

## Indirect upper limits on $\ell_i \rightarrow \ell_j \gamma \gamma$ from $\ell_i \rightarrow \ell_j \gamma$

Fabiola Fortuna<sup>1</sup>, Alejandro Ibarra<sup>2</sup>, Xabier Marcano<sup>3</sup>, Marcela Marín<sup>1</sup>, and Pablo Roig<sup>1</sup>

<sup>1</sup>*Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal 14-740, 07000 Ciudad de México, México*

<sup>2</sup>*Physik-Department, Technische Universität München, James-Frank-Straße, 85748 Garching, Germany*

<sup>3</sup>*Departamento de Física Teórica and Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*



(Received 17 October 2022; accepted 12 January 2023; published 30 January 2023)

We perform an effective field theory analysis to correlate the charged lepton flavor violating processes  $\ell_i \rightarrow \ell_j \gamma \gamma$  and  $\ell_i \rightarrow \ell_j \gamma$ . Using the current upper bounds on the rate for  $\ell_i \rightarrow \ell_j \gamma$ , we derive model-independent upper limits on the rates for  $\ell_i \rightarrow \ell_j \gamma \gamma$ . Our indirect limits are about three orders of magnitude stronger than the direct bounds from current searches for  $\mu \rightarrow e \gamma \gamma$ , and four orders of magnitude better than current bounds for  $\tau \rightarrow \ell \gamma \gamma$ . We also stress the relevance of Belle II or a Super Tau Charm Facility to discover the rare decay  $\tau \rightarrow \ell \gamma \gamma$ .

DOI: [10.1103/PhysRevD.107.015027](https://doi.org/10.1103/PhysRevD.107.015027)

### I. INTRODUCTION

The experimental observation of charged lepton flavor violation (cLFV) would undoubtedly imply the existence of new physics beyond neutrino oscillations [1]. This has motivated a strong experimental program over the last 75 years searching for different cLFV processes, see Fig. 1, each observable providing complementary information about possible beyond the Standard Model (BSM) scenarios.

Here we consider the cLFV decays of leptons to two photons,  $\ell_i \rightarrow \ell_j \gamma \gamma$  [5,7,8], which have been explored in less detail than other cLFV processes such as the single photon process,  $\ell_i \rightarrow \ell_j \gamma$ , especially for the case of  $\tau \rightarrow \ell \gamma \gamma$  [6,9–12].

Experimentally,  $\mu \rightarrow e \gamma \gamma$  was searched for by several experiments aiming also for  $\mu \rightarrow e \gamma$ . The latest of these experiments was the Crystal Box detector, whose result still provides the strongest bound for  $\mu \rightarrow e \gamma \gamma$  [13]. This limit is however two orders of magnitude weaker than present  $\mu \rightarrow e \gamma$  bounds, see Table I, since the MEG experiment was optimized for back-to-back topologies and no new dedicated experiment for  $\mu \rightarrow e \gamma \gamma$  has been carried out since Crystal Box. On the other hand,  $\tau \rightarrow \ell \gamma \gamma$  has rarely been searched for. To the best of our knowledge, the only existing direct experimental search was performed by ATLAS, setting an upper limit of  $\text{BR}(\tau \rightarrow \mu \gamma \gamma) < 1.5 \times 10^{-4}$  after the

LHC run-I [14]. No direct experimental search exists for  $\tau \rightarrow e \gamma \gamma$ .

An alternative for exploring the  $\ell_i \rightarrow \ell_j \gamma \gamma$  channels is to recast the searches for  $\ell_i \rightarrow \ell_j \gamma$ , as some of the events of the former would fall into the signal region defined for the latter [5]. This idea has been recently applied to recast the BABAR search for  $\tau \rightarrow \ell \gamma$  [16], finding that at 90% C.L.  $\text{BR}(\tau \rightarrow \mu \gamma \gamma) < 5.8 \times 10^{-4}$  and  $\text{BR}(\tau \rightarrow e \gamma \gamma) < 2.5 \times 10^{-4}$  [6]. These limits are however several orders of magnitude weaker than the associated ones on  $\tau \rightarrow \ell \gamma$  due to the low acceptance of these searches for  $\tau \rightarrow \ell \gamma \gamma$  events.

