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The instrumental genesis process in future primary teachers using dynamic geometry software

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ABSTRACT

This article, which describes a study undertaken with pairs of future primary teachers using GeoGebra software to solve geometry problems, includes a brief literature review, the theoretical framework and methodology used. An analysis of the instrumental genesis process for a pair participating in the case study is also provided. This analysis addresses the techniques and types of dragging used, the obstacles to learning encountered, a description of the interaction between the pair and their interaction with the teacher, and the type of language used. Based on this analysis, possibilities and limitations of the instrumental genesis process are identified for the development of geometric competencies such as conjecture creation, property checking and problem researching. It is also suggested that the methodology used in the analysis of the problem solving process may be useful for those teachers and researchers who want to integrate Dynamic Geometry Software (DGS) in their classrooms.

Keywords: teacher training; geometric competencies; instrumented activity; DGS; GeoGebra.

Introduction

In the Math education research community, there is an interest in the development of mathematical knowledge of teachers, as this is an area where serious shortcomings have been detected. This article describes some aspects of a study conducted to improve the

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acquisition of didactic geometry competencies in future primary education mathematics teachers. It specifically focuses on an analysed case study in order to delve into the geometry problem solving process using the GeoGebra software.

Dynamic Geometry Software (hereafter, DGS) is designed to dynamically construct and modify Euclidean geometry figures. The geometric properties and relationships between the objects used in a construction are maintained upon manipulating an object and as a result, the objects are modified in a dependent manner.

According to González-López [1], the use of DGS in geometry teaching has changed the nature of general knowledge studied with respect to the pencil and paper context. DGS allows for searching some properties that are maintained by the figure under the action of *dragging*, properties that the user can observe or predict. Assude et al. [2] agree that DGS is not a simple means of interaction between the student and the represented mathematical objects, but may also modify the manner in which the mathematical activity is carried out with respect to the traditional teaching of geometry where pencil and paper are used: it conditions student actions, modifying their conceptualizations and learning.

Studying geometry in the DGS environment is unique due to the dynamic nature of the objects. In order to understand its complexity, it is necessary to differentiate between the concepts of *drawing* and *figure* as well as the importance of *dragging.* Laborde and Capponi [3] define *figure* as a set of established relationships between a geometric object and all of the drawings associated with it. According to Strässer [4], dragging facilitates mediation between the figure and the drawing, in a way that the figure maintains its properties even while subjected to the dragging process. The evolution of the use of dragging to solve geometry problems reveals the cognitive processes that are involved in the problem solving process.

Another intrinsic aspect of DGS is its response to users' actions. Therefore, Laborde [5] introduced the term *retroaction* to define the interaction between a student and the DGS, offering information regarding the student's construction and eliciting a response from the student which enables him/her to progress in the learning acquisition process. It is important to note that retroaction is derived from the DGS and is independent from the teacher.

It has been demonstrated that the integration of technology in mathematics teaching-learning requires that special attention should be paid to a number of aspects. One such aspect is the potential influence of the DGS on abductive reasoning and the ability to justify geometric properties. Some authors affirm that the dynamic geometry environment may promote empirical justifications that discourage the need for formal proofs [1,6], although it offers an environment that promotes free student experimentation, leading to the development of non-traditional learning forms for mathematical procedures and concepts. Other authors have suggested that DGS benefits students by facilitating visualization, exploration and comprehension processes that are necessary to formulate conjectures, thereby improving their argumentation and verification abilities [e.g. 7-10].

In any case, researchers agree that it is important to study the integration of DGS at all education levels. Likewise, close attention should be paid to the design of appropriate tasks and the creation of didactic processes that take the function of the teacher and classroom management [11] into consideration.

As for DGS inclusion in teacher training, Pandiscio [12], who examines future secondary school teacher perceptions, focuses on the need for -- and benefits of -- formal geometric proofs. Evidence reveals that participants do not believe that secondary school students need to undergo formal proof after using a DGS, although they do recognize the difference between repeated property testing and general proofs. Jiang [13] also examines how DGS changes the view of future secondary school teachers regarding mathematics and its teaching. After using DGS to solve geometry problems, students turn out to have improved their reasoning abilities and were more capable of proving properties and theorems. Along the same lines, Haja [14] explores geometric problem solving skills with Cabri II in four future secondary school teachers. The results demonstrate that the future teachers acquire the needed knowledge of geometric content in order to construct dynamic shapes and apply conjecture knowledge to geometric shapes. Evidence also shows that they were capable of using DGS to verify the determined solutions.

In conclusion, considerable research has been conducted as regards the benefits provided by dynamic environments in geometry teaching. The changes occurred as a consequence of the DGS are not only a question of motivating students and making learning more attractive; it also transforms classroom teaching and the characteristics of knowledge constructed by students. However, research studies still have to be elaborated upon these aspects in the initial training of primary school teachers. Most studies focus on secondary school education and teacher training at this level, but we believe that the results can't be extrapolated to pre-service primary school teachers because they lack the necessary geometric knowledge to take advantage of the benefits offered by the DGS. On the other hand, DGS can be used to help them improve their geometric knowledge.

The present research is based on a two-phase study. The first phase examines the development of geometric competency in a GeoGebra environment as compared to competencies obtained during traditional paper and pencil work. The researchers hypothesize that the richness of the dynamic geometry opposed to static geometry could permit the improvement of future primary school teachers' geometric knowledge. The second phase addresses geometric knowledge mediated by GeoGebra DGS. A study with four selected pairs of pre-service teachers with a plane geometry problem was carried out. This paper, which focuses on the case study of one of these pairs, analyses their instrumental genesis process in a shape construction task.

