & \frac{3}{2}\mathcal{K}t^a(1-3\varepsilon) & 0 \\ 0 & \frac{3}{2}\mathcal{K}t^b(1-3\varepsilon) & -\frac{\mathcal{K}^2}{4}(1-3\varepsilon)^2 & 0 \\ \frac{9}{2}\mathcal{K}_b & 0 & 0 & \frac{9}{4}\mathcal{K}_a\mathcal{K}_b \end{pmatrix}\right] \tag{C.18}$$

By using the Hodge duality relations (C.12) we can rewrite the ISD three-form flux in terms of the matrices $\mathbb{R}$ and $\mathbb{Q}$ and the (modified) axion polynomials (3.27) as follows,

$$-\mathrm{Re}\,(\mathrm{G}_3) + \star_6 \mathrm{Im}\,(\mathrm{G}_3) = \begin{pmatrix} \overline{\rho}_0 \\ \overline{\rho}_a + \frac{27\varepsilon}{4(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{\mathcal{K}_a}{\mathcal{K}} s\hat{\rho}_0 \\ -\tilde{\rho} - \frac{6(1-\frac{3}{4}\varepsilon)}{(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{s}{\mathcal{K}}\hat{\rho}_0 \\ \tilde{\rho}^a \end{pmatrix}^t \cdot (\mathbb{R}\cdot\mathbb{Q}^{-1t}) \cdot \begin{pmatrix} \beta^0 \\ \beta^a \\ -\alpha_0 \\ -\alpha_a \end{pmatrix}. \tag{C.19}$$

Next, we evaluate the expression of the scalar potential (C.10) for this flux background and use the Hodge duality relations for the harmonic three-forms (C.12), such that a bilinear structure in terms of the axion polynomials emerges explicitly. After the appropriate Weyl rescaling to 4d Einstein frame we obtain

$$V_{GKP} = e^{K_T+K_Q}\begin{pmatrix} \overline{\rho}_0 \\ \overline{\rho}_a + \frac{27\varepsilon}{4(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{\mathcal{K}_a}{\mathcal{K}} s\hat{\rho}_0 \\ -\tilde{\rho} - \frac{6(1-\frac{3}{4}\varepsilon)}{(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{s}{\mathcal{K}}\hat{\rho}_0 \\ \tilde{\rho}^a \end{pmatrix}^t \cdot \mathcal{G}^{-1} \cdot \begin{pmatrix} \overline{\rho}_0 \\ \overline{\rho}_a + \frac{27\varepsilon}{4(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{\mathcal{K}_a}{\mathcal{K}} s\hat{\rho}_0 \\ -\tilde{\rho} - \frac{6(1-\frac{3}{4}\varepsilon)}{(1-3\varepsilon)(1+\frac{3}{2}\varepsilon)}\frac{s}{\mathcal{K}}\hat{\rho}_0 \\ \tilde{\rho}^a \end{pmatrix}. \tag{C.20}$$

with $\mathcal{G}^{-1} = \mathcal{K}(1+\frac{3}{2}\varepsilon)\mathbb{G}^{-1}$. Note that this expression is equivalent to equation (5.18) upon rotation of the axion basis by the transformation matrix $\mathbb{T}$,

$$\mathbb{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \delta_a^b & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \delta_b^a & 0 \end{pmatrix}, \tag{C.21}$$

which equally allows to switch between the flux quanta basis $(e_0, e_a, -m, m^a)$ and (e_0, e_a, m^a, m).