In this letter, we consider the theoretical correlation between the  $\ell_i \rightarrow \ell_j \gamma \gamma$  and the  $\ell_i \rightarrow \ell_j \gamma$  decays. Clearly, any scenario generating  $\ell_i \rightarrow \ell_j \gamma$  would automatically generate a (model-independent) contribution to  $\ell_i \rightarrow \ell_j \gamma \gamma$ , from the radiation of an additional photon in the final state. Further, any scenario generating  $\ell_i \rightarrow \ell_j \gamma \gamma$  will generate a (model-dependent) contribution to  $\ell_i \rightarrow \ell_j \gamma$  at the quantum level. Barring cancellations, the quantum-induced contribution should not exceed the experimental upper limits on  $\ell_i \rightarrow \ell_j \gamma$ , which in turn allows us to set indirect limits on the rates of  $\ell_i \rightarrow \ell_j \gamma \gamma$ .

In this work, we will pursue an effective field theory (EFT) approach to study these correlations, in order to ensure the generality of our conclusions. Notably, our indirect limits on the process  $\ell_i \rightarrow \ell_j \gamma \gamma$  will turn out to be more stringent than the current direct bounds. Furthermore, these limits do not preclude the possibility of observing the rare decays  $\tau \rightarrow \ell \gamma \gamma$  at Belle II or at a Super Tau Charm Factory, which then represents a competitive probe of cLFV along with the more studied channels  $\tau \rightarrow \ell \gamma$  or  $\tau \rightarrow \ell_j \ell_k \bar{\ell}_k$ .

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP<sup>3</sup>.

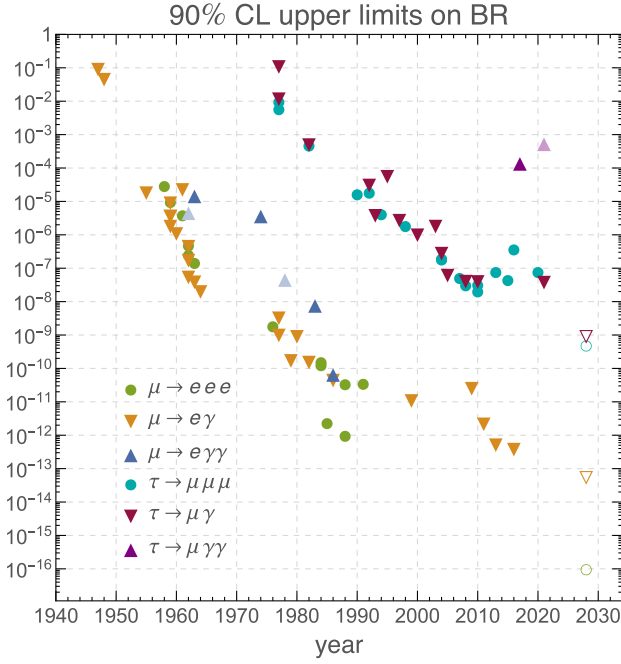


FIG. 1. Historical evolution for the 90% C.L. upper limits on several cLFV leptonic decays. Solid markers correspond to published direct experimental searches, while empty ones indicate future expected sensitivities at MEG II [2], Mu3e [3] and Belle II [4] (arbitrary year). Lighter markers for  $\ell_i \rightarrow \ell_j \gamma \gamma$  were obtained by recasting the available searches for  $\ell_i \rightarrow \ell_j \gamma$  [5,6] or  $\mu e \rightarrow \gamma \gamma$  [7]. The evolution for  $\tau \rightarrow e$  sector is similar to the  $\tau \rightarrow \mu$  one, with the exception that no direct search for  $\tau \rightarrow e \gamma \gamma$  exists.

## II. TOTAL RATE FOR $\ell_i \rightarrow \ell_j \gamma \gamma$ IN THE EFT APPROACH

The effective interaction Lagrangian between two charged leptons of different flavor and one photon has dimension 5 and reads:

$$\mathcal{L}_{\text{dim-5}} = D_R^{ij} \bar{\ell}_{L_i} \sigma_{\mu\nu} \ell_{R_j} F^{\mu\nu} + D_L^{ij} \bar{\ell}_{R_i} \sigma_{\mu\nu} \ell_{L_j} F^{\mu\nu} + \text{H.c.} \quad (1)$$

where the subscripts  $L(R)$  indicate the chirality of the lepton and  $i, j$  are generation indices.