Theoretical framework

This research uses the *theory of instrumented activity* [15] as a theoretical framework. Based on a neo-Vygotskian approach, this theory considers situations in which an instrument is used to measure actions between a subject and an object. The authors define an instrument as any technological system used by a subject in order to complete a task. That is, an instrument, which is an intermediary between the subject and the object, produces and/or modifies the action and has characteristics associated with the operations carried out by the subject and the object in the context of the task undertaken.

It is important to distinguish between two key concepts: artefact, which tends to be a material element, and instrument, which is a psychological construct. An instrument does not exist as such; that is, an artefact (a machine or technological system) is not the subject's instrument, even though it was created for such purposes. The subject should take ownership of it and integrate it into the resulting activity. The instrument is the result of the instrumental relationship established by the subject with the artefact, be it material or not.

In our study, the subjects are the pair student (future primary teacher), the object is the plane geometry problem to be solved and the artefact is the GeoGebra DGS used by the students in the problem solving process.

According to Artigue [16], the subject does not initially consider the artefact to be an instrument; this occurs when instrumental genesis is produced and the subject constructs his/her own personal schemes or incorporates pre-existing social schemes. Thus, the instrument is mixed by nature: part artefact and part cognitive schemes. Here, the concept of scheme is similar to the one described by Vergnaud [17]: an invariant organization of activity for a given class of situations.

The instrumental genesis process works in two directions. First, it is directed from the subject to the artefact. This is the process of *instrumentalization*, in which the subject progressively learns to use the artefact, and may transform it for specific uses developing *usage schemes*. Second, it is directed from the artefact to the subject and is the process of *instrumentation*, in which the subject develops or incorporates so-called *instrumented activity schemes* from cognitive schemes. These are subsequently transformed into techniques (a set of procedures used to solve a determined type of problem) to attend to the task at hand. Rabardel [18,p.29] summarized it as follows:

> Instruments result from a development process (and not only a learning process), which occurs through instrumental geneses. Instruments born of instrumental geneses organize the coordination of the artefact's and the subject's actions, allowing them to be pertinent and efficient mediators for the subject's activity…. Instruments are both private, meaning specific to each individual, and social. The social nature of instruments is due to the social nature of artefacts, usage schemes and instrumented activity schemes. These schemes are social in that they have characteristics that are both shared and widespread in communities and collectivities.

According to Drijvers [19], instrumented activity schemes are defined as significant and coherent mental schemes aimed to resolve a specific type of task with the use of an artefact. The development of such schemes is at the core the of instrumental genesis process. In this paper, according to Drijvers et al. [20], we see techniques as the observable part of subjects' work that should solve a given task, and schemes as the cognitive foundations of these techniques that are not directly observable, but can be inferred from the subjects' activities.

In classroom practice, the development of instrumental genesis is quite complex for students as it is time and effort consuming [20]. Also, it is quite difficult to distinguish between the processes of instrumentation and those of instrumentalization: instrumented activity schemes and usage schemes are interrelated and, in some cases, it is difficult to distinguish between the types of schemes developed. Students may construct schemes that are inappropriate, inefficient or based on erroneous concepts. Therefore, the teacher's role, known as *orchestration* [21], refers to the teacher's intentional and systematic organization and the use of the various artefacts available in a learning environment for a given mathematical task in order to guide the individual and the collective class instrumental genesis process. Orchestration may influence student selection of a number of techniques, and not others.

As for the techniques of instrumental genesis, in order to analyse the use of the GeoGebra dynamism when faced with a geometry construction, in this study we have used the dragging modalities introduced by Arzarello et al. [22]:

- *Wandering dragging:* it moves a draggable point or a random object on the screen, without a specific plan, in order to discover properties or interesting regularities of the figure.
- *Bound dragging:* it moves a semi-draggable point, that is, a point that is already linked to an object and therefore is not free moving. For example, a point belonging to a circumference may move upon it.
- *Guided dragging:* it moves the draggable points of a figure in order to give it a specific shape. For example, the vertices of any quadrilateral may be moved to

turn it into a square. This type of dragging is normally used in the discovery phase, when the student investigates and explores how to solve the task at hand.

- *Lieu muet dragging:* moves a draggable point of a figure in order to make it comply with a determined property. The point is considered to be a hidden geometric site even if the student is not aware of its existence.
- *The dragging test:* it moves draggable or semi-draggable points to check whether the figure maintains its initial properties or not. If the figure passes the test, it has been constructed in accordance with the requested properties. This type of dragging is primarily used to verify or check conjectures.

Purpose and methodology

The purpose of the study is to analyse the instrumental genesis process of a pair student faced to a shape construction task with GeoGebra, demanding the competencies of conjecture formulation and verification. As written in the Introduction, our research was developed in two phases:

First phase

A hundred future primary school teachers received disciplinary and didactic training on the content of Geometry and Measuring in the course of one term. Fifty of them (control group) participated in weekly 90-minute seminars during which problems with traditional paper and pencil methodology needed to be worked put. The other fifty (experimental group) worked in regular pairs solving the same problems with GeoGebra in weekly 90-minute seminars (GeoGebra Workshop). In this way, we could test the hypothesis that the richness of the dynamic geometry opposed to static geometry will enable us to improve our future primary school teachers' geometric knowledge. A report of this phase can be found in Ruiz-López [23].

From the *theory of instrumented activity* perspective [15], the purpose of GeoGebra Workshop was to foster the creation of instrumentalization and instrumentation processes, components of instrumental genesis, as previously explained in section "the theoretical framework." The fundamental rationale behind the selection of future teachers as participants is that teacher training and experience were identified as an important explanatory factor in determining the success or failure of the students' mathematical learning in a technological environment. Future teachers, who should have these skills, should not only develop their own adequate personal instrumental genesis processes but also facilitate the instrumental genesis of their students [19].