Appendix D

Toroidal Orbifolds and matter Kähler Metrics

A typical set of backgrounds suited to test the ideas presented in chapter 6 consist of the orientifold version of $\mathbf{T}^2 \times K^3$ (considered at an orbifold point in moduli space) and toroidal orientifolds (or their $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolded version) with a factorisable ambient six-torus $\mathbf{T}^6$. Each of the three two-tori $\mathbf{T}^2_{(i)}$ is parameterised by periodic coordinates $(x^i, y^i) \sim (x^i + 1, y^i + 1)$ and characterised by a modular parameter τ_i. The ambient space can be equipped with a set of basis three-forms which splits up into a symplectic basis of $\Omega\mathcal{R}$-even $(\alpha_0, \beta^j) \in H^3_+(T^6/\Omega\mathcal{R}, \mathbb{Z})$ and $\Omega\mathcal{R}$-odd $(\beta^0, \alpha_i) \in H^3_-(T^6/\Omega\mathcal{R}, \mathbb{Z})$ three-forms:

$$\begin{aligned}
\alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, & \beta^0 &= -dy^1 \wedge dy^2 \wedge dy^3,\\
\beta^1 &= dx^1 \wedge dy^2 \wedge dy^3, & \alpha_1 &= dy^1 \wedge dx^2 \wedge dx^3,\\
\beta^2 &= dy^1 \wedge dx^2 \wedge dy^3, & \alpha_2 &= dx^1 \wedge dy^2 \wedge dx^3,\\
\beta^3 &= dy^1 \wedge dy^2 \wedge dx^3, & \alpha_3 &= dx^1 \wedge dx^2 \wedge dy^3,
\end{aligned} \tag{D.1}$$

under the orientifold projection $\mathcal{R} : (x^i, y^i) \to (x^i, -y^i)$. In this basis the holomorphic Calabi-Yau three-form Ω_3 reads

$$\begin{aligned}
\Omega_3 &= (dx^1 + i\tau_1 dy^1) \wedge (dx^2 + i\tau_2 dy^2) \wedge (dx^3 + i\tau_3 dy^3)\\
&= \alpha_0 - \tau_2\tau_3\beta^1 - \tau_1\tau_3\beta^2 - \tau_1\tau_2\beta^3 + i\tau_1\tau_2\tau_3\beta^0 + i\tau_1\alpha_1 + i\tau_2\alpha_2 + i\tau_3\alpha_3,
\end{aligned} \tag{D.2}$$

yielding the $\mathcal{N} = 2$ Kähler potential $K_{cs} = -\log\left(i \int \Omega_3 \wedge \overline{\Omega}_3\right) = -\log(8\tau_1\tau_2\tau_3)$ in terms of the modular parameters. The basis of $\Omega\mathcal{R}$-odd two-forms $\omega_a \in H^{1,1}_-(T^6/\Omega\mathcal{R}, \mathbb{Z})$ and their Poincaré dual $\Omega\mathcal{R}$-even four-forms $\tilde{\omega}^a$ are given by

$$\omega_a = \delta_{ai} dx^i \wedge dy^i, \qquad \omega_a \wedge \omega_b = \mathcal{K}_{abc}\tilde{\omega}^c, \tag{D.3}$$

with $\mathcal{K}_{abc} = \mathcal{K}_{123} = 1$ (and permutations thereof) the only non-vanishing triple intersection numbers. In this basis the Kähler form J reads

$$J = i \sum_{j=1}^{3} dz^j \wedge d\bar{z}^j \tag{D.4}$$

Each volume of the three two-tori is measured by the geometric part of the corresponding Kähler moduli and the overall volume of the internal space is the product of the two-tori volumes, i.e. $\mathcal{V} = t_1 t_2 t_3$. The geometric part of the complex structure moduli are given by the periods of $\mathcal{C}\Omega_3$:

$$S_\star = \int \Omega_c \wedge \beta_0 = \xi^0 + i \frac{e^{-D}}{\sqrt{8\tau_1\tau_2\tau_3}}, \qquad U_{\star i} = \int \Omega_c \wedge \alpha_i = \xi^1 + i \frac{e^{-D}}{\sqrt{8\tau_1\tau_2\tau_3}} \tau_j \tau_k. \tag{D.5}$$

with the compensator field $\mathcal{C} = \frac{e^{-D}}{\sqrt{8\tau_1\tau_2\tau_3}}$ following from the definition in the main text. For the factorable toroidal orientifolds, the Kähler potential on the Kähler moduli space and the complex structure moduli space are given respectively by the well-known expressions:

$$K_T = -\sum_{a=1}^{3} \log\left[-i(T^a - \overline{T}^a)\right], \qquad K_Q = -\log\left[-i(S_\star - \overline{S}_\star)\right] - \sum_{i=1}^{3} \log\left[-i(U_{\star i} - \overline{U}_{\star i})\right]. \tag{D.6}$$