On the other hand, the lowest dimensional effective interaction between two charged leptons of different flavor and two photons has mass dimension 7 and is given by [5]

TABLE I. Experimental upper bounds on the rates of the  $\ell_i \rightarrow \ell_j \gamma(\gamma)$  decays.

Decay Mode	Current upper limit on BR (90% C.L.)	
$\mu \rightarrow e \gamma$	$4.2 \times 10^{-13}$	MEG (2016) [15]
$\mu \rightarrow e \gamma \gamma$	$7.2 \times 10^{-11}$	Crystal Box (1986) [13]
$\tau \rightarrow e \gamma$	$3.3 \times 10^{-8}$	BABAR (2010) [16]
$\tau \rightarrow \mu \gamma$	$4.2 \times 10^{-8}$	Belle (2021) [17]
$\tau \rightarrow \mu \gamma \gamma$	$1.5 \times 10^{-4}$	ATLAS (2017) [14]

$$\begin{aligned} \mathcal{L}_{\text{dim-7}} = & (G_{SR}^{ij} \bar{\ell}_{L_i} \ell_{R_j} + G_{SL}^{ij} \bar{\ell}_{R_i} \ell_{L_j}) F_{\mu\nu} F^{\mu\nu} \\ & + (\tilde{G}_{SR}^{ij} \bar{\ell}_{L_i} \ell_{R_j} + \tilde{G}_{SL}^{ij} \bar{\ell}_{R_i} \ell_{L_j}) \tilde{F}_{\mu\nu} F^{\mu\nu} + \text{H.c.} \quad (2) \end{aligned}$$

where  $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\lambda} F^{\sigma\lambda}$  is the dual tensor. In Ref. [5], the Lagrangian also contains the dimension-8 operators  $\bar{\ell}_{L_i} \gamma^\sigma \ell_{L_j} F^{\mu\nu} \partial_\nu F_{\mu\sigma}$  and  $\bar{\ell}_{L_i} \gamma^\sigma \ell_{L_j} F^{\mu\nu} \partial_\nu \tilde{F}_{\mu\sigma}$ , as well as the analogous operators for the right-handed fermions. The effect of these operators is not only suppressed by higher powers of the cutoff scale of the EFT, but also by the mass of the decaying lepton, due to the helicity flip, therefore we will neglect them henceforth.

The effective Lagrangians in Eqs. (1) and (2) generate at tree level the decay  $\ell_i \rightarrow \ell_j \gamma \gamma$ , through the diagrams shown in the upper panel of Fig. 2. The expression for the differential decay rate is complicated and is given in Appendix. For the specific limits where the rate is dominated by the dimension-5 operators, the total decay rate is given by:

$$\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma) = \frac{\alpha m_i^3}{48\pi^2} (|D_R^{ij}|^2 + |D_L^{ij}|^2) \lambda \left( \frac{E_\gamma^{\text{cut}}}{m_i} \right), \quad (3)$$

with  $E_\gamma^{\text{cut}}$  an energy cutoff introduced to regularize the infrared and collinear divergences in the rate (see Appendix), and

$$\begin{aligned} \lambda(x) \simeq & 6 + 2\pi^2 + 6\log^2 2 + 21 \log(2x) + 6 \log(x) \log(4x) \\ & + 18x(2 \log(2x) + 1) + 6x^2(8 \log(2x) - 29) \\ & + \mathcal{O}(x^3). \quad (4) \end{aligned}$$

On the other hand, when it is dominated by the dimension-7 operators, we obtain

$$\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma) = \frac{|G_{ij}|^2}{3840\pi^3} m_i^7, \quad (5)$$

where we have neglected the mass of the final lepton and  $|G_{ij}|^2 = |G_{SL}^{ij}|^2 + |G_{SR}^{ij}|^2 + |\tilde{G}_{SL}^{ij}|^2 + |\tilde{G}_{SR}^{ij}|^2$ .