According to Drijvers [19], students find it quite hard to develop instrumental genesis in classroom practice because it is time and effort consuming. Therefore, a full term GeoGebra Workshop was needed. Orchestration process was especially important in the design of this seminar, and one of the researchers had to be the teacher. The control of instrumental genesis process of each pair of students was intentionally and systematically granted.

Second phase

This paper describes a case study analysed in this part of our research that focuses on the evaluation of the instrumental genesis process.

To assess the depth and limitations of this process, four pairs were chosen from the participants in the first phase. Previous geometry (GS) and digital literacy (DL) skills of both students were taken into consideration before selecting these four pairs [24]. Geometric problems of the standard assessment questionnaire TEDS-M [25] were used to measure GS, while ad hoc instruments (a GeoGebra competence test and qualifications of an ICT course) were applied for DL. A number of aspects were preferred over others on the basis of a potential interest in posterior subsequent analysis:

the pairs had to be heterogeneous (although one homogeneous pair was also included) and had to have the greatest possible range of GS and DL level combinations. Also, it was considered of vital importance that all of the pairs had regularly attended the GeoGebra seminars.

Upon selection, student candidates were informed of the characteristics of the study and they accepted to participate and to be videotaped. The teacher responsible for the GeoGebra Workshop explained the instructions to each pair and videotaped them. Each pair could use a laptop with the GeoGebra software installed in it and a pencil and paper to solve the assigned problem. The teacher allowed time for considering and discussing the problem-solving process. The teacher intervened to check on them for short periods of time, to make suggestions or to see whether the students were having any problems. The four pairs needed approximately one hour to complete the problem.

The rationale behind the use of pairs, both in the GeoGebra seminars as well as in the assessment test, was to allow individuals to interact with each other when they shared a computer resource. We felt that the instrumentation and instrumentalization processes would be more transparent if the individuals had to agree upon their conjectures and knowledge in order to solve the task.

The test consisted of solving a geometry-related problem (classified as a figure construction activity, conjecture and research) as shown below:

(1) Using the GeoGebra 'regular polygon' tool, construct a blue square. Can you inscribe another (red) square inside it? Record all the steps taken with GeoGebra on this paper (indicating the GeoGebra tool used in each case), including the steps taken when creating not useful constructions that had been discarded.

(2) Can you inscribe more squares inside the blue square from the previous activity? Complete the construction, recording all the steps taken conducive to the accomplishment of this (even those that had been discarded).

This problem was proposed in order to consider transfer knowledge from previously-acquired competencies, necessary to solve part 1, and generalization ability as needed for in part 2. We feel that the content-related problem is quite appropriate for students who will be primary school teachers and will therefore have to lead teaching-learning processes involving geometric knowledge – i.e. polygons– with special curricular emphasis on squares.

Data collection instruments considered for the analysis were:

- the construction protocol for the figures obtained in each pair's GeoGebra file
- the personalized protocol written by each pair including notes on the steps followed in the problem-solving process, GeoGebra tools used and trials carried out (even when they were discarded at a later stage)
- video-taping of the interactions between the two members of the pair and between the pair and the teacher

With all of this information, a research protocol was created for each pair, including all previously collected aspects, in order to reconstruct the session as accurately as possible.

As described in the previous section, in the instrumental genesis process, learning is the complementary elaboration of techniques for artefact use and of cognitive schemes that integrate pragmatic and epistemic knowledge. This elaboration arises from the need to solve a mathematical problem when an artefact is used. Trouche [26] claims that when students use artefacts, instrumented action schemes are formed;

for example, in our case, a regular polygon scheme is formed when GeoGebra is used. This scheme is made up of gestures for artefact use, rules of action and conceptual elements.

In addition, Drijvers et al. [20] suggest that techniques are the observable part and schemes are the cognitive foundations inferred from student actions. In accordance with this techniques-schemes duality, in our study, observation of the solving process for the indicated problem was based on the following 6 analysis categories:

(1) Techniques used
(2) Types of dragging
(3) Obstacles to learning
(4) Interaction between the pair
(5) Language used in the personalized protocol
(6) Teacher-pair interaction

The first two categories (Techniques used and Types of dragging) collect information on the techniques used in the interaction between Subjects-Geometric Problem-GeoGebra. Categories (3), (4) and (5) collect information regarding the pair's mental schemes. Schemes are not directly observable but difficulties arising from the same may be seen, in the conceptual obstacles expressed in the pair interaction and in the language used by the students in the personalized protocol.

The theory of instrumented activity is complemented by the idea of instrumental orchestration – i.e. the intentional and systematic behaviour of the teacher in order to promote and guide the student's instrumental genesis process. Thus, a sixth analysis category was created (Teacher-pair interaction). This category permitted the analysis of problem solving behaviour based on the techniques-schemes-orchestration triad, which is the basis of this theory.

These 6 analysis categories have been collected in tables to facilitate the task of summarizing the developed session for each pair, looking at their behaviour in each of the noted areas. From these tables, it was possible to obtain information in order to expand the instrumental genesis process conducted for each particular case studied.

Only a detailed analysis conducted on one of the pairs is presented below. We selected pair 25 since, of the four pairs participating in the research, this pair best represented the most frequent GS and DL category levels of the experimental group (formed by 25 pairs).

Results: Analysis of pair No. 25 activity

Due to the length and complexity of the solving-problem process, the analysis of the process has been divided into three parts: In the parts II and III, we divided the process into phases in order to better analyse the techniques used by pair No. 25.

Some fragments of the protocol, upon which this analysis was conducted, are included. In these, the researcher's comments are written in black; a transcription of the conversation between the students and the teacher is written in blue; the text written by the pair in the personalized protocol for the problem-solving process is written in red; and the red squares include notes made by the researcher while conducting the analysis.