With each $\Omega\mathcal{R}$-even basis three-form (α_0, β^j) in $H^3_+(T^6/\Omega\mathcal{R}, \mathbb{Z})$, we can introduce its de Rahm dual $\Omega\mathcal{R}$-even three-cycle (ρ_0, ρ_i):

$\Omega\mathcal{R}$ − even three-cycle	P.D.	$\Omega\mathcal{R}$ − odd three-cycle	P.D.
$\rho_0 = \pi_1 \otimes \pi_3 \otimes \pi_5$	β^0	$\sigma_0 = \pi_2 \otimes \pi_4 \otimes \pi_6,$	α_0
$\rho_1 = \pi_1 \otimes \pi_4 \otimes \pi_6$	$-\alpha_1$	$\sigma_1 = \pi_2 \otimes \pi_3 \otimes \pi_5$	β^1
$\rho_2 = \pi_2 \otimes \pi_3 \otimes \pi_6$	$-\alpha_2$	$\sigma_2 = \pi_1 \otimes \pi_4 \otimes \pi_5$	β^2
$\rho_3 = \pi_2 \otimes \pi_4 \otimes \pi_5$	$-\alpha_3$	$\sigma_3 = \pi_1 \otimes \pi_3 \otimes \pi_6$	β^3

(D.7)

and repeat the exercise for their $\Omega\mathcal{R}$-odd counterparts, which provide four $\Omega\mathcal{R}$-odd three-cycles (σ_0, σ_i). The choice of the symplectic basis of three-cycles from above also determines the Poincaré dual (P.D.) three-forms for each of the three-cycles. A generic, factorisable three-cycle with topology $S^1 \times S^1 \times S^1$ on T^6 can now be decomposed in terms of this three-cycle basis:

$$\begin{aligned} \Pi_\alpha^{\text{fact}} &= (n_\alpha^1 \pi_1 + m_\alpha^1 \pi_2) \otimes (n_\alpha^2 \pi_3 + m_\alpha^2 \pi_4) \otimes (n_\alpha^3 \pi_5 + m_\alpha^3 \pi_6) \\ &= n_\alpha^1 n_\alpha^2 n_\alpha^3 \,\rho_0 + n_\alpha^1 m_\alpha^2 m_\alpha^3 \,\rho_1 + m_\alpha^1 n_\alpha^2 m_\alpha^3 \,\rho_2 + m_\alpha^1 m_\alpha^2 n_\alpha^3 \,\rho_3 \\ &\quad + m_\alpha^1 m_\alpha^2 m_\alpha^3 \,\sigma_0 + m_\alpha^1 n_\alpha^2 n_\alpha^3 \,\sigma_1 + n_\alpha^1 m_\alpha^2 n_\alpha^3 \,\sigma_2 + n_\alpha^1 n_\alpha^2 m_\alpha^3 \,\sigma_3, \end{aligned} \tag{D.8}$$

by virtue of the torus wrapping numbers $(n_\alpha^i, m_\alpha^i)_{i=1,2,3}$, which encode the one-cycle geometry on the two-torus $T^2_{(i)}$. As reviewed in section 2.6, four-dimensional type IIA orientifold compactifications have to be equipped with spacetime filling D6-branes wrapping such three-cycles fulfilling the special Lagrangian conditions (2.64), such that their combined RR charges cancel the RR charges of the O6-planes. More explicitly, the RR tadpole cancellation conditions 2.68, for D6-branes on factorisable three-cycles read [44]

$$\begin{aligned} \sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 n_\alpha^3 &= 16 \\ \sum_\alpha N_\alpha n_a^1 m_\alpha^2 m_\alpha^3 &= -16 \\ \sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 m_\alpha^3 &= -16 \\ \sum_\alpha N_\alpha m_a^1 m_\alpha^2 n_\alpha^3 &= -16 \end{aligned} \tag{D.9}$$