## III. CORRELATING $\ell_i \rightarrow \ell_j \gamma \gamma$ AND $\ell_i \rightarrow \ell_j \gamma$

Let us first consider scenarios where the dimension-5 operators are not suppressed, so the rate for  $\ell_i \rightarrow \ell_j \gamma \gamma$  is approximately given by Eq. (3). Clearly, the dimension-5 operators also induce the decay  $\ell_i \rightarrow \ell_j \gamma$  (see lower panel of Fig. 2). The rate is given by

$$\Gamma(\ell_i \rightarrow \ell_j \gamma) = \frac{m_i^3}{4\pi} (|D_R^{ij}|^2 + |D_L^{ij}|^2), \quad (6)$$

whence one obtains

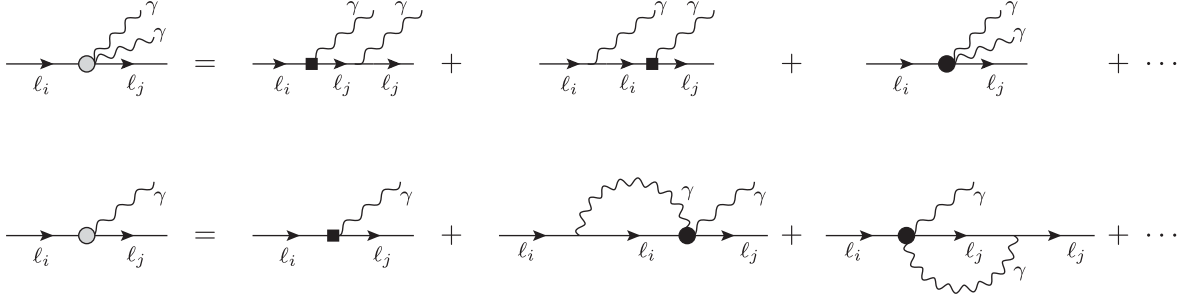


FIG. 2. Feynman diagrams contributing to  $\ell_i \rightarrow \ell_j \gamma \gamma$  (top panel) and  $\ell_i \rightarrow \ell_j \gamma$  (bottom panel) in the effective field theory described by the Lagrangians in Eqs. (1) (black squares) and (2) (black circles). The dots represent higher order contributions.

$$\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma) = \frac{\alpha}{12\pi} \lambda \left( \frac{E_\gamma^{\text{cut}}}{m_i} \right) \Gamma(\ell_i \rightarrow \ell_j \gamma). \quad (7)$$

Using the upper limits on the rates for  $\ell_i \rightarrow \ell_j \gamma$  from Table I and imposing  $E_\gamma^{\text{cut}} = 7(50)$  MeV for  $\mu(\tau)$  decays, one finds the indirect limits:

$$\begin{aligned} \text{BR}(\mu \rightarrow e \gamma \gamma) &\lesssim 2 \times 10^{-16}, \\ \text{BR}(\tau \rightarrow e \gamma \gamma) &\lesssim 8 \times 10^{-11}, \\ \text{BR}(\tau \rightarrow \mu \gamma \gamma) &\lesssim 1 \times 10^{-10}. \end{aligned} \quad (8)$$

Alternatively, there could be scenarios where the dimension-5 operators are suppressed, while not the dimension-7 operators (see Sec. IV). In this case, the decay  $\ell_i \rightarrow \ell_j \gamma$  is induced at the one loop-level (see lower panel of Fig. 2). In this case, and keeping only the leading terms, one finds:

$$\Gamma(\ell_i \rightarrow \ell_j \gamma) \sim \frac{\alpha |G_{ij}|^2}{256\pi^4} m_i^7 \log^2 \left( \frac{\Lambda^2}{m_i^2} \right), \quad (9)$$

where  $\Lambda$  is the cutoff energy scale of the effective field theory. Using Eq. (5) one obtains an approximate correlation between rates

$$\Gamma(\ell_i \rightarrow \ell_j \gamma) \sim \frac{15\alpha}{\pi} \log^2 \left( \frac{\Lambda^2}{m_i^2} \right) \Gamma(\ell_i \rightarrow \ell_j \gamma \gamma), \quad (10)$$

from where one can derive indirect upper limits for  $\ell_i \rightarrow \ell_j \gamma \gamma$  from the upper limits on  $\ell_i \rightarrow \ell_j \gamma$ :

$$\begin{aligned} \text{BR}(\mu \rightarrow e \gamma \gamma) &\lesssim 6.4 \times 10^{-14} \left[ 1 + 0.15 \log \frac{\Lambda}{100 \text{ GeV}} \right]^{-2}, \\ \text{BR}(\tau \rightarrow e \gamma \gamma) &\lesssim 1.5 \times 10^{-8} \left[ 1 + 0.25 \log \frac{\Lambda}{100 \text{ GeV}} \right]^{-2}, \\ \text{BR}(\tau \rightarrow \mu \gamma \gamma) &\lesssim 1.9 \times 10^{-8} \left[ 1 + 0.25 \log \frac{\Lambda}{100 \text{ GeV}} \right]^{-2}, \end{aligned} \quad (11)$$

which have a mild sensitivity to the cutoff scale.