Case study session (1st part). Pair 25: Helena and Lorena

Helena is working at her computer while Lorena is reading the problem. They start the construction:

H: Regular polygon?

L: Yes, blue

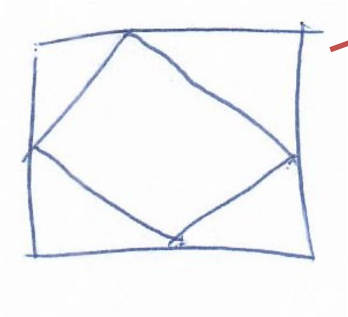
H: Okay

They use 'regular polygon' to construct a blue square

L: Can you inscribe another (red) square inside it? I guess that would be…

H: Do you want a piece of paper?

L: If this is the square, it would be like this (Lorena makes a drawing on a sheet of paper)



L makes a drawing on paper to explain to H the possible solution

H agrees and suggests finding the middle points of each side of the blue square

H: I thought so too. To do that, what I did in a lesson was to find the middle points of each side and …

L: And make segments

H: And make… No!..., of course

L: Then join the segments…

H: Well, I don't know whether we should make segments or make it like this, out of 4 so that it goes together, you see?

H suggests uniting the 4 points by using a 'regular polygon'

L: Yes, so instead of making segments, since we have the points, we will make a regular polygon.

H: Of course

L: Okay then, let's do it.

H: You have to write it down

L: What should I write?

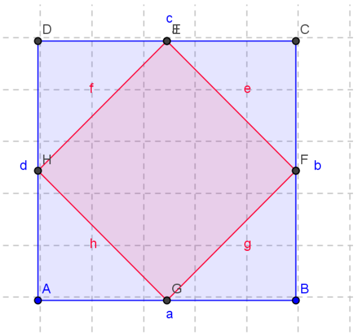
H: Please, write what we said.

L uses more self-assured language than H

Helena makes the construction in silence while Lorena writes the auto-protocol:

The language used is appropriate and concise.

1. To find the inscribed square, we will find the middle point of each side of the square and then we will make a square using the 'regular polygon' tool



The teacher asks them if they have solved it. Suddenly, L is not sure whether the square complies with the conditions, but is quickly convinced.

Teacher: Have you completed the first part? Have you found an inscribed square?

H: Yes, I have.

L: Hold on, I'm not sure whether we did it correctly. It should have its vertices in each of the sides of the blue square, shouldn't it? Yes, okay. Is it like that?

T: Yes, the first part is the easy part.

Figure 1. Protocol of case study session (1st part)

Table 1 summarizes the 6 established categories that permit the analysis of the problem solving process used by the pair:

1st Part of the problem	Using the GeoGebra ‘regular polygon’ tool, construct a blue square. Can you inscribe another (red) square inside it?
Techniques	They construct a blue square from a ‘regular polygon’ tool. They find the middle points of each side and join these points using a ‘regular polygon’. (They select 2 points and enter 4 vertices).
Types of dragging	They do not use any type of dragging.
Obstacles	L proposes that they join the middle points using the ‘segment’ tool but H realizes that they can use a ‘regular polygon’.
Pair interaction	H moves the mouse and L makes a drawing on paper and writes in the personalized protocol. The problem solving process is fast and they work together.
Teacher-pair interaction	The teacher does not intervene in this part. She only confirms that they have come up with a solution.
Language of the personalized protocol	The geometry language used is appropriate and concise. The oral exchange reveals that L is more versed in this kind of language than H.

Table 1. Analysis of the 1st part of the problem Pair No. 25

Like the other pairs studied, pair 25 uses the technique of specifying the middle points of each side of the blue square in order to inscribe the vertices of the red square inside it. Their originality with respect to the other cases is that before constructing the red square with GeoGebra, they used a pencil and paper to draw a possible solution. Apparently, L feels the need to rely on drawings, probably because she is more used to it, and therefore this was more natural for her. Also, this pair selected a different tool to join the middle points and to construct the red square: instead of selecting ‘polygon’ they used ‘regular polygon’. They encountered virtually no obstacles in this part of the problem, only a slight discrepancy when selecting which tool to use to join the points. L first suggested that they should use ‘segment’, and H quickly points out that they should use ‘regular polygon’ and no other problems arise.

Pair 25 did not use any dragging tools in this part, nor did they check the solution at the end of the process. This may account for the fact that they have preferred to construct the square with ‘regular polygon’. This clearly shows that it is a real square, so there is no need to check on this. However, they should make sure that the third and fourth midpoints do indeed lie on the red square.

The interaction between the students is adequate as they work together at all times, although they have clearly divided roles when moving the mouse and writing or drawing. Apparently, L has a greater command of geometry-related language than H, although the latter writes in the personalized protocol and does so quite correctly. The teacher does not intervene in the process at all, just at the end to check whether they have come up with a solution. She does not ask for double checking nor for the use of the ‘dragging test’ technique. She does not ask them either to describe the process that they have followed.

Part II of the Problem		Can you inscribe more squares inside the blue square?
Techniques	Phases	
	1	They try to transfer previously acquired knowledge by drawing (geoboards), without using any GeoGebra tool (Fig.2). They realize that the squares are not inscribed in the blue square.
	2	They begin to use GeoGebra, using a ‘regular polygon’ on 2 points of 2 sides of the blue square and enter 4 vertices. They construct 2 squares in this way, but they see these squares are not always inscribed (Fig.2).
	3	They specify 2 points on 2 sides of the blue square and by using ‘segment between 2 points’, they construct a square. They realize that this does not work and decide to use a ‘regular polygon’. The teacher shows them that they are not inscribed (Fig.2).
	4	They construct many squares, they specify the middle points in the middle of 3 of the sides of the blue square, and remember the ‘geoboards’. The teacher says that they are not inscribed (Fig.2).