D.1 Matter Kähler Metrics on the $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

On the other hand, massless open string excitations arise at the intersection points of two distinct D6-branes wrapping supersymmetric three-cycles and fill out supermultiplets of the supersymmetry algebra generated by the mutually unbroken supercharges. Furthermore, on toroidal orbifold backgrounds the Kähler metrics for these open string states can be computed as a function of the closed string moduli [100–103]. The type of matter (and subsequently the functional dependence of the Kähler metrics) depends on the codimension of the intersection $\Pi_\alpha \cap \Pi_\beta \neq 0$ in the ambient space T^6:

(i) Codimension 3 intersection:
D6-branes wrapping the three-cycles that coincide along each one-cycle on $T^2_{(i)}$ give rise to one $\mathcal{N} = 1$ chiral supermultiplet Φ^i per two-torus. The complex scalar within such a multiplet consists of the three-cycle deformation modulus complexified by the Wilson line along the S^1 cycle on $T^2_{(i)}$, as described in equation (2.69). The three chiral $\mathcal{N} = 1$ supermultiplets transform in the adjoint representation of the gauge group and combine with the $\mathcal{N} = 1$ vector multiplet into an $\mathcal{N} = 4$ vector multiplet, compatible with the maximal number of supercharges preserved by this D6-brane configuration. Two examples of such highly (super)symmetric configurations are depicted in figure D.1.

The Kähler metric for an open string modulus Φ^i along two-torus $T^2_{(i)}$ can be written (at leading order) as a rational function of the closed string

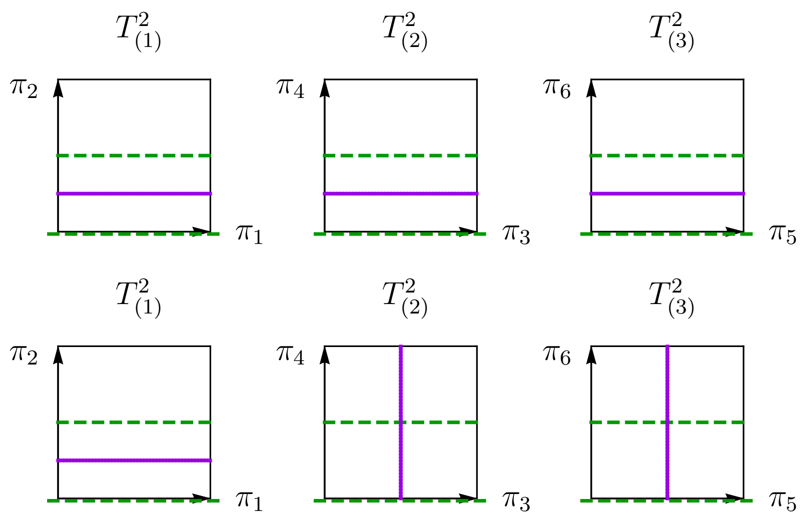

Figure D.1: D6-brane configurations with codimension 3 intersection preserve a local $\mathcal{N} = 4$ supersymmetry: an example of three-cycles with torus wrapping numbers $(1,0;1,0;1,0)$ (above) and an three-cycle example with torus wrapping $(1,0;0,1;0-1)$ (below). The O6-planes are represented by the dashed, green lines.