Regardless of the underlying physics generating the process  $\ell_i \rightarrow \ell_j \gamma \gamma$ , our indirect limits are significantly more stringent than the current direct limits. Concretely, the limit on  $\mu \rightarrow e \gamma \gamma$  is about three orders of magnitude stronger than the direct search using the Crystal Box detector and the limit on  $\tau \rightarrow \mu \gamma \gamma$  is about four orders of magnitude stronger than the direct search performed at ATLAS. Future foreseeable sensitivities of MEG II searching for  $\mu \rightarrow e \gamma$  and of Belle II for  $\tau \rightarrow \ell \gamma$  will improve our indirect limits by about one order of magnitude.

Furthermore, the results in Eqs. (11) motivate a dedicated experimental search for the  $\tau \rightarrow \ell \gamma \gamma$  decays, since this decay might be at the reach of future experiments [18]. Let us consider the specific case of the Belle II experiment. Assuming that Belle II could achieve the same sensitivity for double than for single photon processes, as occurred in the Crystal Box Detector for muon decays [19], Belle II could probe the  $\tau \rightarrow \ell \gamma \gamma$  decays with branching ratios as small as  $\mathcal{O}(10^{-9})$  [18]. If this sensitivity is reached, Belle II will probe uncharted parameter space of the dimension-7 operators, and possibly find evidence for cLFV by the observation of the decay  $\tau \rightarrow \ell \gamma \gamma$ .

#### IV. MODELS WITH ENHANCED $\ell_i \rightarrow \ell_j \gamma \gamma$

There are several scenarios where the dimension-5 operators could be suppressed with respect to the dimension-7 operators, thereby enhancing the rate of  $\ell_i \rightarrow \ell_j \gamma \gamma$  compared to  $\ell_i \rightarrow \ell_j \gamma$ . For instance, it was argued in [5] that in models where cLFV was mediated by charged heavy leptons [20], the GIM suppression [21] could be stronger for  $\ell_i \rightarrow \ell_j \gamma$  than for  $\ell_i \rightarrow \ell_j \gamma \gamma$ . Also, in Ref. [8] it was argued that in some new physics models the dimension-7 operators could actually arise at  $\mathcal{O}(1/\Lambda^2)$ , instead of the naive expectation of  $\mathcal{O}(1/\Lambda^3)$ , so their contributions would be less suppressed than expected.

Another interesting possibility arises in models where the cLFV is mediated by heavy scalars, such as a two Higgs doublet model (2HDM) with off-diagonal Yukawa interactions. In this scenario,  $\ell_i \rightarrow \ell_j \gamma$  decays are induced at one-loop level, however they are suppressed by three chiral flips and therefore the two-loop (Barr-Zee diagrams)

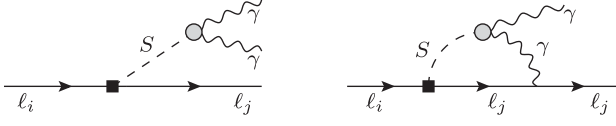


FIG. 3. Example of diagrams generating  $\ell_i \rightarrow \ell_j \gamma \gamma$  and  $\ell_i \rightarrow \ell_j \gamma$  mediated by a scalar with off-diagonal Yukawa couplings and an effective vertex to two photons.

contributions are actually the dominant ones [22]. On the other hand, the  $\ell_i \rightarrow \ell_j \gamma \gamma$  decays do not suffer from this chirality suppression, the dominant contributions are at the one-loop and, consequently, they can have ratios comparable to those of  $\ell_i \rightarrow \ell_j \gamma$ .

More concretely, one can consider a scenario containing a heavy scalar  $S$  with off-diagonal Yukawa couplings to leptons and an effective vertex to two photons (which matches to the framework in Ref. [22] when integrating out the top and  $W$  boson). The double and single photon decays are then generated by diagrams such as those in Fig. 3, which in the heavy scalar limit reduce to a local interaction and Fig. 2, respectively. We have explicitly checked that, in this heavy limit, we recover our EFT result of Eq. (10) with  $\Lambda = m_S$ .