	5	Through the use of the 'guided dragging' technique they explore, once again, the positions that a 'truly inscribed' square may occupy and realize that there are infinite positions (Fig.3).
Types of dragging		They use the dragging test to make sure that the constructions in all the phases of this second part of the problem are OK. They use 'guided dragging' to place the inscribed squares in the blue square in phases 2 and 5, to explore all the positions they can occupy.
Obstacles		In phases 3 and 4 they forget that the constructed figures should be inscribed inside the blue square.
Pair interaction		H handles the software while L writes in the personalized protocol and draws. H also seems to take the initiative when she decides which procedure to use for the problem solving (phase 4).
Teacher-pair interaction		In phase 1, the teacher explains what the fact that the red square is inscribed in the blue one means. She encourages them to use GeoGebra instead of pencil and paper, although both are allowed. She does not intervene until the end of phase 4 when she tells them that the solutions they have come up with are not inscribed squares. The teacher requests them to explore other solutions using 'guided dragging'. In phase 5, she asks how many inscribed squares they can construct and requests that they should make one as a general example.
Language of the personalized protocol		The language used is brief and concise. It is generally correct although it is not clear enough in phase 4. They forget some of the tools used. Their oral language is quite correct; they do not repeatedly use incorrect terms.

Table 2. Analysis of the part II of the problem Pair No. 25

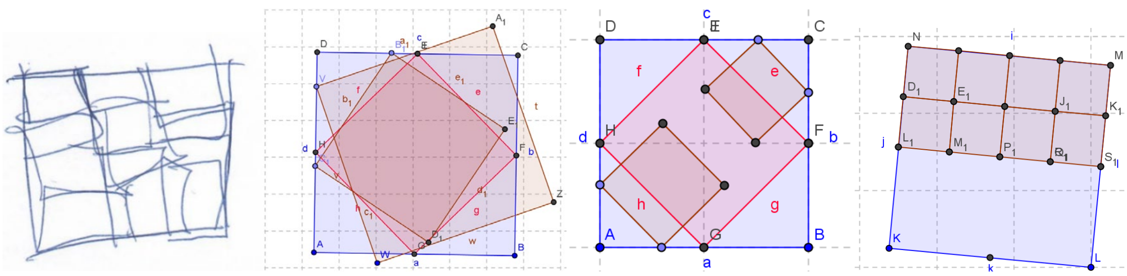


Figure 2. Phases 1, 2, 3 and 4 – part II of the problem Pair No. 25

In this second part of the problem solving process (Table 2), pair 25 develops a number of techniques on their own initiative that show the evolution of their mental schemes activated while solving the problem. The teacher's intervention is limited to an initial explanation, to make sure that they understand the task and in the final part when she shows them that their solutions are not in fact squares inscribed in the blue square. On two occasions, one of the students, L, attempts to transfer procedures from activities with other teaching aid (geoboards). In phase 1, L makes the first pencil-sketch in Figure 2 but her partner has doubts as to whether the squares are inscribed, thus they abandon the idea. Nevertheless, in phase 4 she returns to the same construction with GeoGebra and now it appears that neither of them question the validity of this solution (Figure 2).

However, they do realize that the squares are not inscribed when they are dragged in phase 2, when using 'guided dragging' to inscribe various squares inside the blue one. All the pairs used this procedure, but only in this case did the pair do it without being told, that is, without the teacher's assistance. Whatever the case is, they fail to explore all the possibilities and carry on trying to construct inscribed squares using other methods, without realizing that there is another possibility of coming up with a general solution for the problem.

In phase 3, they look for specific squares once again, overlooking the fact that they should be inscribed in the blue square. They believe that they have found a solution until the teacher intervenes and shows them that the figures are not actually inscribed. They are, however, quick enough to understand the conditions imposed by the problem and, in phase 5, they are able to conjecture that an infinite number of squares may be potentially inscribed in the initial blue square (see Figure 3).

Case study session (2nd part- Phase 5)

Teacher: How is it going?
L: Well, we have tried twice (In relation to phases 3 and 4, see Fig. 2)
H: But we don't know whether…
L: We aren't very sure about one of them
H: We have inscribed these.
T: But these are not inscribed. They should have each vertex in a side of the blue square.
H: Okay, then they're all wrong. These don't have any, either…
T: No, they should be like this: each vertex in a side. We had decided that this was the condition for inscribing.
L: Ah, okay. But, then, it would be the same as this.
T: That's the point, should it be the same as the red one or can we find another square in another position with a different size?
H: A different size is impossible because if it is a square, it should have the same length in order to get from one side to the other square sides.
T: Can't it? Check whether that is the case. Perhaps it is a good idea to have a square and make it comply with that condition. Can this one move?
H: Yes, it can.
Teacher: No, I mean this little one.
H: I don't know what you want us to move.
L: The one in the points of the square.
T: That, if it can move. That's it, look, look at it! There you have it, there is a position, you see? You forced it… erase all the others and leave only one and force it to be well-positioned, touching… Is that a square or it is not? Did you make it using the regular polygon tool?
H and L: Yes, we did.
T: Then, it is a square, you are right. It may look like a Square, but it is not actually one, but if you constructed it using that tool…Look, it is a square and it touches the sides of the blue square, you may be able to construct it. Is that the only possibility? Move it… move it in a different way… try this, for example.
Helena moves the square, following the suggestions made by the teacher and Lorena quietly looks at the screen.
L: It also touches it.
T: It also touches it, doesn't it?
L: Sure.