moduli:

$$K_{\Phi^i\overline{\Phi}^i} = -\frac{\delta_{ai}\delta_{\Lambda i}}{(T^a - \overline{T}^a)(U_{\star\Lambda} - \overline{U}_{\star\Lambda})}\left|\frac{(n^j + i\,\tau_j m^j)(n^k + i\,\tau_k m^k)}{n^i + i\,\tau_i m^i}\right|, \qquad \text{(D.10)}$$

where the last term captures the model-dependent contribution determined by the three-cycle position. In this respect, the model-dependent part of the Kähler metric will be constrained by the special Lagrangian conditions (2.64) imposed on the wrapped three-cycle. More precisely, for the two examples in figure D.1, the Kähler metrics for the two distinguishable D6-brane configurations take the form:

$$\text{ex. 1:}\quad K_{\Phi^i\overline{\Phi}^i} = \frac{-1}{(T^i-\overline{T}^i)(U_{\star i}-\overline{U}_{\star i})}, \quad \text{ex. 2:}\quad K_{\Phi^i\overline{\Phi}^i} = \begin{cases} \frac{-1}{(T^1-\overline{T}^1)(S_\star-\overline{S}_\star)} & (i=1), \\ \frac{-1}{(T^2-\overline{T}^2)(U_{\star 3}-\overline{U}_{\star 3})} & (i=2), \\ \frac{-1}{(T^3-\overline{T}^3)(U_{\star 2}-\overline{U}_{\star 2})} & (i=3). \end{cases} \qquad \text{(D.11)}$$

The main conclusion that one can draw from these examples is that the Kähler metric for a deformation modulus Φ^i is a homogeneous function of degree -1 in the Kähler moduli and of degree -1 in the complex structure moduli (including the dilaton). This statement is true in general for the Kähler metric (D.10), since the model-dependent part is independent of the

Kähler moduli and a homogeneous function of degree zero in the complex structure moduli (upon inclusion of the dilaton).[1]

(ii) Codimension 5 intersection:
D6-brane stacks wrapping two distinct three-cycles Π_α and Π_β that coincide on a one-cycle S^1 along one of the three two-tori and intersect at a point along the remaining four-torus, give rise to a non-chiral pair of $\mathcal{N} = 1$ chiral supermultiplets. The chiral multiplets transform in bifundamental representation and are each others conjugate, such that they combine into a $\mathcal{N} = 2$ hypermultiplet. This feature is a remnant of the local $\mathcal{N} = 2$ supersymmetry preserved by the D6-brane configuration, for which an explicit example is presented in figure D.2.

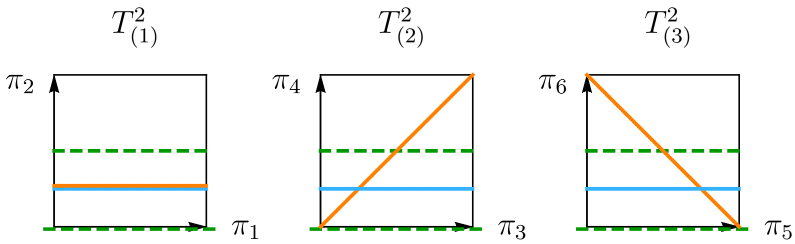

Figure D.2: D6-brane configurations with codimension 5 intersection preserve a local $\mathcal{N} = 2$ supersymmetry. The O6-planes are represented by the dashed, green lines.

The Kähler metric for such an $\mathcal{N} = 2$ hypermultiplet is given (at leading order) by a (non-rational) function of the geometric part of the closed string moduli:

$$K_{\alpha\overline{\beta}} = \frac{|n^i + i\,\tau_i m^i|}{\sqrt{(U_{\star\Lambda} - \overline{U}_{\star\Lambda})(U_{\star\Sigma} - \overline{U}_{\star\Sigma})(T^j - \overline{T}^j)(T^k - \overline{T}^k)}}, \tag{D.12}$$

where (n^i, m^i) denote the wrapping numbers along the two-torus $T^2_{(i)}$ where the two three-cycles coincide on an S^1. The Kähler metric allows for a factorisation in terms of the complex structure moduli and the Kähler moduli, such that it is a homogeneous function of degree -1 in the complex structure moduli (upon inclusion of the dilaton) and a homogeneous function of degree -1 in the Kähler moduli. This case also applies to the Kähler metrics for chiral matter in the symmetric or antisymmetric representation

[1]The same scaling properties can be found in the Kähler metrics for the deformation moduli of non-factorisable three-cycles.

located at the intersection of a D6-brane with its orientifold image, whenever the three-cycle is parallel (or orthogonal) to the O6-plane along one single two-torus.