## V. CONCLUSIONS

Pursuing an effective field theory approach, we have derived model-independent upper limits on the rates of  $\ell_i \rightarrow \ell_j \gamma \gamma$  from the current experimental limits on  $\ell_i \rightarrow \ell_j \gamma$ . Our indirect limits are, even under the most conservative assumptions, significantly more stringent than the current direct limits, concretely three orders of magnitude for  $\mu \rightarrow e \gamma \gamma$  and four orders of magnitude for  $\tau \rightarrow \mu \gamma \gamma$ . When  $\ell_i \rightarrow \ell_j \gamma \gamma$  is dominantly generated by the same dimension-5 operators generating  $\ell_i \rightarrow \ell_j \gamma$ , the stringent limits on the latter preclude the observation of the former in planned experiments. In contrast, in scenarios where the dimension-5 operators are suppressed compared to the dimension-7 operators, the rare decays  $\ell_i \rightarrow \ell_j \gamma \gamma$  can be enhanced compared to  $\ell_i \rightarrow \ell_j \gamma$ , which in our EFT approach is only generated at the one-loop level. In this class of scenarios, the rare decay  $\tau \rightarrow \mu \gamma \gamma$  could be at the reach of the Belle II experiment or at a proposed Super Tau Charm Facility, and could constitute a stringent probe of lepton flavor violation. We also discussed some possible UV-complete scenarios where  $\ell_i \rightarrow \ell_j \gamma \gamma$  is enhanced.

## ACKNOWLEDGMENTS

F. F., M. M. and P. R. are grateful to Eduard de la Cruz, Iván Heredia and Alex Miranda for discussions. This work has been supported by the Collaborative Research Center SFB1258, by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy—EXC-2094—390783311, and by the Science and Technology Facilities Council [Grant No. ST/T004169/1]. F. F. and M. M. acknowledge Conacyt scholarships and P. R. is grateful to funding from “Cátedras Marcos Moshinsky” (Fundación Marcos Moshinsky) and “Paradigmas y Controversias de la Ciencia 2022” (Project No. 319395, Conacyt), which supported M. M. X. M. acknowledges funding from the European Union's HORIZON EUROPE Marie Skłodowska-Curie Actions, Grant Agreement No. 101066105-PheNUmenal.

## APPENDIX: DIFFERENTIAL DECAY RATES

In this appendix we present the double differential decay width for the  $\ell_i \rightarrow \ell_j \gamma \gamma$  processes, including the contributions from effective operators up to dim-7. The differential rate can be cast as:

$$\begin{aligned} \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} &= \left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{dim-5}} \\ &+ \left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{dim-7}} \\ &+ \left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{int}}, \end{aligned} \quad (\text{A1})$$

where in an obvious notation, dim-5 denotes the contribution from the Lagrangian Eq. (1), dim-7 from the Lagrangian Eq. (2), and int is the interference term. Explicitly, and neglecting the mass of the lepton in the final state, we obtain:

$$\begin{aligned} \left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{dim-7}} &= \frac{|G_{ij}|^2}{16\pi^3} m_i^2 (m_i - E_\gamma - E_{\gamma'}) \\ &\times (m_i - 2(E_\gamma + E_{\gamma'}))^2, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{dim-5}} &= \frac{\alpha(|D_R^{ij}|^2 + |D_L^{ij}|^2)}{4E_\gamma E_{\gamma'} + m_i^2 - 2m_i(E_\gamma + E_{\gamma'})} \frac{m_i - 2(E_\gamma + E_{\gamma'})}{4\pi^2 E_\gamma^2 E_{\gamma'}^2} \{48E_\gamma^3 E_{\gamma'}^3 - m_i^4 (E_\gamma - E_{\gamma'})^2 \\ &+ 2E_\gamma E_{\gamma'} m_i^2 (E_\gamma^2 + 6E_\gamma E_{\gamma'} + E_{\gamma'}^2) - E_\gamma E_{\gamma'} m_i (E_\gamma + E_{\gamma'}) (24E_\gamma E_{\gamma'} + m_i^2)\}, \end{aligned} \quad (\text{A3})$$