> The teacher asks them what they have done and tells them that the squares are not inscribed

> H realizes that none of them are inscribed

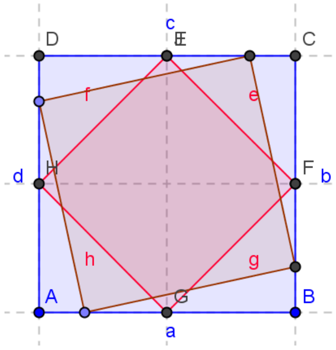
> L thinks that the only inscribed square is the one constructed in the middle points

> The teacher asks if more squares may be inscribed

> The teacher urges them to explore other positions using 'guided dragging'

> The teacher asks if the figure is 'really a square'

> They repeatedly use "guided dragging"



T: You see? How many squares do you think there could be?
H: Well, from this distance to all of that, I mean …

> H points at side AB on the blue square

T: Yes, that's it, move it here, where this other one is, that one… move it here.
L: Not really, because then it goes out
T: Move it here, like this. What happens?
L: There are is an infinite number of squares.

> L answers that there is an infinite number of squares inscribed within the blue square.

T: Ah, you see? Well now go and construct only one square that can move like you are doing, but always with the four vertices in the blue sides. Is that clear?

> The teacher asks them to construct a general inscribed square and they immediately understand

H and L: It is.
T: Great, and the other…
L: We'll erase it.
Helena begins making the changes and Lorena notes what they have been doing throughout the auto-protocol.

Figure 3. Protocol of case study session (part II - Phase 5)

The main obstacle for this pair seems to be that they overlook the condition that the squares should be inscribed. In the personalized protocol, they write the steps as they are taken, with accuracy and using the appropriate language, although they do forget to refer to some GeoGebra tools. As for the interaction between the pair, they continue to behave as they did in the first part, although L seems to be more influential in the selection of the geometric procedure, particularly in her insistence in constructing the squares '*as in the geoboards practice session.*'

In the third part of the solution process, pair 25 encountered considerable difficulties in coming up with a solution. A summary of the 5 phases in which we have divided the process is reported in Table 3.

<table>
<tr><th colspan="2">Part III of the Problem</th><th>Figure generalization: construct a dynamic red square that is inscribed in the blue square</th></tr>
<tr><td rowspan="6">Techniques</td><td>Phases</td><td></td></tr>
<tr><td>1</td><td>They attempt to transfer procedures used in the previous practice sessions, but they are unsuccessful.</td></tr>
<tr><td>2</td><td>They construct 2 points in the 2 sides of the blue square and use ‘regular polygon’. They go back to the case constructed in part 2.</td></tr>
<tr><td>3</td><td>They construct diagonals from a free point using ‘straight line’ and ‘parallels’ to the axes, but are unable to determine the centre of the square. They make sure that it is a square by measuring the angles. They determine the diagonals of the blue square and the free point in one of its sides and erase everything else.</td></tr>
<tr><td>4</td><td>They construct a diagonal of the inscribed square using ‘straight line passing through 2 points’: the free point and the centre point of the blue square. They make various attempts but cannot construct a square that maintains its form after ‘the dragging test’ (Fig. 4).</td></tr>
<tr><td>5</td><td>They make the 2nd diagonal perpendicular to the first, intersecting at the centre. They determine the 4 vertices of the square using ‘intersection between two objects’. They join the vertices with ‘polygon’. To check the solution they use ‘the dragging test’ and measure the 4 angles of the square (Fig. 4).</td></tr>
<tr><td colspan="2">Types of dragging</td><td>In phase 2, ‘guided dragging’ to inscribe the square in the blue one and ‘the dragging test’ to make sure that the figure is maintained.
In all phases, they use ‘the dragging test’ to check the distinct figures.</td></tr>
<tr><td colspan="2">Obstacles</td><td>Ph.1: H has difficulties tracing parallel lines, and wants to make a ‘parallel line to a point’. They have some communication difficulties: L tries to explain something to H who responds “I don’t know what you are saying and I don’t remember.”
Ph.2: They confuse vertex with angle, they have considerable difficulty in identifying the diagonals and in establishing their relationships with opposite vertices.
Ph.3 and 4: They continue to have difficulties with diagonals. L and H forget that they should intersect at the centre of the square.
Ph.5: They want to join the 4 vertices of the solution square using ‘segment’, the teacher suggests that they use the ‘polygon’ tool.</td></tr>
</table>

Pair interaction	Ph. 1: L helps H draw the parallel lines. L suggests other procedures that are not useful. Ph. 2: L answers the teacher and takes the initiative and moves the mouse in order to make the construction. H helps but she seems to be more adept at using the DGS. Ph. 3: The two take turns in using the DGS. L becomes discouraged and H repeats the teacher's instructions. Ph. 4: H answers the teacher and establishes the properties that would allow them to solve the problem, but H still is not capable of applying these properties (the 2 squares must have the same centre). Ph. 5: H makes the construction, answers the teacher and writes the personalized protocol. Apparently, H has taken the initiative when confronted with L's discouragement (although L also participates in the process).
Teacher-pair interaction	Ph. 1: The teacher reminds them that the inscribed square should move within the blue square. Ph. 2: The teacher guides the students to establish relationships between the vertices and the diagonals of the inscribed square. Ph. 3: The teacher directs them to recall that diagonals intersect at the centre of the square. Then, she leaves them alone. Ph. 4: She guides them until H realizes that both squares should have the same centre. Finally, she encourages them to continue because she sees that they have become discouraged saying "*you have almost got it*." Ph. 5: The teacher shows them that the figure that they have constructed in ph. 4 is not a square as one of the diagonals does not cross through the centre. She tries to relax them so that they continue step by step, giving them advice as to which tool to use to join the vertices of the square.
Language of the personalized protocol	They do not record all the procedures followed, but they do record the most relevant ones used. At times, their language is unclear, reflecting the difficulties they have encountered throughout the process. When describing the construction that has led them straight to the solution, they forget to explain that the diagonals intersect at the centre of the square. Their oral language is quite correct. In the end, they do recognize that GeoGebra helped them as it is more precise than traditional pencil and paper work, and permit them to check their solution.