(iii) Codimension 6 intersection:
D6-brane stacks wrapping two distinct three-cycles Π_α and Π_β that intersect point-wise in the ambient space provide for a chiral $\mathcal{N} = 1$ supermultiplet at each independent intersection point of the six-dimensional compactification space. A simple example of a D6-brane configuration for which the intersection set has codimension 6 is presented in figure D.3. The chiral multiplet transforms in the bifundamental representation and its Kähler metric takes the following form:[2]

$$K_{\alpha\overline{\beta}} = \frac{1}{\sqrt[4]{(S_\star - \overline{S}_\star)(U_{\star 1} - \overline{U}_{\star 1})(U_{\star 2} - \overline{U}_{\star 2})(U_{\star 3} - \overline{U}_{\star 3})}} \prod_i \frac{C^{(i)}_{\alpha\beta}}{(T^i - \overline{T}^i)^{\frac{1}{2}}} \tag{D.13}$$

with the model-dependent coefficients $C^{(i)}_{\alpha\beta}$ per two-torus defined as,

$$C^{(i)}_{\alpha\beta} = \left(\frac{\Gamma(|\vartheta^i|)}{\Gamma(1 - |\vartheta^i|)} \right)^{\lambda_i}. \tag{D.14}$$

The parameter $\vartheta^i_{\alpha\beta}$, chosen in the range $0 < |\vartheta^i_{\alpha\beta}| < 1$, measures the angle between the two intersecting one-cycles on two-torus $T^2_{(i)}$ (in units of π), while the constant $\lambda_i = \pm 1$ takes into account the sign of $\vartheta^i_{\alpha\beta}$.

In this case, the Kähler metric factorises into a homogeneous function of degree -1 in the complex structure moduli (upon inclusion of the dilaton) and a homogeneous function of degree $-\frac{3}{2}$ in the Kähler moduli. The model-dependent coefficients $C^{(i)}_{\alpha\beta}$ are homogeneous functions of degree 0 in the complex structure moduli and the Kähler moduli. When a three-cycle intersects with its orientifold image at three non-trivial angles, the corresponding Kähler metrics for the chiral matter states in the symmetric or antisymmetric representation take the same form as (D.13).

[2]In the literature on Kähler metrics one might also stumble on expressions in which the exponents of the Kähler moduli obtain an additional contribution from the angles ϑ^i between the two three-cycles. The (potentially) modified exponents are related to a (potential) four-dimensional field redefinition of the Kähler moduli to arrive at their proper supergravity equivalents. Given the confusion within the literature itself about these ϑ^i-dependent corrections and the fact they do not alter the overall scaling properties of the Kähler metrics, we have decided not to take them into account explicitly.

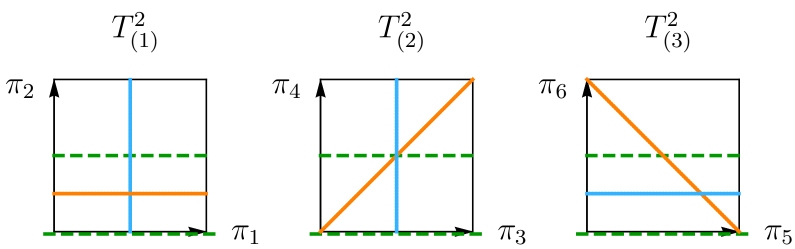

Figure D.3: D6-brane configurations with codimension 6 intersection preserve a local $\mathcal{N} = 1$ supersymmetry. The O6-planes are represented by the dashed, green lines.

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