$$\left. \frac{d^2\Gamma(\ell_i \rightarrow \ell_j \gamma \gamma)}{dE_\gamma dE_{\gamma'}} \right|_{\text{int}} = -\frac{em_i(m_i - 2(E_\gamma + E_{\gamma'}))^2}{4\pi^3} \text{Re}\{D_L^{ij*}(G_{SL}^{ij} + i\tilde{G}_{SL}^{ij}) + D_R^{ij*}(G_{SR}^{ij} - i\tilde{G}_{SR}^{ij})\}, \quad (\text{A4})$$

with kinematical ranges for the photon energies  $0 \leq E_\gamma \leq m_i/2$ ,  $m_i/2 - E_\gamma \leq E_{\gamma'} \leq m_i/2$ . Notice that the dimension-5 contribution suffers from both infrared and collinear singularities, which we can avoid by introducing

a regulator such that  $E_\gamma^{\text{cut}} \leq E_\gamma \leq m_i/2 - E_\gamma^{\text{cut}}$ ,  $m_i/2 - E_\gamma \leq E_{\gamma'} \leq m_i/2 - E_\gamma^{\text{cut}}$ . Strictly, one should introduce different regulators, however we assume them to be the same for simplicity.

- 
- [1] L. Calibbi and G. Signorelli, *Riv. Nuovo Cimento* **41**, 71 (2018).
- [2] A. M. Baldini *et al.* (MEG II Collaboration), *Eur. Phys. J. C* **78**, 380 (2018).
- [3] A. Blondel *et al.*, [arXiv:1301.6113](https://arxiv.org/abs/1301.6113).
- [4] W. Altmannshofer *et al.* (Belle-II Collaboration), *Prog. Theor. Exp. Phys.* **2019**, 123C01 (2019); **2020**, 029201 (E) (2020).
- [5] J. Bowman, T. Cheng, L.-F. Li, and H. Matis, *Phys. Rev. Lett.* **41**, 442 (1978).
- [6] D. A. Bryman, S. Ito, and R. Shrock, *Phys. Rev. D* **104**, 075032 (2021).
- [7] J. Dreitlein and H. Primakoff, *Phys. Rev.* **126**, 375 (1962).
- [8] S. Davidson, Y. Kuno, Y. Uesaka, and M. Yamanaka, *Phys. Rev. D* **102**, 115043 (2020).
- [9] A. Gemintern, S. Bar-Shalom, G. Eilam, and F. Krauss, *Phys. Rev. D* **67**, 115012 (2003).
- [10] A. Cordero-Cid, G. Tavares-Velasco, and J. J. Toscano, *Phys. Rev. D* **72**, 117701 (2005).
- [11] J. Aranda, F. Ramirez-Zavaleta, J. Toscano, and E. Tututi, *Phys. Rev. D* **78**, 017302 (2008).
- [12] J. I. Aranda, A. Flores-Tlalpa, F. Ramirez-Zavaleta, F. J. Tlachino, J. J. Toscano, and E. S. Tututi, *Phys. Rev. D* **79**, 093009 (2009).
- [13] D. Grosnick *et al.*, *Phys. Rev. Lett.* **57**, 3241 (1986).
- [14] I. Angelozzi, In pursuit of lepton flavour violation: A search for the  $\tau \rightarrow \mu \gamma \gamma$  decay with ATLAS at  $\sqrt{s} = 8$  TeV, Ph.D. thesis, University of Amsterdam, IHEF, 2017, <https://inspirehep.net/literature/1650844>.
- [15] A. Baldini *et al.* (MEG Collaboration), *Eur. Phys. J. C* **76**, 434 (2016).
- [16] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **104**, 021802 (2010).
- [17] A. Abdesselam *et al.* (Belle Collaboration), *J. High Energy Phys.* **10** (2021) 19.
- [18] S. Banerjee *et al.*, [arXiv:2203.14919](https://arxiv.org/abs/2203.14919).
- [19] R. Bolton *et al.*, *Phys. Rev. D* **38**, 2077 (1988).
- [20] F. Wilczek and A. Zee, *Phys. Rev. Lett.* **38**, 531 (1977).
- [21] S. L. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
- [22] J. Hisano, S. Sugiyama, M. Yamanaka, and M. J. S. Yang, *Phys. Lett. B* **694**, 380 (2011).