Table 3. Analysis of the part III of the problem Pair No. 25

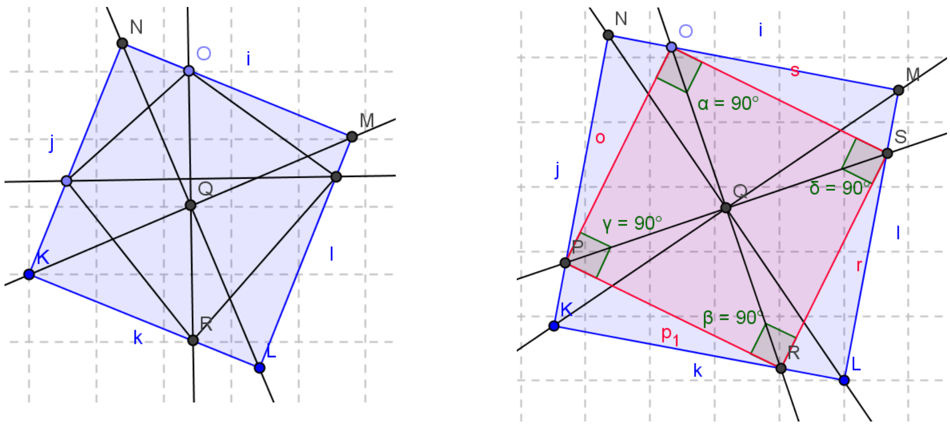


Figure 4. Construction of phase 4 and phase 5 –part III of the problem Pair no.25

In the first phase, we see that they attempt to transfer procedures from previous practice sessions (GeoGebra Workshop). On various occasions, they go back to the process of constructing inscribed squares using ‘guided dragging’, overlapping them on the blue square. They may have found it difficult to assimilate the idea that they should construct a square using their own geometric knowledge, without using the ‘regular polygon’ tool. From phase 3 onward, once they have determined the diagonals of the blue square, they begin to use the properties between vertices and diagonals of a square. However, they are still unable to apply them in order to find the solution (see Figure 4).

We see that they go through difficulties when identifying the diagonal properties: they realize that they are perpendicular, but they are slow to realize that they should intersect at the centre of the square (in fact, at times they forget this only moments after verbalizing it) as well as the fact that the centre of both squares is the same point. The teacher participates less with them than with the other pairs, as she realizes that they understand that they should construct a generally inscribed square and remember the properties necessary to do so. But at the end of the process, upon seeing the pair’s discouragement from failing to find the solution, she guides them until they finally realize that the second diagonal must be perpendicular to the first, crossing through the centre of the blue square.

The students show geometry-based obstacles (difficulties in identifying the diagonals, confusing vertex with angle, constructing diagonals that do not intersect at the centre of the square) as well as technical obstacles (using ‘segment’ to join the vertices of the square, tracing parallel lines). The most surprising obstacle occurs when, after having verbalized determined properties that would allow the successful construction (upon guidance by the teacher), the pair proceeds to ignore these properties and continues to attempt to solve the problem using the same methods that have already proven ineffective. For example, H answers in phase 4 that diagonals of a square are “perpendicular to each other” and later says that “the centre of big square is the same than the centre of the red one,” but then the pair constructs the non-square quadrilateral of Figure 4.

The use of geometric language in the conversation between the students is quite appropriate. However, in this part of the process it is evident that the written language of the personalized protocol is less precise than in the first and second parts. This seems to reflect their fatigue and the confusion produced due to the considerable amount of time that they have spent trying to determine the solution. The pair forgot to go through some of the procedures used and does not write them in the protocol.

Pair interaction is modified in phase 3. Up to this point, L had taken the initiative in making the geometry-related decisions. This includes moving the mouse at times (although it consistently appeared that H had the better grasp of GeoGebra). But upon failing to come up with the solution, L became quite discouraged and H began taking control in all of the tasks: constructing the figure, answering the teacher’s questions and writing the personalized protocol. L worked with H, but she adopted a secondary role in this final phase of the project.

A confession made by the students in the end (Figure5) seems noteworthy: L recognizes that GeoGebra was difficult to use at times, but that on this occasion it had helped her to 'see better.' H added that it is easier to check results using GeoGebra than with pencil and paper:

L: … We had even done that already …look, sometimes GeoGebra is hard for me because I forget how it works…how to use the tools or something like that, and then when I succeed, I say: Jeez, we did that one last week!
Teacher: And drawing, with pencil and paper, do you think that would have been easier for you?
L: Not really, the truth is you can see better with GeoGebra.
H: But that's because sometimes with a pencil and paper it's more inexact, so here since it's always a square, I think it's easier to see it than if you draw it.
L: Because when you draw… you may see a square but it isn't a square. You can see it better with GeoGebra.
H: Like what happened to us here (she shows the paper with their drawings). It's drawn by hand but it's an example. You draw and you don't know precisely if it is a square because you don't know if it complies with the conditions for being a square, but with GeoGebra you can check it very easily: measuring the angles or things that when using your hands isn't so easy.

Figure 5. Extract of the conversation between pair 25 and the teacher

Discussion

In light of the pair's activity during the proposed problem solving process, it is now possible to establish the main study findings, based on the *instrumentation theory*, our theoretical framework. Particularly, we present the most relevant conclusions derived from the triangle techniques-cognitive schemes-orchestration, which is the core of the theory.

Regarding the purpose of the study, we may conclude that the instrumental genesis process carried out by pair No. 25 was complex from the co-emergence perspective of Kieran and Drijvers [27]. They affirm that instrumental techniques have both a pragmatic role (allowing the student to use the tool to solve the task) as well as an epistemic role (contributing to the student's conceptual understanding). Co-emergence is the need and the possibility that both of these roles will develop while supporting each other.

In our study, we have found indications that the first role may, in fact, hinder or hide the emergence of the second. A co-emerging conflict was detected regarding the mental scheme of the square and the GeoGebra tools used to construct it. In fact, the students tended to believe that there was a concrete GeoGebra tool that would allow them to solve the problem and that they needed to find it, trying out different tools of the software. The problem was actually to be solved by increasing conceptual comprehension of the square: by extending beyond the polygon scheme characterizing it by its four equal sides and four right angles alone, to view it as a shape with two diagonals that intersect perpendicularly at the centre.

Regarding techniques, we also found that participants were natural and systematic when using the ´dragging test´ method to check whether the constructions maintained their form. They used 'guided dragging' as an exploration tool for inscribing the red square in the blue one and also to investigate whether the existence of 'various' squares may be the solution to the problem. These results are consistent with the uses of these types of dragging found in the majority of the research involving DGS.

The subjects participating in the study recognized that GeoGebra was difficult to use at times but that it did help them to 'see better'. They also added that it was easier to check their results using GeoGebra than using pencil and paper. Let us remember that one student used the pencil and paper resource as a complementary or alternative tool to the GeoGebra resource. She does this as a support mechanism, in order to explain to her colleague the procedure that she wished to carry out, although in the end she recognizes that the drawings made on paper were not as useful as GeoGebra due to their inaccuracy and because the properties of the figures cannot be checked as easily as with the software.

As for mental scheme construction, the study offers various findings. First, the behaviour of the pair in the problem solving process demonstrates that there is a deliberate attempt to use previously studied geometric knowledge, but they have not delimited the validity or use of their knowledge. One immediate instructional implication of this is the need to work on mathematical tasks, not as isolated exercises, but rather, as systems of tasks covering a wide field of problems where they arise and where the application of the developed mathematical concept is to be applied. Second, the language found in the personalized protocols is unclear at times, particularly when the pairs discussed procedures that did not result in the desired solution. The inappropriate use of terminology may have led the pairs to make mistakes in selecting the appropriate GeoGebra tool.

The study also offers suggestions regarding the orchestration process. This is the process that was initiated in the GeoGebra seminars: weekly, hour and a half long sessions of geometry problem solving extending over a term. In the register analysis of pair No. 25 it was possible to verify that the intervention of the teacher was a key factor in the solution process for the generalization part of the problem and in the participants' selection of techniques and procedures to use. From this information, we can highlight various determining factors in the quality of the instrumental genesis process that were related to orchestration.

First, there is the didactic-technological design of the teaching process, including the corresponding learning tasks and activities. When this process develops in a digital environment, students tend to focus primarily on the tools and functionalities of the technology. For this reason, the teaching process design should be guided by didactic considerations emphasizing conceptual comprehension of the task's underlying mathematic principles, with software to the advantage of this basic pencil-and-paper comprehension.

The second factor concerns the role of the teacher in the teaching-learning practices. The integration of technology in mathematics education is not a panacea that reduces the essential role of the teacher. The teacher must review mathematical ideas and other issues related to problem solving, suggest tools or functionalities of the software as they relate to other resources such as pencil and paper, and motivate, creating an appropriate activity environment. Both factors will guide us in the review and improvement of future GeoGebra seminars teaching practice.

Thus, the fact that the subjects participating in this study are future primary school teachers clears up some important aspects regarding their specific training: They will have to lead orchestration processes with their students and, therefore, they should attain their own complete and thorough instrumental genesis process. According to Fortuny, Iranzo and Morera [28], the model of instrumental orchestration could be a useful framework for the analysis of teaching practices in computer-assisted geometry teaching. More elaborated examples of this kind of practices are needed in order to develop a repertoire of instrumental orchestrations. These may not only help better understand teaching practices but also improve teachers' professional development.

In this sense, another contribution of this study which may be of potential interest to teachers and researchers, is the methodology used to analyse the GeoGebra problem solving process in the case study. The tables, which include information collected through the different tools, describe the following elements: techniques and dragging types used, obstacles encountered, pair interaction, teacher-pair interaction and language used. Once analysed, it will be possible to continue to examine the instrumental genesis process [29] for the students as well as to design appropriate problems in order to carry on with the process and facilitate an improved development of their geometric competencies.

To summarize, results of the research demonstrate the difficulty of attaining complete instrumental genesis due to three interlinked factors: deficient conceptual understanding of the required mathematics on the part of the students, limited development of instrumentalization in the use of the different GeoGebra tools and an imperfect instrumental orchestration process. However, we conclude that the instrumental perspective is an effective and successful theoretical approach that may provide guidelines for the instructional design of tasks as well as for the analysis of student behaviour in a technological environment. Therefore, we believe that this study may be of interest to educators of future primary school teachers.

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Table 1. Analysis of the 1st part of the problem Pair No. 25

Table 2. Analysis of the part II of the problem Pair No. 25

Table 3. Analysis of the part III of the problem Pair No. 25

Figure 1. Protocol of case study session (1st part)

Figure 2. Phases 1, 2, 3 and 4 – part II of the problem Pair No. 25

Figure 3. Protocol of case study session (part II - Phase 5)

Figure 4. Construction of phase 4 and phase 5 –part III of the problem Pair No.25

Figure 5. Extract of the conversation between pair 25 and the teacher